HOW RATIONAL IS THE MARKET? TESTING ALTERNATIVE HYPOTHESES ON FINANCIAL MARKET EQUILIBRIUM

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How Rational is the Market? Testing Alternative Hypotheses on Financial Market Equilibrium*

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ABSTRACT

It is widely recognized that heterogeneous information across traders plays an important role in generating financial market activity. However, the predictions of any model of financial markets depend completely on the equilibrium concept used to solve the model. The choice of equilibrium subsumes assumptions and implications concerning: the degree to which traders make use of price as information, the amount of noise trading in the market, the information content of prices, the effect on prices of differences in beliefs, etc. An empirical analysis of equilibrium formation serves as an implicit test of the relative importance of these different properties.

We devise tests that distinguish between competitive (Walrasian), fully revealing rational expectations, and noisy rational expectations equilibria based on their comparative static predictions concerning trading volume around public information signals. These tests are implemented using data on stock market trading volume, price changes, and changes in analysts' earnings forecasts around interim earnings announcements. Empirical results strongly support the noisy rational expectations hypothesis. This indicates that a significant amount of noise trading exists (so that private information has value) but not enough to obfuscate entirely the information content of price. Our analysis also indicates that the dispersion of private information across traders has an impact on trading volume, but not on price. Finally, we explore the implications of our results for asset pricing and volatility, as well as for certain "anomalous" phenomena observed in financial markets.

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I. Introduction

The proposition that differences in opinion make a horse race now has strong analytical support. In financial markets, divergent beliefs arising from asymmetric information play an important role in generating activity. Without these divergent beliefs, speculative trading in assets would not exist.[1] These results have spawned (and are a product of) the significant theoretical progress in the analysis of markets where traders have heterogeneous information.[2] However, no empirical research has yet studied which equilibrium concept is most appropriate in modelling these markets. This issue is significant because the choice of equilibrium notion determines the model's predictive outcomes. This paper is an attempt to take a first step in resolving this problem.

Another important issue is that the choice of equilibrium solution is a specification of the behavior of traders and of the structure of information and trading. Does noise trading exist? Do traders look only at their private information or at prices as well? Do prices convey information? Does the dispersion of information affect price? An empirical analysis would provide insight on the roles these phenomena actually play in financial markets.

Consider a market for a risky asset in which individuals who trade in the asset possess heterogeneous private information concerning its future value. Three alternative equilibrium hypotheses have been introduced to derive testable implications from such a model: competitive (Walrasian) equilibrium, fully revealing rational expectations and noisy rational expectations. Under the competitive equilibrium hypothesis (Debreu (1959), Lintner (1969)), traders condition trading decisions on their private information but do not try to extract additional information from market price. The fully revealing rational expectations hypothesis (Grossman (1976)), which has antecedents in the work on efficient markets (Fama (1970)), postulates that traders make use of both their private information and the information contained in price. In addition, it assumes that, in equilibrium, prices reflect all relevant private information of traders. Because of this property of equilibrium prices, a trader would make an identical decision by looking only at price and ignoring his own private information. On the other hand, in a noisy rational expectations equilibrium (Grossman and Stiglitz (1980)), traders again condition decisions on both private information and price but noise in the market prevents price from being a sufficient statistic for all relevant market information. Thus, a trader's private information is not redundant.

We derive comparative static implications concerning trading volume which distinguish between the three equilibrium hypotheses. Trading volume is a logical choice of variable to study since
equilibrium prices and traders' demands depends on the equilibrium concept used. We focus on the

"speculative" component of volume, in contrast to the "noise trading" component. Speculative trading is performed by agents whose valuation of the asset, given their information, differs from its current price. Noise traders possess non-speculative (e.g., liquidity) incentives to buy or sell the asset. If noise trading activity is considered (observationally) random, then the speculative component is the predictable quantity of volume. In particular, around information events, the expectation of the noise trading component of abnormal volume should be zero.

Thus, the response of trading volume to a public information signal allows us to construct testable implications of different equilibrium concepts in the following way. Each equilibrium hypothesis corresponds to a particular assumption concerning traders' information sets. Equilibrium posterior beliefs of traders regarding the future value of the asset are conditioned on these information sets. We decompose (speculative) volume around a public information event into (1) a component due to the mean of squared changes in the differences of traders' posterior beliefs from their average (the "dispersion of beliefs") and (2) a residual term which depends on the difference between average posterior expectation and price. The residual term can be reduced to different expressions depending on the equilibrium notion used to solve the model.

Under competitive (Walrasian) equilibrium, traders do not make use of the information contained in prices. We show that, in this case, price changes equal changes in the average of traders' posterior expectations. This implies that volume can be completely explained by changes in the dispersion of traders' posterior beliefs around their average.

In a fully revealing rational expectations equilibrium, all traders' posterior expectations collapse to the same value. Price must also equal this value. Changes in traders' expectations are identical and equal changes in price. Expected trading volume is therefore equal to zero.

In a noisy rational expectations equilibrium, traders extract information from prices but prices do not fully reveal information. Noise induces traders to base their decisions on their own private information as well as on price. This leads traders to form heterogeneous posterior beliefs and price to diverge from the average of these beliefs. This implies that changes in price, as well as in the dispersion of beliefs, is a determinant of trading volume.

These comparative static implications concerning volume form the basis of a test which nests the three equilibrium hypotheses. Each equilibrium hypothesis implies that a different set of variables has explanatory power for trading volume. We then consider a specific information event, firms' interim earnings announcements. Traders' expectations will adjust following this public information signal. The consequent trading (at the new market price) allows us to analyze the empirical associations between post-announcement volume and the different sets of explanatory variables. We can then study which equilibrium hypothesis is supported by the data.

However, these empirical tests require the measurement of expectational variables. To provide proxies for these variables, we use a previously unexplored data set containing survey data on the forecasts of individual analysts of annual earnings of various firms listed on the NYSE. These forecasts display a significant amount of divergence and therefore seem to reflect the differences in (information generated) beliefs among the population of informed traders in the stock market.[3] The data base comprises the raw data of individual analysts' forecasts and the dates these forecasts were reported. Thus, observable changes in the forecast sample can be used to calculate movements in expectational variables.

Our tests reject the fully revealing rational expectations hypothesis. An interesting result is that a test of the predictions of competitive equilibrium against an unspecified alternative supports this hypothesis. However, using a test which nests both the competitive equilibrium and noisy rational expectations hypotheses, competitive equilibrium is strongly rejected against the specific alternative of noisy rational expectations.

This result indicates that a significant amount of noise trading is present in markets, but not enough to entirely obfuscate the information content of prices. Private information retains some value, so that there exist incentives to spend resources for its acquisition. A consequence of this is that traders' equilibrium beliefs remain asymmetric even after observing price. This is why both changes in price and in the dispersion of beliefs are significant in explaining trading volume. Nevertheless, we find that the price change and dispersion change variables are orthogonal. This implies that price is affected by aggregate quantities, but not by the magnitude of the diversity of information or beliefs.

A further implication of these results is that movements in price underestimate (on average) movements in fundamental values. Nevertheless, the variance of price exceeds its efficient markets magnitude. This is because equilibrium beliefs of speculators are more volatile, and not only because of the direct presence of noise trading. The greater volatility of trader's expectations has implications for certain phenomena which have been observed in financial markets such as the small firm effect, the Cragg-Malkiel study, the excess volatility of asset prices, and others which we discuss in the fourth section of this paper.

This paper is organized as follows: in section two, we define a basic model of a market for a risky asset in which traders are asymmetrically informed. The model is along the lines of Hellwig (1981) and Diamond and Verrecchia (1982). We consider the solutions of the model under each of the three equilibrium concepts and derive the various implications concerning volume and price behavior. In section three, we discuss sample and estimation methods. We then present the empirical results. In this section we also discuss how our analysis can be interpreted as describing the predictable or speculative component of volume and the residuals in our empirical model as the noise trading component. Implications of our empirical results for financial market activity are discussed in sections three and four. Conclusions are made in the final section.

II. The Model

We consider the following basic model and develop solutions of the model under three different equilibrium hypotheses. We then study the implications of each equilibrium for price and trading volume behavior.

Consider a one period model with a riskless asset and a single risky asset with (stochastic) liquidation value v. v is a realization of the random variable $\tilde{v} \sim N(\overline{v}, \sigma_v^2)$. We interpret v as the expected value of the risky asset conditional on all public and private information in the economy. There are I traders in the market. Traders have heterogeneous private information. We assume that, in the beginning of the period, trader i receives a private signal

$$y^i = v + \epsilon^i,$$

where $\epsilon^i \sim N(0, \sigma_\epsilon^2)$. Since we interpret v as the full-information value of the asset, it is reasonable to assume that ϵ^i is independent of v and of ϵ^j for $j \neq i$.

Trader i has preferences over end of period wealth W^i described by the constant absolute risk aversion utility function

$$U_i(W^i) = -\,exp\,(-r_i W^i)$$

(where r_i is trader i's coefficient of absolute risk aversion). Note that if the ith trader holds X^i units of the risky asset, R is the return on the riskless asset, $\overline{W}^i$ is i's initial wealth endowment, and the price of the risky asset is p then end-of-period wealth is

$$W^i = vX^i + R(\overline{W}^i - pX^i).$$

Let I^i denote the information set on which trader i conditions his decisions. (In the following analysis I^i will, alternately, consist of price, private information y^i, or the vector of price and private information). Since W^i is normally distributed conditional on I^i, trader i will maximize

$$E(\tilde{W}^i \mid I^i) + \frac{r_i}{2} var (\tilde{W}^i \mid I^i) = E(\tilde{v}X^i + R(\overline{W}^i - pX^i) \mid I^i) + \frac{r_i}{2} var (\tilde{v}X^i + R(\overline{W}^i - pX^i) \mid I^i),$$

and i's demand X^i for the risky asset will be

$$X^i = \frac{E(\tilde{v} \mid I^i) - Rp}{r_i \, var (\tilde{v} \mid I^i)} \tag{1}$$

Volume is defined as

$$V = \frac{1}{2} \sum_i |\Delta X^i| \tag{2}$$

Using (1), V can be rewritten as

$$V = \frac{1}{2} \sum_i \left| \Delta \frac{(f^i - \bar{f}) + (\bar{f} - Rp)}{r_i \, var (\tilde{v} \mid I^i)} \right| \tag{3}$$

where $f^i = E(\tilde{v} \mid I^i)$, $\bar{f} = \frac{\sum f^i}{I}$. It turns out that $\Delta(\bar{f} - Rp)$ can be reduced to different functions of the change in price according to the equilibrium concept used to solve the model. Our tests are based on examining the alternative expressions for $\underset{I \to \infty}{plim} V^2$ as the equilibrium hypothesis is varied.[4]

A. Competitive (Walrasian) Equilibrium:

Traders condition their decisions solely on their own private information y^i. Thus, from (1) above (with $I^i = y^i$)

$$X^i = \frac{E(\tilde{v} \mid y^i) - Rp}{r_i var (\tilde{v} \mid y^i)} \tag{1'}$$

Using the projection theorem, we have

$$f^i = E(\tilde{v} \mid y^i) = \beta y^i + (1 - \beta)\bar{v}, \tag{4}$$

$$s = var (\tilde{v} \mid y^i) = \sigma_v^2 (1 - \beta), \text{ where } \beta = \frac{\sigma_v^2}{\sigma_v^2 + \sigma_\epsilon^2}.$$

If A is the supply of the risky asset, equilibrium price is given by

$$\sum_i \frac{f^i - Rp}{r_i s} = A$$

so that we can solve for p as

$$p = \sum \frac{f^i}{RMr_i} - \frac{s}{MR} A,$$

where

$$M = \sum_i (\frac{1}{r_i})$$

And, from (2), volume is given by

$$V = \frac{1}{2} \sum | \Delta X^i | = \frac{1}{2} \sum_i | \frac{\Delta f^i - R \Delta p}{r_i s} | . \tag{2'}$$

(note that $var(\tilde{v} \mid y^i)$ does not depend on y^i.)

However, it will be useful to look at a different expression for (2'). As in (3) above, let us decompose trading volume into two components: the amount due to the shift in the dispersion of traders' beliefs around the average and the amount due to "price effects." Price effects are present if the real price of the asset is not equal to the average of traders' expectations of the asset's value. Thus, write ΔX^i as

$$\Delta X^i = \Delta \frac{(f^i - \bar{f}) + (\bar{f} - Rp)}{r_i s}$$

Using the definition of y^i and (4), the change in price Δp is given by

$$\Delta p = \sum_i \frac{\beta \Delta v + \beta \Delta \epsilon_i}{RMr_i} = \frac{\beta}{R} \Delta v + \frac{\beta}{R} \sum_i \frac{\Delta \epsilon_i}{r_i M}$$

Thus, assuming that traders' risk aversion coefficients are uniformly bounded, as the number of traders I becomes large, $\Delta p \rightarrow \frac{\beta}{R} \Delta v$. Similarly as I increases, $\bar{f} \equiv \frac{\sum f^i}{I} \rightarrow \beta v$ so that $\Delta \bar{f} = R \Delta p$. Thus, in a market with a large number of traders,

$$\Delta X^i = \Delta \frac{(f^i - \bar{f}) + (\bar{f} - Rp)}{r_i s} = \Delta \frac{(f^i - \bar{f})}{r_i s} \tag{5}$$

and

$$V = \frac{1}{2} \sum_i | \Delta \frac{(f^i - \bar{f})}{r_i s} | \tag{3'}$$

Note that, in the case of competitive equilibrium, price effects are zero.

However, for the purpose of nesting our alternative hypotheses later on, it will be useful to consider an expression for the square of volume. From (4) and the definition of y^i, $f^i - \bar{f} = \beta \epsilon^i$. Thus, using (5) we can write

$$\left[\sum_i |\Delta X^i|\right]^2 = \left[\sum |\frac{\beta \Delta \epsilon^i}{s r_i}|\right]^2$$

$$= \sum_i \left[\frac{\beta \Delta \epsilon^i}{s r_i}\right]^2 + \sum_i \sum_{j \neq i} |\frac{\beta \Delta \epsilon^i}{s r_i}| \; |\frac{\beta \Delta \epsilon^j}{s r_j}|$$

As a sample, $|\Delta \epsilon^i||\Delta \epsilon^j|$ would behave as a product of randomly chosen independent variables so that, for I large,

$$\sum \frac{|\Delta \epsilon^i||\Delta \epsilon^j|}{I(I-1)} \approx E|\tilde{U}_1||\tilde{U}_2| = E|\tilde{U}_1| \, E|\tilde{U}_2| = \frac{4}{\pi} \sigma_\epsilon^2 \tag{6}$$

where $U_i \sim N(0, 2\sigma_\epsilon^2)$, and where the last equality follows from Mosteller (1946). Thus, assuming that traders' risk aversion coefficients are ex-ante independent of $\tilde{v}$ and $\tilde{\epsilon}$,

$$p\,lim \sum_i \left[\frac{\beta \Delta \epsilon^i}{s r_i}\right]^2 = p\,lim \frac{\sum \left[\frac{1}{r_i}\right]^2 \sum \left[\frac{\beta \Delta \epsilon^i}{s}\right]^2}{I}$$

Volume, V, is defined by $V = \frac{1}{2} \sum |\Delta X^i|$. Thus, since $f^i - \bar{f} = \beta \epsilon^i$, for I large,

$$V^2 \approx \frac{N}{4} \sum \frac{\left[\Delta \frac{f^i - \bar{f}}{s}\right]^2}{I} + N(I^2 - I) \frac{\beta^2}{s^2} \frac{\sigma_\epsilon^2}{\pi}, \tag{7}$$

where $N = \sum (\frac{1}{r_i})^2$

Thus, in a competitive or Walrasian model, agents trade on the basis of changes in differences in beliefs concerning the asset's future value. The aggregation of these differences is measured by the dispersion of beliefs around the average. Since traders do not condition beliefs on prices, quantity demanded (and price) is proportional to the average value of beliefs. Changes in price only occur when there are changes in average beliefs. Since these effects counterbalance each other, changes in price cannot help predict volume.

B. Fully Revealing Rational Expectations (Efficient Markets)

Traders observe y^i and the price of the risky asset, and condition posterior beliefs on both these pieces of information. Since there is no source of noise, prices convey all relevant information. That is, price is a sufficient statistic (in this case, the mean of private signals $\bar{y} = \frac{\sum y^i}{I}$) for the set of all traders' private information: a trader's signal is redundant given price and the market reaches a full-information allocation of assets.

We follow Grossman (1976) in computing the equilibrium of the model. From (1), trader i's demand for the risky asset is

$$X^i = \frac{E(\tilde{v} \mid y^i, p) - Rp}{r_i \, var\,(\tilde{v} \mid y^i, p)} \tag{1''}$$

Conjecture a price of the form

$$p = a_0 + a_1 \bar{y}$$

where $\bar{y} = \sum \frac{y^i}{I}$. Thus, since $E(\tilde{v} \mid p) = E(\tilde{v} \mid \bar{y}) = \frac{\bar{v}}{1 + I\sigma_v^2} + \frac{I\sigma_v^2 \bar{y}}{1 + I\sigma_{v_2}}$, and $var\,(\tilde{v} \mid p) = var\,(\tilde{v} \mid \bar{y}) = \frac{\sigma_v^2}{1 + I\sigma_v^2}$, it must be that

$$f^i \equiv E(\tilde{v} \mid y^i, p) = E(\tilde{v} \mid y^i, \bar{y}) = E(\tilde{v} \mid \bar{y}) \quad \text{for all } i$$

(assuming $a_1 > 0$). If A is the supply of the risky asset, equilibrium requires that

$$\sum_i \frac{f^i - Rp}{r_i s} = A$$

so that a direct computation yields (for $M = \sum \frac{1}{r_i}$),

$$a_0 = \frac{\bar{v}M - \sigma_v^2 A}{(1 + I\sigma_v^2)RM} \ , \ a_1 = \frac{I\sigma_v^2}{(1 + I\sigma_v^2)\,R}$$

and trader i's demand for the risky asset can be reduced to

$$X^i = \frac{\sigma_v^2 A}{M r_i \sigma_v^2}.$$

Thus, we can immediately deduce that volume $\frac{1}{2}\sum |\Delta X_i|$ is zero whenever there is no change in supply.

However, to provide a contrast with the other hypotheses, it will be instructive to apply the method developed in section IIA above and which we will use again in section IIC. As in (3), let us decompose volume into a component which depends on the dispersion of traders' beliefs and a "price effect" term. Change in demand is

$$\Delta X^i = \Delta \frac{(f^i - \bar{f}) + (\bar{f} - Rp)}{r_i s} = 0 \qquad (5')$$

and volume V satisfies

$$V^2 = 0 \qquad (7')$$

This is because $f^i = E(\tilde{v} \mid p)$ for all i so that $f^i = \bar{f}$. Also, $\Delta\bar{f} = Is\Delta\bar{y} = R\Delta p$, so that the price effect is also zero. Thus, the absence of dispersion in traders' expectations of the asset's value makes the equilibrium price collapse to this common value. The absence of "gains from trade" from any trader's viewpoint leads to zero expected volume. If there is positive volume in an efficient or fully revealing market, this volume is unpredictable from data on prices, traders' expectations, etc.

C. Noisy Rational Expectations

Traders condition their decisions on price as well as on their private information.[5] Thus, if trader i receives private information y^i, then from (1) (with $I^i = (y^i, p)$), trader i's demand for the risky asset is given by

$$X_i = \frac{E(\tilde{v} \mid y^i, p) - Rp}{r_i \, var\, (\tilde{v} \mid y^i, p)} \qquad (1''')$$

Following Hellwig (1980), Diamond and Verrecchia (1982), and Pfleiderer (1982), however, the supply of the risky asset is stochastic and not directly observable.[6] Thus, price will not reveal a sufficient statistic for the vector of traders' private signals. Since markets are not efficient, the private information of a trader will not be redundant given price. Let Z denote the supply of the risky asset; Z is a realization of the random variable $\tilde{Z} \sim N(\bar{Z}, \sigma_Z^2)$.

Hellwig (1980) proves the following theorem. There exists an equilibrium with prices linear in the "state variables"; that is, equilibrium price functions are of the form $\tilde{p} = a_0 + a_1\tilde{v} + a_2\tilde{Z}$, where a_0, a_1, a_2 are given in figure 1. By the projection theorem, trader i's posterior belief is given by $f^i \equiv E(\tilde{v} \mid \tilde{y}^i, \tilde{p}) = b_0 + b_1\tilde{y}^i + b_2\tilde{p}$ where b_0, b_1, b_2 are given in figure 1. We will use these results to construct an expression for trading volume.

First, let us examine the relationship between the coefficients of the equilibrium price function and the coefficients of the equilibrium belief function. Let $s = var(\tilde{v} \mid y^i, p)$. Equating aggregate demand with aggregate supply

$$\sum_{i=1}^{I} \frac{f^i - Rp}{r_i s} = \sum_{i=1}^{I} \frac{b_0 + b_1 y^i + b_2 p - Rp}{r_i s} = Z$$

allows us to write price as

$$p = \frac{1}{R - b_2}\left[b_0 + b_1\left[v + \sum_{i=1}^{I} \frac{\epsilon_i}{r_i M}\right] - \frac{sZ}{M}\right] \tag{8}$$

where $M = \sum(\frac{1}{r_i})$. If risk aversion coefficients are uniformly bounded across traders, then $plim \sum_{i=1}^{I} \frac{\epsilon_i}{r_i M} = 0$. Since $p = a_0 + a_1 v + a_2 Z$, (8) implies that

$$a_0 = \frac{b_0}{R - b_2}\ ,\ a_1 = \frac{b_1}{R - b_2}\ ,\ a_2 = \frac{-s}{M(R - b_2)} \tag{9}$$

The relations in (9) will prove to be useful below.

Now consider traders' demands and volume. Once again, as in (3) and (3'), let us decompose trading volume into a component depending on the dispersion of traders' beliefs around the average and a component due to the divergence of the asset's price from traders' average assessment of its value (the "price effect"). Let $f^i = E(\tilde{v} \mid y^i, p)$, $\bar{f} = \sum \frac{f^i}{I}$. Write trader i's demand for the risky asset as

$$X^i = \frac{(f^i - \bar{f}) + (\bar{f} - Rp)}{r_i s}$$

where

$$s = var\ (\tilde{v} \mid p, y^i) = \sigma_v^2(1 - Ra_1).$$

As $I \rightarrow \infty$, since $y^i = v + \epsilon^i$,

$$\bar{f} = \frac{\sum_i (b_0 + b_1 y^i + b_2 p)}{I} \rightarrow b_0 + b_1 v + b_2 p. \tag{10}$$

Using (9) above,

$$plim\,(\bar{f} - Rp) = b_0 + b_1 v + (b_2 - R)p$$

$$= b_0 + b_1 v + (b_2 - R)(a_0 + a_1 v + a_2 Z) = \frac{sZ}{M}.$$

Thus, in a market with a large number of traders, the change in trader i's holdings is

$$\Delta X^i = \Delta \frac{(f^i + \bar{f}) + \frac{sZ}{M}}{r_i s} \tag{5''}$$

so that trading volume is

$$V = \frac{1}{2} \sum_i |\Delta \frac{(f^i - \bar{f})}{r_i s} + \frac{\Delta Z}{r_i M}| \tag{3''}$$

(note that s does not depend on p or y^i).

Note that, unlike in the case of competitive equilibrium (see (7)), the price effect is non-zero. Again, for the purposes of nesting our hypotheses, it will be useful to consider an expression for the square of volume using the methods of section II A above. The procedure is analogous to that used in deriving (7).

First, note that, as I becomes large, $f^i - \bar{f} \rightarrow b_2 \in_i$. Thus, from (3'') above,

$$V^2 = \frac{1}{4} (\sum_i \frac{1}{r_i} |k_1 \Delta \in^i + k_2 \Delta Z|)^2, \quad \text{where } k_1 = \frac{b_2}{s}, \; k_2 = \frac{1}{M}.$$

We can then write

$$V^2 = \frac{1}{4} \sum_i \frac{1}{r_i^2} (k_1 \Delta \in^i + k_2 \Delta Z)^2 + \frac{1}{4} \sum_{i,j} \frac{1}{r_i r_j} |k_1 \Delta \in^i + k_2 \Delta Z| \, |k_1 \Delta \in^j + k_2 \Delta Z|.$$

Assuming that traders' risk aversion coefficients are ex-ante independent of $\tilde{v}, \tilde{\in}, \tilde{Z}$,

$$\underset{I \rightarrow \infty}{p\,lim} \sum_{i,j} \frac{1}{r_i r_j} |k_1 \Delta \in^i + k_2 \Delta Z| \, |k_1 \Delta \in^j + k_2 \Delta Z|$$

$$= p \lim_{I \to \infty} \sum_{i,j} \frac{1}{r_i r_j} \sum \frac{|k_1 \Delta\epsilon^i + k_2 \Delta Z| \, |k_1 \epsilon^j + k_2 \Delta Z|}{I(I-1)}$$

For large samples, given the realization ΔZ, we would expect $\sum |k_1 \Delta\epsilon^i + k_2 \Delta Z| \, | k_1 \Delta\epsilon^j + k_2 \Delta Z \,|$ to behave approximately as a sum of independent random variables so that

$$\sum \frac{|k_1 \Delta\epsilon^i + k_2 \Delta Z| \, |k_1 \Delta \epsilon^j + k_2 \Delta Z|}{I(I-1)} \approx E|W_1| E|W_2|$$

where $W_i \sim N(k_2 \Delta Z, 2k_1^2 \sigma_\epsilon^2)$. From Leone et.al. (1961),

$$E|W_i| = \sqrt{\frac{4}{\pi} k_1^2 \sigma_\epsilon^2} \; exp\left[-\frac{1}{2}\left[\frac{k_2 \Delta Z}{2k_1 \sigma_\epsilon}\right]^2\right] + k_2 \Delta Z \left[1 - \Phi\left[-\frac{k_2 \Delta Z}{\sqrt{2} k_1 \sigma_\epsilon}\right]\right],$$

where Φ is the cumulative normal distribution. Note that (from Figure 1), $k_1 = \frac{b_2}{s} = \frac{b_2}{\sigma_v^2 (1 - a_1 R)} = \frac{1}{\sigma_\epsilon^2}$. Thus,

$$E|W_i| = \sqrt{\frac{4}{\pi} \frac{1}{\sigma_\epsilon^2}} \; exp\left[-\frac{1}{2}\left[\frac{\Delta Z \sigma_\epsilon}{\sqrt{2} M}\right]^2\right] + \Delta Z \left[1 - \Phi\left[-\frac{\Delta Z \sigma_\epsilon}{\sqrt{2} M}\right]\right].$$

We hypothesize that $\Delta Z \sigma_\epsilon$ is large enough that $E|W_i| \approx k_2 \Delta Z$; this is, of course, a specification of the model. Under this specification, for $N = \sum_i (\frac{1}{r_i^2})$, $N' = \sum_{i,j} \frac{1}{r_i r_j}$,

•

$$p\,lim\, V^2 = p\,lim\, \frac{1}{4} \sum_i \left[\frac{1}{r_i^2}\right] \sum \frac{\left[\left[\frac{b_2}{s} \Delta\epsilon^i\right]^2 + \left[\frac{\Delta Z}{M}\right]^2 + 2\frac{b_2}{sM} (\Delta Z \, \Delta\epsilon^i)\right]}{I} + \frac{N'}{4M^2} (\Delta Z)^2$$

$$= p\,lim\, \frac{N}{4M^2} \sum_i \frac{\left[\left[\frac{b_2}{s} \Delta\epsilon^i\right]^2 + \left[\frac{\Delta Z}{M}\right]^2\right]}{I} + \frac{N'}{4M^2} (\Delta Z)^2. \qquad (11)$$

Using Figure 1 and recalling that $f^i - \bar{f} = b_2 \epsilon^i$ for large I, we can write (11) as

$$V^2 = \frac{N}{4} \frac{\sum_i \left[\frac{\Delta(f^i - \bar{f})}{s}\right]^2}{I} + \frac{(\frac{N'}{M^2} + \frac{N}{M^2})}{4} (\Delta Z)^2. \qquad (12)$$

ΔZ, however is unobservable; we want to find an expression for the last term in (12) in terms of observable variables. From the price equation,

$$\Delta Z = c_1 \Delta p + c_2 \Delta v$$

where

$$c_1 = \frac{1}{a_2} \quad c_2 = -\frac{a_1}{a_2}.$$

Since $\Delta p \, \Delta v = a_1(\Delta v)^2 + a_2 \, \Delta v \, \Delta Z$, we have that

$$(\Delta Z)^2 = c_1^2 \, (\Delta p)^2 + c_2^2 (\Delta v)^2 + c_1 c_2 (a_1 (\Delta v)^2 + a_2 \Delta v \, \Delta Z)$$

$$= c_1^2 (\Delta p)^2 - c_2^2 (\Delta v)^2 + e_1 \tag{13}$$

where we used $c_1 c_2 a_1 = -c_2^2$ and where $Ee_1 = 0$.

Using the expression for traders' average beliefs, $\Delta \bar{f} = b_1 \Delta v + b_2 \Delta p$, we can replace $(\Delta v)^2$ in (13) as follows. Since

$$(\Delta v)^2 = (\frac{1}{b_1})^2 \, (\Delta \bar{f})^2 + (\frac{b_2}{b_1})^2 \, (\Delta p)^2 - 2\frac{b_2}{b_1^2} \, \Delta p \Delta \bar{f} \text{ and}$$

$$\Delta p \Delta \bar{f} = \Delta p (b_1 \Delta v + b_2 \Delta p) = b_1 (a_1 (\Delta v)^2 + a_2 \Delta v \, \Delta Z) + b_2 (\Delta p)^2, \tag{14}$$

we can solve for $(\Delta v)^2$ as

$$(\Delta v)^2 = \left[\frac{1}{(b_1 + 2b_2 a_1)} \right] (\Delta \bar{f})^2 - \left[\frac{b_2^2}{b_1 (b_1 + 2b_2 a_1)} \right] (\Delta p)^2 + e_2 \tag{15}$$

where $Ee_2 = 0$.

The expression for $(\Delta v)^2$ in (15) allows us to write $(\Delta Z)^2$ (see (13)) in terms of $(\Delta p)^2$ and $(\Delta \bar{f})^2$. Thus, from (12), volume is described by

$$V^2 = \frac{N}{4} \sum_i \frac{\left[\dfrac{\Delta (f^i - \bar{f})}{s} \right]^2}{I} + \frac{\dfrac{N'}{M^2} + \dfrac{N}{M^2}}{4} (q_1 \, (\Delta p)^2 + q_2 (\Delta \bar{f})^2) + e \tag{7''}$$

where

$$q_1 = \frac{1}{a_2^2} (1 + \frac{a_1^2 b_2^2}{b_1 (b_1 + 2b_2 a_1)}) > 0 \tag{16a}$$

$$q_2 = -\frac{1}{a_2^2} (\frac{a_1^2}{b_1 + 2b_2 a_1}) < 0 \tag{16b}$$

and $Ee = 0$. Under noisy rational expectations, volume is affected by price changes. This captures the following phenomenon. Suppose traders condition decisions on price. The price change term then measures the effect on volume of change in the amount of the noise in prices, and the propagation of this effect through expectations: noise drives a wedge between traders mean expectation and price, and increases the divergence of traders' (posterior) expectations around price. The magnitude of this wedge is proportional to the change in (noisy) supply ΔZ. The second term in (7'') extracts this change in supply from changes in price and in average beliefs.

III. Empirical Analysis

Our aim is to test the three alternative hypotheses, presented in the last section, on equilibrium pricing in financial markets where traders have heterogeneous beliefs. The tests are based on examining the determinants of stock market trading volume for a sample of firms listed on the NYSE. The expectational variables are measured using survey data on individual analysts' forecasts for each firm in the sample. We formulate the structure of the test in section A below. Section B discusses our sampling methods and estimation procedure. The empirical results are presented in section C.

A. Formulation of the Test

Equations (7), (7') and (7'') allow us to formulate a structural econometric model which nests the three candidate hypotheses.[7] Consider the equation

$$V^2 = \beta_1 \frac{1}{I} \sum_i \left[\Delta \frac{(f^i - \bar{f})}{\sigma_v^2} \right]^2 + \beta_2 \left[\frac{\Delta p}{p^*} \right]^2 + \beta_3 \left[\frac{\Delta \bar{f}}{\bar{f}} \right]^2 + \beta_0 \tag{17}$$

where

V is the trading volume of the asset

I is the number of traders

f^i is trader i's posterior expectation of the asset's future value

$\bar{f}$ is the average of traders' posterior expectations

σ_v^2 is the variance of the asset's future value

p is the asset's (current) price

p^* is an adjusted price scaling variable

The first term in (17) explains the amount of volume due to a change in the dispersion of expectations across traders. We call the first term of (17) the *"change in dispersion" term.* The second term, which we call the *"price change" term,* measures the impact of price changes on volume. The third term in (17) measures how volume changes with the magnitude of the average of traders' expectations. We call this term the *"average belief change"* term. The value of the coefficient vector $\beta' = (\beta_0, \beta_1, \beta_2, \beta_3)$ depends upon the choice of equilibrium. Under competitive equilibrium (see equation (7)), $\beta_1 > 0$, $\beta_2 = \beta_3 = 0$, and $\beta_0 > 0$. Under noisy rational expectations (see equation (7'')), $\beta_1 > 0$, $\beta_2 > 0$, $\beta_3 < 0$, $\beta_0 = 0$ and $|\beta_2| > |\beta_3|$. Finally, the fully revealing efficient markets hypothesis predicts (equation (7')) that volume is unpredictable from price and expectations data so that $\beta_0 = \beta_1 = \beta_2 = \beta_3 = 0$. This last case is just a specialization to our context of the "random walk hypothesis."

The scaling factors for the independent variables in (17) were chosen in order to minimize the instability of coefficients β' across firms and over time. The empirical support for our choices are discussed in subsection C below. Briefly, the theoretical considerations are as follows. Scaling $\Delta(f^i - \bar{f})$ by σ_v^2 in (17) and (18) is more appropriate than using an unscaled variable if $\frac{\sigma_v^2 + \sigma_\epsilon^2}{\sigma_\epsilon^2}$ is more stable than $\frac{\sigma_v^2 + \sigma_\epsilon^2}{\sigma_\epsilon^2 \sigma_v^2}$ (see (7) and (7'')). This case would be true if, for example, σ_v^2 varies across firms and time less than σ_ϵ^2 does. Note that the conditions under which β_1 would be stable are similar across hypotheses. Thus, tests of the invariance of β_1 across firms will be consistent under any assumed hypothesis.

Suppose we scale Δp by p. Recall that $\beta_2 \neq 0$ only under noisy rational expectations. The coefficient of the unscaled variable in this case is proportional to $c_1^2 = \frac{1}{a_2^2}$, where a_2 is the coefficient of supply in the (linear) price function. Since p is linear in a_2, p will tend to rise as c_1^2 falls. This particular effect would tend to make the coefficient of the variable scaled by p more stable. However, under all three equilibrium hypotheses, p depends on v, the full information valuation of the asset. This effect would tend to make the coefficient of the scaled variable increase with v. To adjust for this, we consider the scaling factor $p^* = kp$, where k is a variable which we expect to be positively correlated with v. Thus, k should be decreasing with the relative capitalized market value of the firm. Empirically, we find

$k = (\frac{\bar{E}}{E})^{1/4}$ to be optimal, where E is the firm's number of shares outstanding times its current stock price and $\bar{E}$ is the average value of E across all firms in our sample. The stability of coefficients across firms is further discussed along with the empirical results in subsection C below.

One important consideration involves the conditions under which forecasts of annual earnings represent expectations of (future) stock liquidation values, as our model requires. Expected present values of future profits are one-to-one in earnings forecasts if earnings follow a Markov process. Note that the conditions we need for our model to be consistent with our empirical analysis are the assumptions often used in asset pricing models (see Constantinides (1988)).

Interpretation of Residuals

Residuals in the competitive equilibrium model (see (7)) can be interpreted as measurement error in the standard way, or as deviations of the sample cross product forecast errors from the large sample constant value $\frac{4}{\pi}\sigma_\epsilon^2$ (see (6)). However, because of the presence of noise trading, the interpretation of residuals in the noisy rational expectations model may seem less straightforward. We will show here how, if residuals represent noise trading volume, they will not necessarily be correlated with the independent variables.

Below we derive (unsystematic) trading volume V by taking the residuals of the market model which regresses individual firms' trading volume V^T on the market trading volume V^M (see sub section III B below for estimation procedure):

$$V^T = \alpha_0 + \alpha_1 V^M + V$$

Suppose we split systematic (market) and unsystematic volume into components corresponding to speculative trading volume and noise trading volume.

$$V \equiv V^s + V^N$$

$$\alpha_0 + \alpha_1 V^M \equiv W^s + W^N$$

$$V^T \equiv U^s + U^N$$

(where the superscripts s and N refers to the speculative component and noise trading component respectively). Then,

$$V = V^s + V^N = (U^s - W^s) + (U^N - W^N)$$

Traders' demands in our model depend on their private information and not on liquidity or other noise trading motives; thus, our model describes V^s, speculative trading volume. The variable V^2 in (17) therefore equals $V^2 = ((U^s - W^s) + (U^N - W^N))^2 = (V^s + V^N)^2 = (V^s)^2 + 2V^N V^s + (V^N)^2$, where $V_s^2 = \beta_1(\Delta H)^2 + \beta_2(\frac{\Delta p}{p^*})^2 + \beta_3(\Delta\bar{f}/\bar{f})^2 + \beta_0$, the right hand side of (17) (where under rational expectations $\beta_0 = 0$).

Is it possible for V^N and V^s to be uncorrelated, and for $2V^N V^s + (V^N)^2$ (appropriately scaled) to be interpreted as the residual of (17)? The reason why one might think that positive correlation would have to exist is the following. We are essentially interpreting the residuals in (17) as noise trading volume. Recall that supply, Z, in the noisy rational expectations version of the model is random. Since our model describes speculative trading, Z can be thought of as the net supply of the asset held by non-noise traders. Z is random because of the "random" behavior of noise traders. Thus, will price, which depends on Z, not have to be correlated with the residual? The answer is no. ΔZ represents the net trade between speculators and noise traders, while noise trading volume is the sum of the quantities bought and sold by all noise traders. If noise trading behavior is determined by non-speculative motives and can be described through time by a stationary random process, then noise trading volume should be uncorrelated with speculative trading centered around specific information events.[8]

B. Sample and Estimation Procedures:

a. Sampling

To estimate the change in dispersion term in (17),

$$(\Delta H)^2 \equiv \frac{1}{I}\sum_i \left[\Delta \frac{(f^i - \bar{f})}{\sigma_v^2}\right]^2, \tag{18}$$

we consider changes in analysts' forecasts surrounding interim quarterly earnings announcements. Two separate data sets are used. Note that to calculate $\Delta(f^i - \bar{f})$ requires tracking the change in individual analysts' forecasts of each company before and after the information event. Thus, it is necessary to identify and date each analysts' forecast. For this purpose, we use the first data set. This is comprised of detailed survey data on individual analysts' forecasts obtained from the IBES (Institutional Brokers Estimate System) data base provided by Lynch, Jones and Ryan Co., for the period 1983 through 1986. Included in this data base are the raw data of forecasts of individual analysts and the dates in which they were reported. On the other hand, accurate measurement of the denominator of the change in dispersion (the unsystematic variance of earnings σ_v^2) requires a sufficiently long time series of data. Thus, we estimate the denominator using a second data base, the IBES consensus data tape which covers the period from 1976 to 1987. This data base only reports the mean, variance and median of monthly forecasts across analysts. Thus, the aggregated data in this consensus sample could not be used to measure the numerator of $(\Delta H)^2$.

We now turn to estimation procedure. Additional details of the sampling methods we use are discussed in the Appendix.

b. Estimation Procedure

We estimate the change in dispersion term by

$$(\Delta H)^2 \equiv \frac{\sum_{i=1}^{I_{kt}} \left[\Delta \frac{(f_i(v_{kt}) - \bar{f}(v_{kt}))}{var\ (v_{kt})} \right]^2}{I_{kt}},$$

where v_{kt} is the earnings per share (EPS) of the kth firm, $f_i(v_{kt})$ is the ith analyst's forecast of the EPS of firm k at time t, and $var(v_{kt})$ is the estimated variance of the EPS of firm k. I_{kt} is the number of analysts forecasting the EPS of firm k at time t.

Specifically, suppose company k in our sample announces earnings at time t (measured by month and year). We then estimate $\Delta \left[\frac{f_i(v_{kt}) - \bar{f}(v_{kt})}{var(v_{kt})} \right]$ by matching individual analysts' forecasts before and after the earnings announcement:

$$\left[\frac{f_i(v_{k(t+1)}) - \bar{f}(v_{k(t+1)})}{var\ (v_{k(t+1)})} \right] - \left[\frac{f_i(v_{k(t-1)}) - \bar{f}(v_{k(t-1)}))}{var\ (v_{k(t-1)})} \right]$$

where $var\ (v_{ks})$ is estimated as follows. Let y_j denote the month of the jth earnings announcement of firm k in year y and let $y_j + 1, y_j - 1$ denote the months before and after y_j. Let v_{ky} denote the end-of-year y EPS of firm k (what we call v_{kt} elsewhere for arbitrary dates t). We calculate the time series averages

$$var\ (v_{j-1}) = \frac{\sum_{y=76}^{y=84} (v_{ky} - \bar{f}(v_{k(y_j-1)})^2}{9}$$

$$var\ (v_{j+1}) = \frac{\sum_{y=76}^{y=84} (v_{ky} - \bar{f}(v_{k(y_j+1)})^2}{9}$$

Thus, in estimating variances we calculate $var(v_{j+1})$ and $var(v_{j-1})$ around earnings announcements occurring in the same month every year, and calculate variances for each announcement month.[9] If t is the date of the jth earnings announcement of firm k for that year, then we estimate $var\,(v_{k(t-1)})$ by $var\,(v_{j-1})$ and $var\,(v_{k(t+1)})$ by $var\,(v_{j+1})$ for all k and t.

Similarly, we estimate the change in average belief term by

$$(\frac{\Delta \bar{f}}{\bar{f}})^2 = \left[\frac{\bar{f}(v_{k(t+1)}) - \bar{f}(v_{k(t-1)})}{\bar{f}(v_{k(t-1)})} \right]^2$$

To estimate the price change term, we calculate the square of the percentage change in stock price $(\Delta p/p)^2$ from the two days before the Wall Street Journal Index day to the day of the Wall Street Journal Index day.

Total daily trading volume in a given stock is measured as the (market value of) shares traded divided by the (market value of) shares outstanding. To remove market movements, trading volume is measured as the residual term of the market model which regresses individual firms' trading volume on the market trading volume over the 90 trading days ending 10 trading days prior to the earnings announcement day. (Total daily market trading volume is measured as the market value of all stocks traded on the NYSE divided by the total market value of all firms' listed in the NYSE). We add (abnormal) trading volume on the day that a given interim earnings report appeared in the Wall Street Journal to (abnormal) trading volume on the previous trading day in order to account for the possibility that the announcement appeared in the Dow Wire Service that day.[10]

Note that the maximum values of the change in dispersion and the average belief change terms are (10 and 16 standard deviations from the mean. This supports our modelling assumption that beliefs are dispersed over a large range; however, this outlier problem may distort our results. To reduce the impact of large measurement errors on our estimates of expectational variables, outliers for $(\Delta H)^2$ and $(\Delta \bar{f}/\bar{f})^2$ beyond two standard deviations from the mean are winsorized to two standard deviations. Since similar results are obtained with and without winsorizing, we only report estimates based on the winsorized variables. Summary statistics of the above variables are presented in Panel A of Table 1.

B. Empirical Results

a. Regression Results

As a preliminary result, note that the correlation coefficient between the change in dispersion, and the squared trading volume is significantly positive at the 1 percent level (see panel B, Table 1). The correlation coefficient between the squared percentage change in stock price and the square of trading volume is also significant at the 1 percent level. Note that all three equilibrium hypotheses predict an (asymptotically) zero correlation between price and the change in dispersion term. Panel B shows that the data is consistent with this conclusion.

Consider now the multivariate regression results.[11] We first examine the regression of trading volume on the change in dispersion, that is we look at

$$V^2 = \delta_1 (\Delta H)^2 + \delta_0$$

where $(\Delta H)^2$ is the first term in (17). The results are reported in Regression 1 in Table 2. Regression 1 shows a significantly positive impact of the change in dispersion at the 1 percent level, while the intercept is also significantly positive at the 1 percent level. Thus, a test against an unspecified hypothesis supports the competitive equilibrium concept.

Regression 3 in Table 2 A reports the full results for equation (17). In Regression 2, we also study the effect of excluding the average belief change variable $(\frac{\Delta \bar{f}}{\bar{f}})^2$. Note that the coefficient of $(\frac{\Delta p}{p^*})^2$ is significant at the 1 percent level. Also, the inclusion of the price change term does not change the significance level of the change in dispersion, $(\Delta H)^2$ while the t-statistic for the intercept is reduced down to an insignificant level. Thus, the (significant) intercept term in Regression 1 can be decomposed and explained as tracking the average effect of average price changes on volume.

[Table 1]

[Table 2]

A surprising result is that the coefficient of $(\Delta H)^2$ (as well as the t-statistic of the coefficient) is essentially unchanged after the inclusion of the price term. This indicates that $(\Delta H)^2$ and $(\Delta p)^2$ are orthogonal (as the correlation table in Table 1, Panel B already indicates). Of course, this orthogonality is an implication for the model of all three equilibrium concepts, since $\Delta\epsilon^i$ and changes in the equilibrium price Δp are independent. This result does not help distinguish between the three hypotheses but serves as a test of our modelling specifications.

However, the orthogonality of $(\Delta H)^2$ and $(\Delta p/p^*)^2$ across combinations of regressions in Table 2 implies that the dispersion of private information across traders has an impact on trading volume, but not on the level of prices. Price may be determined by aggregate quantities (there is some correlation between price changes and changes in average expectations), but not by the magnitude of differences in beliefs.

A problematic result is the insignificance of the coefficient of $(\frac{\Delta\bar{f}}{\bar{f}})^2$, the average belief change (in non-winsorized samples, the sign of the coefficient is negative but still insignificant).[12] Note from (16b) that, in a noisy rational expectations world, the coefficient of $(\frac{\Delta\bar{f}}{\bar{f}})^2$ would be of smaller magnitude than the coefficient of the price change term and would approach zero as the variance of the forecast error σ_ϵ^2 increased. This is one possible explanation for the insignificance of the average belief term. Nonetheless, we offer another explanation. Our linear model does not allow for wealth effects; CARA preferences imply that proportions of assets held in a portfolio are invariant to changes in wealth. If risk preferences changed with wealth, then a change in the average valuation of the asset $\bar{f}$ would increase trading volume. This effect generates a positive relation between $(\Delta\bar{f}/\bar{f})^2$ and V^2. If the "information effect" were zero (as in competitive equilibrium) then the net effect of a change in average beliefs would be positive. However, if the information effect were negative (as in the noisy rational expectations case) then the wealth effect would have an offsetting influence.

In panel B of Table 2, we consider the same regressions but suppress the intercept. This may be instructive because the noisy rational expectations hypothesis predicts a zero intercept. The results are essentially unchanged.

Since we use cross-section data on earnings announcements which occur over several years, it is useful to check for heteroskedasticity of the disturbances. For this purpose, we plotted a graph of residuals against the independent variables $(\Delta H)^2$ and $(\frac{\Delta p}{p})^2$ and found no evidence of heteroskedasticity. Calculation of sample correlation coefficients and applications of Goldfeld-Quandt tests also found no evidence of heteroskedasticity.

An error-in-variable problem may arise with respect to the change in dispersion term $(\Delta H)^2$. However, one can show that, even with squared variables, any error will bias the coefficient of $(\Delta H)^2$ toward zero.[13] Since $(\Delta H)^2$ and $(\Delta p)^2$ are orthogonal, bias in the coefficient of $(\Delta H)^2$ will not affect the coefficient of $(\Delta p)^2$, and the estimated intercept term will be biased upwards. Thus, the significance of the coefficients of the independent variables and the lack of significance of the intercept cannot be attributed to this type of measurement error.

Under the interpretation of residuals offered in the end of section IIIA above, the $1 - R^2$ of the regression based on (17) is a measure of the proportion of volume due to noise trading activity. Our results (see, in particular, regression 3 in Table IIA and IIB) indicate that a minimum of about 17 percent of unsystematic volume is due to speculative trading. In any case, a large fraction of unsystematic trading seems to be due to noise trading. This is significant in the light of the crucial role noise traders may play in financial markets (Black (1986), Shleifer and Summers (1990)).

Note that, under noisy rational expectations, changes in price underestimate changes in the fundamental (liquidation) value of the asset. This follows from the price equation $\Delta p = a_1 \Delta v + a_2 \Delta Z$, where $a_1 < 1$ from Figure 1. In particular, if we assume that $E\Delta Z = 0$ then changes in average price undervalue changes in fundamental values relative to efficient markets. Below, we show that the variance of equilibrium price is generally greater than σ_v^2, the variance of price under efficient markets (see expression (19) below). This underlines the idea that in markets with heterogeneous information and noise, prices are more volatile because posterior equilibrium beliefs of speculative traders are more volatile, and not only because of the direct presence of noise trading.

Unfortunately, this "underestimation effect" is not directly testable. One related property which we can check is how price varies with the average change in beliefs. Under noisy rational expectations, $\Delta \bar{f} = b_1 \Delta v + b_2 \Delta p$ so that one can substitute average beliefs in the price equation to get $\Delta p = d_1 \Delta \bar{f} + d_2 \Delta Z$. Empirically, d_1 should be significant and the elasticity of price with respect to $\bar{f}$ should be less than one. Our regression results are consistent with this prediction:

$$\frac{\Delta p}{p} = 0.0317 + 0.0227 \frac{\Delta \bar{f}}{\bar{f}} \qquad (t-\text{statistics in parentheses})$$

(1.709) (2.774)

c. Stability of Coefficients

We chose scaling variables (see section IIIA) to minimize the variation of coefficient values across members of the sample. Nonconstancy of the coefficients β_1 and β_2 in (17) will bias the intercept term away from zero in an OLS regression such as that reported in Table 2. Since the results in Table 2 support the rational expectations hypothesis, which predicts a zero intercept value, failing to completely correct for changing coefficient values does not weaken this conclusion.

Nevertheless, it would be instructive to check for the presence of coefficient variability. In particular, variability may arise if the scaling factors do not adequately correct for correlation between firm value and other variables in the model (A positive correlation between firm value and price, for example, is suggested by all three equilibrium hypotheses). We therefore divide the sample into two parts according to the stock market valuation of firms. We run separate regressions for the high value and low value groups of firms. Results of the regressions as well as of F -tests for equivalence of coefficients reported in Table 3 indicates no significant variation across groups.

[Table 3]

IV. Further Implications of Heterogeneous Information

Let us summarize the empirical findings so far. Trading volume and prices are best explained by a rational expectations model with asymmetrically informed traders and noise. This allows us to calculate an upper bound for the amount of volume due to noise trading. We also find that changes in the dispersion of information have an effect on volume but not on price. The main implication for asset pricing is that when equilibrium posterior beliefs are asymmetric, asset price changes do not reflect changes in fundamental value as fully as when markets are efficient. However, price volatility is greater (see (19) below). We will now explore how these results bear on certain observed phenomena in financial markets.

1. Asset Price Volatility

Beginning with Shiller, there now exists a substantial body of research documenting the "excess volatility" of stock prices.[14] These studies show that the arrival of new public information ("news") cannot account for all the volatility of stock prices within the context of an efficient markets model. With efficient markets (see section IIB above), the asymptotic variance of asset prices is given by $\hat{\sigma}_p^2 = \frac{\sigma_v^2}{R}$. This is, of course, the hypothetical variance in the Shiller tests, where v is the present value of expected future dividends conditional on all public information today. However, under noisy rational expectations, price variance is given by (using Figure 1):

$$\sigma_p^2 = a_1^2\sigma_v^2 + a_2^2\sigma_z^2 = \sigma_v^4 \frac{(1+\sigma_\epsilon^2 \frac{\sigma_z^2}{N^2})^2 (\sigma_v^2 + \frac{\sigma_\epsilon^4 \sigma_z^2}{N^2})}{((1+\sigma_\epsilon^2 \frac{\sigma_z^2}{N^2})\sigma_v^2 + \sigma_\epsilon^4 \frac{\sigma_z^2}{N^2})^2} \qquad (19)$$

so that the variance ratio $\frac{\sigma_p^2}{\hat{\sigma}_p^2} > 1$ whenever

$$\sigma_v^2\sigma_\epsilon^4 (\frac{\sigma_z^2}{N^2}) > (\frac{\sigma_z^2}{N^2}) + \sigma_v^2$$

The price variance ratio will tend to be large as the variance of private signals and the variance of "risk adjusted per capita supply" $\frac{\sigma_z^2}{N^2}$ increases. Heterogeneous information can play an important role in explaining asset price variability. The empirical support we find for the noisy rational expectations hypothesis may offer a resolution to the "excess volatility" puzzle.

2. The Small Firm Effect.

Banz (1981), Keim (1983), Brown, Kleidon and Marsh (1983) and others find strong empirical evidence on the negative relationship between market value and stock returns. Subsequent research (Chan and Chen (1988), Friend and Lang (1988)) has indicated that this apparent anomaly may be due to a risk effect not captured by traditional measures. Here we provide additional evidence for this hypothesis as well as shed some light on the source and nature of this unexplained "risk effect."

We divided the 266 firms in our sample into two groups along the median market size. The average change in dispersion (18) for the small firm subgroup is significantly larger than for the large firm subgroup (for small firms $(\Delta H)^2 = 58.89$, for large firms $(\Delta H)^2 = 11.33$; the t-statistic for the difference is $t = 2.64$).

Using (10) and Figure 1, the change in dispersion $(\Delta H)^2$ is, in the limit, equal to

$$(\Delta H)^2 = \frac{2b_1^2\sigma_\epsilon^2}{\sigma_v^4} = \frac{2\sigma_\epsilon^6(\frac{\sigma_z^2}{N^2})^2}{((1+\sigma_\epsilon^2\frac{\sigma_z^2}{N^2})\sigma_v^2+\sigma_\epsilon^4(\frac{\sigma_z^2}{N^2}))^2} \tag{20}$$

On the other hand, the conditional variance of v is

$$s = \sigma_v^2(1-Ra_1) = \frac{\sigma_\epsilon^4(\frac{\sigma_z^2}{N^2})\sigma_v^2}{(1+\sigma_\epsilon^2\frac{\sigma_z^2}{N^2})\sigma_v^2+\sigma_\epsilon^4(\frac{\sigma_z^2}{N^2})} \tag{21}$$

Thus, $(\Delta H)^2$ and s will generally move together in response to changes in underlying variables. Changes in σ_ϵ^2 and $\frac{\sigma_z^2}{N^2}$ will always move $(\Delta H)^2$ and s in the same direction, while a change in σ_v^2 will vary $(\Delta H)^2$ and s in the same direction for certain parameter values. This provides a link between firm size and asset return variance through the empirical negative effect of $(\Delta H)^2$ on market value.

3. The Cragg-Malkiel Study

Cragg and Malkiel (1982) find the variance of forecasts of earnings growth to have a significant positive effect on stock returns even after traditional risk variables (such as beta) have been included. Moreover, they find the relative strengths of betas and forecast variances as explanatory variables to vary across different time subperiods. Thus, Cragg and Malkiel conjecture that "there is another risk concept provided by [the forecast variance measure]" (p. 147). This conjecture is further supported by their finding that forecast variance is a better predictor of price-earnings ratios than beta and is strongly negatively significant even after dividend-earnings ratios and expected growth rates are introduced (p. 152).

Here, we propose an explanation for these effects based on heterogeneous information. The variance of growth forecasts used in the Cragg and Malkiel study can be interpreted as the variance of posterior beliefs of future earnings in a multiperiod model (since growth will determine the present value of expected future earnings). Now consider the effects of changes in the variance of beliefs f^i in the context of our model.

Recall that under noisy rational expectations, equilibrium posterior beliefs are given by $f^i = b_1 y^i + b_2 p$, where b_1 and b_2 are defined in Figure 1. Thus, from (10), the variance of posterior beliefs σ_f^2 is $b_1^2\ \sigma_\epsilon^2$ in the limit. The conditional variance of the asset's payoff (given in (21) above) can be written as $s = \sigma_v^2(1 - Ra_1) = b_1\sigma_\epsilon^2$. Thus, the variance of posterior beliefs is positively related to the asset's payoff variance (and so will tend to be negatively related to the asset's expected returns).

To explain the effect on price-earnings ratios, note that expected equilibrium price can be written as

$$\bar{p} = b_1 \frac{(1 + \sigma_\epsilon^2 (\frac{\sigma_z^2}{N}))}{\sigma_\epsilon^2 (\frac{\sigma_z^2}{N})} \ (\bar{v} - (\frac{\sigma_\epsilon^2}{N})\bar{Z}).$$

Thus, a change in σ_ϵ^2 will move σ_f^2 and $\bar{p}$ in opposite directions. An increase in σ_v^2 will raise σ_f^2 and will decrease $\bar{p}$ on average if $\bar{v} < \frac{\sigma_\epsilon^2}{N}\ \bar{Z}$. A similar result holds concerning changes in $\frac{\sigma_z^2}{N}$.

4. Day-of-the-week Patterns

French and Roll (1986) and Barclay, Litzenberger and Warner (1990) find that asset prices and returns have much greater variance during trading than during non-trading hours. With heterogeneous expectations and noise, price variance σ_p^2 is given by equation (19) above. As long as the variance magnitudes are not too low, σ_p^2 is increasing in σ_ϵ^2 and σ_z^2. Thus, if during non-trading hours, the variance of the private signals of the population of traders present is lower than during trading hours, asset price variance will be lower. Similarly, asset price volatility will be lower in non-trading periods if the variance of noise trading is lower at these times.[15]

5. The Equity Premium Puzzle

Finally, we briefly point out that the greater variance of asset prices and greater conditional variance of asset payoffs under noisy rational expectations than under efficient markets has relevance for the Prescott-Mehra (1985) results.

V. Conclusions

The conclusions drawn from any analysis of asset markets will depend on the assumptions made concerning the structure of market information and how traders process information. The equilibrium paradigms used in most studies of asset markets differ in their assumptions concerning the amount of information conveyed by price and what information sets traders' base portfolio decisions on. In this paper we showed that the equilibrium hypotheses of Walrasian competition, fully revealing rational expectations and noisy rational expectations each led to different comparative static restrictions concerning the relation of trading volume to changes in the dispersion of traders' expectations, in market price and in mean expectations. These restrictions allowed us to construct a test which nested all three equilibrium hypotheses. Using individual analysts' forecasts to estimate expectational variables, we found the empirical results to be consistent with the implications of the noisy rational expectations model and to reject the other two hypotheses.

In fact, it is quite surprising to find the results so uniformly consistent with the predictions of the noisy rational expectations model, since these predictions relied on particular specifications of our basic model. This indicates that the distributional and preference assumptions often used in models of securities markets may be good approximate building blocks, at least as far as generating general hypotheses from these models are concerned.

There is a sense in which noisy rational expectations is intermediate between the two other equilibrium hypotheses. If the noise in the market is very unpredictable (i.e., high variance of noise trading quantities) then traders will rely heavily on their private information, since price will not convey market information very well. On the other hand, if non-speculative trading is predictable (negligible noise trading variance), then the actions of traders will reveal their information by affecting price. Our findings imply that there is considerable noise in the market, but not enough to completely dilute the information content of prices. These are the conditions under which Beja's paradox (Grossman (1977)) does not exist, while still requiring the modelling of market rationality. That is, our results describe an environment under which it is not contradictory to assume both traders' rationality and private incentives to acquire information.

It is interesting that this last property coincides with greater volatility of trader's equilibrium beliefs. As we discuss in sections three and four of this paper, this has important implications for asset pricing and market volatility.

Appendix

We impose the following four additional criteria for including firms and their interim earnings announcements in our sample. These criteria were chosen to improve the accuracy of estimates of the change in dispersion and the average belief change variables.

1. We include only firms with complete data for the month before and the month after (the month of) earnings announcements (i.e. three consecutive months).
2. Firms/interim earnings announcements are included in our sample if 10 or more active analysts make forecasts both within the period of the two months before and within the period of two months after the announcement date.
3. The sample in the fourth quarter is excluded.
4. Firms whose fiscal year end do not correspond to the calendar year end are excluded.

Detailed individual analysts' forecasts are available for about three fifths of the 48 months in the sample period, with some missing months throughout the sample period. We impose the first criterion to account for this missing month problem. The second criterion is necessary because a small number of pairs of pre-announcement and post-announcement analysts' forecasts would lead to unreliable estimates of the change in dispersion. This criterion also assures that the average expectation is based only on recently updated analysts' forecasts. Since analysts start to forecast the following fiscal year end earnings after the beginning of the fourth quarter, the third criterion is required to ensure that relationships between (current fiscal year end) earnings forecasts and volume are always comparable. The fourth criterion is necessary for the consistent calculation of the non-systematic earnings variance term in the change in dispersion term.

After screening the data based on these criteria, the final sample consists of 266 events for 101 firms covering the period from April 1984 to March 1986.

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Figure 1

Hellwig's coefficient solutions for the noisy rational expectations model

$$a_0 = \frac{\bar{v}\sigma_z^2 N + \sigma_v^2 \bar{Z} \dfrac{N^2}{\sigma_\epsilon^2}}{N\sigma_z^2 + \sigma_v^2 \dfrac{N}{\sigma_\epsilon^2}\sigma_z^2 + \sigma_v^2 N \left[\dfrac{N^2}{\sigma_\epsilon^4} \right]}$$

$$a_1 = \frac{\sigma_v^2 \dfrac{N}{\sigma_\epsilon^2}\sigma_z^2 + \sigma_v^2 \dfrac{N^2}{\sigma_\epsilon^4} N}{R\left[N\sigma_z^2 + \sigma_v^2 \dfrac{N}{\sigma_\epsilon^2}\sigma_z^2 + \sigma_v^2 N \dfrac{N^2}{\sigma_\epsilon^4} \right]}$$

$$a_2 = -\frac{\sigma_v^2\sigma_z^2 + \sigma_v^2 N \left[\dfrac{N}{\sigma_\epsilon^2} \right]}{R[N\sigma_z^2 + \sigma_v^2 \dfrac{N}{\sigma_\epsilon^2}\sigma_z^2 + \sigma_v^2 N \dfrac{N^2}{\sigma_\epsilon^4}]}$$

$$b_1 = \frac{R\sigma_\epsilon^2 \sigma_z^2}{(N^2 + \sigma_\epsilon^2 \sigma_z^2)} a_1$$

$$b_2 = \frac{R}{(1 + \sigma_\epsilon^2 \dfrac{\sigma_z^2}{N^2})}$$

Table 1

Summary statistics of data on squared trading volume[a] V^2, change in dispersion[b] $(\Delta H)^2$, price change $(\Delta p/p^*)^2$ and average belief change[d] $(\Delta\bar{f}/\bar{f})^2$. The sample size is 266 which covers the period of 1983-1986. For the calculations in Panel B, $(\Delta H)^2$ and $(\Delta\bar{f}/\bar{f})^2$ were winsorized to the mean plus or minus 2 standard derivations to avoid distortionary effects of outliers. Similar results are obtained for non-winsorized variables.

Panel A

Variables	Mean	Stan. Dev.	Minimum	Maximum
V^2	0.0482	0.1716	0.0000	2.0156
$(\Delta H)^2$	75.24	475.44	0.0008	4679.69
$(\Delta p/p^*)^2$	0.0007	0.0012	0.0000	0.0077
$(\Delta\bar{f}/\bar{f})^2$	0.0513	0.6779	0.0000	11.0327

Panel B

	Correlation	Coefficient	Table	
	V^2	$(\Delta H)^2$	$(\Delta p/p^*)^2$	$(\Delta\bar{f}/\bar{f})^2$
V^2	1	0.330 [e]	0.267 [e]	0.062
$(\Delta H)^2$		1	-.013	-0.0134
$(\Delta p/p^*)^2$			1	0.113[f]
$(\Delta\bar{f}/\bar{f})^2$				1

a. V^2 (the square of (abnormal) volume) is estimated as the square of the residual term of the market model which is used to remove the market movements.

b. $(\Delta H)^2$ is estimated by IBES individual and consensus (average) forecasts. $(\Delta H)^2$ measures the squared difference between the post earnings announcements period and the pre-earnings-announcements period of the deviation of individual forecasts from the average.

c. $(\Delta p/p^*)^2$ is calculated as $\sqrt{E/\bar{E}}*(\Delta p/p)^2$ where $(\Delta p/p)^2$ is computed by using the stock price from two days before the Wall Street Journal Index day to the day of the Wall Street Journal Index day. E is the number of shares outstanding times stock price. $\bar{E}$ is the total average E of those 266 firms.

d. $(\Delta\bar{f}/\bar{f})^2$ is estimated by IBES individual analysts' forecasts. It measures the squared percentage change of post-announcement average forecasts from pre-announcement average forecasts.

e. Significant at 1% level.

f. Significant at 10% level.

Table 2.

Regressions of squared trading volume[a] (V^2) on the change in dispersion[b] $(\Delta H)^2$, the price change[c] $(\Delta p/p^*)^2$ and average belief change[d] $(\Delta\bar{f}/\bar{f})^2$ for a total sample of 266 events from 1983 to 1986. $(\Delta H)^2$ and $(\Delta\bar{f}/\bar{f})^2$ are winsorized to the mean plus or minus 2 standard deviations to avoid distortionary effects of outliers. Similar results are obtained for non-winsorized variables. Panel A reports regression results with intercept, while Panel B reports those results with suppressed intercept terms.

Panel A

	Intercept	$(\Delta H)^2$	$(\Delta p/p^*)^2$	$(\Delta\bar{f}/\bar{f})^2$	R^2
1.	0.035	0.00038			0.105
	(3.41)[e]	(5.67)[e]			
2.	0.007	0.00039	38.78		0.176
	(0.65)	(5.97)[e]	(4.87)[e]		
3.	0.0068	0.00039	38.20	0.062	0.174
	(0.60)	(5.98)[e]	(4.76)[e]	(0.65)	
4.	0.033	0.00038		0.114	0.106
	(3.20)[e]	(5.69)[e]		(1.15)	

Panel B

	$(\Delta H)^2$	$(\Delta p/p^*)^2$	$(\Delta\bar{f}/\bar{f})^2$	R^2
1.	0.00043			0.135
	(6.51)[e]			
2.	0.00039	41.37		0.236
	(6.25)[e]	(6.00)[e]		
3.	0.00039	40.54	0.067	0.234
	(6.24)[e]	(5.79)[e]	(0.70)	
4.	0.00043		0.161	0.140
	(6.49)[e]		(1.61)	

a. V^2 (the square of (abnormal) volume) is estimated as the square of the residual term of the market model which is used to remove the market movements.

b. $(\Delta H)^2$ is estimated by IBES individual and average (consensus) forecasts. $(\Delta H)^2$ measures the squared difference between the post-earnings-announcement period and the pre-earnings-announcements period of the deviation of individual forecasts from the average.

c. $(\Delta p/p^*)^2$ is calculated as $\sqrt{E/\bar{E}}*(\Delta p/p)^2$ where $(\Delta p/p)^2$ is computed by using the stock price from two days before the Wall Street Journal Index day to the day of the Wall Street Journal Index day. E is the number of shares outstanding times stock price. $\bar{E}$ is the total average E of those 266 firms.

d. $(\Delta \bar{f}/\bar{f})^2$ is estimated by IBES individual analysts' forecasts. It measures the squared percentage change of post-announcement average forecasts from pre-announcement average forecasts.

e. Significant at 1% level.

Table 3

Regressions of squared trading volume[a] V^2 on the change in dispersion[b] $(\Delta H)^2$, the price change[c] $(\Delta p/p^*)^2$ and average belief change $(\Delta \bar{f}/\bar{f})^2$ for a total sample of 266 events from 1983 to 1986. The sample is divided along the median (2642.29 million) into high value firms (regression 1) and low value firms (regression 2). $(\Delta H)^2$ and $(\Delta \bar{f}/\bar{f})^2$ are winsorized to the mean plus or minus 2 standard deviations to avoid distortionary effects of outliers. Similar results are obtained for non-winsorized variables.

		Intercept	$(\Delta H)^2$	$(\Delta p/p^*)^2$	$(\Delta \bar{f}/\bar{f})^2$	R^2
1.	High Value Firms	0.0067	0.0006	38.11	0.0576	
	(N=133)	(0.443)	(1.891)[g]	(4.362)[e]	(0.581)	0.133
	Mean Value=8,240.3 Million					
2.	Low Value Firms	0.0040	0.0004	38.79	0.0755	0.189
	(N=133)	(0.222)	(5.24)[e]	(2.32)[f]	(0.335)	
	Mean Value=1,436.2 Million					
	F-Statistics For The Difference of Coefficients in Regressions 1 and 2		F=0.546	F=0.001	F=0.740	

a. V^2 (the square of (abnormal) volume) is estimated as the square of the residual term of the market model which is used to remove the market movements.

b. $(\Delta H)^2$ is estimated by IBES individual and consensus (average) forecasts. $(\Delta H)^2$ measures the squared difference between the post-earnings-announcement period and the pre-earnings-announcements period of the deviation of individual forecasts from the average.

c. $(\Delta p/p^*)^2$ is calculated as $\sqrt{E/\bar{E}}*(\Delta p/p)^2$ where $(\Delta p/p)^2$ is computed by using the stock price from two days before the Wall Street Journal Index day to the day of the Wall Street Journal Index day. E is the number of shares outstanding times stock price, and $\bar{E}$ is the total average E of those 266 firms.

d. $(\Delta \bar{f}/\bar{f})^2$ is estimated by IBES individual analysts' forecasts. It measures the squared percentage change of post-announcement average forecasts from pre-announcement average forecasts.

e. Significant at 1% level.

f. Significant at the 5% level.

g. Significant at the 10% level.

FOOTNOTES

[1]See Milgrom and Stokey (1982), Varian (1985).

[2]For a comprehensive overview of the many directions of work in this area, see the papers in Bhattacharya and Constantinides (1989). Recent research can also be found in the current work in insider trading and market-making such as Kyle (1985), Glosten and Milgrom (1985), Admati and Pfleiderer (1988).

[3]Note, however, that an unrepresentative lack of divergence would only bias the test toward finding our measure of the dispersion of expectations to be insignificant, which is not the case (see the empirical analysis in section III):

[4]For a discussion of the robustness of the model we use, see footnote 7.

[5]Pfleiderer (1982), who adopts a different approach from that used here, was the first to study a rational expectations model of trading volume. Also, he studies some interesting comparative static results we do not consider here. For example, it is found that expected volume increases with the precision of private signals.

[6]Stochastic supply can be explained by noise trading. Z is the net supply held by speculators. Our theory then describes the predictable speculative component of volume; the component of volume due to unsystematic noise trading is explained by the disturbance term in our linear model (see section IIIc below). Madrigal and Scheinkman (1989) derive noisy linear prices (analogous to the price functions derived below) in which the amount of volume due to noise trading is constant and known. Since the model we analyze below can be rationalized as the reduced form of the model in Madrigal and Scheinkman, this presents another way of justifying our approach to deriving volume under rational expectations. Madrigal and Scheinkman also show how the model below can be put in the framework of a market-making model.

Noise trading can emerge from a variety of sources. For our purposes, it is sufficient to think of idiosyncratic liquidity demands as the cause. DeLong et al. (1990) arrive at interesting results by studying an asset pricing model which explicitly specifies noise trading behavior. Their model does not rely on liquidity reasons but instead introduces a group of investors who trade on the basis of random, "erroneous" expectations.

[7]The structural equation we derive to nest the alternative hypotheses (see (17)), as well as the empirical results, turn out to be quite robust to changes in this basic theoretical model. For example, results are not altered by adding a common error component to traders' signals or by allowing new and old signals to be correlated when considering the comparative static results which generate the volume equations. Also, results do not change if, instead of using the comparative static method, we derive testable restrictions by examining the effect of the arrival of new information, holding the value v of the asset constant. In fact, we chose to use the above model because, with our interpretation of v as the full information value of the asset, it yields testable restrictions which are robust both to application and to changes in the basic structure. The reason for this robustness is that changes in the model's specifications are generally subsumed in posterior belief functions, which is a data variable. For a further discussion of alternative models of volume, see Lang, Litzenberger and Madrigal (1990).

[8]Unfortunately since noise trading volume cannot be separately observed, this hypothesis is not directly testable. Nevertheless, note that, even if price and true residuals were positively correlated under

noisy rational expectations, the test would remain valid. This correlation would bias β_2 upward. Since the correlation exists only under noisy rational expectations, the prediction of this hypothesis (that β_2 is significant and positive) would still be tested.

[9]In calculating $var(v_k)$, similar results are obtained if $v_k - \bar{f}(v_k)$ is instead estimated using data of one month before and of the current month of earnings announcement.

[10]Daily trading volume and shares outstanding data for individual stocks and for NYSE as a whole are provided by the Rodney White Center, University of Pennsylvania.

[11]Note that in the noisy rational expectations case, the equilibrium conditions are recursive (equilibrium price is a function of exogenous variables) so that there is no theoretical basis for simultaneity bias in the estimation.

[12]Note that, as predicted by the noisy rational expectations model, there exists some correlation between $(\frac{\Delta \bar{f}}{f})^2$ and $(\frac{\Delta \bar{p}}{p^*})^2$ (see Table 1, Panel B). The correlation does not seem strong enough, however, to cause problems in interpreting the multivariate regressions.

[13]We consider errors-in-variables of the form

$$Y^2 = aX^2 + bW^2 + \delta \tag{1}$$

$$\hat{X} = X + \eta \tag{2}$$

where X, W are the independent variables, (1) is the true model and $\hat{X}$ is the observed variable. Y and W are measured correctly. $\delta \sim N(0,\sigma^2)$, $\eta \sim N(0,t^2)$ are independent of each other and of X and W. For an interpretation of δ which justifies (1), see section III D below (see also Griliches and Ringstad (1970)).

For the model described by (1) and (2), one can show that $\hat{a}$, the estimate of a, is biased in the limit by

$$plim\ \hat{a} = a - plim \frac{\sum a(z\hat{X}'\eta - \eta^2 - t^2)^2}{\sum(\hat{X}^2 - \bar{\hat{X}})^2},$$

so that the sign of the bias is opposite the sign of the coefficient a. The estimate $\hat{b}$ of the coefficient of W is unbiased if $\hat{X}$ is uncorrelated with W (see Levi (1973)).

[14]Examples of further work in this area include Campbell and Shiller (1987, 1988), Campbell and Kyle (1987), Mankiw, Romer and Shapiro (1985), and West (1988).

[15]See Admati and Pfleiderer (1988) for a theory of trading patterns.