THE SUSTAINABILITY OF BUDGET DEFICITS WITH LUMP-SUM AND WITH INCOME-BASED TAXATION

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Abstract

The paper examines the sustainability of fiscal policies in a stochastic economy with a particular focus on two benchmark policies, balanced budgets and tax smoothing. These policies are typically sustainable if lump-sum taxes are available, but they are generally not sustainable in a stochastic environment, if taxation is constrained by the size of the economy. The sustainability problems arise because the debt-income ratio becomes excessive whenever there are sufficiently low realizations of aggregate income.

I also compute the probability of reaching high debt-income ratios with different policies and I discuss the role of debt management for sustainability. It turns out that balanced budgets can be sustained for ever with very high probability (but less than one), but so can policies with permanent primary or with-interest deficits. I argue that debt management is important for sustainability in a stochastic environment and show that the use state-contingent government liabilities can be helpful in designing sustainable versions of tax-smoothing and balanced budget polices.

1. Introduction

Though we are living in an uncertain world, much of the positive as well as normative theory of public finance has been developed in the context of certainty models. To be sure that the policies being studied are actually feasible in the real, stochastic world, one should (at a minimum) verify that they are sustainable in such a stochastic environment. However, the sustainability question has not been systematically examined in stochastic models, not even for such basic, well-known policy rules as balanced budgets and tax smoothing.[1] This paper shows that their sustainability cannot be taken for granted.

The sustainability of fiscal policies is analyzed in a simple stochastic Lucas (1978) type exchange economy. The Lucas-economy provides a simple equilibrium setting, in which all budget constraints can be derived rigorously. The model imposes dynamic efficiency as maintained assumption, which seems justified by the results of Abel et al. (1989). Two versions of sustainability are considered, depending on the availability of lump-sum taxes. With lump-sum taxes, sustainability imposes no restrictions on the level of government debt and on its relation to the size of the economy. But since lump-sum taxes are rarely observed in practice, I also consider a version of the model where tax revenues are limited by aggregate income. The limit is derived from an incentive constraint. Then sustainability requires a bounded debt-income ratio and other restrictions on government policy that are stronger than in the case of lump-sum taxes.

The stochastic setting has interesting implications for the sustainability of simple, well-known policy rules that are commonly considered sustainable in any sense of the word (and that are sustainable in a certainty setting under weak conditions). Balanced budgets and tax smoothing policies will be analyzed as benchmark examples, because they have received considerable political and academic attention.

In the popular discussion, balanced budgets are widely seen as a prudent, fiscally conservative, and unquestionably sustainable policy, while the sustainability of the 1980s-sized deficits is viewed, at best, with suspicion.[2] This paper will show, however, that the sustainability of a balanced budget policy is highly questionable, if one considers a stochastic environment with income-based taxation and maintains that the government uses essentially[3] safe debt to finance its deficits. The main problem is that constant debt leads to a variable debt-income ratio, which will become excessive with positive probability if aggregate income in the far future is sufficiently uncertain. For example, an income process with unit root component is enough to prove non-sustainability.

In the academic literature, the policy of tax smoothing has acquired a benchmark status primarily because of its welfare properties. Under some conditions, it is the solution to a Ramsey-type optimal taxation problem (see Barro 1979, Blanchard and Fischer 1989). While the tax smoothing policy is clearly sustainable when derived in a certainty setting, it has often been extended to a stochastic environment by interpreting tax smoothing as smoothing of expected tax rates.[4] The paper shows that a policy rule of smoothing expected tax rates is generally unsustainable with income based-taxation.

The results for balanced budgets and tax smoothing are initially derived in a simplified scenario, which highlights the common root of their sustainability problems. Assuming i.i.d. income growth and a constant government spending to income ratio, both balanced budgets and tax smoothing call for taxes that equal government spending plus debt service, where debt service is a constant fraction of outstanding debt. (This paper is not about how fiscal policy should respond to temporary fluctuation in government spending or to business cycles. The simplified setting helps to abstract from such differences.) The two policies can be interpreted as two alternative norms for the long-term level of debt service and they can be analyzed by studying the class of constant debt service policies.

Under certainty, constant debt service policies are sustainable with income taxes (and therefore also with lump-sum taxes), if debt service equals or exceeds the difference of interest and growth rate. Tax smoothing, which sets debt service equal to this difference, is just sustainable. Provided income growth is positive, the balanced budget rule is also sustainable and, since it sets a higher debt service than the tax smoothing policy, it can be considered a cautious, "prudent" policy, just as popular intuition suggests. But with income uncertainty, constant debt service policies are typically unsustainable with income taxes, namely unless debt service is so high that the debt-income ratio does not increase even for the "worst" realization of income growth. Being borderline without uncertainty, the policy of smoothing expected tax rates is always unsustainable. Balanced budgets are unsustainable whenever there is a non-zero probability that income can decline.

Three extensions are considered, concerning balanced budgets, tax smoothing, and nominal debt, respectively. First, since the probability of substantial income declines may be very small, one might try to defend a balanced budget policy with the argument that it is sustainable with very high probability. The paper shows that the probability of sustaining a balanced budget policy is indeed near one for plausible parameters. But it is difficult to use this as an argument in favor of balanced budgets, because the criterion is so weak: Policies with much lower debt service and even policies with permanent primary deficits can be sustained with a probability near one. In other words, if one is satisfied with a policy that is sustainable with a probability less than one, one cannot complain about budget deficits on sustainability grounds. But if one demands sustainability with probability one (which I would prefer), balanced budgets fail the sustainability test.

Second, I show that there is a sustainable stochastic version of the tax smoothing rule, which differs from smoothing expected tax rates. It is obtained if one abandons the assumptions of safe government debt and instead uses state contingent debt to stabilize tax rates perfectly over all time periods and all states of nature. The standard assumption that government debt is safe debt—or similar ad-hoc assumptions on debt management—seem to be important for making simple policy rules unsustainable in a stochastic world.

Third, I discuss a potential role for nominal debt as state-contingent claim. If inflation is exogenous and not perfectly correlated with income, the results on the non-sustainability of a real balanced budget policy with safe real debt hold analogously for a nominally balanced budget with nominally safe debt. Indeed, the results on balanced budgets and on tax smoothing are generic for any type of debt (the exception being a case of degenerate distributions). On the other hand, if inflation is state-contingent in a very specific way (enforced, e.g., by monetary policy), nominal debt may help to sustain tax smoothing and (for different inflation processes) balanced budget policies.

The paper is organized as follows. Section 2 develops the general stochastic model and derives the intertemporal budget constraints. Section 3 motivates a restriction to income-based taxation and derives the additional constraints on government policy. Section 4 examines the sustainability of tax-smoothing and balanced budgets in general and under simplifying assumptions, under which both are members of a class of constant debt service policies. Section 5 shows that the balanced budget rule is sustainable with very high probability for plausible parameter values and that this probability is almost as high for policies that promise much lower debt service, including some policies that promise "permanent" primary deficits. Section 6 shows that the tax smoothing rule can be generalized from a certainty to a stochastic environment in a way that maintains sustainability, if one allows state-contingent government liabilities. The sustainable tax smoothing policy is one of perfectly stable tax rates for all times and all states of nature. Section 7 provides generalizations and discusses nominal debt as one type of state-contingent liability. Results are summarized in Section 8.

2. The Model

Government policy will be analyzed in a Lucas (1978) type exchange economy with government sector. This section specifies the model and determines the set of sustainable policies, assuming lump-sum taxes are available.

The economy is populated by identical, infinitely-lived consumers (or dynasties) who are each endowed with a stochastic stream of dividends Y_t from a "fruit tree." To emphasize the dependence of Y_t on the state of nature at time t, denoted by s_t, is will sometimes write it as $Y(s_t)$. Conditional on period-t information, the state of nature s_n is assumed to have a distribution $F(s_n|s_t)$, n>t. (In applications where there is no ambiguity about the conditioning, expectations with respect to $F(\cdot|s_t)$ will be denoted by $E_t[\cdot]$.) Endowments are units of perishable goods that can be used for private consumption C_t and for government spending G_t, subject to the resource constraint $C_t + G_t = Y_t$. To purchase G_t from individuals, the government levies taxes T_t on individuals and/or borrows on financial markets. For this section, taxes are lump-sum, i.e., can take any value on the real line. I assume that financial markets are complete and that the government has the ability to trade on all markets.

The latter assumption deserves some emphasis, because it is more general than the usual assumption in the public finance literature that government debt is essentially safe debt. In contrast to certainty models, a government in a stochastic economy has a variety of choices which type of safe or "risky" claims to issue or to hold (see Lucas and Stokey 1983). Of course, a government can ignore the availability of state-contingent claims and issue only safe, non-contingent debt securities. But I will argue below that this would be a significant restriction with important implications for sustainability.

I general, let the value of government obligations payable in state s_{t+1} be denoted by $D(s_{t+1})$ and let $p_t(s_{t+1})$ be the pricing function of contingent claims on period t+1 in period t. (For convenience, I will still refer to D as "government debt" even if government liabilities are general state-contingent claims.) Then the basic government budget equation in period t is

$$(1) \qquad \int_{s_{t+1}} p_t(s_{t+1})\cdot D(s_{t+1})\, dF(s_{t+1}|s_t) - D(s_t) = G(s_t) - T(s_t).$$

The budget equations imply an intertemporal budget constraint, if a transversality condition restricts the asymptotic path of government debt. It would of course beg the issue of sustainability, if one simply assumed such a constraint. The correct form of the transversality constraint depends on individuals' willingness to hold government debt,[5] which I will consider next.

Individuals maximize expected utility

$$(2) \qquad \sum_{n\geq 0} \beta^n \cdot \left[\int_{s_{t+n}} U(C(s_{t+n}))\, dF(s_{t+n}|s_t) \right]$$

subject to budget equations

$$(3) \qquad \int_{s_{t+n+1}} p_{t+n}(s_{t+n+1})\cdot A(s_{t+n+1})\, dF(s_{t+n+1}|s_{t+n}) - A_{t+n} = Y_{t+n} - T_{t+n} - C_{t+n}.$$

for all states s_{t+n}, and subject to a No-Ponzi-Game (NPG) condition to be specified below. Utility is strictly increasing and (weakly) concave with continuous derivatives, and $A(s_{t+n})$ denotes individual asset holdings, i.e., contingent claims payable to the individual in state s_{t+n}. To have all maximization problems well defined and to prevent technical complications, I assume that government spending satisfies $0 \leq G_t < Y_t$ and that the infinite sums

$$(4a) \qquad \sum_{n\geq 0} \beta^n \cdot \left[\int_{s_{t+n}} U(Y(s_{t+n}) - G(s_{t+n}))\, dF(s_{t+n}|s_t) \right] \text{ and}$$

$$(4b) \qquad \sum_{n\geq 0} \beta^n \cdot \left[\int_{s_{t+n}} U'(Y(s_{t+n}) - G(s_{t+n}))\cdot(Y(s_{t+n}) - G(s_{t+n}))\, dF(s_{t+n}|s_t) \right]$$

are finite in all states of nature.[6]

The specification of NPG-conditions under uncertainty and with an arbitrarily behaving government is a non-trivial issue. By extension from the certainty and finite horizon cases, one should suspect that the period-by-period constraints (1) and (3) translate into intertemporal budget constraints of the form

$$(5) \qquad D_t = \sum_{n\geq 0} \Big[\int_{s_{t+n}} P(s_{t+n}|s_t)\cdot[T(s_{t+n})-G(s_{t+n})]\, dF(s_{t+n+1}|s_{t+n})\Big]$$

and

$$(6) \qquad A_t = \sum_{n\geq 0} \Big[\int_{s_{t+n}} P(s_{t+n}|s_t)\cdot[C(s_{t+n})+T(s_{t+n})-Y(s_{t+n})]\, dF(s_{t+n+1}|s_{t+n})\Big]$$

for each state of nature s_t, where

$$(7) \qquad P(s_{t+n}|s_t) = \prod_{i=0}^{n-1} p(s_{i+1}|s_i)$$

denotes the period-t pricing function for period t+n claims. In the appendix, I show that these stochastic versions of the "standard" budget constraints indeed apply, provided some regularity conditions hold.

The individual first order conditions then imply that the pricing function $P(\cdot)$ is given almost surely by

$$(8) \qquad P(s_{t+n}|s_t) = \beta^n \cdot \frac{U'(C(s_{t+n}))}{U'(C(s_t))},$$

where $C(\cdot)=Y(\cdot)-G(\cdot)$ holds in equilibrium. (From now on, obvious technical qualifications like "almost surely" or "only for states that may occur with positive probability" will be omitted to save space.) Thus, $P(\cdot)$ is a function of the exogenous endowment and government spending processes. Other equilibrium conditions are $D(s_t)=A(s_t)$ for all s_t.

Government policies that satisfy the budget constraint (5) for all states of nature will be called sustainable. It should be noted that the constraint (5) imposes no restrictions on government policy over any finite time interval. That is, given a government policy over a finite time interval, a continuation of the policy can be defined in a way that satisfies the budget constraint, because future taxes can be set arbitrarily high. Additional assumptions must be imposed to make the sustainability requirement a testable restriction on fiscal policy.

Two types of additional restrictions seem interesting. First, one might restrict the analysis to policy rules that are time-invariant in the sense that a finite sample provides enough information to determine unambiguously how fiscal variables will evolve if the policy is "continued in the same way." The policies considered below will be defined by such simple, time-invariant rules. Second, one might explore if they are additional restrictions on taxation that might make it impossible to continue a policy in a sustainable way when debt becomes "excessive." Intuitively, the concept of sustainability seems to be associated with both time-invariance and feasibility aspects (see, e.g. Kremers 1989). Here I will first formalize a feasibility constraint motivated by non lump-sum taxation. Then the sustainability of simple, timeless policy rules will be examined with and without the additional constraint.

3. The Capacity to Tax

It is well known from the certainty case that the intertemporal budget constraint is consistent with a variety of policies that appear somewhat implausible in practice. Most importantly, sustainable policies may have debt-income ratios that diverge to infinity (McCallum 1984). Such policies are usually by supported by rising ratios of tax revenues to income. With lump-sum taxes, such arbitrarily high income tax rates are feasible. But since lump sum taxes are almost non-existent in practice, it is doubtful whether all policies that satisfy the intertemporal budget constraint are practically feasible.

In the literature, the notion of "capacity to tax" or "collateral" (Kremers 1989) has been discussed as a stronger constraint on policy. This section will present a slight modification of the basic model that motivates such an upper bound on tax rates. The assumptions on taxation are intentionally kept simple so that the basic structure of the model remains largely unchanged.

Let lump-sum taxes be replaced by an income tax that is levied on income from the fruit tree at the tax rate τ_t.[7] The tax rate can be limited as follows. Assume that labor is needed to "harvest" the fruit tree. Let L_t be labor supply in period t and let time be normalized to one so that $0 \leq L_t \leq 1$. Let $\tilde{Y}_t = Y_t \cdot L_t$ be the quantity of consumption goods harvested with labor input L_t, where Y_t indicates the total yield of the tree. The utility function (2) remains unchanged, i.e., there is no disutility of labor. Then the individual labor supply and production problem is straightforward: If $\tau_t<1$, $L_t=1$ and pre-tax income is $\tilde{Y}_t=Y_t$. If $\tau_t>1$, $L_t=0$ and pre-tax income is $\tilde{Y}_t=0$. As tie-breaking convention, assume $L_t=1$ for $\tau_t=1$.

The key policy implication is that tax revenues are limited to $T_t\leq Y_t$. It is not feasible to raise additional tax revenues by increasing tax rates above one. Assuming the government does not want to shut down the economy and receive no tax revenue, one can restrict the analysis to tax rates $\tau_t\leq 1$. Since tax rates below one have no incentive effects, the economy with income taxes is equivalent to the lump-sum tax economy described in Section 2 subject to the additional constraint $\tau_t\leq 1$ on the government. If endowment is interpreted as labor income, all results of Section 2 can be maintained, in particular those on the intertemporal budget constraint.

Since fiscal policy is now subject to an additional feasibility constraint, it makes sense to define the sustainability of fiscal policy in a more narrow sense. I will call a policy <u>sustainable with income taxes</u> or <u>feasible</u>, if it satisfies $\tau_t(s_t)\leq 1$ for all states s_t as well as the budget constraint (5). In contrast, I will refer to a policy that merely satisfies (5) as <u>sustainable with lump-sum taxes</u>.

Under some conditions, the ratio of debt to income (endowments) is a useful criterion for assessing sustainability. Unless the value of the fruit tree is infinite, any policy that maintains a bounded debt-income ratio in all states of nature satisfies the intertemporal budget constraint (is sustainable with lump-sum taxes). As in the certainty setting (McCallum 1984), a bounded debt-income ratio is then sufficient but not necessary for sustainability with lump-sum taxes. On the other hand, a bounded debt-income ratio turns out to be a necessary condition for sustainability with income taxes, provided the fruit tree has a bounded price-dividend ratio. To be precise, let

(9) $$V_t = \sum_{n\geq 1} \left[\int_{s_{t+n}} P(s_{t+n}|s_t)\cdot Y(s_{t+n})\, dF(s_{t+n+1}|s_{t+n})\right]$$

be the value of the fruit tree, $v_t=Y_t/V_t$ the dividend-price ratio, and $d_t = D_t/Y_t$ the debt-income ratio. Then the following holds.

Proposition 1:

(a) If $V(s_t) < \infty$, and if the debt-income ratio has an upper bound $\bar{d}$, $d(s_t) < \bar{d}$, both applying to all states s_t that occur with positive probability, then the policy is sustainable with lump-sum taxes.

(b) If the dividend-price ratio has a lower bound $\bar{v} > 0$, then a policy that is sustainable with income taxes must have a bounded debt-income ratio with upper bound $\bar{d} \le (1+1/\bar{v}) < \infty$, $d(s_t) < \bar{d}\ \forall s_t$.

Proof: If $V(s_t) < \infty\ \forall s_t$, then $\lim_{n\to\infty} \int_{s_{t+n}} P(s_{t+n}|s_{t+k}) \cdot Y(s_{t+n})\, dF(s_{t+n}|s_{t+k}) = 0\ \forall s_{t+k}$. Hence

$$0 \le \lim_{n\to\infty} \int_{s_{t+n}} P(s_{t+n}|s_{t+k}) \cdot D(s_{t+n})\, dF(s_{t+n}|s_{t+k}) \le \bar{d} \cdot \lim_{n\to\infty} \int_{s_{t+n}} P(s_{t+n}|s_{t+k}) \cdot Y(s_{t+n})\, dF(s_{t+n}|s_{t+k}) = 0,$$

which means that the path of government debt satisfies the transversality condition for all s_{t+k}. Together with equation (1), this implies (5), proving (a). For (b), note that $T_t \le Y_t$ and $G_t \ge 0$ implies

$$\sum_{n\ge 0} \left[\int_{s_{t+n}} P(s_{t+n}|s_t) \cdot [T(s_{t+n}) - G(s_{t+n})]\, dF(s_{t+n+1}|s_{t+n}) \right] \le Y_t + V_t \le (1+1/\bar{v}) \cdot Y_t.$$

Thus, if d_t exceeds $(1+1/\bar{v})$, (5) must be violated. ||

Given a finite value of consumption (assumption (4b)), the finite value of the fruit tree assumed in (a) does not seem restrictive. It is sufficient to assume, e.g., that the ratio of government spending to income is bounded away from one. Then a bounded debt-income ratio is an easily verifiable sufficient condition for sustainability with lump-sum taxes. Proposition 1(b) is useful as a negative criterion, even though the bound $\bar{v} > 0$ may be less easily verifiable in practice: Anybody defending a policy with growing, unbounded debt-income ratio as sustainable with income taxes would have to make a claim that $\bar{v} = 0$ applies to the economy in question. (Such a claim would be difficult to believe even for Japan.) From now on, a finite value of the fruit tree and a bounded price-dividend ratio will be assumed so that both parts of Proposition 1 apply.

A bounded debt-income ratio is not sufficient for sustainability with income taxes, because even policies with a low bound on debt may postulate high tax rates in some states of nature. In general, tax rates in all periods and all states of nature must be examined to verify sustainability. However, if one has policy observations for a finite time interval, there will always be <u>some</u> continuation that is sustainable with income taxes, provided debt is less or equal to the present value of consumption at the end of the sample period.

Propositions 2:

Given policy realizations up to the start of period t, there is a continuation of the policy for all periods t+k, $k\geq 0$, if and only if initial debt D_t does not exceed the present value of consumption at time t, i.e.,

$$(10) \qquad D_t \leq \sum_{n\geq 0} \Big[\int_{s_{t+n}} P(s_{t+n}|s_t)\cdot C(s_{t+n})\, dF(s_{t+n+1}|s_{t+n}) \Big].$$

Proof: Since $T_t \leq Y_t$, all policies must satisfy $T_t - G_t \leq Y_t - G_t = C_t$. Therefore, if (10) were violated, (5) could not hold. On the other hand, if (10) is satisfied, setting $\tau(s_t)=1\ \forall s_t$ (or somewhat lower in case of a strict inequality in (10)) is enough to satisfy (5). ||

Proposition 2 is particularly useful when policies are specified in a way that guarantees $\tau_t \leq 1$ by construction. To prove that such policies are sustainable (or not) with income taxes, one has to verify only that the path of debt remains below the present value of consumption at all times (or is above it in at least one state of nature).

This section has introduced a unit bound on tax rates as the only additional constraint on government policy. The unit bound follows naturally from my simple assumption about labor supply incentives. In more general models of taxation, incentive effects on labor supply may arise at much lower tax rates, which might lead to a lower bound on tax rates. But since the paper is concerned with qualitative features of different policy, the numerical value of the bound will not be important. It could be replaced by whatever number comes out of a more elaborate model of taxation without changing the results below. Note that default considerations would have similar implications for sustainability as limited taxation if they also impose a bound on the debt-income ratio.

4. Balanced Budgets, Tax Smoothing, and Other Simple Policy Rules

In this section, I will examine the sustainability of some simple fiscal policy rules, the rules of balanced budgets and tax smoothing in particular. Their sustainability properties can be illustrated most easily under a set of simplifying assumptions, under which both balanced budgets and tax smoothing are members of a family of policies characterized by a single debt service parameter.

4.1. Policy Definitions

The policy of balancing the budget specifies that tax revenues should equal current government spending plus the interest charge on debt outstanding from the previous period. Assuming safe debt, this is

$$T_t = G_t + r_{t-1} \cdot D^*_{t-1},$$

where $r_{t-1}=E_{t-1}[p(s_t)]^{-1}-1$ is the period t-1 safe interest rate (with E_{t-1} referring to the expectation conditional on one specific state s_{t-1}) and D^*_{t-1} is debt at the end of period t-1. Since debt at the end of a period is related to debt at the start of the next one by $D_t = (1+r_{t-1}) \cdot D^*_{t-1}$, this policy can also be written in as

(11) $$T_t = G_t + r_{t-1}/(1+r_{t-1}) \cdot D_t.$$

A distinctive feature of the balanced budget policy is that debt stays constant,

(12) $$D_{t+1} = (1+r_t) \cdot [G_t - T_t + D_t] = D_t.$$

Note that all these variables are defined in real terms. Generalizations to uncertain real returns on government debt, to stochastic inflation, and to nominally balanced budgets are in Section 7. (The case of nominal debt and deterministic inflation is also included in Section 4.2.) I focus on strictly safe debt here, because it will turn out to be a generic case.

The policy of tax smoothing is most easily defined for a deterministic economy. Without uncertainty, the policy rule of tax smoothing sets tax rates equal to a constant τ that satisfies the intertemporal budget constant, which is

$$\tau = [\sum_{n\geq 0} P(s_{t+n}|s_t) \cdot G_{t+n} + D_t] / \sum_{n\geq 0} P(s_{t+n}|s_t) \cdot Y_{t+n},$$

where $P(s_{t+n}|s_t) = \prod_{k=0}^{n-1} (1+r_{t+k})^{-1}$ is the product of discount factors. For a stochastic economy, this rule is commonly generalized to a policy of smoothing tax rates in expectation,[8] which requires

(13) $$\tau_t = [\sum_{n\geq 0} E_t[P(s_{t+n}|s_t) \cdot G_{t+n}] + D_t] / \sum_{n\geq 0} E_t[P(s_{t+n}|s_t) \cdot Y_{t+n}].$$

One can easily verify that formula (13) implies $E_t[\tau_{t+n}]=\tau_t$ for all n>0.

The formulas show clearly that balanced budgets and tax smoothing policies prescribe very different tax responses to temporary changes in government spending and to changes in interest rates (see also Barro 1979). But the emphasis here will be on a common feature: Both policies are typically defined as tax policies without reference to debt management, to the type of liabilities that the government issues.

The issue of debt management must be addressed, because a tax rule does not provide a complete specification of fiscal policy in a world with uncertainty. For theoretical analysis, debt management is of course not an issue if a deterministic model is used. But some assumption is made at least implicitly whenever policy analysis is applied to the real, stochastic world. The assumption implicit in most of public finance seems to be that governments issue default-free, nominal bonds, as they have done historically. Government debt is therefore either assumed to be safe, having state-independent returns, or if some uncertainty in real returns is recognized, the return distribution on government debt is taken as exogenously given.[9] I will argue that debt management is important for sustainability in a stochastic setting. Policies that are sustainable in a certainty model can easily become non-sustainable in a stochastic environment, in particular when simple—straightforward but ad-hoc—assumptions about debt management are made.

The argument why even simple, apparently straightforward policies may run into sustainability problems under uncertainty can be explained most easily in examples and under simplifying assumptions about the stochastic environment. As examples, I will examine the benchmark polices of balanced budgets and of tax smoothing in a simplified setting that abstracts from inessential features of these policies, assuming the government uses strictly safe debt. Later sections will show that the arguments are applicable under more general assumptions on the stochastic setting and for other types of government debt.

4.2. A Simplified Setting

For now, I will abstract from autocorrelation in time series and from temporary variations in government spending. Both should be inessential for sustainability, because the issue is primarily a long-term one. To be precise, assume that the growth rate of income, $y_t = Y_t/Y_{t-1}-1$, is i.i.d. with mean $\bar{y}$ and that the spending-income ratio is an exogenous constant, $g_t = \bar{g}$, $0 \leq \bar{g} < 1$. In a growing economy it seems also reasonable to assume homothetic utility, so that growth per se does not affect interest rates. The safe interest rate is then a constant $r_t = r$, and the dividend-price ratio is a positive constant $v_t = v > 0$.[10]

Under these assumptions, balanced budgets and tax smoothing are both tax rules of the form

$$\tau_t = g + \rho \cdot d_t, \tag{14}$$

for different debt service parameters ρ. Debt is assumed to be safe, real risk-free debt. Its initial value is assumed to be positive, $d_0>0$, but low enough that sustainability does not fail just because of an excessive initial value. The balanced budget rule is obtained for $\rho = \rho_{BB} \equiv r/(1+r)$. The tax smoothing rule is obtained for a ρ-parameter that is implicitly determined by the requirement of constant tax rates. If growth is deterministic, setting $\tau_{t+1}=\tau_t$ in (14) implies $d_{t+1}=d_t$, which is obtained by setting $\rho=\rho_{TS} \equiv (r-\bar{y})/(1+r)$. If growth is stochastic, the condition $E_t[\tau_{t+1}] = \tau_t$ for constant tax rates in expectation implies

$$\rho = \rho_{TSE} \equiv 1 - E[(1+r)/(1+y_{t+1})]^{-1}.$$

Other cases may be interesting, too. For example, if inflation is deterministic and if i is the constant nominal interest rate, policy (14) with $\rho=i/(1+i)$ yields a nominally balanced budget.

Under certainty, the analysis of balanced budget and tax smoothing policies and their comparison is straightforward. Policies of the form (14) are sustainable with income taxes, if and only if $\rho \geq \underline{\rho} = (r-\bar{y})/(1+r)$.[11] Since $\rho_{TS} = \underline{\rho}$ satisfied this condition, the tax-smoothing rule is always sustainable with income taxes (which should not surprise, since the rule was originally derived by Barro (1979) in an optimization problem that includes a budget constraint and an excess burden function penalizing high tax rates).

The balanced budget policy is sustainable with income taxes, if and only if $\bar{y}\geq 0$. This condition is non-trivial, but since most economies on average tend to grow over time, it may seem like a reasonable, empirically justifiable assumption. If $\bar{y}>0$, balanced budgets provide for even more debt service than tax smoothing and more than is needed for sustainability ($\rho_{BB}>\underline{\rho}=\rho_{TS}$), supporting for the notion that it is a prudent, "fiscally conservative" policy. Similarly, nominally balanced budgets require non-negative nominal growth and they provide more debt service than needed, if nominal growth is strictly positive. For completeness, it should be noted that sustainability with lump-sum taxes only requires $\rho>0$ (as noted by McCallum 1984). This is always satisfied for tax smoothing and real balanced budgets (since $r>\bar{y}$ and $r>0$ under certainty) and it is satisfied for nominally balanced budgets, if $i>0$.

Overall, analysis under certainty suggests that the benchmark policies are sustainable in any sense of the word. Unfortunately, this conclusion does not seem to be robust with respect to adding a realistic dose of uncertainty. Under uncertainty, the key observation about policy rules of the form (14) is that they do not have any feature that limits movements in the debt-income ratio.[12] Taxes are specified in terms of tax rates. Debt evolves deterministically in level terms. This mismatch implies that the debt-income ratio under income uncertainty is a geometric random walk,

$$(15) \qquad d_{t+1} = \frac{1+r}{1+y_{t+1}} \cdot (1-\rho) \cdot d_t.$$

Whenever there is a positive probability that $1+y_{t+1}<(1+r)\cdot(1-\rho)$, the path of d_t will exceed any positive upper bound with positive probability, proving non-sustainability according to Proposition 1(b). Denoting the lower bound of the support of y_{t+1} by $\underline{y}$, policies of the type (14) are sustainable with income taxes, if and only if debt service exceeds a lower bound $\underline{\rho}$, i.e., if

$$\rho \geq \underline{\rho} \equiv 1 - [(1+r)/(1+\underline{y})]^{-1} = (r-\underline{y})/(1+r).$$

Sustainability with lump-sum taxes again requires only $\rho>0$.[13]

A comparison of ρ_{TSE} and $\underline{\rho}$ shows that tax smoothing is <u>never</u> sustainable with income taxes, if there is any amount of income uncertainty. (A non-zero variance of income growth implies $1/(1+\underline{y})>E[1/(1+y_{t+1})]$, hence $\rho_{TSE} < \underline{\rho}$.) For the balanced budget policy, the condition for sustainability with income taxes is $\underline{y} \geq 0$. In other words, if there is any positive probability that real incomes may fall, the balanced budget policy is unsustainable. (If budgets were balanced in nominal terms, the analogous condition would require no declines in nominal income.) The condition $\underline{y} \geq 0$ requires much more than just a positive growth rate on average, and since we have observed recessions, it does not seem to be satisfied in practice. To be sustainable with income taxes, a policy of the type (14) would have to have such high ρ-value that the debt income ratio does not increase in the worst-case scenario for income. It would be a policy that rapidly paid down the debt under "normal" circumstances. To summarize:

Proposition 3: Under the assumptions of this section,

(a) if the variance of income growth is non-zero, a policy of smoothing expected tax rates is not sustainable with income taxes.

(b) If there is a positive probability of a decline in income, a balanced budget policy is not sustainable with income taxes.

Proof: see above. ||

Of course, the simplicity of these sustainability conditions is to some extent due to the special assumptions of this subsection. It seems worth examining how general the conditions are.

4.3 Sustainability under more General Conditions

When one examines the assumptions for the non-sustainability results in Proposition 3, the unit root assumption on the income process stands out. It implies that whenever there is a positive probability of any decline in income, then there is also a positive probability of an arbitrarily large decline in income in the longer run. Since the movement of real debt under the balanced budget rule is deterministic, a sufficiently large decline in income will push the debt income ratio above any given upper bound, leading to non-sustainability.

To see that the income process is critical, a brief review of the general balanced budget and tax smoothing formulas of Section 4.1 is sufficient. As equation (12) shows, a constant level of government debt is a general feature of balanced budgets that is valid without the simplifying assumptions of Section 4.2. Therefore (using Proposition 1(b)), a lower bound on the path of income—inconsistent with any unit root process—is generally necessary for sustainability with income taxes. In case of trend-stationary income, it would depend on the precise specification of the income process whether or not income has a lower bound that is high enough to assure sustainability. In case of a unit root process, the balanced budget policy is never sustainable.

Concerning the tax smoothing policy, it is sufficient to note that tax smoothing—in the sense defined in Section 4.1—implies a martingale process for tax rates. Thus, the condition $\tau_t \leq 1$ will generally be violated, provided only that the income and government spending processes are such that the variance of tax rate innovations is bounded away from zero (see Bizer and Durlauf 1989).

Overall, the tax smoothing and balanced budget policies seems to be non-sustainable under fairly general conditions. Assuming that sustainability is considered a reasonable normative requirement for fiscal policies, politicians who wanted to defend a policy recommendation in favor of balanced budgets or in favor of tax smoothing would have to claim either (1) that income does not have a unit root or (2) that the assumed upper bound on taxes is unwarranted, i.e., that lump-sum taxes are available.

A defense of balanced budgets or tax smoothing based on doubts about the unit root assumption cannot be ruled out, since the empirical debate about the time series properties of aggregate income (GNP) seems inconclusive at this point.[14] But even if the aggregate income process has no unit root and is highly autocorrelated instead, the variance of far-ahead income may still be high enough to yield non-sustainability. In any case, the burden of proof should—in the interest of prudence—be on those who recommend balanced budgets as sound public policy. Since Christiano and Eichenbaum (1989) doubted the relevance of the unit root controversy, it may be worth noting that sustainability is an issue for which the long-term properties of the aggregate income process seem to be very important.

A defense based on the availability of lump-sum taxes may also be problematic, not only because lump-sum taxes are rarely used in practice, but because the sustainability of balanced budgets and tax smoothing is not even assured with lump-sum taxes. In a stochastic dynamically efficient economy, the relevant conditions $r>0$ (for $\rho_{BB}>0$) and $E[(1+r)/(1+y_{t+1})]>1$ (for $\rho_{TSE}>0$) are not assured, even if the average growth rate is positive (see Abel et al. 1989). For the U.S., the average real interest rate over the period 1929-1988 has been 0.23% and the sample average of $(1+r)/(1+y_{t+1})$ has been 0.9734.[15] Thus, even with lump sum taxes the sustainability of balanced budgets cannot be taken for granted (r being so close to zero) and the sustainability of tax smoothing seems highly questionable.

Another way to respond to the non-sustainability result is to argue that the definition of sustainability is too strict. If ρ is high enough that the debt income ratio falls in expectation, the probability of ever reaching a debt-income value that creates feasibility problems may be very small. This line of argument is explored in the next section.

5. Modified Policy Rules and Probabilistic Arguments

Since simple, popular policy rules such as balanced budgets turn out to be unsustainable with income taxes, one might wonder what happened if the government simply pursued such a policy as long as possible and abandoned it only in states of nature when it becomes infeasible. This section will show that balanced budgets and many other policy rules that were found to be non-sustainable in Section 4 are indeed "sustainable with very high probability." But the section also shows that many other polices that impose substantially lower debt service, even some that even allow persistent primary deficits, are "sustainable" in the same sense. Thus, probabilistic arguments do not provide a convincing argument in favor of balanced budgets.

Proposition 2 shows that any policy has some feasible continuation as long as debt does not exceed the present value of consumption. If one again imposes the simplifying assumptions of Section 4.2, where $C_t=(1-\bar{g})\cdot Y_t$ and $v_t=v>0$ is constant, the present value of consumption is simply $(1-\bar{g})\cdot(1+v)\cdot Y_t$. Thus, is feasible to service government debt as long as $d_t \leq (1-\bar{g})\cdot(1+v)$. A simple tax rate formula such as (14) will have to be abandoned in states of nature in which the debt-income ratio threatens to exceed this bound. But a complete specification of a policy must include a course of action in such states of nature. A natural way to keep the debt-income ratio bounded is to assume a tax increase in states of nature with excessive debt-income ratio. To be specific, I will replace tax rule (14) by a tax rule of the form

$$(16) \qquad \tau_t = \begin{cases} g+\rho\cdot d_t & \text{if } d_t < d^* \\ g+d_t-d^*\cdot(1-\rho) & \text{if } d_t \geq d^*, \end{cases}$$

where d^* is a policy parameter. Provided

$$(17) \qquad d^* \leq \bar{d} = \frac{(1-\bar{g})\cdot(1+v)\cdot(1-\underline{y})}{(1+r)\cdot(1-\rho)},$$

all policies of the form (16) are sustainable with income taxes as well as with lump sum taxes[16] and therefore provide a complete specification of policy. Provided $0<d_0<\bar{d}$ and $\underline{y} \geq -1.0$,[17] one can chose 0 <

$d_0 < d^* \leq \bar{d}$, so that the government starts out with positive debt (making it an interesting problem) but below the upper bound (providing some scope for discretion).

For $\rho = \rho_{BB}$ and $\rho = \rho_{TSE}$, equation (16) can be interpreted as generalized balanced budget and tax smoothing policies, respectively, which coincide with the original policy rule (14) as long as debt is at or below the upper limit d^* but assume a tax increase whenever debt exceeds d^*. The state-contingent switch to higher taxes "fixes" the sustainability problem associated with (14) and it is necessary to provide a complete policy specification. Casual observation of the political process suggests that it may not be unrealistic to model policy in this way as alternating between periods of "business as usual" (following the standard rule) and "crisis" (when debt hits the bound). This section explores what such an interpretation of policy rules implies for sustainability.

Only weak restrictions are needed to make the case of debt being strictly below d^* the "normal," highly likely case: Starting at $d_t < d^*$, we have $E_t[d_{t+1}] \leq d_t < d^*$ for all policies with $\rho \geq \rho_{TSE}$. In some cases, depending on the distribution of output, one can even show that it is very likely that the debt-income ratio will never reach d^*, provided ρ is somewhat above ρ_{TSE}. One example is the log-normal case:

Proposition 4:

Assume $(1+y_t)$ is log-normal with $E[\log(1+y_t)]=m$ and $var[\log(1+y_t)]=s^2$, and assume $d_0 \leq d^*$. Then

$$P \equiv \text{Prob}\{\, d_t < d^* \ \forall\ t \geq 0 \,\} \geq 1 - (d_0/d^*)^{\lambda},$$

where $\lambda = 2/s^2 \cdot [m - \log(1+r) - \log(1-\rho)]$. If $\lambda > 0$, then $P > 0$.

Proof: If d_t were a geometric Brownian motion with drift $[m - \log(1+r) - \log(1-\rho)]$ and variance s^2, $\text{Prob}\{d_t < d^* \ \forall\ t \geq 0\} = 1-(d_0/d^*)^{\lambda}$ would be an application of the optional stopping theorem (see Karlin/Taylor 1975, p.361). An inequality holds, because $d_t < d^*$ is only required for integer values of t.||

The interesting parameter combinations are those that yield $\lambda > 0$, i.e., for which the debt-income ratio has a downward drift. The proposition shows that the probability of ever hitting the upper bound d^* declines exponentially with that drift. For the balanced budget rule, which sets $\log(1-\rho) = \log(1+r)$, the proposition applies whenever average output growth is positive, $m > 0$. (Recalling that non-negative output growth was the condition for sustainability in the certainty model, Proposition 4 seems to provide some indication of how certainty arguments generalize in a stochastic economy.) Taking GNP as empirical counterpart to income, approximate U.S. data are m=3% and s=6% (see Bohn 1990), which yields λ=16.67. Noting that the current U.S. debt-GNP ratio of about 0.50 is less than half its historical peak of 1.13 in 1946 (which must have been feasible), we have $P \geq 1-0.5^{16.67} = 99.999\%$. (Intuitively, the number is one minus the probability the GNP will drop by more than 0.50. For comparison, a 50% fall in real income would be substantially worse than the Great Depression, in which income fell only by about 30%.) Thus, the probability of being able to continue with balanced budgets for ever seems to be extremely close to one.[18]

Those who start to wonder whether this result may be taken as argument in favor of balanced budgets should note that the probability of sustaining a policy with much lower debt service is also close to one. For example, if the interest rate is r=1% and debt is $2500 billion,[19] the balanced budget calls for a primary surplus of about $25 billion. If the primary surplus were set to zero instead, $\rho=0$, one would still have $\lambda=2\cdot(3\%-1\%)/6\%^2=11.11$ and $P \geq 99.955\%$, and if the government ran a $25 billion primary deficit, setting $\rho=-1\%$, one would have $\lambda=5.55$ and $P \geq 97.87\%$.[20] In other words, there is a high probability—but no certainty—that economic growth alone will keep the debt-income ratio bounded, even if taxes pegged equal to or somewhat below current spending.[21] Thus, weakening the sustainability requirement does not seem to provide an argument for balanced budgets: If one considers policies that can be sustained with probability close to but not equal to one (say, 95% confidence) to be normatively acceptable, one does not even have an argument against running primary deficits, much less an argument in favor of balanced with-interest budgets.

Overall, this section documents that many polices that were shown to be unsustainable in Section 4 can be sustained for ever with very high probability. But if one falls for the temptation to call such policies sustainable, i.e., to omit the explicit reference to the need for a policy change in very rare "worst case" scenarios, one would also remove much of the argument in favor of balanced budgets.

6. Perfect Tax Smoothing and State-Contingent Debt

This section returns to the issue of debt management. I show that tax rules of the form (14) can be made sustainable in a stochastic environment, if the government issues a specific type of state contingent liabilities instead of safe debt. The basic reason for the sustainability problems observed in Sections 4 and 5 was the fact that the debt-income ratio moved stochastically. The problem can be eliminated, if debt is indexed to income. Such indexing is particularly important for tax smoothing, because it removes the need for any changes in tax rates, i.e, allows perfect tax smoothing instead of just holding tax rates constant in expectation.

The virtues of income indexation are easy to see under the simplifying assumptions of Section 4.2. The economy is one in which all real variables (Y_t, C_t, G_t) expand stochastically at the rate y_t. This suggests that government debt also be made proportional to income, which can be done by making promised payments on debt contingent on economic growth.[22] In terms of state-contingent claims, that means that the government issues $D(s_{t+1}) = d_t^* \cdot Y(s_{t+1})$ claims on state s_{t+1}, where d_t^* is a proportionality constant determined by the period-t budget equation. With this debt management policy, the debt-income ratio at the start of period t+1 is $d_{t+1}=d_t^*$ for all states of nature. To determine d_t^*, note that tax policy determines the end-of-period debt to which the value of all contingent claims must add up. Equation (1) becomes

$$d_t^* \cdot \int_{s_{t+1}} p_t(s_{t+1}) \cdot Y(s_{t+1})\, dF(s_{t+1}|s_t) = g \cdot Y_t - \tau_t \cdot Y_t + d_{t-1}^* \cdot Y_t \tag{18}$$

Assuming tax policy again follows a simple rule of the form (14), the debt-income ratio evolves deterministically as

$$d_{t+1} = d_t^* = (1 + 1/v) \cdot (1-\rho) \cdot d_t,$$

using the fact that the discounted value of next period's income is $v/(1+v) \cdot Y_t$. A tax policy of the form (14) combined with income-indexed debt is sustainable with income taxes, if and only if ρ is such that $(1 + 1/v) \cdot (1-\rho) \leq 1$, i.e., $\rho \geq 1/(1+v)$. As before, it is sustainable with lump-sum taxes, if and only if $\rho > 0$.[23]

Note that setting $\rho = 1/(1+v)$ yields $d_{t+1}=d_t=d_0$ and $\tau_{t+1}=\tau_t=\tau_0$ for all states of nature, which is a policy that stabilizes tax rates perfectly. In contrast to the policy of smoothing expected tax rates defined in Section 4, this policy is sustainable in any sense of the word. More generally, income-indexed debt in an economy with income uncertainty resurrects some of the intuition and simplicity of the certainty case. In particular, perfect tax smoothing is feasible, it is sustainable, and it has a positive debt service parameter (here because $v>0$), which is as low as possible (here $\rho = 1/(1+v)$). In analogy to the balanced budget rule under certainty, a policy that sets ρ equal to the expected return on income-indexed debt, $\rho = E[(1+1/v)\cdot(1+y_{t+1})] = (1+1/v)\cdot(1+\bar{y})$), is sustainable if $\bar{y}\geq 0$, and it is "more conservative" than tax-smoothing if $\bar{y}>0$, just like the balanced budget rule under certainty.

This analysis under simplified assumptions should suffice to show that state-contingent liabilities may be important for sustainability questions. In general, the appropriate type of indexation would not be as obvious, but the existence of a perfect tax smoothing policy generalizes easily, as follows. Starting in period t, set τ_t as indicated in equation (13). In period t+1, the tax rate can be held constant at that level for all states of nature, $\tau_t = \tau(s_{t+1}) = \tau$, provided the government sells contingent claims

$$D(s_{t+1}) = \sum_{t\geq n} \tau \cdot \sum_{n\geq 1} E_{t+1}[P(s_{t+n}|s_{t+1}) \cdot G_{t+n}] - \sum_{n\geq 1} E_{t+1}[P(s_{t+n}|s_{t+1}) \cdot G_{t+n}]$$

on states s_{t+1}. Proceeding analogously in subsequent periods, the tax rate will stay unchanged at τ for all times and all states of nature. Thus, there is always a debt management policy that supports perfect tax smoothing. The policy is sustainable with income taxes as well as with lump sum taxes. The next section will show that other cases of state-contingent debt are also interesting.

7. Nominal Government Debt and other Extensions

This section will emphasize the distinction between arbitrary and purposeful debt management policies and their implications for sustainability. I show that the results of Sections 4 and 5 remain generically valid, if government debt is not strictly safe, but if it has an arbitrary, exogenously given return distribution. This includes the practically relevant case of nominal debt and shows that the policies of Section 6 are a special cases.

The distinction between arbitrary and purposeful debt management is particularly important for nominal government debt. If inflation is an exogenous random variable that is not perfectly correlated with income, the results of Sections 4 and 5 remain essentially unchanged if safe real debt is replaced by safe nominal debt. But if inflation is a government (monetary policy) instrument, nominal debt may be used to implement either switching rules of the type discussed in Section 5 or a perfect tax smoothing policy of the type defined in Section 6.

In general, the government in period t can choose to sell an arbitrary set of state contingent securities $D(s_{t+1})$ at prices $p(s_{t+1})$. If a tax rule is specified first, the budget equation (1) determines the present value of this debt, $E_t[p_{t+1} \cdot D_{t+1}]=(G_t-T_t+D_t)$, which reduces the degrees of freedom by one, but does not eliminate debt management as an issue. Safe debt is the particular debt management policy that sets equal values $D(s_{t+1})=(1+r_t)\cdot(G_t-T_t+D_t)$ across all states of nature. Income-indexed debt is the special case that sets $D(s_{t+1})=d_t^* \cdot Y(s_{t+1})$. No matter what securities are issued, one can characterize the promised payoffs on the portfolio of government liabilities in terms of its (stochastic) rate of return $R(s_{t+1})$, defined as

$$(19) \qquad R(s_{t+1}) = \frac{D(s_{t+1})}{G_t-T_t+D_t},$$

assuming $G_t-T_t+D_t \neq 0$. This notation is convenient because individual optimization implies that

$$(20) \qquad E_t[\,\beta \cdot \frac{U'(C(s_{t+1}))}{U'(C(s_t))} \cdot R(s_{t+1})\,] = 1$$

In case of safe debt, $R(s_{t+1})=1+r_t$, and in case of income-indexed debt, $R(s_{t+1})$ is proportional to $(1+y_{t+1})$. Another well known special case is nominal debt, which is $R(s_{t+1})=(1+i_t)/(1+\pi_{t+1})$, where π_{t+1} is an exogenous stochastic process of inflation and where i_t is implicitly given by (20). Combined with a tax rule of the form $T_t = G_t + i_{t-1}/(1+i_{t-1}) \cdot D_t$, it defines the policy of balancing the budget in nominal terms.

For the analysis, consider again the simplified scenario of Section 4.2 with i.i.d. growth and a constant ratio of government spending to income, but now consider debt with an arbitrary return distribution. For a tax rule of the form (14), the debt-income ratio evolves according to

$$d_{t+1} = \frac{R_{t+1}}{1+y_{t+1}} \cdot (1-\rho) \cdot d_t. \tag{21}$$

It is apparent that this is also geometric random walk with non-zero variance, as in Section 4.2, equation (15), unless R_{t+1} is exactly proportional to $1+y_t$. Since the results of Section 4 and 5 all followed from this random walk property, they are generic (applicable unless $R_{t+1}/(1+y_t)$ is a constant) and can be replicated analogously for any debt policy (by replacing $(1+r)/(1+y_{t+1})$ by $R_{t+1}/(1+y_{t+1})$ everywhere). Thus, the policies of smoothing expected rates and balanced budgets are generally non-sustainable, with one exception: Nominal debt would implement perfect indexation and make policies of the form (14) sustainable as in Section 6, if inflation were perfectly negatively correlated with income (e.g., through a quantity equation).

In the numerical examples of Section 5, the variance of $\log(1+y_{t+1})$ would have to be replaced by the variance of $\log[R_{t+1}/(1+y_{t+1})]$, so that the probability of sustaining a policy would depend on the the variance of R_{t+1} and its covariance with income. If inflation is negatively (positively) correlated with income growth, the probability of sustaining balanced budgets and the probability of sustaining zero primary surpluses will be higher (lower) than indicated in Section 5.

Consideration of state-contingent debt may also provide a new perspective on switching-rules of the type discussed in Section 5. A policy switch may not (or not only) be a change in the tax rule, but a change in debt management. For example, suppose the government "normally" issues safe debt and follows a simple tax rule of the type (14) with $\rho < \underline{\rho}$. The policy is not sustainable with income taxes, but it can be made part of a sustainable policy, if the government just promises to switch to income-indexed debt, whenever the debt-income ratio reaches a critical value d^*. If debt is nominal and the government controls the rate of inflation, a common understanding that the government normally maintains price stability but that it is allowed to inflate in "crises" would constitute such a switching rule.[24] (Of course, the expected real return on such debt would exceed the safe rate and approach the rate of return on equity as the debt-income ratio approached the trigger level. If policy "normally" sets debt service at a value $\rho<1/(1+v)$, an increase in debt service to $1/(1+v)$ would have to be part of the policy switch.) Thus, a balanced budget policy with nominal debt may be sustainable with income taxes, confirming the notion that a government cannot go bankrupt if all its debt is denominated in its own fiat currency. But the switching-rule interpretation shows that such a policy effectively involves a conditional monetization of the debt, which may stretch the notion of sustainability. (See Sargent and Wallace 1981 for a scenario where "crisis" and monetization occur with certainty.)

Finally, it should be noted that the assumptions of i.i.d. growth and $g_t = \bar{g}$ again seem to be inessential. If g_t varies, the concepts of balanced budgets and tax smoothing become more distinct and the contingent claims valuation is more complicated, since consumption growth differs from income growth.[25] But the policy of balancing the budget in nominal (real) terms combined with safe nominal (real) debt always implies a constant level of debt in nominal (real) terms and is therefore not sustainable with income taxes, unless the nominal (real) income process has a suitable lower bound that prevents the debt-income ratio from becoming large.[26]

8. Conclusions

The paper has shows that the sustainability even of simple policy rules like balanced budgets or tax rate smoothing should not be taken for granted in a stochastic economy and that sustainability is often sensitive to assumptions about debt management. The main results are as follows.

With no uncertainty, balanced budgets and tax smoothing are sustainable in any sense of the terms. With stochastic aggregate income, complications arise for a variety of tax polices when the government borrows in terms safe debt, because low realizations of income will then raise the debt-income ratio. Neither tax smoothing nor balanced budgets are sustainable, if there is an upper bound on feasible tax rates that limit's the government ability to service debt. It is true, though, that balanced budgets can be maintained for ever with a probability that is very close to one. But if one lowers the sustainability standard to require only that a policy can be maintained with high probability, there is little reason to balance budgets: For plausible parameters, a policy of running small permanent primary budget deficits would be sustainable with almost as high a probability as balanced budgets.

The non-sustainability result for balanced budgets and tax smoothing with safe real debt turns out to be more generally valid for generic debt financing methods, with the debt policy that supports perfect tax smoothing being the unique exception. In particular, balanced budgets in nominal terms and tax smoothing policies are non-sustainable when debt is nominal debt, unless inflation and income growth are correlated is specific ways.

One source of sustainability problems under uncertainty seems to be the use of ad-hoc, exogenously specified debt management policies. If state-contingent debt is used to support a tax smoothing objective, the resulting policy of perfect tax smoothing is sustainable under general condition. This result highlights the fact that, in contrast to deterministic models, tax policy does not completely specify the government's financial policy under uncertainty. In a model with more than one security, debt management remains an issue even when a tax rule has been specified, and it seems to be important for the sustainability of the complete debt-and-tax policy. This is an aspect of fiscal policy that perhaps deserves more attention in future research and in policy making.

Footnotes

[1] See McCallum (1984) and Sargent and Wallace (1981) for theoretical analyses under certainty. Empirical tests for the sustainability of actual U.S. fiscal policy have been executed by Hakkio and Rush (1986), Hamilton and Flavin (1986), Kremers (1989), Trehan and Walsh (1988), Wilcox (1989), but under restrictive assumptions on interest rates that essentially assume away the effect of uncertainty on asset prices. This paper studies the sustainability of policies in general; a discussion of the empirical tests is in Bohn (1990).

[2] These beliefs are most clearly reflected in the Gramm-Rudman-Hollings laws and in various proposals for a balanced budget amendment. The most recent Economic Report of the President (1990) repeatedly mentions a movement towards balanced budgets as major goal of U.S. fiscal policy. Doubts about the sustainability of recent policies have been raised by Kremers (1989) and Wilcox (1989).

[3] In the analysis, I initially assume safe real debt and show later that the results are generic for general debt securities, including nominal debt.

[4] This is implicitly done when an error term is added in the empirical analysis to an optimal debt or tax rate equation derived under certainty, e.g., as in Barro (1979) and Sahasakul (1986). Blanchard and Fischer (1989) correctly recognize this as "shortcut" (p.620, footnote 36).

[5] This is one reason why a fully specified equilibrium model is needed to analyze sustainability questions.

[6] Since $C(\cdot)=Y(\cdot)-G(\cdot)$ in equilibrium, the first condition guarantees finite utility. The pricing function $P(\cdot)$ below will show that the second condition assures a finite present value of consumption. The assumptions merely require that endowment growth is not too fast in relation to the rate of time preference.

[7] If one wants to justify the absence of lump-sum taxes more formally, a slight departure from the representative agent setting would be enough. For example, assume agents (indexed by i) own land of different "quality" q(i) so that individual income is $Y_t(i) = Y_t \cdot q(i)$ for all t. If q(i) has a unit average and if utility functions are homothetic, nothing significant changes on aggregate. (Individual agents' activities

are simply scaled by the factor q(i), but they aggregate to the variables used in the text.) If q(i) is not observed by the government and if q(i) has support on the entire positive real line, excluding q=0, the government cannot levy a lump sum tax, because for any non-zero value of such a tax, there are agents with sufficiently small q(i) values that will be unable to pay. The paper ignores taxes on interest income from government bonds, because they would not affect the equilibrium after-tax return on bonds but might create time consistency problems, which are not the topic of this paper.

[8] See Blanchard and Fischer (1989). In the literature, the tax smoothing policy is has usually been developed in a certainty setting without reference to uncertainty. The extension to smoothing expected tax rates is made implicitly when error terms are added in empirical analysis (see, e.g., Barro 1979 and 1986, Sahasakul 1986). In comparison to Blanchard and Fischer, note that the discounting in (12) generally involves state-contingent claims prices and not the safe or any other common interest rate. Even if all government debt is safe, government spending and income are stochastic and generally have to be discounted at a rate different from the safe interest rate.

[9] For example, the safe debt assumption has been made in the sustainability literature by Hamilton and Flavin (1986) and Trehan and Walsh (1988), the exogeneity assumption has been made by Wilcox (1989) and it is implicit in most empirical studies of tax smoothing, e.g., Barro (1979) and (1986), Sahasakul (1986). An important exception is Lucas and Stokey's (1983) paper, which considers optimal policies with state-contingent debt.

[10] Note that the safe interest rate r is not necessarily positive and that it does not necessarily exceed the average growth rate $\bar{y}$, if individuals are sufficiently risk averse (see Abel et al. 1989). If growth is deterministic, $r > \bar{y}$ does apply, since the economy is by construction dynamically efficient.

[11] The debt-income is non-increasing for this range of ρ-values, hence bounded by d_0. A bounded debt-income ratio is necessary because of Proposition 1(b) and sufficient because both sides of (10) are proportional to income under the simplifying assumptions of this section. That is, if D_0 satisfies Proposition 2—as assumed—and if $d_t \leq d_0$, then D_t will also satisfy Proposition 2.

[12] The fact that tax-smoothing may be problematic for this reason was noted in Barro's original article (1979, p.950). See also Bizer and Durlauf (1989) and (1990). My point is that the problem is much more pervasive, applicable not just to tax smoothing policies.

[13] For a proof, apply Proposition A1 in the appendix: In (*), $D(s_{t+n})=[(1+r)\cdot(1-\rho)]^{n-k}\cdot D(s_{t+k})$ is discounted at the safe interest rate r, so that $LD(s_{t+k}) = D(s_{t+k}) \cdot \lim_{n\to\infty} (1-\rho)^{n-k}$, which equals zero if and only if $|1-\rho|<1$ or $0<\rho<2$. The restriction $\rho<2$ is omitted in the discussion to save space and since it is difficult to imagine a realistic policy that would violate $\rho<2$; in a high deficit environment, the critical condition is $\rho>0$.

[14] See Nelson and Plosser (1982), Campbell and Mankiw (1987), Christiano and Eichenbaum (1989), Durlauf (1990).

[15] Using one-month Treasury bills and GNP-growth; see Bohn (1990) for a more detailed discussion of the data.

[16] The constraint with income taxes is that $d_{t+1}(s_{t+1}) \leq (1-\bar{g})\cdot(1+v)$ with probability one, starting with any value of d_t. Since (16) bounds the ratio of end-of-period debt to income by $(1-\rho)\cdot\bar{d}$, condition (17) is sufficient. Sustainability with lump-sum taxes follows from Proposition 1(a).

[17] If $\underline{y} = -1.0$, then $Y_t=0 \; \forall t$ would have positive probability. Then safe, default-free government debt could not exist.

[18] Proposition 3 has less striking implications for tax smoothing. The equation for ρ_{TSE} implies $\lambda=1$ regardless of the income parameters, so that $P\geq 1-d_0/d^*>0$. It is positive, but lower than in the case of balanced budgets (about 56% if $d_0=0.50$ and $d^*=1.13$). But regardless of the probability, the fact that tax rates will have to be increased sharply at the debt boundary suggests that "tax smoothing" is a somewhat questionable label for this policy, though that seems to be label used in the literature; the next section will introduce a more restrictive notion of tax smoothing. Moreover, the exact specification is important, due to Jensen's inequality: If tax smoothing were defined in terms of logarithms instead of levels as the policy that satisfies $E_t[\log(d_{t+1})]=\log(d_t)$ (see Trehan and Walsh 1990), tax smoothing would be the policy with $\lambda=0$.

[19] The average real interest rate on one-month Treasury bills has been 0.23% for 1929-88 (see Bohn (1990). In the numerical example, I take the higher value of 1%, because the real rate has been much higher recently. A higher (lower) value would reduce (increase) all probabilities.

[20] To have cases with $\lambda>0$ and $\rho<0$, it is critical that average growth is above the safe interest rate, as in this example. This is possible in a dynamically efficient economy under uncertainty, but not under certainty (see Abel et al. 1989), and it seems to be relevant for the U.S. See Bohn (1990) for the U.S. data and a comment on the high real interest rates of the 1980s.

[21] An analysis of the welfare implications (of requiring polices to be sustainable with probability one or less and of relying on growth) is beyond the scope of this paper, but should be an interesting topic for future research. Bohn (1990) suggests that even a small positive probability of not being able to sustain a policy may impose risks on future tax payers that are not insignificant in utility terms.

[22] Without using contingent debt, one could stabilize the debt-income ratio at the end of a period by varying tax rates, namely by setting taxes high whenever growth was low, and conversely. The key feature of income indexation is that debt at the start of a period is automatically proportional to income, so that there is no need for changes in tax rates.

[23] For a proof, apply Proposition A1 in the appendix again: In (*), $D(s_{t+n})=[(1+1/v)\cdot(1-\rho)]^{n-k}\cdot D(s_{t+k})$ is discounted at rate 1/v, so that $LD(s_{t+k}) = D(s_{t+k}) \cdot \lim_{n\to\infty} (1-\rho)^{n-k} = 0$, if and only if $|1-\rho|<1$ or $0<\rho<2$. The condition $\rho<2$ is again omitted.

[24] Monetary history provides some evidence for such an understanding, e.g., in the context of suspension and resumption of a gold parity (see Hepburn 1915 on the greenback period and Barro 1987 on Britain). Historically, variations in government spending (wars) were obviously important triggers of "crisis" periods and could be included easily, though a formal inclusion would require a lot of notation.

[25] In addition, the balanced budget rules may require modifications, if g_t takes very high values that might push tax rates above one even for moderate levels of debt.

[26] The policy is sustainable with lump sum taxes whenever the average discount rate on safe nominal (real) claims is positive.

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Appendix: Transversality Conditions

Here is why and under what regularity conditions the intertemporal budget constraints (5) and (6) apply. The derivation is essentially an extension of Cass (1972) and O'Connell and Zeldes (1988) to a stochastic model.

Regularity conditions are needed, because private Ponzi games could not be ruled out, if the government volunteered to be on the lending side of a Ponzi game, and because some limits might fail to exists, if tax policy were too "erratic." Since our main concern is about government deficits and not about the government's possible over-accumulation of financial assets, it seems reasonable to restrict the the analysis of policies to those that do not permit Ponzi-games against the government. (In light of the S&L-crisis, one might wonder, though, whether the possibility of the government allowing private agent to run Ponzi-games against it should be taken more serious. But since a permit to run a Ponzi scheme against the government—nowadays perhaps called S&L charter—can always be interpreted as a tax break in the amount of the present value of the Ponzi scheme, even then a formal treatment of this case may not be needed.) Similarly, the non-existence of limits seems to be a technical and economically uninteresting complication that can be ruled out in the interest of simplicity (see Bohn 1990 for more comments). If such economically rather uninteresting cases are excluded, one obtains the following.

Proposition A1:

If the limit

$$(*) \qquad LD(s_{t+k}) = \lim_{n\to\infty} \int_{s_{t+n}} P(s_{t+n}|s_{t+k}) \cdot D(s_{t+n})\, dF(s_{t+n}|s_{t+k})$$

exists conditional on every state s_{t+k} that may occur with positive probability, $k \geq 0$, and if one only considers policies with $LD(s_{t+k}) \geq 0$, then:

(a) The government must satisfy the transversality constraints $LD(s_{t+k}) = 0\ \forall s_{t+k}$. Equivalently, the government must satisfy the intertemporal budget constraint (5) for all states of nature.

(b) Individuals must satisfy equation (6) for all states of nature.

Proof: The conditions $LD(s_{t+k})\geq 0$ formalize the notion that the government does not allow others to run Ponzi-games against it, not now (in state s_t) or in any future state (s_{t+k},k>0). Necessary conditions for individual optimality then rule out individual lending in Ponzi-games, as in the certainty case (Cass 1972, O'Connell and Zeldes 1988). Thus, Ponzi-games are ruled out in general, which means that individuals are constrained by the NPG-conditions

$$\limsup_{n\to\infty} \int_{s_{t+n}} P(s_{t+n}|s_{t+k})\cdot A(s_{t+n})\, dF(s_{t+n}|s_{t+k})\geq 0 \ \forall s_{t+k}$$

and the government is constrained by $LD(s_{t+k})\leq 0$. Combined with the assumption $LD(s_{t+k})\geq 0$, this implies $LD(s_{t+k})=0 \ \forall s_{t+k}$. Equation (1) combined with $LD(s_{t+k})=0$ is equivalent to (5). For individuals, first-order conditions, NPG-conditions, and the existence of the LD-limits imply that $\lim_{n\to\infty} \int_{s_{t+n}} P(s_{t+n}|s_{t+k})\cdot A(s_{t+n})\, dF(s_{t+n}|s_{t+k})$ exists and equals zero for all s_{t+k}, which, combined with (3), is equivalent to (6). ||

A discussion of Ponzi-games in stochastic economies with and without government and a more detailed proof of the statements in Proposition A1 for the case of a discrete state space can be found in Bohn (1990).