STOCK PRICE MANIPULATION

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ABSTRACT

Prior to the Securities Exchange Act of 1934, manipulation of stock prices was an issue of great concern. The Act reduced the possibilities for manipulation by, among other things, making it illegal for a manager to sell short his firm's shares or for false information about a firm to be released. This paper asks whether an uninformed raider can profitably manipulate stock prices simply by buying and selling shares. It is shown that in a rational expectations framework where all agents maximize utility, it is possible for an uninformed raider to manipulate stock prices profitably, provided investors attach a positive probability to the raider being informed.

1. Introduction

Historically, the possibility of artificially influencing stock prices has been an important issue.[1] Soon after the Amsterdam Stock Exchange was founded at the beginning of the seventeenth century, brokers discovered that they could profitably manipulate stock prices. They would engage in a concentrated bout of selling. Frightened investors would then also sell, prices would fall and the brokers could buy back stock to restore their original positions at a lower price. The brokers also found that the profitability of these "bear raids" could be increased by spreading false rumors about the poor prospects of the firm.

This type of manipulation occurred in all stock markets that were established during the following years. In some countries such activities were judged to be illegal. During the Napoleonic Wars, the prices of bonds and stocks on the London Stock Exchange were sensitive to the progress of the fighting. Manipulators would operate in conjunction with newspapers to spread false information about the war and profit from the resulting changes in prices. A number of people were tried for a conspiracy to manipulate prices in this way and were convicted of fraudulently creating fictitious prices.

There are many colorful accounts of how fortunes were made on Wall street during the nineteenth century through stock price manipulation. Jacob Little, who was nicknamed the "Great Bear of Wall Street", would sell short shares that he did not own and then spread rumors about the insolvency of the company. After he had forced the price down he would cover his short position. Stedman (1905; p.101) recounts that Little "had been known to gorge and digest more stock in one day than the weight of the bulk of his whole body in certificates."[2]

Bear raids were not always successful, since they created the possibility for another speculator to corner the market. One widely recounted example is the Harlem Railway corner.[3] At the beginning of 1863, Commodore Cornelius Vanderbilt bought stock in the Harlem Railway at around $8 to $9 a share. He took an interest in running the company and its stock price advanced to $50 per share. In April 1863, the New York City Council passed an ordinance allowing the Harlem Railway to build a streetcar the length of Broadway and as a result the stock price went to $75. Members of the council then conspired to sell the stock short, repeal the ordinance and thus force the price down. However, Vanderbilt discovered the plot and managed to buy the entire stock of the company in secret. When the members of the council tried to cover their short positions after the repeal of the ordinance, they discovered that none of the stock could be purchased. Vanderbilt forced them to settle at $179 per share.

In cases like the the Harlem Railway corner, third parties such as the New York City Council took actions to affect the value of a stock. On occasion, managers of firms would also manipulate the value of their own stock. For example, in 1901 the managers of American Steel and Wire Company (later to become U.S. Steel) shorted the firm's stock and then closed its steel mills. When the closure was announced, the stock price fell from around $60 to around $40 per share. The managers then covered their short positions and reopened the mills, at which point the stock price rose to its previous level.[4]

The many instances of stock price manipulation that were revealed over the years lead to considerable discussion of the issue. In fact, Huebner (1934; p. 397) argued that stock price manipulation was the most widely discussed aspect of stock markets. After the Great Crash of 1929, there was widespread public concern that the fall in prices had been caused by bear raids. As a result of this concern, the Senate Committee on Banking and

Currency conducted extensive investigations into the operations of the security markets. Although they uncovered little evidence of bear raids during the Great Crash, they did uncover extensive evidence of other types of manipulation. In particular, many witnesses outlined the operation of "trading pools" during the 1920's. A group of investors would combine, first to buy a stock, then to spread favorable rumors about the firm and then finally to sell out at a profit. Among other things, "bulling" stocks in this way had the advantage of avoiding a corner, since with margin trading, which is the counterpart of short selling in a bear raid, settlement is made in cash rather than shares. On a number of occasions trading pools would act in concert with journalists who would write favorable stories about the stock being manipulated in return for a share in the profits. For example, Sobel (1968; pp. 248-249) recounts how John J. Levinson, a pool manager and Raleigh T. Curtis, who wrote a column entitled "The Trader" in the New York Daily News, made profits over one million dollars a year in this way.

The evidence uncovered by the Senate Committee led to extensive provisions in the Securities Exchange Act of 1934 to eliminate manipulation. The kinds of manipulation that the Act sought to outlaw fall naturally into two categories. The first can be described as *action-based manipulation*, that is, manipulation based on actions that change the actual or perceived value of the assets. Examples of action-based manipulation are the Harlem Railway and American Steel and Wire Company cases described above. The second category can be described as *information-based manipulation*, that is, manipulation based on releasing false information or spreading false rumors. The trading pools run by John J. Levinson are examples of information-based manipulation.

The Securities Exchange Act attempted to eradicate action-based manipulation by, among other things, making it illegal for directors and officers to sell short the securities of their own firm. There were also a number of provisions of the Act designed to eliminate information-based manipulation. Firms were required to issue information to the public on a regular basis so that the spreading of rumors would be more difficult. The Act also made it illegal for anybody to attempt to raise or depress stock prices by making statements which they knew to be false.

Although there have been a number of well-publicized exceptions, the Act has by and large been fairly successful in eradicating these two categories of manipulation.[5] However, there is a third category of manipulation which the Securities Exchange Act did not attempt to eradicate. We refer to this third category as *trade-based manipulation*. It occurs when a trader attempts to manipulate a stock simply by buying and then selling, without taking any publicly observable actions to alter the value of the firm or releasing false information to change the price.

On the face of it, it would seem that trade-based manipulation cannot be profitable. The argument is simple. When a raider tries to buy a stock, he drives up the price. When he tries to sell it, he drives down the price. Thus, any attempt to manipulate the price of a stock simply by buying and selling requires the trader to "buy high" and "sell low". This is the reverse of what is required to make a profit. Jarrow (1989) has formalized this argument, showing that, under certain conditions, profitable manipulation is impossible in an efficient market. The purpose of the current paper is to investigate whether profitable, trade-based manipulation is theoretically possible in a model where all agents are rational.

Hart (1977) and Jarrow (1989) have analyzed manipulation formally in the context of dynamic models of asset markets. Hart considers conditions under which profitable speculation is possible in a deterministic setting. He finds that if the stationary equilibrium is unstable or demand functions are non-linear and satisfy some technical conditions, speculators can trade profitably. Jarrow (1989) extends Hart's analysis to a stochastic setting and derives similar results. He shows that profitable speculation is possible if there is "price momentum" so that an increase in price caused by the speculator's trade at one date tends to increase prices at future dates. In addition, he is able to show that profitable manipulation is possible if the speculator can corner the market. In both papers, the form of the investors' demand functions is taken as exogenous, rather than being derived from expected-utility-maximizing behavior. So it is not clear whether and under what conditions manipulation is consistent with rationality.

In contrast, we develop a model with asymmetric information where all agents have rational expectations and maximize expected utility. Also, we work in a finite horizon framework, where bubbles are ruled out by construction. It is shown that profitable price manipulation is possible, even though there is no price momentum and no possibility of a corner. Incomplete information is crucial to our argument. Investors are uncertain whether a raider who buys the share does so because he knows it is undervalued or because he intends to manipulate the price. For the usual reasons, there are multiple equilibria in this model and a few of them do not involve manipulation in our sense. However, we argue that the most plausible equilibria involve trade-based manipulation.

In another important strand of the literature, Vila (1989) considers a model of action-based manipulation. The manipulator can take actions which alter the true value of the firm. To provide camouflage for the manipulator's trades, it is assumed that "noise traders" sell for some exogenous reason, such as a need for liquidity. In this paper, we only consider trade-based manipulation, in which the manipulator simply buys and sells the stock. We do not allow the manipulator to take actions that alter the value of the firm. In another paper, Vila (1987) considers a model of corners and squeezes in a futures market. The price can be manipulated by anyone who obtains a sufficiently large fraction of the supply. Corners and squeezes are rather special phenomena, which depend on different factors from trade-based manipulation in our sense. In any case, corners do not play a role in our analysis.

The paper proceeds as follows. Section 2 describes a three-period asset market. Characterization of the third-period equilibrium is trivial, since the value of the assets is revealed by then. The second-period equilibrium is characterized in Section 3. In Section 4, we characterize those first-period equilibria in which profitable trade-based manipulation occurs. In Section 5 we consider equilibria in which manipulation cannot occur and argue that these equilibria are less plausible. A numerical example is presented in Section 6. Section 7 considers an alternative model in which a stronger form of profitable manipulation is possible. Section 8 contains concluding remarks.

2. THE BASIC MODEL

As we pointed out in the Introduction, our primary interest is in demonstrating the *theoretical* possibility of profitable stock price manipulation. To this end, we make a number of simplifying assumptions that render the subsequent analysis tractable. The resulting model is not intended to be a realistic description of an actual stock market. Nonetheless, it may help the reader follow the analysis if we provide a "real world" interpretation of the formal model.

The setting we have in mind comes from the world of mergers and acquisitions. We start with a company whose shares are not very actively traded. Most of the stockholders are small "passive" investors. They hold a sufficiently large part of their wealth in this stock so that they are effectively risk averse. At some point, the stock attracts the attention of a speculator, whom we shall refer to as the "raider". The raider starts to buy the stock. In the minds of the pre-existing investors, there may be many reasons for the raid. The raider may think the stock is undervalued, he may expect future announcements that will affect the value of the stock, he may intend to acquire control of the company or he may simply be manipulating the price. Whatever the motivation for the raid, the investors do not know it. This uncertainty will affect the price at which they are willing to sell the stock.

Once the stock has been "put into play", other speculators become interested. They swarm into the market in search of arbitrage profits. These agents are collectively referred to as the "arbitrageurs". To keep things simple, we assume that they are uninformed and that they each take sufficiently small individual positions (relative to their wealth) that they can be treated as approximately risk neutral. If the raider is indeed a mere manipulator of the stock price, these are the ideal candidates on whom to unload the stock.

We claim that, in this setting, the raider can make a profit simply from buying and selling the stock. Since the investors and arbitrageurs do not know his motivation in any particular instance, they infer that, on average, the raider has information about the true value of the stock. The mere attempt to trade the stock may affect the market-clearing price. In order to make a profit, however, the manipulator must buy low and sell high. How is this possible in a world of rational expectations? The answer is that since the initial investors are risk averse and uncertain about the future progress of the raid, they may be willing to sell at a price which is lower than the price at which the arbitrageurs will eventually buy. The difference is the raider's profit.

Although this setting is highly stylized, we do not feel that it is implausible. Perhaps the most restrictive aspect of the model is the assumption of two separate clienteles for the stock, one that holds the stock before it has "come into play" and one that holds the stock afterwards. In fact, in the mergers and acquisitions context, the clienteles that are active in a stock do tend to change after it has "come into play". In any case, these assumptions, along with many others, can be relaxed at the cost of somewhat greater analytical difficulty.

This, in brief, is the setting for the analysis. It can be described more formally as follows. There are three dates indexed by $t = 0,1,2$. There are two securities. One is a risk-free asset with a zero yield (*money*). The other is a risky *stock* with a return of $\tilde{R}$ at date 2. Money serves as the numeraire at each date.

There are three types of agents in the market. Initially, the stock is held by a large number of risk averse *investors*. The investors are assumed to be active only at the first two dates, that is, dates 0 and 1. Because there is a large number of investors, they all behave as price takers. There is a single *raider* who has no initial endowment of the stock. He is active in all three periods and seeks to maximize the expected utility of his wealth at date 2. Finally, there is a large number of *arbitrageurs*, who are active only at the last two dates, that is, dates 1 and 2. The arbitrageurs are risk neutral and seek to maximize their expected wealth at date 2. Although there is a large number of investors and arbitrageurs, for most purposes we shall write as if there were a single representative investor and a single representative arbitrageur.

As we shall soon see, the existence of a risk neutral arbitrageur greatly simplifies the analysis. It implies that security prices at date 1 are equal to expected returns.

All endowments of money are normalized to zero. The representative investor has an endowment of $E > 0$ units of the risky asset. The other agents have no endowment of the risky asset.

The representative investor and the raider maximize the expected utility of their final wealth. The investor's preferences are represented by a von Neumann-Morgenstern utility function V and the raider's preferences are represented by a von Neumann-Morgenstern utility function U. We assume that U and V are twice continuously differentiable, strictly increasing and strictly concave.

The stock has a positive return only at the final date. The return takes on two values, denoted by R_H and R_L, with $R_H > R_L > 0$. All agents attach the same prior probability distribution to stock returns. The prior probability of a high return is $0 < p_M < 1$.

We model the motivation for the raid in a particularly simple way. With probability α, the raider will receive further information about the value of the stock at date 1. If the raider receives information at date 1 he is called *informed*; otherwise, he is called *uninformed*. The raider's information takes the form of a posterior probability of a high stock return. The posterior probabilities at date 1 take on two values, denoted by p_H and p_L, with $0 < p_L < p_M < p_H < 1$. The high signal is received with probability π. If the raider is uninformed his posterior probability is equal to his prior p_M. Thus,

$$p_M = \pi p_H + (1 - \pi)p_L. \tag{1}$$

Although the information only arrives at date 1, the raider already knows at date 0 whether he is going to be informed at date 1. Thus, at the first date (date 0) there are two types of raider, the informed raider, distinguished by the subscript I and the uninformed raider (the "manipulator") distinguished by the subscript M. At the second date (date 1) there are three types of raider, the uninformed raider (M), the informed raider with the high signal (H) and the informed raider with the low signal (L).

The general pattern of trade in equilibrium is as follows. At the first date (date 0), the raider buys some of risky asset. He does this whether he knows that he will receive information at the second date (date 1) or whether he is simply manipulating the price. In other words, the two types "pool" at the first date. At the second date, the investor will liquidate any remaining holdings of the risky asset and consume his wealth. The raider would like to unload his holdings of the risky asset to the arbitrageurs and he will do so in some states. But in others, he will hold on to some of the asset in order to signal its quality. In other words, there is a signaling equilibrium at the second date. At the third date (date 2), asset positions are liquidated and agents consume their wealth.

3. SIGNALING EQUILIBRIUM AT THE SECOND DATE

It is convenient to describe the equilibrium backwards, beginning with the second date. The trading process at date 1 is represented by the following game. The raider moves first, choosing a quantity S of the stock which he offers to sell. The arbitrageurs then bid for the stock in Bertrand fashion. The arbitrageurs condition their beliefs on the quantity of stock the raider places on the market. Since the arbitrageurs are risk neutral, the price of the stock must equal its expected value. In other words, their "best response" to the raider's action is uniquely determined by their beliefs. This game is a signaling game of the kind discussed by Cho and Kreps (1987) and can be analyzed in the same way.

The game at the second date has many equilibria. We shall focus on a unique distinguished signaling equilibrium, which Cho-Kreps (1987) refer to as the Riley equilibrium. This is the equilibrium focussed on by Riley (1979). It is selected by a number of well known refinements of Nash equilibrium, for example, Universal Divinity (Banks and Sobel, 1988) and Strategic Stability (Kohlberg and Mertens, 1986). For a fuller discussion see Cho and Kreps (1987; pp. 208-214) or Cho and Sobel (1990).[6]

Note that in analyzing the game, we can ignore the behavior of the investors. They will unload the rest of their shares but cannot affect the price, since they have no information. The raider, on the other hand, is a "big" agent. The equilibrium price will be a function of his action.[7]

Equilibrium with Informed and Uninformed Raiders

The Riley equilibrium depends on the *support* of the distribution of types of raider. We begin by analyzing the case in which there are three types in the support, the informed high type (H), the informed low type (L) and the uninformed manipulator (M). The other cases are simple variants of this one.

Suppose that at date 0, the raider bought $B_0 > 0$ units of the stock at a price of $q_0 > 0$. These are the initial conditions for the game at date 1. They are treated as parameters in what follows.

The expected utility of a raider of type T = L,M,H is denoted by $u_T(x,m)$ and defined by putting

$$u_T(x,m) = p_T U(xR_H + m) + (1 - p_T)U(xR_L + m), \qquad (2)$$

where x is the quantity of the stock held and m is the amount money. Let $\mu_1(S)$ denote investors' beliefs concerning the probability of the high return when the raider sells S. Let $q_1(S)$ denote the price at which S units can be sold. Let S_H, S_M and S_L denote the quantities sold by types H, M and L, respectively. Then a *separating equilibrium* consists of an array $\left\{(S_L,S_M,S_H),\mu_1(\cdot),q_1(\cdot)\right\}$ satisfying the following conditions:

(i) $\mu_1(S_L) = p_L$, $\mu_1(S_M) = p_M$, and $\mu_1(S_H) = p_H$;

(ii) $q_1(S) = \mu_1(S)R_H + (1 - \mu_1(S))R_L$, $\forall S$;

(iii) S_T maximizes the raider's expected utility $u_T(B_0 - S, q_1(S)S - q_0B_0)$ for $T = L,M,H$.

The first condition requires that beliefs be consistent with the equilibrium strategies. Implicit in condition (i) is the requirement that the raider's choice of S reveals his type. The second condition requires that the equilibrium price be equal to the expected value; this follows from the risk neutrality of the arbitrageurs. The last condition requires that the raider choose his supply optimally, taking as given the best response of the market. Note that only pure strategies are allowed.

The *Riley equilibrium* is a special kind of separating equilibrium which satisfies the following conditions:

(i) S_L maximizes $u_L(B_0 - S, q_LS - q_0B_0)$;

(ii) S_M maximizes $u_M(B_0 - S, q_MS - q_0B_0)$

subject to $u_L(B_0 - S, q_MS - q_0B_0) \leq u_L(B_0 - S_L, q_LS_L - q_0B_0)$;

(iii) S_H maximizes $u_H(B_0 - S, q_HS - q_0B_0)$

subject to $u_M(B_0 - S, q_HS - q_0B_0) \leq u_M(B_0 - S_M, q_MS_M - q_0B_0)$

and $u_L(B_0 - S, q_HS - q_0B_0) \leq u_L(0, (q_L - q_0)B_0)$.

The low type chooses S_L to maximize his expected utility taking the fair price for his stock as given (condition (i)). The manipulator chooses S_M to maximize his expected utility, subject to the condition that the low type may not envy him and taking as given the fair price for his stock (condition (ii)). The high informed type chooses S_H to maximize his expected utility, subject to the condition that the manipulator and the low type may not envy him and taking as given the fair price for his stock (condition (iii)).

We distinguish the *equilibrium* (which includes a specification of prices and beliefs off the equilibrium path) from the equilibrium *outcome* (which only specifies the prices and quantities that will be observed in equilibrium). Under the maintained assumptions, it is straightforward to show that there is a unique Riley outcome for each choice of (B_0, q_0). (See Section B of the Appendix for details). Furthermore, the equilibrium outcome is characterized by the following conditions:

$$S_L = B_0; \tag{3}$$

$$u_L(B_0 - S_M, q_M S_M - q_0 B_0) = u_L(B_0 - S_L, q_L S_L - q_0 B_0); \tag{4}$$

$$u_M(B_0 - S_H, q_H S_H - q_0 B_0) = u_M(B_0 - S_M, q_M S_M - q_0 B_0); \tag{5}$$

Because the arbitrageurs are risk neutral, they are willing to buy the stock at its expected value. Since the raider is risk averse, he will want to sell as much as possible at this price. This means that the low informed type will sell his entire holding. The manipulator will choose to sell an amount that just makes the low type indifferent between selling S_L at a price q_L and selling S_M at a price q_M. Similarly, the high type will choose to sell an amount that just makes the manipulator indifferent between selling S_M at a price q_M and selling S_H at a price q_H. From the maintained assumptions, it also follows that $0 < S_H < S_M < S_L$.

In this model, the raider always reduces his holdings at date 1. This is true even when the raider receives the good signal. In some contexts, such as mergers and acquisitions, such rapid profit-taking may seem unrealistic. In this case, it occurs simply because the arbitrageurs are risk neutral. If they were risk averse, then the raider might increase his holdings of stock when he receives the good signal.

Equilibrium with an Informed Raider Only

An exactly analogous argument leads to the conclusion that there is a unique Riley outcome in the case where there is no uninformed raider. It is important to note that this is not the limiting case where α converges to unity — we are ruling out the *possibility* of an uninformed type here. As before, the low type sells his entire holding for its conditional expected value ($S'_L = B_0$). The high type sells as much as he can at a price equal to expected value, subject to the condition that he not attract the low type. The amount he sells (S'_H) is determined by the self-selection constraint

$$u_L(B_0 - S'_H, q_H S'_H - q_0 B_0) = u_L(0, (q_L - q_0)B_0). \tag{6}$$

Equilibrium with an Uninformed Raider Only

If the raider is known to be uninformed, there is no signaling and the raider sells all of his holding of the risky asset at the expected value. Note that the expected value of the risky asset is q_M for every value of S in this equilibrium.

4. POOLING EQUILIBRIUM AT THE FIRST DATE

In this section we start by characterizing pooling equilibrium at date 0 and then demonstrate the existence of such equilibria. In these equilibria, the uninformed raider is able to manipulate the stock price by imitating the behavior of the informed raider.

For each value of (B_0, q_0) and for each assumption about the support of the distribution of raider types, we have selected a unique equilibrium outcome at date 1. Using this mapping from first-period outcomes to second-period outcomes, we can express the expected utilities of the raider and investor as functions of the equilibrium outcome at date 0. These functions allow us to define a reduced form signaling game at date 0, which can be analyzed in a straightforward way.

The Reduced Form Game

Let μ_0 denote the beliefs concerning the raider's type at date 0. The *support* of agents' beliefs at date 1 is assumed to be determined by the common beliefs of the agents at date 0. If $\mu_0 = 1$, the raider is informed with probability one and the support of beliefs at date 1 is assumed to be $\{L,H\}$. This means that the stock price only takes on the values q_H and q_L at date 1. If $\mu_0 = 0$, the raider is uninformed with probability one and the support of beliefs at date 1 is $\{M\}$. The only price observed at date 1 is q_M. If $0 < \mu_0 < 1$, the support of beliefs at date 1 is $\{L,M,H\}$ and the prices q_L, q_M and q_H can be observed at date 1.

For each specification of B_0, q_0 and μ_0, there is a unique equilibrium outcome at date 1 and hence a unique expected utility for each agent. However, the distribution of prices observed at date 1 is determined by μ_0 alone. If $\tilde{q}_1$ denotes the (random) stock price at date 1 then

$$\tilde{q}_1 = \begin{cases} q_L \text{ w. pr. } (1 - \pi)\mu_0 \\ q_M \text{ w. pr. } (1 - \mu_0) \\ q_H \text{ w. pr. } \pi\mu_0 \end{cases} . \tag{7}$$

The investor chooses the quantity of stock to sell in order to maximize his expected utility, taking as given the stock price and his expectations about future prices.

Suppose that the representative investor chooses to sell S_0 units of the stock at date 0, at a price of q_0 per unit. Then the investor's expected utility is given by the formula

$$E\{V[(E - S_0)\tilde{q}_1 + q_0S_0]\} = (1 - \pi)\mu_0V[(E - S_0)q_L + q_0S_0] \tag{8}$$

$$+ (1 - \mu_0)V[(E - S_0)q_M + q_0S_0] + \pi\mu_0V[(E - S_0)q_H + q_0S_0].$$

Let $\mu_0(B)$ denote beliefs about the raider's type and let $q_0(B)$ denote the stock price when the amount of stock purchased is B. Then the investor's indirect demand function $q_0(B)$ is implicitly defined by the first-order condition:

$$E\{V'[(E - B)\tilde{q}_1 + q_0(B)B](q_0(B)B - \tilde{q}_1)|\mu_0 = \mu_0(B)\} = 0, \tag{9}$$

where the conditional distribution of $\tilde{q}_1$ is given by (7) above.

For any choice of (B_0,q_0,μ_0) at date 0 and any sale of stock S by the raider at date 1, let $q_1(S;B_0,q_0,\mu_0)$ denote the equilibrium price at date 1. Suppose the raider is informed. Then his expected utility at date 0, given he is choosing S optimally at date 1, can be written

$$u_I(B_0,q_0,\mu_0) = \pi \max_S u_H(B_0 - S,q_1(S;B_0,q_0,\mu_0)S - q_0B_0) \tag{10}$$

$$+ (1 - \pi) \max_S u_L(B_0 - S,q_1(S;B_0,q_0,\mu_0)S - q_0B_0).$$

Similarly, if the raider is uninformed at date 0, his expected utility is

$$u_M(B_0,q_0,\mu_0) = \max_S u_M(B_0 - S,q_1(S;B_0,q_0,\mu_0)S - q_0B_0). \tag{11}$$

The raider takes the indirect demand function $q_0(B)$ and the belief function $\mu_0(B)$ as given and chooses the quantity of stock to buy in order to maximize his expected utility. For example, if he is an informed raider, he chooses B to maximize $u_I(B,q_0(B),\mu_0(B))$. Implicit in the definition of the reduced form function u_I is the selection of an equilibrium outcome at date 1 corresponding to each choice of B, $q_0(B)$ and $\mu_0(B)$. In other words, the raider's choice of B uniquely determines everything else, given the functions

$q_0(\cdot)$ and $\mu_0(\cdot)$.

Using these reduced form functions, we can define a reduced form game at date 0 as follows. The raider moves first, choosing the quantity of stock B_0 that he wants to buy. Then the investors' demands determine the market-clearing price q_0 as a function of their beliefs and the raider's action. In effect, this is another signaling game where the investor's "best response" is uniquely determined as a function of his beliefs and the action of the raider.

A *pooling equilibrium* of this game is defined by an array $(B_0, \mu_0(\cdot), q_0(\cdot))$ satisfying the following conditions:

(i) B_0 maximizes both $u_I(B, q_0(B), \mu_0(B))$ and $u_M(B, q_0(B), \mu_0(B))$;

(ii) $q_0(\cdot)$ satisfies (9);

(iii) $\mu_0(B_0) = \alpha$.

Existence of Pooling Equilibrium at Date 0

To show that a pooling equilibrium exists at the first date, we proceed as follows.

First, choose $B_0 > 0$ arbitrarily. Put

$$\mu_0(B) = \begin{cases} \alpha \text{ if } B = B_0 \\ 0 \text{ if } B \neq B_0 \end{cases}. \qquad (12)$$

Then it is easy to see that $q_0(B)$ is uniquely defined by (9). Note that for any $B \neq B_0$, the raider is revealed to be a manipulator. Then at date 1, the equilibrium stock price will reflect this belief: $q_1(S;B,\mu_0(B),q_0(B)) = q_M$ for any S. Then (9) implies that $q_0(B) = q_M$ as well. Since the price is uniquely determined, independently of the raider's action, $u_I(B,\mu_0(B),q_0(B)) = u_M(B,\mu_0(B),q_0(B)) = U(0)$ for any $B \neq B_0$.

Now consider the equilibrium outcome at date 1 corresponding to the choice of $B = B_0$ at date 0. Since $0 < \mu_0(B_0) < 1$, the equilibrium stock price will be random at date 1. In fact, the support of the stock price distribution will be $\{q_L, q_M, q_H\}$. If $B_0 > 0$ is sufficiently small, the raider can be treated as approximately risk-neutral. Since, on average the raider receives the same revenue whether he sells his holdings of the stock at date 1 or date 2, the cost of signaling at date 1 is second-order. This means it is possible to focus on the profit that the raider makes from trading the asset which is of first-order magnitude. Since he is approximately risk neutral, whether informed or uninformed, the raider will make a profit from trade if and only if the price he pays at date 0 is less than the expected value of the stock at dates 1 and 2. Formally,

$$\frac{d}{dB_0} u_I(B_0, q_0(B_0), \mu_0(B_0))\Big|_{B_0=0} = \frac{d}{dB_0} u_M(B_0, q_0(B_0), \mu_0(B_0))\Big|_{B_0=0}$$

$$= (q_0(0) - q_M)U'(0) \qquad (13)$$

(See Section C of the Appendix). Thus, for B_0 sufficiently small, $u_I(B_0, q_0(B_0), \mu_0(B_0)) > U(0)$ and $u_M(B_0, q_0(B_0), \mu_0(B_0)) > U(0)$ if and only if $q_0(0) < q_M$. But this last condition must be satisfied since the investor is bearing positive risk if $E > B_0$. Thus, a pooling equilibrium has been shown to exist for every $B_0 > 0$ sufficiently small.

Stability of the Pooling Equilibrium

There is a continuum of pooling equilibria, one for each value of B_0 that is sufficiently small. This fact alone calls out for some attempt at refinement. There is an additional reason for questioning the stability of the pooling equilibria, however. In analyzing equilibrium at date 1, we argued that the Riley outcome was the unique plausible outcome on the basis of standard refinements. If separation is plausible at date 1, it behooves us to explain why it is not plausible at date 0.

The model we have analyzed is not a signaling game and so, strictly speaking, the standard refinements do not apply. However, the reduced form game that we have analyzed does look rather like a signaling game, so we can apply standard refinements, with some modification, to this "game". There is one big difference, however. In defining the reduced form "game", we have already built in a good deal of selection. In particular, we have assumed that the support of beliefs at date 0 determines the support of beliefs at date 1. Thus, "small" changes in beliefs at date 0 can lead to "big" changes in the equilibrium outcome at date 1.

The refinement we use is an adaptation of the Cho-Kreps Intuitive Criterion. Let $(B_0, q_0(\cdot), \mu_0(\cdot))$ be a fixed but arbitrary equilibrium of the reduced form "game". Let $u_M(B)$ and $u_I(B)$ denote the equilibrium expected utilities of the uninformed and the informed raider, respectively, when B is chosen. Let $u_M(B;\gamma)$ and $u_H(B;\gamma)$ denote the expected utilities if the equilibrium beliefs were replaced by $\hat{\mu}_0$, where $\hat{\mu}_0(B') = \mu_0(B')$ for $B' \neq B$ and $\hat{\mu}_0(B) = \gamma$. These expected utilities are uniquely defined under the assumptions that we have made about the selection of equilibrium at the second date.

The question is whether the raider can credibly signal his type and, if so, whether he would want to do so. We say that a pooling equilibrium *fails to satisfy the Intuitive Criterion* if, for some $B \neq B_0$ and some type T,

(i) $u_{\sim T}(B;\gamma) < u_{\sim T}(B_0)$ for any γ;

and

(ii) $u_T(B;T) > u_T(B_0)$,

where $u_T(B;T)$ denotes the expected utility of type T when he is known to be of type T and ~T denotes the other type.

If an uninformed raider can credibly signal his type, it is clear that he will never want to do so. In any equilibrium in which the raider is believed to be uninformed for sure, his expected utility will be U(0), i.e, less than he obtains in the pooling equilibrium. Thus, we only need to check that the criterion is satisfied for the informed type.

The informed type may benefit from revealing himself to be informed. But it is not clear that he can do so credibly. To show this we argue in two steps. First, we show that for any $B > 0$, if the investor believes that the choice of B reveals the raider to be informed, then both types of raider, the informed and the manipulator, will get the same expected utility from choosing B. Second, we show that in the pooling equilibrium, the informed raider's expected utility is greater than the uninformed raider's expected utility. Putting these two facts together, we see that any deviation which is (weakly) preferred by the informed raider must be strictly preferred by the manipulator, if the deviation leads the investors to believe that the deviation comes from the informed raider. Then condition (i) above cannot be satisfied for $T = I$ and the Intuitive Criterion will be satisfied.

To see the first result, suppose that the raider's choice of B is interpreted by investors as a signal that the raider is informed. The equilibrium at the second date will be a separating equilibrium in which the support of the distribution of types will be {L,H}. In this equilibrium, the low type will be indifferent between his type's outcome (q_L, S_L), say, and choosing the high type's outcome (q_H, S_H). The uninformed type will prefer the high type's outcome (q_H, S_H) to anything else he can obtain. Thus, the expected utilities of the informed and uninformed types at date 0 can be calculated on the assumption that they both choose (q_H, S_H) at date 1. Since both types have the same beliefs at date 0, they must have the same expected utility. In the notation introduced above $u_I(B;1) = u_M(B;1)$ for any B. (See

Section C of the Appendix).

The second result we need is that $u_I(B_0) > u_M(B_0)$. From the self-selection constraints used in the construction of the separating equilibrium with three types at date 1, we know that the low informed type is indifferent between his trade and that of the manipulator and that the high informed type strictly prefers his trade to that of the manipulator. Since their beliefs are the same at date 0, the equilibrium expected utility of the informed type is greater than the expected utility he would get from choosing (q_M, S_M) at date 1, which is equal to the manipulator's expected utility. In the notation introduced above, $u_I(B_0) > u_M(B_0)$. (See Section C of the Appendix).

This proves that any deviation preferred by the informed type will also be preferred by the manipulator, if the deviation is believed to come from the informed raider. In other words, *any pooling equilibrium at date 0 satisfies the Intuitive Criterion.*

Given that a continuum of equilibria satisfy the Intuitive Criterion, it is natural to ask whether there are any further arguments that can be used to choose between the equilibria in this particular context. Suppose the equilibrium value of B that gives the maximum utility to an informed raider from all possible pooling equilibria is B_{0I}, then investors might reason as follows. "The raider moves first. If he could determine the equilibrium and he was informed he would choose B_{0I} since this gives him the highest level of utility. The uninformed raider would also have an incentive to choose B_{0I} since he would not want to be identified as uninformed. Therefore it is sensible to believe that if the raider chooses B_{0I} he is informed with probability α." Of course this argument is not definitive, but it does suggest that the equilibrium where the raider chooses B_{0I} may be of particular interest.

The Possibility of Profitable Manipulation

It is important to notice in what sense the pooling equilibrium represents an example of successful stock price manipulation. The manipulator is a speculator in the traditional sense that he begins with a zero holding of the stock and eventually unwinds the total position (if we include trade at date 2). Furthermore, by pooling, the manipulator has an effect on the stock price that he could not have had if his type had been revealed at date 0. If his type had been revealed, the price profile would have been $q_M - q_M - \begin{Bmatrix} R_H \\ R_L \end{Bmatrix}$ rather than $q_0 - q_M - \begin{Bmatrix} R_H \\ R_L \end{Bmatrix}$. Finally, the manipulator makes a profit, in the sense that his expected utility at date 0 in the pooling equilibrium is higher than his expected utility in any equilibrium in which he reveals his type. However, it may not be the case that the manipulator makes money in every state of nature. If the stock return is low, he will be worse off than if he had not traded at all. In Section 7 we consider an alternative scenario in which the manipulator makes money in every state.

5. SEPARATING EQUILIBRIUM AT THE FIRST DATE

We have emphasized the pooling equilibria because it is only in a pooling equilibrium that stock price manipulation can occur. However, there can also be a separating equilibrium at date 0, although it does not seem to us a very plausible one.

The definition of separating equilibrium at date 0 is similar to the definition of pooling equilibrium; the only difference is that there are two quantities of the stock traded by the two types of raider. Call them B_I and B_M. The equilibrium conditions must be altered to

(i) B_I maximizes $u_I(B,q_0(B),\mu_0(B))$ and B_M maximizes $u_M(B,q_0(B),\mu_0(B))$;

(ii) $q_0(\cdot)$ satisfies (9);

(iii) $\mu_0(B_I) = 1$ and $\mu_0(B_M) = 0$.

Since $\mu_0(B_M) = 0$, $u_M(B,q_0(B),\mu_0(B)) = U(0)$ by a previous argument. By an earlier argument, $u_M(B,1) = u_I(B,1)$ so if $B = B_I$ then equilibrium requires that $u_I(B_I,q_0(B_I),\mu_0(B_I)) = U(0)$ too.

To see that a non-trivial separating equilibrium exists, it is enough to show that for some value of $B_I > 0$, $u_I(B_I,1) = U(0)$. This value of B_I can be supported as an equilibrium if investors' beliefs are given by $\mu_0(B) = 0$ for all $B \neq B_I$. To see that such a value of B_I does exist, note that by an earlier argument, $u_I(B,1) > U(0)$ for B sufficiently small (see Section C of the Appendix). For B = E, $q_0(B) = q_M$ since the investor is risk neutral at the margin when his holding of the stock is zero. If the raider purchases any of the stock at a price equal to its expected value, he must be made worse off, that is, $u_I(E,1) < U(0)$. Then for some intermediate value, $0 < B_I < E$, we have $u_I(B_I,1) = U(0)$.

Although a separating equilibrium exists, it is not a very satisfactory one. There are two unattractive aspects to this equilibrium. The first is that it is not a very good equilibrium from the point of view of the raider. Although there may exist extreme pooling equilibria in which one or more type receives U(0), in almost every pooling equilibrium all types of raider receive strictly more than U(0). Thus, if the raider's preferences have any role in the selection of an equilibrium, the separating equilibrium will not be chosen. Second, the only beliefs that will support this equilibrium are extreme ones. In the analysis of the pooling equilibrium at date 0, it was assumed in (12) that $\mu_0(B) = \alpha$ if $B = B_0$ and 0 otherwise. This assumption was for analytical convenience; there are many other sets of beliefs which involve $\mu_0(B) > 0$ for a range of B which would support a pooling equilibrium. However, the *only* way to support the separating equilibrium is to assign the beliefs $\mu_0(B) = 0$ to all values of $B \neq B_I$. Such extreme beliefs seem rather arbitrary.

The separating equilibrium satisfies the Intuitive Criterion, since any deviation by the uninformed manipulator is unprofitable and any deviation by the informed raider is bound to attract the uninformed raider for some beliefs. However, by an earlier argument, for any $0 < \gamma < 1$, $u_I(B,\gamma) > u_M(B,\gamma)$ and for $\gamma = 0,1$, $u_I(B,\gamma) = u_M(B,\gamma)$. Thus, for any $B \neq B_I, B_M$ and any beliefs, the informed raider has at least as strong an incentive to defect as the manipulator and typically a stronger incentive. One could argue, therefore, that reasonable beliefs should satisfy $\mu_0(B) = 1$ for $B \neq B_I, B_M$, in the spirit of Universal Divinity. Such beliefs are inconsistent with the existence of a separating equilibrium.

6. AN EXAMPLE

In this section we present an example to illustrate some of the properties of the model presented above.

In the example the investor and raider both have exponential utility,

$$U = -\exp(-C), \tag{14}$$

where C is consumption at date 1 for the investor and consumption at date 2 for the raider. The other values of the parameters are $E = 10$, $R_H = 2$, $R_L = 0$, $\alpha = 0.8$, $\pi = 0.5$, $p_H = 0.6$, and $p_L = 0.4$ so that $p_M = 0.5$, $q_H = 1.2$, $q_M = 1.0$ and $q_L = 0.8$. A date 0 pooling equilibrium exists for values of B_0 between 0 and 3.31.

As argued above, a natural date 0 pooling equilibrium to consider is the one where the informed raider's utility u_I is maximized. The value of B_0

that maximizes u_I is $B_{0I} = 1.46$. The amounts of the risky asset that are sold in the date 1 signaling equilibrium by the manipulator and the informed raider if he receives the high signal are 0.831 and 0.628, respectively; the informed raider that receives the low signal sells his entire holding, of course. The price at time 0 is 0.827. This compares to an expected price of 1.000 at date 1. The expected utility of the informed raider is - 0.925 and the expected utility of the manipulator is - 0.937 compared to a level of -1.000 if they did nothing (i.e. $U(0) = -1.000$).

The possible price paths are illustrated by the solid lines in Figure 1. At date 0 the price is 0.827. At date 1 it rises to 1.2 if the raider is informed and receives the high signal and to 1.0 if the raider is uninformed but it falls to 0.8 if the raider is informed and receives the low signal.

The value of B_0 corresponding to the equilibrium where u_M is maximized is 1.41. At first sight this is perhaps surprising, since between dates 0 and 1 the manipulator bears no risk; he knows that the price at date 1 will be 1.00. However, in order to signal that the price should be $q_M = 1.00$ rather than $q_L = 0.80$ at date 1, he can only sell 0.795 of the risky security. This means he must bear considerable risk between dates 1 and 2. The informed raider on the other hand can sell everything when he receives the low signal and must only bear risk between dates 1 and 2 when he receives the high signal, even though in the latter case he must bear more risk than the manipulator.

Another case of interest is where the investor assigns a zero probability to the raider being uninformed. Note that this case is not the same as the limit when $\alpha \to 1$ since now the relevant constraint determining S_H is (6) rather than (5). In the pooling equilibrium at date 0 with $B_0 = 1.46$ the first difference is that $q_0 = 0.813$; it is lower since information is always acquired so the risk borne by investors is greater. The other main difference is that $S_H = 0.651$ since the raider now has to separate himself from the person with a low signal rather than the manipulator. The informed raider's level of expected utility at date 0 is now - 0.899. The path of prices is illustrated by the long dotted lines in Figure 1. At date 1 it goes to 1.2 if the raider gets the high signal and 0.8 if he gets the low signal.

The equilibrium with $B_0 = 1.46$ is, of course, no longer the one where u_I is maximized. This now occurs at $B_0 = 1.98$. Now $q_0 = 0.815$ which is higher than when $B_0 = 1.46$ since the raider is bearing more risk between date 0 and date 1 and the investors are bearing less. The value of S_H at date 1 is 0.964 and finally the raider's level of utility is - 0.894.

The final case of interest is where the raider is known to be uninformed with probability 1. Here the raider's actions have no effect and the price is 1.0 as illustrated by the short dotted line in Figure 1.

7. AN ALTERNATIVE FORMULATION

As discussed at the end of Section 4, the uninformed raider is better off on average from manipulating the price; ex post he will be better off when the return on the stock is R_H but worse off when it is R_L. By changing the assumptions of the model, it is possible to give examples where the uninformed raider is better off in every state from manipulating the price. Instead of assuming that the return to the risky asset at date 2 is always R_H or R_L, irrespective of whether the raider is informed or uninformed, it is now assumed that the return to the risky asset is R_H or R_L if the raider is informed but R_M with probability 1 if the raider is uninformed. The cost of this change in assumptions is that the model is no longer as tractable; a general analysis is very complex and it is only practicable to give numerical examples.

In this version of the model, an equilibrium can exist at date 1 where the uninformed raider pools with the informed raider that receives a high signal and both separate from the informed raider that receives a low signal. In contrast to the previous model where only the Riley signaling equilibrium at date 1 is stable, this partial pooling equilibrium is the unique stable equilibrium. Any deviation which is weakly preferred by the high informed type will be strictly preferred by at least one of the other types.

An equilibrium where the manipulator and the informed raider with a high signal pool exists in an example similar to that in Section 6 except that $R_M = 0.8$. In this case, the informed raider's utility is maximized at $B_{0I} = 1.26$. The path of prices in this case is illustrated in Figure 2. The price at date 0 is 0.834. If the raider is uninformed or receives the high signal at date 1 the price raises to 1.067; if the raider receives the low signal the price falls to 0.8. It can be seen that even though the final price when there is manipulation is below the initial price (i.e. $R_M = 0.8 < q_0 = 0.834$) the uninformed raider is always better off manipulating the price than not manipulating it. In the latter case the price would be equal to $R_M = 0.8$ at all three dates and his profit would always be zero. However, when he manipulates he makes a profit for sure. He buys $B_0 = 1.26$ at date 0 at a price of $q_0 = 0.834$; the price then rises to $q_1 = 1.067$ since the manipulator pools with the informed type by selling 0.628 of the security at date 1. At date 2 the manipulator sells the remaining 1.26 - 0.628 = 0.632 at $R_M = 0.8$. The net profit is therefore $- 1.26 \times 0.834 + 0.628 \times 1.067 + 0.632 \times 0.8 = 0.125$ with probability 1.

In the particular case analyzed it is possible to show that the equilibrium of the date 1 subgame and the equilibrium of the entire game are stable in the sense used in the previous sections. However, in general it is not possible to do this and it is for this reason that only a numerical example is given in this section.

8. CONCLUDING REMARKS

A number of authors have argued that stock price manipulation was an important phenomenon in U. S. stock markets up until the 1930's. Concern about the harmful effects of manipulation led to the passage of the Securities Exchange Act of 1934. This made a number of practices that facilitated manipulation, such as short selling by managers and the announcement of false information, illegal. There is some evidence that these restrictions have largely eliminated action-based and information-based manipulation. Stock price manipulation has therefore been considered to be a phenomenon which is mainly of historical interest.

The results presented above, show that trade-based stock price manipulation, which was not made illegal by the Securities Exchange Act, is consistent with rational utility maximizing behavior. In the particular model considered, the raider finds out whether or not he will receive information about a stock. If he finds out he will not receive information, he is still able to make a profit by manipulating the stock price to his advantage provided investors assign some probability to him being informed. Thus manipulation is possible without actions to alter the true value of the firm or the release of false information. To the extent these are possible they will increase investors' beliefs the raider is informed and make manipulation more profitable.

One interpretation of the model, discussed in Section 2, is in the context of mergers and acquisitions. Another example is when an analyst or firm (such as Valueline) is known to be able to acquire information about whether firms are over- or under-valued. In general, whenever there is a possibility for acquiring private information profitable manipulation may be possible.

Thus, although the Securities Exchange Act of 1934 may have eliminated action-based and trade-based manipulation schemes, it did nothing to limit trade-based manipulation schemes. The importance of such schemes is of course an empirical question. However, casual observation suggests they might be important. For example, the trading activities of mergers and acquisitions specialists in the stock of firms that they do not subsequently acquire is substantial. Some part of the profits from this trade may be the result of manipulation of the type discussed in this paper.

In addition to the empirical importance of trade-based manipulation, another important issue which remains to be analyzed is whether such manipulation is undesirable or not. Although a number of authors regard all types of manipulation as undesirable,[8] this was not the position that lay behind the framing of the Securities Exchange Act of 1934. This did not rule out all types of manipulation per se, since this would have prevented the type of "price pegging" sometimes done by underwriting syndicates when new securities are issued (see Twentieth Century Fund (1935) pp. 499-502 for a discussion of the desirability of this type of manipulation). It thus remains an open question whether or not the type of trade-based manipulation considered is undesirable.

APPENDIX

In this Appendix, we prove some elementary facts about the equilibrium discussed in the text.

A. *Monotonicity*

We start by showing that the preferences of the different raider types satisfy the usual monotonicity condition (see, e.g., Cho and Kreps (1987)). At date 1, the raider has preferences on (x,m) pairs, where x denotes the holding of stock and m the holding of money. If the probability of the high return is p then expected utility is $pU(xR_H + m) + (1 - p)U(xR_L + m)$. Taking a total differential of expected utility gives

$$dU = pU'(xR_H + m)(R_H dx + dm) + (1 - p)U'(xR_L + m)(R_L dx + dm).$$

Consider the effect of reducing x and increasing m, i.e, put $dx < 0$ and $dm > 0$. Suppose that, for some p, the differential is equal to zero. The first term must be negative and the second positive. Then an increase in p must make the differential negative. Thus the indifference curves of the different types of raider are strictly ordered by steepness in (x,m) - space. More precisely, for any (x,m) and (x',m') with $m < m'$ and $x > x'$,

$u_L(x,m) = u_L(x',m')$ implies $u_T(x,m) > u_T(x',m')$ for T = M,H;

$u_M(x,m) = u_M(x',m')$ implies $u_H(x,m) > u_H(x',m')$ and $u_L(x,m) < u_L(x',m')$;

$u_H(x,m) = u_H(x',m')$ implies $u_T(x,m) < u_T(x',m')$ for T = L,M.

B. *Uniqueness of the Riley Equilibrium Outcome*

We start by considering the equilibrium with three-type support. Suppose the initial conditions at date 1 are given by (B_0, q_0, μ_0) where $B_0 >$

0, $q_0 > 0$ and $0 < \mu_0 < 1$. The Riley outcome is uniquely determined as follows. The low informed type (L) chooses S_L to maximize $u_L(B_0 - S, q_L S - q_0 B_0)$. Since q_L equals the low type's expected value of the stock, there is a unique solution to this problem: $S_L = B_0$.

The manipulator chooses S_M to maximize $u_M(B_0 - S, q_M S - q_0 B_0)$ subject to the self-selection constraint

$$u_L(B_0 - S, q_M S - q_0 B_0) \leq u_L(B_0 - S_L, q_L S_L - q_0 B_0).$$

The unconstrained maximum again requires $S_M = B_0$, which violates the self-selection constraint. Since $u_M(\cdot)$ is strictly concave in S, the self-selection constraint must hold as an equation. The self-selection constraint is satisfied as an inequality at S = 0 so there must exist $0 < S_M < B_0$ that satisfies the constraint as an equation. It follows from the monotonicity property on indifference curves that this is the unique value of S_M that satisfies the constrained maximization problem. That is, if S and S′ both satisfy the constraint above, then monotonicity implies that the manipulator will prefer the smaller value.

The high type chooses S_H to maximize $u_H(B_0 - S, q_H S - q_0 B_0)$ subject to the self-selection constraints

$$u_T(B_0 - S, q_H S - q_0 B_0) \leq u_T(B_0 - S_T, q_T S_T - q_0 B_0) \text{ for } T = L, M.$$

Again, the unconstrained optimum requires $S = B_0$, which violates the self-selection constraints, so at least one constraint is binding. Both self-selection constraints are satisfied as inequalities at S = 0, so there must exist $0 < S_H < B_0$ that satisfies both constraints with one of them holding as an equation. From the monotonicity conditions derived earlier, the binding constraint corresponds to T = M. Thus,

$$u_M(B_0 - S_H, q_H S_H - q_0 B_0) = u_M(B_0 - S_M, q_M S_M - q_0 B_0).$$

There are at most two values of S_H satisfying this equation and monotonicity implies the high informed raider will prefer the smaller one. In fact, monotonicity and the fact that

$$u_H(B_0 - S_H, q_H S_H - q_0 B_0) \geq u_H(B_0 - S_M, q_M S_M - q_0 B_0)$$

shows that $S_H < S_M$. Thus, there is a unique value of S_H that solves the constrained maximization problem.

To sum up, in the case with three-type support we have found a unique set of values $0 < S_H < S_M < S_L = B_0$ determining the Riley outcome and satisfying

$$u_M(B_0 - S_H, q_H S_H - q_0 B_0) = u_M(B_0 - S_M, q_M S_M - q_0 B_0).$$

and

$$u_L(B_0 - S_M, q_M S_M - q_0 B_0) = u_L(B_0, q_L S_L - q_0 B_0).$$

Next consider the equilibria with two-type support. The case where $B_0 > 0$, $q_0 > 0$ and $\mu_0 = 1$ is solved in an exactly similar way. We uniquely derive the values $0 < S'_H < S'_L = B_0$ that determine the Riley outcome and satisfy

$$u_L(B_0 - S'_H, q_H S'_H - q_0 B_0) = u_L(B_0, q_L S'_L - q_0 B_0).$$

In the case where $\mu_0 = 0$, the amount of stock traded by the manipulator at date 1 is $S''_M = B_0$.

Note that in each case, the amounts of stock traded depend on the initial conditions (B_0, q_0, μ_0).

C. The Reduced Form Game

The first fact that we derive is used in the proof of existence of the pooling equilibrium at date 0. It is shown that, for some B_0 sufficiently small, both types can make a positive profit in the pooling equilibrium of the reduced form game at the first date. Consider first the informed raider. Suppose that $\mu_0(B_0) = \alpha$, for all B_0 sufficiently, small and that $q_0(B_0)$ is derived using these beliefs. Suppressing the reference to $\mu_0(B_0)$, the informed raider's expected utility can be written

$$u_I(B_0,q_0(B_0)) = \pi\Big\{p_H U\Big((B_0 - S(B_0))R_H + q_H S(B_0) - q_0(B_0)B_0\Big)$$

$$+ (1 - p_H)U\Big((B_0 - S(B_0))R_L + q_H S(B_0) - q_0(B_0)B_0\Big)\Big\}$$

$$+ (1 - \pi)U(q_L - q_0(B_0)B_0),$$

where $S(B_0)$ is the amount sold by the high informed type at date 1. The derivative of the informed's expected utility, evaluated at $B_0 = 0$, is

$$\frac{d}{dB_0}u_I(B_0,q_0(B_0))\Big|_{B_0=0} = \pi\Big\{p_H U'(0)\Big((1 - S'(0))R_H + q_H S'(0) - q_0(0)\Big)$$

$$+ (1 - p_H)U'(0)\Big((1 - S'(0))R_L + q_H S'(0) - q_0(0)\Big)\Big\}$$

$$+ (1 - \pi)U'(0)\Big(q_L - q_0(0)\Big).$$

Using the facts that $p_T R_H + (1 - p_T)R_L = q_T$ and $\pi q_H + (1 - \pi)q_L = q_M$ we can rewrite the right-hand side as

$$\pi U'(0)\Big((1 - S'(0))q_H + q_H S'(0) - q_0(0)\Big) + (1 - \pi)U'(0)\Big(q_L - q_0(0)\Big)$$

$$= U'(0)(q_M - q_0(0)),$$

as required. The calculation for $T = M$ is similar.

The remaining results are used in the analysis of stability of the pooling equilibrium at date 0. Having solved for the equilibrium outcome at date 1 for each set of initial conditions, we can associate an expected utility for each type uniquely to each set of initial conditions, as shown in Section 4. Two relations among these expected utilities are important in the analysis of equilibrium at date 0. The first says that the informed raider is always better off than the uninformed raider at date 0 when $0 < \mu_0 < 1$. Let $(B_0,q_0(\cdot),\mu_0(\cdot))$ be the pooling equilibrium, so that 0

$< \mu_0(B_0) < 1$. Let (S_L, S_M, S_H) denote the equilibrium quantities traded by the raider at the second date. Then the expected utility of the informed type at date 0 is

$$\pi u_H(B_0 - S_H, q_H S_H - q_0(B_0)B_0)$$

$$+ (1 - \pi)u_L(B_0 - S_L, q_L S_L - q_0(B_0)B_0)$$

$$> \pi u_H(B_0 - S_M, q_M S_M - q_0(B_0)B_0) + (1 - \pi)u_L(B_0 - S_M, q_M S_M - q_0(B_0)B_0)$$

from monotonicity and the self-selection constraints. Since $p_M = \pi p_H + (1 - \pi)p_L$, this last expression can be rewritten as

$$p_M U((B_0 - S_M)R_H + q_M S_M - q_0(B_0)B_0) +$$

$$(1 - p_M)U((B_0 - S_M)R_L + q_M S_M - q_0(B_0)B_0),$$

which is precisely the expected utility of the manipulator. Thus, whenever the two types pool at date 0 and expect a separating equilibrium with three types at date 1, the informed type must be better off than the uninformed.

By a similar argument, we can show that when the equilibrium expected at date 1 has only two types (H and L), the two types at date 0 (I and M) have the same expected utility if they pool at B. Let $(B_0, q_0(\cdot), \mu_0(\cdot))$ be an equilibrium and suppose that $\mu_0(B) = 1$. The expected utility of the high type is

$$\pi u_H(B - S_H, q_H S_H - q_0(B)B) + (1 - \pi)u_L(B - S_L, q_L S_L - q_0(B)B)$$

$$= \pi u_H(B - S_H, q_H S_H - q_0(B)B) + (1 - \pi)u_L(B - S_H, q_H S_H - q_0(B)B)$$

$$= p_M U((B_0 - S_M)R_H + q_M S_M - q_L B_0) + (1 - p_M)U((B_0 - S_M)R_L + q_M S_M - q_L B_0)$$

from the self-selection constraints and the fact that $p_M = \pi p_H + (1 - \pi)p_L$. If the manipulator finds himself in the second-period equilibrium corresponding to two types, his most preferred choice must be (q_H, S_H). This follows from the monotonicity properties, since if he weakly preferred any pair (q,S) to (q_H,S_H) one of the other types would strictly prefer it, contradicting the equilibrium conditions. Thus, the expected utility of the manipulator, if he pools with the informed type at date 0 and expects a two type equilibrium at date 1 must be

$$p_M U((B_0 - S_M)R_H + q_M S_M - q_L B_0) + (1 - p_M)U((B_0 - S_M)R_L + q_M S_M - q_L B_0)$$

as required.

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Notes

[1]For accounts of the history of stock price manipulation see Sobel (1965) and Twentieth Century Fund (1935).

[2]Quoted by Sobel (1965; p. 60).

[3]See Eiteman, Dice and Eiteman (1965) p. 562 for an account of this episode.

[4]See Wycoff (1968) pp. 77-78 for an account of this incident.

[5]See, for example, Securities and Exchange Commission (1959).

[6] Note that we are applying these refinements to a subgame so our refinement of the entire game will be to the extensive form, rather than the normal form as advocated by Kohlberg and Mertens (1986).

[7]Note that largeness need not be literally interpreted as meaning that the raider has positive measure; another possibility is that the raider is recognized as possibly having information so that his actions have a non-negligible effect; cf. Gale and Hellwig (1987).

[8]For example, in his discussion of the Securities Exchange Act, Flynn (1934; p. 284) states "It is, of course, absolutely essential that all forms of manipulation shall be eliminated from the securities markets."

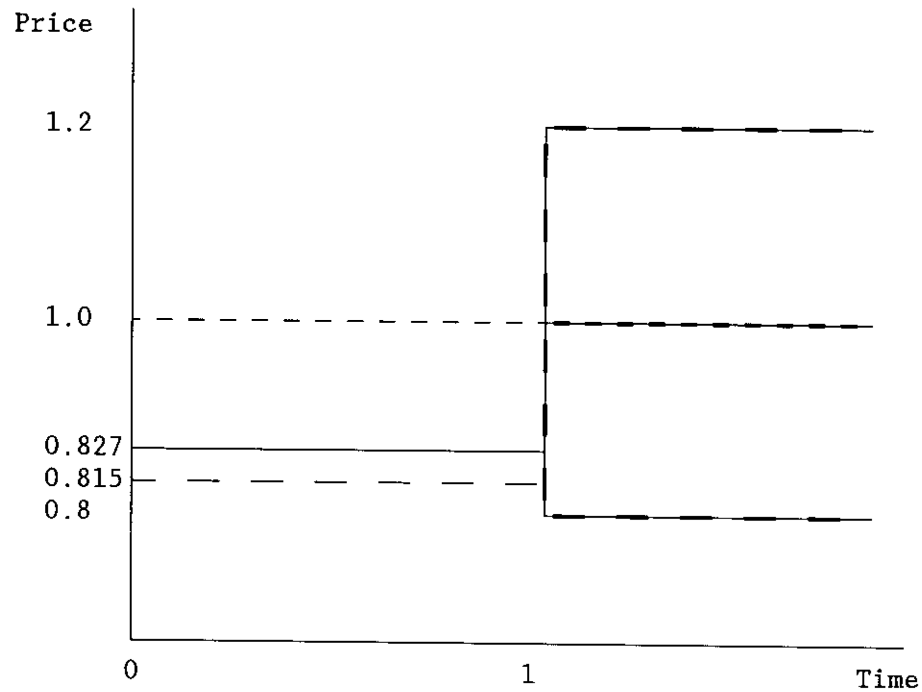

Figure 1

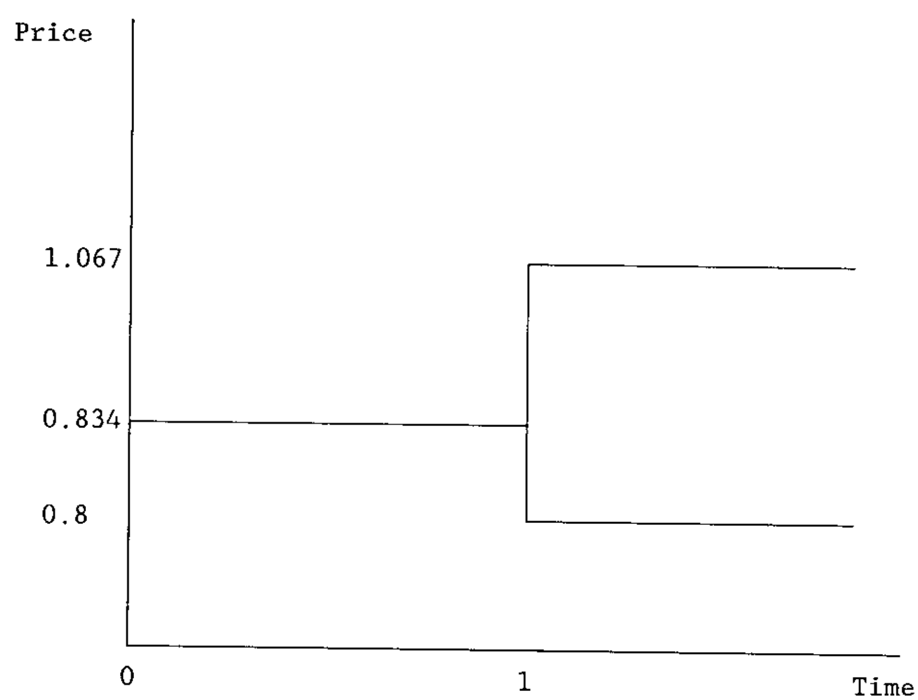

Figure 2