SECURITY PRICES AND MARKET TRANSPARENCY

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Abstract

This paper analyzes a market where investors observe the intermediate stages of price formation and can revise their orders as prices are determined. A trading mechanism that exhibits this property is said to be *transparent.* The issue of market transparency arises in many current policy issues such as the timing and quality of trade reporting, sunshine trading, and the effectiveness of publicizing order imbalances to reduce price volatility.

We first examine a non-transparent where traders submit written orders before market clearing. We then contrast this system with a transparent market mechanism where traders can revise their orders during the price formation process. This system is shown to have the same equilibrium as a system where information on shocks to order flow is directly disclosed to market participants. Throughout, trading is modeled as a game between strategic traders with rational expectations. Contrary to popular intuition, transparency can increase price variability and lower liquidity.

1 Introduction

The relationship between trading arrangements and security prices has been the subject of great interest following the events of October, 1987. A major concern is the ability of market mechanisms to accommodate substantial variation in order flow without increased price volatility. After the crash, several investigative commissions proposed trading halts following large price movements ('circuit breakers') to reduce instability. A trading halt, it is argued, would allow information on transitory order imbalances to be publicized, providing opportunities for speculative traders to place stabilizing orders. Similar logic applies to 'sunshine trading,' where traders can pre-disclose their trading intentions with a view to reducing the price impact of a large order.

Underlying these policy recommendations is the belief that price variability would be reduced by making the market more *transparent* to investors. In a transparent market traders can observe the process of price formation and revise their orders while prices are being determined.[1] This paper investigates the impact of disclosing market information during the process of price formation on security prices and market quality. Stock price volatility provides an obvious source of motivation for a study of the effects of information disclosure, but the issue arises in many other contexts, including the timing and quality of trade reporting, inter-market communications, and access to information about public limit orders. To focus attention on transparency, we consider two mechanisms at opposite ends of the spectrum in terms of the provision of market information to traders. The first mechanism operates as a batch market where traders submit written orders for simultaneous execution at a single price. This system is not transparent because traders cannot observe how the equilibrium price is determined. The second mechanism is transparent because traders are

[1] A verbal call auction exhibits transparency because traders observe the intermediate stages of price formation. By contrast, a system where orders are submitted in writing before clearing is non-transparent since traders cannot observe how the equilibrium price is determined.

permitted to revise their orders as prices are determined. The transparent mechanism we analyze functions as a call auction, but the model readily extends to the analysis of other transparent market mechanisms. In particular, we analyze a mechanism where information on shocks to order flow is directly disclosed to market participants, as in sunshine trading or a circuit breaker activated by large order imbalances.

Throughout the paper, trading is modeled as a imperfect information game between strategic traders with rational expectations. We demonstrate that a transparent system can exacerbate the price volatility generated by temporary order imbalances. Market quality can also suffer with lower liquidity and higher implicit transaction costs; in an extreme case, transparency can induce market failure. This result appears counter-intuitive at first glance. In a transparent system, speculative traders will absorb a portion of any order imbalance, tending to reduce the magnitude of temporary price swings. However, traders' strategic behavior can offset this effect. Transparency allows traders condition on both price and quantity. We show that a marginal increase in a trader's order quantity will lead to an increase in the demand of all other traders. As a result, traders tend to scale back their orders to minimize their price impact. If the market is sufficiently thin, the reduction in order size is associated with an increase in the absolute deviation between the transaction price and the full information value of the security. This increases unconditional price variability and reduces liquidity. Intuitively, a mechanism that permits traders to condition on price and volume can generate a different equilibrium allocation than one where traders can condition only on price.

The paper contributes to the growing literature on market microstructure. Recent empirical research suggests that market microstructure can have important effects on the properties of asset prices. Amihud and Mendelson (1987) find significant differences in the distribution of NYSE stock returns from open-to-open and close-to-close, which they attribute to differences between trading arrangements at the opening and closing. Stoll and Whaley (1990) confirm these results, concluding that NYSE opening practices affect stock price volatility.[2] Kamara (1988) argues that price disparities between treasury-bill futures and spot markets can be explained by the differences in trading mechanisms in the two markets.

Recent theoretical papers also suggest a link between trading structures and asset prices. In the models of Leach and Madhavan (1989) and Admati and Pfleiderer (1989), trading patterns arise endogenously from market makers' attempts to learn the private information of insiders. Glosten (1989) contrasts a monopolistic and competitive dealer system, and shows a monopolist can open the market in situations where competitive dealers cannot.[3] Finally, studies by Ho, Schwartz, and Whitcomb (1985), Easley and O'Hara (1988), and Rock (1990) show that the kinds of orders permitted and the way they are processed affect the character of equilibrium. This paper adds to this literature by demonstrating that seemingly subtle differences in the information made available to investors can have substantial effects on asset prices.

The paper is organized as follows: Section 2 develops the theoretical framework is described and in Section 3, we analyze a non-transparent trading mechanism. Section 4 describes a transparent market mechanism based on a call auction with interactive order submission. Trading is modeled as an imperfect information game between strategic players. Section 5 compares the two equilibria, and demonstrates conditions under which transparency affects market performance. We apply the model to analyze sunshine trading, where traders can pre-disclose their trades. Contrary to popular belief, transparency can increase price variability and decrease market liquidity. Finally, Section 6 concludes the paper.

[2] By contrast, Roll (1988) finds differences in trading structures across nations had little or no impact on the size of the price decline on October 19, 1987. However, it is unclear whether this finding is specific to the day of the crash.

[3] Kyle (1985), Admati and Pfleiderer (1988), and Grossman and Miller (1988) provide dynamic models with strategic informed traders.

2 The Analytical Framework

Consider a two-period model where investors trade claims to a single risky asset. We will discuss two trading mechanisms that differ in the amount of market information provided to traders at the time of order submission. The exact details of the two trading protocols will be explained after we establish the basic framework and notation. The value of the risky asset in period 1 is represented by a random variable, $\tilde{v}$.[4] The stochastic payoff, $\tilde{v}$, can be thought of either as a liquidating dividend or the full information price following a public announcement. In this market, there are N traders (indexed by $i = 1, \ldots, N$), each of whom receives a private information signal concerning the payoff of the risky asset. Trader i 's information signal, $\tilde{y}_i$, is a random variable:

$$\tilde{y}_i = v + \tilde{\epsilon}_i$$

where v is the realized value of the risky asset and $\tilde{\epsilon}_i$ is normally distributed with mean 0 and precision $\rho > 0$. The realization of the information signal $\tilde{y}_i$ is denoted y_i. Assuming all traders have diffuse prior beliefs, trader i's prior distribution of $\tilde{v}$ is normal with mean y_i and precision ρ.[5] Trader i enters the market with an initial endowment of $x_i \in \Re$ of the risky asset and initial wealth W_{0i}. Traders can be individual investors or broker-dealers, and in the latter case, x_i is interpreted as initial inventory. Endowments of the risky asset are independently normally distributed across traders with mean 0 and precision $\psi > 0$.[6] The private information of trader i is the pair (x_i, y_i). Let Φ_i represent the information set of trader i given her private signal.[7]

[4] All random variables are defined on a common probability space. The technical details, although straightforward, are omitted for simplicity.

[5] The assumption of independent signals is mathematically equivalent to specifying a diffuse prior distribution for traders. This assumption can be relaxed to allow correlation among conditional expectations at the cost of considerable complexity.

[6] The supply of the risky asset is "widely" distributed, so that the initial endowment gives no information about the endowments of other traders.

[7] Formally, this can be thought of as the σ-algebra generated by (y_i, x_i).

Traders maximize the expected utility of final period wealth, W_{1i}. We assume traders have constant absolute risk aversion β. Let q_i represent the order quantity of trader i, with the convention that security purchases are represented by positive numbers and trader sales by negative numbers.[8] Information is not the sole motivation for trade in this model. Since traders are risk averse and enter the market with non-zero endowments, a portion of transaction volume arises from portfolio hedging. As a result, liquidity trading arises endogenously.

In addition to portfolio hedging, volume may contain other components uncorrelated with the fundamental value of the asset. Let $\tilde{Z}$ represent the stochastic shock to the supply of the risky asset.[9] The noise shock is a convenient summary statistic for extrinsic uncertainty, and can be interpreted in several different ways. The most obvious interpretation is that $\tilde{Z}$ represents the aggregate trades of uninformed traders who trade for exogenous liquidity or life-cycle reasons. We can easily extend our results to allow the mean of $\tilde{Z}$ to be a linear function of price. In this case, $\tilde{Z}$ can be thought of as the consolidation of limit orders, perhaps originating from rational traders who are unaware that an information event has occurred. An alternative interpretation, which we will pursue later on, is that the shock originates from a single liquidity-motivated trader. In this case, prior disclosure of the size of Z corresponds to *sunshine trading.* We assume that $\tilde{Z}$ is normally distributed with zero mean and variance $\sigma_z^2 \geq 0$.

This paper follows the approach of Kyle (1989) to model trading with strategic rational agents, but makes liquidity trading partly endogenous. Kyle (1989) demonstrates the existence of a rational expectations equilibrium under imperfect competition, and compares this to the case of perfect competition. Pagano (1989) provides a similar model to explain the

[8]The market has no imperfections in that there are no short sale restrictions, no taxes, and no transaction costs. However, traders are not 'schizophrenic' because they recognize their influence on price.

[9]The realized order imbalance is denoted Z, with the convention that $Z > 0$ represents excess demand and $Z < 0$ represents excess supply of the risky asset. We will analyze the case where there is no exogenous noise.

variation in volume across markets. The particular mechanism used to select the equilibrium price is not described in these models, but their equilibria resemble the equilibrium for the written order entry system we analyze in the next section. The equilibrium for the transparent mechanism, however, is very different. This difference arises because of traders can condition their beliefs on both prices and volume. We turn now to the specifics of market organization.

3 Market Clearing Without Transparency

Consider a trading mechanism where traders submit written orders that are accumulated for simultaneous execution at a single market clearing price. Real-world examples include batch markets for inactive stocks in some European stock exchanges, as well as the process of competitive bidding for U.S. Treasury bills.[10] This type of system is non-transparent because traders cannot observe the equilibrium price and volume until after the market has cleared. Trading is modeled as a game characterized by the number of players, their reward functions, information signals, endowments, and beliefs. Let W_{0i} denote the cash holdings (initial wealth) of trader i. The private information of trader i is represented by $I_i = (W_{0i}, x_i, y_i) \in \Re^3$ for $i = 1, \ldots, N$.[11] The trader's (pure) strategy is a mapping $q_i : \Re \times \Re^3 \rightarrow \Re$ is the demand function for trader i, mapping price and the initial state into desired order quantity.[12] The trading mechanism is described as a game $\Gamma = (N, u(\cdot), \{I_i\}, \{q_i(\cdot)\})$. We define an equilibrium for this trading mechanism using a Bayes-Nash solution concept.

Definition 1 *An equilibrium for the game* Γ *is a price* p^*, *and a set of strategy functions,* $\{q_i(p; I_i)\}$, *such that:*

[10] See, e.g., Haller and Stoll (1989) who describe the operation of the Frankfurt exchange, where stocks are traded in a single price auction at the opening, noon, and the close.

[11] We assume throughout the paper that $N > 2$, to eliminate trivial cases where, say, a monopolist insider sets arbitrarily high prices. This would not be a problem if the mean of $\tilde{Z}$ were price dependent.

[12] Contrast this with Ho, Schwartz, and Whitcomb (1985) who model a batch market where traders strategies are restricted to 'rectangular' demand functions, causing a divergence from the Walrasian equilibrium.

(i) Excess demand is zero at the equilibrium price:

$$\sum_{i=1}^{N} q_i(p^*; I_i) + Z = 0$$

(ii) Trader i maximizes the expected utility of final period wealth, W_{1i}, given the strategies of other traders:

$$q_i(p^*; I_i) \in \underset{\{q_i\}}{\operatorname{argmax}} \{E[u(\tilde{W}_{1i}) \mid \Phi_i \wedge p^*]\}$$

where:

$$\tilde{W}_{1i} = (\tilde{v} - p^*)q_i + \tilde{v}x_i + W_{0i}$$

for $i = 1, \ldots, N$.

Condition (i) requires that the equilibrium price clears the market. Condition (ii) requires that the strategy function selected by trader i be a best response to the conjectured strategy functions of other traders. In forming the best-response, traders use Bayes' rule to form their beliefs using the statistical information generated by the game, including the price. Thus, traders' probability assessments are determined endogenously in equilibrium. Implicit in the formulation of condition (ii) is that traders choose strategies knowing that not only do they influence prices directly through their order size and indirectly through the effect their actions have on the beliefs of other players. Theorem 1 demonstrates that Γ has a solution with linear strategy functions. All proofs are contained in the appendix.

Theorem 1 (Equilibrium with Written Order Entry) *There exists an equilibrium for the exchange game Γ described above where the optimal strategy functions, $\{q_i(\,\cdot\,; I_i)\}$, and market clearing price, p^*, are given by:*

$$\begin{aligned} q_i(p; I_i) &= \gamma[(1-\delta)(y_i - p) - \alpha x_i] \\ p^* &= \frac{1}{N}\left(\sum_{i=1}^{N}\left[y_i - \left(\frac{\beta}{\rho}\right) x_i\right] + (N-1)\lambda Z\right) \end{aligned}$$

for $i = 1, \ldots, N$, where α, δ, γ, and λ are positive constants described in the appendix.

Corollary 1 *If $\sigma_z^2 = 0$, a linear equilibrium exists if $\rho < \rho^* \equiv \frac{\beta^2(N-2)}{\psi N}$.*

Theorem 1 provides a closed-form solution for the game Γ. The theorem does not rule out the existence of equilibria with non-linear strategies. A trader's optimal strategy has two components: the first component is $\gamma(1-\delta)(y_i - p)$, i.e., a constant proportion of the difference between the information signal and price. This represents the *speculative* or information-motivated part of trade. The second component, $-\alpha\gamma x_i$, is a negative fraction of the endowment, and represents *portfolio hedging.* This component is not based on private information signals. When portfolio hedging is the only source of non-information trading ($\sigma_z^2 = 0$) the existence of a linear equilibrium requires the precision of private information, ρ, be bounded above. Even in a large market (i.e., with large numbers of traders), existence is not assured since even in the limit ρ^* is positive. To see this, note that $\lim_{N\to\infty} \rho^* = \beta^2/\psi > 0$. Interestingly, this result obtains even though agents are symmetric as far as the *quality* of their information signals is concerned. Intuitively, if traders obtain high quality information signals and there is very little endogenous liquidity trading (i.e., high values of ψ and low β), traders are unwilling to reveal their information to others and less willing to share risk by trading.

3.1 Price Variability and Noise Trading

From the discussion above, liquidity trading has two dimensions: endogenous portfolio hedging by traders with non-zero endowments of the risky asset and exogenous noise trading. The expected volume of portfolio trading is proportional to the standard deviation of risky asset endowments, $1/\sqrt{\psi}$, while the expected volume of noise trading is proportional to σ_z.[13] An increase in the volume of noise trading leads to larger expected order imbalances and hence larger absolute deviations from the full information price because traders are risk-averse.

[13]This follows from the normality of $\tilde{x}_i$ and $\tilde{Z}$.

Theorem 2 (Noise Trading and Price Variability): *Price variability increases with the volume of liquidity (portfolio hedging and noise) trading.*

Theorem 2 implies the correlation between price variability and volume is positive since total transaction volume increases with the amount of noise trading. The result also provides a partial explanation for the empirical evidence of a relationship between transitory order imbalances and price movements in auction markets.[14] We turn now to a description of an alternative trading mechanism that exhibits transparency.

4 Market Clearing With Transparency

In this section we analyze a *transparent* market where traders can observe the process of price formation and have the opportunity to revise their orders before market clearing. Examples include verbal call auctions and the opening procedures for continuous trading systems (e.g., Toronto CATS) where traders receive indicated prices based on the current excess demand prior to the opening. The NYSE opening provides a limited amount of transparency in that the specialist is required to provide notifications to floor traders and regional specialists if there appears to be a significant price movement.[15]

We will begin by analyzing a system whose prices act as predictors of the market clearing price given current demand conditions. Later, we will consider an alternative system based on direct disclosure of information. We will show that the equilibria of the two systems coincide. Consider a trading mechanism which operates as follows: the market opens when an exchange designated auctioneer (whom we assume is not permitted to trade for his own account) posts an arbitrary trial price, denoted p_0. Traders then submit their demands at

[14]Haller and Stoll (1989), using data from the Frankfurt Stock Exchange, conclude that "even in auction markets, prices are driven away from their true underlying value by temporary imbalances of orders." See also Blume, MacKinlay, and Terker (1989).

[15]In addition, information on the overnight accumulation of market orders is displayed through the Opening Automated Report Service (OARS).

this 'warm-up' price. On the basis of observed excess demand the auctioneer then announces an *indicated* or *anticipatory* price.[16] Traders then revise their orders given the indication, and the process is repeated until in round τ, say, an indicated price, p_τ, is found where excess demand is exactly zero. If the indicated price for the next period equals the p_τ, all orders are executed at this price and the market clears.

What differentiates this mechanism from the usual Walrasian *tâtonnement* process is that there can be shifts in traders' demand schedules in addition to movements along these demand schedules as traders revise their orders during the process of price formation. In a transparent market, these adjustments take place without generating trade.[17]

Let q_i^t be the order of trader i (where $i = 1, \ldots, N$) in round t when the trial price is p_t, and let $q^t = (q_1^t, \ldots, q_N^t)$. The history of trading in round t is $h_t = \{(p_0, q^0), \ldots, (p_t, q^t)\}$. Let Φ_i^t represent the information set of trader i at round t of the trading process.[18] The N floor traders do not observe the exogenous shock Z – this is consistent with Z representing small orders routed automatically to the market or simply some sort of extrinsic uncertainty. The action taken by trader i in round t is q_i^t, which depends on h_t and p_t. The auctioneer's price revision, $(p_{t+1} - p_t)$, is a function, denoted $\phi(Q_t, h_t)$, where Q_t is the excess demand in round t. We place no restrictions on ϕ except that it is a continuous and increasing function of excess demand with $\phi(0, h_t) = 0$. Traders need not know the specific rule for generating indicated prices but do know it satisfies these properties. We represent this mechanism as a trading game Γ_1. We define an equilibrium analogous to that for the game Γ that requires traders' final round actions to be best responses to the final round actions of others given the history of trading. Formally:

[16]The particular rule the auctioneer uses to generate indicated prices need not concern us for the moment. In the appendix we will show that an algorithm exists to generate indicated prices, and that this algorithm leads to (almost sure) convergence to the market clearing price.

[17]See also Hellwig (1982), Jordan (1982), and Dubey, Geanakoplos, and Shubik (1989) for models of the dynamics of trading in auction markets.

[18]Formally, Φ_i^t is the σ-algebra generated by the set $\{h_t \wedge \Phi_i\}$, with the convention $\Phi_i^0 \equiv \Phi_i$, so that $\Phi_i^{t-1} \subset \Phi_i^t$.

Definition 2 *An equilibrium allocation for the trading game* Γ_1 *is a price,* p_τ*, and a vector of order quantities* $q^\tau = (q_1^\tau, \ldots, q_N^\tau)$ *such that:*

(i) Excess demand is zero at the price p_τ*:*

$$\sum_{i=1}^{N} q_i^\tau(p_\tau) + Z = 0.$$

(ii) The optimal action of a trader maximizes expected utility given the strategies of other traders and the history of trading:

$$q_i^\tau \in \operatorname*{argmax}_{q \in \Re} \{E[u(W_i) \mid \Phi_i^\tau \wedge p_\tau]\}.$$

Trader i's beliefs are formed in accordance with Bayes rule given the strategies of other traders.

(iii) p_τ *is a measurable function of the history of trading,* h_τ*, for* $\tau < \infty$*.*

Remarks: In Definition 2 (ii), the trader conditions on the equilibrium price and the vector of demands at that price. In addition, any information revealed through the bidding process, as represented by h_t is embodied in the trader's information set Φ_i^τ. The condition also requires that a trader's expectations be based on prior information and the information revealed through the history of trading. Since the trader's action is a best-response to the actions of other traders, given the history, it follows that a rational trader only conditions on information that is credible. For example, a strategy of bidding up prices by submitting large buy orders in the early stages and then selling heavily at the proposed equilibrium price is not credible since other traders recognize the dynamic inconsistency in the trader's actions. The last condition is the requirement that there exists a rule to determine the equilibrium price from the history of trading, so that the proposed equilibrium is attainable in a finite number of iterations. The definition of the equilibrium does not place restrictions on the actions of traders in rounds where the market does not clear.[19] In the equilibrium

[19]A formal definition of a sequential equilibrium entails several technical difficulties because the action space, $\{q_i\}$, and player types, $\{I_i\}$, are infinite dimensional. The approach taken here is to focus only on the final round treating fictitious (non-binding) trading as analogous to pre-game communication.

allocation that we solve for, this is not a problem since traders' strategies are *policies* which depend only on the current state. This implies that they disregard all previous information generated by fictitious trading and condition their beliefs only on the credible (or binding) actions of others. To justify these strategies it is necessary to show that traders do not gain by conditioning on 'past events.' This is shown in the appendix, where we also consider the beliefs of traders more formally. Theorem 3 shows that a linear equilibrium is attainable in finite time given that traders' strategies are policies.

Theorem 3 (Equilibrium in a Transparent System): *If $\rho < \beta^2(N-2)/\psi N$, there exists an equilibrium allocation for the exchange game Γ_1 where the price, p_τ, and optimal order quantities $\{q_i^\tau\}$, are given by:*

$$\begin{aligned} p_\tau &= \frac{1}{N}\left(\sum_{i=1}^{N}\left[y_i - \left(\frac{\beta}{\rho}\right)x_i\right] - \lambda_1[N(1-\delta_1)-1]Z\right) \\ q_i^\tau(p_\tau) &= \frac{\gamma_1(1-\delta_1)(y_i-p_\tau)-\gamma_1\alpha_1 x_i}{1-\zeta} + \frac{\zeta}{1-\zeta}\left(\sum_{h\neq i} q_h^\tau\right) \end{aligned}$$

where $\gamma_1, \delta_1, \lambda_1$, and ζ are constants defined in the appendix. The equilibrium allocation $(p^*, \{q_i(p^*)\})$ under Γ_1 and Γ are equivalent if $\sigma_z^2 = 0$.

Theorem 3 characterizes equilibrium under market transparency. We prove in the appendix that equilibrium can be achieved in a finite number of rounds and provide a specific rule to generate indicated prices. In short, the proposed equilibrium is attainable.[20] There are two major differences between this equilibrium and the solution to Γ. First, the conditions necessary for the existence of a linear equilibrium are more stringent. The quality of private information must be bounded above to ensure existence. This upper bound is equal to ρ^* which is defined in Theorem 1, so that the remarks following Theorem 1 apply here. In particular, a large market is not sufficient to ensure existence. Second, unlike

[20]Our objective is to find examples where the equilibrium is generically different under transparency, so we do not examine other equilibria. The linear equilibrium is a natural object of attention since it imposes the fewest computational burdens on agents, exhibits stability, and yields a closed-form solution.

Theorem 1, traders' actions depend not only on the price but also on the actions of others. In a non-transparent system the price not only determines the equilibrium allocation but also conveys information to traders with rational expectations. Here, quantities supplement price as signals to traders. To formalize the nature of the interdependence of actions, we introduce the concept of a trader's *conjectural variation.* Let C_{ij} represent the conjectural variation of trader i, i.e., trader i's beliefs regarding the responsiveness of trader j's quantity to a marginal increase in q_i.[21] We write:

$$C_{ij} = \frac{\partial q_j(p, q_{-j})}{\partial q_i}$$

where q_{-i} denotes the vector of demand functions of traders other than i. From Theorem 3, $C_{ij} = \zeta/(1-\zeta)$ for all $i, j = 1, \ldots, N; i \neq j$. Observe that $C_{ij} = C$, independently of i and j because of traders have symmetric information. In the appendix we show $0 < \zeta < \frac{1}{2}$ so that $0 < C < 1$. This implies that trader's equilibrium order quantity is an increasing function of the equilibrium order quantities of others, a fact which has important implications for market quality.

5 Trading Arrangements and Performance

We turn now to a comparison of market quality and welfare under the two regimes. Transparency resolves some of the uncertainty facing traders through the process of price formation. In a transparent mechanism, strategic traders can condition on quantities and prices. The iterative price formation process permits traders to infer the magnitude of shocks to the supply of the risky asset.[22] By contrast, traders in a non-transparent system do not observe the equilibrium price until after the market has cleared. Since these traders can submit price

[21]Conjectural variations refer to the *local* responses of other players; in a neighborhood around the equilibrium these conjectures will be satisfied.

[22]This fact is established in the appendix in the proof of Theorem 3. The resolution of Z through fictitious trading also serves as a proxy for the resolution of uncertainty not modeled here, such as uncertainty concerning the number and identity of market participants.

contingent orders, they can effectively condition on the price, but not on quantities. We would expect the difference in the information available to traders in the two mechanisms to affect the equilibrium allocation. The next result contrasts price volatility in the two systems.

Theorem 4 *There exists a constant $k_N \in (0,1)$ such that price variability is lower in a transparent mechanism if:*

$$\frac{\rho\psi}{\beta^2} < k_N.$$

Price variability is higher in a transparent mechanism if:

$$k_N < \frac{\rho\psi}{\beta^2} < 1$$

and σ_z^2 is sufficiently high.

Corollary 2 (Large Market): *In a large market price variability is always lower under transparency.*

The theorem provides conditions under which price variability is higher in a transparent mechanism. Strategic traders profit from the deviation between the transaction price and the expected value of the security. Since $C_{ij} > 0$, strategic traders scale back the size of their trades, including the portfolio hedging component, and in a thin market this can increase the absolute size of the deviation of price from expected value. Since price is distributed normally, higher price volatility is equivalent to larger absolute price deviations. We demonstrate below that the information provided through the interactive price formation process in a transparent system effectively permits traders to infer the size of the order imbalance. Rational traders accommodate part of the imbalance, tending to reduce the size of the deviation between the full information price and the clearing price.

This argument can be made more formal by noting that from the proof of Theorem 5, q_i^T can be written as:

$$q_i^T = \gamma_1(1-\delta_1)(y_i - p_\tau) - \gamma_1\alpha_1 x_i - \gamma_1\delta_1\lambda_2 Z.$$

Since the coefficient of Z is negative, traders' speculative actions partly accommodate the shock to the net supply of the risky asset. This effect is offset by changes in the strategies of traders that tend to increase the price impact of noise shocks. Using Lemma 2 (in the appendix), we observe that:

$$\gamma_1(1-\delta_1) < \gamma(1-\delta).$$

This condition implies that the strategy functions in a transparent market are less price sensitive than the strategies in a non-transparent system. Informed traders, who condition on quantities as well as prices, scale back the size of their order for any given discrepency between their signal and the price. As a result, the information component of trading becomes a smaller fraction of total volume. As traders' demand schedules become less price sensitive, the sensitivity of the market price to a given noise shock increases. This effect can dominate the speculative effect so that price variability can be higher in a transparent market. This can occur even though (from Theorem 2) noise trading itself is destabilizing.

This argument suggests that information disclosure can reduce price variability if the market is sufficiently competitive. This is easily verified. From the proof of Theorem 4, $\lim_{N\to\infty} k_N = 1$ so that price variability is always lower in a transparent mechanism if the market is sufficiently competitive. Transparency increases stability in markets that are already large and liquid. This is consistent with casual observation regarding the link between market arrangements and trading activity. For example, in many European exchanges it is common for active issues to trade in a semi-continuous call auction while inactive issues trade once or twice a day in a written-entry batch market. Given a finite number of traders, however, there is a *range* of values for ρ, β, σ_z^2, and ψ under which transparency is destabilizing. This implies that there is no critical size above which transparency leads to greater stability. Finally, the result shows that the equilibria for Γ and Γ_1 are generically different unless $\sigma_z^2 = 0$. The key point is that a written entry system can operate in economies where a transparent system may not be viable. With the results at hand, we are now in a position to examine a specific policy designed to increase transparency.

5.1 Information Disclosure

An alternative method of achieving market transparency is to disseminate information on current market conditions directly to traders. In this section we show that we can readily accommodate such a system into the current framework. Consider for example sunshine trading, where a trader can pre-announce his trading intentions.[23] In this case, we now interpret Z as the order of a large uninformed trader who has pre-committed to trading for liquidity reasons. This interpretation can be formally modeled within the current framework. It is easily shown that the optimal action of a risk-averse uninformed trader, given that prices have mean v, is to hedge a constant fraction of his endowment of the risky security by a market order, so that Z represents this fraction.[24] It can also be shown that none of the informed traders will choose to disclose their trades if given the opportunity to do so, since this means revealing valuable information to competitors. Direct disclosure also arises in other contexts. For example, if Z is interpreted instead as the order imbalance arising from consolidating small trades through an automated system, transparency corresponds to reporting the net effect of automated trading to market participants.

Consider a trading mechanism, denoted Γ_2, where Z is displayed to floor traders before they submit their demands. Demands are submitted in writing, so the only difference between this system and the written order entry system Γ is that trader i ($i = 1, \ldots, N$)

[23]Admati and Pfleiderer (1990) provide a model of sunshine trading, and examine its impacts on a variety of performance and welfare measures.

[24]If the uninformed trader is 'sophisticated' in that he recognizes the price impact of the trade, it is still optimal to hedge a constant fraction of endowment if the price functional is linear.

observes Z before choosing $q_i(p)$. There is no interactive order revision as in the game Γ_1, so we retain the Bayes-Nash equilibrium concept of Definition 1. The next result establishes the existence of a well-defined solution to the game Γ_2.

Theorem 5 *If $\rho < \rho^*$, there exists a well-defined Bayes-Nash equilibrium for the game Γ_2. The equilibrium allocation (i.e., the price and vector of order quantities) in Γ_2 coincides with the allocation for Γ_1.*

The result demonstrates that the equilibria for the games Γ_2 and Γ_1 are equivalent if the quality of private information is bounded above. In other words, the interactive order submission process of Γ_1 indirectly resolves the uncertainty concerning the realization of Z, uncertainty that is directly resolved by information disclosure in Γ_2. The result helps us understand the nature of the difference between a transparent and non-transparent market mechanism. Transparency matters because it helps resolve uncertainty; a transparent market protocol is, in some sense, equivalent to direct information disclosure. From Theorem 4, it follows that the expected absolute pricing error (the deviation of the clearing price from the full information price of the security, $E[|\tilde{p} - v|]$) can be larger if traders are permitted to submit their demands after information on order imbalances is disclosed. If we view the game Γ_2 as describing the rules for sunshine trading, it is clear that the expected losses of the uninformed trader can, depending on the parameters, be larger than the losses from non-disclosure. Traders with large liquidity-based trades can be worse off through sunshine trading.

5.2 Liquidity, Transaction Costs, and Welfare

Price variability is only one aspect of market performance. In this section, we consider the impact of transparency on liquidity, implicit transaction costs, and social welfare. A measure of market liquidity is *market depth*, which Kyle (1985) defines as the order flow necessary to induce a unit price change.[25] Following Kyle, let Υ represent market depth, where:

$$\Upsilon = \left(\frac{dp^*}{dZ}\right)^{-1}.$$

A related issue concerns traders' perceptions of their impact on the equilibrium price. Since prices move in the direction of the trade there is a bid-ask spread implicit in a single-price auction. Suppose we can express the equilibrium price as an increasing function of trader i 's demand, which we denote $p(q_i)$.[26] Then we can define an implicit spread $s(\cdot)$ as the difference between the market clearing price if an order of size q_i was to buy and the price if the order was to sell:

$$s(q_i) = p(|q_i|) - p(-|q_i|).$$

Theorem 5 shows that price variability and market depth are inversely related, but the implicit spread is higher in a transparent market

Theorem 6 *Price variability and market depth are inversely related; the mechanism with greater price volatility also provides less liquidity. However, the implicit bid-ask spread for an order of given size is strictly greater in a transparent market.*

A more liquid market offers greater price stability. If we regard uninformed investors as the source of orders that are not price contingent, these investors suffer lower expected losses in deeper markets, a fact that is consistent with the emphasis on price variability in public policy debates. The implicit spread, however, is strictly greater in a transparent market. In a transparent market the actions of an informed trader convey more information than under a non-transparent system. As a result, a trader's action has an indirect effect through the

[25]The definition applies only to anonymous market orders since price contingent orders effectively determine the price. This qualification is unnecessary in Kyle (1985) where all traders submit market orders and the price is determined by competitive market makers who observe only the net order flow.

[26]The technical details are provided in the appendix.

actions of others (represented by the conjectural variation) in addition to the direct effect on prices, leading to wider implicit spreads.[27]

Although the measures of market quality discussed so far figure prominently in policy discussions, they are not necessarily relevant from a welfare perspective. As the impact of transparency depends on the parameters of the model we would expect the same to be true for welfare. Under our assumptions, traders care about only the mean and the variance of future wealth. However, improved information need not increase utility because it can eliminate opportunities for risk-sharing. In particular, we can show that the volume of endogenous liquidity trading is strictly lower in a transparent market. As a result, we cannot make unambiguous welfare statements about market organization. Finally, note that transparency is costly, requiring either the physical presence of traders or complex communications networks to permit interactive order submission. Cost considerations would favor a non-transparent system.

6 Conclusions

This paper investigates the impact of disclosing information on current market conditions to traders. We compare two trading mechanisms at opposite extremes in terms of market transparency: the first system is a closed auction where orders are submitted before prices are determined. The second system is transparent in that traders observe the intermediate stages of price formation. In both cases, trading is modeled as an imperfect information game between strategic agents with rational expectations. In the closed auction, traders who submit price contingent orders can condition on price. By contrast, traders in the transparent system can condition on both price and volume. This is a logical extension to

[27]This proposition is testable since implicit spreads can be estimated from the serial covariance of transaction prices using a formula due to Roll (1984). Haller and Stoll (1989) compute implied spreads for stocks on the Frankfurt Stock Exchange (which is organized as an auction market) in this manner and find that implicit spreads are significantly positive.

rational expectations models where traders condition only on price. The paper contributes to this literature by providing an explicit model of the mechanism that allows such conditioning to take place.

The difference in the information available to traders can affect the equilibrium allocation through the differences in their strategic responses. We provide conditions under which transparency can increase price instability and reduce liquidity, possibly even to the extent of market failure. This can occur even though transparency resolves some of the uncertainty about the magnitude of non-information based trading or noise trading that is the source of price instability. The model can be readily applied to the analysis of current policy issues concerning transparency, such as sunshine trading. We show that pre-disclosure of trade size can actually increase the price impact of a large order. Similarly, publicizing the size of order imbalances originating from automated order routing can exacerbate the absolute deviation of price from its equilibrium value. If the market is sufficiently competitive, however, information disclosure always reduces volatility. Seemingly subtle differences in trading protocols can have a substantial affect on the existence and character of equilibrium.

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Appendix

Proof of Theorem 1:

The proof constructs the Bayes-Nash equilibrium by solving for a trader's best response to the conjectured strategies adopted by other traders and then shows the conjectures are consistent.

Step 1: (Traders' Conjectures and the Clearing Price) For notational convenience index traders other than i by $h = 1, \ldots, i-1, i+1, \ldots, N$. Suppose trader i conjectures that all other traders adopt linear strategy functions:

$$q_h(p; I_h) = A_h(I_h) - Bp \tag{1}$$

where $I_h = (W_{0h}, x_h, y_h)$ and B is a constant. Note that I_h is private information and is known only to trader h, so that A_h is also private information. Suppose trader i places an order for q_i. Using (1) and the market clearing condition we obtain:

$$\sum_{h \neq i}^{N} A_h - (N-1)Bp + q_i + Z = 0. \tag{2}$$

From equation (2) we can express price as a function of q_i, denoted $p(q_i)$:

$$p(q_i) = p_{-i} + \lambda q_i \tag{3}$$

where p_{-i} and λ are defined as follows:

$$p_{-i} = \frac{\sum_{h \neq i}^{N} A_h + Z}{(N-1)B} \tag{4}$$

$$\lambda = \frac{1}{(N-1)B}. \tag{5}$$

For trader i, p_{-i} is a random variable (denoted $\tilde{p}_{-i}$) because neither A_h nor Z is observable at the time of order submission. To construct the trader's strategy, we first consider the case where trader i actually observes the realization of $\tilde{p}_{-i}$. We relax this in Step 2 below. Trader i then chooses her optimal order quantity q_i^* to maximize her expected utility of wealth. With $p^* = p(q_i^*)$, final period wealth is:

$$W_{1i} = (v - p^*)q_i^* + vx_i + W_{0i} \tag{6}$$

where W_{0i} is the initial cash holdings of trader i. Grossman (1976) shows that maximizing the expected utility of wealth (assuming $\tilde{W}_{1i}$ is normally distributed) is equivalent to maximizing:

$$E[\tilde{W}_{1i} \mid \Phi_i] - \frac{\beta}{2}\sigma^2(\tilde{W}_{1i} \mid \Phi_i) \tag{7}$$

where $E[\cdot \mid \cdot]$ and $\sigma^2(\cdot \mid \cdot)$ are the conditional expectation and variance operators. Substituting (6) into equation (7), we obtain:

$$E[\tilde{v}|\Phi_i \wedge p^*](q_i + x_i) - p(q_i)q_i + W_{0i} - (\beta/2)\sigma^2(\tilde{v}|\Phi_i, p^*)(q_i + x_i)^2. \tag{8}$$

Trader i's optimal order quantity, q_i^* is found by differentiating this expression with respect to q_i^*:

$$E[\tilde{v} \mid \Phi_i \wedge p^*] = p(q_i^*) + p'(q_i^*)q_i^* + \alpha_i(q_i^* + x_i) \tag{9}$$

where $\alpha_i \equiv \beta\sigma^2(v|\Phi_i \wedge p^*)$. The second order condition for a maximum is:

$$2p'(q_i^*) + p''(q_i^*)q_i^* + \alpha_i < 0. \tag{10}$$

Step 2: (Construction of the Best Response Function) Suppose trader i conjectures that $\tilde{p}_{-i}$ is normally distributed with mean v and variance $1/\pi$ and that $Cov(p_{-i}, y_i) = Cov(p_{-i}, x_i) = 0$. We will show that in equilibrium trader i's conjectures concerning $q_h(\cdot)$ and p_{-i} are correct. Trader i cannot invert the market clearing price p^* to infer the realization of $\tilde{p}_{-i}$ because p^* is not observed at the time of order submission. We claim that trader i can effectively condition on p^* by submitting a demand schedule that specifies her demand at every price. To show this is feasible, suppose the realization of $\tilde{p}_{-i}$ is p^*_{-i}. Then, trader i's optimal order quantity q_i^* is determined from (9) with trader i acting as the 'marginal' trader who determines the price. Rationality also requires trader i condition upon the statistical information present in the price. This yields a point $(p^*_{-i}, q_i^*) \in \Re^2$. From (3), the corresponding price is $p(q_i^*)$ so that uniquely associated with the point (p^*_{-i}, q_i^*) is the pair $(p(q_i^*), q_i^*)$, a point on trader i's demand schedule. Repeating this exercise for $p^*_{-i} \in \Re$ leads to a curve in $\Re^2$, the graph of $(p(q_i^*), q_i^*)$, trader i 's demand function in (p, q)-space.

The next step is to construct the trader's demand function, denoted $q_i(p; I_i)$, as outlined above, given one point on it. Suppose trader i believes p_{-i} is the realization of $\tilde{p}_{-i}$. Under our assumptions concerning the distribution of y_i and p_{-i}, trader i's conditional expectation of $\tilde{v}$, is (see DeGroot (1970), page 167):

$$E[\tilde{v}\,|\Phi_i \wedge p_{-i}] = (1-\delta)y_i + \delta p_{-i} \tag{11}$$

where $\delta = \pi/(\pi+\rho)$ is a constant. From (9), the optimal order quantity $q_i^* = q_i(p^*)$ satisfies:

$$q_i^* = \frac{E[\tilde{v}|\Phi_i \wedge p^*] - p^* - \alpha_i x_i}{\alpha_i + \lambda} \tag{12}$$

where $\alpha_i = \beta\sigma^2(\tilde{v}|\Phi_i \wedge p_{-i})$. Since $\sigma^2(\tilde{v}|\Phi_i \wedge p_{-i}) = (\pi+\rho)^{-1}$, α_i is a constant for all i, denoted $\alpha = \beta/(\pi+\rho)$. In (12), q_i^* depends upon $E[\tilde{v}|\cdot]$ which depends the conjectured value of p_{-i}. We must ensure that each price- quantity pair on the demand schedule is consistent with the posterior beliefs generated by observing the realization of that particular trade. To do this, substitute the value of $E[\tilde{v}|\cdot]$ given by (11) into equation (12). Since $p_{-i} = p^* - \lambda q_i^*$, we obtain:

$$q_i(p^*; I_i) = \gamma(1-\delta)(y_i - p^*) - \gamma\alpha x_i \tag{13}$$

where:

$$\gamma \equiv \frac{1}{(\alpha + \lambda + \delta\lambda)}. \tag{14}$$

Equation (13) is the optimal demand given $p = p^*$; the demand schedule, $q_i(p; I_i)$, is the graph of $(q_i(p^*), p^*)$ for $p^* \in \Re_+$. Inspection of (13) shows that the optimal strategy of trader i is of the form conjectured in (1) with:

$$A_i(I_i) = \gamma[(1-\delta)y_i - \alpha x_i] \tag{15}$$

$$B = \gamma(1-\delta). \tag{16}$$

Step 4: (Existence) Traders have the same price sensitivity although their reservation prices A_i vary with the initial state I_i. To show the strategy functions are well- defined, we must express the unknown values of the strategy functions, $\alpha, \lambda, \gamma, \delta$, and π in terms of the parameters N, β, ρ, and ψ, and verify that the initial conjectures are satisfied. From the definition of B and λ write:

$$\lambda = \frac{1}{(N-1)\gamma(1-\delta)}. \tag{17}$$

Since $\alpha = \beta/(\pi+\rho)$ and $\delta = \pi/(\pi+\rho)$, $\alpha/(1-\delta) = (\beta/\rho)$. Then, from (2), and the conjectured values of A_i and B, the market clearing price is:

$$p^* = \frac{1}{N}\left(\sum_{i=1}^{N}\left[y_i - \left(\frac{\beta}{\rho}\right)x_i\right] + \frac{Z}{\gamma(1-\delta)}\right). \tag{18}$$

The formula for p_{-i} is identical to (18), except that the summation excludes trader i:

$$p_{-i} = \frac{1}{N-1}\left(\sum_{h\neq i}^{N}\left[y_h - \left(\frac{\beta}{\rho}\right)x_h\right] + \frac{Z}{\gamma(1-\delta)}\right). \tag{19}$$

The variance of p_{-i} is given by:

$$\frac{1}{\pi} = \omega_0 + \frac{\omega_1}{\gamma^2(1-\delta)^2} \tag{20}$$

where:

$$\omega_0 = \frac{\rho^{-1} + \psi^{-1}(\beta/\rho)^2}{(N-1)} \tag{21}$$

$$\omega_1 = \frac{\sigma_z^2}{(N-1)^2} \tag{22}$$

are constants. When $\sigma_z^2 > 0, \omega_1 > 0$ and (20) can be inverted to express γ as a function of π:

$$\gamma = \left(\frac{\pi+\rho}{\rho}\right)\sqrt{\frac{\omega_1}{(\pi^{-1}-\omega_0)}}. \tag{23}$$

Write (23) as $\gamma = g(\pi)$. Next, we derive a second function relating γ to π. The equilibrium values of γ and π are the solutions to these two non-linear equations. Substituting (17) into (14), we obtain:

$$\frac{1}{\gamma} = \alpha + \frac{(1+\delta)}{(N-1)\gamma(1-\delta)}.$$

Write this equation as:

$$\gamma = \frac{(\pi+\rho)[(N-2)\rho - 2\pi]}{(N-1)\beta\rho}. \tag{24}$$

Equation (24) expresses γ as a function of π. Denote this function by $\gamma = f(\pi)$. A solution to the game Γ exists if there exists π^*, where $\pi^* > 0$, solving $f(\pi^*) = g(\pi^*) > 0$.
Step 5: (Equilibrium) From (23), $g(\pi)$ is a continuous function of π on the interval $(0, \pi_0)$, where π_0 is:

$$\pi_0 = \frac{1}{\omega_0} \tag{25}$$

where ω_0 is defined in (21). Then, $g(\pi) \geq 0$ and $g'(\pi) > 0$. It is easy to show that $g(0) = 0$ and that $g(\pi) \to \infty$ as $\pi \to \pi_0$. Examining (24), $f(\pi)$ is continuous in π, for $\pi \in [0, \pi_0]$. Putting $\pi = 0$ in (24) yields $f(0) = (\rho/\beta)(N-2)/(N-1)$, which is strictly positive for $N > 2$.

When $N = 2$, $f(\pi) = 0$ for $\pi \in (0, \pi_0)$, and we obtain the (trivial) no-trade equilibrium. From the definition of $f(\cdot)$, it is clear that $f(\pi)$ is well-defined, i.e., $|f(\pi)| < \infty$, for $\pi \in [0, \pi_0]$. Define $R(\pi) \equiv f(\pi) - g(\pi)$. Clearly, $R(\pi)$ is continuous in π on the interval $[0, \pi_0)$. Note that $R(0) > 0$ and that the limit of $R(\pi)$, as $\pi \uparrow \pi_0$ is $-\infty$. Applying the Intermediate Value Theorem, there exists $\pi^* \in (0, \pi_0)$ such that $R(\pi^*) = 0$. Since $g(\pi)$ is positive for $\pi \in (0, \pi_0)$, $\gamma^* > 0$. Finally, it is easy to verify the initial conjectures that p_{-i} is normally distributed with $E[p_{-i}] = v$ and $Cov(p_{-i}, y_i) = Cov(p_{-i}, x_i) = 0$.

Step 6: (Endogenous Liquidity Trading) In the case where $\sigma_z^2 = 0$, the only source of non-information based trading is portfolio hedging. Assume without loss of generality that $Z = 0$. Then equations (4)–(18) are unchanged. By definition, $\omega_1 = 0$ in this case, and using (18) the precision of p_{-i} is π_0, given by equation (25). From (13) and (17), we obtain:

$$\gamma_0 = \frac{(\pi_0 + \rho)[(N-2)\rho - 2\pi_0]}{(N-1)\beta\rho}. \tag{26}$$

Equation (26) alone now determines γ_0. To ensure the equilibrium is meaningful, γ_0 must be positive. From (26), $\gamma_0 > 0$ implies:

$$\frac{(N-2)\rho}{2} > \frac{1}{\omega_0}.$$

We can rearrange this inequality to obtain:

$$\rho < \frac{(N-2)\beta^2}{N\psi}. \tag{27}$$

Let $\rho^* \equiv \frac{(N-2)\beta^2}{N\psi}$, so that the second order condition (10) is satisfied if $\rho < \rho^*$. Given (27) holds and $\sigma_z^2 = 0$, we obtain:

$$q_i(p^*) = \gamma_0(1 - \delta_0)(y_i - p^*) - \gamma_0\alpha_0 x_i \tag{28}$$

where γ_0 is given by (26), $\delta_0 = \pi_0/(\rho + \pi_0)$, and $\alpha_0 = \beta/(\rho + \pi_0)$. This argument establishes Corollary 1. ∎

Proof of Theorem 2:
First, we establish a useful Lemma.

Lemma 1 *The unconditional variance of the equilibrium price is a linear function of the variance of* $\tilde{p}_{-i}$:

$$\sigma^2(p^*) = \frac{N-1}{N}\left(\omega_0 + \frac{N-1}{N}(\pi^{-1} - \omega_0)\right) \tag{29}$$

where ω_0 *is a constant defined in equation (21).*

Proof of Lemma 1: Taking the variance of p^* in equation (18) and substituting the definition of ω_0 in (21) and the definition of π in (20) into this expression yields the Lemma. ∎

Now consider two possible values for σ_z^2 say σ_1^2 and σ_2^2, where $\sigma_2^2 > \sigma_1^2 > 0$. From (24), it follows that $f(\cdot)$ does not depend on σ_z^2. However, the function $g(\cdot)$ depends on σ_z^2 through the constant ω_1. Let $g_1(\pi)$ represent equation (23) when $\sigma_z^2 = \sigma_1^2$ and similarly define $g_2(\pi)$. From (23), it is clear that:

$$g_2(\pi) > g_1(\pi) \tag{30}$$

for $\pi \in (0, \pi_0)$. Suppose π_1 is the solution to $f(\pi) = g_1(\pi)$, where $f(\cdot)$ is defined by (24). Consider the function $R_2(\pi) \equiv f(\pi) - g_2(\pi)$. From (23), $R_2(\pi_1) < 0$. As $R_2(0) > 0$, the Intermediate Value Theorem implies there exists $\pi_2 \in (0, \pi_1)$ such that $R_2(\pi_2) = 0$. Since $\pi_2 < \pi_1$, it follows from Lemma 1 that $\sigma^2(p^*)$ is higher when $\sigma_z^2 = \sigma_2^2$. Next, observe that an increase in ψ reduces ω_0, but does not affect $f(\cdot)$ in (24). A fall in ω_0 decreases the value of $g(\pi)$ for all $\pi \in (0, \pi_0)$. Since $f(\cdot)$ is unaffected by a change in ψ, it follows from the arguments above that π^* increases as ψ increases. Applying Lemma 1, the variance of the market clearing price, $\sigma^2(p^*)$, falls as π^* rises and ω_0 decreases. ∎

Proof of Theorem 3: Suppose that in round $t \geq 1$, trader i believes that q_h^t is a linear policy:

$$q_h^t(p_t; q_{-h}^t) = A_h(I_h) - Bp_t + C\sum_{j \neq h} q_j^t \tag{31}$$

for $h, j = 1, \ldots, N; j \neq h$. Here we assume traders can condition on the current order quantities of floor traders. We need to make certain modifications when traders can observe only quantities traded in the previous period.[27] If round τ is the final or equilibrium round (we will consider the dynamics by which this equilibrium was reached below), it follows that $p_{\tau+1} = p_\tau$. Market clearing in round τ implies that:

$$\sum_{h \neq i}^{N} A_h - (N-1)Bp_\tau + C\sum_{h \neq i}\sum_{j \neq h} q_j^\tau(p_\tau) + q_i^\tau + Z = 0. \tag{32}$$

The market clearing constraint is implicit in the double summation above since:

$$\sum_{h \neq i}\sum_{j \neq h} q_j^\tau = -\sum_{h \neq i} q_h^\tau - (N-1)Z. \tag{33}$$

In this mechanism q_h conveys statistical information concerning I_h to other traders. To make this explicit, suppose trader i regards (A_h/B) as a draw from an independent normal distribution with mean v and precision $\pi_1/(N-1)$. This will be shown to be correct in equilibrium. Define:

$$\mu_{-i} = \frac{\sum_{h \neq i}^{N} A_h}{B(N-1)}. \tag{34}$$

Under our assumptions, trader i conjectures that μ_{-i} is the realization of a normally distributed random variable $\tilde{\mu}_{-i}$ with mean v and precision π_1 that is uncorrelated with y_i and x_i. Substituting equation (33) into (32), we can express price in terms of q_i^τ and Z:

$$p_\tau = \mu_{-i} + \lambda_1 q_i^\tau + \lambda_2 Z \tag{35}$$

[27] In this case, equation (31) is simply a linear function of the previous round of quantities. Other modifications, where necessary, are discussed below.

where:

$$\lambda_1 = \frac{1+C}{B(N-1)}$$
$$\lambda_2 = \frac{1-C(N-2)}{B(N-1)}.$$

The trader's conditional expectation, given the realization of $\tilde{\mu}_{-i}$, is:

$$E[\tilde{v}|\Phi_i \wedge \mu_{-i}] = (1-\delta_1)y_i + \delta_1\mu_{-i} \tag{36}$$

where $\delta_1 = \pi_1/(\pi_1 + \rho)$ is a constant. From (9) trader i acts as the marginal trader who actually determines the price:

$$E[\tilde{v}|\Phi_i \wedge p_\tau] = p_\tau + \lambda_1 q_i^\tau + \alpha_1(q_i^\tau + x_i). \tag{37}$$

where $\alpha_1 = \beta/(\rho+\pi_1)$. Substitute $\mu_{-i} = p_\tau - \lambda_1 q_i^\tau - \lambda_2 Z$ and (36) into this equation. The demand at the price p_τ is given by $q_i^\tau(p_\tau)$, where:

$$q_i^\tau(p_\tau) = \gamma_1(1-\delta_1)(y_i - p_\tau) - \gamma_1\alpha_1 x_i - \gamma_1\delta_1\lambda_2 Z \tag{38}$$

where

$$\gamma_1 \equiv \frac{1}{\alpha_1 + \lambda_1 + \delta_1\lambda_1}. \tag{39}$$

Substituting $Z = -\sum_{j=1}^{N} q_j^\tau$ into (38), we obtain:

$$q_i(p_\tau) = \frac{\gamma_1(1-\delta_1)(y_i - p_\tau) - \gamma_1\alpha_1 x_i + \gamma_1\delta_1\lambda_2\left(\sum_{h\neq i} q_h^\tau\right)}{1-\gamma_1\delta_1\lambda_2}. \tag{40}$$

From equation (40), the best-response strategy of a trader i has the same form as conjectured with:

$$A_i = \frac{\gamma_1[(1-\delta_1)y_i - \alpha_1 x_i]}{1-\gamma_1\delta_1\lambda_2} \tag{41}$$
$$B = \frac{\gamma_1(1-\delta_1)}{1-\gamma_1\delta_1\lambda_2} \tag{42}$$
$$C = \frac{\gamma_1\delta_1\lambda_2}{1-\gamma_1\delta_1\lambda_2}. \tag{43}$$

The precision of μ_{-i} can be calculated directly:

$$\pi_1 = \frac{1}{\omega_0} \tag{44}$$

where ω_0 was defined above in equation (21). Using the definition of λ_1, we obtain:

$$\lambda_1 = \frac{1}{(N-1)(1-\delta_1)\gamma_1}. \tag{45}$$

Since the value of π_1 is identical the value of π_0 in equation (25), the remainder of the proof follows from Corollary 1 of Theorem 1. It follows that γ_1 and δ_1 take the same values as γ_0 and δ_0, defined in Theorem 1 for the case where $\sigma_z^2 = 0$. In other words, the equilibrium allocation here is identical to the equilibrium allocation under a written-entry system where the realization of $\tilde{Z}$ has been disclosed. Next, we verify that λ_2 is well-defined. From equation (45):

$$\lambda_2 = \lambda_1 \left(1 - \frac{(N-1)C}{1+C}\right)$$

Substituting (43) and the definition of λ_1 into this equation we obtain:

$$\lambda_2 = \frac{1}{(N-1)\gamma_1}. \tag{46}$$

which is always positive. Given the existence of the parameters $\delta_1, \gamma_1, \pi_1, \lambda_1$, and λ_2, a linear equilibrium of the type postulated exists. Substituting (43) into (38) we obtain:

$$q_i^{\tau}(p_{\tau}) = \gamma_1[(1-\delta_1)y_i - \alpha_1 x_i] - \zeta Z - \gamma_1(1-\delta_1)p_{\tau} \tag{47}$$

where $\zeta \equiv \delta_1/(N-1)$. Since $\zeta \in (0, \frac{1}{2})$ traders willingly absorb a part of the order imbalance Z, tending to stabilize prices. From (47), the equilibrium price is:

$$p_{\tau} = \frac{1}{N}\left\{\sum_{i=1}^{N}\left[y_i - \left(\frac{\beta}{\rho}\right)x_i\right] + \left[\frac{1-N\zeta}{\gamma_1(1-\delta_1)}\right]Z\right\}. \tag{48}$$

With $q_i(p, q_{-i})$ as given above in (31), we can verify that the intermediate demands are well-defined and then show that the proposed solution does not unravel. For period $0 < t \leq \tau$, $q^t \in \Re^N$ is the solution to the system of N linear equations:

$$\begin{aligned} q_1^t &= q_1(p_t, q_{-1}^t) \\ &\vdots \\ q_N^t &= q_N(p_t, q_{-N}^t). \end{aligned}$$

Since $C \in (0,1)$ and $A_i \neq A_j$ except on a set of Lebesgue measure zero, a solution (a.s) exists. Note that Walras' Law does not apply here since the market is not required to clear at intermediate trading rounds.[28] Next, we establish the existence of a measurable function of h_{τ} that yields p_{τ}. Consider the following algorithm (Newton's method) for determining indicated prices:

$$p_{t+1} - p_t = \phi(Q_t, h_t) = -\left[\frac{p_t - p_{t-1}}{Q_t - Q_{t-1}}\right]Q_t. \tag{49}$$

This iterative rule computes the zero of the excess demand function and can be shown to assure the convergence of the price process in a finite number of rounds.[29] Next, we must show that the

[28] If traders cannot observe current actions, for $0 < t \leq \tau$ we have:

$$q_i^t = q_i(p_t, q_{-i}^{t-1})$$

for $i = 1, \ldots, N$. For any fixed p, this dynamical system converges asymptotically to a finite q, although not necessarily the market clearing quantities. The function ϕ can be chosen to insure the Liapunov condition is satisfied.

[29] The minimum required is three rounds. By contrast, if traders condition on lagged quantities, convergence occurs only at the limit once the equilibrium price has been discovered. To avoid this difficulty, we should not require exact market clearing but rather clearing within a small tolerance bound. The excess could be accommodated by the auctioneer or by rationing on the short side of the market.

proposed solution is an equilibrium when considering the set of all history-dependent functions. This follows because if a set of strategies are best-responses to each other when agents use only the current 'state' variables (p_t, q^t), then this set of strategies are still best-responses to each other when the considering history-dependent strategies as well.[30] A stronger statement can be made. In this game, the history of trading provides no additional information to traders if traders adopt linear strategies. To prove this, note that:

$$A_i(I_i) = q_i^\tau + \gamma_1(1-\delta_1)p_\tau - \frac{\gamma_1\delta_1\lambda_2}{1-\gamma_1\delta_1\lambda_2}\left(\sum_{h\neq i} q_h^\tau\right) \quad (50)$$

so the sets $\{(q^\tau, p_\tau)\}$ and $\{h_t, (q^\tau, p_\tau)\}$ support identical beliefs. This shows that there is no advantage to reneging on a proposed equilibrium once other traders have revealed their demands. This is intuitively obvious because orders at intermediate or non-equilibrium prices are not binding so traders will not reveal more information than that imparted by the equilibrium allocation. Traders place no weight on the actions of others at disequilibrium prices, since trading is fictitious except in the final round. The intermediate stages of price formation (i.e., agents' actions for $t < \tau$) can be thought of as representing a form of pre-game communication. Suppose in round t', trader j *defects* from the equilibrium strategy by submitting a demand $q_j^{t'}$ that is inconsistent with equation (31). This defection cannot be optimal in round τ since, by definition:

$$q_j^{t'} \neq q_j^\tau = \underset{\{q_i\}}{\operatorname{argmax}}\{E[u(\tilde{W}_{1i}) \mid \Phi_i \wedge p_\tau]\}.$$

When t' is not the final round, off-equilibrium path actions have no effect on the eventual equilibrium allocation which depends only on credible (i.e., binding) actions. If traders follow linear policies, an attainable equilibrium allocation exists. ∎

Proof of Theorem 4: The unconditional variance of p_τ under Γ_1 can be determined directly from (48):

$$\sigma_1^2(p_\tau) = \frac{N-1}{N}\omega_0 + \left[\frac{\sigma_z(1-\zeta N)}{N\gamma_1(1-\delta_1)}\right]^2. \quad (51)$$

The constant ζ is defined to be $\delta_1/(N-1)$. The variability of prices under the game Γ is given by Lemma 1. The difference in price variability under Γ and Γ_1 is:

$$\sigma^2(p^*) - \sigma_1^2(p_\tau) = \left[\frac{\sigma_z}{N\gamma(1-\delta)}\right]^2 - \left[\frac{\sigma_z(1-\zeta N)}{N\gamma_1(1-\delta_1)}\right]^2. \quad (52)$$

To compare the two variances, we need an additional result.

Lemma 2 *If $\sigma_z^2 > 0$, traders' strategy functions are less price sensitive in a transparent mechanism:*

$$\gamma_1(1-\delta_1) < \gamma(1-\delta).$$

Otherwise, if $\sigma_z^2 = 0$, the slope coefficients are identical.

[30] A formal statement is provided in a lemma due to Blackwell (1965). Of course, when the set of strategies is expanded to history dependent strategies, there may exist a Pareto dominating equilibrium for this game.

Proof of Lemma 2:

Recall that $\gamma_1 = \gamma_0$ and $\delta_1 = \delta_0$, where γ_0 and δ_0 were defined above. To establish the lemma we must prove that $\gamma_0(1-\delta_0) < \gamma(1-\delta)$. From equation (24) and the definition of δ, we can write $\gamma(1-\delta)$ as:

$$\gamma(1-\delta) = \frac{(N-2)\rho - 2\pi}{\beta(N-1)}. \tag{53}$$

Now, γ_0 is given by equation (26), and using the definition of δ_0, we can write:

$$\gamma_0(1-\delta_0) = \frac{(N-2)\rho - 2\pi_0}{\beta(N-1)}. \tag{54}$$

The proof of Lemma 2 follows immediately upon comparing (53) and (54) since $\pi < \pi_0$ if $\sigma_z^2 > 0$. If $\sigma_z^2 = 0$, $\gamma_0(1-\delta_0) = \gamma(1-\delta)$. ∎

From (52), $\sigma^2(p^*) < \sigma_1^2(p_T)$ if and only if:

$$\gamma_0(1-\delta_0) < (1-\zeta N)\gamma(1-\delta). \tag{55}$$

Using equations (53) and (54), equation (55) can be written as:

$$(N-2)\rho - 2\pi_0 < (1-\zeta N)[(N-2)\rho - 2\pi]. \tag{56}$$

Rearrange this to obtain:

$$2\pi < (N-2)\rho - \frac{(N-2)\rho - 2\pi_0}{1-\zeta N}. \tag{57}$$

Observe that $\delta_1 = \delta_0 = \pi_0/(\rho+\pi_0) = 1/(1+\rho/\pi_0)$. Then $\zeta = \delta_1/(N-1)$ is:

$$\zeta = \frac{1}{N + (\beta^2/\rho\psi)} \tag{58}$$

using the definition of π_0. Note that $(1-\zeta N)$ is independent of σ_z^2. Let

$$\Xi \equiv (N-2)\rho - \frac{(N-2)\rho - 2\pi_0}{1-\zeta N}. \tag{59}$$

Ξ is the right hand side of (57). Lemma 3 shows that $\Xi > 0$ for a range of parameter values.

Lemma 3 *There exists a constant, $k_N > 0$, which is independent of β, ρ, and ψ such that $\Xi > 0$ in equilibrium if $\frac{\rho\psi}{\beta^2} > k_N$.*

Proof of Lemma 3: Observe that Ξ is independent of the level of noise σ_z^2. Now $\Xi > 0$ is equivalent to:

$$(N-2)\rho > \frac{(N-2)\rho - 2\pi_0}{1-\zeta N}. \tag{60}$$

Using the definition of ζ in (58) and the definition of π_0, this reduces to:

$$\frac{N^2 - 4N + 2}{N^2} < \frac{\rho\psi}{\beta^2}. \tag{61}$$

From Theorem 1, when $\sigma_z^2 = 0$, existence requires:

$$\frac{\rho\psi}{\beta^2} < \frac{(N-2)}{N}. \tag{62}$$

Comparing (62) with (61), we see that $\Xi > 0$ in equilibrium if the parameter values satisfy:

$$\frac{(N^2-4N+2)}{N^2} < \frac{\rho\psi}{\beta^2} < \frac{(N-2)}{N}. \tag{63}$$

Define k_N by $k_N \equiv (N^2-4N+2)/N^2$. Note that for all $N \geq 2$, $k_N < (N-2)/N$. Only if the precision of endowments or private information is sufficiently low, i.e., $\frac{\rho\psi}{\beta^2} < k_N$, is $\Xi < 0$. ∎

To complete the proof of the theorem note that by Lemma 3, if ψ (or ρ) is sufficiently high, i.e., if $\rho\psi > (\beta^2/k_N)$, then $\Xi > 0$. Combining Theorem 2 and Lemma 1, we see that π is strictly decreasing in σ_z^2. Therefore, there exists a critical value for σ_z^2 such that if σ_z^2 is sufficiently high, the inequality (57) is satisfied and $\sigma_1^2(p_T) > \sigma^2(p^*)$. This proves the theorem. ∎

Proof of Theorem 5:

The proof parallels the proof of Theorem 1 (Step 6), so we provide a partial sketch. Suppose that trader i conjectures the demand functions of other traders in the second stage of the game are linear in price as described in (38):

$$q_i(p) = \gamma_1(1-\delta_1)(y_i - p) - \gamma_1\alpha_1 x_i - \gamma_1\delta_1\lambda_2 Z$$

The market clearing condition (32) then implies that p^* is a linear function of q_i, as given by equation (35). Observing p^* is equivalent to observing p_{-i}. Therefore, the conditional expectation is given by (36), and following the derivation of the solution to the game Γ_1, we see that the conjectures are satisfied in equilibrium, and the demands and price are as given by Theorem 3. In this case, Γ_2 is equivalent to Γ_1. ∎

Proof of Theorem 6:

In the non-transparent system, depth is the inverse of the derivative of the price functional (18) with respect to Z:

$$\Upsilon = N\gamma(1-\delta). \tag{64}$$

Under transparency, we use equation (48) to obtain:

$$\Upsilon_1 = \frac{N\gamma_1(1-\delta_1)}{1-\zeta N}. \tag{65}$$

From equations (52) and (55), $\sigma^2(p^*) < \sigma_1^2(p_T)$ if and only if

$$\gamma_1(1-\delta_1) < \gamma(1-\delta)(1-\zeta N). \tag{66}$$

Dividing both sides of this expression by $(1-\zeta N)$, we see that $\sigma^2(p^*) < \sigma_1^2(p_T)$ if and only if $\Upsilon_1 < \Upsilon$. Therefore, price variability and market depth are inversely related. Turning now to the implicit costs of trading, the implicit spread function follows from equation (3):

$$s(q_i) = 2\lambda|q_i|.$$

Similarly, from (35) the implicit spread in a transparent market is:

$$s_1(q_i) = 2\lambda_1|q_i|.$$

From Lemma 2, it follows that $\lambda_1 > \lambda$ so that for all $q_i \in \Re$, $s_1(q_i) > s(q_i)$. ∎