ASSET RETURNS, INVESTMENT HORIZONS, AND INTERTEMPORAL PREFERENCES

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Comments Welcome

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ABSTRACT

A representative-agent pricing model with time-varying moments of consumption growth is used to analyze implications about means and volatilities of equity returns and interest rates, first-order autocorrelations of equity returns for various investment horizons, and R^2's in projections of equity returns for various horizons on predetermined financial variables. An analysis using non-expected-utility preferences reveals that high risk aversion is key in matching empirical benchmarks for average returns, but low intertemporal substitution is important in obtaining implications corresponding to estimates of volatilities, autocorrelations, and the predictability of returns.

1. Introduction

Empirical evidence indicates that expected returns on stocks and bonds vary through time. Much of this evidence is characterized either by autocorrelations of returns or by regressions of returns on various predetermined variables.[1] The length of the holding period over which a return is computed, or the return "horizon," seems to affect the nature of this evidence in significant ways. For example, the sample autocorrelations of returns on indexes of NYSE stocks are positive and in the range of 0.1 to 0.2 for horizons of one month [e.g., Fama and Schwert (1977)], but sample autocorrelations for horizons of five years are negative and in the range of -0.2 to -0.5 [e.g., Stambaugh (1986) and Fama and French (1988)]. In essence, the pattern of autocorrelations is U-shaped with respect to return horizon.[2] Regressions of one-month stock returns on predetermined variables often produce R-squared values less than 0.02 [e.g., Keim and Stambaugh (1986)], whereas regressions of longer horizon returns (several years) on similar predetermined variables can produce R-squared values in excess of 0.30 [e.g., Fama and French (1989)].

The estimated behaviors of time-varying expected returns across horizons of various lengths enlarge the collection of empirical regularities confronting models of asset pricing. Previous studies have examined magnitudes of unconditional first and second moments of returns within the standard model that discounts payoffs by the marginal rates of substitution for a representative investor whose consumption equals the estimated average

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[1] A partial list of the studies reporting such evidence includes Fama and Schwert (1977), Hall (1981), Huizinga and Mishkin (1984), Fama (1984), Rozeff (1984), Keim and Stambaugh (1986), Campbell (1987), Fama and Bliss (1987), Fama and French (1988a, 1988b), Poterba and Summers (1988), Lo and MacKinlay (1988), and Huberman and Kandel (1990).

[2] This U-shaped pattern is also discussed by Poterba and Summers (1988). Lo and MacKinlay (1988) find positive autocorrelation in one-week index returns.

per capita. For example, Mehra and Prescott (1985) illustrate this model's difficulty in obtaining both a sufficiently low mean interest rate and a sufficiently high mean equity return, and Grossman and Shiller (1981) describe the model's difficulty in obtaining a sufficiently high variance of equity returns.

This study explores implications about moments of asset returns in a model having the standard feature of an infinitely-lived representative agent. We specify the preferences of the representative agent as being those modeled previously by Epstein and Zin (1989a, 1989b) and Weil (1989), wherein the expected utility axioms do not obtain and the agent's risk aversion is separated from his elasticity of intertemporal substitution. We examine implications about unconditional means and variances of asset returns, but we also explore implications about the predictability of future asset returns for various investment horizons. Section 2 develops the model and its implications about asset returns.

By constructing numerical examples, we explore the model's ability to mimic sample properties of asset returns, and we investigate the roles played by parameters of both the preference function and the consumption process. Our objective here is not to investigate whether various sample estimates, such as autocorrelations of long-horizon returns, are significantly different from zero or from another value implied by a given choice of the model's parameters.[3] We simply use the sample estimates as convenient benchmarks for exploring the model's capabilities and limitations.

Section 3 begins the numerical investigation by considering examples in the special case of time-additive expected utility. It appears that this case

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[3]For example, the statistical precision and reliability of autocorrelations of long-horizon returns is considered by Fama and French (1988a), Richardson (1988), and Cecchetti, Lam, and Mark (1988).

can accommodate unconditional first and second moments of consumption growth and asset returns as well as the general U-shaped pattern in estimated autocorrelations of equity returns, as long as risk aversion is allowed to assume a value that is high by traditional modeling standards. Since, in the time-additive case, specifying high risk aversion is equivalent to specifying (the inverse of) the elasticity of intertemporal substitution, it is difficult to determine which of the two effects plays the key role in obtaining the desired implication about a given moment.

In section 4 we turn to the more general specification of preferences in which the effects of risk aversion and intertermporal substitution can be gauged independently. It appears that high risk aversion is necessary to obtain implied first moments of the interest rate and the equity return that correspond to sample estimates. Our results here confirm those of Weil (1989), who finds that separating risk aversion from intertemporal substitution does not improve the ability to match first moments with a low value of risk aversion. It appears, however, that a relatively low value of intertemporal substitution is necessary to obtain an implied volatility of equity returns as high as that observed empirically. In addition, low intertemporal substitution appears to be the key requirement in obtaining implied patterns with respect to investment horizon of the first-order autocorrelation and the predictability of equity returns.

In order to match empirical benchmarks for first moments of returns, the representative-agent model appears to require relative risk aversion exceeding values traditionally thought to be reasonable. In section 5, we briefly review and reconsider several of the arguments often made against such high values, and we suggest that these arguments be viewed as less than conclusive.

2. The Pricing Model

2.1 Intertemporal Preferences

We develop a pricing model in the endowment framework of Lucas (1978), where the physical stock of capital is fixed and aggregate consumption equals aggregate output in each period. Let c_t denote aggregate consumption at time t.

The representative consumer's utility for future consumption is specified in the recursive form analyzed previously in studies by Epstein and Zin (1989a, 1989b), Epstein (1988), and Weil (1989), who build on the earlier work of Kreps and Porteus (1978). These preferences relax the standard assumptions that consumers maximize the expected value of a time-additive utility function. The infinitely lived consumer maximizes lifetime utility U_t, the recursive structure of which is given by

$$U_t = \left[c_t^{\frac{\eta-1}{\eta}} + \beta \, [E_t\{U_{t+1}^{(1-\alpha)}\}]^{\frac{\eta-1}{\eta(1-\alpha)}} \right]^{\frac{\eta}{\eta-1}} \tag{1}$$

where E_t is the conditional expectation operator given the information available at time t, $0 < \beta < 1$, $0 < \alpha \neq 1$, and $0 < \eta \neq 1$.[4]

The parameter α can be interpreted as the coefficient of relative risk aversion appropriate for computing certainty equivalents in atemporal gambles. The parameters η and β reflect intertemporal substitution and time preference. When future utility is deterministic, then β is the rate of time preference and η is the elasticity of intertemporal substitution. Throughout the paper we will refer to α as "risk aversion" and η as "intertemporal substitution."

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[4] Although (1) is not defined for $\alpha = 1$ and $\eta = 1$, these special cases can be included within the same general framework [e.g., Epstein and Zin (1989b) and Weil (1989)].

We investigate asset prices as determined by the Euler equation,

$$\beta^{\frac{(\alpha-1)\eta}{1-\eta}} \; E_t \left\{ (c_{t+1}/c_t)^{\frac{1-\alpha}{1-\eta}} (1 + R_{A,t+1,1})^{\frac{\alpha\eta-1}{1-\eta}} (1 + R_{k,t+1,1}) \right\} = 1 \quad , \quad (2)$$

where $R_{A,t+1,1}$ is the one-period rate of return from time t to time t+1 on the consumer's optimal portfolio (aggregate wealth), $R_{k,t+1,1}$ is the rate of return on any asset k, and E_t denotes the expectation conditioned on information at time t. In the case of time-additive expected utility, where $\alpha = 1/\eta$, equation (2) simplifies to the more familiar expression,

$$\beta \, E_t\{ (c_{t+1}/c_t)^{-\alpha} (1 + R_{k,t+1,1})\} = 1 \quad . \quad (3)$$

Our numerical investigations in later sections will first analyze the special case in (3) and then consider the general form in (2).

2.2 The Consumption Process

We specify a time-varying conditional distribution of consumption growth, and our modeling approach includes features similar to those used by Mehra and Prescott (1985) and Abel (1988). The representative consumer's consumption, c_t, is a nonstationary process in which the conditional distribution at time t of c_{t+1} is lognormal. Let λ_{t+1} denote unity plus the one-period growth rate in consumption, i.e., $\lambda_{t+1} = c_{t+1}/c_t$. Assume

(i) Conditional on information at time t, the logarithmic consumption growth rate, $\ln(\lambda_{t+1})$, is distributed normally with mean μ_t and standard deviation σ_t.

(ii) The pair $(\mu_t \; \sigma_t)$ follows a joint stationary Markov process with a finite number of states, S. That is $(\mu_t\ \sigma_t) \in \{(\mu_i, \sigma_i),\ i = 1, \ldots, S\}$. Let s_t denote the state for $(\mu_t\ \sigma_t)$ at time t. Let Φ denote the $S \times S$ transition matrix with (i,j) element

$$\phi_{ij} = \text{Prob}(s_{t+1} = j \mid s_t = i) \tag{4}$$

Let π denote the S-vector of steady-state probabilities.

(iii) Given s_t, the distribution of s_{t+1} is independent of λ_{t+1} (which is drawn from the distribution defined by s_t).

Given the assumptions above, the state of the economy follows a Markov process. There are an infinite number of states, each represented as (c, i), where, at time t, $c = c_t$ and $i = s_t$. There are an infinite number of values for consumption $(0 \leq c \leq \infty)$, but only a finite number of values for i, which represents the state for the conditional moments of consumption growth.

2.3 Asset Prices

We use this model to derive prices of various types of financial claims as well as conditional and unconditional moments of returns on various assets. We first define the following (shadow) prices when the economy is in state (c, i) at time t.

$P_F(c, i; N)$: the price of an N-period riskless claim on one unit of consumption at time t+N.

$P_A(c, i)$: the price of aggregate wealth (the claim on total future consumption).

$P_B(c, i; \theta)$: the price of a risky bond that pays at time t+1 either (i) a fraction θ of aggregate wealth at time t or (ii) total aggregate wealth plus consumption at time t+1, whichever is less.

$P_L(c, i)$: the price of levered equity, which is the claim on aggregate wealth net of a one-period risky bond with $\theta = \theta_L$, the degree of leverage in the economy.

The financial claims considered here have the properties that, in state (c, i), either the prices depend only on i and not on c, which is true for the price of the N-period riskless asset, $P_F(c, i; N)$, or the prices are homogeneous of degree one in c, which is true for the risky assets. For the latter cases we define

$$P_A(c, i) = w(i) \cdot c \tag{5}$$

$$P_B(c, i; \theta) = g(i; \theta) \cdot c \tag{6}$$

$$P_L(c, i) = q(i) \cdot c \tag{7}$$

Given the definition of levered equity, $q(i) = w(i) - g(i; \theta_L)$. Propositions 1 through 3 in the Appendix provide analytic expressions for the above asset prices in terms of the model's parameters.

The price of a risky asset is expressed as the product of current consumption and a quantity depending only on i, the state for (μ_t, σ_t). This multiplicative form suggests a decomposition of the natural logarithm of the price as the sum of two components, one stationary and the other nonstationary. With levered equity, for example,

$$\ln P_L(c, i) = \ln c + \ln q(i) \quad , \tag{8}$$

and it is clear from the assumptions and the previous discussion of the model that $\ln c$ is nonstationary, $\ln q(i)$ is stationary, and the increments in $\ln q(i)$ are independent of λ, the increments in $\ln c$. The same type of decomposition holds for the prices of aggregate wealth and risky debt.[5] Fama and French (1988a) and Poterba and Summers (1988) consider similar decompositions of prices into stationary and nonstationary components. Those studies consider a first-order autoregressive representation for the stationary component, but $\ln q(i)$ will not, in general, admit this type of linear representation.

2.4 Rates of Return

We now consider properties of rates of return for investment horizons of various lengths. Our starting point is the one-period rate of return on levered equity. Consider this return when the economy moves from state (c, i) at time t to state $(c\lambda_{t+1}, j)$ at time t+1. Given the definition of levered equity, coupled with the simplified representations of prices in equations (5) through (7), this one-period rate of return can be written as

$$R_{L,t+1,1} = \frac{\max(0,\ \lambda_{t+1}[1 + w(j)] - \theta_L w(i))}{q(i)} - 1 \quad . \tag{9}$$

A special case of (9) occurs for $\theta_L = 0$, in which case the return simplifies to the one-period return on total aggregate wealth,

[5] The model of Mehra and Prescott (1985) also admits this sort of decomposition for the price of aggregate wealth. In their model, both components depend on realized consumption growth.

$$R_{A,t+1,1} = \frac{\lambda_{t+1}[1 + w(j)]}{w(i)} - 1 \tag{10}$$

The N-period return on levered equity, covering the investment horizon from time t to t+N, is defined as

$$R_{L,t+N,N} = \prod_{n=1}^{N} (1 + R_{L,t+n,1}) - 1 \quad . \tag{11}$$

Denote the expected value of the N-period return in (11) as $E_L(c, i; N)$. It follows from (9) and the assumed consumption process that the conditional distribution of the rate of return when the economy is in state (c, i) depends only on i, the state for (μ_t, σ_t). Let the S-vectors $\mathbf{E}_L(N)$ and $\mathbf{V}_L(N)$ contain the conditional means and variances of the N-period returns on levered equity in each of the S states for (μ_t, σ_t). Propositions 4 and 5 in the Appendix provide analytical expressions for these conditional moments.

Unconditional moments of returns are obtained by combining the conditional moments of returns, $\mathbf{E}_L(N)$ and $\mathbf{V}_L(N)$, with the steady-state probabilities. The unconditional mean and variance of $R_{L,t+1,N}$ are given by

$$\bar{E}_L(N) \equiv E\{R_{L,t+N,N}\} = \pi'\mathbf{E}_L(N) \tag{12}$$

$$\bar{V}_L(N) \equiv \text{var}\{R_{L,t+N,N}\} = \pi'\mathbf{V}_L(N) + \pi'\Big[[\mathbf{E}_L(N) - \bar{E}_L(N)\iota]*[\mathbf{E}_L(N) - \bar{E}_L(N)\iota]\Big] \tag{13}$$

where ι is an S-vector of ones and "*" denotes a Hadamard matrix product.[6]

The first-order autocorrelation of equity returns for various investment

[6] If A, B and C are m×n matrices and A*B = C, then $c_{ij} = a_{ij} \cdot b_{ij}$.

horizons can be calculated from the above unconditional moments as

$$\mathrm{corr}(R_{L,t+N,N},\ R_{L,t,N}) = \frac{E\{1 + R_{L,t+N,2N}\} - [E\{1 + R_{L,t+N,N}\}]^2}{\mathrm{var}\{R_{L,t+N,N}\}}$$

$$= \frac{\bar{E}_L(2N) - 2\bar{E}_L(N) - [\bar{E}_L(N)]^2}{\bar{V}_L(N)} \tag{14}$$

2.5 <u>Projecting Returns on Predetermined Financial Variables</u>

The model also provides implications about the goodness of fit, or R-squared, in linear projections of equity returns for various horizons on predetermined financial variables. For this analysis we define three financial variables that depend only on i when the economy is in state (c, i). These variables correspond roughly to those used to predict equity returns in various empirical studies [e.g., Rozeff (1984), Keim and Stambaugh (1986) and Fama and French (1988b, 1989)].

The dividend-price ratio when the economy is in state (c,i) at time t is defined as the ratio of conditional expected consumption to the price of levered equity:

$$dp(i) = \frac{c \cdot E\{\lambda_{t+1}|i\}}{P_L(c,\ i)} = \frac{e^{\mu_i + \frac{1}{2}\sigma_i^2}}{q(i)} \tag{15}$$

The N-period term spread in state (c, i) is defined as the difference between the N-period riskless rate and the one-period riskless rate:

$$yterm(i) = R_F(i; N) - R_F(i; 1) \tag{16}$$

where

$$R_F(i; n) \equiv P_F(c, i; n)^{-1/n} - 1 \tag{17}$$

Recall, as noted earlier, that $P_F(c, i; n)$ does not depend on c. In the numerical examples considered below, we set N = 240 (months).

The default spread in state (c, i), is defined as the difference between the yield on a risky bond with leverage ratio $\theta = \theta_J$ minus the one-period riskless rate:

$$\begin{aligned} ydef(i) &= \frac{\theta_J P_A(c, i)}{P_B(c, i; \theta_J)} - R_F(i; 1) \\ &= \theta_J \frac{w(i)}{g(i; \theta_J)} - R_F(i; 1) \end{aligned} \tag{18}$$

In the examples considered below, we set $\theta_J = .95$. That is, the default spread reflects a risky bond that promises to pay 95% of current aggregate wealth.

Since the above three financial variables depend only on i when the economy is in state (c, i) at time t, it is straightforward to compute, $\bar{\rho}_N$, the multiple correlation coefficient (across i) between the expected levered-equity return for an N-period horizon, $E\{R_{L,t+N,N}|i\}$, and the three financial variables, dp(i), yterm(i), and ydef(i). The implied R-squared in a projection of $R_{L,t+N,N}$ on these three variables is then equal to $(\bar{\rho}_N)^2$ times the ratio of the variance of the conditional expected return to the variance of the return.

3. An Example with Time-Additive Utility

Our objective is to use the pricing model to investigate the properties of asset returns implied by alternative specifications of preferences and consumption processes. We begin by constructing an example in the more familiar framework of time-additive utility. The consumption process and the preference parameters in this example are chosen to produce implications that appear to be consistent with a number of previously reported empirical findings, including several of the benchmarks set forth by Mehra and Prescott (1985). Using this initial example as a point of departure, we first consider the effects of persistence in the consumption moments. Then in section 4, we use the more general form of the preference function to consider the separate effects of risk aversion and intertemporal substitution.

3.1 The Consumption Process

The consumption process in the model is defined by the Markov process for the conditional moments of consumption growth, (μ_t, σ_t). We construct our example with a four-state process in which each moment can take two distinct values: μ_t equals μ^+ or μ^- and σ_t equals σ^+ or σ^-. For simplicity, we assume equal unconditional probabilities of the four states, and we assume that μ_t and σ_t evolve independently of each other. The transition matrix Φ is then specified by two values, ρ_μ and ρ_σ, the autocorrelations of μ_t and σ_t. If the states, as numbered in ascending order, are (μ^+, σ^-), (μ^-, σ^-), (μ^+, σ^+), and (μ^-, σ^+), then

$$\Phi = .25 \left[\begin{pmatrix} 1+\rho_\sigma & 1-\rho_\sigma \\ 1-\rho_\sigma & 1+\rho_\sigma \end{pmatrix} \otimes \begin{pmatrix} 1+\rho_\mu & 1-\rho_\mu \\ 1-\rho_\mu & 1+\rho_\mu \end{pmatrix} \right] \quad . \tag{19}$$

In this example we initially set $\rho_\mu = 0.94$ and $\rho_\sigma = 0.20$, and these values allow the pricing model to produce implications about autocorrelations of equity returns that correspond roughly to the patterns found in the data. We then consider alternative specifications of ρ_μ and ρ_σ.

The example is constructed with a single period corresponding to one month. We specify the values μ^+, μ^-, σ^+, and σ^-, the conditional moments for the monthly logarithmic growth rates, so that the implied mean and the implied standard deviation for the annual simple growth rates equal those reported by Mehra and Prescott (1985) based on data for the period from 1889 through 1978. Panel A of table 1 reports the values for the monthly Markov process for (μ_t and σ_t), and panel B reports the implied moments for simple annual growth rates.

The values μ^+, μ^-, σ^+, and σ^- are chosen not only to match the two unconditional annual moments but also to allow the model to produce various implications about moments of asset returns that conform to empirical estimates. For example, many choices of μ^+ and μ^- are consistent with a given mean of the simple growth rate (which also depends on the σ's). For a given choice of preferences, however, changing μ^+ and μ^- changes the implied means and variances of asset returns, and some choices of μ^+ and μ^- may not even allow an equilibrium with positive prices for aggregate wealth. The process for consumption growth constructed in the example has conditional moments that do not vary greatly in relation to realized growth rates. For example, annual growth rates have an R-squared of about .0016 (the variance of the mean rate divided by the variance of the realized rate). Nevertheless, the implications about moments of asset returns will differ significantly from the case in which consumption growth rates are identically and independently distributed (i.i.d.).

3.2 The Equity Premium

Although our primary focus in this study is not the "equity premium puzzle" described by Mehra and Prescott (1985), the problem they pose presents a natural point at which to begin our investigation of the moments of asset returns implied by the model. To complete the specification of the numerical example in the case of time-additive utility, we must give values for the preference parameters α and β. In addition, since the "equity" in our model is levered, we must also specify the degree of leverage, θ_L. This first example is constructed with $\alpha = 29$, $\beta = .9978$, and $\theta_L = 0.44$.

We select these parameters in order that, when coupled with the assumed consumption process, the implied average one-year interest rate (0.80%) and one-year equity return (6.98%) match the values used by Mehra and Prescott as empirical benchmarks. (Since our model is specified on a monthly basis, a one-year return is a 12-period return.) Table 2 summarizes the moments of the returns on these assets implied in this example.

Mehra and Prescott also report estimates of the standard deviations of the annual real rate and the equity return. When we match the equity-return standard deviation of 16.54%, we obtain a real-rate standard deviation of 4.28%, somewhat less than the estimate of 5.67% reported by Mehra and Prescott. We return to this point below in our discussion of more general preferences.

Specifying relative risk aversion (α) equal to 29 violates the upper bound of 10 that Mehra and Prescott set for this parameter a priori, and they recognize that values of α greater than 10 allow one to match the first moments of interest rates and equity returns by making "small" changes in the consumption process. They do not address the feasibility of matching other moments of returns when making such changes. Section 5 will discuss the issue of "high" relative risk aversion in more detail.

3.3 Properties of Equity Returns for Long and Short Horizons

Figure 1 displays the implied first-order autocorrelations of equity returns for investment horizons ranging from one month through ten years [equation (14)]. The implied autocorrelations begin at -.08 for a one-month horizon, decline to -.21 at a 30-month horizon, and then increase toward zero for longer horizons. This U-shaped pattern is roughly consistent with the pattern of autocorrelations estimated by Fama and French (1988a).

Figure 1 plots two sets of estimated first-order autocorrelations. The first of these uses monthly returns on the value-weighted portfolio of stocks on the New York Stock Exchange for the period from December 1926 through December 1985.[7] The estimated autocorrelation is the slope coefficient in a regression of the current return on the lagged return, and the regression uses overlapping observations in the same manner as Fama and French (1988a).[8] The second set of sample autocorrelations, shown for investment horizons of 12 months, 24 months, etc., uses annual returns on Standard & Poor's Composite Index for the period from 1891 through 1985 (again using regressions with

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[7]These data are obtained from the Center for Research in Security Prices (CRSP) at the University of Chicago.

[8]The estimates are not bias adjusted. Fama and French (1988a) report simulation evidence suggesting that the bias in the estimated autocorrelations is, in general, not severe when the true autocorrelations are similar to those displayed in figure 1.

overlapping observations).[9]

As figure 1 demonstrates, the pricing model considered here can imply a pattern of negative autocorrelations for investment horizons of two years or more that appear to coincide reasonably well with estimates obtained from the data. The data also produce, however, positive sample autocorrelations for short investment horizons, such as one month.[10] We were unable to find parameter specifications for the model that result in positive autocorrelations at short horizons but negative autocorrelations at longer horizons. One feature of the model that could be important in this regard is the assumption that unexpected consumption growth does not impact the change in the conditional moments of consumption growth. [A similar assumption appears in the model of Abel (1988).] The independence between μ_t and σ_t used in this example does not appear to play a critical role. Kandel and Stambaugh (1990) obtain a similar pattern of implied autocorrelations from the time-additive expected-utility version of the model using a process in which μ_t and σ_t are correlated.

Figure 2 displays the implied R-squared values in projections of equity returns on the three predetermined financial variables defined in section 2.5, dp(i), yterm(i), and ydef(i). The implied R-squared value begins at .038 for one-month returns, rises to .091 for 29-month returns, and then declines gradually toward zero for longer return horizons. We also find that, for all investment horizons, the explanatory power of this projection is very close to the theoretical maximum--that provided by the true expected return. In other

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[9] These data are obtained from Wilson and Jones (1987) for the period prior to 1926 and from CRSP for the period thereafter.

[10] Previous evidence indicating positive autocorrelation in short-horizon stock-market returns includes Fama and Schwert (1977) and Lo and MacKinlay (1988).

words, the multiple correlation coefficient (computed across i) between the conditional expected return and the three financial variables is generally greater than 0.99.

We also show in figure 2 the sample values obtained in regressions of returns on the value-weighted portfolio of NYSE stocks on three financial variables. These variables are essentially the same as those used by Fama and French (1989):[11]

D/P : for the value-weighted portfolio of NYSE stocks, the ratio of dividends paid for the twelve months ending at t to the price at the end of month t.

$y_{Aaa} - y_{TB}$: the difference between the Aaa yield and the yield on a U.S. Treasury Bill with maturity closest to one month.

$y_{Baa} - y_{Aaa}$: the difference between Moody's average yield on bonds rated Baa and bonds rated Aaa.

The regressions are estimated using overlapping return horizons. As observed in previous studies, the R-squared is small in regressions using monthly returns, approximately .02. As the return horizon increases, however, the R-squared increases steadily, to more than .25 at a 48-month horizon. Although the R-squared implied in this example of the model does not reach values as high as the sample estimates, the pattern of lower values for short

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[11]Similar variables have also been used by other researchers to predict asset returns. For example, Rozeff (1984) finds that dividend-price ratios predict stock returns, and Keim and Stambaugh (1986) find that (among other variables) the difference in yields between low-grade bonds and Treasury Bills predicts stock and bond returns. Contemporaneous changes in similar variables have also been used as common risk factors in empirical investigations of multifactor pricing models. In the latter context, Chen, Roll, and Ross (1986) use return spreads between (i) low-grade and high-grade bonds and (ii) long-term high-grade bonds and Treasury Bills.

horizons and higher values at longer horizons is similar to that found in the sample values.[12]

3.4 The Role of Persistence in the Consumption Moments

In the above example, the autocorrelations of the consumption moments, ρ_μ and ρ_σ, were selected to produce implied autocorrelations of returns that correspond roughly to the patterns that appear in the data. We now take a closer look at the roles of ρ_μ and ρ_σ in determining the autocorrelations of returns.

When the model specifies variation in μ_t, the conditional mean of the logarithmic consumption growth rate, then the effects of ρ_σ on the return autocorrelations are negligible as compared to the effects of ρ_μ. In the above example, $\rho_\mu = 0.94$ and $\rho_\sigma = 0.20$, but only the first of these choices plays an important role in affecting the pattern of implied autocorrelations of returns displayed in figure 1. Figure 3 displays implied autocorrelations of returns for the original value of $\rho_\mu = 0.94$ as well as for $\rho_\mu = 0.5$ and $\rho_\mu = 0$. For $\rho_\mu = 0.5$, the return autocorrelations begin at -0.21 for a one-month horizon, reach a minimum of -0.30 at a three-month horizon, and then increase toward zero for longer horizons. For $\rho_\mu = 0$, the autocorrelations begin at -0.41 for a one-month horizon and then increase monotonically toward zero. In general, lowering ρ_μ tends to lower the (negative) short-horizon autocorrelations and raise the autocorrelations for longer horizons. The value of ρ_σ is held constant at 0.2 for these three values of ρ_μ, but very similar patterns are obtained for different values of ρ_σ.

When the model specifies virtually no variation in μ, then ρ_σ appears to

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[12] The sample R-squared values provide upward biased estimates of the true R-squared, due primarily to the autocorrelation in the residuals caused by the use of overlapping observations.

play much the same role as does ρ_μ in the previous examples. Figure 4 displays implied return autocorrelations when μ_t is held constant ($\mu^+ = \mu^- =$ .146%). A relatively high value of $\rho_\sigma = .9$ produces a pattern similar to that previously obtained with the high value of ρ_μ. The autocorrelations begin at -0.04 for a one-month horizon, decline to -0.24 for a horizon of 19 months, and then increase toward zero. Similarly, the values of $\rho_\sigma = 0.5$ and $\rho_\sigma = 0$ produce return autocorrelations very similar to those obtained previously when ρ_μ assumed those values.

We conclude that persistence in the conditional mean is the most important property of the consumption process for determining the pattern of return autocorrelations. Persistence in the conditional variance of consumption growth appears to play a secondary but similar role.

4. Separating Risk Aversion and Intertemporal Substitution

In the time additive case, since risk aversion α is linked to intertemporal substitution η by the relation $\alpha = 1/\eta$, the high risk aversion specified in the previous example necessarily implies a correspondingly low value of intertemporal substitution. In order to discover the roles that each of these parameters plays in obtaining the desired moments of asset returns, we analyze in this section the general non-expected-utility versions of the preferences in (1). As discussed earlier, these preferences break the link between risk aversion and intertemporal substitution that characterizes the time-additive case.[13]

In this section we construct examples intended to illustrate the separate

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[13] Constantinides (1989) considers time-non-separable "habit-forming" preferences that maintain the expected-utility framework but also separate risk aversion from intertemporal substitution. See also Bergman (1985) and Sundaresan (1989) for analyses of time-non-separable preferences in the expected-utility framework.

effects of risk aversion and intertemporal substitution on the moments of returns analyzed in the previous sections. We consider four values of risk aversion (α = 1/2, 2, 10, 29) and four values for intertemporal substitution (η = 2, 1/2, 1/10, 1/29). Four of the sixteen combinations represent time-additive utility where $\alpha = 1/\eta$. The last pairing (α = 29, η = 1/29) is the one used in the example of section 3. The consumption process, the rate of time preference (β = 0.9978), and the degree of leverage (θ_L = 0.44) are also taken from that example.

Table 3 presents the unconditional means and standard deviations of the annual interest rate and the equity return as implied by the model for the various combinations of α and η. The two columns under "unconditional mean" correspond to tables 1 and 2 of Weil (1989), who reports the unconditional means of the interest rate and the equity premium for various combinations of α and η.[14] As Weil observes, lowering η for a given α raises the interest rate. The time-additive version of the model appears incapable of delivering a sufficiently low interest rate with traditionally "moderate" levels of risk aversion, and Weil concludes that lowering intertemporal substitution only aggravates this problem. Stated differently, and as confirmed by table 3, it appears that high risk aversion is required to match both of the desired first moments of interest rates and equity returns, whether or not intertemporal substitution is high or low.

Weil (1989) in fact makes the stronger statement that "there is no way to fit both the level of the risk-free rate and the risk premium when the [Von Neumann-Morgenstern] restrictions are imposed" (p. 412). He then observes that separating α and η allows such a fit, but only for an "implausibly" high

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[14] Weil uses the two-state process for realized consumption growth first specified by Mehra and Prescott (1985).

value of α. As we see here in tables 2 and 3, a high value of α is indeed required, but the first moments of both returns can be fit within the framework of time-additive expected utility (with $\beta < 1$, as Weil requires also).[15]

In addition to the challenges presented by observed means of asset returns, observed volatilities of returns have also presented challenges to standard models of asset pricing. For example, Grossman and Shiller (1981) conclude that it is difficult to reconcile the observed volatility of equity returns with the observed volatilities of growth rates in consumption and dividends. The last column of table 3 indicates that the implied volatility of equity returns is decreasing in η but virtually unaffected by α. Thus, a low value of intertemporal substitution appears to be important in obtaining a sufficiently high volatility of equity returns. We must caution that the separate effects of these preference parameters described here are probably better viewed as relative than as absolute. "Dividends" in this simple model are assumed to equal consumption, and sample estimates suggest that such an assumption understates the volatility of the growth rate in dividends relative to that of consumption. Nevertheless, even if dividends are assumed to be as smooth as consumption, it appears that the observed volatility of equity returns can be matched by specifying a sufficiently low value for intertemporal substitution, and that value appears to be virtually independent of risk aversion.

The implied volatility of the interest rate appears to depend importantly on both α and η, and neither parameter has a monotonic effect

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[15] The reader should also note that our consumption process differs from the Mehra-Prescott (1985) process used by Weil. In addition, the equity in our model is levered, whereas Weil treats equity as an unlevered claim on aggregate consumption.

across the ranges considered. The highest interest-rate volatilities occur for the highest values of risk aversion, and in those cases the volatility is increasing in η. For the level of risk aversion that appears necessary to match the desired first moments, that is $\alpha = 29$, we see that low intertemporal substitution is required to obtain both a sufficiently low interest-rate volatility and a sufficiently high equity-return volatility. Recall that, in the time-additive example in section 3, the implied volatility of the interest rate (4.3%) is slightly lower than the empirical benchmark (5.7%). By making η higher than $1/\alpha$ and making slight changes to the other parameters of the model, we can match the first and second moments of both the interest rate and the equity return.

The separate effects of α and η on the autocorrelations of equity returns are displayed in figure 5. For higher values of η, the autocorrelations are negative but close to zero for all horizons. As η is lowered, the pattern of autocorrelations becomes more U-shaped with respect to investment horizon. In all cases, the effects of α appear to be negligible. For example, the values giving the U-shaped pattern in the time-additive example of section 3 (cf. figure 1) obtain whether $\alpha = 1/2$ or $\alpha = 29$. Similarly, the autocorrelations are essentially flat at zero for $\eta = 1/2$, whether $\alpha = 1/2$ or $\alpha = 29$. Thus, it appears that low intertemporal substitution is the key to generating the patterns of negative autocorrelations found in sample estimates.

Figure 6 displays the effects of α and η on the implied R-squared in projections of equity returns on the three predetermined financial variables. Both α and η exert significant effects, although η appears to be more important. For $\eta = 1/2$, the implied R-squared is virtually zero for all investment horizons, whether $\alpha = 1/2$ or $\alpha = 29$. Lower values of η produce larger R-squared values, particularly at the longer horizons, and α has a greater effect in these cases. For a given η, the R-squared value is decreasing in α. With $\alpha = 1/2$ and $\eta = 1/29$, the implied R-squared is only 0.02 for a one-month horizon but reaches a maximum of 0.23 for a thirty-month horizon.

We can summarize much of this section by observing that, while high risk aversion seems necessary to match estimated first moments of equity returns and interest rates, low intertemporal substitution generally appears to play the more important role in the matching estimates of the equity return's volatility, autocorrelation, and predictability.

5. On High Relative Risk Aversion

The value for relative risk aversion of $\alpha = 29$ used in the previous examples exceeds levels traditionally thought to be reasonable. As noted earlier, for example, Mehra and Prescott (1985) restrict α to be 10 or less in their investigation. A central argument for a lower value of α, cited by Mehra and Prescott and many others, relies on the estimated "price of risk," as computed for example by Friend and Blume (1975). Friend and Blume use the relation wherein ω, the fraction of wealth placed in risky assets, is given by

$$\omega = \frac{1}{\alpha} \cdot \left[\frac{E(R_S) - R_F}{\operatorname{var}(R_S)} \right] , \qquad (20)$$

where the bracketed quantity, the price of risk, is the ratio of the expected excess return on the risky asset (stocks) to the variance of the risky asset's return. With an estimated price of risk of about 1.7, and the proportion of wealth in risky assets between 0.5 and 0.8, Friend and Blume arrive at an estimate of α between 2 and 3. While this calculation is appropriate in the case where consumption growth rates (and thus returns) are independently and identically distributed (i.i.d.), equation (20) generally does not obtain in other settings.[16]

In fact, the implied price of risk for equity in our original example equals the sample estimate of 2.26 for the Mehra-Prescott sample period of 1889-1978, since the model is calibrated to match the average interest rate, the average equity return, and the variance of the equity return (cf. table 2). In other words, the Friend and Blume approach applied to interest rates and stock returns generated by our example with $\alpha = 29$ would lead one to infer that α is in the range of 3 to 4. Kocherlakota (1988) demonstrates a similar result using $\alpha = 13.7$ and a value of β greater than unity ($\beta = 1.14$ for annual periods).

A closely related argument often made against higher values of α is that investors with this degree of risk aversion would, based on (20), place fractions of wealth in the risky asset that would be much lower than what we observe in practice. For example, with a price of risk equal to 2 and $\alpha = 29$, the fraction of wealth placed in risky assets would be only 0.07. Again, since (20) generally fails when returns are not i.i.d., this calculation is misleading. In an i.i.d. environment, an investor will (optimally) consume a fixed fraction of wealth, thereby equating the variances of the growth rates of consumption and wealth. A high α leads the investor to prefer a low variance of consumption, thereby dictating, in this i.i.d. setting, a low variance of wealth and a low choice of ω. In other settings, however, optimal consumption will be smoothed relative to wealth and thereby be consistent with higher fractions of wealth in the risky asset. Black

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[16] Brown and Gibbons (1985) also assume i.i.d. consumption growth rates in their approach to estimating the coefficient of relative risk aversion.

(1988) makes this point in the context of a continuous time production economy. Numerical examples of risky-asset demand in such production-type models have yet to be obtained. The model developed here is that of an exchange economy, and the relative demand for risky assets is not endogenous. Without further analysis, however, it is at least premature to use (20) to argue against a high α in models with time-varying moments.

High α values might also appear to be inconsistent with patterns across countries in interest rates and average rates of consumption growth. A familiar relation from neoclassical growth theory expresses the interest rate R_F as

$$R_F = \rho + \alpha \mu \tag{21}$$

where ρ is the discount factor $[= (1-\beta)/\beta]$ and μ is the expected consumption growth rate. Consider that, based on sample averages, expected consumption growth in Korea is about 4% per year higher than in the U.S.[17] With α and ρ identical in both countries, if $\alpha = 29$ then equation (21) implies a difference in interest rates between the two countries of 116%. However, equation (21) ignores the effects of uncertainty in growth rates. If, for example, growth rates in a given country are independently distributed lognormal with mean μ and variance σ^2, then (21) is replaced by

$$R_F = \rho + \alpha \mu - \frac{1}{2} (\alpha \sigma)^2 \tag{22}$$

[e.g., Rubinstein (1974)]. If interest rates were the same in both countries, then a 4% difference in expected growth rates would require a difference in

[17]We are grateful to Robert Lucas for suggesting this example.

σ's of about 2.8% (using the Mehra-Prescott σ of 3.57% for the U.S.). A cursory examination of consumption data for both countries published by the International Monetary Fund reveals that, for the period from 1957 to 1987, Korea's estimated σ exceeds that of the U.S. by about 2.3%. Although this exercise is not intended to confirm a higher value of α, it does illustrate the pitfalls in arguments based on the more familiar expression in (21).

Numerous empirical investigations of asset pricing models report estimates of relative risk aversion that vary over a wide range, depending on (i) the specification of the asset pricing model's implications, (ii) the sample period, (iii) the frequency of the data (monthly, quarterly, or annual), (iv) the use of real or nominal quantities, and (v) adjustments for temporal aggregation. Studies using monthly consumption data often obtain estimates of relative risk aversion below unity [e.g., Hansen and Singleton (1982, 1983)], while other studies that use quarterly or annual data and attempt to account for temporal aggregation have obtained estimates of 100 or more. Grossman, Melino, and Shiller (1987) obtain large estimates in some cases and describe their estimates over 20 as "too large to be plausible," but they do observe that their highest estimates of risk aversion are accompanied by their weakest evidence against the pricing model's overidentifying restrictions. Naik and Ronn (1988) argue that the higher estimated values of relative risk aversion are consistent with reasonable real interest rates and equity premiums. Hall (1988) concludes, based on an empirical examination of consumption and asset returns, that the intertemporal elasticity of substitution is probably quite low. If utility is additively separable, this result also implies a high degree of relative risk aversion (although Hall does not argue for such a conclusion).

Inferences about α are perhaps most elusive when pursued in the introspective context of thought experiments. An inherent difficulty in such experiments is the assumption that relative risk aversion is globally constant. When relative risk aversion is assumed to be constant over a wide range of variation in wealth, it seems possible to construct a gamble that makes any given value of α seem unreasonable. To illustrate the difficulty, consider an investor with current wealth of \$75,000 who is faced with a gamble in which X dollars are won or lost with equal probability. As Mankiw and Zeldes (1989) note, if X were \$25,000, then an investor with relative risk aversion of 30 would pay about \$24,000 to avoid taking the gamble. This might seem like a large payment, suggesting that relative risk aversion is much lower. Suppose we entertain a value of $\alpha = 2$, which would imply a more reasonable payment of \$8,333 to avoid this gamble. Now suppose X were instead 74,250. The investor with $\alpha = 2$ would pay 73,507 to avoid this gamble. This might again seem like a large payment, suggesting an even lower value for α. On the other hand, if X were \$375, then an investor with $\alpha = 2$ would pay \$1.88 to avoid the gamble, which would seem to be too small a payment. Alternatively, the investor with $\alpha = 30$ would pay \$28 to avoid the gamble of \$375, which seems more reasonable.

Figures 7 and 8 illustrate, for small and large gambles, the relation between the fraction of X paid to avoid the gamble and the ratio of X to the investor's wealth. The fraction of X paid to avoid the gamble approaches zero as X becomes small and, for $\alpha \geq 1$, approaches unity as X approaches the investor's total wealth.[18] Clearly, the larger the gamble, the greater the reliance placed on the constancy of relative risk aversion. Figure 7 is relevant for gambles that make low values of α seem too low. This figure,

[18] For $\alpha < 1$, the fraction of X paid approaches $1 - (1/2)^{[\alpha/(1-\alpha)]}$ as X approaches total wealth.

representing small gambles, is robust to alternative specifications of expected-utility preferences.[19] On the other hand, figure 8, representing large gambles, depends importantly on the assumption that relative risk aversion is globally constant, and this figure is relevant for gambles that make high values of α seem too high.

Obviously, we follow others in specifying preferences with constant relative risk aversion due to analytical tractability. Although this assumption is important to the outcome of thought experiments with large gambles, the extent to which this assumption is crucial to the implications of the pricing model is less clear. Some clues may lie in a recent study by Epstein and Zin (1989c). They obtain a higher implied equity premium using a form of preferences in which investors pay more to avoid small gambles, similar to what a high value of α would imply in our context, while investors pay less to avoid large gambles than a high value of α would imply here. Their results viewed together with ours suggest that the key ingredient in obtaining a high equity premium is a high aversion to small gambles, such as in the graph for $\alpha = 30$ in figure 7, whether or not the aversion to large gambles is as great as the graph for $\alpha = 30$ in figure 8.

6. Conclusions

A pricing model in which a representative investor maximizes time-additive expected utility can produce implications about various moments of asset returns that correspond to sample estimates. The model analyzed here assumes that the conditional distribution of the monthly consumption growth rate is lognormal with moments that follow a finite-state Markov process. We analyze numerical examples of the model in which the implied unconditional

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[19] See Pratt (1964).

mean and variance of annual consumption growth rates match empirical benchmarks. Appropriate choices of the model's parameters allow us to match unconditional first and second moments of asset returns as well as the patterns with respect to investment horizons of the first-order autocorrelation of equity returns and the R-squared in projections of equity returns on financial variables.

Matching the various moments of returns when utility is time-additive appears to require relative risk aversion that is high by traditional modeling standards. Since high risk aversion is equivalent to a low elasticity of intertemporal substitution, it is not clear which of these properties is important in obtaining the desired implication about a given moment of asset returns. When risk aversion and intertemporal substitution are allowed to enter preferences separately, it appears that matching the first moments of interest rates and equity returns requires high risk aversion. On the other hand, low intertemporal substitution appears necessary to obtain the desired implications about the volatility, the autocorrelation, and the predictability of equity returns.

This study illustrates both capabilities and limitations of the standard representative-agent framework in producing desired implications about unconditional means and variances of asset returns as well as the behavior of autocorrelations of returns for investment horizons of various lengths. Foremost among the limitations is that high relative risk aversion, equal to 29 in our numerical examples, is necessary to obtain the desired unconditional mean returns. Although our analysis may not remove objections to such a value, we believe that precluding such values a priori is more difficult than suggested by arguments often made in this regard.

APPENDIX

Proposition 1.

$$P_A(c,\ i) = c \cdot w(i) \tag{A1}$$

where w(i) is the i-th element of the vector **w** satisfying

$$w(i)^{\frac{\eta(\alpha-1)}{\eta-1}} = \sum_{j=1}^{S} \phi_{ij} \cdot E\{\lambda^{(1-\alpha)}|i\} \cdot \{\beta\cdot[1+w(j)]\}^{\frac{\eta(\alpha-1)}{1-\eta}} \quad . \tag{A2}$$

and $E\{\ \cdot \mid i\ \}$ denotes the conditional expectation given that $(\mu\ \sigma)$ is in state i.

Proposition 2.

$$P_B(c,\ i;\ \theta) = g(i;\ \theta)\cdot c \tag{A3}$$

where

$$g(i;\ \theta) =$$

$$= \sum_{j=1}^{S} \beta^{\frac{\eta(\alpha-1)}{1-\eta}} \cdot \phi_{ij} \cdot [(1+w(j))/w(i)]^{\frac{1-\eta\alpha}{\eta-1}} \cdot$$

$$\cdot \quad E\{\ \min\ [\lambda^{1-\alpha} \cdot (1+w(j))\ ,\ \lambda^{-\alpha} \cdot \theta \cdot w(i)]\ |\ i\}. \tag{A4}$$

Proposition 3.

$$P_F(c,\ i;\ N) = (\Psi^N \iota)_i \quad , \tag{A5}$$

where Ψ is the $S \times S$ matrix with (i,j) element

$$\psi_{ij} = \beta^{\frac{\eta(\alpha-1)}{1-\eta}} \cdot \phi_{ij} \cdot E\{\lambda^{-\alpha}|i\} \cdot [(w(j)+1)/w(i)\ ]^{\frac{\eta\alpha-1}{1-\eta}} \quad . \tag{A6}$$

Proposition 4.

$$E_L(N) = \Gamma^N \iota - \iota \tag{A7}$$

where ι denotes an S-vector of ones, Γ^N denotes the N-th matrix power of Γ, Γ is an S×S matrix with (i, j) element

$$\gamma_{ij} = \frac{E\{\max[0, \lambda \cdot (1 + w(j)) - \theta_L \cdot w(i)] \mid i\}}{q(i)} , \tag{A8}$$

and

$$q(i) = w(i) - g(i; \theta_L) . \tag{A9}$$

Proposition 5.

$$V_L(N) = \Xi^N \iota - [(\Gamma^N \iota)*(\Gamma^N \iota)] , \tag{A10}$$

where Ξ is an S×S matrix with (i, j) element

$$\xi_{ij} = \phi_{ij} \frac{E\{(\max[0, \lambda \cdot (1 + w(j)) - \theta_L \cdot w(i) \cdot (1+w(j)])^2 \mid i\}}{[q(i)]^2} . \tag{A11}$$

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Table 1

The Consumption Process in the Numerical Example

A. The Markov Process for the Conditional Moments of the Monthly Growth Rate of Consumption

State	Logarithmic Growth Rate (%) Mean (μ)	Logarithmic Growth Rate (%) Std. Dev. (σ)	Probability of moving to state 1	2	3	4	Unconditional Probability
1	0.162	0.875	0.582	0.018	0.388	0.012	0.25
2	0.130	0.875	0.018	0.582	0.012	0.388	0.25
3	0.162	1.130	0.388	0.012	0.582	0.018	0.25
4	0.130	1.130	0.012	0.388	0.018	0.582	0.25

Autocorrelation of μ : 0.94
Autocorrelation of σ : 0.20

Correlation between μ and σ : 0

B. Implied Unconditional Moments for Annual Series

	Mean (%)	Standard Deviation (%)	First-Order Autocorrelation
Growth Rate (Simple)	1.830	3.570	0.001
Conditional Expected Growth Rate	1.830	0.142	0.476
Conditional Standard Deviation of the Growth Rate	3.567	0.047	0.005

Table 2

Unconditional Moments of Annual Asset Returns Implied in the Example with Time-Additive Utility[a]

	Mean (%)	Standard Deviation (%)	First-Order Autocorrelation
Riskless rate	0.80	4.28	0.87
Return on levered equity[b]	6.98	16.54	-0.17

[a]The representative consumer is assumed to maximize

$$E_t\left\{ \sum_{\tau=t}^{\infty} \beta^{\tau-t} \frac{c_\tau^{1-\alpha} - 1}{1 - \alpha} \right\}.$$

The numerical example is constructed using $\alpha = 29$ and $\beta = .9978$ (monthly). The logarithmic consumption growth rate obeys a conditional normal distribution with moments following the Markov process given in panel A of table 1.

[b]Levered equity is defined with $\theta_L = 0.44$. (That is, levered equity accounts for roughly sixty percent of aggregate wealth.)

Table 3

Moments of Annual Returns Implied for Alternative Specifications of Risk Aversion and Intertemporal Substitution

Preference Parameters		Unconditional Mean (%)		Unconditional Std. Deviation (%)	
Risk Aversion	Intertemporal Substitution	Interest Rate	Equity Return	Interest Rate	Equity Return
1/2	2	2.45	2.56	0.07	6.42
1/2	1/2	5.23	5.34	0.13	6.59
1/2	1/10	21.34	21.55	0.90	8.62
1/2	1/29	70.08	70.85	1.11	16.63
2	2	2.31	2.76	0.49	6.43
2	1/2	4.94	5.40	0.29	6.59
2	1/10	20.12	20.70	0.51	8.58
2	1/29	65.44	66.72	0.83	16.47
10	2	1.63	3.81	2.74	6.49
10	1/2	3.46	5.68	2.52	6.61
10	1/10	13.84	16.30	1.59	8.43
10	1/29	42.80	46.32	0.65	15.91
29	2	0.40	6.36	8.27	6.65
29	1/2	0.43	6.35	8.06	6.67
29	1/10	0.56	6.39	6.94	8.24
29	1/29	0.80	6.98	4.28	16.54

Note: The consumer is assumed to maximize lifetime utility U_t, the recursive structure of which is given by

$$U_t = \left[c_t^{\frac{\eta-1}{\eta}} + \beta\, [E_t\{U_{t+1}^{(1-\alpha)}\}]^{\frac{\eta-1}{\eta(1-\alpha)}} \right]^{\frac{\eta}{\eta-1}},$$

where risk aversion is denoted by α, and intertemporal substitution is denoted by η. The logarithmic consumption growth rate obeys a conditional normal distribution with moments following the Markov process given in panel A of table 1. All of the cases set the rate of time preference $\beta = .9978$ (monthly), and equity is defined with the leverage parameter $\theta_L = 0.44$. (That is, levered equity accounts for roughly sixty percent of aggregate wealth.)

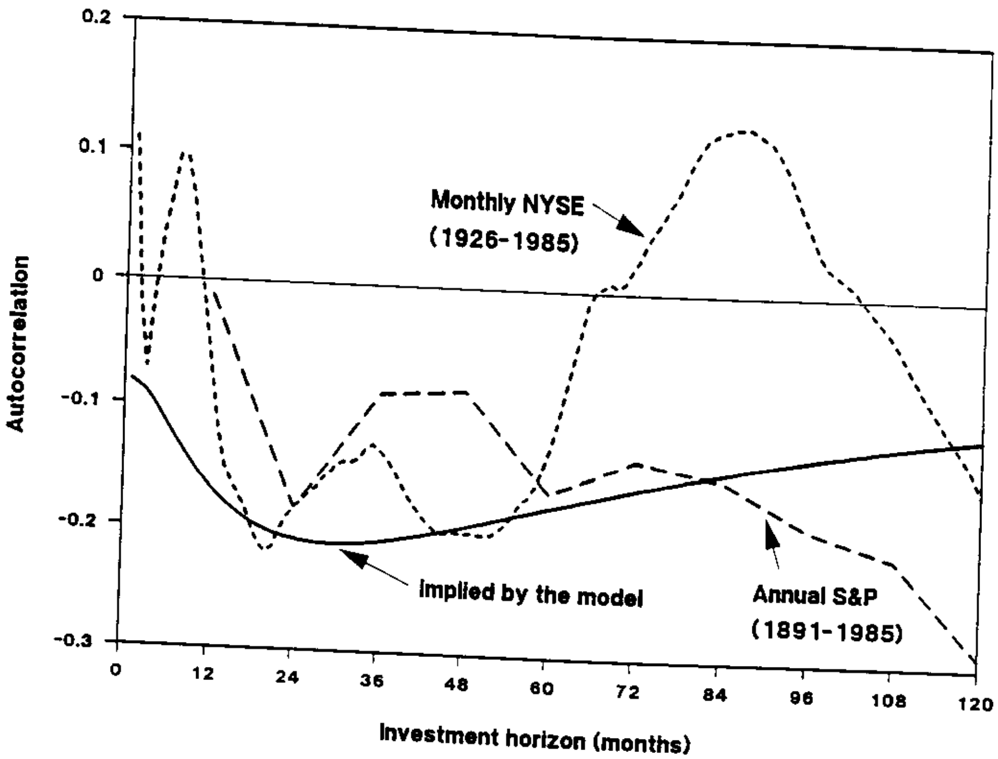

Figure 1. First-order autocorrelations of equity returns for various investment horizons.

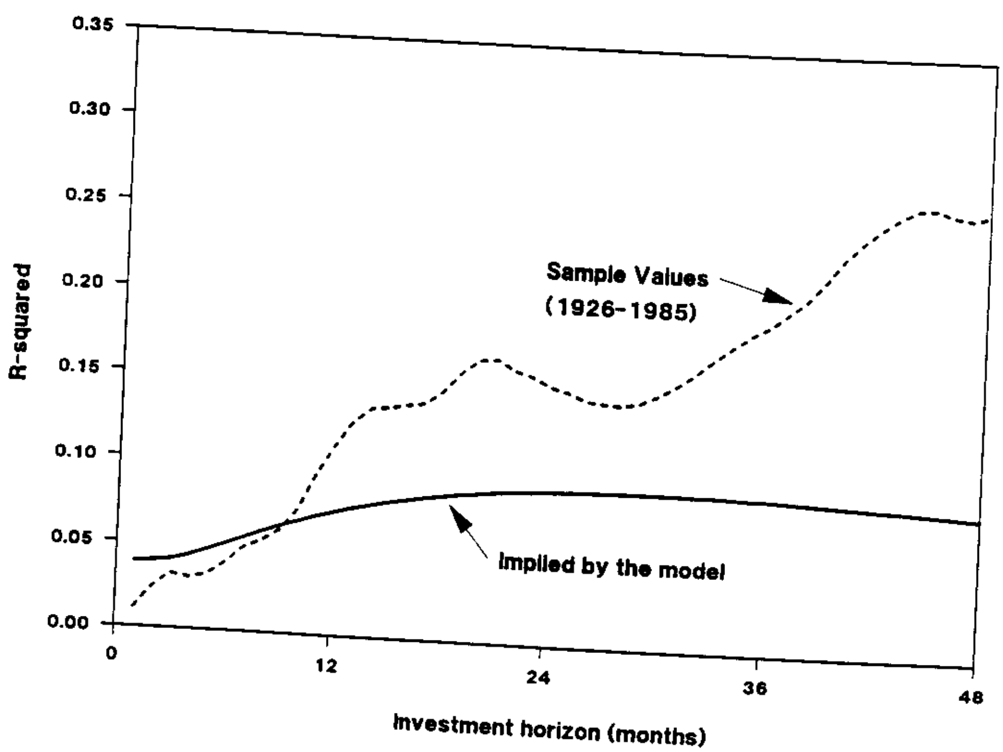

Figure 2. R-squared in projections of equity returns for various investment horizons on a dividend-price ratio, a term spread, and a default spread.

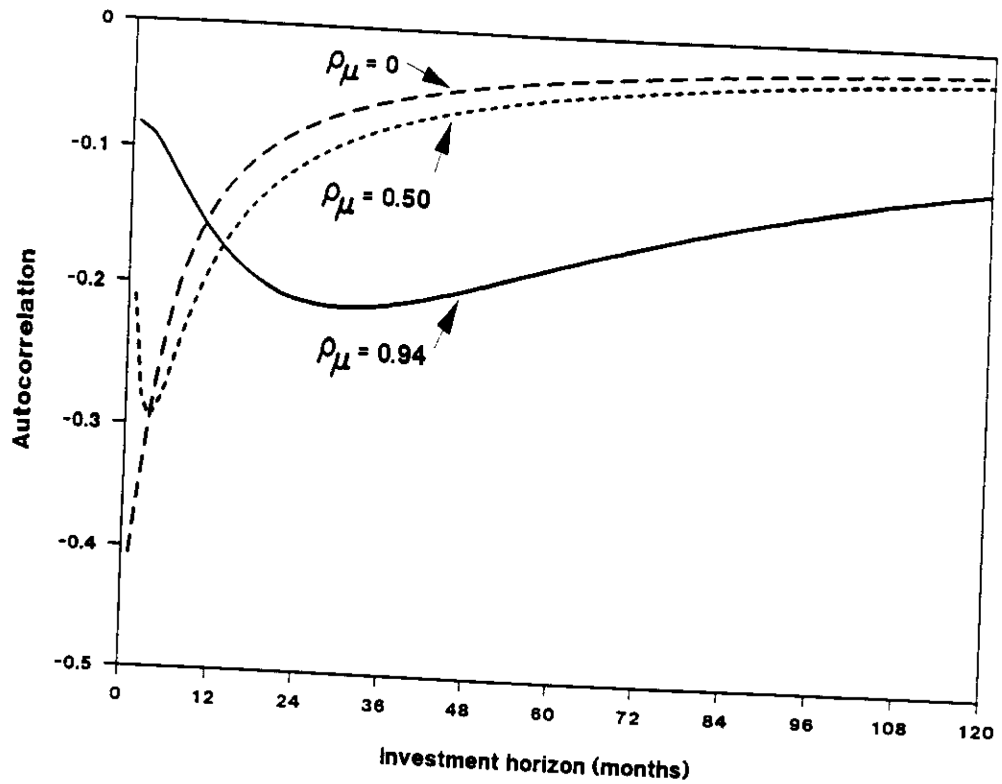

Figure 3. First-order autocorrelations of equity returns for various investment horizons. The values plotted are those implied by the equilibrium model for alternative values of ρ_μ, the autocorrelation of the expected monthly consumption growth rate.

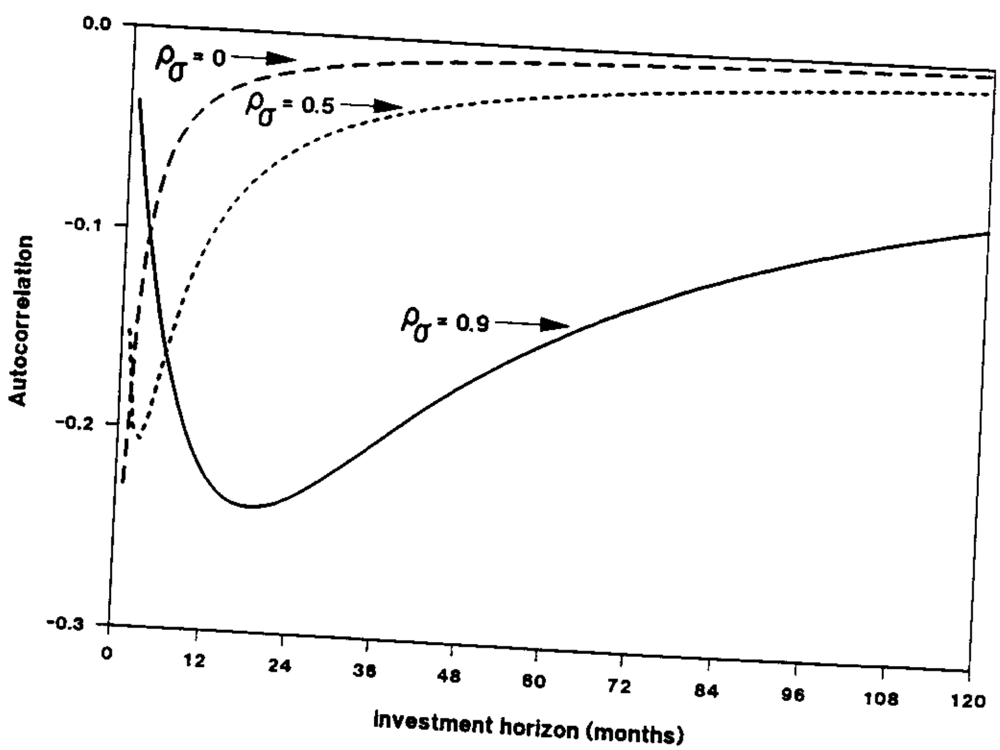

Figure 4. First-order autocorrelations of equity returns for various investment horizons. The values plotted are those implied by the equilibrium model for alternative values of ρ_σ, the autocorrelation of the standard deviation of the monthly consumption growth rate.

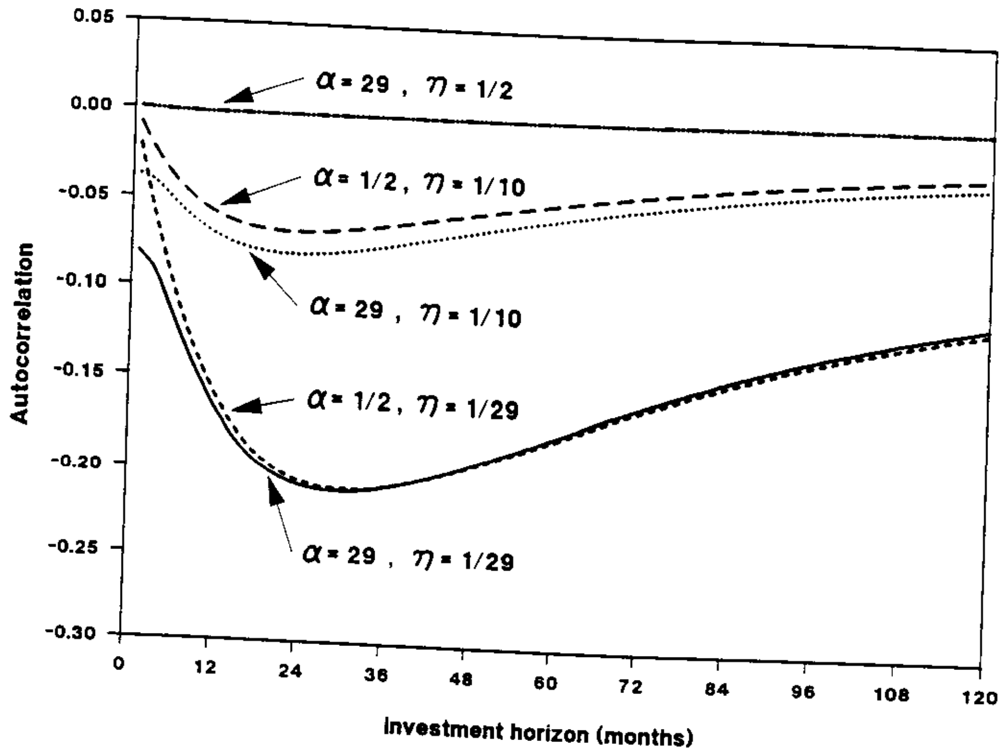

Figure 5. First order autocorrelations of equity returns for various investment horizons. The values plotted are those implied by the equilibrium model for alternative values of risk aversion (α) and intertemporal substitution (η).

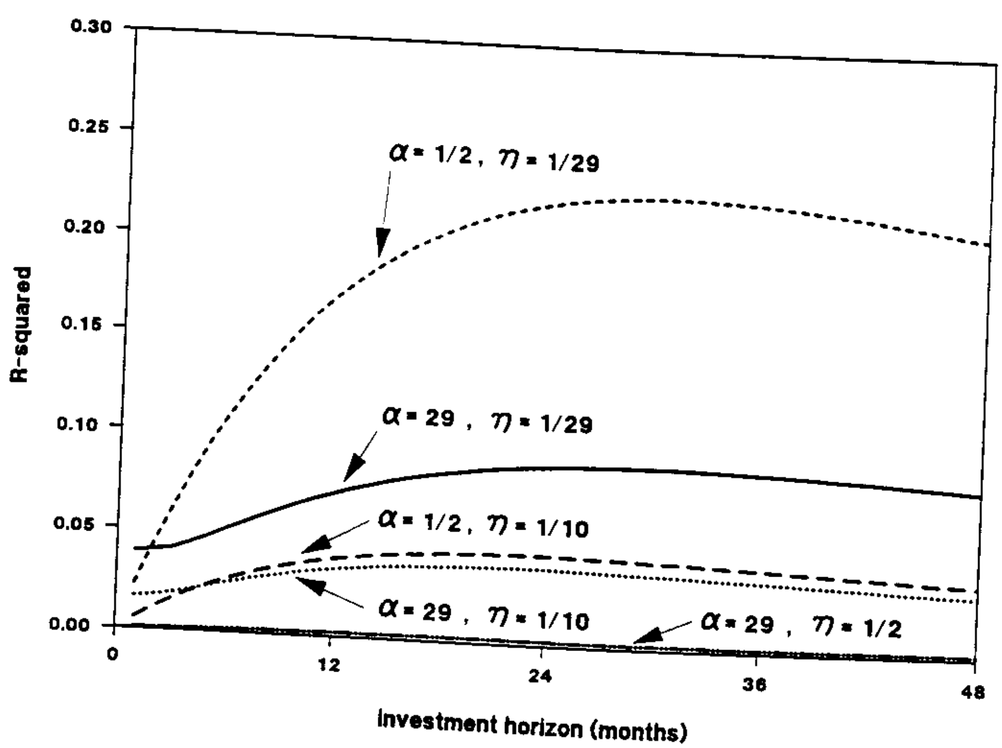

Figure 6. R-squared in projections of equity returns for various investment horizons on a dividend-price ratio, a term spread, and a default spread. The values plotted are those implied by the equilibrium model for alternative values of risk aversion (α) and intertemporal substitution (η).

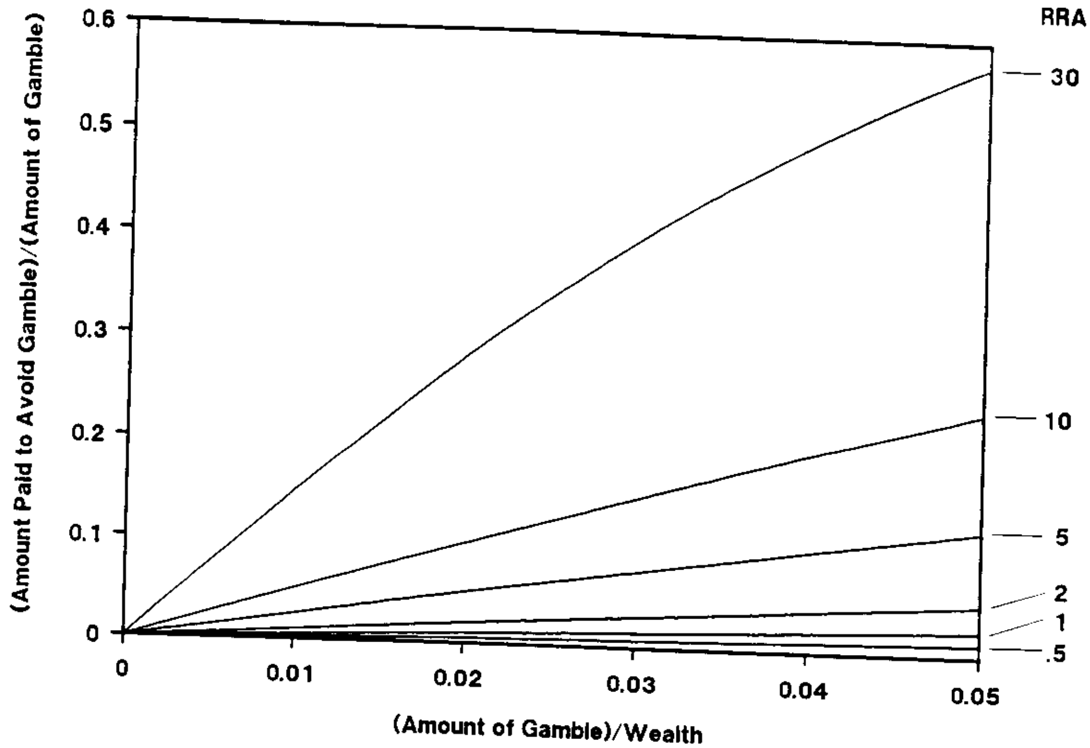

Figure 7. Small gambles with constant relative risk aversion (RRA). The amount gambled is won or lost with equal probabilities.

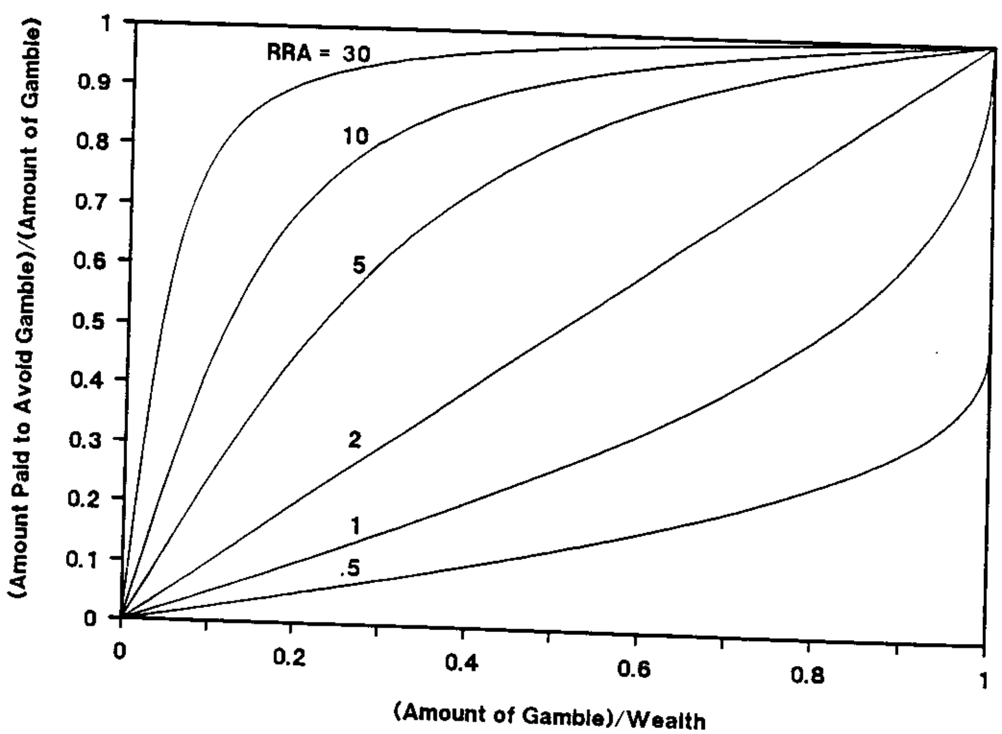

Figure 8. Large gambles with constant relative risk aversion (RRA). The amount gambled is won or lost with equal probabilities.