EXCHANGE RATE UNCERTAINTY, FORWARD CONTRACTS AND THE PERFORMANCE OF GLOBAL EQUITY PORTFOLIOS

by

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Abstract: A empirical analysis of the effects of foreign exchange risk on global equity portfolios is performed. For typical levels of foreign equity holdings, exchange risk is not found to be a substantial component of the overall risk of a portfolio and the value of hedging that risk is found to be very sample period specific. During periods of rapid currency appreciation, for example 1980-85, hedged portfolios did outperform unhedged portfolios; however, this result is reversed during periods of rapid currency depreciation. Overall, while hedging does reduce the riskiness of a global portfolio, it also reduces return so that no statistically significant improvement in portfolio performance is obtained.

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Introduction

The benefits of international diversification to investors has been known since the early studies of Grubel (1968) and Levy and Sarnat (1970). While those benefits still remain, recently Eun and Resnick (1988) found that further improvements in portfolio performance could be obtained in internationally diversified portfolio by hedging exchange risk through the use of forward contracts. Their analysis, however, suffers from at least two shortcomings which this paper addresses. First, any comparison of investment strategies can be very dependent upon the sample period over which the comparison is made. This paper looks at several sample periods for both dollar-denominated and pound-denominated returns. Second, any comparison involving statistics must incorporate the distribution of those statistics. Comparing only point estimates of statistics assumes a precision of estimation that most financial economists are uncomfortable with. This paper borrows tests from the asset pricing literature to test the mean-variance efficiency of portfolios which are hedged against foreign exchange risk.

The organization of the paper is as follows. Section I introduces the concept of hedging in a portfolio context and provides descriptive statistics of the data. Test of the benefits from hedging exchange risk are conducted in Section II. An extension of the analysis to incorporate optimal hedging of exchange risk is presented in Section III. Section IV provides a summary of the results and makes suggestions on further research in this area.

I. The Environment

Consider an investor who purchases a portfolio of assets, some of which are denominated in foreign currencies. In each period define the return on this portfolio to be R_t. Over n periods the cumulative return on the portfolio is given by

$$(1) \quad R_{t,t+n} = R_t R_{t+1} \ldots R_{t+n},$$

where

$$R_t = P_t' S_t / P_{t-1}' S_{t-1},$$

P_t is the vector of asset prices at time t in the foreign currency and S_t is the appropriate exchange rate vector expressed in terms of local currency units (which is taken to be the US$) per foreign currency (FC). It is clear from the definition that the return over any horizon will be influenced by changes in both the foreign currency value of the asset, as well as changes in the value of the foreign currency against the dollar. It is this second component involving the change in the value of the foreign currency which will be referred to as exchange risk. This paper will examine the effect of exchange risk on overall portfolio performance.

Now consider an investment in this same portfolio, but combined with a strategy of foreign exchange hedging through the use of forward contracts. A forward contract allows an investor to buy/sell a specified amount of foreign currency at a given future date for a predetermined price. These contracts are widely traded for some currencies and provide a convenient means of hedging exchange risk because there is no cash flow associated with a contract until the date at which the currency trade actually takes place. To keep things simple, the hedging strategy will be to sell forward each period the amount of foreign currency invested. The statistical tests in Sections II and III will relax this assumption and consider a more general case. If we define the forward rate to be F_t($/FC), then the gain/loss on the hedging position each period is given by

$$Y_{t+1} = (F_t - S_{t+1})/S_t.$$

For a single period, the total return on the hedged portfolio is therefore

$$(2) \quad R^h_{t+1} = R_{t+1} + Y_{t+1}.$$

Over multiple periods the returns to hedging will also be compounded into the total return so that the return over n periods is

$$(3) \quad R^h_{t,t+n} = (R_{t+1} + Y_{t+1}) \ldots (R_{t+n} + Y_{t+n}).$$

The desirability of hedging exchange risk will be evaluated by comparing the ex post values of (1) and (3) over a particular holding period in a mean-variance framework.

Most previous work on hedging has been confined to the case of single period returns. Unless it is expected that there is some interesting correlation patterns in returns, looking at single period returns is instructive and this paper will also follow that approach, where a period is taken to be a month.[1] When n=1, equation (2) defines the hedged returns. The variance of the return is given by

$$var(R^h_t) = var(R_t) + var(Y_t) + 2cov(R_t, Y_t).$$

Substituting the definition for Y_t and noting that F_{t-1} is not a random

1 Mean reversion in exchange rates due to purchasing power parity might be one reason to consider longer holding period returns. Most evidence, however, indicates that in the short run exchange rates can be characterized by a random walk. Glen (1989) provides evidence in favor of serial correlation in real exchange rate changes, but only for holding periods in excess of one year.

variable at time t, this expression reduces to

$$(4)\quad var(R_t^h) = var(R_t) + var(S_t/S_{t-1}) - 2cov(P_tS_t/P_{t-1}S_{t-1}, S_t/S_{t-1}).$$

Thus, the ability of hedging to reduce risk depends on the relationship between the covariance between dollar returns on the portfolio and the returns on foreign currencies and the variance of the exchange rate.

Before proceeding to an analysis of the behavior of portfolios of foreign and domestic equities, it is useful to examine the performance of the components which will be used in forming those portfolios. Throughout this paper, the empirical analysis will be conducted using a set of value-weighted country stock market indices.[2] The choice of value-weighted indices and portfolios is a natural one since most asset pricing theories predict that such portfolios should be mean-variance efficient. Also, buy and hold strategies minimize transaction costs and are easy to implement by individual investors. Alternative strategies which require portfolio rebalancing each period were analyzed in this context by Eun and Resnick (1988). They found little difference between the ex post behavior of using equally-weighted portfolios and portfolios formed through more complicated mean-variance optimizing strategies.

Summary statistics for the country indices and the return on forward contracts are presented in Table 1. Means and standard deviations are reported on an annualized monthly dollar return basis, with statistics for the stock indices reported in excess of the U.S. treasury bill rate. A certain amount of risk-return tradeoff is readily apparent, with the U.S. market having both the lowest excess return and standard deviation over the

[2] The data appendix contains a complete description of the data used.

sample period. Japan produced the highest ex post sample average return, but was surpassed by the U.K. in terms of standard deviation. With the exception of the German mark, all of the forward contracts produced negative average returns, which is the first evidence indicating a cost involved with hedging exchange risk. Tests for normality of the time series are represented by the Studentized Range Statistic presented in the third column. Generally, these series exhibit sufficient kurtosis that normality can be rejected.[3] One potential source of this rejection will be investigated below and the implications of possible nonnormality will be considered when testing for the benefits of hedging.

Table 1 also presents evidence on the variance and covariance terms in equation (4) for the individual country stock market indices. It is evident that, at least for investments in the individual indices, substantial risk reduction is possible through the use of forward contracts. Of course, it may be that the reduction in risk is completely offset by the accompanying reduction in returns which the negative returns on forward contracts imply. This risk-return tradeoff would be expected if the risk due to variance in exchange rates is priced by the market in a manner similar to the other assets.

Another aspect of the overall environment which is worthwhile investigating before proceeding with the analysis of the benefits of hedging is the general behavior of the exchange rate during the sample period. Figure 1 presents the U.S. dollar effective exchange rate over most of the sample period. The effective exchange rate is calculated by the

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[3] Eliminating the observation for October, 1987 does reduce these statistics considerably.

International Monetary Fund and weights the value of the dollar against all of its major trading partners on the basis of the value of trade conducted. What the graph shows is that the sample period has been largely dominated by the rise in the value of the dollar during the early eighties, followed by its subsequent fall in value back to its original level. The implications of this behavior for the present study could be enormous. If, for example, the period of rapid appreciation of the dollar consisted of generally unexpected appreciations of the dollar, where unexpected is taken to mean in excess of the interest rate differential between the two countries, then this will manifest itself in unusually high (positive) returns on forward contracts. Conversely, to the extent that the depreciation of the dollar was largely unexpected, forward contracts will produce large negative returns during that period. Given these implications of the behavior of the dollar, it is important to be careful not to draw conclusions about the value of hedging by looking at a single short sample period, especially if that period was 1980-85 or 1985-89.

Because the analysis that follows will look at the value of hedging from the perspective of both a U.S. and a U.K. investor, the effective exchange rate of the pound is presented in Figure 2. Here the behavior is much different than that of the dollar, with less evidence of a single dominating episode of appreciation/depreciation.

The analysis so far looked only at portfolios composed of assets entirely denominated in the same currency. Portfolio theory, however, tells us that whenever assets are less than perfectly correlated it is better to hold diversified portfolios. Table 2 provides evidence on this by presenting the correlation matrix of foreign currency stock returns and changes in the value of the foreign currencies for the sample countries. One source of risk reduction is readily apparent in the generally low correlations between foreign currency stock returns. Over the sample period, for example, the correlation between the Japanese market and the US market was only 0.33. The largest correlation was 0.71 between the US and Canadian markets. As an overall measure, the (equally weighted) average correlation over this group of stocks for this period was 0.44.

Panel B of Table 2 presents the correlations between foreign currency changes. Here there is much more variation in the correlations than there is for the stock returns, with the numbers ranging from a low of 0.12 between the Canadian dollar and the Japanese yen and a high of 0.90 between the DM and the Swiss franc. The overall average correlation was 0.53, not too much higher than that of the stocks themselves.

Perhaps the most interesting part of the table is Panel C, which presents the cross-correlations between the foreign currency stock returns and the various foreign currency changes. A total of 19 of the entries are reported as negative; however, these are mostly small in absolute value and statistically insignificant. Note, however, that most of the positive entries are also insignificant. Generally, these low and sometimes negative correlations make it possible to substantially reduce the overall variance of a portfolio.[4]

The total variance on a portfolio containing assets denominated in more

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[4] These numbers are substantially at odds with the significantly positive correlations that Eun and Resnick (1988) found for (essentially) all of the stocks and currencies in their sample. The reason for this is that they used a short sample period, 1980-85, which was a period over which both the dollar and the stock market appreciated significantly. This highlights the danger of looking at too short a sample period.

than one currency is given (approximately) by

$$(5)\quad var(R_t) = \sum_i\sum_j \omega_i\omega_j cov(r_i,r_j) + \sum_i\sum_j\omega_i\omega_j cov(e_i,e_j) + 2\sum_i\sum_j\omega_i\omega_j cov(r_i,e_j)$$

where ω_i is the weight placed on asset i in the portfolio, $\Sigma_i\omega_i = 1$, r_i is the local currency return on asset i and e_i is the return on holding currency i.[5] Writing the variance in this manner allows for decomposition of the overall variance into its constituent components, which then gives a better idea of the impact of exchange rate changes on portfolio variance and the desirability of hedging in a multi-currency portfolio.

Table 3 presents the percent of the total variance that can be attributed to each of the three components in relation (5), where the portfolio is given by a fraction δ invested in the U.S. market portfolio and the remaining fraction, $1-\delta$, is placed in a value-weighted portfolio of non-U.S. stocks from the six countries. Two points emerge from this table. First, when only foreign assets are held, $\delta=0$, the exchange rate component of dollar returns is approximately 30%. In this case, hedging of foreign exchange risk could substantially reduce the riskiness of the portfolio. Second, because of the insignificant correlations between currency returns and stock returns, for low levels of foreign asset holdings the currency component of the overall variance of the portfolio is negligible. For example, in a portfolio containing 20% foreign equities, which is a large

[5] Writing the variance this way ignores the variances and covariances of the cross-product terms involving stock returns and currency changes. These terms are generally of a smaller magnitude than the covariances in equation (5) since they involve cross-products. These terms will be included in the empirical tests that compare the performance of hedged and unhedged portfolios.

fraction for most U.S. investors, the variance added to the portfolio due to exchange rate movements was less than 1.4% of the total variance. For only 10% foreign stock holdings, the exchange component increased overall variance by less than 1%. Table 4 presents similar numbers from the perspective of a UK investor with generally similar findings.[6] The UK investor observed similar contributions to variance from exchange rate changes for low levels of foreign equity holdings, but at the 100% level a slightly higher fraction of overall variance was due to the currency factor.

In addition to the variance component, evaluating the desirability of hedging requires an evaluation of the risk-return tradeoff that hedging involves. One statistic which allows such a comparison is the Sharpe measure, which is the ratio of the excess return of an asset to its standard deviation. This is a widely used measure and is reasonable in a mean-variance framework because efficient portfolios will involve holdings of the market portfolio and the risk-free asset and will all have the same Sharpe measure. Thus, any portfolio with a lower Sharpe measure is inefficient.

Table 5 presents Sharpe measures, as well as the means and variances of the returns for annualized monthly dollar returns for various degrees of investment in foreign equities. Results are presented for the entire sample period, as well as for the three five-year subperiods.[7] Comparing mean returns only, it is readily apparent that any risk reduction achieved through the use of forward contracts is bought through a reduction in average return.

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[6] The non-UK portfolio includes four countries not included in the non-US portfolio: Australia, Italy, Hong Kong and the Netherlands. The reason for this is that the index values for these four countries were unavailable prior to 1976.

[7] The final subperiod actually contains 64 monthly observations

For example, a 10% investment in foreign equities over the entire sample period reduced variance by 0.16%, but at a cost of 0.18% in average return. The result is that, to the second decimal place, there was no effect on the Sharpe measure.

While there was little difference in the behavior of the hedged and unhedged portfolios over the entire sample period, the behavior in the subperiods is more varied. In the first subperiod, 1974-78, the reduction in average return on the hedged portfolio was sufficiently large that the unhedged portfolio appears to perform substantially better. Not surprisingly, during the second subperiod, which corresponds closely to the substantial appreciation of the dollar pointed out in Figure 1, the hedged portfolio actually has both a higher average return, as well as lower variance.[8] When the dollar was depreciating, as it was during the final subperiod, the results are again reversed, with the hedged portfolio having a lower average return, but only a slightly lower variance. These results point out the danger of drawing conclusions from a single, short sample period.

Similar statistics for monthly pound denominated returns are presented in Table 6. In this case, with the exception of very large positions in foreign equities during the 1983-89 subperiod, the unhedged portfolio did at least as well or better than the hedged counterpart.

A graphical interpretation of the results is given in Figure 3, which shows in mean-standard deviation space for the 1974-89 period the ex post

[8] In order to correspond more closely to Eun and Resnick (1988), I also calculated these statistics for the period 1980-85. The Sharpe measures for the unhedged and hedged portfolios were 0.14 and 0.29 respectively for a 100% investment in the foreign portfolio.

performance in dollars of the value-weighted foreign equity portfolio both hedged and unhedged. The Sharpe measure of a portfolio is equal to the slope of a line connecting the corresponding portfolio with the origin. In this case, the proximity of the two portfolios is manifest in nearly equal Sharpe measures. Also presented in the graph are the efficient frontiers for the two sets of assets: the set composed only of foreign equities and the set which also includes the forward contracts. It is evident that the difference between the two frontiers is greater than the difference between the two value-weighted portfolios. Thus, it may be the case that restricting the analysis to a particular weighting scheme is unduly restrictive. Fortunately, research from empirical tests of asset pricing in a mean-variance framework provides an approach to determining whether the shift in the efficient frontiers seen in Figure 3 is in fact statistically significant. It is this matter which is pursued in the next section.

II. Tests of Mean-Variance Efficiency

There are at least two shortcomings to the analysis conducted in the previous section. First, it was restricted to a particular set of weights being imposed on individual assets. It may be that some other set of weights would have changed the results dramatically. Second, while it is possible to compare the point estimates of Sharpe measures for any two portfolios, comparisons of the estimates without consideration of their distributions is not very meaningful in a statistical sense. It is these two problems which this section will address.

Roll (1977) showed that tests of the capital asset pricing model are equivalent to tests of the mean-variance efficiency of the benchmark portfolio. Grinblatt and Titman (1987) extended this idea to cover multiple factor models, with the implication that in a multi-factor framework some linear combination of the factors must be mean-variance efficient. In a mean-variance framework, efficiency of a set of factors can be determined through the regression

$$(5)\quad R_t = \alpha + \beta' X_t + \varepsilon_t$$

where R_t is an excess return, β is a vector of regression parameters, X_t is the set of factors and ε_t is the regression residual. The factors, X_t, are efficient if and only if $\alpha = 0$.

These concepts apply to the present context in the following way. In a mean-variance framework, the expected return on an asset will be determined by the riskiness of the asset, where risk is determined by the systematic risk that the asset exhibits. To the extent that the portfolio of country indices is well-diversified and the risk inherent in forward contracts is systematic risk, adding forward contracts to the portfolio of country indices should not effect its expected performance. If, however, the risk inherent in forward contracts is non-systematic risk so that there is no risk premium required to obtain them and the portfolio of country indices is not sufficiently diversified in terms of currency risk, then their addition to a portfolio may effect its performance.

The argument that there is no risk premium associated with forward contracts has been accepted in the present context by Eun and Resnick (1988). More generally, however, both theoretical models of asset pricing and empirical work on exchange rates indicate the existence of a risk premium.[9] What is important empirically is not only the question of the existence of a risk premium, but also, whether the magnitude of the risk premium is similar in the equity markets and the forward exchange markets. It is this question which will be addressed in this section.

Gibbons, Ross and Shanken (GRS) (1989) and Kandel and Stambaugh (1989) develop statistical tests of mean-variance efficiency in a multivariate setting. One appeal of their analysis for the present study is that it relates tests of mean-variance efficiency to Sharpe measures. Following Kandel and Stambaugh (1989), consider a portfolio consisting of K1 assets and a second portfolio consisting of K2 assets. In this case the set of K1 assets will consist of the country indices, whereas both the forward contracts and the country indices will be included in the set of K2 assets. Let H0 denote the hypothesis that the portfolio of K1 assets is mean variance efficient and let HA denote the hypothesis that the portfolio of K2 assets is mean-variance efficient. Kandel and Stambaugh consider the statistic

$$Q = \frac{\theta_{K2}^2 - \theta_{K1}^2}{1 + \theta_{K1}^2} - 1$$

where θ_{Ki} is the maximal Sharpe measure obtainable from the set of assets Ki. Under joint normality of the asset returns and the null hypothesis that the set of K1 assets is mean-variance efficient, they show that [T/(T-2)][(T-N-1)/N]*Q has an F distribution with degrees of freedom N and (T-N-1), where T is the number of observations and N is the number of forward contracts

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[9] Hodrick (1987) provides an extensive survey on the literature of risk premiums in forward exchange rates.

included in K2. This statistic is equivalent to a likelihood ratio test that the intercepts are zero in a system of regressions of the N assets on the set of K1 factors.

To relate the statistic to Figure 3 and the previous section, the statistic compares the slopes of the tangent lines to the efficient frontiers of the set of K2 and K1 assets. If the set of K1 assets is efficient then the two slopes should not be statistically different. As discussed above, the slopes of these tangent lines are given by the Sharpe measures. In this case, however, no restrictions are imposed on the weight of each asset in the portfolio. Instead, the optimal portfolios are calculated and their are Sharpe measures are compared. Note, however, that the comparison is made between squared Sharpe measures, which implies that the test will identify intercepts which are statistically negative as well as positive.

The values of the adjusted Q statistics and their corresponding probability values are presented in Table 7, where they are identified as F. In this case the null hypothesis, H0, consists of holding only the set of possible equities (both foreign and U.S.), whereas the alternative hypothesis allows the hedging instruments to be added to this set. In no case, either for the dollar or pound returns is the null hypothesis rejected. The implication is that even though in some cases differences between the Sharpe measures of the hedged and unhedged portfolios occurred, these differences are not sufficiently large that the hypothesis that they are the same can be rejected.

A possibly serious shortcoming of this likelihood ratio test is that it assumes that all excess returns are multivariate normal and homoscedastic. While this has positive implications in that it produces a known finite sample distribution for the statistic, the assumptions may be somewhat over restrictive. Given the evidence in Table 1 against normality, it may be that under the sample distribution this particular statistic is not sufficiently powerful to reject the null even when it should. One possible cause of the fat tails observed in these series is conditional heteroskedasticity. Mixing observations from distributions which are normal, but with different variances can produce distributions with high degrees of kurtosis. One test for the presence of heteroskedasticity in the data is given by examining TR^2 from the regression

$$\varepsilon_t^2 = \alpha' Z_t + \tau_t$$

where T is the number of observations, ε_t is obtained from the regression given in equation (5), and Z_t consists of the squared values of the factors taken from regression (5). Breusch and Pagan (1979) show that TR^2 from this regression is asymptotically distributed as $\chi^2(s-1)$, where s is the number of instruments. The results of these tests are presented in Table 8. The test rejects homoskedasticity at the 5% level for five out of six of the forward rates for the dollar denominated returns, with five out of eight tests rejecting for the pound denominated returns. The importance of this finding is pointed out by Richardson (1989) who compares the performance of tests of mean-variance efficiency when returns are not normally distributed. His finding is that the F statistic may be biased in favor of the null hypothesis.[10] Fortunately, an alternative statistic is available which is

[10] Conversely, Affleck-Graves and McDonald (1989) conduct simulations and find little evidence that the type of departures from normality that typically observed in security returns will lead to substantial changes in the size or power of the GRS statistic. Richardson uses substantially more observations than Affleck-Graves and McDonald, which could account for the

robust under heteroscedasticity.

The consistency of ordinary least squares (OLS) parameter estimates is preserved under heteroskedasticity, however, OLS standard errors are no longer consistent. White (1980) develops a heteroskedasticity-consistent covariance matrix estimator which is useful in situations such as this. To draw inferences, one need only estimate the desired parameters by OLS, and then substitute the corrected variance-covariance matrix using White's estimator. In the current situation, this is accomplished by estimating the complete set of regression intercepts in equation (5) and then jointly testing that all intercepts are zero using the heteroskedasticity-consistent covariance matrix and the appropriate set of linear restrictions on the parameters.[11] This is commonly referred to as a Wald test. This test statistic, ChiH(#), is also presented in Table 7. For the overall period mean-variance efficiency of the country indices can not be rejected; however, for the period 1979-83 a rejection is observed.[12] For comparative purposes, the same test statistic calculated without correcting for heteroskedasticity, Chi(#), is also presented in Table 7. The effect of the correction is most dramatic during the last subperiod, where inferences are changed from rejecting the null to accepting it. It should be pointed out that for this subperiod half of the estimated intercepts were negative and of the same order of magnitude as the positive intercepts. Because the statistic places equal weight on both positive and negative deviations from zero, it is

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difference.

[11] Richardson (1989) finds that this approach to inference is more likely to reject than the F statistic described above.

[12] I also calculated the statistic for the 1980-85 period. The resulting $\chi^2(6)$ statistic had a value of 24.36, which is highly significant.

possible that this rejection does not indicate any superior performance on the part of the hedged portfolio.

The results for the pound investor are substantially different from the dollar investor's in one respect. While the null hypothesis is not rejected for the overall sample period, rejection does occur for the 1977-83 subperiod for both chisquared statistics. In this case, however, seven out of eight of the estimated intercepts were negative, implying that the risk premium paid for hedging exchange risk in this period was significantly more than what was justified ex post by the amount of risk reduction obtained. In this case, the investor was statistically better off not hedging; in fact, the investor would have improved performance by actually buying the currencies forward rather than selling them. Again, the value of considering heteroskedasticity is evident in the behavior of the statistics during the final subperiod, where six out the eight estimated intercepts were negative.

III. Optimal Forward Positions

The analysis thus far has been restricted to the simple hedging strategy which involves selling forward only the amount invested. It may be, however, that because of the correlation structure between assets a somewhat different hedging strategy is superior. It is this question that is investigated in this section.

Consider an investor making a portfolio decision involving n+1 assets: n foreign securities whose returns involve foreign exchange components and the domestic portfolio. In addition to the n foreign securities which involve positive (or negative) investments, the investor also has available forward contracts for foreign exchange which require no investment and allow a known foreign exchange exposure to be hedged. The investor's portfolio problem is defined as follows. Let

W_t = dollar value of wealth invested at time t

and

ω_{it} = fraction of wealth invested in asset i, with $\Sigma_i \omega_{it} = 1$.

The payoff on the investment in the n+1 risky securities is given by

$$\sum_i \omega_{it} W_t (P_{it+1} S_{it+1})/(P_{it} S_{it}) = \sum_i \omega_{it} W_t R_{it+1}.$$

Now define $\alpha_{it}\omega_{it}W_t/S_{it}$ as the amount of foreign currency i sold forward at time t at the forward exchange rate F_{it}. Let

$h_{it} = \alpha_{it}\omega_{it}$ = fraction of wealth sold forward

then the payoff on the portfolio is given by

$$P_{t+1} = \sum_i [\omega_{it} W_t R_{it+1} + h_{it} W_t (F_{it+1} - S_{it+1})/S_{it}].$$

The return on the portfolio is equal to

$$R_{t+1} = P_{t+1}/W_t - 1 = \sum_i [\omega_{it} R_{it+1} + h_{it} f_{it+1}] - 1.$$

where

$$f_{it+1} = (F_{it} - S_{it+1})/S_{it}.$$

Imposing the constraint that the sum of the portfolio weights, ω_i, must equal one, the return on the portfolio can also be written as

$$R_{t+1} = y_t' r_{t+1}$$

where

$$y_t' = (1-\Sigma_i\omega_{it},\omega_{1t},\ldots,\omega_{nt},h_{1t},\ldots,h_{nt})$$

$$r_{t+1}' = (R_{1t+1},\ldots,R_{n+1t+1},f_{1t+1},\ldots,f_{nt+1}).$$

With this vector notation the variance of the portfolio return can be written as

$$\mathrm{var}(R_{t+1}) = y'\Sigma\, y$$

where, for convenience, the time subscripts have been dropped and the covariance matrix Σ is given by

$$\Sigma = r_{t+1}r_{t+1}' = \begin{bmatrix} \Sigma_{ij} & \Sigma_{ifj} \\ \Sigma_{fij} & \Sigma_{fifj} \end{bmatrix}.$$

The overall covariance matrix Σ can thus be partitioned into four parts: the covariance matrix of dollar returns on the n+1 assets, the covariance matrix of the n forward currencies, and the two matrices which give the covariances between dollar returns on the assets and the foreign currencies.

The investor's portfolio problem can now be written as

$$\min_{\omega_i,h_i} \quad y'\Sigma\, y$$

$$\text{s.t.} \quad {}_tr_{t+1}'y = \gamma$$

where ${}_tr_{t+1}$ is the time t expectation of r_{t+1} and γ is an arbitrarily chosen minimum return constant. By varying the value of γ, the efficient frontier of portfolios will be traced out. The first-order conditions for this problem are given by

$$y' = \lambda r_{t+1}\Sigma^{-1}/2$$

$$_t r'_{t+1} y = \gamma$$

where λ is the scalar Lagrange multiplier on the single constraint. This represents a system of 2n+2 linear equations in 2n+2 variables which, given knowledge of $_t r_{t+1}$ and Σ, can be solved explicitly for y.

This section will consider the special case where the investor has already made the portfolio decision to invest in foreign securities, i.e. the weights ω_i have been chosen, and the decision now is on the optimal amounts, h_i, of each foreign currency to sell forward. It is easy to show that the solution to this problem is

$$(6) \quad h' = \lambda\, {}_t f_{t+1} \Sigma^{-1}_{fifj}/2 - \omega' \Sigma_{ifj} \Sigma^{-1}_{fifj},$$

where $_t f_{t+1}$ is the time t expected return vector for the set of forward contracts. The optimal position thus involves two components, a speculative component involving the expected return on the position and a hedging component which minimizes portfolio risk.[13] Note that the second component consisting of the product of the two matrices is just the multiple regression coefficients from regressions of the n+1 dollar asset returns on the n forward contract returns. Thus, unlike naive strategies which consider only the effect of exchange rate changes on the returns of assets denominated in that currency, the optimal hedging strategy considers the effect of an exchange rate change on all assets and, given the weight that they represent in the portfolio, chooses the hedged amount of foreign currency in order to

[13] This is similar to the result in Danthine and Anderson (1981), except that they assume a specific form for the investor's preferences so that there is a risk parameter instead of a Lagrange multiplier involved.

minimize overall risk. It is also interesting that even in the case where the matrix Σ_{ifj} is diagonal, unless it is the identity matrix it is not optimal to choose a hedge ratio equal to one due to the correlation between asset returns and exchange rates.

While investments in forward contracts did not produce zero returns over any of the sample periods, generally, the returns that they produced were not significantly different from zero. As a result of this, together with the idea that the main thrust of this study is to look at the hedging motivation for forward contracts, implementation of the optimal hedging strategy will be limited to the hedging component of equation (6). In order to avoid using information unavailable to an investor the following procedure is employed: using a sample of 60 monthly returns for the foreign equities and forward contracts a multivariate regression is run. The dependent variable is a global portfolio with δ% invested in the US portfolio and the remainder invested in a value-weighted non-US portfolio. The independent variables are the set of forward contract returns on the six currencies in our sample. The regression coefficients from this regression are then taken as the hedge ratios in the subsequent period. This strategy is then repeated using only the most recent 60 observations to estimate the regression coefficients. The result is a time series of hedge ratios which covers the period 1979-89.

Figure 4 presents the estimated hedge ratios for the pound. It is interesting that in no case is the ratio equal to one. Furthermore, the sign of the ratio changes from positive to negative and back over the sample period, indicating that during some periods the pound was actually bought forward, not sold. Thus, while this strategy theoretically minimizes the overall risk of the portfolio, it produces a strategy which is at times at odds with the general approach to hedging exchange risk.

The results of statistical tests using the optimal hedge ratios are presented in Table 9. Results are presented for the overall period, 1979-89, as well as for two subperiods. For comparison with the discussion in Section II, Sharpe measures for the various strategies are presented. Generally, there is very little difference between the performance of the hedged and unhedged strategies using this measure.

Table 9 also presents the results of tests of mean-variance efficiency. For these tests the following approach is adopted. For each period, optimal hedge ratios are calculated for portfolios with various levels of foreign investment. The returns on selling the foreign currencies forward are then multiplied by these ratios, thus generating a set of time series, one for each currency. These time series are then tested for efficiency using the two Wald tests described in the previous section.[14] This is one approach to addressing the question of whether an investor could have used existing information to adopt a hedging strategy that would have produced superior performance. For the overall period, only one of the strategies produced statistically significant results, the 100% foreign portfolio optimal hedge ratios. Again, however, three of the six estimated regression intercepts were negative, not positive as required for rejection of mean-variance efficiency.

The importance of this finding for the main topic of this study should

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[14] To see if it is meaningful to compare performance using different levels of foreign investment, I calculated the correlation of the time series generated using the optimal hedge ratios. As an example, the correlation between the French franc time series calculated with a 10% foreign investment and a 100% foreign investment was 0.783. For some currencies the correlation was higher, but for others the correlation was lower.

not be underestimated. As Figure 4 shows, the optimal hedge ratios which produced significantly superior performance during the 1979-89 sample period were not ratios which minimized exchange risk, but rather minimized overall portfolio risk. In order to do this the strategy involved periods in which exchange risk was actively assumed, not hedged. This result is particularly critical of the view that the reduction of exchange risk alone is something that investors should consider.

IV. Conclusions

The value of hedging exchange risk is that it reduces the variance of a portfolio's return. This is true in every sample period for both dollar and pound denominated returns. However, if this insurance is priced by the market in a manner similar to that for other types of risk, then the risk reduction will also be offset by a reduction in return. This paper compares the tradeoff between these two factors in a mean-variance framework. The conclusion is that when sufficiently long periods of time are considered there is no improvement in the risk-adjusted performance of a hedged portfolio over an unhedged portfolio. The hedged portfolio does have lower risk, but it also has a lower return which exactly offsets the risk reduction. For some time periods this tradeoff was not exact; during periods when the dollar was appreciating hedged portfolios outperformed unhedged portfolios, but the opposite occurred during periods when the dollar depreciated in value.

To confirm that these results were not restricted to the movement in the dollar, a similar analysis was performed from the perspective of a pound investor. In this case, for reasonable levels of investment in foreign equities the unhedged portfolio also performed as well as the hedged portfolio.

For comparative purposes, a strategy of optimal hedging was also investigated. The results indicated that superior performance could have been achieved over the sample period. Surprisingly, this result came not through the use of forward contracts to hedge exchange risk, but rather through the use of their ability to reduce the overall variance of the portfolio, which often meant taking long positions in foreign currencies.

Much work remains to be done. First, consideration of alternative hedging strategies which include the use of additional market information would seem appropriate. These strategies might attempt to identify, ex ante, those periods, such as 1980-85, when hedging exchange risk was profitable. Second, inclusion of foreign currency bonds may have important consequences for the results. Given that the returns on holding bonds move inversely to interest rates, as do exchange rates, holding foreign currency denominated bonds in a portfolio may provide a natural hedge against (at least short term) exchange rate changes. Unfortunately, data on foreign bonds is not as readily available as are returns on foreign market indices.

Data Appendix

Equity returns used throughout are country index returns, including dividend yields, taken form Morgan Stanley's Capital International Perspective. Each country index is a value-weighted index representing approximately 60% of its respective market. The foreign equity portfolio used in the analysis wad formed from the individual country portfolios using market capitalization weights from January, 1980. For the dollar investor those weights were: Canada - 0.15, France - 0.08, Germany - 0.11, Japan - 0.39, Switzerland - 0.06, U.K. - 0.21. For the pound investor the weights were: Australia - 0.03, Canada - 0.07, France - 0.03, Germany - 0.05, Hong Kong - 0.01, Italy - 0.01, Japan - 0.17, Netherlands - 0.01, Switzerland - 0.03, U.S. - 0.59.

Exchange rates come from multiple sources. Dollar forward exchange rates for the period 1974-83 were obtained from Data Resources Incorporated, while those for the period 1984-89 were calculated from one month eurodeposit rates obtained from Morgan Guarantee Trust's World Financial Markets. Dollar spot rates come from the International Monetary Fund's International Financial Statistics. Sterling exchange rates, both spot and forward were obtained from Datastream. The Sterling rates consist of bid and ask rates for both the forward and spot rates, whereas the forward rates for the dollar are bid rates, and the spot rates are averages of the bid and ask rates. Thus, the Sterling rates incorporate the complete round-trip bid/ask spread, while the dollar rates incorporate only half of the bid/ask spread.

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Table 1
Summary Statistics
Annualized Monthly Excess Dollar Returns
1974:01 - 1989:04

Country	Mean	Std Dev	SR*	Var(e)	2*Cov(R,e)
Canada	5.40	76.08	6.35	2.52	12.05
France	9.84	92.28	6.65	15.39	34.63
Germany	8.40	74.16	7.50	16.85	29.79
Japan	15.12	71.76	5.51	16.95	38.72
Switzerland	6.00	69.84	7.40	21.77	36.77
U. K.	12.00	101.28	9.33	15.18	32.92
U. S.	5.04	58.44	7.88		
Canada $	-0.24	16.20	6.47		
French Franc	-2.04	39.72	5.89		
German Mark	0.12	41.40	6.49		
Japanese Yen	-3.36	41.28	6.35		
Swiss Franc	-0.36	47.40	7.01		
U.K. Pound	-1.44	39.84	6.71		

* Studentized Range Statistic: the 5% critical value for the sample size is approximately 6.30.

Table 2
Correlation Matrix
1974:01 - 1989:04

A: Local Currency Stock Correlations

	FRAN	GERM	JAPAN	SWIT	UK	US
CAN	0.44	0.24	0.26	0.46	0.51	0.71
FRAN		0.47	0.34	0.52	0.47	0.49
GERM			0.29	0.62	0.35	0.38
JAPAN				0.31	0.30	0.33
SWIT					0.52	0.57
UK						0.56

B: Currency Correlations

	FF	DM	YEN	SF	UKP
C$	0.21	0.22	0.12	0.19	0.25
FF		0.89	0.63	0.82	0.65
DM			0.62	0.90	0.64
YEN				0.63	0.52
SF					0.61

C: Currency/Stock Cross-Correlations

	C$	FF	DM	YEN	SF	UKP
CAN	0.32	0.10	0.03	-0.01	0.05	0.10
FRAN	0.12	0.06	0.01	0.04	0.01	0.04
GERM	0.04	-0.10	-0.07	-0.07	-0.07	-0.03
JAPAN	0.19	-0.01	0.00	0.10	-0.00	0.01
SWIT	0.13	-0.06	-0.08	-0.08	-0.13	-0.05
UK	0.15	0.02	-0.05	-0.04	-0.06	0.03
US	0.23	0.04	0.02	-0.03	-0.02	-0.03

Table 3
Decomposition of Portfolio Variance
Dollar Denominated Monthly Returns
1974:01 - 1989:04

% Held in Non-US	% of Total Variance Due to LC Stock Return	Exchange Rate	Cov
10	99.7	0.3	0.1
20	98.6	1.3	0.2
50	90.3	8.7	1.0
100	69.3	27.8	2.9

1 month returns using Canada, France, Germany, Japan, Switzerland, UK and US MSCIP indices. Non-US portfolio is value-weighted using the January 1980 market capitalizations.

Table 4
Decomposition of Portfolio Variance
Pound Denominated Monthly Returns
1977:01 - 1989:04

% Held in Non-UK	% of Total Variance Due to LC Stock Return	Exchange Rate	Cov
10	99.3	0.3	0.4
20	97.9	1.2	0.9
50	87.8	9.4	2.7
100	61.4	33.6	5.0

1 month returns using Australia, Canada, France, Germany, Hong Kong, Italy, Japan, Netherlands, Switzerland, UK and US MSCIP indices. Non-UK portfolio is value-weighted using the January 1986 market capitalizations.

Table 5
Annualized Monthly Dollar Returns
1974:01 - 1989:04

	% Held in Foreign Portfolio				
	10	20	50	100	Hedge Port.
Mean Unhedged	5.63	6.27	8.18	11.37	
Mean Hedged	5.45	5.89	7.26	9.52	-1.85
Var U	31.61	29.61	27.02	34.15	
Var H	31.45	29.08	24.15	23.24	9.41
Sharpe U	0.10	0.12	0.16	0.19	
Sharpe H	0.10	0.11	0.15	0.20	

Table 5A
Annualized Monthly Dollar Returns
1974:01 - 1978:12

	% Held in Foreign Portfolio				
1974:01-1978:12	10	20	50	100	Hedge Port.
Mean U	0.22	1.32	4.63	10.16	
Mean H	-0.38	0.12	1.63	4.16	-6.00
Var U	35.72	33.29	30.18	38.89	
Var H	35.52	32.70	27.19	27.83	6.78
Sharpe U	0.00	0.02	0.08	0.16	
Sharpe H	-0.01	0.00	0.03	0.08	
1979:01 - 1983:12					
Mean U	5.25	4.86	3.70	1.76	
Mean H	5.93	6.23	7.13	8.63	6.87
Var U	24.56	23.20	21.84	28.67	
Var H	23.64	21.22	15.86	13.28	8.72
Sharpe U	0.11	0.10	0.08	0.03	
Sharpe H	0.12	0.14	0.18	0.24	
1984:01 - 1989:04					
Mean U	11.07	12.23	15.72	21.52	
Mean H	10.46	11.01	12.65	15.40	-6.12
Var U	34.80	32.48	28.87	33.98	11.75
Var H	35.38	33.39	29.23	28.37	
Sharpe U	0.19	0.21	0.29	0.37	
Sharpe H	0.18	0.19	0.23	0.29	

Table 6
Annualized Monthly Pound Returns
1977:01 - 1989:04

	% Held in Foreign Portfolio				
	10	20	50	100	Hedge Port.
Mean U	10.09	9.49	7.71	4.76	
Mean H	9.92	9.17	6.90	3.13	-1.63
Var U	38.06	34.47	27.92	31.23	
Var H	37.88	33.89	24.78	19.35	10.17
Sharpe U	0.16	0.16	0.15	0.09	
Sharpe H	0.16	0.16	0.14	0.07	

Table 6A
Annualized Monthly Pound Returns

1977:01-1989:04	% Held in Foreign Portfolio 10	20	50	100	Hedge Port.
Mean U	9.92	8.94	6.02	1.15	
Mean H	9.72	8.55	5.05	-0.79	-1.94
Var U	32.21	27.89	19.88	23.07	
Var H	32.48	28.16	18.66	14.27	10.02
Sharpe U	0.17	0.17	0.13	0.02	
Sharpe H	0.15	0.16	0.12	-0.02	
1983:07 - 1989:04					
Mean U	10.28	10.11	9.61	8.78	
Mean H	10.15	9.85	8.96	7.49	-1.29
Var U	45.15	42.30	37.24	40.48	
Var H	44.46	40.74	31.89	24.93	10.48
Sharpe U	0.15	0.16	0.16	0.14	
Sharpe H	0.15	0.15	0.16	0.15	

Table 7
Tests of Mean-Variance Efficiency

Dollar	1974-89	1974-78	1979-83	1984-89
F	0.69 (0.66)	0.78 (0.59)	1.49 (0.20)	1.95 (0.09)
CHI(6)	4.51 (0.61)	6.68 (0.35)	12.77 (0.05)	15.64 (0.02)
CHIH(6)	4.96 (0.55)	4.94 (0.55)	21.52 (0.00)	2.92 (0.82)

Pounds	1977-89	1977-83	1983-89
F	1.19 (0.43)	1.60 (0.25)	1.97 (0.15)
CHI(8)	11.01 (0.20)	17.84 (0.02)	23.02 (0.00)
CHIH(8)	8.48 (0.39)	17.31 (0.03)	4.26 (0.83)

Table 8
Tests for Heteroskedasticity

Dollar Denominated Returns
1974:01 - 1989:04

C$	FF	DM	Yen	SF	Pound
12.05 (0.10)	21.55 (0.00)	48.48 (0.00)	23.10 (0.00)	43.16 (0.00)	34.52 (0.00)

Pound Denominated Returns
1977:01 - 1989:04

C$	FF	DM	IL	Yen	HF	SF	$
7.77 (0.73)	21.86 (0.03)	21.03 (0.03)	20.95 (0.03)	7.08 (0.79)	22.07 (0.02)	22.17 (0.02)	5.71 (0.89)

Table 9
Optimal Hedge Ratios
Dollar Monthly Returns

1979:01 - 1989:04	10	20	50	100
Sharpe Unhedged	0.15	0.16	0.20	0.21
Sharpe Hedged	0.15	0.16	0.19	0.20
Chi(6)	7.97 (0.24)	7.79 (0.25)	8.04 (0.24)	12.25 (0.06)
ChiH(6)	9.61 (0.14)	9.58 (0.14)	10.16 (0.12)	13.75 (0.03)

1979:01 - 1983:12				
Sharpe U	0.11	0.10	0.08	0.03
Sharpe H	0.11	0.11	0.08	0.03
Chi(6)	11.44 (0.08)	11.49 (0.07)	11.35 (0.08)	10.32 (0.11)
ChiH(6)	9.33 (0.16)	9.16 (0.16)	8.05 (0.23)	8.39 (0.21)

1984:01 - 1989:04				
Sharpe U	0.19	0.21	0.29	0.37
Sharpe H	0.18	0.20	0.27	0.36
Chi(6)	1.63 (0.95)	1.45 (0.96)	1.36 (0.97)	2.04 (0.92)
ChiH(6)	3.59 (0.73)	3.27 (0.77)	2.72 (0.84)	2.95 (0.82)

US Dollar Effective Exchange Rate

1974:01 - 1988:12

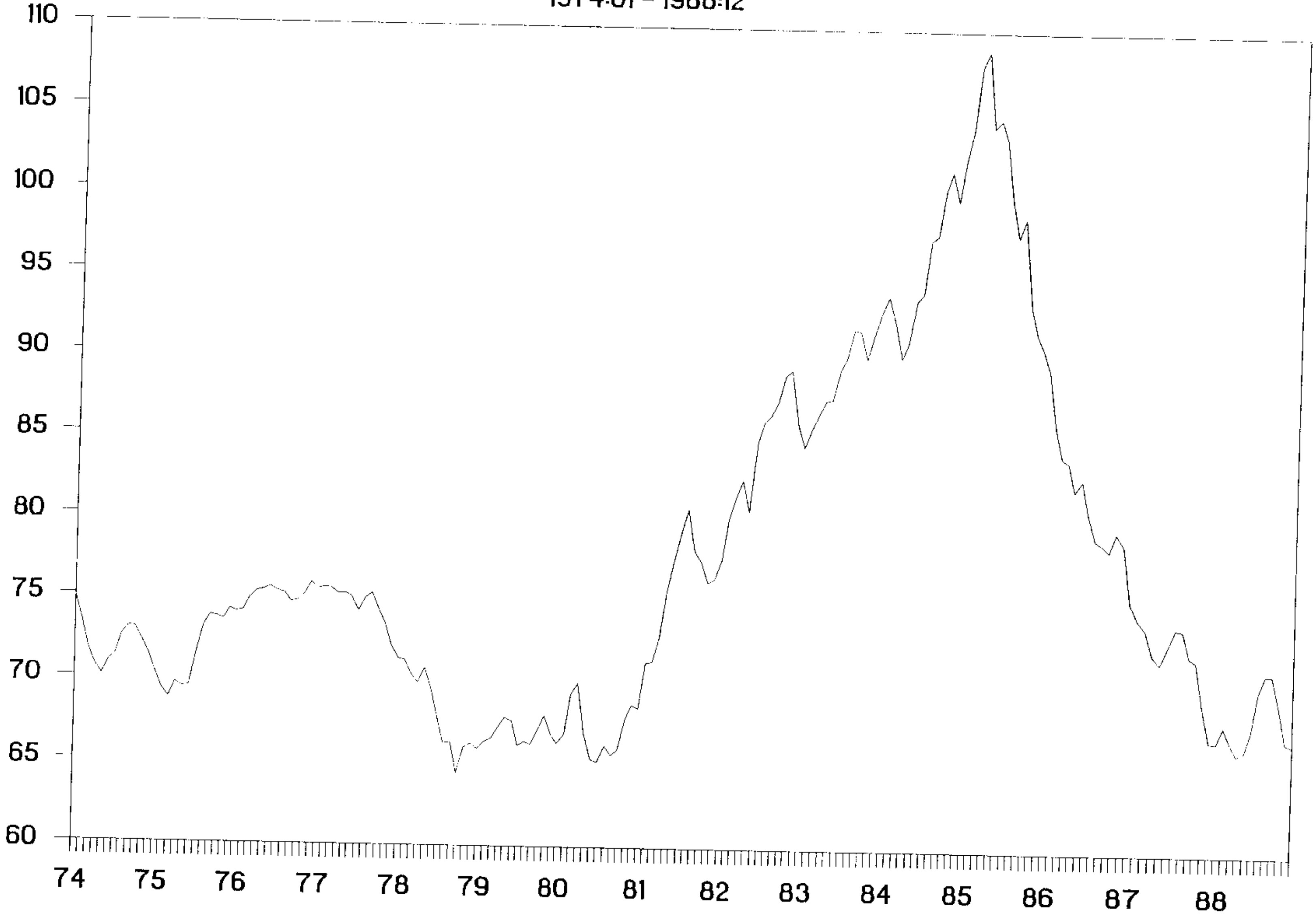

Figure 1

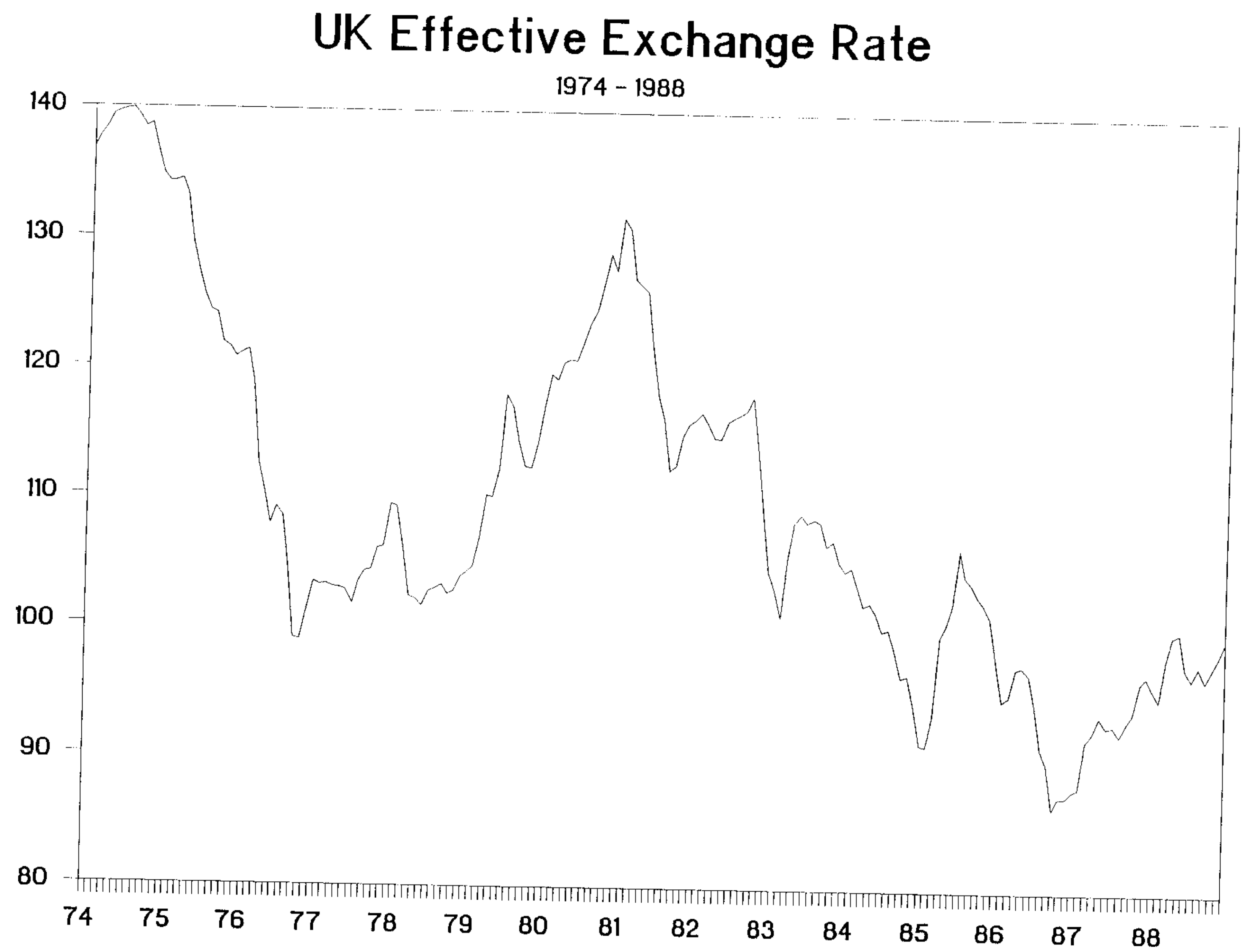
UK Effective Exchange Rate
1974 - 1988
140
130
120
110
100
90
80
74
75
76
77
78
79
80
81
82
83
84
85
86
87
88

Figure 2

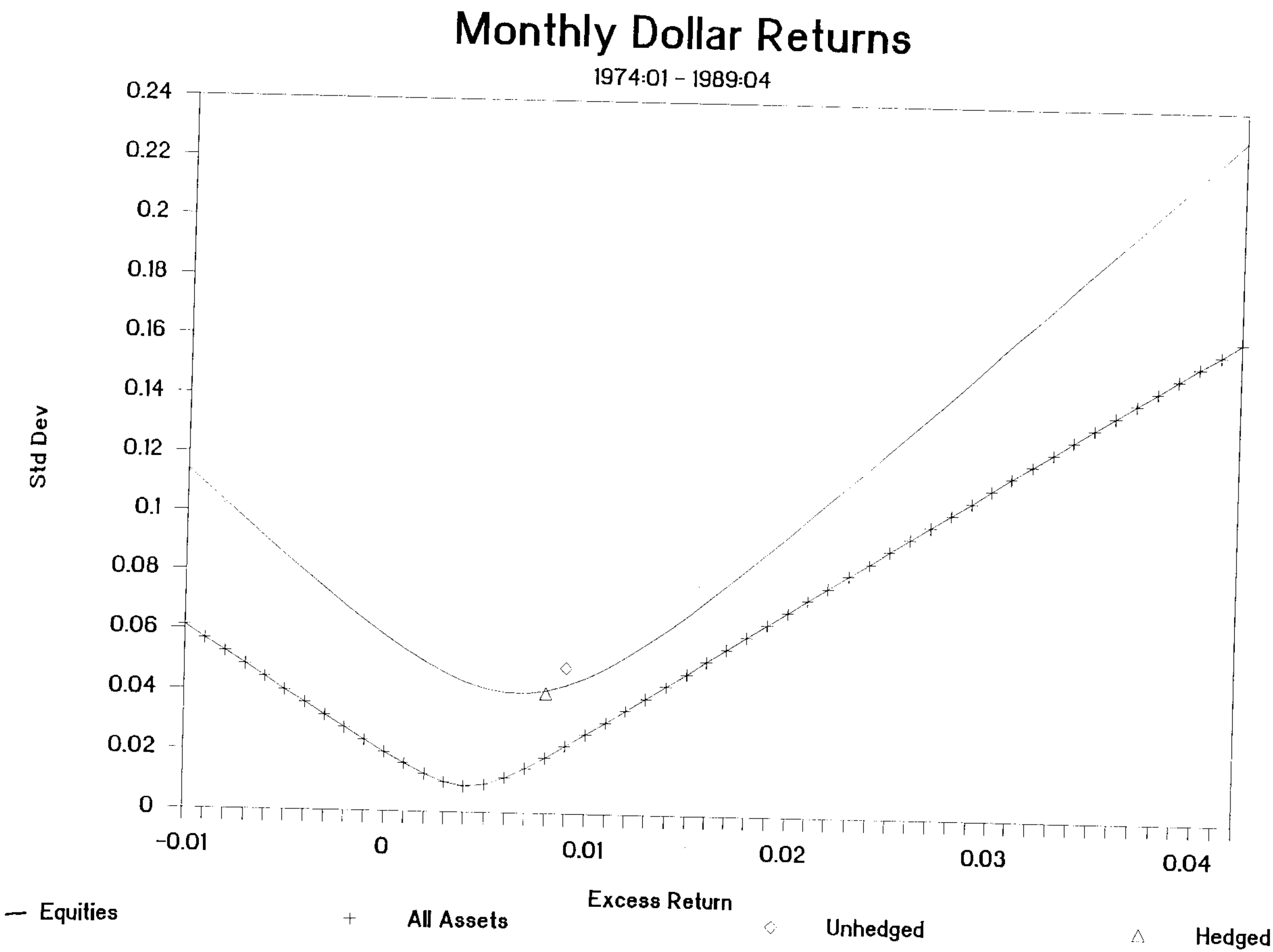
Monthly Dollar Returns
1974:01 - 1989:04
Std Dev
0.24
0.22
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.01
0
0.01
0.02
0.03
0.04
Excess Return
Equities
All Assets
Unhedged
Hedged

Figure 3

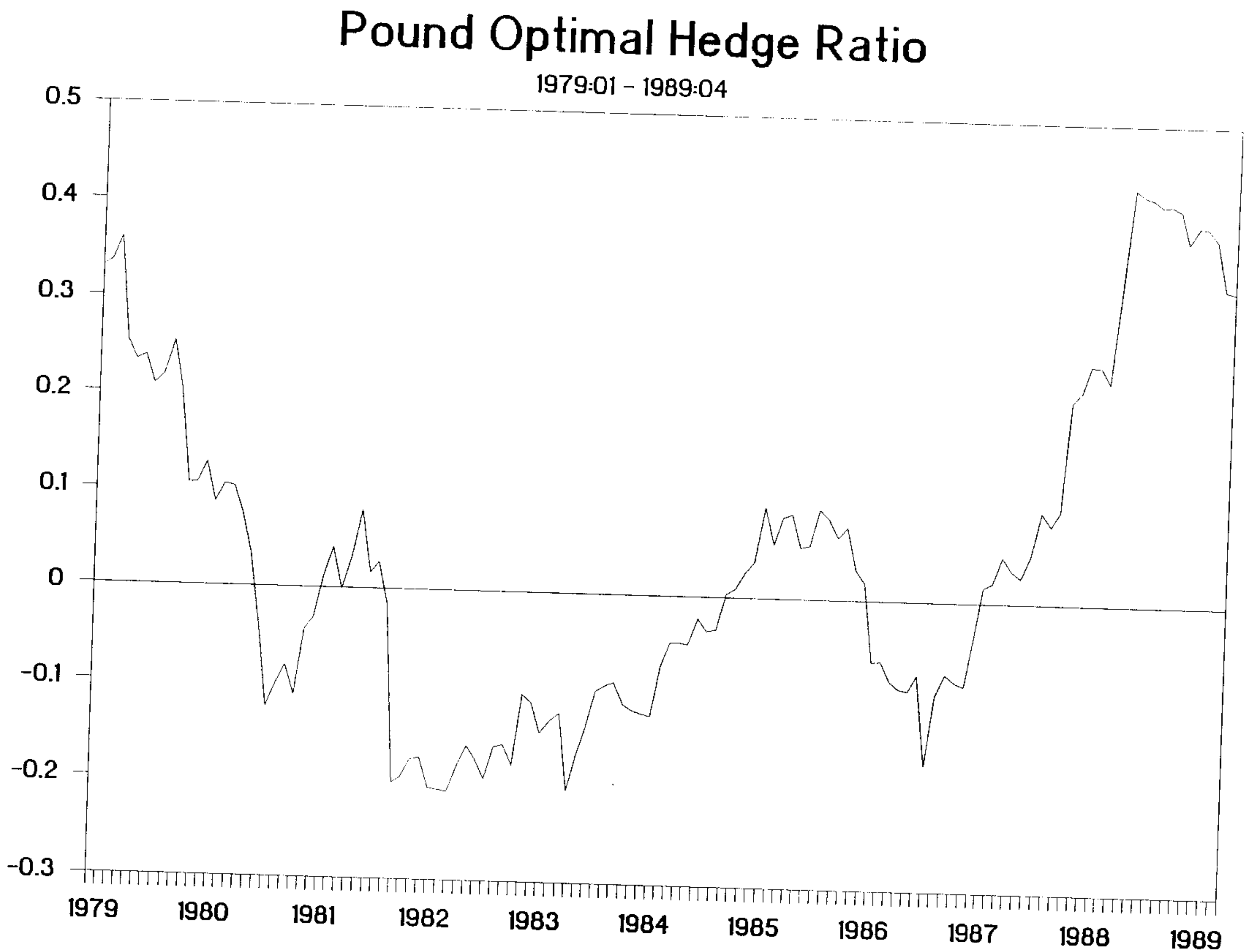
Pound Optimal Hedge Ratio
1979:01 - 1989:04
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
1979
1980
1981
1982
1983
1984
1985
1986
1987
1988
1989

Figure 4