THE VALUATION OF CORPORATE FIXED INCOME SECURITIES

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Abstract

We develop contingent claims valuation models for corporate bonds that are capable of generating yield spreads consistent with the levels observed in practice. We incorporate important features in the valuation related to the occurrence of and payoff upon bankruptcy and focus on the default risk of coupons in the presence of dividends and interest rate uncertainty. Numerical solutions are employed to show that the resulting yield spreads are sensitive to interest rate expectations but not to the volatility of the interest rates. Interaction between call provisions and default risk in determining yield spreads is explicitly analyzed to show that the call provision has a differential effect on Treasury issues relative to corporate issues.

In their path-breaking papers, Black and Scholes [3] and Merton [16] emphasized the correspondence between corporate liabilities and options, and indicated how the theory of option pricing might be used to value corporate liabilities. This correspondence has been the cornerstone of a number of studies : Merton [17] examined the risk structure of interest rates, Black and Cox [2] provided significant extensions by explicitly modelling some indenture provisions, and Brennan and Schwartz [5] and Ingersoll [13] used this correspondence to value convertible and callable corporate liabilities. This list is only partial, but it illustrates the range of issues which may be addressed using option pricing theory.

While the insights offered by this research are beyond doubt, the ability of this approach to explain the yield spreads between corporate securities and comparable default-free Treasury securities has been questioned in recent papers. In a paper which is closely related to our work, Jones, Mason and Rosenfeld [14] sought to test the predictive power of a "contingent claims pricing" model based on some simplifying assumptions which included nonstochastic interest rates, strict "me-first" rules and the sale of assets to fund bond related payments; they also permitted interaction of multiple call and sinking fund provisions. The empirical findings of Jones, Mason and Rosenfeld [14] indicate that such versions of contingent claims pricing models do not generate the levels of yield spreads which one observes in practice.[1] Over the 1926-86 period, the yield spreads on high grade corporates (AAA rated) ranged from 15 to 215 basis points and averaged 77 basis points ; and the yield spreads on BAAs (also investment grade) ranged from 51 to 787 basis points, and averaged 198 basis points. We show later in the paper that the conventional contingent claims model due to Merton [17] is unable to generate default premiums in excess of 120 basis points, *even* when excessive debt ratios and volatility parameters are used in the numerical simulation. The inability of contingent claims pricing models to account for the magnitude of the yield spreads between corporate and Treasury instruments provides the motivation for this paper. The focus is on two issues central to the valuation of corporate claims.

First, we make explicit assumptions about how and when bankruptcy occurs and we discuss the nature of the payoffs with regard to indenture provisions. Previous studies have generally placed the burden of bankruptcy on the principal payment at maturity, and not on the coupon obligations along the way. The focus here, by contrast, is on (i) the possibility of the firm defaulting on its coupon obligations and on (ii) the interaction between dividends and default risk.

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[1]Similar conclusions have been reported by Ramaswamy and Sundaresan [19] for corporate floating rate instruments.

Second, the values of Treasury and corporate fixed income securities are influenced significantly by interest rate risk. Jones, Mason and Rosenfeld [14] concluded that the introduction of stochastic interest rates might improve the performance of contingent claims pricing models. We model this source of uncertainty by specifying a stochastic process for the evolution of the short rate which serves to summarize the interest rate risk. We find that although the yields on both Treasury and corporate issues are significantly influenced by the uncertainty in interest rates, the yield spreads were quite insensitive to interest rate uncertainty. The role of call provisions in corporate and Treasury fixed income securities is investigated in the same setting. The call provision has a differential effect on Treasury issues relative to corporate issues : we find that the call feature is relatively more valuable in Treasury issues than it is in corporate issues. The differential effect of call provisions is a significant factor in explaining the observed yield spreads between noncallable ("straight") corporates and straight Treasuries on the one hand and callable corporates and callable Treasuries on the other.

Our paper, by incorporating these features in a simple partial equilibrium setting, makes two contributions. First, it builds a contingent claims model with stochastic interest rates to accommodate the risk of default in the coupons in the presence of dividends, and examines the effect of the call provision in this more realistic setting. Second, it provides evidence that these models are capable of generating yield spreads that are consistent with the levels observed in practice. To be sure, all the models here study firms that have relatively simple capital structures, with a single issue of debt outstanding. Given the results, however, we are hopeful that contingent claims models will be useful in studying the more complex liabilities of firms with complicated capital structures.[2]

The paper is organized as follows. In Section I, we build the contingent claims valuation framework for pricing corporate and Treasury fixed income securities. We discuss the differences between the models we study and the model in Merton [17]. Section II provides a numerical analysis of straight, noncallable corporate and Treasury debt issues. We characterize the behavior of yield spreads with respect to changes in maturity (the term structure of yield spreads), with respect to shifts in the debt ratios of the firm, and with respect to the parameters that govern the stochastic process that drives interest rates. In Section III we extend the model to callable bonds and examine optimal call policies in a stochastic term structure environment. We conclude the paper in Section IV.

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[2]See Jones, Mason and Rosenfeld [14, 15] for a discussion of complicated capital structures. A useful discussion of this technique is in Cox and Rubinstein [10, Ch. 7] ; see also Fisher [12].

I. The Valuation of Noncallable Corporate Bonds

Corporate fixed income securities are priced to yield a higher expected return than comparable Treasury issues due to the possibility of default. The credit quality of the issuing corporation, the nature of the issue (senior or subordinated, secured or unsecured, callable or noncallable, with or without sinking fund provisions, etc.), the liquidity of the market place and the contractual provisions specifying the payoffs in the event of a default all serve to determine the market value and hence the promised yields on corporate fixed income securities. We take up these issues now in the context of a contingent claims valuation model. The model constructed here differs from the standard contingent claims model in the way in which we specify the occurrence and implications of bankruptcy. In other respects, however, it is similar to the models in Brennan and Schwartz [4] and Jones, Mason and Rosenfeld [14]. We study the valuation of the single debt issue of a firm whose total market value follows a diffusion process, in a world in which the Miller-Modigliani [18] theorems are understood to apply.[3]

The following assumptions are employed :

(A0) Trading occurs continuously in perfect and frictionless financial markets with no taxes, transaction costs or informational asymmetries. Investors act as price takers.

(A1) The value of the firm, denoted V, follows the lognormal diffusion process

$$dV = (\alpha - \gamma) V \, dt + \sigma_1 V \, dZ_1 \quad , \tag{1}$$

where α is the instantaneous expected rate of return on the firm gross of all payouts, $\sigma_1{}^2$ is the instantaneous variance of the return on the firm and Z_1 is a standard Wiener process. The dynamics of the value of the firm are such that γV has a natural interpretation : it is the *net cash outflow* from the firm resulting from optimal investment decisions. An important implication of this interpretation of γV is that it is independent of the capital structure of the firm. The firm's capital structure consists of equity and a single, coupon-bearing bond issue with principal P. This bond issue may be either callable or noncallable (in this section we assume it is noncallable).

(A2) The uncertainty in the term structure of interest rates is captured by the process on the short rate, r, which is given by

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[3]Black and Cox [2] and Jones, Mason and Rosenfeld [14] have discussed the implications of multiple issues of debt and "me-first" rules in this context.

$$dr = \kappa(\mu - r)\,dt + \sigma_2 \sqrt{r}\,dZ_2 \quad . \tag{2}$$

The scalar μ is the long-run mean rate of interest, κ is the speed with which the interest rate r approaches the long-run mean rate and the instantaneous variance of change in r is proportional to its level. Z_2 is a standard Wiener process. The instantaneous correlation between dZ_1 and dZ_2 is given by ρ. Default-free bonds are priced according to the Local Expectations Hypothesis, i.e. the expected rate of return on every bond over the next instantaneous holding period is equal to r.

(A3) The bond's indenture provisions prohibit the stockholders from selling the assets of the firm to pay dividends. The bondholders have priority and must be paid their coupon continuously at the rate of \$ c. Otherwise the firm is forced into bankruptcy, which is assumed to be costlessly enforced. Thus the lower reorganization boundary for the firm's value, denoted by V^*, is given by c/γ. At this level, the total net cash flow per unit time will be just sufficient to pay the contractual coupon.

(A4) The payoffs to the bondholders upon bankruptcy are defined as follows : when $V = V^*$, the payoff is

$$\text{Min}\,[\delta(\tau)\,B(r,\tau;\,c),\,V^*] \quad , \tag{3}$$

where $B(r,\tau;\,c)$ is the value of a comparable Treasury bond, and $\delta(\tau)$ is the fraction of the value of that equivalent Treasury bond. Relation (3) says that bondholders will recover upon bankruptcy either the total value of the firm or $\delta(\tau)$ fraction of the value of a comparable Treasury obligation (which is free of default risk), whichever is less. By assuming that $\delta(0) = 1$, we ensure that the bondholders are promised P or V, whichever is less, at maturity if the bond is still alive at that time.

Assumption (A0) is standard. The goal of our effort is to demonstrate that the observed yield spreads between corporates and Treasuries can be generated in the context of models where market imperfections play no part. Assumption (A1) is a convenient way to represent the random evolution of the firm's value. In a more general setting, with (perhaps) stochastic investment opportunities, the value dynamics will depend on the optimal investment decisions made by the managers. We have abstracted from those issues for simplicity. The economic content of assumption (A2) is that the uncertainty in the term structure can be modelled using a single state variable. The particular stochastic process in relation (2) has been used by Cox, Ingersoll and Ross [7] and Richard [20] and these papers provide the properties of the conditional moments of the process as well as the stationary distribution implied by (2).[4] This approach differs from that in Brennan and Schwartz [5], who employ two state variables to capture the uncertainty in interest rates. Since we are attempting to simultaneously model the stochastic evolution of the value of the firm and the the evolution of interest rates, the problem already has two state variables. The introduction of an extra state variable such as the long rate would render the valuation problem quite intractable. The assumption on the Local Expectations Hypothesis enables us to avoid specifying investors' preferences and the nature of risk premiums ; a complete discussion of this issue is in Cox, Ingersoll and Ross [8].

The assumptions embodied in (A3) distinguish our paper from the earlier contributions in the literature : we have modelled the fact that if and when the firm's cash flows are unable to cover its interest obligations, bankruptcy is precipitated. An essential feature of corporate bonds is that omission of a coupon payment precipitates bankruptcy (for a detailed treatment of the standard bond indenture provisions, see Smith and Warner [21]). To incorporate default risk of coupons and payment of common dividends, we recognize the firm's net cash flow as a fundamental variable in the analysis. Net cash flow should be interpreted as the total cash flow *less* the optimal investment outlay. We have implicitly assumed that the process for the net cash flow of the firm and the preferences of investors are such that the value of the firm follows a lognormal diffusion process.

We offer a rather simple but internally consistent account of how bankruptcy occurs and how it is settled between bondholders and shareholders. The firm by assumption (A1) has equity and a single bond issue with bondholders receiving a coupon rate of \$ c and a promised final payment of P. The bond indentures prohibit the firm from selling assets. Otherwise, it may be optimal for shareholders to sell the assets and pay themselves a liquidating dividend. We assume that the net cash flow is continuously distributed to both shareholders and bondholders. The bondholders are entitled to the contractual coupon and shareholders receive any residual cash flow if the net cash flow is large enough. Otherwise the firm is forced into bankruptcy. The key to our approach, then, is the recognition that the cash flow problem is the source that precipitates bankruptcy. This approach is broadly consistent with the fact that bond indenture provisions found in practice are almost always specified in terms of cash flows. Since bankruptcy is defined in terms of cash flow alone, we do not rule out the possibility that at the time of bankruptcy, the value of the firm can be *higher* than the value

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[4]An added advantage of this choice is that it permits the use of known solutions for the price of default-free, pure discount bonds to arrive at the value of coupon bearing Treasuries, which are needed in defining the boundary conditions to our valuation problem. The parameters for this process have been estimated by Cox, Ingersoll and Ross [9] and by Brown and Dybvig [6].

of the remaining debt obligations evaluated as if they were default-free. These considerations lead to the definition of the lower reorganization boundary V^*.[5]

In practice, due to the difficulty of objectively defining the optimal investment policy, managers acting on behalf of shareholders have an incentive to reduce the amount of investment to pay off the coupon in order to avoid bankruptcy -- and this may not be observable to bondholders. The deviation from the optimal investment policy will reduce the value of the firm and in turn the value of the corporate bond. In this model, however, there are no informational asymmetries, and both shareholders and bondholders agree to the optimal investment policy and hence agree as to the net cash flows of the firm.

A complete description of bankruptcy requires a description of rules on how the assets of the firm will be divided between bondholders and shareholders. The lower reorganization boundary in relation (3) keeps the bondholders from reaping benefits from bankruptcy because they receive only a fraction of the value of a comparable default-free bond. One can justify this by appealing to the illiquidity of the market for real assets ; the available evidence indicates (see Altman and Nammacher [1]) that on average the market values of low-grade corporate bonds ("junk" bonds) are approximately 40% of their par values immediately after filing for bankruptcy. To be consistent with these observations, we have chosen the lower reorganization boundary presented in equation (3). Brennan and Schwartz [5] employ a similar distribution rule : they assume bondholders will take over the firm when the firm's value falls below a fraction of the par value of the bond. Under their rule, however, bankruptcy is not cash-flow driven, and it may work in favor of the bondholders because the fraction of par value can be larger than the value of an equivalent default-free bond when interest rates are high.

Assumption (A3) and relation (3) play an important role in our contingent claims pricing model, and therefore merit further discussion. The model we have chosen requires an *ex ante* specification of the occurrence of and payoffs from bankruptcy ; it specifies the fraction $\delta(\tau)$ exogenously ; and it does not allow for either new equity issues or side-payments (for example, in a mutually agreeable reorganization, perhaps with changes in future investment policy) from stockholders to bondholders to alter this specification along the way. This rules out strategic decisions leading to agreements on the part of both bondholders and stockholders. Such a situation might arise, for example, if the payoff received by bondholders

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[5]It is sometimes argued that these models should permit the sale of assets in order to make coupon payments. This clearly alters the firm's investment policy and hence its future net cash flows. As a practical matter, the sale of assets is restricted with some exclusions ; see Smith and Warner [21, p. 126].

in relation (3), $\delta(\tau)\, B(\tau; c)$ was less than they would receive if they agreed to forego coupons for the next n months and receive a higher level of coupons thereafter. This is a limitation of the model presented here ; our concern, however, was to see how well the model might perform in the absence of considerations regarding reorganization.[6]

The Valuation Equation

The underlying state variables in this model are the value of the firm V and the interest rate r, so we represent the total value of the corporate bond as $W(V,r,\tau; c)$ where τ is the time to maturity of the bond. Given assumptions (A1) and (A2), Brennan and Schwartz [5] show that the value of the corporate bond must satisfy the following partial differential equation :[7]

$$\frac{1}{2}\sigma_1^2 V^2 W_{vv} + \rho\sigma_1 \sigma_2 \sqrt{r}\, V\, W_{rv} + \frac{1}{2}\sigma_2^2\, r\, W_{rr}$$

$$+ \kappa(\mu - r)\, W_r + (r - \gamma)\, V\, W_v - r\, W + c = W_\tau \quad . \tag{4}$$

We need to append boundary conditions (in addition to the bankruptcy-related condition (3)) in order to complete the description of the bond's value. As V approaches infinity, the payoff function approaches the value of a comparable default-free bond. Thus,

$$\lim_{V \to \infty} W(V,r,\tau; c) = B(r,\tau; c) \quad . \tag{4a}$$

Because we are considering a single (hence, seniormost) noncallable corporate bond, the bondholders will get the lower of the face value (P) or the market value of the firm at maturity. This leads to the following terminal condition :

$$W(V,r,\tau{=}0; c) = \text{Min}[\, V, P\,] \quad . \tag{4b}$$

In order to solve the valuation equation (4) subject to the boundary conditions (3), (4a) and (4b), one must first compute the prices of the default-free coupon bonds $B(r,\tau; c)$ since these appear as boundary values in (3) and (4a). We can compute these prices by using the formula for default-free, zero coupon bonds provided in Cox, Ingersoll and Ross [7] and computing the value of a coupon bearing, default-free bond as the sum of the values of the

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[6]Cox and Rubinstein [10, pp 402-403] discuss the use of call provisions to circumvent some indenture provisions that might be restrictive on shareholders.

[7]In the development of (4), we can admit a proportional factor risk premium for interest rate risk. We have set the factor risk premium to zero although our simulations indicate that our results are not sensitive to this specification.

coupons and principal.

The valuation equation described here differs from Merton's equation for a firm with a single debt issue (see Merton [17], and extensions in Black and Cox [2]) by (i) incorporating interest rate risk, (ii) accounting for coupons and the attendant risk of default prior to maturity. It differs from the valuation equation in Brennan and Schwartz [5] in the way in which default arises and the specification of the payoffs upon default, as discussed earlier.

There is no known closed-form solution to the valuation equation (4), so we will display numerical solutions in Section II. Before we do so, it will be useful to review the evidence on yield spreads for corporate bonds. Figure 1 displays the yields on AAA and BAA rated debt over long-term Treasury yields ; the monthly data series are taken from the *Federal Reserve Bulletin*. The yield spreads on high grade corporates ranged from 15 to 215 basis points and averaged 77 basis points ; and the yield spreads on BAAs (also investment grade) ranged from 51 to 787 basis points, and averaged 198 basis points, over the 1926-1986 period. These yields are not separately available for noncallable and callable bonds -- in fact, the majority of the existing corporate issues of long term bonds are callable. For this reason, we are unable to construct a series of noncallable coupon-bearing bonds' yield spreads, and also to provide evidence on the relation between these yield spreads and the maturities of the bonds. However, Fama [11] reports a negative relationship between average default premiums and maturities for money market securities, which are pure discount instruments.

Subordinated Debt Issues and LBOs

The framework developed in this paper may be used to examine another topical issue in the market for corporate bonds: deeply subordinated debt issues and their use in the funding of management buyouts. Although only a single debt issue was considered in the model presented, we can value subordinated debt in the *same setting* by using the insights of Black and Cox [2]. In their paper, Black and Cox [2] showed that junior (subordinated) debt may be represented as a portfolio of a prespecified number of suitably specified debt issues (or equivalently, as a vertical spread[8]). The same logic extends to our setting as well with two important modifications.

- First, the senior and junior bonds in our model pay coupons and the portfolio arguments of Black and Cox [2] will have to be modified to account for that feature.

[8]The face amount of the senior debt issues in the portfolio must be specified carefully. See, for an example, Cox and Rubinstein [10].

- Second, the lower reorganization boundary will depend on the *sum of the coupon cash flows of the two outstanding layers of debt.* The lower reorganization boundary may be specified so that it depends on the *promised cash flows of either the senior bondholders alone or both bondholders -- and the manner in which one specifies the boundary will affect the qualitative results.*

With these modifications, we can proceed with the pricing of junior or junk debt in the same manner as Black and Cox [2] have done in their paper.

The treatment of the valuation of junior debt issue in our setting resolves a key difficulty that Black and Cox [2] found and noted in their setting: the issuance of junior debt actually benefitted the senior debt holders. Briefly, this problem arose because Black and Cox [2] only considered zero coupon debt issues. As a result, the existence of a junior debt issue involved no additional cash flows from the firm but precipitated an earlier lower reorganization which could only help the senior bond holders, because they would control the firm sooner, *ceteris paribus.*

In the setting suggested in our model, the existence of junior debt forces the firm to pay periodic coupons to the junior as well as senior bondholders and thus leave the senior bondholders with a smaller firm if and when the lower organization boundary is reached prior to the maturity of the bond issues. Thus, in our model there will be scenarios in which senior bondholders will be worse off with the presence of junior debt issues. This feature resolves the difficulty that Black and Cox [2] had in their setting and is consistent with what one observes in real life: the issuance of junk bonds typically leads to the downgrading of existing senior debt issues.[9] We hope in future work to pursue the valuation of junior debt with these features.

II. Numerical Solutions to the Valuation Equation for Noncallable Bonds

In this section we present solutions to the valuation equation, obtained by employing the alternating directions implicit method. The following parameter values are chosen for the stochastic process (in relation (2)) for the interest rate : $\kappa = 0.5$, $\mu = 9\%$, $\sigma_2 = 0.078$. At these parameter values, the yield to maturity of long term, pure discount bonds approaches 8.89%

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[9]From the standpoint of valuing LBOs, our model may be used in the following manner. As junior debt is issued to repurchase equity, wealth transfer begins to take effect across different security holders. By the Modigliani-Miller theorem, the total value of the issuing firm must remain the same. Therefore, the amount of junior debt that must be issued to effect an LBO is the outcome of an iterative process. Computationally, we must iterate with varying levels of junior debt issues and look for the fixed-point where the value of junior debt issued just equals the value of equity that is outstanding.

as maturity increases ; the steady state density of r has a mean of $\mu = 9\%$ and a standard deviation of 2.34%.[10] The speed of mean reversion in r is captured by the value of κ, and it indicates that if we start at a current value of r at 6%, then the conditional mean and conditional standard deviation of r in 1 year's time is 7.18% and 4.27% respectively. The values of κ, μ, and σ were chosen so that by varying the current value of r (as we do below, from 7% to 11%), we obtain rising, relatively flat, and declining term structure scenarios for default free bonds. The parameter σ_1 for the standard deviation of the process on the firm's total value was varied around 0.15, the net cash flow rate γ was fixed[11] at 0.05, and we used values of ρ of −0.2, 0, and 0.2. We report results with ρ of −0.2, consistent with the slight negative relationship between the returns of common stock indices and nominal Treasury Bill returns ; the results for ρ of 0 and 0.2 are virtually the same. The value of the "fraction to recover", $\delta(\tau)$, was kept constant (i.e. time-invariant) at 0.8[12] ; this is a relatively conservative figure, given that Altman and Nammacher [1] report a figure close to 0.4. We vary the capital structures by varying the ratio (P/V) between 0.5 and 0.25, which straddles observed debt ratios of 30-35%.[13] These are the central parameter values employed ; we also examined the effects due to variation in these parameters, and we report on the relevant results below.

To fix matters, Table I shows the default premiums on a 9%, noncallable 10 year corporate bond with a face value (P) of \$100 for (i) Merton's valuation equation (with no interest rate uncertainty),[14] and (ii) for our model (also with no interest rate uncertainty by fixing $\sigma_2 = 0.0$, $r = \mu = 9\%$, and a flat term structure). Thus, Table I serves as a useful point of departure for the subsequent results (which report on stochastic term structure settings), and it *isolates* the influence of the assumption on premature default and the lower boundary condition in relation (3). As the table shows, Merton's model with a standard deviation σ_1 of 0.15 displays a default premium ranging from 7 basis points (at V = 200, or a debt ratio of 50%) to 1 basis point (at V = 400). By contrast, the numerical solutions to the valuation

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[10]The steady state density of r is a gamma ; and the conditional distribution of r(s) given r(t), s > t, is a non-central Chi-square.

[11]This value of γ is appropriate for a firm with a 30% debt ratio, a coupon rate of 9%, and a dividend yield of 3.5%. That is, $\gamma = (0.3)(0.09) + (0.7)(0.035) = 0.0515$. We report below the effect of varying γ around 0.05.

[12]By keeping $\delta(\tau) = 0.8$ and retaining assumption (A4) and equation (4b), which relate to the boundary conditions at bankruptcy and expiration will give rise to a discontinuity at expiration. We have found however, that shapes and levels of yield curves are relatively unaffected by this specification especially at longer maturities.

[13]The ratio B/V might be an economically more meaningful measure of capital structure. We choose the ratio P/V as a measure of capital structure in order to examine the term structure of yield spreads for a given capital structure because the ratio B/V is dependent on r and τ, and will not be constant as these variables change.

[14]Note that this model will permit the sale of assets to meet coupon payments in order to remain feasible.

equation (4), assuming $\sigma_2 = 0$ and $r = \mu = 9\%$ (a flat term structure) display default premiums ranging from 205 basis points to 5 basis points, with a 26 basis point "spread" at a debt ratio of 33%. In order to increase the spread to reasonable levels in Merton's model (48 basis points at a debt ratio of 33%), we had to *double* the riskiness of the firm's assets.

We now turn to the results for the case with a stochastic term structures. We show these results for three initial values of r, of 7, 9, and 11%. Figures 2, 3 and 4 plot the term structure of yield spreads[15] for debt ratios corresponding to P/V = 0.42, 0.33 and 0.28. As expected, the capital structure of the firm has a significant effect on the shape of the term structure of yield spreads. For firms with a low debt ratio, the spreads on corporate bonds increase as time to maturity increases. In this case, long term bonds are riskier than short term bonds because more coupons are subject to default risk. With a high debt ratio, the spreads increase in the first place and then decrease as time to maturity increases. In this case investors holding short term bonds are exposed to the more significant possibility of default on the balloon payment.[16] The levels of the yield spreads, however, decline significantly with reduced debt ratios across all maturities. For instance, with P/V = 0.42, a 10 year corporate bond may command a yield spread of 70 to 92 basis points depending on the level of interest rates. However at a P/V ratio of 0.28, the range becomes 7 to 10 basis points.

These results are fairly insensitive to variations in the values selected for the correlation coefficient (ρ) between the process for the value of the firm and the interest rate process.[17] The uncertainty in the interest rates (σ_2) significantly influenced the yields on *both* Treasury and corporate issues. Interest rate risk, however, is relatively independent of default risk so that the *spreads* are quite unaffected by the interest rate uncertainty. This can be seen by comparing Table II (which incorporates stochastic interest rates) to Table I. Furthermore, as the value for σ_2 is *doubled* to 0.156, the default premium for a 10 year, 9% corporate bond increases by only 4 basis points to 86 basis points when the P/V = 0.42 and $r = 9\%$. However, the location of the interest rate r relative to its long-run mean rate μ

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[15]It might be more useful to construct the term structure of yield spreads using Treasury and corporate bonds of various maturities all selling at par. For this purpose, it is necessary to find coupon rates that give rise to par value for bonds. In the context of our model, it is not feasible because the value of corporate bonds for a given short term interest rate and a given capital structure, is not a monotonic function of coupon rates. We have varied the coupon levels, and we find that the shape of the term structure of yield spreads is not sensitive to coupon rates around the coupon rate of 9%.

[16]This phenomenon is sometimes called the "crisis at maturity".

[17]In order to save space, we do not report the results from changing various parameter values. These tables are available from the authors upon request.

makes a significant difference as noted in Figure 2. Comparing Figure 2 with Figures 3 and 4 suggests that interest rate expectations play an important role in the determination of default premiums when default risk is relatively high (a high P/V ratio).[18]

The yield spread is sensitive to changes in the recovery factor (δ) and the net cash flow ratio (γ). For a firm with $\gamma = 0.06$ and P/V = 0.42, the yield spread would be 37 basis points for a 10 year, 9% bond when the short rate is 9%. This is 44 basis points lower than the yield spread for a firm with $\gamma = 0.05$. That is, the higher is the net cash flow ratio, the lower is the yield spread. This makes sense, because given a firm's value, a higher net cash flow means the firm is more likely to meet coupon obligations. For Tables I and II and Figures 2, 3 and 4, we maintained δ, the recovery factor at 0.8. We show the yields to maturity of a 9%, 10 year, noncallable bond when $\delta = 0.4$ in Table III. In this case the yield spread is as large as 823 basis points given P/V = 0.5 and r = 7%. This suggests that the yield spread can be very large for low grade bonds with a small recovery factor. Given the work of Altman and Nammacher [1] these estimates of spreads are consistent with the evidence on the spreads of junk bonds.

These results are encouraging : they indicate that our model is capable of generating default-premiums in the range observed in practice. We now study yield spreads for callable corporate securities. This is an important task, for the majority of corporate debt is subject to call provisions. The force of the call provision will be to raise the yields, *ceteris paribus*, so that the comparisons to real world bonds will actually come closer.

III. The Numerical Solutions for Callable Corporate Bonds

Corporations typically issue debt with call features. One motivation is, of course, to have the necessary flexibility to refinance the debt should rates go down. Rational investors will naturally recognize this and anticipate the issuing firms to refinance in periods of low interest rates. They will pay a lower price for a corporate bond with a call feature than for an otherwise identical corporate issue without the call feature. In examining callable corporate bonds, we define the *total spread* as the yield differential between a callable corporate bond and an otherwise similar but noncallable Treasury bond ; this is in contrast to the *call premium*, which is the difference in the yields of callable and noncallable, but similar (either

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[18]We have also computed yield spreads on noncallable (and callable) bonds generated from Merton's model with stochastic interest rates but with standard asset sales and bankruptcy rules. A comparison of these results with those reported in Table I shows that stochastic interest rates serve to increase the yields-to-maturity of both corporate and Treasury bonds but leave the yield spread relatively unchanged. This conclusion is also warranted if the bonds are callable. The relevant tables and figures are available from the authors upon request.

corporate or Treasury) bonds. In the following analysis, interactions between default risk and the call provision play an important role in determining the total spread defined in this way.

We might expect the issuing firm to act rationally and call the debt issue when the rates reach a critical value.[19] The optimal call policy, however, depends not only on interest rates but also upon the value of the firm. It is beneficial for the issuing firm to retire the bond when the interest rate is low and the value of the firm is high. But if during periods of low interest rates, the value of the firm is also lower, then the firm may not be able to call the bonds and as a result we might expect the yield spreads of callable corporates to differ from those of noncallable corporates. The foregoing arguments should also make it clear that the optimal call policy will depend on both the state variables V and r, and will represent a critical surface of firm values and interest rates for each maturity $\{C(\tau), R(\tau)\}$, which must be found endogenously.

Suppose that the bond is called by the firm when the value of the firm is $C(\tau)$ and the interest rate is $R(\tau)$. The value of callable corporate bonds, denoted $G(V,r,\tau; c)$, corresponding to a call policy, $\{C(\tau), R(\tau)\}$, also satisfies the valuation equation (4) in Section I. The conditions specified in equations (3) and (4b) also apply to the callable corporate bond. The upper boundary condition, however, will change to

$$G(C(\tau),R(\tau),\tau; c) = K \quad . \tag{5}$$

In (5), we have assumed K to be the single fixed call price at which the issuing firm has the right to call the bond.[20]

The valuation equation (4) subject to (3), (4a), (4b) and (5) is solved numerically by the alternating direction implicit method, and the values of callable corporate bonds are presented in Table IV. The parameter values chosen are the same as those for noncallable bonds. The call price K was assumed to be equal to the par value of the bond. From Table IV, one can see that the total yield spread between a 10 year, 9% corporate bond and a comparable noncallable Treasury bond is 103 basis points when the debt ratio is 42% and the interest rate is 9%. For the same parameter configuration, as Table II shows, the yield

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[19]Ingersoll [13] pointed out that in practice firms do not seem to follow the theoretically optimal policy in calling convertible bonds. This issue has been examined in recent years. However, no evidence has been documented whether similar behavior is shown by firms in calling bonds without the conversion feature.

[20]The optimality of the call policy $\{C(\tau), R(\tau)\}$ is ensured by the smooth-pasting condition,

$$\lim_{\{V,r\} \to \{C(\tau),R(\tau)\}} G_V = \lim_{\{V,r\} \to \{C(\tau),R(\tau)\}} G_r = 0 \quad .$$

spread between a noncallable corporate bond and a noncallable Treasury bond is 81 basis points. Therefore, we may conclude that out of 103 basis points, only 22 basis points can be attributable to the call provision of the bond. A significant part of the yield spread on corporate bonds, therefore, appears to be determined by default risk.

For debt ratios of 42 to 28%, the critical interest rate below which the corporate bond is optimally called is plotted in Figure 5. These are cross-sectional views of the critical surface for each debt ratio (this is made necessary because as the firm's value V changes, the debt ratio changes). The optimal call policy of the firm appears to be more sensitive to the interest rate than to the value of the issuing firm. When the interest rate is higher than 7%, it is suboptimal to call 10 year or longer maturity bonds regardless of the value of the firm. As the value of the firm falls the critical interest rate also declines. This is consistent with one's intuition : at low firm values the issuing firm waits much longer before calling the bonds. The trade-off here is that a decision to call forces an immediate cash outflow but relieves the firm of its (now) high coupon obligations.

The term structure of total yield spreads on a callable corporate bond (defined as the difference between the yield on the callable corporate bond and that on the straight default-free bond) is plotted in Figures 6, 7 and 8 for different debt ratios. The shape of the curves is the same regardless of the debt ratios although the level of yield spreads is high when the debt ratio is high. This is in contrast to the results for noncallable bonds (see, for example, Figure 4) where the yield spread increases with maturity over a wider range. In Figures 6, 7 and 8 the yield spreads go through a maximum, hinting at a stronger interaction between call provisions and default risk at longer maturities.

We can compare this total spread to the default premium on the noncallable corporate bonds that were analyzed in Section II, and hence study the effect of the call provision. In order to isolate the effect of the call provision, however, it is necessary to estimate the impact of call provisions on default-free Treasury bonds. Given assumptions (A0) and (A2) in Section II, the value of a callable Treasury bond will depend on the single state variable r, and also the call policy employed by the Treasury. Just as in the case of the callable corporate bond, the value of the callable, coupon bearing Treasury bond, denoted $H(r,\tau; c)$, will satisfy a valuation equation and reflect an optimal call policy $R^*(\tau)$. This valuation equation (which is equivalent to setting H_V to zero in relation (4)) and the associated boundary conditions are[21]

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[21]Equation (6a) says that, at maturity, the bond must sell at par. As $r \to \infty$ the bond becomes worthless, which is reflected in (6b). Finally, at the critical interest rate, the bond must sell for the call price as shown in (6c).

$$\frac{1}{2}\sigma_2^2 r H_{rr} + \kappa(\mu - r) H_r - r H + c = H_\tau \ , \tag{6}$$

$$H(r,\tau{=}0; c) = P \ , \tag{6a}$$

$$\lim_{r \to \infty} H(r,\tau; c) = 0 \ , \tag{6b}$$

and

$$H(R^*(\tau),\tau; c) = K \ . \tag{6c}$$

The market value of the callable government bond is obtained by choosing $R^*(\tau)$ that minimizes $H(r,\tau; c)$. The optimal call policy is, obviously, endogenous to the valuation problem.[22]

The valuation equation (6) subject to the boundary conditions (6a) - (6c) is solved by the finite difference implicit method to obtain the value of the callable government bond. The parameter values chosen are the same as those for the noncallable government bond. The contribution of the Treasury's call provision to the promised yield to maturity is measured by the difference in the yield to maturity between the callable government bond and the straight government bond. This turned out to be equal to about 45 basis points for a 10 year, 9% coupon bond. This is *larger* than the yield differential between the callable corporate bond and the noncallable corporate bond which is 22 basis points for comparable coupon and maturity. Thus the call feature interacts with the risk of default in determining the total yield spread on a callable corporate bond. Our analysis suggests that the call feature reduces the value of government bonds by more than it reduces that of a corporate bond, *ceteris paribus*.

Figure 5 plots the critical level of interest rates below which the government bond is called optimally, and shows this to be a monotone decreasing function of time to maturity. With longer time to maturity, default-free bonds are called at lower interest rates. Comparing it with the critical interest rates for a callable corporate bond, we find that the critical level of interest rates is lower for the corporate bond. As default risk serves to *reduce* the value of the corporate bond, it takes lower interest rates to raise the value of the corporate bond to the call price.

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[22]The smooth-pasting condition which guarantees the optimality of the call policy is

$$\lim_{r \to R^*(\tau)} H_r = 0 \ .$$

All callable Treasury bonds have a call protection period which is in effect until the bonds have 5 years or less to maturity. To study its impact, we solved the valuation equation for Treasury bonds with and without the call protection period. Table V summarizes the results when the bonds have a call protection feature. Corresponding to interest rates of 7%, 9% and 11%, these results for a 10 year Treasury bond may be contrasted with the yield to maturity of Treasury bonds without the protection feature in Table IV. With the call protection, yields are lower, *ceteris paribus*, since the protection feature works to the advantage of the buyers. The bond may sell *above* par (which is the call price) at sufficiently low interest rates : at 7%, the bond sells at 102.08 whereas the bond without call protection sells at 99.78. The yield spreads widen as the interest rates drop. Again, this is intuitive : the protection feature is most valuable under such circumstances.

We are now in a position to isolate the interaction between the call provision and default risk. It is instructive to do this for an example : we consider a Treasury bond and a corporate bond, both with 10 years to maturity when the current short rate is 9%. For the corporate bond, the debt ratio is 42%. Here is a summary of the yields to maturity for callable and noncallable bonds :

Security	Yield To Maturity
Straight Government Bond	8.93%
Callable Government Bond	9.38%
Straight Corporate Bond	9.74%
Callable Corporate Bond	9.96%

The total spread defined as the the yield differential between the callable corporate bond and the straight government bond is 103 basis points. The difference in the yield to maturity between the straight corporate bond and the straight government bond measures the portion of total spread that can be attributable to default risk. It is 81 basis points. The contribution of the call provision to the total premiums is 45 basis points and is the yield differential between the callable government bond and the straight government bond. Therefore 81 + 45 – 103 = 23 basis points is attributable to interaction between the call provision and default risk. These reductions in yield differentials due to interaction between the call provision and default risk are summarized in Table VI. They are larger, the higher is the debt ratio and the lower is the interest rate.

IV. Summary

In this paper, we have developed a corporate bond valuation model which incorporates some important real world features. We have modelled the coupon risk and the importance of cash flow shortages in precipitating bankruptcy. The approach presented here allows us to propose a definition of default that is internally consistent and plausible in terms of generating the magnitude of premiums on corporate bonds empirically observed. The results also imply some testable implications for the shape of the term structure of yield spreads.

Stochastic interest rates seem to play an important role in determining the yield differentials between a callable corporate bond and an equivalent government bond due to the interactions between call provisions and default risk. This suggests that care should be taken in interpreting empirical results regarding the effect of default risk on the values of callable corporate bonds.

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Table I

Default Premiums (basis points) on 9%, 10 Year Corporate Bonds under a Flat, Nonstochastic Term Structure

	Merton's Model		Our Model[a]	
Firm Value (V)	σ_1=0.15	σ_1=0.30	σ_1=0.15	σ_1=0.30
200	7	120	205	294
220	4	98	127	251
240	2	80	82	217
260	1	67	55	189
280	1	56	37	167
300	1	48	26	148
320	1	41	18	132
340	1	35	13	119
360	1	31	10	108
380	1	28	7	98
400	1	25	5	89

[a]Solution to the valuation equation (4) subject to (3), (4a) and (4b) with $\sigma_2 = 0$ (nonstochastic interest rate case), net cash flow ratio (γ) = 0.05 and recovery factor (δ) = 0.8.

Merton's model has nonstochastic interest rate. The yield curve is flat, at 9%, for both models.

Table II

Yield Spreads (basis points) on 9%, 10 Year Noncallable Corporate Bonds[a] under a Stochastic Term Structure

	Interest Rate (r)		
Firm Value (V)	7%	9%	11%
200	217	204	191
220	139	126	112
240	92	81	70
260	62	53	45
280	42	36	26
300	29	25	21
320	20	17	14
340	14	12	10
360	10	9	7
380	8	7	5
400	5	5	4

[a]Solution to the valuation equation (4) subject to (3), (4a) and (4b) with the following parameter values :

Face value (P) : 100, Recovery factor (δ) = 0.8,
Volatility parameter for the firm value (σ_1) : 0.15, Net cash flow ratio (γ) : 0.05,
Long-run mean rate of interest (μ) : 9%, Speed of adjustment parameter (κ) : 0.5,
Volatility parameter for the interest rate (σ_2) : 0.078,
Correlation coefficient (ρ) : –0.2.

Table III

Yield Spreads (basis pints) on 9%, 10 Year Noncallable Corporate Bonds[a] under a Stochastic Term Structure (Recovery Factor (δ) = 0.4)

	Interest Rate (r)		
Firm Value (V)	7%	9%	11%
200	823	760	696
220	480	426	375
240	301	262	224
260	196	164	141
280	132	111	93
300	91	76	63
320	63	53	43
340	45	37	30
360	32	27	21
380	23	19	15
400	17	14	11

[a]Solution to the valuation equation (4) subject to (3), (4a) and (4b) with the following parameter values :

Face value (P) : 100, Recovery factor (δ) = 0.4,
Volatility parameter for the firm value (σ_1) : 0.15, Net cash flow ratio (γ) : 0.05,
Long-run mean rate of interest (μ) : 9%, Speed of adjustment parameter (κ) : 0.5,
Volatility parameter for the interest rate (σ_2) : 0.078,
Correlation coefficient (ρ) : –0.2.

Table IV

The Value[a] of and Yield Spreads[b] (basis points) on 9%, 10 Year Callable Corporate Bonds (Callable at Par)

	Interest Rate (r)		
Firm Value (V)	7%	9%	11%
200	89.60	87.46	85.40
	226	214	201
220	93.71	91.55	89.39
	156	143	129
240	96.24	93.94	91.58
	115	103	91
260	97.75	95.35	92.84
	92	80	70
280	98.62	96.19	93.58
	78	66	58
300	99.12	96.70	94.03
	70	58	50
320	99.40	97.01	94.30
	66	53	46
340	99.55	97.20	94.47
	64	50	43
360	99.64	97.32	94.58
	63	48	41
380	99.69	97.39	94.65
	62	47	40
400	99.71	97.44	94.70
	61	47	39
Callable Treasury Bond	99.78	97.56	94.81
	60	44	37

[a]Solution to the valuation equation (4) subject to (3), (4a), (4b) and (5) with the following parameter values :

Face value (P) : 100, Recovery factor (δ) = 0.8, Call price (K) : 100, Volatility parameter for the firm value (σ_1) : 0.15, Net cash flow ratio (γ) : 0.05 Long-run mean rate of interest (μ) : 9%, Speed of adjustment parameter (κ) : 0.5, Volatility parameter for the interest rate (σ_2) : 0.078, Correlation coefficient (ρ) : –0.2.

[b]Yield spreads between 9%, 10 year callable corporate bonds and comparable noncallable Treasury bonds.

Table V

The Value[a] and Yield Spreads[b] (basis points) on 9% Callable Treasury Bonds with Call Protection

Interest Rate (r)	Time to Maturity (yrs) 5	10	15	20
5%	n.a.[c]	105.50	106.48	107.02
	n.a.	49	13	7
6%	n.a.	103.77	104.70	105.23
	n.a.	26	13	7
7%	n.a.	102.08	102.95	103.48
	n.a.	26	13	7
8%	99.26	100.42	101.24	101.75
	62	26	13	7
9%	98.20	98.79	99.56	100.06
	50	26	13	7
10%	96.98	97.18	97.90	98.39
	41	25	13	8
11%	95.69	95.60	96.28	96.76
	35	25	13	7
12%	94.37	94.05	94.69	95.15
	31	24	13	7
13%	93.03	92.52	93.12	93.57
	21	23	13	7
14%	91.68	91.02	91.58	92.03
	23	24	13	8
15%	90.33	89.55	90.07	90.51
	22	23	13	7

[a]Solution to the valuation equations (6), (6a), (6b) and (6c) with the following parameter values :

Face value (P) : \$ 100, Call price (K) : \$100,
Long-run mean rate of interest (μ) : 9%, Speed of adjustment parameter (κ) : 0.5,
Volatility parameter of the interest rate (σ_2) : 0.078.

[b]Yield spreads between 9%, 10 year callable Treasury bonds which are callable during the last five years and comparable noncallable Treasury bonds.

[c]It is optimal to call bonds.

Table VI

The Effect of Interaction[a] (basis points) between Default Risk and Call Provision on the Yield Spread of 9%, 10 Year Callable Corporate Bonds[b]

	Interest Rate (r)		
Firm Value (V)	7%	9%	11%
200	51	34	27
220	43	28	20
240	37	23	16
260	30	18	12
280	24	15	9
300	19	12	8
320	14	9	5
340	10	7	4
360	7	6	3
380	6	5	2
400	4	3	2

[a]The interaction is the reduction in the total yield spread of a callable corporate bond due to the presence of the call provision. It is computed by subtracting the corporate bond's call premium from the Treasury bond's call premium.

[b]The following parameter values are used to compute the yields to maturity.

Face value (P) : 100, Recovery factor (δ) = 0.8, Call price (K) : 100,
Volatility parameter for the firm value (σ_1) : 0.15, Net cash flow ratio (γ) : 0.05,
Long-run mean rate of interest (μ) : 9%, Speed of adjustment parameter (κ) : 0.5,
Volatility parameter for the interest rate (σ_2) : 0.078,
Correlation coefficient (ρ) : –0.2.

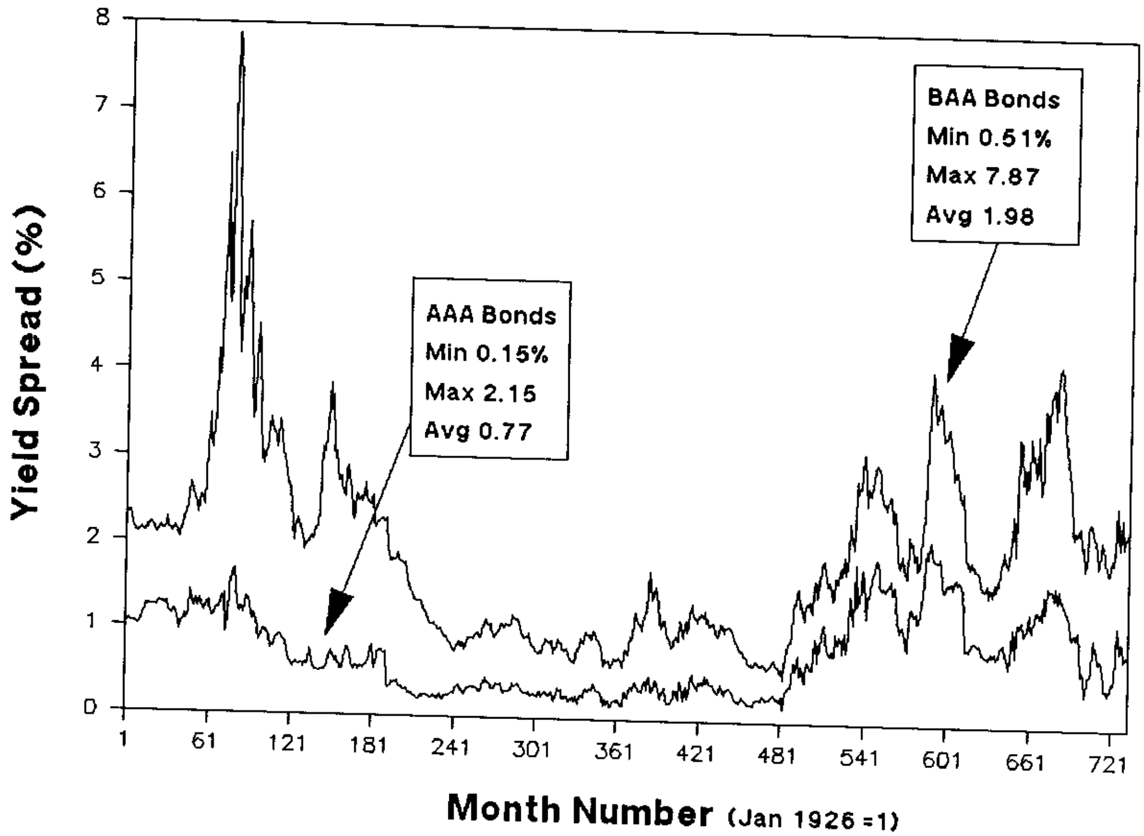

Figure 1. Monthly Yield Spreads, 1926-86: Average Corporate Yields minus Treasury Yields.

(Source: Federal Reserve Bulletin)

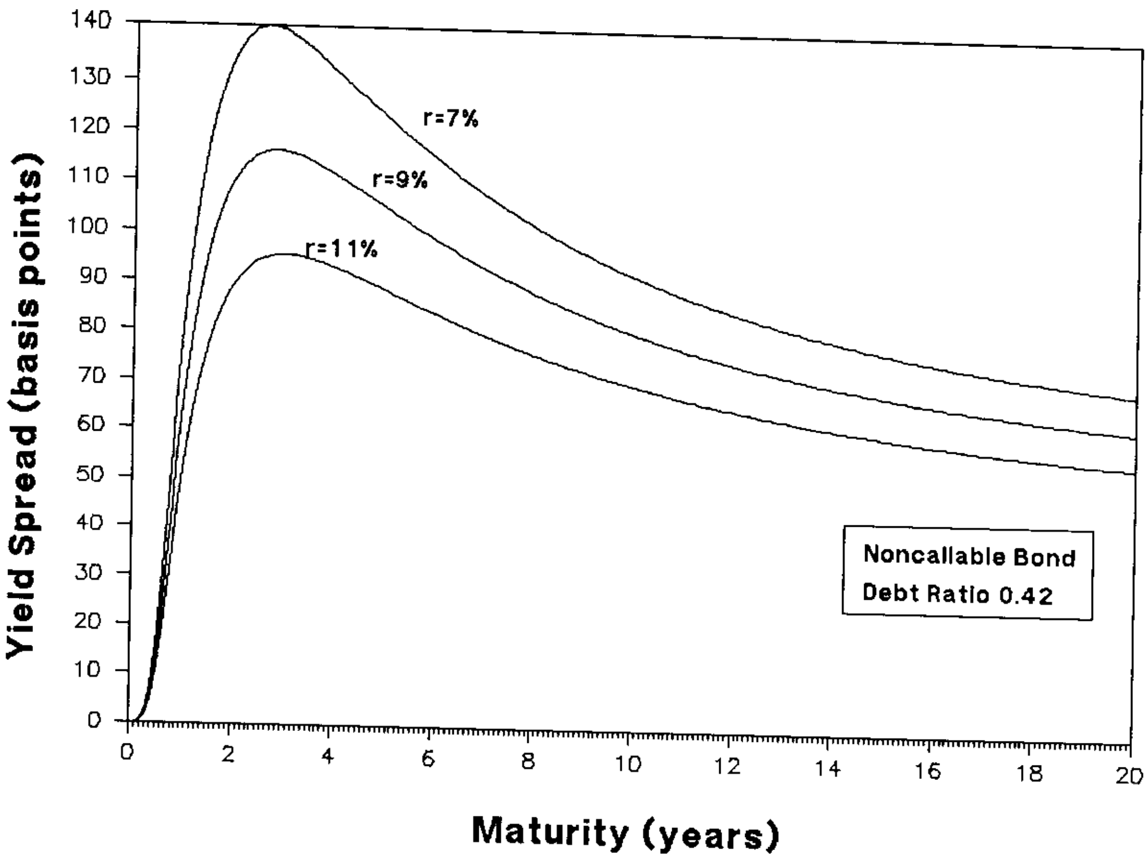

Figure 2. Yield Spread vs. Maturity for a noncallable, 9% coupon bond. The debt-to-value ratio is fixed at 0.42. r is the current interest rate.

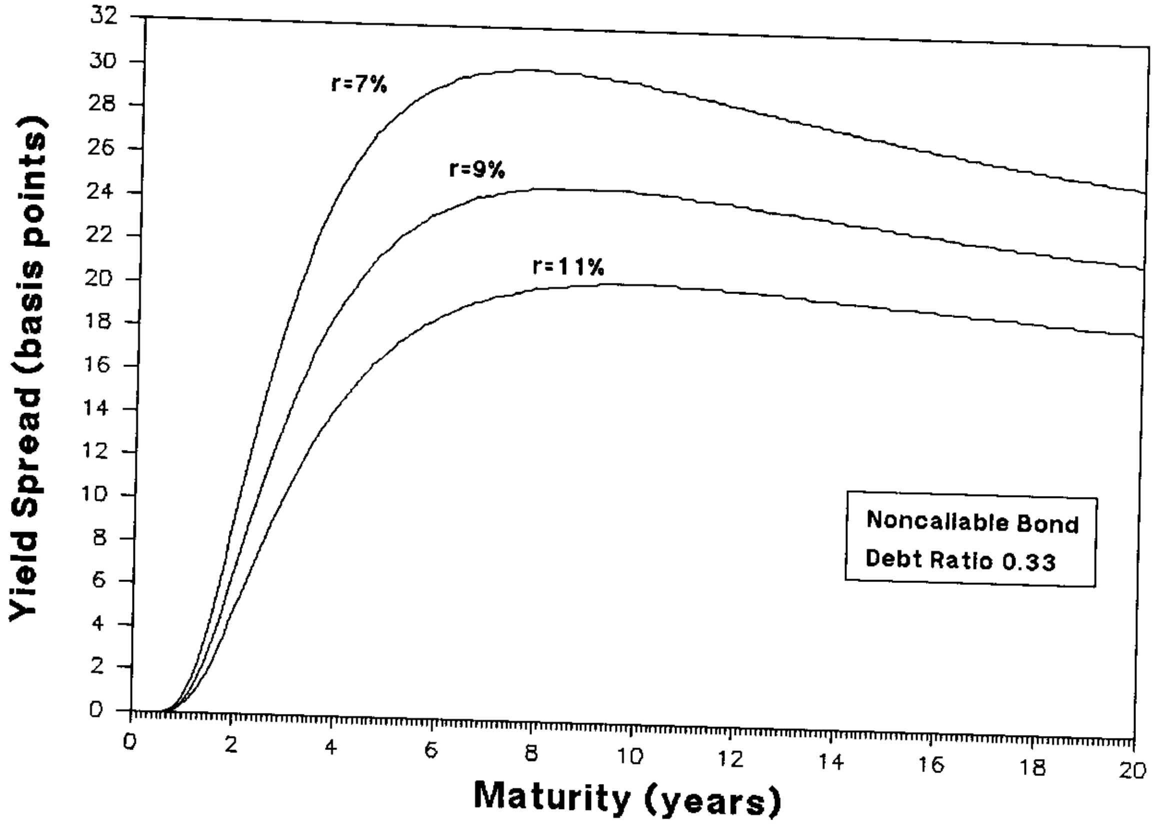

Figure 3. Yield Spread vs. Maturity for a noncallable, 9% coupon bond. The debt-to-value ratio is fixed at 0.33. r is the current interest rate.

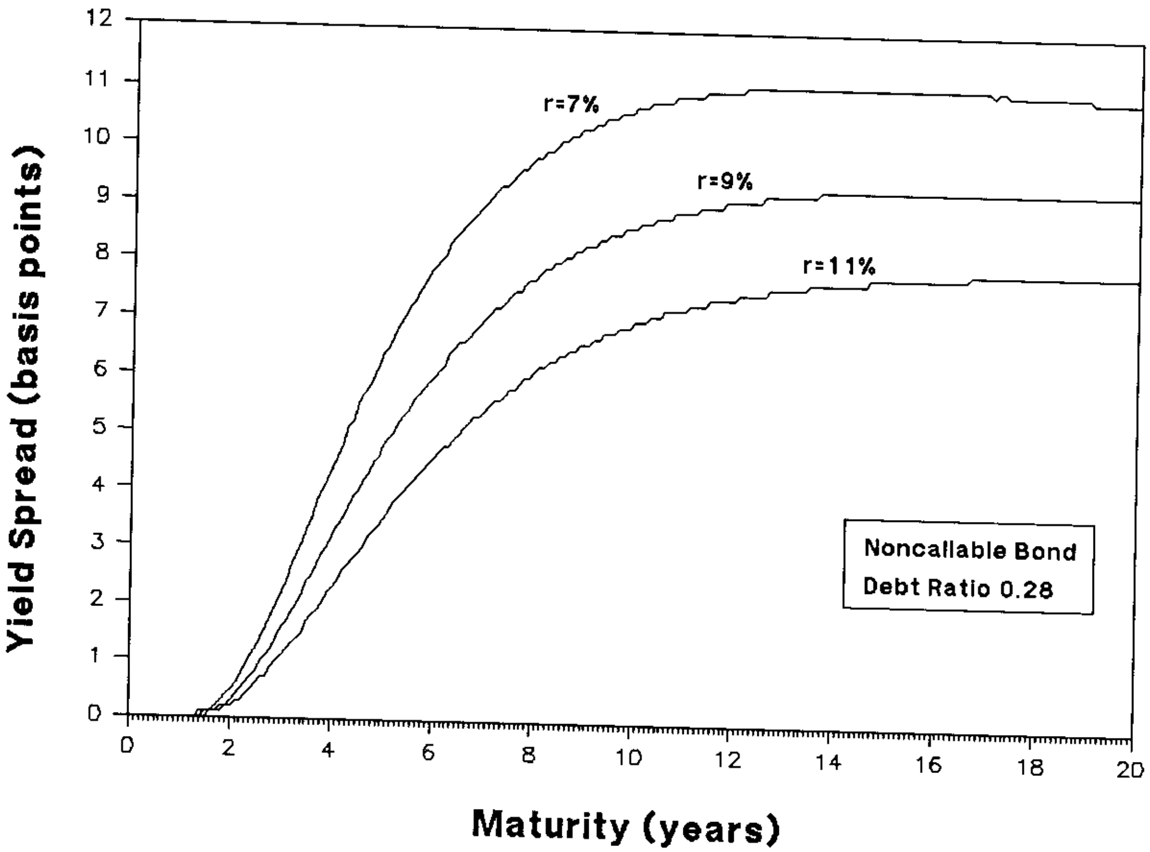

Figure 4. Yield Spread vs. Maturity for a noncallable, 9% coupon bond. The debt-to-value ratio is fixed at 0.28. r is the current interest rate.

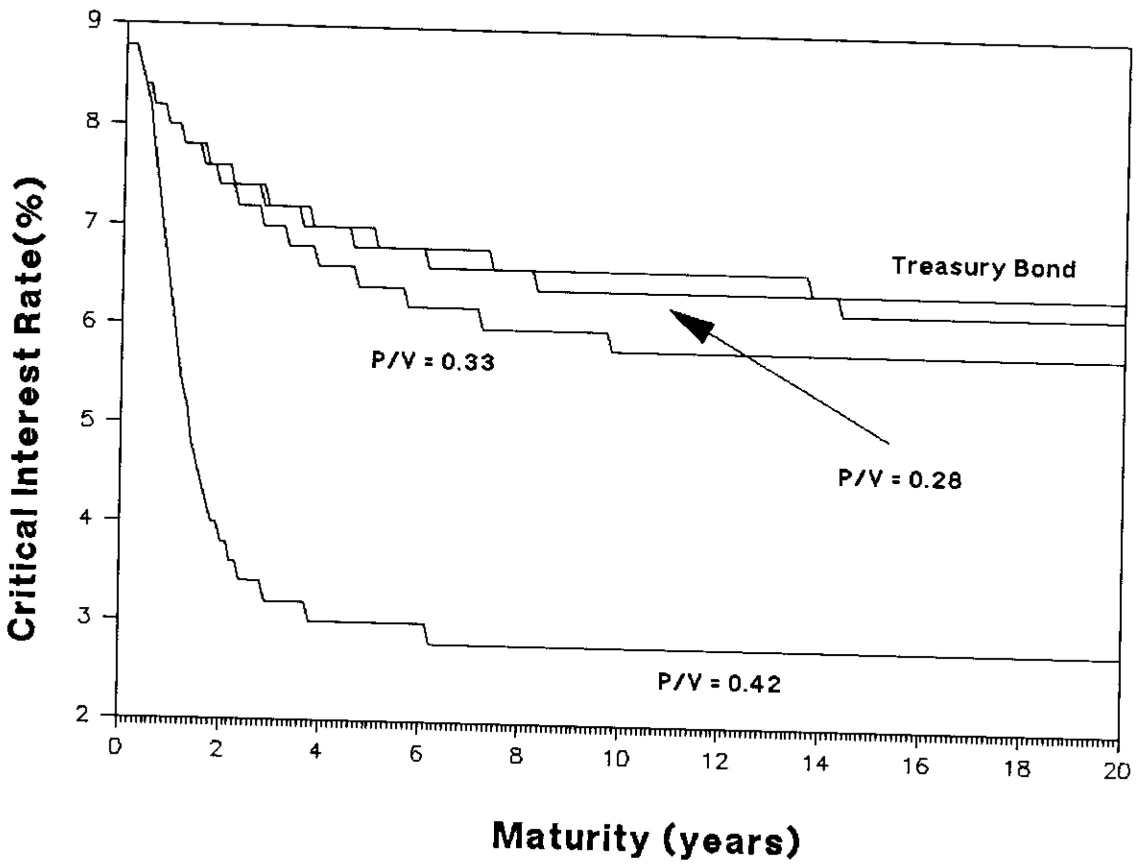

Figure 5. Critical Boundary for the optimal call policy for a 9% coupon bond. The bond will be called if the current interest rate falls to the boundary at the appropriate debt ratio (P/V) for the corporate bond.

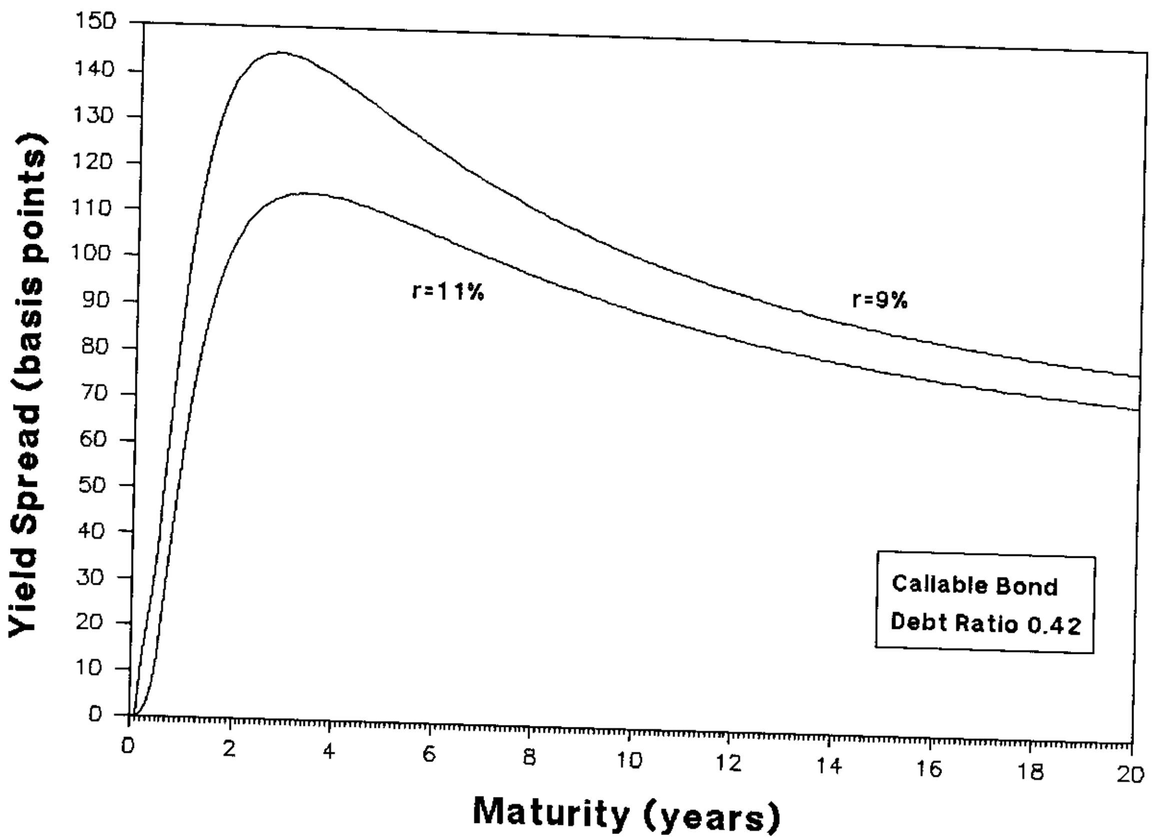

Figure 6. Yield Spread vs. Maturity for a callable, 9% coupon bond. The debt-to-value ratio is fixed at 0.42. r is the current interest rate.

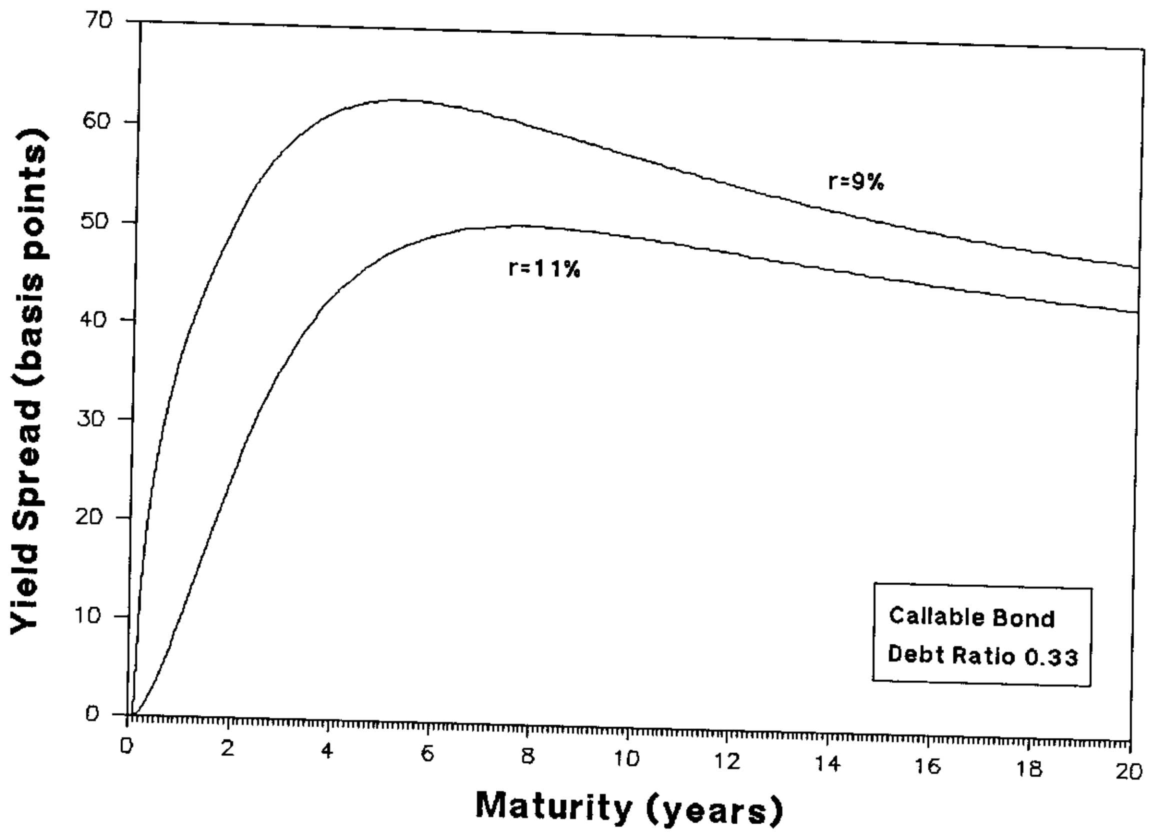

Figure 7. Yield Spread vs. Maturity for a callable, 9% coupon bond. The debt-to-value ratio is fixed at 0.33. r is the current interest rate.

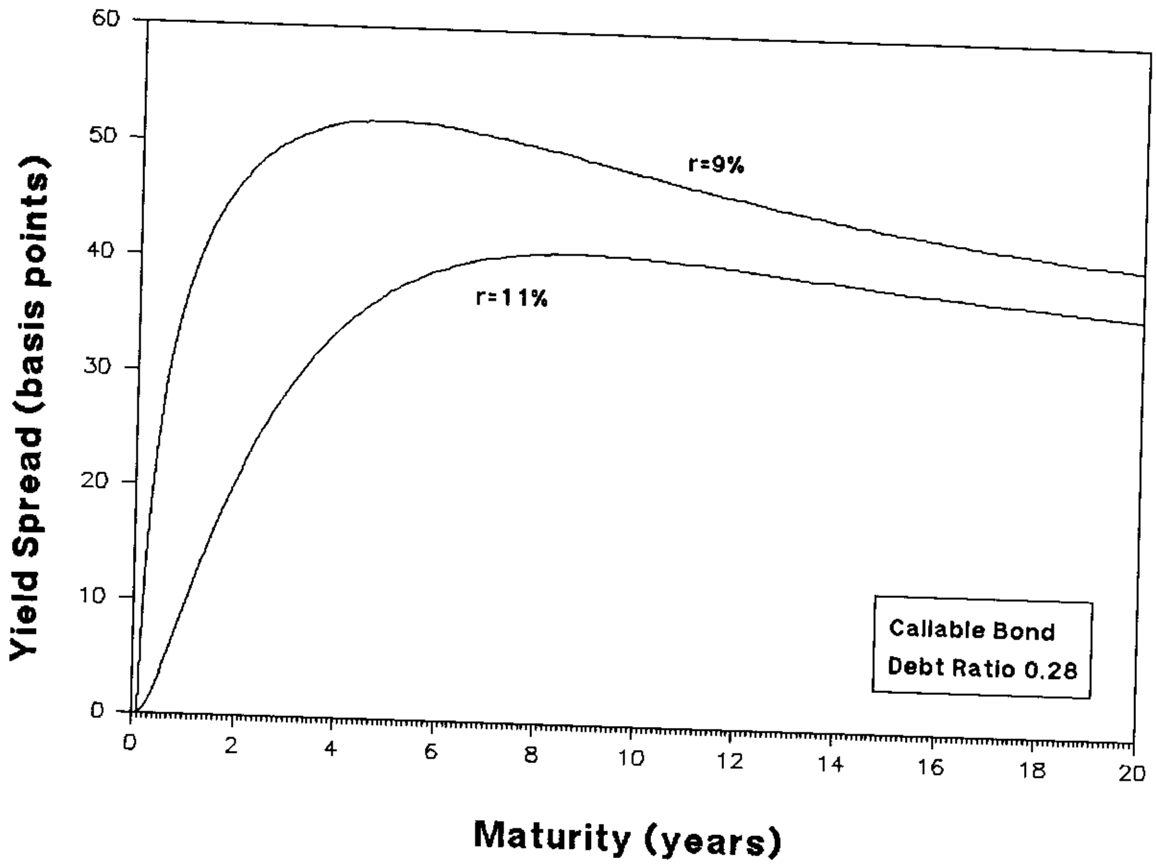

Figure 8. Yield Spread vs. Maturity for a callable, 9% coupon bond. The debt-to-value ratio is fixed at 0.28. r is the current interest rate.