SECURITY BASKETS AND INDEX-LINKED SECURITIES

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Abstract

Security baskets and index-linked securities are securities whose values are functions of the cash flows or values of other assets. Intermediaries create security baskets by pooling or bundling more primitive assets such as mortgages, credit card receivables and other loans, or equities as in the case of closed-end mutual funds. Index-linked securities, such as index participations and stock index futures, are created by stock and futures exchanges. Creation of these "composite" securities would appear to be redundant if investors could individually purchase the securities that compose the security basket or index, thus creating their own diversified portfolios. However, we show that when some investors possess inside information, composite securities are not redundant. They provide superior "liquidity" for uninformed investors. By holding these securities, uninformed investors with unexpected needs to trade can reduce their expected losses to investors with inside information. Moreover, the existence of these securities affects real investment decisions and equilibrium rates of return. We provide an application of our model to the problem of international portfolio choice and mutual fund design.

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I. Introduction

Security baskets and index-linked securities, which we will refer to as composite securities, are securities whose values are functions of the cash flows or values of other assets. There are numerous examples of such composite securities which are created by intermediaries or stock and futures exchanges. Intermediaries bundle assets to create new securities whose payoffs depend on the cash flows of the underlying asset pool. Straightforward examples of such security baskets include mortgage- and asset-backed securities, and closed-end mutual funds. While intermediaries generally create composite securities with positive net supply, exchanges can create composite securities with zero net supply, such as stock index futures and index participations. Obviously, the creation of these composite securities gives investors a trading vehicle that is an alternative to buying or selling the underlying assets which compose the basket or index.

The outstanding volume of composite securities is very large. For example, securitization in the form of mortgage-backed securities (residential mortgages) is currently about $800 billion. Securitization of credit card receivables is over $12 billion as of March 1989. In addition to baskets of securities, index linked securities are also very popular. The S&P 500 stock index futures contract is now one of the highest volume futures contracts. Index participations, which are securities whose value is determined by a stock index, have recently begun trading on the Philadelphia Stock Exchange and on the American Stock Exchange (Financial Times, May 16,1989). The Chicago Board Options Exchange has also proposed trading an index participation, and recently the New York Stock Exchange began trading in baskets of stocks, called "exchange stock portfolios," (New York Times, June 5, 1989).[1]

Creating security bundles or index-linked securities appears to be a redundant activity. Consumers, on their own, could apparently accomplish the same resulting cash flow by holding a diversified portfolio of the same securities in the same proportions. Thus, it is difficult to explain the existence of a large number of securities that are simply repackagings of existing securities. In this essay we argue that the activity of creating composite securities is not redundant. It affects real investment decisions and equilibrium rates of return. Consumers cannot replicate the returns from the bundles or index-linked securities by holding portfolios of the same securities in the same proportions.

The reason that these securities are not superfluous is that consumers who trade using them, rather than the underlying assets, reduce their expected losses to investors possessing superior information (insiders). To explain this we study a model environment with many primitive assets in which some traders must trade for liquidity reasons. When they trade, they trade in markets in which there are insiders, and in which prices are not fully revealing. The trading situation is similar to Kyle (1985) in that insiders can profit at the expense of uninformed liquidity traders. We focus on the response of optimizing liquidity traders to the prospects of losses to insiders. That is, we ask whether the liquidity traders can package the given primitive assets to prevent or, at least, minimize their expected losses to the insiders.

In previous work, Gorton and Pennacchi (1989), we showed that uninformed liquidity traders could design securities (transform primitive cash flows) that could prevent trading losses to insiders. In that case, composite

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[1] See Kupiec (1989) for a description of these index participations.

securities (i.e., security bundles or index-linked securities) were not created, rather primitive cash flows were split (into debt and equity) to create a portion (the debt) which was less prone to incurring losses to insiders because it was (relatively) riskless, that is, its value was known.[2] Here we consider whether liquidity traders can prevent or minimize losses to insiders by combining securities, rather than creating new securities by splitting the cashflows of more primitive securities. Thus, here we take the menu of securities as given and ask whether combining them in various ways can improve the situation of the liquidity traders.

To answer this question we embed the Kyle model in a larger, multi-asset model which allows us to explicitly consider the optimizing response of the liquidity traders to the presence of insiders. Kyle (1985) showed that for a given variance of liquidity traders' net demand for a security, the insider's profit is larger when the variance of the security's return is larger. Essentially, the insider camouflages his trading activity using the randomness of the liquidity traders' activity and the uncertainty about the true value of the security. For simplicity, Kyle took the liquidity traders to be non-optimizing agents whose net demands were represented as a normally distributed shock. He focused on characterizing the insiders' optimal strategy.

In Kyle's model the security's price is set by a market maker. The market maker understands that the order flow reflects both the liquidity traders' demands and the activity of the insider. The market maker cannot distinguish between these, however, since he only observes the total net order

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[2]As will be seen below, the environment here does not admit the possibility of intermediation as a solution. Unlike Gorton and Pennacchi (1989), here there are no agents willing to hold bank equity at the initial date. The environment here is less restrictive since it does not require that informed traders know that they will become informed by date 1.

flow. Since the market maker understands the adverse selection problem, he sets the price in such a way as to optimally disentangle the two influences. But, this is harder to accomplish when the true value of the security has a larger variance or when the variance of the liquidity traders' demands is higher. Thus, the expected profits of the insiders, which equals the expected losses to the liquidity traders, increase when the security's variance is higher.

Now it is clear that the rationale for creating composite securities, i.e., security bundles or index-linked securities, is simply that rate of return variance can be reduced when primitive securities are combined. By trading a composite security rather than the primitive securities that underlie the composite, the liquidity traders can reduce their expected losses to insiders because they are trading a lower variance security. Therefore, the creation of composite securities changes the returns received by liquidity traders. Furthermore, the demand for primitive securities that make up these composite securities will be different than the demand for primitive securites that would result if markets for composite securities did not exist. In a production economy, firms' real investment decisions and equilibrium asset supplies are affected.

The model used in this paper allows us to explicitly solve the portfolio problem of liquidity traders who are the only traders with security demands at the beginning of the model. Because security returns are assumed to be normally distributed, we can derive the liquidity traders' efficient portfolio frontier. We show that the efficient frontier in the presence of insiders is everywhere dominated by the efficient frontier without insiders. However, when composite securities are created, the efficient frontier unambiguously improves the investment opportunities of liquidity traders.

The paper proceeds as follows. Section II is concerned with specifying the model environment. We introduce a model that allows us to explicitly consider the optimizing decisions of the liquidity traders. They will choose an initial portfolio knowing that subsequently they may have to trade in markets in which insiders are present. The model is constructed so that each of the subsequent security markets are similar to Kyle's model. With this larger framework we derive the expected returns and covariances of asset returns in the presence of insiders and show that the liquidity traders are disadvantaged. In Section III these results are presented in terms of an efficient frontier when insiders are present. Section IV then considers the creation of composite securities and demonstrates the improvement in expected utility. Again, this is characterized in terms of an efficient frontier. Section V gives an illustrative example of the possible gains from security bundling in the context of international portfolio choice. Section VI concludes.

II. The Model Environment

The model economy we will develop embeds a Kyle-type model in a larger structure that allows us to consider the optimal decisions of some traders usually taken as nonoptimizing in this style of model. However, as in other models of this type there will also be so-called noise traders. These traders should be interpreted as trading for exogenous reasons, and their decisions are not modelled.[3]

There are three dates in the economy, t = 0, 1, and 2. Firms issue

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[3]As explained below, the existence of noise traders is not a critical factor in reaching our qualitative results. Noise is not intrinsic to the model, but leads to tractable solutions to the model's portfolio choice problem.

securities to agents in exchange for capital at date 0. At date 1, a forward market for the securities is open. Agents can trade forward contracts on firm shares. The contracts are settled at date 2 at which time firms pay a liquidating dividend in the form of units of the consumption good.

A. Liquidity and Noise Traders

The economy is populated by informed and uninformed agents. There are two kinds of uninformed agents, liquidity traders and noise traders. There is a large number, μ, of liquidity traders who receive an endowment of capital at date 0. They have utility defined at either date 1 or date 2 with a concave and twice differentiable utility function given by U(W), where W refers to final wealth for either date 1 or date 2. Liquidity traders with utility at date 1 will be referred to as "early" consumers, while those with utility at date 2 will be referred to as "late" consumers. There is also a large number of noise traders who receive an endowment of consumption units at date 1 and who have utility from consumption at date 2.

Liquidity trader preferences are uncertain initially, that is, at date 0 each liquidity trader does not know if he will be an early consumer or a late consumer. The number of liquidity traders who will be early consumers is a random variable, $\tilde{n}_0$. It is assumed that there is a probability k that a given liquidity trader will turn out to be an early consumer, so that the random variable $\tilde{n}_0$ will have a binomial distribution with mean equal to $k\mu$ and a variance equal to $\mu k(1 - k)$. Likewise, the number of noise traders is assumed to be the result of a drawing from another population of agents of size μ. The total number of noise traders will be a random variable, $\tilde{n}_1$, with mean equal to $k\mu$ and variance equal to $\mu k(1 - k)$. In addition, it is assumed that the random number of noise traders is uncorrelated with the random number of early consumers.

Each liquidity trader receives an endowment of non-storable capital, at date 0, equal to e. At date 0, liquidity traders may choose to invest their capital in up to M different technologies in the economy, each owned by a firm. Each technology is in perfectly elastic supply. For each unit of capital, a firm issues a share, which for the i^{th} firm, has a date 2 random rate of return, $\tilde{v}_i$, which is distributed $N(\bar{p}_i, \Sigma_i)$.[4] The return on each firm's shares is assumed to be uncorrelated with $\tilde{n}_0$ and $\tilde{n}_1$.

Each noise trader receives an endowment of non-storable consumption goods, at date 1, equal to e. Noise traders are assumed to sell their endowments of consumption units and buy securities for forward delivery in proportion to the amount in which they were issued. In other words, they purchase the 'market' portfolio. This assumption simplifies the analysis and will be discussed later.[5]

At date 1 a set of forward markets open where claims on the M firms' shares are traded.[6] The forward market for security i works in the following manner. A short seller may sell forward shares of security i, receiving p_i units of the consumption good at date 2 in return for delivery of one share of firm i (or $\tilde{v}_i$ units of the consumption good) at date 2. Alternatively, the

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[4]Negative consumption is permitted at date 2.

[5]Without this assumption, the noise traders would optimally choose a portfolio which differed from that being sold by the liquidity traders, since noise traders will choose a portfolio knowing they will always be trading in a market where insiders are present whereas liquidity traders initially choose a portfolio knowing there is only a probability k that they must trade with insiders. This would make aggregation of net liquidity and noise trader demands more difficult, but is not a critical factor for our qualitative results.

[6]We consider forward markets, rather than spot markets, in order to allow the possibility of very large (infinite) long or short positions in the security at date 1. This will be convenient when the behavior of insiders and market makers is considered.

short seller can receive p_i units of the consumption good at date 1 if one share of firm i is delivered at time 1.[7] An agent taking a long position in a forward contract agrees to take delivery of one share of security i at date 2 (or $\tilde{v}_i$ units of the consumption good) in exchange for paying p_i units of the consumption good at date 1.

At date 1, early consumers will sell forward all their shares that, in total, equal $k\tilde{n}_0 e$, receiving p_i units of the consumption good for each share of firm i sold. A noise trader, who at date 1 will receive an endowment of non-storable consumption goods will use this to purchase securities forward in this same forward market. The following lemma explains how individual liquidity and noise trader demands can be aggregated. It shows that by increasing the number of liquidity and noise traders, while shrinking their endowment in an appropriate manner, net forward purchases will be approximately normally distributed.

Lemma 1: Let $e = \sigma/(2\mu k(1 - k))^{\frac{1}{2}}$. Then as $\mu \to \infty$, the net aggregate forward purchase of securities by liquidity traders and noise traders at date $t = 1$ equals:

$$\tilde{u} \sim N(0,\sigma^2).$$

Proof: The proof is an application of the Central Limit Theorem. The level of net forward purchases, u, can be viewed as the sum of μ independent drawings, X_i, $i = 1,\ldots, \mu$ with a drawing being the result of a simultaneous determination of early or late consumption for a given liquidity trader with the event of whether or not a given agent will be a noise trader from the other population of agents of size μ. Therefore, for a given drawing, the net

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[7] This implies that the forward and spot price for shares will be the same at date 1.

purchase of securities at date 1, X_i, will be one of the following three outcomes:

1) $X_i = +e$, corresponding to the event {late consumer, noise trader}, which occurs with probability $k(1 - k)$.

2) $X_i = 0$, corresponding to the event {late consumer, no noise trader}, which occurs with probability $(1 - k)^2$, or to the event {early consumer, noise trader} which occurs with probability k^2.

3) $X_i = -e$, corresponding to the event {early consumer, no noise trader}, which occurs with probability $k(1 - k)$.

Therefore the expected value of X_i is zero and its variance is $2e^2k(1 - k) = \sigma^2/\mu$. Now we can consider the limiting distribution of u, which is equal to the sum of the X_i's, as $\mu \to \infty$. By the Central Limit Theorem, the limiting distribution of:

$$\frac{u}{\sigma} = \left(\sum_{i=1}^{\mu} X_i\right)/\left((\mu)^{\frac{1}{2}}(\sigma^2/\mu)^{\frac{1}{2}}\right)$$

is normal with mean zero and variance one.||

Lemma 1 provides the distribution for the net aggregate forward purchases of the liquidity and noise traders at date 1. Define the proportion of liquidity traders' aggregate initial capital invested in the i^{th} market as w_i. Since noise traders are assumed to purchase the 'market,' the proportion of their wealth invested in security i is also w_i. Therefore, an obvious extension of Lemma 1 gives the aggregate net forward purchase of security i at date 1. It is $\tilde{u}_i \sim N(0, w_i^2\sigma^2)$.

The point of the lemma is that while our economy has a finite number of potential traders with finite endowments, as the number of agents becomes large while their per capita endowments shrink at an appropriate rate, the distribution of aggregate net purchases can be made approximately normal with finite variance.

B. Informed Traders

There are M informed traders, each observing the realized liquidation value of one of the M securities, $\tilde{v}_i$, after the date 0 securities market closes, but prior to the opening of the forward market at date 1. A trader who has information about the liquidation value of security i is assumed to optimally choose a quantity of firm i's shares to purchase or sell forward. Let $x_i(\tilde{v}_i)$, be the net forward purchase of firm i's shares by the i^{th} informed trader that maximizes expected profits. In choosing x_i, the informed trader does not know the net liquidity and noise trader demand for security i. However, it is known that this net demand is distributed as above, and is independent of the distribution of $\tilde{v}_i$.

C. Trading Equilibrium

Given the distribution of the net liquidity and noise demands in each of the M markets, each market is similar to the single market analyzed by Kyle (1985). Kyle (1985) assumes the existence of a competitive market maker who observes the order flow $\tilde{x}_i + \tilde{u}_i$, then determines a price in terms of consumption goods, $p_i = p_i(\tilde{x}_i + \tilde{u}_i)$, and a market position that clears the market. Both the market maker and the informed traders are assumed to be endowed with a sufficient quantity of consumption goods such that their budget constraints are not binding in what follows.[8]

An equilibrium at date 1 is:

(i) a trading strategy for the informed agent in each market, i, that maximizes his profits, knowing how the market maker sets the price and;

[8]This is the same assumption as Kyle.

(ii) a price setting rule, chosen by the market maker, that is constrained to earn zero profits knowing how the informed agent behaves.

Kyle solves for an informed trader's optimal trading strategy, and shows that, in equilibrium, the informed trader makes expected profits at the expense of the uninformed traders. Kyle shows in his Theorem 1 that there exists a unique equilibrium in which the informed trader's trading strategy, $x_i(\tilde{v}_i)$, and the market maker's price setting rule, $p_i(\tilde{x}_i + \tilde{u}_i)$, are linear functions:

$$x_i = \beta_i(\tilde{v}_i - \bar{p}_i) \tag{1}$$

$$p_i = \bar{p}_i + \lambda_i(\tilde{x}_i + \tilde{u}_i) \tag{2}$$

where $\beta_i = w_i\sigma/\Sigma_i^{1/2}$, and $\lambda_i = \Sigma_i^{1/2}/(2w_i\sigma)$.

The market maker's price setting rule, given by Kyle, is based on the market maker's inference of the level of insider trading. When the uninformed agents' net order flow, u_i, is normally distributed, then Kyle's linear price setting rule is optimal. In the present environment, order flow is only normally distributed in the limit, when $\mu \to \infty$. When μ is less than infinity, Kyle's linear pricing rule is still the best linear pricing rule in the sense of minimizing mean squared errors. We are concerned with situations in which μ is less than infinity, but sufficiently large such that agents use linear least squares decision and inference rules.

From the results given by (1) and (2), we can now calculate a liquidity trader's date 0 expected return on asset i and the covariance of asset i and asset j. These unconditional moments account for the possibility that the asset will need to be sold at time 1, with a possible loss to insiders, if the liquidity trader is an early consumer, whereas if the liquidity trader is a late consumer, the asset will be liquidated at date 2.

Proposition 1: A liquidity trader's expected return on asset i and the covariance between a liquidity trader's returns on asset i and asset j are given by:

$$E[r_i] = \bar{p}_i - \frac{k(1-k)e}{2\sigma} \Sigma_i^{\frac{1}{2}} \tag{3}$$

$$= \bar{p}_i - \phi \Sigma_i^{\frac{1}{2}}$$

where $\phi = \frac{1}{2} \left(\frac{k(1-k)}{2\mu}\right)^{\frac{1}{2}}$.

$$Cov(r_i, r_j) = \Sigma_i^{\frac{1}{2}} \Sigma_j^{\frac{1}{2}} \left[(1 - \frac{3k}{4})\rho_{ij} + (\frac{k}{8(1-k)})(1 - \frac{(1-k)^2}{\mu})\right] \tag{4}$$

Proof: See Appendix.||

As a benchmark, compare these moments to the moments which would prevail if no insiders were present. In that case, we would have:

$$E[r_i] = \bar{p}_i \tag{5}$$

$$Cov(r_i, r_j) = \Sigma_i^{\frac{1}{2}} \Sigma_j^{\frac{1}{2}} \rho_{ij}(1 - k) \tag{6}$$

Note that if k = 0 in (3) and (4), so that all liquidity traders will be late consumers, then the expected returns and variances of the assets will be the same as for the case when no insiders are present, since they never would need to trade with insiders. For k > 0, insiders unambiguously decrease the expected return on any asset, i, by a factor which is linear in the asset's standard deviation. Comparing equation (4) with equation (6), the asset variance with insiders is unambiguously larger in absolute value when insiders are present.

We can now state:

Proposition 2: The presence of insiders diminishes the expected utility of liquidity traders.

Proof: Since individual securities have normally distributed returns, portfolios of these securities (being linear combinations) will also be normally distributed. Therefore, the expected utilities of holding two portfolios can be compared by simply comparing the first two moments of the portfolios' return distributions. The proof consists of showing that for any set of portfolio weights (i.e., both efficient and inefficient), liquidity traders receive a lower expected portfolio return and face a higher portfolio variance when insiders are present. Recall that the expected return on any portfolio is lowered by the presence of insiders, by Proposition 1.[9] Now consider any set of portfolio weights, w. We must show that: $w'\Sigma^*w > w'\Sigma w$, where Σ^* is the covariance matrix when insiders are present, and Σ is the covariance matrix when insiders are not present.

Define $K \equiv \Sigma^* - \Sigma$ and note that:

$$w'Kw = (k/4)w'\Sigma_i^{\frac{1}{2}}\Sigma_j^{\frac{1}{2}}\rho_{ij}w + \left(\frac{k}{8(1-k)}\right)\left(1 - \left(\frac{(1-k)^2}{\mu}\right)\right)w'\Sigma_i^{\frac{1}{2}}\Sigma_j^{\frac{1}{2}}w$$

The first term is just a multiple of the quadratic form for the case of no insiders, and is, therefore, positive definite. The second term equals is a positive constant times

$$[w_1\Sigma_1^{\frac{1}{2}} + w_2\Sigma_2^{\frac{1}{2}} + \cdots + w_M\Sigma_M^{\frac{1}{2}}]^2 \geq 0 .$$

Therefore, the covariance matrix when insiders are present exceeds the covariance matrix without insiders by a positive definite matrix.||

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[9]Note that this assumes that liquidity traders do not engage in short sales at the initial date. Since liquidity traders are identical and assets must be in positive supply, this rules out the feasibility of short sales.

$$\Sigma_i^{\frac{1}{2}}\Sigma_j^{\frac{1}{2}}[(1 - \frac{3k}{4})\rho_{ij} + (\frac{k}{8(1 - k)})(1 - \frac{(1 - k)^2}{\mu})];$$

ε = the M dimensional unit vector.

The efficient frontier is defined by an M dimensional vector of portfolio weights, w, which solves:

$$\underset{W}{\text{Min}}\ (1/2)\ w'\Sigma^* w \tag{7}$$

subject to: (i) $w'R^* = \bar{R}$; and (ii) $w'\varepsilon = 1$.

where $\bar{R}$ is the individual's required rate of return. Following Merton (1972) the solution to this problem, w_p, is given by:

$$w_p = g + h\bar{R} \tag{8}$$

where:

$$g \equiv \frac{1}{D}\left[B(\Sigma^{*-1}\varepsilon) - A(\Sigma^{*-1}R^*)\right]$$

$$h \equiv \frac{1}{D}\left[C(\Sigma^{*-1}R^*) - A(\Sigma^{*-1}\varepsilon)\right]$$

and where:

$$A \equiv \varepsilon'\Sigma^{*-1}R^* \tag{9a}$$

$$B \equiv R^{*\prime}\Sigma^{*-1}\ R^* \tag{9b}$$

$$C \equiv \varepsilon'\Sigma^{*-1}\varepsilon \tag{9c}$$

$$D \equiv BC - A^2 \tag{9d}$$

It is straightforward to show that the variance of this frontier portfolio can

$$\sigma_p^2 \equiv w_p'\Sigma^* w_p = (C\bar{R}^2 - 2A\bar{R} + B)/D \tag{10}$$

which is obtained by substituting in the portfolio weights given by (8). This is a hyperbola in standard deviation-expected return space with center (0, A/C), vertex $[1/C]^{\frac{1}{2}}$, and asymptotes given by $\bar{R} = A/C \pm [D/C]^{\frac{1}{2}}\sigma_p$.

Recall from Proposition 2 that the welfare of the liquidity traders is reduced in the presence of insiders. The proof involved showing that the presence of insiders resulted in a lower expected return and higher variance for <u>any</u> portfolio held by the liquidity traders. Hence, it must be the case that the presence of insiders results in lower expected return and higher variance of all <u>efficient</u> portfolios held by the liquidity traders. Therefore, the efficient portfolio frontier in the presence of insiders lies below the efficient frontier without insiders.

IV. Creating Composite Securities

In this section we will consider an intermediary which seeks to design a composite security that maximizes the utility of a representative liquidity trader. At time 0, the balance sheet of the intermediary is composed of assets consisting of the newly issued shares of firms. The intermediary finances these assets by issuing shares to liquidity traders in return for the liquidity traders' endowments. The intermediary agrees to liquidate its shares at time 2 when it receives the proceeds from the stock of the firms it has purchased. Hence, this intermediary can be thought of as a closed-end mutual fund or a grantor trust issuing an asset-backed security. Liquidity traders purchase "closed-end mutual fund shares" or "asset-backed securities" that can be traded in a secondary market at time 1. To start with, we show optimally designed.

As in the case of primitive security markets, it is assumed that at time 1 there is a market maker and an insider present in the market for the composite security. At time 1, the insider is assumed to know the time 2 liquidation value of the composite security, which equals the liquidation values of the primitive securities underlying the composite security.[10] Thus, we have:

Proposition 3: A liquidity trader's expected return and variance of a composite security with vector of portfolio weights c is given by:

$$E[r_c] = c'\bar{p} - \phi(c'\Sigma c)^{\frac{1}{2}} \tag{11}$$

$$Var(r_c) = c'\Sigma c\left((1 - \frac{3k}{4}) + (\frac{k}{8(1 - k)})\right) \tag{12}$$

Proof: The proof is an application of Proposition 1 for the single, composite security.||

The following proposition gives the rationale for creation of a composite security.

Proposition 4: A composite security can always be created which increases the expected utility of the liquidity traders.

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[10] We assume the information set of the insider operating in the composite security market is the union of the information sets of all individual insiders operating in each individual security market. This assumption regarding the amount of inside information in the composite market is the worst possible case from the point of view of the liquidity traders. It implicitly assumes that insiders could collude to optimally utilize their

the advantage of creating a composite security.

Proof: The proof consists of showing that for <u>any</u> set of individual security portfolio weights that would be chosen by a liquidity trader, if a composite security was constructed with these same portfolio weights, then the liquidity trader would receive a higher expected return, and face a lower variance, by holding this composite security rather than the individual securities that make up the portfolio. The first step is to compare a liquidity trader's expected return on the composite security and the portfolio of individual securities. The second step is to compare variances.

Step 1: When securities are unbundled, the expected return is given by:

$$w'\bar{p} - \phi[w_1\Sigma_1^{\frac{1}{2}} + \ldots + w_M\Sigma_M^{\frac{1}{2}}] \tag{13}$$

From Proposition 3, we can compute the expected return on a composite security with these same weights. The expected return on the composite security will then exceed that on the portfolio of individual securities if:

$$\sum_{i=1}^{M} w_i\Sigma_i^{\frac{1}{2}} > [\sum_{i=1}^{M}\sum_{j=1}^{M} w_i w_j \Sigma_{ij}]^{\frac{1}{2}} \tag{14}$$

Since both sides of the above inequality are positive, we can square both sides to obtain:

$$[\sum_{i=1}^{M} w_i\Sigma_i^{\frac{1}{2}}]^2 > \sum_{i=1}^{M}\sum_{j=1}^{M} w_i w_j \Sigma_{ij} \tag{15}$$

Expanding the squared term on the left hand side, and subtracting the right hand side from the left, we obtain:

$$2\sum_{i=1}^{M}\sum_{j=1}^{M} w_i w_j (1-\rho_{ij})\Sigma_i^{\frac{1}{2}}\Sigma_j^{\frac{1}{2}} \tag{16}$$

which is greater than zero if at least one of the asset correlation securities chosen by the representative liquidity

trader must have positive security weights, w_i.)

Step 2: Using Proposition 1, we know that when securities are unbundled, the variance of a liquidity trader's return is:

$$(1 - \frac{3k}{4})w'\Sigma w + \frac{k}{8(1 - k)} \left(1 - \frac{(1 - k)^2}{\mu}\right) \left[\sum_{i=1}^{M} w_i \Sigma_i^{\frac{1}{2}} \right]^2 \qquad (17)$$

while from Proposition 3, the variance of the return on the composite security is:

$$w'\Sigma w\left[1 - \frac{3k}{4} + \frac{k}{8(1 - k)} \left(1 - \frac{(1 - k)^2}{\mu}\right)\right] \qquad (18)$$

Note that the variance on the composite security will be less than that on the portfolio of individual securities if:

$$w'\Sigma w < \left[\sum_{i=1}^{M} w_i \Sigma_i^{\frac{1}{2}} \right]^2 \qquad (19)$$

In Step 1, we proved that this inequality must hold.||

Using the results of Proposition 3, we can also compute a set of asset weights to create an optimal composite security, i.e., that composite security that would maximize the utility of a liquidity trader.

Proposition 5: The optimal composite security is a set of portfolio weights, c, which solves:

$$\min_{\{c\}} c'\Sigma c \qquad (20)$$

subject to: (i) $c'\bar{p} - \phi[c'\Sigma c]^{\frac{1}{2}} = \bar{R}$; and (ii) $c'\varepsilon = 1$.

Proof: This follows from the results of Proposition 3 giving the expected return and variance of a composite security for a given set of portfolio depends only on the mean and variance of the return distribution are those

minimizing the variance of the composite security return subject to a given expected return.||

A closed form solution does not exist to Problem (20), the constrained minimization problem to determine the optimal portfolio weights. However, a numerical solution can be computed. Of course these portfolio weights will, in general, be different from those of the representative liquidity trader for the case in which a composite security was not available, i.e., equation (8). Hence, if the M primitive production technologies are in perfectly elastic supply, the equilibrium supplies of these primitive securities will differ when a composite security is available versus when it is not. Alternatively, if the M production technologies possess completely inelastic supplies, equilibrium rates of return must adjust to the point where liquidity traders are content to hold the available stocks. In this case the equilibrium rates of return on these primitive securites will differ when a composite security is available versus when it is not.

V. An Illustrative Example

We now consider a numerical example that illustrates how potential losses to insiders and the creation of composite securities can effect the optimal portfolio choice of uninformed individuals. Our example is the portfolio choice problem of a U.S. investor who may have an unexpected need to trade. The U.S. investor is assumed to select a portfolio of foreign and domestic stocks. This problem is relevant to the model environment considered in the previous sections of the paper because foreign investment is likely to be an instance where information asymmetries can be particularly severe.

One can interpret this example as illustrating how uninformed investors stocks. It also measures the gain that results from creating a multi-country

"world" stock mutual fund relative to the situation where only individual country stock index funds were previously available. The example uses data on the monthly U.S. dollar returns of stock markets in 17 major countries over the period 1971 to 1985. The data was compiled by Morgan Stanley Capital International and is reported in Solnik (1988). We assume that the sample means and the covariance matrix of these country stock returns represent the parameters $\bar{p}_i$, i=1,...,17, and the 17x17 covariance matrix, Σ. In other words, we are implicitly assuming that there exists a stock index fund for each of the 17 countries that is available to a U.S. investor. Further, it is assumed that there is an equal probability that the U.S. investor will need to liquidate his or her portfolio by the end of one year, i.e., k = 0.5, and also that the liquidity and noise traders each have a population size of μ= 10.

In Figure 1 we have graphed the efficient frontiers for three different cases. The curve with "diamonds" is the portfolio frontier for the case of no insiders, where the distribution of security returns are given by equations (5) and (6) in the text. For the case in which insiders are present and investors hold "single country" mutual funds of stocks, the efficient frontier is traced out by using equation (10) and varying the expected portfolio return, $\bar{R}$. This frontier is represented by the curve with "rectangles" in Figure 1. As was proved in the text, the expected utility of liquidity traders is lowered in the presence of insiders, as the portfolio frontier without insiders dominates that when insiders are present.

Finally, when multi-country stock funds are available, the optimal fund portfolio weights (the optimal composite security) can be computed by numerically solving the problem in (20). The efficient composite security that while the efficient portfolio frontier when a composite security is held is still dominated by the frontier for the case in which no insiders are present, it unambigously improves the liquidity traders situation relative to the case of individually holding single country mutual funds.

VI. Conclusions and Further Research

We have shown that the presence of insiders in securities markets will affect the portfolio decisions of risk-averse investors who have uncertain needs to trade. The expected rate of return on an asset falls as an asset's standard deviation and the probability that the investor will need to liquidate the security rises. This lowered expected rate of return represents the expected loss to insiders. Because insiders were assumed to know perfectly the liquidation value of the security, their informational advantage was proportional to the asset's risk (standard deviation). However, at the cost of further complication, the model might be made more realistic if an insider was assumed to possess only a noisy signal of a security's liquidation value. Insiders may have noisier signals in some asset markets than in others for reasons unrelated to the primitive asset's return distribution (e.g. industry structure, regulation of information disclosure, etc.). In this case, we would expect that the loss to insiders would not be simply related to the asset's standard deviation but would depend on the accuracy of the insider's signal.

A more realistic model might also consider liquidity traders with heterogeneous liquidation probabilities. Liquidity traders would likely hold portfolios that differ depending on their ex-ante probability of liquidation. In addition, one would expect somewhat different results if liquidity traders were only required to liquidate a portion of their wealth in the intermediate period model. Liquidity traders might minimize their losses to insiders by liquidating those assets with the lowest risk, refraining from trade in more risky assets. This could lead to liquidity traders being penalized less (in terms of lower expected return) from holding some higher risk assets.

This paper has given one explanation for the existence of composite securities. It does so by modeling the "liquidity" services these securities provide. Trading in composite securities reduces liquidity traders' expected losses to insiders relative to trading the assets which compose the security basket or index. While our model implies that there will exist a single optimal composite security held by all liquidity traders, extensions of the model to consider heterogenous liquidation probabilities would likely result in a multiple composite securites being optimal.

Our model has assumed that each primitive security is represented by a single firm or technology and that intermediaries or exchanges can create composite securities by issuing claims that pool or index these primitive securities. However, we should point out that this pooling of securities could be accomplished at the firm level rather than the intermediary level. In other words, a conglomerate merger of firms that then issue a single (composite) security would result in the same benefits accruing to liquidity traders. The equilibrium allocation and levels of physical investment in the multiple technologies would be the same as in the case of composite securities being created by intermediaries. In the absence of intermediaries, this benefit to firm size provides an explanation for the emergence of large firms that is unrelated to physical scale economies.

Appendix

Proof of Proposition 1

Step 1: Calculation of the liquidity traders' expected return on asset i when insiders are present. The date 0 unconditional expected return on asset i, $E[r_i]$, equals:

$$E[r_i] = E[\tilde{p}_i(\frac{\tilde{n}_0}{\mu}) + \tilde{v}_i(1 - \frac{\tilde{n}_0}{\mu})] \qquad (A1)$$

Substituting in for $\tilde{p}_i$ from equation (2) in the text:

$$E[r_i] = E[(\bar{p}_i + \lambda_i\beta_i(\tilde{v}_i - \bar{p}_i) + \lambda_i ew_i(\tilde{n}_1 - \tilde{n}_0)(\frac{\tilde{n}_0}{\mu}) + \tilde{v}_i(1 - \frac{\tilde{n}_0}{\mu})]$$

$$= \bar{p}k + \frac{\lambda_i ew_i}{\mu} E[\tilde{n}_1\tilde{n}_0 - \tilde{n}_0\tilde{n}_0] + \bar{p}_i(1 - k)$$

$$= \bar{p}_i + \frac{\lambda_i ew_i}{\mu} [(k\mu)^2 - ((k\mu)^2 + \mu k(1-k))] \qquad (A2)$$

Substituting in for λ_i, we have:

$$E[r_i] = \bar{p}_i - \frac{k(1 - k)e}{2\sigma} \Sigma_i^{\frac{1}{2}} \qquad (A3)$$

Finally, substituting in for e, we arrive at:

$$E[r_i] = \bar{p}_i - \frac{1}{2} (\frac{k(1 - k)}{2\mu} \Sigma_i)^{\frac{1}{2}} \equiv \bar{p}_i - \phi\Sigma_i^{\frac{1}{2}} \qquad (A4)$$

<u>Step 2:</u> Calculation of the covariance between the returns to asset i and asset j. This covariance is given by:

$$E[(r_i - E(r_i))(r_j - E(r_j))] = E\{[\lambda_i\lambda_j\beta_i\beta_j(\tilde{v}_i - \bar{p}_i)(\tilde{v}_j - \bar{p}_j) + \lambda_i\lambda_j\tilde{u}_i\tilde{u}_j +$$

$$\lambda_i\tilde{u}_i\phi\Sigma_j^{\frac{1}{2}} + \lambda_j\tilde{u}_j\phi\Sigma_i^{\frac{1}{2}} + \phi^2\Sigma_i^{\frac{1}{2}}\Sigma_j^{\frac{1}{2}}](\frac{\tilde{n}_0}{\mu}) + [\Sigma_{ij} + \phi\Sigma_i^{\frac{1}{2}}\Sigma_j^{\frac{1}{2}}](1 - \frac{\tilde{n}_0}{\mu})\}$$

$$= \frac{k\Sigma_{ij}}{4} + \frac{\lambda_i\lambda_j w_i w_j e^2}{\mu} E[(\tilde{n}_1 - \tilde{n}_0)^2\tilde{n}_0] + \frac{\lambda_i\phi\Sigma_j^{\frac{1}{2}} w_i e}{\mu} E[(\tilde{n}_1 - \tilde{n}_0)\tilde{n}_0]$$

$$+ \frac{\lambda_j\phi\Sigma_i^{\frac{1}{2}} w_j e}{\mu} E[(\tilde{n}_1 - \tilde{n}_0)\tilde{n}_0] + \phi^2 k\Sigma_i^{\frac{1}{2}}\Sigma_j^{\frac{1}{2}} + (\Sigma_{ij} + \phi^2\Sigma_i^{\frac{1}{2}}\Sigma_j^{\frac{1}{2}})(1 - k)$$

$$= \Sigma_{ij}(1 - \frac{3k}{4}) + \Sigma_i^{\frac{1}{2}}\Sigma_j^{\frac{1}{2}}[\frac{e^2k^2u}{4\sigma^2} - \frac{\phi ek(1 - k)}{\sigma} + \phi^2]$$

Recalling that $e = \sigma/(2\mu k(1 - k))^{\frac{1}{2}}$, the above expression equals:

$$= \Sigma_{ij}(1 - \frac{3k}{4}) + \frac{k}{8(1 - k)} \Sigma_i^{\frac{1}{2}}\Sigma_j^{\frac{1}{2}} [1 - \frac{(1 - k)^2}{\mu}]$$

$$= \Sigma_i^{\frac{1}{2}}\Sigma_j^{\frac{1}{2}} [(1 - \frac{3k}{4})\rho_{ij} + \frac{k}{8(1 - k)} [1 - \frac{(1 - k)^2}{\mu}]] \qquad (A6)$$

In the above computation we have used the fact that:

$$E[\tilde{n}_i^3] = (k\mu)^3 + 3(k\mu)^2/2.$$

This can be computed from noting that:

$$E[\tilde{n}_i - E(n_i)]^3 = 0 \tag{A7}$$

since $\tilde{n}_i$ is normally distributed. Then (A7) can be solved for $E[\tilde{n}_i]$. Also, we used the fact that $E[(n_1 - n_0)^2 n_0] = (k\mu)^2$. This can be verfified by direct computation.||

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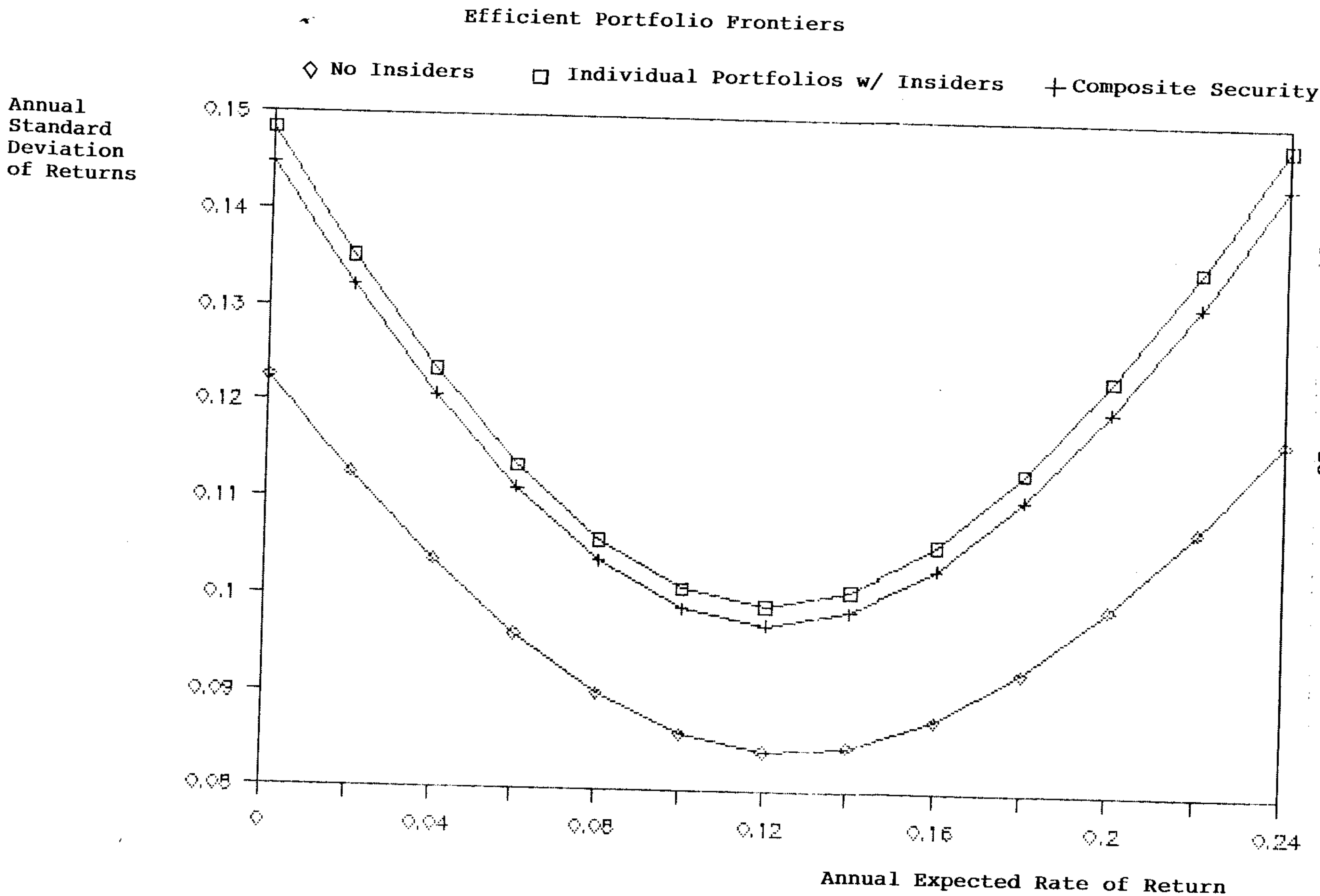
<u>Figure 1</u>


Efficient Portfolio Frontiers
◇ No Insiders
□ Individual Portfolios w/ Insiders
+ Composite Security
Annual Standard Deviation of Returns
0.15
0.14
0.13
0.12
0.11
0.1
0.09
0.08
0
0.04
0.08
0.12
0.16
0.2
0.24
Annual Expected Rate of Return