VARIANCE RATIO TESTS OF A RANDOM WALK IN REAL EXCHANGE RATES

by

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Abstract: Under certain conditions, efficient markets imply random walk behavior in real exchange rates. Much of international finance theory, however, is based on the idea of purchasing power parity, which implies mean reversion in real exchange rates. This paper uses variance ratio statistics to test for random walk behavior in real exchange rates. Unlike most previous tests of this hypothesis, the tests do reject a random walk for monthly data; however, the monthly statistics do not provide evidence in favor of mean reversion. Interestingly, tests using annual data for the twentieth century are unable to reject a random walk despite evidence of mean reversion. This appears to be due to the relatively small number of observations available.

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I. Introduction

Considerable effort has been spent by international financial economists on testing the theoretical concept of Purchasing Power Parity (PPP) and its implication that real exchange rates should exhibit mean reversion. The alternative view of ex ante PPP and random walk behavior in realized exchange rates was first put forward by Roll (1979). When Roll was unable to reject random walk behavior using a simple regression technique with short-term changes in the real exchange rate, subsequent analysis attempted to use more powerful approaches to reinstate the idea that PPP should dominate exchange rates and mean reversion in real exchange rates should occur. Subsequent regression tests by Adler and Lehman (1983) using both monthly and annual data were also unable to reject the random walk hypothesis. Mishkin (1984) was unable to reject either ex ante PPP or uncovered interest parity individually; however, joint tests of the two hypotheses were rejected, showing that with more powerful tests rejection of the random walk hypothesis might be achieved. Cumby and Obstfeld (1984) also conducted joint tests on the slope and intercept in a regression involving real exchange rates, as well as allowing for heteroscedasticity. Their results are strongly at variance with the hypothesis of ex ante PPP. Huizinga (1987) explored the idea that it might take some time for mean reversion to occur by looking at various longer term approaches to the problem, including one which employed a ratio of the variances of the exchange rate when measured at different frequencies. This ratio seemed to indicate that over substantially long periods of time mean reversion did occur, but with no distribution theory behind his statistics no inferences could be drawn. More recently, Abuaf and Jorion (1989) employ a combination of long-term mean reversion and the possibility of increased power from multivariate tests. Using annual data starting from the beginning of this century they find significant mean reversion in real exchange rates.

A similar question regarding random walks has also persisted in the literature on stock prices. Recently, Lo and McKinley (1988a and 1988b) derived both the asymptotic and empirical distributions for a variance ratio test of the random walk hypothesis which they employed in tests using stock market prices. Using this test they were able to reject the random walk hypothesis. The advantages of the test are two. First, although the statistic requires sampling at (possibly) large intervals, because overlapping observations are allowed no degrees of freedom are lost and relatively long-term results can be obtained with a relatively small data set. Second, the statistic has been defined so as to be consistent under general forms of heteroscedasticity, which is attractive in tests of both stock prices and exchange rates.

In this paper I employ the Lo and McKinley variance ratio test on a series of both monthly and annual real exchange rates. Kaminsky (1987) also applied the variance ratio test to real exchange rates, but for only 4 currencies against the dollar using monthly observations, with no allowance for heteroscedasticity and over a shorter sample period. Kaminsky's results are mixed, with some currencies deviating from random walk behavior, but only for selected lag intervals. In this paper, tests using short term differences of from 2 to 8 months are generally unable to reject the random walk hypothesis, which agrees with the previous tests discussed above. At differences of 16 and 32 months, however, significant deviations from random walk behavior are found in most countries tested, the most significant exception being the Canadian dollar. At odds with these monthly results, however, the annual time series are unable to reject the random walk hypothesis at intervals of up to 8 years.

The outline of the paper is as follows. The next section describes the statistics used in the tests. Section III discusses the results, with conclusions presented in the final section.

II. The Statistics

Let X_t be a random variable which follows a diffusion process

$$dX_t = \tau dt + \sigma dW(t).$$

If we sample X_t at discrete intervals, the variance of the increments is linear in the observation interval. That is, the variance of $X_t - X_{t-2}$ is twice the variance of $X_t - X_{t-1}$. Lo and McKinley (1988a) use this to develop a test of the random walk hypothesis. Under an assumption of homoscedastic increments they show that the statistic

$$(1) \quad M_r(q) = \sigma^2(q)/\sigma^2(1) - 1$$

is asymptotically normally distributed, where

$$(2) \quad \sigma^2(q) = \frac{1}{m} \sum_{k=q}^{nq} [X_k - X_{k-q} - q\mu]^2$$

$$m = q\ (nq - q + 1)(1 - q/nq)$$

nq is the total number of observations and the usual maximum-likelihood estimator of μ is employed. Inferences may be drawn through the use of the statistic

$$(3) \quad Z(q) \equiv \sqrt{nq}\ M_r(q) \left[\frac{2(2q-1)(q-1)}{3q} \right]^{-\frac{1}{2}}$$

which has a standard normal distribution. Monte Carlo experiments in Lo and McKinley (1988b) provide evidence on the small sample properties of Z(q) under various assumptions. For the values of q and nq used in the tests that follow, the empirical critical values of Z(q) are reasonably close to their asymptotic values. Consequently, the asymptotic distribution will be used for inferences.

In order to allow for general forms of heteroscedasticity in the variance of the increments of X_t, Lo and McKinley propose using the same statistic $M_r(q)$, but altering the estimate of its variance to allow for heteroscedasticity. The proposed statistic now becomes

(4) $Z(q) = M_r(q)/\sqrt{\theta}$

where

$$\theta(q) = \sum_{j=1}^{q-1} \left[\frac{2(q-j)}{q} \right]^2 \delta(j)$$

$$\delta(j) = \frac{\sum_{k=j+1}^{nq} (X_k - X_{k-1} - \mu)^2 (X_{k-j} - X_{k-j-1} - \mu)}{\left[\sum_{k=1}^{nq} (X_k - X_{k-1} - \tau)^2 \right]} .$$

Both of these versions of the Z(q) statistic are utilized in the next section to test the hypothesis that the real exchange rate follows a random walk.

III. Empirical Results

Monthly exchange rates for nine currencies against the U.S dollar were obtained from the International Financial Statistics (IFS) data base.

Deflating these by the corresponding consumer price indices, also from the IFS, produced the desired real exchange rates. In the results reported below, the natural logarithm of the real exchange rate was used; however, the tests were also conducted on levels of the real exchange rate with similar results. The monthly time series used in the tests all end on December, 1988. The starting dates vary from June, 1973 to August, 1975 since the total number of observations used, nq+1, is dependent on the value of q.

The results of the tests for the monthly data are given in Table 1 where the variance ratio, $M_r(q)+1$, as well as both measures of Z(q) described above are presented. The homoscedastic measures of Z(q) are presented in parentheses, while the heteroscedasticity-consistent measures are presented in brackets. At small increments, q = 2,4 and 8 months, virtually no rejections of the random walk hypothesis are found. Given the inability of other short term tests of this hypothesis to reject this is not surprising. At longer increments, however, the advantage of these statistics becomes evident. When the variance ratio is calculated using increments of 16 months, 6 out of nine currencies in the sample reject the random walk hypothesis at the 5% level. Using increments of 32 months all currencies, except for the Canadian dollar, reject the null. Generally, the rejections do not depend on which version of Z(q) is employed. Unreported results for q=64 are similar to those for q=32.

These results are somewhat in agreement with the long term tests conducted by Huizinga (1987), who found that the variance ratio for most currencies included in that study increased as the value of q increased, until a maximum was reached after which the ratio declined. The one exception to this rule was the Canadian dollar, whose behavior differs in this study as well.

For large values of q, the variance ratios in table 1 generally have values in excess of 1. Lo and McKinley show that the variance ratio is approximately equal to 1 plus a weighted average of the first q-1 autocorrelation coefficients of $X_t - X_{t-1}$. Thus, for q=2 the variance ratio gives us an estimate of the first-order autocorrelation coefficient. Mean reversion requires negative serial correlation in changes in the exchange rate so that the variance ratio for q=2 should be less than one. Examining Table 1 shows that in fact 4 out 9 countries have variance ratios less than 1, although none of the ratios are significantly different than 1. Unreported estimates confirm this with 5 out of 9 countries producing (insignificantly) negative first-order autocorrelation coefficients. The increase in value of the variance ratio as q increases comes about due to some rather large positive higher-order autocorrelation coefficients. Thus, for periods of up to 32 months there is enough positive correlation in real exchange rate movements that random walk behavior is rejected. This may reflect the swings in the value of the dollar that have occurred since the collapse of Bretton Woods, a good example of which is the general upward trend in the dollar from early 1980 until early 1985, which was then followed by over two years of decline. Positive serial correlation of this sort is at odds with random walk behavior, as well the theory of PPP. It may be, however, that 32 months is not a sufficiently long enough period to test PPP. Rather than attempt to test for even longer term reversion using the monthly data set, we choose instead to examine long-run behavior using annual data.

Both Adler and Lehmann (1983) and Abuaf and Jorion (1989) incorporate more long term tests of PPP by using a set of annual time series on real exchange rates collected by Lee (1978). Table 2 presents the variance ratio tests using these annual time series, where the time series has been extended by Abuaf and Jorion (1989) to incorporate annual observations over the period 1900-1987.[1] The results here are surprisingly different from those in Table 1. First, in no instance is the variance ratio significantly different from unity. Thus, the null hypothesis of a random walk can not be rejected for intervals of up to 16 years. Also at odds with the monthly results is the magnitude of the ratios themselves. Nearly all of them are less than 1, which is clearly consistent with mean reversion and long-term PPP. Finally, the difference between the homoscedastic and heteroscedastic Z-statistics is much larger for the annual data than for the monthly data. This is perhaps not so surprising given that the annual data cover such different regimes as floating exchange rates, Bretton Woods, the gold standard and two world wars. It should also be mentioned that while they are not significantly different from 1, the variance ratios in Table 2 taken together with those in Table 1 display behavior similar to those calculated by Huizinga (1987).

The reasons for these contradictory results may be any (or all) of the following. First, the annual time series are relatively short compared to the monthly time series. Thus, despite the relatively low values of the variance ratio, the Z-statistic is not sufficiently large (in absolute value) to be able to reject the null hypothesis that they are equal to zero. It is likely that the imprecision of the estimates for the annual series is due to the small number of observations available. Unfortunately, it will be some time before this can be remedied.

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[1] The author would like to thank Philippe Jorion for providing this data set.

Perhaps even more important, the annual observations are based on annual average exchange rates for each period, whereas the data used in Table 1 are based on end of month exchange rates. The use of averages will tend to smooth the series and to the extent that this reduces variance may cause the variance ratio to be reduced. Also, it is possible that the annual data is subject to measurement error. Certainly the composition of the basket used in calculating the price index has changed over the sample period. It is also possible that the precision with which the data is compiled has changed, either for better or for worse.

A more intuitive argument for the difference which would appeal to adherents of PPP is that both sets of results are correct. In the short and medium-term the exchange markets are dominated by investors who move capital to take advantage of their beliefs in the way exchange rates will move. This behavior is also consistent with overshooting of the sort described by Dornbusch (1976). Ultimately, however, any over (under) valuation of a currency causes a country to become less (more) competitive in trading its products abroad and it is trade in goods which dominates, forcing the exchange rate towards its steady state level. The tests which employ only the monthly data allow us to view the effects of the short-term trends in the exchange rate and reject a random walk; however, we have an insufficient number of annual observations to be able to reject the hypothesis that over the long-term the exchange rate reverts to its mean level.

IV. Conclusion

Previous tests of the random walk hypothesis for real exchange rates have been unconvincing for proponents of PPP. Unfortunately, the tests involving only short-term approaches appear not to be sufficiently powerful to overcome the problems inherent in tests using near unit-root time series. Using the variance ratio tests developed in Lo and McKinley (1988a), however, this paper finds significant deviations from random walk behavior in real exchange rates measured on a monthly basis. These deviations, however, do not provide evidence in favor of mean reversion in real exchange rates, at least not for periods of up to 32 months. In fact, the evidence seems to be more in favor of medium-term trends in the real exchange rate, something which is visually apparent from graphs of the period under consideration.

Tests employing annual data produce strikingly different results. First, in no case is the random walk hypothesis rejected. Second, at sufficiently large intervals the variance ratios indicate negative serial correlation in real exchange rate changes, albeit statistically insignificant. This agrees with the findings of Abuaf and Jorion (1989), who employ multivariate tests on this annual data and find evidence of mean reversion. The correlations between currencies seems to add power to their tests which are not present in these univariate statistics.

For those who believe that the random walk hypothesis is a bad theory of exchange rate behavior this paper provides some hopeful evidence. However, proponents of PPP will be discouraged to find that despite rejection of the random walk, mean reversion in exchange rates is not proven. These results may lend support to the theory of Dumas (1988), who describes a process for the real exchange rate which is not a random walk, but which also will not necessarily display mean reversion. Neither, in that model, will the real exchange rate ever deviate too far from its PPP level, something readily apparent from the behavior of the annual exchange rates analyzed here.

Future research should attempt to incorporate more from theories like Dumas (1988) in tests of the process followed by the real exchange rate.

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Table 1

Variance Ratio Tests - monthly observations ending 1988:12

q =	2	4	8	16	32
Belgium	1.010 (0.138) [0.112]	1.182 (1.321) [1.153]	1.317 (1.452) [1.383]	1.913 (2.752)* [2.709]*	3.419 (4.797)* [4.818]*
Canada	0.934 (0.898) [0.767]	0.844 (1.129) [1.018]	0.913 (0.399) [0.381]	1.157 (0.472) [0.476]	1.459 (0.911) [0.970]
France	0.982 (0.257) [0.227]	1.137 (0.996) [0.934]	1.360 (1.651) [1.647]	1.809 (2.439)* [2.482]*	3.046 (4.057)* [4.115]*
Germany	0.994 (0.077) [0.065]	1.148 (1.070) [0.962]	1.254 (1.165) [1.131]	1.758 (2.284)* [2.285]*	2.834 (3.637)* [3.615]*
Italy	1.042 (0.576) [0.530]	1.203 (1.470) [1.326]	1.426 (1.954) [1.831]	1.953 (2.871)* [2.765]*	2.560 (3.094)* [3.226]*
Japan	1.098 (1.336) [1.211]	1.240 (1.739) [1.635]	1.502 (2.304)* [2.163]*	1.935 (2.818)* [2.622]*	2.324 (2.626)* [2.548]*
Netherlands	0.993 (0.095) [0.076]	1.145 (1.054) [0.915]	1.257 (1.178) [1.116]	1.738 (2.224)* [2.185]*	2.804 (3.578)* [3.596]*
Switzerland	1.019 (0.265) [0.244]	1.168 (1.217) [1.129]	1.307 (1.407) [1.338]	1.472 (1.423) [1.396]	2.136 (2.253)* [2.203]*
UK	1.095 (1.298) [1.334]	1.133 (0.962) [1.018]	1.215 (0.987) [1.006]	1.527 (1.587) [1.594]	2.205 (2.389)* [2.471]*
nq =	186	184	184	176	160

The first row for each country presents the value of the variance ratio for the given value of q. The second row presents, in parentheses, the Z(q)-statistic under the assumption of homoscedasticity. The third row presents, in brackets, the Z(q)-statistic under the assumption of heteroscedasticity.

Table 2

Variance Ratios - Annual Data 1900 - 1987

q =	2	4	8	16
Canada	1.057 (.527) [.504]	0.833 (.818) [.736]	0.742 (.779) [.742]	0.427 (1.164) [1.176]
France	0.857 (1.325) [.823]	0.779 (1.082) [.682]	0.448 (1.670) [1.208]	0.195 (1.635) [1.387]
Germany	1.105 (.970) [.690]	1.180 (.884) [.614]	1.068 (.204) [.149]	0.934 (.134) [.109]
Italy	0.933 (.623) [.255]	0.735 (1.300) [.605]	0.561 (1.329) [.750]	0.449 (1.120) [.777]
Japan	1.133 (1.230) [.875]	0.867 (.651) [.411]	0.491 (1.540) [1.074]	0.321 (1.379) [1.137]
Netherlands	0.992 (.073) [.049]	0.921 (.385) [.279]	0.703 (.898) [.704]	0.393 (1.233) [1.074]
United Kingdom	1.144 (1.334) [1.154]	0.897 (.504) [.456]	0.537 (1.401) [1.339]	0.318 (1.386) [1.406]
nq =	86	84	80	64