ON THE ECONOMETRICS OF PREDICTING INFLATION FROM THE NOMINAL INTEREST RATE

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(24-89)

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ABSTRACT

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The problems of aggregating divergent individual forecasts of inflation, each subject to error on a number of grounds, into a "market forecast" are considered; and it is shown to be extremely unlikely that such an aggregate will always reproduce precisely the optimal forecast based on all relevant prior events, whether or not these are accurately observable. If this is not the case, then regressions relating the inflation rate to prior values of the nominal interest rate generate parameter estimates that are subject to bias, even if the real rate is constant. Inferences based on those parameter estimates are likely to be invalid.

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I

In econometric analyses a good deal of grief arises from the confusion of a theoretical construct, which may or may not be capable of measurement in a thought experiment, and the measured value available to economic decision makers. Even if the variable measured corresponds conceptually to the theoretical construct, there is almost certainly some measurement error in the data actually available. This may do relatively little damage to hypothesis testing or parameter estimation if the expected value of the measurement error is zero and its variance is small. But it behooves the investigator to give some thought to the characteristics of this error and to qualify his inferences in a way that makes allowance for probable biases.

Of particular interest in this connection is the construct of a rational expectations forecast. This is an optimal forecast at time t for some future period based on all relevant events prior to t and fully utilizing all predictive power they contain. It presumes a correct formulation of the relationships between these events and the forecasted variable.[1] The actual value of the forecasted variable depends also on events subsequent to t and uncorrelated with the information incorporated in the optimal forecast.

For example, let π_{t+1} be the inflation rate in period t + 1:

$$\pi_{t+1} = \frac{P_{t+1}}{P_t} - 1 ,$$

where P_t and P_{t+1} are, respectively, the price levels at the end of periods t and t + 1. Let $\bar{\pi}^e_{t+1}$ be the optimal forecast of π_{t+1} as of the end of period t. Then the forecast error,

(1) $$\epsilon_{t+1} = \pi_{t+1} - \bar{\pi}^e_{t+1} ,$$

is by construction uncorrelated with $\bar{\pi}^e_{t+1}$.[2]

Now consider the "market forecast" of inflation, π^e_{t+1}, for some market in which participants' behavior is affected by their expectations of the inflation rate. This is not an unambiguous concept, except in the remarkable circumstance that all participants have identical expectations. Presumably the market forecast, however defined, is some function of individual forecasts:

(2) $$\pi^e_{t+1} = f\ (\pi^e_{1,t+1}, \ldots \pi^e_{N,t+1})$$

where N is the number of market participants. While many methods of aggregating individual expectations are possible, the market forecast, to be useful, should have the property that, if all participants had agreed on that forecast, the market outcome would have been the same as that resulting from the varying expectations actually held.

In the next section a unique specification of the market forecast satisfying this condition is derived for the market in a one-period unindexed financial asset, and the plausibility of the behavioral hypothesis that this forecast is identically equal to the rational expectations forecast is explored. The following section discusses the bias that arises in least squares estimates of the relationship of future inflation to the current nominal interest rate if the market forecast of inflation is an imperfect measure of the rational expectations forecast. The final section briefly summarizes the implications of this bias for certain empirical studies.

II

Inflationary expectations are generally believed to be an important determinant in markets for (unindexed) financial assets. We define the market forecast of inflation in terms of a one-period financial asset for which the supply at the end of period t is determined independently of the interest rate prevailing at that time.[3]

Under certainty, the demand for such an asset is generally modeled as a function of the real rate of return, r_{t+1}, over period t + 1 and other variables. If $\bar{S}_t$ is the pretermined aggregate supply and if the demand function for the jth market participant is given by

$$D_{jt} = D^j (r_{t+1}, Z_{jt}) ,$$

where Z_{jt} stands for the set of all variables affecting demand for the asset and observable to the jth individual, then r_{t+1} is determined as the solution of

(3) $$D_t = \sum_j D^j(r_{t+1}, Z_{jt}) = \bar{S}_t .$$

It is the real rate that produces market equilibrium, given $\bar{S}_t$ and the Z_{jt}.[4]

Under uncertainty, the jth market participant acts in terms of his expected real rate of return, $r^e_{j',t+1}$, which, given his expectation as to the inflation rate, stands in a one-to-one correspondence with the nominal rate, i_t:

$$r^e_{j',t+1} \equiv \frac{1 + i_t}{1 + \pi^e_{j',t+1}} - 1 .$$

Dropping higher order terms for notational convenience, this may be approximated by

(4) $$r^e_{j,t+1} = i_t - \pi^e_{j,t+1} .$$

The demand function of the jth market participant may therefore be written

(5) $$D_{jt} = D^j (i_t - \pi^e_{j',t+1}, Z_{jt}) .$$

For given values of expected inflation and other relevant variables, the individual's demand depends only on the nominal rate. It follows that aggregate demand for the asset, D_t, is a function only of the nominal rate, which is then determined by setting D_t equal to the given supply $\bar{S}_t$.

The market forecast of the inflation rate with respect to the particular market considered may now be defined as the expectation, π^e_{t+1}, which if held by all participants would produce the nominal rate observed. If $\bar{i}_t$ is the value of i_t that satisfies

(6) $$\Sigma\ D^j\ (i_t - \pi^e_{j,t+1}\ Z_{jt}) = \bar{S}_t\ ,$$

then π^e_{t+1} may be unambiguously defined as the inflation forecast that satisfies

(7) $$\Sigma\ D^j\ (\bar{i}_t - \pi^e_{t+1},\ Z_{jt}) = \bar{S}_t\ .$$

Comparing (7) with (3) it is obvious that

(8) $$\pi^e_{t+1} = \bar{i}_t - r_{t+1} = i_t - r_{t+1}\ ,$$

assuming that the market clears.

Unique solutions of (6) and (7) for i_t and π^e_{t+1} exist so long as all the functions, D^j, are monotonic in the expected real rate of return.[5] These equations implicitly determine the function f in (2). Note that there is no reason to expect f to remain constant over time since $\bar{S}_t$ and the Z_{jt} are likely to change.

In empirical analyses it is frequently assumed that the market forecast of inflation is identically equal to the rational expectations forecast:

(9) $$\pi^e_{t+1} \equiv \bar{\pi}^e_{t+1}\ .$$

This is a very strong behavioral assumption. How plausible is it? That inflation expectations differ among market participants is demonstrably true.[6] If inflation forecasts depend on publicly available data and the same data are available to all, then the differences in forecasts must be accounted for by differences in the way individuals process the data -- in their implicit models of the relationships determining the inflation rate. Obviously not all of these models can be correct.

Let the true structural relationship be

$$\pi_{t+1} = \phi(X_t) + \varepsilon_{t+1} , \tag{10}$$

where X_t represents the set of all relevant variables known or conceptually knowable at t and ε_{t+1} reflects other relevant variables uncorrelated with those in X_t and has expected value of zero. Then the rational expectations forecast, $\bar{\pi}^e_{t+1}$, is simply $\phi(X_t)$. Even if (measured values of) all the elements of X_t are available and the functional form of ϕ is known, the parameters of ϕ cannot be estimated without sampling error from the small times-series samples available. Errors would still occur in the best possible forecasts for any given period due to (1) errors of measurement in the reported values of the X_t and (2) sampling errors in the estimated parameters of ϕ.

These errors affect similarly all "well informed" forecasts. Thus they do not cancel out when such forecasts are averaged at a point of time. Even if optimally informed forecasters exist and consistently dominate the margin at which the nominal rate, i_t, is determined, the "market forecast" does not provide an error-free measure of the theoretical construct, $\bar{\pi}^e_{t+1}$. The behavioral hypothesis (9) then depends on the heroic assumption that at every point in time the aggregate errors of less informed market participants exactly offset the (relatively small) error of well-informed participants. While this may be true on average over many time periods, it defies credibility to maintain that it holds in every period.

The case for rational expectations as a behavioral hypothesis rests essentially on the premise that, if the market did not process all currently available information in the most efficient way, profit opportunities would exist. Efficient markets would then quickly eliminate all such profit opportunities. But rationality cannot guarantee that the market forecast is identical at all times with the function $\phi(X_t)$ in the structural relationship (10), which fully reflects the implications for future inflation of all events prior to the forecast. First, there are time lags in the learning process that must occur after any change in structure; historical data do not represent a steady state. Second, some relevant information is not observable. In particular, the market forecast itself is not observable. Each market participant knows only his own forecast (and some subset of other forecasts that are made public). Nor can the market forecast be inferred from the nominal interest rate. Since this depends on two nonobservables, r_{t+1} and π^e_{t+1}, it cannot be used to identify either one.

Even if the market forecast were observable for comparison with the inflation rate subsequently observed, that information could be used only to eliminate systematic errors (due for example to systematic measurement errors in the data used as inputs). But there will also be stochastic errors because some of the X_t that enter into the structural relationship may be non-observable, because sampling error in parameter estimates will produce different effects at different times depending on the value of the associated variables, and because the function (2), which relates π^e_{t+1} to individual forecasts, may vary over time for reasons previously discussed. Thus rationality cannot eliminate the discrepancy that exists historically between the market forecast and the rational expectations forecast, $\phi(X_t)$, and there is no obvious reason to believe its variance is small. It is, of course, the market forecast, rather than the rational expectations forecast, that is reflected in the nominal interest rate.

III

Econometric analyses that depend on the identity of the two forecasts as a maintained hypothesis are subject to bias. We consider, as a particular example, the work of Fama [1975] and others who use nominal interest rates to predict inflation and to test hypotheses about real and nominal rates. Let v be the measurement error contained in the market forecast at the end of period t:

$$\pi^e_{t+1} = \bar{\pi}^e_{t+1} + v_t \;. \tag{11}$$

From (10) it follows that

$$\pi_{t+1} = \bar{\pi}^e_{t+1} + \varepsilon_{t+1} \;, \tag{10a}$$

where ε and $\bar{\pi}^e$ are uncorrelated. Substituting from (11), and (8) we have

$$\begin{aligned} \pi_{t+1} &= \pi^e_{t+1} - v_t + \varepsilon_{t+1} \\ &= i_t - r_{t+1} - v_t + \varepsilon_{t+1} \;, \end{aligned} \tag{12}$$

From the same relationships, we know that

$$i_t = r_{t+1} + \bar{\pi}^e_{t+1} + v_t \;. \tag{13}$$

For purposes of hypothesis testing, equation (12) may be rewritten as

$$\pi_{t+1} = a + bi_t + u_t \;, \tag{12a}$$

where u_t is a stochastic disturbance. If the real rate is constant over time and the disturbance, u_t, is uncorrelated with i_t, then ordinary least squares (OLS) may be used to provide a consistent estimate $(\hat{a})$ of the constant real rate, while the estimate of the slope coefficient $(\hat{b})$ should be close to 1.

This is the model tested by Fama [1975] for one- to six-month Treasury bills. It is attractive because it provides not only a measure of the nonobservable real rate, but also a method for utilizing the information contained in the nonobservable aggregate expectation, π^e_{t+1}, to forecast one-period inflation. Furthermore, under the maintained hypothesis that the market expectation is rational $(\pi^e_{t+1} \equiv \bar{\pi}^e_{t+1})$, an estimate of b insignificantly different from 1 provides confirmation of the joint hypothesis that the real rate is constant and that the market's inflation forecast is fully incorporated in the nominal interest rate (the Fisher effect [1930]).

Note, however, that the logic of the test depends crucially on the hypothesis, which we have shown to be highly implausible, that the market forecast is identical with the optimal forecast at each point in time (or at least that the measurement error is constant). Otherwise, even if the real rate of interest is constant so that $a = -r_{t+1}$ in (12a), the stochastic disturbance

(14) $$u_t = -v_t + \varepsilon_{t+1}$$

must from (13) be negatively correlated with the independent variable, i_t, producing downward bias in the least squares estimate of b based on (12a).[7]

If the real rate of interest is not in fact constant, then the stochastic disturbance in (12a) is

(14a) $$u_t = -(r_{t+1} - \bar{r}) - v_t + \varepsilon_{t+1} ,$$

where $\bar{r}$ is the expected value of the real rate. Since the deviation of r_{t+1} from its expected value is positively correlated with i_t from (13), this results in a further downward bias in the OLS estimate of b.

IV

Fama's 1975 finding of a value close to 1 for the 1953-71 period, which is not confirmed for other sample periods,[8] could derive from peculiarities of his sample. The variance of both the real rate and the measurement error may have been unusually small or they may have been negatively correlated with each other. The latter alternative is not implausible. In the 1958 recession, a rise in the unemployment rate of 2½ percentage points was accompanied by continued inflation. This was contrary to prior experience of price declines in recession. In the 1970-71 recession, a rise in the unemployment rate of 2.4 percentage points over two years failed to produce even a slowing of the inflation rate (as measured by the GNP deflator), which remained well above anything experienced in the 1950-68 period. It seems likely that inflation was substantially underestimated in these years ($v_t > 0$) relative to a more enlightened analysis based on, say, the expectations-augmented Phillips curve. The underanticipation of realized inflation is confirmed by comment at the time. If it is hypothesized that the real rate tends to be below average in depressed periods ($r_{t+1} - \bar{r} < 0$), then there is some basis for expecting a negative correlation between the first two components of the disturbance term in (14a) over the 1953-71 period.

In later work Fama and Gibbons [1982] find slope coefficients of about 1.3, using periods of one month and one quarter for the period 1953-77. After correcting for the problem introduced by changes in the real rate by assuming that the constant term in the inflation regression (12a) follows a random walk, the slope coefficient is reduced to something close to 1. However, if (nonconstant) measurement error exists the market's expectation of inflation (relative to the optimal forecast), then their assumptions about the disturbance term obtained by differencing (12a) are invalid. In this case the appropriate inference from their results might be

(1) Changes in inflation tend to be underestimated by the market.

(2) There is some negative correlation between deviations of the market forecast from the optimal forecast and deviations of the real rate from its long-term average value.

In summary, we have considered the problem of aggregating a variety of divergent individual forecasts of inflation, each subject to error on a number of grounds, into a "market forecast" and have shown that it is extremely unlikely that such an aggregate can reproduce at every point in time the optimal forecast based on all relevant prior events, whether or not they are accurately observable. If measurement error exists in the market's expectation, then regressions relating the inflation rate to prior values of the nominal interest rate produce parameter estimates that are subject to bias, perhaps of substantial magnitude, and the inferences drawn by Fama, Mishkin and others from such estimates are invalid.

Footnotes

[1]This concept is due to Muth [1960].

[2]The optimal forecast might alternatively be defined as the best that is possible based on variables that are generally observable at t. But then it can no longer be assumed that the forecast error is uncorrelated with the optimal forecast so defined since information available to some market participants but not to others may affect not only the observable variables entering into the forecst at t, but also subsequent actions affecting realized inflation in t + 1.

[3]The period might be a month, a quarter, a year or some other time span. A one-month Treasury bill is an example of an asset meeting the specification.

[4]Note that we use here the inflation rate, rather than its inverse as in Fama [1975]. Furthermore, r_{t+1} is here the market-clearing real rate at the end of period t and thus it differs conceptually from an expected value based on lagged values of the real rate or other generally observable prior information. The market-clearing real rate as determined at t need not, of course, equal the ex post real rate.

[5]Note, however, that if supply, as well as demand, were permitted to depend on i_t, it would not generally be true that a solution exists for π^e_{t+1} that produces the same outcomes for both yield and quantity that in fact occur.

[6]A recent paper by Zarnowitz and Lambros [1987] analyzes the dispersion of inflation forecasts in the quarterly survey conducted by the American Statistical Association and the National Bureau of Economic Research.

[7]The only escape from the conclusion of bias in the OLS estimate $\hat{b}$ lies in the possibility that v_t and ε_{t+1} are linearly related with a slope coefficient of 1:

$$\varepsilon_{t+1} = v_t + r_{t+1} ,$$

where η_{t+1} is uncorrelated with v_t. It is hard to imagine a rationale for such a relationship.

[8]See for example Fama and Gibbons [1982] and, most recently, Mishkin [1988] for the period 2/64-12/86.

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