SAVING IN THE TWENTY FIRST CENTURY

by

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ABSTRACT

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Population and labor force growth are expected to approach zero in the first half of the next century and this will tend to reduce further an already unsatisfactory level of aggregate saving. This paper investigates the determinants of aggregate saving under these demographic assumptions, with particular emphasis on workers' strategies for dealing with the risk of outliving their resources after retirement. The permanent income and life cycle models represent different strategies for addressing this risk and are found to differ substantially in their implications for savings.

Aggregate saving is found to be unfavorably affected by taxes on labor income, while the effect of taxes on property income is ambiguous. Public and private pension plans have a negative aggregate impact, even though employer plus employee contributions fully support workers' post retirement benefits. Inheritance taxes have no effect in the cases considered.

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Saving in the Twenty-First Century

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I. Introduction

Recent concern over high Federal deficits has been intensified by concern over low rates of private saving. The United States cannot afford as high a deficit (relative to NNP) as other developed nations because its private savings rate is so much lower. As deficits have risen in the 1980's, personal savings rates have fallen sharply from levels that were already low by international standards.

It is a remarkable fact that real per capita consumption in this country rose by one-third from 1972 to 1988 (a growth rate of about 2 percent per year), while at the same time the real wages of civilian workers fell by about 7 percent (or at an average rate of almost 1/2 of one percent per year) and the unemployment rate rose from an average of 4 3/4 percent in 1950-72 to 6.6 percent in 1973-80 and 7 1/2 percent in the 1980s.[1] This consumption growth was financed in part by higher property income and by tax reductions, but the funds came primarily from the earnings of wives who entered the labor force and from the growth of household debt. In the 1980s personal savings rates fell in spite of tax cuts and abnormally high real interest rates (both of which might be expected to increase saving incentives) and in spite of the continuing rise in two-earner families.

Real per capita consumption on the other hand continued to grow at close to its long-term rate throughout the 1972-87 period. This might be

[1]Fringe benefits have, of course, risen considerably over this period, largely reflecting increases in employers' social security contributions and in the cost of employer-financed health plans, which has risen much faster than general inflation.

interpreted as evidence of the power of consumption norms (not only those living standards already achieved, but the expected growth in these standards) to influence consumption behavior independently of the short-term (or even the long-term) behavior of income. In any case, the record of the last 15 years makes it hard to ignore the possibility that there may have been a significant shift in the fundamental relationship of consumption to income.

This paper examines the implications of certain long-term socioeconomic trends for aggregate saving, not only in respect to behavior in the 1980s but also in terms of prospects for the next twenty or thirty years. The trends of concern are the slowing of productivity growth that occurred after 1973 and the continuing decline in the population growth rate since the mid 1960s. Both have unfavorable consequences for aggregate saving. Low saving in its turn may have further unfavorable repercussions on productivity growth.

Productivity growth has averaged about 1 percent per year over the last 15 years (with substantial cyclical variation), as compared with about 2 1/4 percent per year in the 1950s and 1960s. Based on current trends, population growth will become zero or negative sometime early in the next century. Labor force growth, which is more critical for income growth, will remain somewhat insulated from the slowing trend of population growth so long as the female labor force participation rate continues to rise. While this rate rose from 39 percent to 56 percent in the 22 years from 1965 to 1987, this still leaves women some 20 percentage points below men; the rising trend may still have some way to go. However, it cannot continue indefinitely, and some slowing in the annual increase was already apparent in the 1980s.

The significance of productivity and labor force growth for aggregate saving is well known. Even if individuals consume after their retirement all that they save while working (i.e., bequests planned equal bequests received), aggregate saving will still exceed zero so long as labor force and productivity are growing. A simple example will clarify the argument. Suppose that all workers work and save for 45 years (say age 20 to age 65) and then spend 15 years in retirement, exhausting their assets by the time of death. Then retirees dissave at three times the annual rate at which they saved while working. However, if the number of workers at any given time (ages 20-65) is more than three times the number of retired as the result of labor force growth and/or if their (real) earnings are higher due to productivity growth, then the aggregate saving of the workers will exceed the dissaving of the retired.

The impetus to aggregate saving from this source has diminished already and may plausibly be expected to diminish much further. In this context the cumulative lifetime saving of the individual worker becomes a critical consideration. We examine the circumstances under which this will exceed zero and the policies that might enhance it.

To sharpen the analysis we will consider the case of zero population and labor force growth (age distribution constant), with modest productivity growth dependent on the level of saving. We will not deal at any length with intergenerational altruism as a motivation for nonzero lifetime saving. We do not know much about the determinants of such altruism, but if they are stable over time and if real wages as well as population are held constant, then it is reasonable to expect that planned bequests would remain the same from one generation to another in real terms.[2] In the absence of an estate or inheritance tax, this means that planned bequests would equal bequests received and

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[2]Note that if productivity growth is small and most of the resulting increment in output is captured by the suppliers of capital, growth in real wages will be minimal.

cumulative lifetime saving would be zero. With an estate tax, some saving might occur as the burden of the tax was distributed in some way between the consumption of the bequeather during his lifetime and the after-tax benefit to his heirs.

More interesting are bequests that arise, not from altruism, but from consumer strategies for dealing with risk. In the next section the impact of uncertainty as to length of life will be discussed. The following section deals with the potential for increasing saving by policies that raise the after-tax yield on financial assets (which may be accomplished either by raising interest rates or by tax incentives). Section IV deals briefly with uncertainties as to labor income and asset yields and draws some tentative conclusions.

II. Effects of Uncertainty as to Length of Life

How much do the retired actually dissave? The answer hinges in large part on questions of measurement. The "social security wealth" and "pension fund wealth" that an individual possesses at retirement disappear largely or entirely at death. To what extent should this count as dissaving from the individual point of view? From the societal point of view? A second measurement question relates to the treatment of owned homes. These depreciate over the retirement years in the sense that their remaining useful life is reduced. This may or may not be offset by capital gains, to the extent that housing prices rise relative to other prices. Physical depreciation should be counted as dissaving, although it is not clear to what extent this actually is done, while capital gains should be and ordinarily are excluded from the savings calculation.

This paper deals with the simpler question of how much of the (net) financial wealth that is built up voluntarily by an optimizing consumer and is entirely subject to his control is liquidated to support consumption after retirement. Casual observation suggests that most of it is consumed in the case of individuals whose lifetime earnings were below the median and who lived well beyond their life expectancy. Certainly many of these end up in their last years with almost nothing beyond their owned homes and their pension and Social Security incomes. On the other hand many wealthy individuals are able to support their desired level of consumption indefinitely out of property income and even to increase their wealth after retirement.

We consider two strategies for dealing with the risk of outliving one's resources. In the first case the individual behaves as if he expects to live forever. This requires the accumulation of a fund at retirement that yields a return sufficient to support the desired consumption stream without any dissaving. Such behavior is in the spirit of the permanent income hypothesis and we refer to one who follows this strategy as "homo Friedman." The second strategy is to provide for consumption up to an age such that the probability of living beyond that age is acceptably small, say 5 or 10 percent, depending on risk aversion. Someone following this strategy is referred to as "homo Modigliani." Some dissaving occurs in this case for (nearly) everyone, but cumulative lifetime saving remains positive for all but a small percentage of the population. In general we might expect to find "homo Friedman" more frequently among high earners and "homo Modigliani" more frequently among middle or low earners.

The Case of "Homo Friedman." We abstract for the moment from the complications introduced by pension fund and social security income and we consider a very simple utility function. In this section we take labor income and asset yield as certain and consider first the case of "homo Friedman." All economic variables are in real terms.

Let r be the personal rate of time preference; i, the after-tax rate of return on financial assets; C_t and Y_t, consumption and income in period t (assumed to occur at the beginning of the period); A_t, net financial assets at the end of period t; and R, the number of periods to retirement. Let the utility function be

(1) $$U \sim \sum_{t=1}^{\infty} \frac{u(C_t)}{(1+r)^{t-1}}, \text{ where}$$

$$u(C) = -\beta(C)^{-\gamma}, \quad \beta,\gamma > 0 .$$

Maximizing (1) subject to the constraint

(2) $$\sum_{t=1}^{\infty} \frac{C_t}{(1+i)^{t-1}} = A_0 + \sum_{t=1}^{R} \frac{Y_t}{(1+i)^{t-1}}$$

and taking note of the accounting identities

$$A_t = (1+i)(A_{t-1} - C_t + Y_t), \quad t = 1, 2 \ldots ,$$

which jointly imply (2), we obtain the optimizing conditions

$$C_t = C_{t-1}\left(\frac{1+i}{1+r}\right)^{\frac{1}{1+\gamma}}$$

$$= C_1\left(\frac{1+i}{1+r}\right)^{\frac{t-1}{1+\gamma}} .$$

Substituting in (2) we have

(3) $$C_1 = \left[A_0 + \sum_{t=1}^{R} \frac{Y_t}{(1+i)^{t-1}}\right] \div \sum_{t=1}^{\infty} \left[\left(\frac{1+i}{1+r}\right)^{\frac{1}{1+\gamma}}\left(\frac{1}{1+i}\right)\right]^{t-1} .$$

If the rate of time preference is equal to the after-tax yield on assets, this simplifies to the familiar relationship

(4) $$C_1 = \frac{i}{1 + i} [A_0 + \sum_{t=1}^{R} \frac{Y_t}{(1 + i)^{t-1}}] = i[\frac{A_0}{1 + i} + \sum_{t=1}^{R} \frac{Y_t}{(1 + i)^t}] .$$

In this case the worker's plan is to maintain consumption constant over his lifetime. Each period during his working life he accumulates enough nonhuman wealth to exactly offset the reduction in human wealth as he moves one period closer to retirement.

A young "homo Friedman," just entering the labor force with no assets (i.e., no bequest from parents), plans to accumulate, by the time he retires R^* periods later, an amount, AF_{R^*}, sufficient to support consumption indefinitely thereafter at the chosen level. Using (4) and setting $A_0 = 0$, this implies

(5) $$AF_{R^*} = \sum_{t=1}^{\infty} \frac{C_1}{(1 + i)^{t-1}} = \sum_{t=1}^{R^*} \frac{Y_t}{(1 + i)^{t-1}} .$$

The amount, AF_{R^*}, is maintained unchanged by the retired worker until he dies; and it then passes to his heirs. This bequest occurs as the result of a strategy for protecting the bequeathor against the risk of outliving his resources and is quite independent of any motive of intergenerational altruism. If the labor force is N and average labor income remains constant at $\bar{Y}$,[3] then aggregate annual saving would be

(6) $$SF = \frac{N\bar{Y}}{R^*} \sum_{t=1}^{R^*} \frac{1}{(1 + i)^{t-1}} = \frac{N\bar{Y}}{R^*} \frac{1 + i}{i} [1 - (\frac{1}{1 + i})^{R^*}] .$$

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[3]This assumption does not necessarily rule out output growth. If the marginal-efficiency-of-investment schedule is flat, marginal and average return on investment are the same and any increment in output resulting from new investment is captured by the suppliers of capital. Workers do not share in productivity gains except to the extent that they save. As a practical matter, we note that real wages in 1987 were no higher than in 1967 so that labor income does not appear to have benefited significantly from productivity gains over this period.

This amount is potentially available for expansion of the capital stock.

The case just discussed, in which there is motivation for saving but bequests received are zero, is plausible only with an estate tax of 100 percent. While such a tax would obviously discourage saving based on intergenerational altruism, it has no effect on the saving behavior of workers under present assumptions. The aggregate saving of workers remains as in (6). Since the adult population (labor force plus the retired) is constant, the number of retired who die each year is equal to the number of new entrants to the labor force, or $\frac{N}{R*}$. Multiplying this by the average size of estate, AF_{R*}, we have an aggregate value exactly equal to aggregate annual saving by workers. If this full amount is collected in estate taxes, the assets of the private sector remain unchanged, with workers using their saving to buy up assets from estates as these are sold for taxes.

What happens to investment in this case depends on what government does with the proceeds of the estate tax. If all of these are spent, then in real terms the resources which are released from production of consumer goods by workers' saving are redirected to the goods demanded by government and not to investment goods. There is no growth in the capital stock or in aggregate output. On the other hand if the government uses all of the estate tax proceeds to reduce debt, then the full amount of current saving becomes available to expand the capital stock and the output stream is increased accordingly. Households which relinquish government debt offer the proceeds to business firms to finance investment. If the rate of return on real investment is equal to the (before-tax) yield on financial assets[4] and the income tax rate is τ, then the increment in output amounts to

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[4] I.e., if the marginal and average return on real investment are equal. See previous footnote.

$$(7) \qquad \Delta Q = \frac{i(SF)}{1 - \tau} = \frac{(1 + i)N\bar{Y}}{(1 - \tau)R^*}\left[1 - \left(\frac{1}{1 + i}\right)^{R^*}\right] .$$

This is exactly equal to the net interest saving for the government that results from the retirement of debt.[5] Households have simply shifted their wealth from one form to another, with no increase in the total and no claim on additional income. The government may either spend the net interest saving or use it to reduce the debt further (providing additional funds for investment) or cut income taxes, with favorable effects both on work effort and on saving.

Next consider the effect of a zero inheritance tax. In this case the assets of the private sector grow by the amount SF per year as in (6) and consumption by $\frac{i}{1 + i}$ SF, as recipients of bequests adjust their consumption according to (4). Note that, according to (4), recipients of bequests immediately raise their consumption, leaving only SF/(1 + i) as a permanent increment to their wealth. If all of this flows into investment (i.e., the government debt remains constant), then output rises by $\frac{i}{1 - \tau}\frac{SF}{1 + i}$. Since all of this extra output (after income tax) flows into consumption, aggregate saving remains as in (6). Saving is no higher than with a 100 percent estate tax.[6]

<u>The Case of "Homo Modigliani</u>." We turn now to the case of "<u>homo</u> Modigliani." He will save less than "<u>homo</u> Friedman," but if he is risk averse--i.e., willing to accept only a small probability of outliving his

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[5]Presumably the government pays a rate of $i/(1 - \tau)$ on its debt in order to yield i after tax; but the gross interest saving due to reduction of debt is partially offset by the loss of tax revenue that would have resulted from interest income that would otherwise have been earned.

[6]This result differs markedly from that of Kotlikoff and Summers, who argue that planned bequests motivated by altruism are a powerful determinant of saving.

resources--then he will leave some positive bequest at death in a high proportion of the cases.

Let retirement occur after a work life extending over R* periods, and let L* be so chosen that the probability of living more than L* years beyond retirement is acceptably small. Again we take after-tax labor income, Y, and asset yield, i, to be given with certainty. A young "homo Modigliani" just entering the labor force then maximizes the utility function

$$(1a) \qquad U \sim \sum_{t=1}^{R^*+L^*} \frac{-\beta C_t^{-\gamma}}{(1+r)^{t-1}}$$

subject to the constraint

$$(2a) \qquad \sum_{t=1}^{R^*+L^*} \frac{C_t}{(1+i)^{t-1}} = A_0 + \sum_{t=1}^{R^*} \frac{Y_t}{(1+i)^{t-1}},$$

where A_0 is the bequest received.[7] Again we obtain the optimizing conditions

$$C_t = C_1\left(\frac{1+i}{1+r}\right)^{\frac{t-1}{1+\gamma}}, \quad t = 1, 2, \ldots R^*+L^* .$$

Substituting in (2a), we have at entry into the labor force

$$(3a) \qquad C_1 = \left[A_0 + \sum_{t=1}^{R^*} \frac{Y_t}{(1+i)^{t-1}}\right] \div \sum_{t=1}^{R^*+L^*} \left(\frac{1+i}{1+r}\right)^{\frac{t-1}{1+\gamma}}\left(\frac{1}{1+i}\right)^{t-1} .$$

Assuming for the moment that i = r, this simplifies to

$$(4a) \qquad C_1 = \frac{i}{1-\left(\frac{1}{1+i}\right)^{R^*+L^*}}\left[\frac{A_0}{1+i} + \sum_{t=1}^{R^*} \frac{Y_t}{(1+i)^t}\right] .$$

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[7] For present purposes we ignore the complications introduced by the fact that in practice bequests may well be received 10 or 20 or 30 years after entry into the labor force.

Clearly this is larger than (4). Assuming $R^* = 45$ and $L^* = 20$, annual consumption for "*homo* Modigliani" is 17 percent higher than for "*homo* Friedman" at an interest rate of 3 percent, 4 1/2 percent higher at $i = 5$ percent and 1 1/4 percent higher at $i = 7$ percent.

Assets at retirement, AM_{R*}, and aggregate annual saving by workers, SM, are a fraction of the corresponding values for "*homo* Friedman," AF_{R*} and SF. Specifically

(5a) $$AM_{R*} = \frac{1 - \left(\frac{1}{1+i}\right)^{L*}}{1 - \left(\frac{1}{1+i}\right)^{R*+L*}} AF_{R*} .$$

(6a) $$SM = \frac{1 - \left(\frac{1}{1+i}\right)^{L*}}{1 - \left(\frac{1}{1+i}\right)^{R*+L*}} SF .$$

At the assumed values of L^* and R^* this fraction amounts to .52 for $i = 3$ percent, .65 for $i = 5$ percent and .75 for $i = 7$ percent. Thus saving is significantly lower for "*homo* Modigliani" during the work life. Furthermore, substantial dissaving occurs after retirement, whereas "*homo* Friedman" maintains wealth unchanged after retirement, consuming only the income earned on assets.

It is an interesting exercise to evaluate the relationships (6) and (6a) at recent levels of the wage bill, $N\bar{Y}$. Since no account is taken of the impact on aggregate saving of labor force growth and since this has been quite high in recent years, we might expect these relationships to *understate* the savings levels actually experienced. For $N\bar{Y}$ we use after-tax labor income exclusive of nondiscretionary savings items, such as social security contributions and employer contributions to pension funds, so that the variable estimated is discretionary saving.[8]

Aggregate levels of discretionary saving, as estimated from (6) and (6a) for reasonable values of R* and L* and a range of (real) interest rates, are shown in the table below for two recent periods. In spite of the presumed understatement, the values produced by the permanent income model are found to be larger than actual saving by an order of magnitude, not only in the spendthrift days of 1986-87, but even in the more normal period of 1978-80.

Savings estimates come much closer to actual values when the life cycle model is used, with the (quite arbitrary) assumption that half of what a worker has acquired at retirement is dissaved on average, by the time of death.[9] We note that the permanent income model estimates are very sensitive to the time preference/interest rate utilized by workers in formulating their consumption plans, while the life cycle estimates are much less so.

Estimates of Personal Saving in Recent Periods Based on Initial Model (Billions of dollars)

	1986-87	1978-80
Permanent Income Model		
i = .03	966	560
i = .05	713	413
i = .07	557	323
Life Cycle Model*		
i = .03	252	146
i = .05	232	135
i = .07	209	121
Actual	125	122

* Assuming that 1/2 of the assets accumulated at retirement are subsequently dissaved.

[8] Apart from the growth factors already mentioned, nondiscretionary saving by workers should offset the dissaving of the retired (as pensions and Social Security benefits are spent) in a fully funded retirement system.

[9] Whether or not this is consistent with the assumption L* = 20 depends on the precise probabilities of death in each of the 20 years following retirement. However the numerical results are not very sensitive to modest changes in L*.

<u>Effect of Pensions and Social Security Benefits</u>. Even though pension payments and Social Security Benefits must be fully supported by prior contributions (nondiscretionary saving) in an actuarilly sound retirement system, so that their average impact on cumulative lifetime saving is zero, nevertheless their existence affects discretionary saving in two ways:

> First, there is a simple transfer from discretionary to nondiscretionary saving. Since nondiscretionary saving is sufficient on average to provide for some part of post-retirement consumption, say $\bar{C}$, for a number of periods determined by life expectancy at the date of retirement, there is no need to undertake discretionary saving to support this portion of consumption over this period.
>
> Second, the need to insure against living beyond one's life expectancy is reduced, since the amount $\bar{C}$ is guaranteed for life. This reduces the cost of protecting consumption against the contingency of living beyond one's life expectancy and so reduces total--as well as discretionary--saving.

Let us see how these considerations affect the consumption and cumulative lifetime saving of "<u>homo</u> Friedman" and "<u>homo</u> Modigliani."

We assume a fully funded retirement system. While it may be possible temporarily, in a period of labor force growth, for the retired to capture benefits in excess of their prior nondiscretionary saving, this is not feasible for any extended period in a situation of zero labor force growth. It is convenient to divide labor income, Y, into two components: Y', which is immediately available to the worker to spend or save at his discretion; and Y", which consists of employer and employee contributions to pension funds and Social Security, is not subject to the worker's discretion, and is entirely saved. If $\bar{C}$ represents the level of post-retirement benefits and E(L) is the life expectancy at retirement, then a fully funded system requires that on average

$$(8)\quad \sum_{t=1}^{R^*} \frac{Y''_t}{(1+i)^{t-1}} = \sum_{t=R^*+1}^{R^*+E(L)} \frac{\bar{C}}{(1+i)^{t-1}} = \frac{\bar{C}}{(1+i)^{R^*}} \frac{1+i}{i} \left(1 - \frac{1}{(1+i)^{E(L)}}\right) .$$

For a "<u>homo</u> Friedman" just entering the labor force, the constraint (2), becomes

$$(2b)\quad \sum_{t=1}^{\infty} \frac{C_t}{(1+i)^{t-1}} = A_0 + \sum_{t=1}^{R^*} \frac{Y'_t}{(1+i)^{t-1}} + \sum_{t=R^*+1}^{\infty} \frac{\bar{C}}{(1+i)^{t-1}}$$

$$= A_0 + \sum_{t=1}^{R^*} \frac{Y'_t}{(1+i)^{t-1}} + \sum_{t=1}^{R^*} \frac{Y''_t}{(1+i)^{t-1}} + \sum_{t=R^*+E(L)+1}^{\infty} \frac{\bar{C}}{(1+i)^{t-1}} .$$

Then under the assumptions of this section, the optimum consumption level is

$$(4b)\qquad C_1 = \frac{i}{1+i} \left(A_0 + \sum_{t=1}^{R^*} \frac{Y'_t}{(1+i)^{t-1}}\right) + \frac{\bar{C}}{(1+i)^{R^*}} .$$

This is larger by $\bar{C}/(1+i)^{R^*+E(L)}$ than the value that could be supported by total labor income, $Y' + Y''$, in the absence of social funding of the retirement system.

In order to achieve consumption of C_1 after retirement, the worker must accumulate, out of discretionary labor income, Y'_t, an amount, AF'_{R^*}, such that at retirement

$$AF'_{R^*} = \sum_{t=1}^{\infty} \frac{C_1 - \bar{C}}{(1+i)^{t-1}} - A_0 .$$

Substituting from (4b) and assuming $A_0 = 0$ and $Y'_t = \bar{Y}'$, we obtain for aggregate discretionary saving

$$(9)\qquad SF' = \frac{N}{R^*} \frac{1+i}{i} \left(1 - \frac{1}{(1+i)^{R^*}}\right) (\bar{Y}' - \bar{C}) .$$

4.8.3

We note that socially funded retirement benefits reduce discretionary saving by a factor $(1 - \frac{\bar{C}}{\bar{Y}'})$, dependent on the size of retirement benefits relative to the average level of discretionary income. For $\bar{C} = \bar{Y}'$ there is no discretionary saving and thus for a fully funded retirement system aggregate saving is zero. Workers consume at the level $\bar{Y}'$ throughout their lives, with nondiscretionary savings during the working years just sufficient on average to support consumption at the level $\bar{Y}'$ after retirement.

For a "*homo* Modigliani" just entering the labor force, with a planning horizon of $R^* + L^*$, the constraint is

$$\text{(2c)} \quad \sum_{t=1}^{R^*+L^*} \frac{C_t}{(1+i)^{t-1}} = A_0 + \sum_{t=1}^{R^*} \frac{Y'_t}{(1+i)^{t-1}} + \sum_{t=R^*+1}^{R^*+L^*} \frac{\bar{C}}{(1+i)^{t-1}}$$

$$= A_0 + \sum_{t=1}^{R^*} \frac{Y'_t + Y''_t}{(1+i)^{t-1}} + \sum_{t=R^*+E(L)+1}^{R^*+L^*} \frac{\bar{C}}{(1+i)^{t-1}} .$$

The optimum consumption level is

$$\text{(4c)} \quad C_1 = \frac{i}{1+i} \frac{1}{1 - (\frac{1}{1+i})^{R^*+L^*}} \Big[A_0 + \sum_{t=1}^{R^*} \frac{Y'_t}{(1+i)^{t-1}}$$

$$+ \frac{\bar{C}}{(1+i)^{R^*}} \frac{1+i}{i} \Big(1 - \frac{1}{(1+i)^{L^*}}\Big)\Big] .$$

This is larger by the amount

$$\left[\frac{1 - (\frac{1}{1+i})^{L^*-E(L)}}{1 - (\frac{1}{1+i})^{R^*+L^*}}\right] \left[\frac{\bar{C}}{(1+i)^{R^*+E(L)}}\right]$$

than the level that could be supported by total labor income without social funding of the retirement benefits. Since the first factor is substantially less than 1 for reasonable values of $L^* - E(L)$, this consumption increment is much less than the case of "homo Friedman."

The retirement fund, AM'_{R^*}, that must be accumulated out of discretionary savings in this case is given by

$$AM'_{R^*} = \sum_{t=1}^{L^*} \frac{C_1 - \bar{C}}{(1 + i)^{t-1}} - A_0 .$$

Substituting from (4c) and assuming $A_0 = 0$ and $Y'_t = \bar{Y}'$, we obtain for aggregate discretionary saving by workers:

$$\text{(9a)} \qquad SM' = \frac{N}{R^*} \frac{1 + i}{i} \left(1 - \frac{1}{(1 + i)^{L^*}}\right)\left\{\frac{1 - \frac{1}{(1 + i)^{R^*}}}{1 - \left(\frac{1}{1 + i}\right)^{R^*+L^*}} \bar{Y}' + \frac{1 - \frac{1}{(1 + i)^{L^*}}}{1 - \left(\frac{1}{1 + i}\right)^{R^*+L^*}} \frac{\bar{C}}{(1 + i)^{R^*}} - \bar{C}\right\} .$$

Of this, some substantial part is dissaved after retirement, so that aggregate saving for the population as a whole is some fraction of (9a).

Again we may wish to examine the implications of these models for aggregate saving in recent years, subject to the same caveats as before. For purposes of example, we assume that $\bar{C} = \frac{1}{2} \bar{Y}'$ and that "homo Modigliani" on average dissaves half of the assets accumulated out of discretionary saving by the time of death.

We recall that estimates based on models that assume zero labor force growth are expected to understate actual saving when applied to periods in which labor growth was in fact substantial. Nevertheless, estimates from the permanent income model remain far above actual saving. Only if the average level of retirement benefits is assumed to be a very high proportion of average after-tax discretionary labor income could the model be made consistent with the behavior observed in 1978-80, let alone 1986-87. The life cycle model on the other hand underestimates saving for the 1978-80 period and, at time preference/interest rates above 3 percent, for 1986-87 also. The results obtained are therefore not implausible on their face.

Model Estimates of Personal Saving, Adjusted for
the Impact of a Fully Funded Retirement System
(Billions of dollars)

	1986-87	1978-80
Permanent Income Model		
i = .03	483	280
i = .05	357	207
i = .07	278	162
Life Cycle Model*		
i = .03	126	73
i = .05	116	67
i = .07	105	61
Actual	125	122

*Assuming that 1/2 of the assets accumulated at retirement are subsequently dissaved.

The models described in this section have two implications regarding the effects of specific policies on aggregate saving in the absence of labor force growth.

1. Unlike a tax on labor income, which discourages saving by workers, an inheritance tax has no negative impact on workers' savings behavior. Any tax may, of course, divert resources from investment spending to government spending, but that depends entirely on the use to which tax proceeds are put.

2. Pension plans and the Social Security system, even if fully funded, reduce the cumulative lifetime saving of individual workers (and therefore aggregate saving) by reducing their need to insure themselves against the risk of living beyond their life expectancy. The impact is substantial so long as

the guaranteed post-retirement benefit is a substantial fraction of discretionary labor income during the working years. Results under the permanent income hypothesis are quite sensitive to the value chosen for the time preference rate and after-tax asset yield, which are held equal to each other throughout this section.

In the next section we consider the impact on aggregate saving of policies intended to raise the after-tax interest rate, presumably without affecting individuals' rate of time preference. Since saving is primarily desired in order to raise investment (without creating inflationary pressures) and since monetary policy that raises before-tax interest rates will serve to discourage investment, we direct our attention specifically to tax incentives that raise the after-tax yield on saving.

III. Effects of Tax Incentives for Saving

In the models of the previous section the effect of a high after-tax interest rate was to discourage saving. (Note tables). This result reflects the fact that any given level of retirement income can be achieved with less current saving when the interest rate is high than when it is low. Because high asset yields raise the total dollars available for lifetime consumption, they have a favorable "income effect" on current consumption. In the previous section there was no offsetting "substitution effect" because equality was maintained between the interest rate and the rate of time preference. Thus there was no incentive at high interest rates to tilt the consumption path by shifting expenditures from the current to subsequent periods. We now consider the effect of changes in after-tax asset yields with the rate of time preference held constant, in order to investigate the probable impact of tax policies that increase rewards to saving. In this case substitution effects favorable to saving are expected to occur as yields rise, and these may partially offset, or even overwhelm, the unfavorable income effect noted in the previous section. The presumption of much of the current policy discussion relating to consumption taxes and capital gains taxes appears to be that the substitution effect will dominate.

To take the simplest case, we examine the consumption of "homo Friedman" with $A_0 = 0$, $Y'_t = \bar{Y}'$ and no socially funded retirement programs. From (3) this implies

$$(10) \qquad C_1 = \frac{\bar{Y} \sum_{t=1}^{R^*} \frac{1}{(1+i)^{t-1}}}{\sum_{t=1}^{\infty} \left[\left(\frac{1}{1+r}\right)^{\frac{1}{1+\gamma}} \left(\frac{1}{1+i}\right)^{\frac{\gamma}{1+\gamma}} \right]^{t-1}}$$

where r, the rate of time preference, is constant and i varies with tax policies that serve to increase the after-tax return on financial assets. We note that the numerator in (10) declines as i increases, while the denominator also falls. The behavior of the ratio is ambiguous depending primarily on the utility function parameter γ. In this case the fund accumulated at retirement is

$$(11) \qquad AF''_{R^*} = C_1 \left(\frac{1+i}{1+r}\right)^{\frac{R^*}{1+\gamma}} \sum_{1}^{\infty} \left[\left(\frac{1}{1+r}\right)^{\frac{1}{1+\gamma}} \left(\frac{1}{1+i}\right)^{\frac{\gamma}{1+\gamma}} \right]^{t-1}$$

$$= \bar{Y} \left(\frac{1+i}{1+r}\right)^{\frac{R^*}{1+\gamma}} \sum_{t=1}^{R^*} \frac{1}{(1+i)^{t-1}}$$

and aggregate annual saving is

$$(12) \qquad SF'' = \frac{N\bar{Y}(1+i)}{iR^*} \left[1 - \left(\frac{1}{1+i}\right)^{R^*}\right] \left(\frac{1+i}{1+r}\right)^{\frac{R^*}{1+\gamma}} .$$

For "homo Modigliani" in this case, saving by workers, SM", is a fraction of that for "homo Friedman" and is partially offset by dissaving of the retired. Specifically

$$(12a) \qquad SM'' = SF''\left\{\frac{1 - \left[\left(\frac{1}{1+r}\right)^{\frac{1}{1+\gamma}}\left(\frac{1}{1+i}\right)^{\frac{\gamma}{1+\gamma}}\right]^{L^*}}{1 - \left[\left(\frac{1}{1+r}\right)^{\frac{1}{1+\gamma}}\left(\frac{1}{1+i}\right)^{\frac{\gamma}{1+\gamma}}\right]^{R^*+L^*}}\right\} .$$

Assuming that the average "homo Modigliani" dissaves half of his retirement fund by the time of death, we show below the impact of the utility function parameter, γ, on aggregate saving under the two models. We set r, the rate of time preference, at 3 percent, R* at 45 and L* at 20, and show a range of interest rates from 5 to 7 percent.

For $\gamma = 1/2$, which implies that marginal utility diminishes very slowly as consumption rises, we find high savings ratios--obviously unrealistic in the case of "homo Friedman"--and a strong positive effect of the interest rate on saving. For $\gamma = 3$, which implies that marginal utility diminishes at a considerably more rapid rate, we find savings rates reduced in both models, while the interest rate effect is very small and positive under the life cycle model and very small and negative under the permanent income model. Still higher values of γ will further reduce savings rates and will lead to a negative interest rate effect on saving for both models.

Effect on Aggregate Saving of Policy-Induced Changes in After-Tax Interest Rates

	Ratio of Aggregate Saving to the Wage Bill			
	$\gamma = 1/2$		$\gamma = 3$	
Interest Rate	Permanent Income	Life Cycle	Permanent Income	Life Cycle
.05	.737	.209	.514	.161
.06	.861	.254	.503	.167
.07	1.014	.309	.497	.175

There is, of course, no general agreement as to the functional form of the utility function of consumers. The point we make is that the effect of interest rates on saving depends crucially on the degree of concavity of the function, since that determines the strength of the substitution effect of the interest rate on consumption and saving. The income effect is always favorable to consumption and unfavorable to saving. Whether tax incentives--substituting consumption taxes for income taxes, reducing capital gains taxes--will stimulate saving or discourage it depends on the particular characteristics of utility functions. Rational consumers could go either way.

IV. Conclusions and Directions for Further Research

It is beyond the scope of this paper to consider in any detail the implications for aggregate saving of uncertainty about future labor income or about the real rate of return that will in fact be realized on financial assets. Individuals just entering the labor force undoubtedly experience substantial uncertainty on both counts.

Apart from liquidity constraints, short-term fluctuations of labor income and asset yield around their expected values should have little effect on consumption behavior under the standard models, and should produce only short-run effects on aggregate saving. When individuals experience sufficiently large deviations of assets from the growth path implicit in their long-run consumption plan, this will, of course, trigger a recalculation of the optimum plan.[10] Even after such corrections, workers who initially overestimated labor income and asset yield may well end up with a smaller retirement fund

[10] We do not assume that recalculation will occur for trivial deviations of assets from their planned path because the effort involved represents a cost and because there is a psychological cost to modifying consumption habits, especially in the downward direction.

than if these variables had been accurately predicted, while those who initially underestimated are likely to end up with a larger than optimal accumulation of assets at retirement. But unless there is some bias in workers' estimates of future income, there is no reason to expect the average level of assets at retirement or aggregate saving to be affected.

A more serious question relates, not to short-term variability of income, but to uncertainty about the average levels of labor income and asset yield that will be realized over the working life. To what extent, if at all, do workers try to compensate for this uncertainty by planning initially for a larger retirement fund than would be optimal under income certainty, in order to reduce the probability of falling substantially below the optimum? To what extent would deviations from the optimum, which should be positive on average, be dissaved after retirement? Further work is needed to deal with these questions.

Further work is also needed to determine realistically the pattern of saving/dissaving over the years of life after retirement, using a socially meaningful measure of dissaving, and to see how (if at all) this pattern is related to the average level of labor income during the work life. Much more reliable evidence is needed on the magnitude of utility function parameters and in particular on how rapidly marginal utility diminishes as consumption grows, holding other things constant. The numerical calculations in this paper, while based on plausible assumptions, can be at most suggestive, indicating the kind of information required for a serious empirical analysis of future saving behavior. Alternative models should be investigated in which consumption is liquidity-constrained up until, say, age 45 and life cycle saving begins in earnest at that point.

Abstracting from the complications related to income uncertainty and utilizing plausible values or a range of values for unknown parameters, our tentative conclusions are:

1. Under the permanent income model specified here, workers protect themselves completely against the possibility of outliving their resources by planning to accumulate sufficient assets at retirement to support the planned consumption stream indefinitely--i.e., out of return on assets, with no liquidation of capital. This requires aggregate levels of discretionary saving that appear to be unrealistically high even when public and private pension plans, funded by nondiscretionary saving, guarantee a substantial proportion of the post-retirement consumption stream.

2. Under the life cycle model specified here, workers still protect themselves to a substantial degree against the risk of outliving their resources by choosing a planning horizon significantly beyond their life expectancy. The retirement fund required is considerably smaller in this case, implying lower saving during the work life; and much of what is accumulated is liquidated to support consumption after retirement. For reasonable assumptions as to the fraction of the retirement fund that is dissaved on average after retirement and the fraction of the planned post-retirement consumption stream that is guaranteed through pension plans, this model is capable of producing reasonable values for discretionary saving.

3. Under the permanent income model the full amount of assets accumulated at retirement flows through into bequests. Under the life cycle model, bequests are a random variable, inversely dependent on the number of periods the worker survives after retirement; but on average they are substantially greater than zero if workers chose a planning horizon substantially greater than their life expectancy. If the demands of

intergenerational altruism are satisfied by the bequests automatically generated by risk strategies, then aggregate saving is unaffected by an inheritance tax.

4. Aggregate saving is positively related to after-tax labor income, exclusive of non-discretionary savings items, and negatively related to the level of post-retirement consumption guaranteed by public and private pension funds.

5. The after-tax yield on financial assets has two offsetting effects on aggregate saving. High asset yields tend to stimulate consumption and discourage saving because they increase the total dollars available for lifetime consumption. On the other hand, they tend to discourage current consumption, holding the rate of time preference constant, because they make future consumption more attractive relative to current consumption; and this encourages current saving. The characteristics of the utility function determine which effect will dominate.