ON CASH-IN-ADVANCE MODELS OF MONEY DEMAND AND ASSET PRICING

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Abstract

The paper shows how a cash-in-advance model of money demand can be written in a way that combines a simple, yet empirically defensible, money demand equation with tractability in asset pricing. Return premia are determined as in the standard barter exchange model, except that a short-term risk-free nominal interest rate enters into the first order condition. In special cases, asset prices satisfy the familiar barter-economy Euler equations exactly. Thus, contrary to much of the literature, money may not significantly affect asset pricing. Simple barter-economy Euler equations are approximately valid even in the presence of money.

1. Introduction

A cash-in-advance constraint has become a popular device for introducing money in macroeconomic equilibrium models.[1] Unfortunately, researchers modeling money demand with a cash-in-advance constraint found that a "reasonable" money demand function could only be derived at the cost of setting up a relatively complicated model. One important complication is that asset prices usually did not satisfy the familiar barter-economy Euler equation:[2]

$$E_t\left[\frac{\beta \cdot u'(c_{t+1})}{u'(c_t)} \cdot r_{kt+1}\right] = 0, \quad \forall k \,, \tag{1}$$

where $u'(c_t)$ is the marginal utility of consumption, β the rate of time preference, and r_{kt+1} the return on asset k. On the other hand, in light of the disappointing results of consumption based asset pricing models,[3] models that do not impose (1) might be considered promising.

The point of this paper is to show how a cash-in-advance model of money demand can be written in a way that combines tractability in asset pricing with a simple, yet empirically defensible money demand equation. Asset pricing proceeds as in the standard barter exchange model, except that a short-term risk-free nominal interest rate enters into the first order condition. In special cases, asset prices satisfy the barter-economy Euler equation (1) exactly. Empirically, the interest rate effect is found to be small, though significant. Thus, contrary to much of the literature cited above, monetary considerations are apparently not very important for asset pricing. The performance of consumption based asset pricing models with or without money remains unsatisfactory.

Empirical evidence, e.g., in Judd and Scadding (1982) and Mankiw and Summers (1986), suggests that a money demand function should have at least the following properties: (1) Money demand is positively related to a scale variable, like aggregate output or consumption. (2) The income velocity of money is variable. (3) Money is demanded even if it is dominated as a store of value. (4) Money demand is negatively related to interest rates on securities that compete as stores of value.

Cash-in-advance models have generally no problem with requirements (1) and (3). But simple versions lead to a quantity equation with unit income velocity of money (see Lucas (1982)), which violate (2) and (4). While variable velocity can be accommodated with additional assumptions,[4] interest elasticity seems difficult to motivate. Two ways around this problem have been proposed. Svensson (1985) adds uncertainty to the model in a way that generates precautionary demand for money. Then the elasticity of precautionary money demand with respect to the opportunity cost of holding money generates interest elasticity, but it leads to equations for money demand and asset prices in terms of Lagrange multipliers that are not easy to interpret. Lucas (1984) introduces a distinction between money goods and credit goods and imposes the cash-in-advance constraint only on money goods. Then substitution between the two types of goods should make money demand income and interest elastic. But the model is too general to yield precise results on the determinants of money demand.

This paper develops a cash-in-advance model that has features (1)-(4) but does not complicate money demand and asset pricing equations excessively. The analysis adopts Lucas' distinction of money and credit goods with some simplifying assumptions. In every period, money, securities, and some goods are traded "early," while some other goods can only be traded "late," when all other markets have closed. (Alternatively, one could distinguish goods by location instead of time, but I adopt Lucas' labels.) The key simplification compared to Lucas is that there is no uncertainty between the early and late phases of the period.

Late purchases must be made with money, which cannot be used by the seller until the start of the next period. The goods traded early correspond to Lucas' credit goods in that consumers do not need money for their purchases. In contrast to Lucas, it is even unnecessary to assume credit transactions for the early goods (though I will still refer to them as credit goods). The key difference to money goods is that sales revenues can be used immediately.

Provided nominal interest rates are positive, buyers will acquire just enough money in early trading to make the planned late purchases. There is no precautionary demand for money. The opportunity cost of money is the interest rate on a one-period nominal bond. Hence, one plus this interest rate is the relative price of money and credit goods. If demand for money goods (which creates money demand) depends on their relative price, the interest rate enters into money demand in a simple and direct way. Consumption of money goods, endowments, or total consumption can be used as scale variables.

With this approach, the Euler equations for asset pricing look similar to the standard equations obtained for barter economies. They can be written in two versions. In one version they involve marginal utility with respect to any one good--as one would obtain in a barter economy with many commodities. In the other version they involve a marginal utility of aggregate consumption--as in barter-exchange models. The only difference to barter-models is that the marginal utility is computed from an indirect utility function, which also depends on the nominal interest rate. The analysis suggests that simple barter-economy Euler equations are approximately valid even in the presence of money.

The paper is organized as follows. Section 2 describes the model. Strong assumptions on government, firms, and stochastic structure are made to keep the exposition as simple as possible. The model may be generalized as in any barter-economy. In Section 3, a money demand function is derived. Sections 4 and 5 explore implications for asset pricing in partial and general equilibrium. Section 6 briefly summarizes the conclusions.

2. The Model

Consider a discrete-time, infinite horizon, representative agent model of a closed economy without capital. In every period (indexed by t), individuals have preferences over a good c_{1t} that is traded "early," when securities markets are open, and a good c_{2t} that is purchased "late," when securities markets have closed, against money M_t. Both goods are perishable. The key assumption is that sellers cannot use money received "late" until markets open in the following period. Early purchases (of good 1) are paid in a way that the seller can immediately use the receipts.[5]

The representative individual's utility function is

$$E_t \sum_{j\geq 0} \beta^j \cdot u(c_{1t+j}, c_{2t+j}) \ , \tag{2}$$

where $u(\cdot)$ is increasing in both arguments and concave. Individuals have endowments of inputs y_t that they sell to firms, they obtain money transfers from the government, and they can trade securities. Securities (indexed by k) are either shares of firms or any other claims. Let Q_{kt} and R_{kt+1} be the (nominal) price and return, respectively. To describe the individual budget constraints one has to specify the rest of the economy.

The government's function is to print (or destroy) money at a rate ω_t and to distribute new money $(\omega_t - 1) \cdot M^s_{t-1}$ in a lump-sum fashion to individuals, where ω_t can be any stochastic process of money growth.

Firms transform endowments into consumption goods c_{1t} or c_{2t} subject to the constraint[6]

$$y_t \geq c_{1t} + c_{2t} \tag{3}$$

Let p_t be the price of endowments y_t, p_{1t} the price of c_{1t}, and p_{2t} the price of c_{2t}. While $p_{1t} = p_t$ is obvious, the relative price of the money good must reflect the fact that cash cannot be used until the next period. But assuming that there is a short-term, nominally risk-free asset and that firms can trade securities, there is a simple solution for p_{2t}. Let security $k = 0$ be the short-term nominal bond, which has a price of $Q_{0t} = 1$ and a nominal return $R_{0t+1} = (1 + i_t)$ in period t+1. Then any (t+1)-dated nominal claim is discounted by $1/(1 + i_t)$, so that the firms must charge

$$p_{2t} = (1 + i_t) \cdot p_t \tag{4}$$

for good c_{2t}. Since this is a critical result, a formal proof is provided in the appendix.

Individual budget constraints require that "early" consumption plus money holdings M_t for "late" consumption must be financed by endowments, money transfers, and cash flow from trading and ownership of securities and money,

$$c_{1t} + \frac{M_t}{p_t} \leq y_t + (\omega_t - 1) \cdot \frac{M^s_{t-1}}{p_t} + \frac{M_{t-1} - p_{2t-1} \cdot c_{2t-1}}{p_t} + \sum_k \left(\frac{R_{kt}}{p_t} \cdot X_{kt-1} - \frac{Q_{kt}}{p_t} \cdot X_{kt} \right)$$

where X_{kt} are net holdings of securities. Notice that money transfers are exogenous to individuals (although $M = M^s$ in equilibrium). Consumption of c_{2t} is constrained by

$$p_{2t} \cdot c_{2t} \leq M_t \; . \tag{5}$$

The focus of analysis is on paths of money supply for which nominal interest rates are positive, i.e. $R_{0t+1} = 1 + i_t > 1$.[7] Then (5) holds with equality and the individuals' optimization problem reduces to maximizing (2) subject to

$$c_{1t} + (1 + i_t) \cdot c_{2t} \leq y_t + (\omega_t - 1) \cdot \frac{M^s_{t-1}}{p_t} + \sum_k \left(\frac{R_{kt}}{p_t} \cdot X_{kt-1} - \frac{Q_{kt}}{p_t} \cdot X_{kt} \right). \tag{6}$$

For later reference, notice that money demand is simply

$$M_t = (1 + i_t) \cdot p_t \cdot c_{2t} \; . \tag{7}$$

The first order conditions of optimal policy are (denoting partial derivatives by subscripts)

$$(1 + i_t) \cdot u_1(c_{1t}, c_{2t}) - u_2(c_{1t}, c_{2t}) = 0 \tag{8}$$

$$E_t[u_1(c_{1t}, c_{2t}) - \beta \cdot u_1(c_{1t+1}, c_{2t+1}) \cdot r_{kt+1}] = 0, \quad \forall k \tag{9}$$

where $r_{kt+1} = R_{kt+1}/Q_{kt} \cdot p_t/p_{t+1}$ is the real return on asset k.

Equation (8) in a standard first order condition for consumption demand in a multi-good model: Allocation of consumption is determined by relative prices. Equations (9) are the set of Euler equations for asset prices; they will be discussed in Section 4. The solution of the individual's problem are demand functions for consumption, money, and securities in terms of exogenous variables.

The only special feature that distinguishes this optimization problem from one in a barter economy is that $R_{0t+1} = 1 + i_t$ appears in (9) as part of the asset return r_{0t+1} and simultaneously in (8) as a relative price.[8] In contrast to Lucas' (1984) and Svensson's (1985) papers, money does not add more complications. This is because at the time when consumers decide how much money to hold there is no uncertainty about the need for money and about its purchasing power. Purchasing power risk is taken by sellers who can lay off this risk on the bond market. The following sections will show how this separation of uncertainty from the opportunity cost of money can be exploited to obtain simple money demand and asset pricing equations.

3. Money Demand

The basic question of money demand analysis is whether there is a stable relation between money demand and a small set of other macroeconomic variables.[9] Unless the nominal interest rate i_t is zero, equation (7) indicates that money demand is a simple function of prices, the nominal interest rate, and consumption of money goods. But since the category of money goods is not easily measured empirically, it cannot be used in an operational money demand equation.

Fortunately, good-2 consumption is closely related to two measurable macro-variables, namely total consumption spending and endowments, which may serve as proxies for transactions volume instead. The key to replacing c_{2t} in money demand is the tight link between c_{2t} and c_{1t} expressed in equation (8). The fact that the interest rate i_t appears in equation (8) causes no complications, since i_t is already a determinant of money demand in equation (7). Money demand is therefore a function of total consumption spending, prices, and the interest rate,

$$M = M_t^d(C_t, p_t, i_t) \ , \tag{10}$$

where total consumption spending, C_t, is defined as

$$C_t = p_{1t} \cdot c_{1t} + p_{2t} \cdot c_{2t} = p_t \cdot (c_{1t} + (1 + i_t) \cdot c_{2t}) \ . \tag{11}$$

Alternatively, using equations (3), (7), and (8), money demand can be written as a function of endowments, prices, and the interest rate,

$$M = M_t^d(y_t, p_t, i_t) \ . \tag{12}$$

But since the derivation (especially equation (3)) relies heavily on the simplifying assumptions of the model (no investment, government spending, or net exports), the analysis will concentrate on equation (10).[10] Notice that equation (10) and (12) are not identical: Nominal consumption differs from nominal endowments, which are $p_t \cdot y_t = p_t \cdot (c_{1t} + c_{2t})$, because positive nominal interest rates distort consumer prices (p_{1t}, p_{2t}) relative to production cost (p_t for both goods). The distortionary seignorage $i_t \cdot p_t \cdot c_{2t}$ is returned to individuals as a lump-sum transfer.[11] In equilibrium, this transfer matches the amount by which the sum of individual funds needed for consumption at consumer prices, $p_t \cdot (y_t + i_t \cdot c_{2t})$, exceeds aggregate endowments.[12]

Returning to equation (10), notice that consumption and money demand are proportional to prices p_t. If one defines

$$m_t^d = \frac{M_t^d}{p_t} = (1 + i_t) \cdot c_{2t} \tag{13}$$

$$c_t = \frac{C_t}{p_t} = c_{1t} + (1 + i_t) \cdot c_{2t} \ , \tag{14}$$

s real money demand and real consumption,[13] real money demand is a function of real consumption and the nominal interest rate, $m = m_t^d(c_t, i_t)$. The derivatives are easily obtained as

$$\frac{dm^d}{dc} = \frac{(1 + i_t) \cdot (-u_{11} \cdot (1 + i_t) + u_{12})}{\Delta} \qquad \text{and}$$

$$\frac{dm^d}{di} = \frac{-(1 + i_t) \cdot u_1 + c_{2t} \cdot (u_{21} \cdot (1 + i_t) - u_{22})}{\Delta} \ ,$$

where $\Delta = -u_{11} \cdot (1 + i_t)^2 + (u_{12} + u_{21}) \cdot (1 + i_t) - u_{22} > 0$.[14] Using endowments instead of consumption (i.e., starting with (12) instead of (10)) would lead to a similar demand function.

The derivatives show that money demand is increasing in consumption c_t, provided the utility function has sufficiently small mixed second partial derivatives (e.g., $|u_{11}| > |u_{12}|$ is sufficient), while the effect of interest rates is ambiguous. The derivative with respect to interest rates consists of an income and a substitution effect. If nominal interest rates rise, more money is needed to purchase each money good, but the purchase of money goods will be reduced ($dc_{2t}/di_t < 0$ is unambiguous). A negative effect of interest rates on money is obtained, if money and credit goods are close substitutes or if the income effect is small (e.g., if the amount of c_{2t} is small). But there is a possibility that money demand may not be interest elastic (or even increasing in i_t) even when there are substitutes for money goods.

An example may illustrate these effects.

Example 1: CES-utility, general risk preferences: Suppose individual utility has constant elasticity of substitution between the two goods and suppose individuals are risk averse over time,

$$u(c_{1t}, c_{2t}) = v\left[\left(\alpha \cdot c_{1t}^{-\rho} + (1-\alpha) \cdot c_{2t}^{-\rho}\right)^{-1/\rho}\right], \tag{15}$$

where $v(\cdot)$ is an increasing concave function and $\sigma = 1/(1+\rho) \geq 0$ is the elasticity of substitution. These assumptions imply a money demand function

$$m_t^d = \frac{c_t}{1 + (\alpha/(1-\alpha))^{\sigma} \cdot (1+i)^{\sigma-1}}$$

Thus, money demand has a unit consumption elasticity. It has a negative interest elasticity, if and only if the elasticity of substitution σ exceeds one. ||

Overall, there is an exact relationship between money, consumption, and an interest rate that can be interpreted as money demand function. Specifically, a short-term, nominal, riskfree interest rate enters, but no other asset returns.

4. A Partial Equilibrium Approach to Asset Pricing

There are two ways to approach asset pricing. First, prices of all securities are characterized by individuals' first order conditions. Second, one may specify stochastic processes for all exogenous variables and solve for assets prices in general equilibrium. The general equilibrium results necessarily depend on the complete model, while the Euler equations rely only on the specification of consumption. To focus on the more robust implications of the model, this section will exploit the first order conditions (8) and (9) without relying on other assumptions about the macroeconomic structure. The next section comments on general equilibrium issues.

The first insight follows from a simple observation: The Euler conditions (9) look exactly like first order conditions of multi-good asset pricing models without money. As in other multi-good models, first order conditions for asset prices can be written in terms of the derivative of utility with respect to any one consumption good (with returns expressed in terms of its price).

Here the credit good is used (with $p_{1t} = p_t$), but one could rewrite the first order condition in terms of marginal utility and price of the money good, $u_2(\cdot)$ and p_{2t}. Thus, money does not necessarily upset standard asset pricing theory (c.f. Townsend (1987) and Lucas (1984)[15]).

As in other multi-good models, one cannot write the first order conditions in terms of aggregate consumption alone, except under restrictive assumptions.[16] But they can be rewritten in terms of an indirect utility function over total real consumption c_t (or endowments y_t) and the interest rate i_t, exploiting the special role of the nominal interest rate as asset return and relative price, as follows.

In equilibrium, real consumption c_t and the interest rate i_t determine c_{1t} and c_{2t} from equations (8) and (14). An indirect utility function $U(\cdot)$ is defined by substituting these relations into $u(\cdot)$,[17] i.e.

$$U(c_t, i_t) = u(c_{1t}(c_t, i_t), c_{2t}(c_t, i_t)) \ . \tag{16}$$

Straightforward calculus shows that the partial derivative of $U(\cdot)$ with respect to c_t, denoted by $U_c(c_t, i_t)$, is equal to the marginal utility $u_1(c_{1t}, c_{2t})$ that appears in the Euler equations. Thus, the first order conditions (9) are equivalent to

$$E_t\left[\frac{\beta \cdot U_c(c_{t+1}, i_{t+1})}{U_c(c_t, i_t)} \cdot r_{kt+1}\right] = 1, \quad \forall k \ . \tag{17}$$

Financial assets can be priced off equation (17) in the usual way. A risk-free real asset must offer a real return of $\frac{1}{\beta} \cdot U_c(c_t, i_t) / E_t[U_c(c_{t+1}, i_{t+1})]$. Return premia on risky assets depend on the covariance of real returns with the indirect marginal rate of substitution, $U_c(c_{t+1}, i_{t+1}) / U_c(c_t, i_t)$.

Compared to barter-economy asset pricing characterized by equation (1), the only difference is that the interest rate appears as an argument in indirect marginal utility. The nominal interest rate plays a special role, because the need for money distorts the choice of consumption goods. Therefore marginal utility U_c does not just decline with higher consumption (one can verify $U_{cc} < 0$), but also changes with the nominal interest rate, depending on the sign of U_{ci}. I will summarize this result as a theorem and then turn to some illustrative examples:

Theorem: Asset prices are characterized by the first order conditions (17).

Example 2: CES-utility, general risk preferences: If preferences are characterized by equation (15), the indirect utility function is multiplicative in consumption and interest rates and can be written as

$$U(c_t, i_t) = v(c_t \cdot f(i_t)) , \quad \text{where}$$

$$f(i_t) = \left[\alpha^{\sigma} + (1-\alpha)^{\sigma} \cdot (1+i_t)^{-(\sigma-1)}\right]^{\frac{1}{(\sigma-1)}} . \tag{18}$$

Notice that $f'(i_t) < 0$ and that $(\sigma - 1) > 0$ is required for negative interest elasticity of money demand. In addition, one can compute

$$U_c(c_t, i_t) = f(i_t) \cdot v'(c_t \cdot f(i_t)) > 0 ,$$

$$U_i(c_t, i_t) = f'(i_t) \cdot c_t \cdot v'(c_t \cdot f(i_t)) < 0 ,$$

$$U_{cc}(c_t, i_t) = f(i_t)^2 \cdot v''(c_t \cdot f(i_t)) < 0 , \quad \text{and}$$

$$U_{ci}(c_t, i_t) = f'(i_t) \cdot (1-\delta) \cdot v'(c_t \cdot f(i_t)) ,$$

where δ is the (not necessarily constant) degree of relative risk aversion. Thus, $U_{ci} > 0$ if and only if $\delta > 1$. ||

Intuitively, high interest rates reduce absolute utility, since they increase the cost of money goods. The reason why δ matters is that, with time-separable utility, a risk aversion parameter $\delta > 1$ is equivalent to an elasticity of intertemporal substitution below one (see Hall (1988)). In case of low intertemporal substitution ($\delta > 1$), consumers respond to a high price of money goods (i.e., a high interest rate) by increasing current consumption spending; the opposite happens in case of high elasticity of substitution.

Example 2 suggests that money does not complicate asset pricing at all in the case of log-utility, which is so widely used in financial economics:

Example 3: CES-utility, logarithmic intertemporal preferences: This is the case of $\delta = 1$, $U_{ci} = 0$ in the previous example. Since $v'(x) = 1/x$, $U_c(c_t, i_t) = 1/c_t$ which does not involve the interest rate. Equation (17) is equivalent to equation (1). ||

If utility is not logarithmic, quantitative differences between asset pricing in a monetary versus barter economy depend on the volatility of interest rates and on the parameters of $u(\cdot)$. To see the approximate effect of money, consider the class of CRRA-utility functions:

Example 4: CES-utility with constant relative risk aversion: This is the case of $v(x) = x^{1-\delta}/(1-\delta)$ in Example 2. The logarithm of the marginal rate of substitution, $U_c(c_{t+1}, i_{t+1})/U_c(c_t, i_t) = (c_{t+1}/c_t)^{-\delta} \cdot (f(i_{t+1})/f(i_t))^{1-\delta}$, can be approximated by

$$\log\left[\frac{U_c(c_{t+1}, i_{t+1})}{U_c(c_t, i_t)}\right] \cong -\delta \cdot \log\left[\frac{c_{t+1}}{c_t}\right] - (1-\delta) \cdot g(\alpha, \sigma) \cdot \log\left[\frac{1+i_{t+1}}{1+i_t}\right], \qquad (19)$$

where $g(\alpha, \sigma) = ((1-\alpha)/\alpha)^{\sigma}$. ||

Notice that interest rate changes have a minor effect, if goods 1 and 2 are close substitutes, if "most" goods are purchased without money or if interest rates vary little. That is, $g(\alpha, \sigma)$ is small, if σ is large or if α is close to one. Moreover, if monetary authorities smooth short-term nominal interest rates, as they often do (with the exception of distinct periods like 1979-82), the difference to barter-economy asset pricing may be negligible for a wide range of preferences.

Assuming that consumption and returns (including $1+i_t$) are jointly log-normal,[18] equation (19) and (17) imply a testable asset pricing model. It differs from barter economy models (see, e.g., Hansen and Singleton (1983) and Mankiw and Shapiro (1986)) by giving nominal interest rates a role in addition to consumption.

It is ultimately an empirical question whether the rate of change in short-term nominal interest rates adds significant explanatory power. If the answer is yes, asset pricing with money may improve the performance of consumption-based asset pricing models. But the preceding paragraphs suggest that the interest rate effect is weak.[19]

As a simple test, I replicated Mankiw and Shapiro's tests of capital asset pricing (CAPM) models. Their objective was to test the significance of the market-beta and the consumption-beta for stock returns. As suggested by equation (19), I add an "interest-rate beta," defined as the covariance of the three-month T-bill rate with a stock return dividend by the covariance with the market portfolio, to the basic CAPM regressions.

The results are displayed in Table 1.[20] Regressions 1 - 3 estimate the basic models (CAPM, consumption-CAPM, and a combined model, respectively) and yield results similar to those obtained by Mankiw and Shapiro (1986). Regressions 4 - 6 add interest rate changes.[21] One can see that the interest rate beta has a significant effect in the consumption model (Regression 5). Its negative sign is consistent with an elasticity of substitution $\delta > 1$. The economic significance of the interest rate variable, however, is minor:

(1) The simple market model still appears to be far superior to the consumption model even when the interest rate is added (Compare the standard errors of regressions 1 and 5 and note the insignificance of the consumption-beta in regressions 3 and 6).

(2) Adding the interest rate beta leaves the standard error of the market model essentially unchanged (Compare regression 4 versus 1).[22]

Thus, there is good news and bad news: The results suggest that simple barter-economy Euler equations can be applied even in the presence of money (as approximations that ignore the interest rate effect). But there is little hope that monetary arguments can improve asset pricing models in an economically significant way.

Many of the simplifying assumptions on the macroeconomic environment that were made in Section 2 are clearly not essential for equations (17) and (19). Whenever i_t and c_t follow some known (estimated) stochastic processes, return premia can be determined immediately from these equations. This may justify the partial equilibrium interpretation. On the other hand, i_t and c_t

are endogenous variables that can be determined from assumptions on basic driving variables. This is done in the next section.

5. Asset Pricing in General Equilibrium

In the exchange economy described in Section 2, endowments are an exogenous variable, but consumption is not.[23] The asset pricing equations (9) and (17) can be rewritten in terms of endowments as follows.

Equations (3) and (8) imply that c_{1t} and c_{2t} are functions of y_t and i_t. Marginal utility $u_1(c_{1t}, c_{2t})$ in equation (9) can therefore be replaced by a function $u*(y_t, i_t) = u_1(c_{1t}(y_t, i_t), c_{2t}(y_t, i_t))$. It has a negative partial derivative u_y^* with respect to endowments y_t. Analogous to (17), return premia are determined by

$$E_t\left[\frac{\beta \cdot u*(y_{t+1}, i_{t+1})}{u*(y_t, i_t)} \cdot r_{kt+1}\right] = 1, \quad \forall k . \tag{20}$$

An example may be helpful to illustrate the similarity of this asset pricing equation to the Euler equation (17):

Example 5: CES-utility, general risk preference: With the CES-utility defined in equation (15), the function $u*(y_t, i_t) = f*(i_t) \cdot v(y_t \cdot f*(i_t))$ has the same functional form as $U_c(\cdot)$, except that $f(i_t)$ defined in (18) is replaced by

$$f*(i_t) = (1-\alpha)^{-1/\rho} \cdot \frac{\left[1 + (\alpha/(1-\alpha))^{\sigma} \cdot (1+i_t)^{\sigma-1}\right]^{-1/\rho}}{1 + (\alpha/(1-\alpha))^{\sigma} \cdot (1+i_t)^{\sigma}} . \tag{21}$$

If the first "1" in the denominator were replaced by "$1 + i_t$," this would be equivalent to $f(\cdot)$. Unless the absolute level of interest rates is large, the differences between $U_c(\cdot)$ and $u*(\cdot)$, $f(\cdot)$ and $f*(\cdot)$, and y_t and c_t are minor. ||

Given endowments, the procedure for valuing assets depends on the type of monetary policy. If monetary policy is formulated in terms of money stocks, the paths of nominal interest rates and prices are determined jointly by the first order condition

$$E_t\left[\frac{\beta \cdot u*(y_{t+1}, i_{t+1})}{u*(y_t, i_t)} \cdot (1+i_t) \cdot \frac{p_t}{p_{t+1}}\right] = 1 \tag{22}$$

(which is equation (20) for asset k = 0) and the money market equilibrium condition

$$\frac{M_t}{p_t} = (1+i_t) \cdot c_{2t}(y_t, i_t) \ . \tag{23}$$

To obtain solutions, assumptions on the exogenous paths of money supplies and endowments would have to be made. Asset pricing then proceeds in two steps: First, prices and short-term nominal interest rates are determined in (22) and (23). Then, returns on other assets are determined from equation (20) evaluated at the computed levels of (y_t, i_t).

If monetary policy is formulated in terms of nominal interest rates, however, the procedure simplifies: Given the target path i_t, all asset prices can be determined immediately from the first order conditions (20). The path of money supplies and prices required to implement this path of interest rates can then be obtained from (22) and (23). If monetary policy is interpreted in this way, solving (22) and (23) will be unnecessary.[24]

6. Summary

The paper has shown how to write a cash-in-advance model of money demand in a way that real money demand is a function of variables are empirically important: A nominal interest rate enters as the opportunity cost and aggregate consumption enters as a scale variable. This model of money is consistent with asset pricing equations that resemble the Euler equations familiar from barter economies. The analysis suggests that simple barter-economy Euler equations are approximately valid even in the presence of money.

Footnotes

[1]See, e.g., Clower (1967), Lucas (1984), Svensson (1985), Lucas and Stokey (1987), Townsend (1987), Cole and Stockman (1988), Lucas (1988), Helpman and Razin (1984) and the references therein. The main alternatives of cash-in-advance are money in the utility function, overlapping generation models and "shopping time" models of the type considered, e.g., by Kimbrough (1986). Money in utility seems inherently implausible, because a piece of paper called money is clearly not wanted as a final good (though the analysis would be formally similar to cash-in-advance). Overlapping generations models, in which money is a store of value, have difficulties in explaining why money exists in the presence of interest bearing nominal assets. In contrast, the cash-in-advance approach is consistent with financial assets that dominate money as a store of value. A comparison to "shopping-time" models is left for future research, but preliminary results indicate that this approach would lead to asset pricing equations similar to those obtained here.

[2]See, e.g., Rubinstein (1976), Lucas (1978), Breeden (1979), Hansen and Singleton (1983), and Breeden, Gibbons, and Litzenberger (1989).

[3]See, e.g., Mankiw and Shapiro (1986), Hansen and Singleton (1983), and Mehra and Prescott (1985).

[4]For example, property (2) would be satisfied if only a fraction of all purchases are made with money, where the fraction is determined by some transactions technology that varies exogenously over time.

[5]The key characteristic of good 1 is that there is no gap between payment by the buyer and use of the receipts by the seller. Given that, date and means of payment are irrelevant. Here, immediate payment and receipt of payment is assumed. Lucas (1984) assumes payment in the next period (credit), which implicitly includes an interest charge. In present value terms there is no difference, provided buyers and sellers use the same discount rate.

[6]Without loss of generality, one may assume that c_{1t}, and c_{2t} are the same good from the perspective of production and only distinguished by the time dimension. One could also model production and add a labor-leisure choice for individuals, but that would add unnecessary notation.

[7]If $i_t = 0$, money would be a perfect substitute for nominal bonds. Equation (8) below would still determine the relative demand for c_{1t} and c_{2t}. Equation (9) for $k = 0$ would determine the demand for $(X_{0t} + M_t)$, but the demand for money demand by itself would become indeterminate, subject only to equation (5).

[8]Also, the model has only one credit and one money-good. But extensions to more than one money and credit good are straightforward.

[9]See Lucas (1988, section III) for a related discussion of money demand analysis.

[10]The use of consumption as scale variable is consistent with extensions of the model in which other sectors of the economy do not need money. Otherwise, the sum of consumption and variables characterizing the volume of monetary transactions in the other sectors would have to be used.

[11]Optimal monetary policy would of course set $i_t = 0$; but monetary policy is considered exogenous.

[12]Equilibrium requires $X_{kt} = 0$ for all $k \geq 1$, $X_{0t} = N_t = M_t/(1+i_t)$, and $R_{0t} \cdot X_{0t-1} = M_{t-1} = M^s_{t-1}$. Hence, the right hand side of (6) can be rewritten as follows: $(\omega_t - 1) \cdot M_{t-1}/p_t + \Sigma_k (R_{kt} \cdot X_{kt-1} - Q_{kt} \cdot X_{kt})/p_t = M_t/p_t/(1+i_t) - \omega_t \cdot M_{t-1}/p_t = i_t/(1+i_t) \cdot M_t/p_t = i_t \cdot c_{2t}$.

[13]Any other price index could be used, provided real money and consumption are defined with the same index. Even if different indices of p_{1t} and p_{2t} were used, the variables determining real money demand would not change, since any price index consisting of p_{1t} and p_{2t} would only involve i_t.

[14]This is obtained by taking the total differential of (8) and (14).

[15]Lucas writes that "... modifications are required for a monetary system, so that pricing formulas differ in important ways from the barter versions..." (1984, p. 10). Here, the only difference is that the relative price of goods involves an interest rate.

[16]A fixed weight price index exists only under the assumption of Cobb-Douglas utility, which has a unit elasticity of substitution. This assumption is too strong for our purposes, because it implies a zero interest elasticity of money demand, see Example 1.

[17]The term indirect utility is used to highlight the role of i_t even though $U(\cdot)$ still involves a choice variable, c_t.

[18]These are assumptions commonly made in non-monetary models; I will not attempt to justify them.

[19]In terms of equation (19), one would have to estimate a value for $g(\alpha, \sigma)$. But even if the effect is statistically significant, the improvement may be small in periods of stable short-term interest rates.

[20]See Mankiw and Shapiro (1986) for methodological details. The sample consists of 365 firms with complete return data on the CRSP file from 1959 to 1987. Coefficients are estimated by generalized least squares (GLS), assuming that excess returns have one common factor plus uncorrelated components. Analogous regressions were run with OLS and weighted least squares (WLS) for 1959-87 and with GLS, OLS, and WLS for 1959-82 (which is Mankiw and Shapiro's sample period). The results are very similar to those in Table 1 and therefore not reported; details are available from the author.

[21]Since market returns do not appear in equations (1) and (17), one may question the theoretical justification for regressions 1, 3, 4, and 6. But if consumption data contain measure-

ment error, the market return may be interpreted as a proxy for consumption services. In addition, Giovanni and Weil (1988) show that the market return has a role in models with more general preferences.

[22]Moreover, one might argue that even a statistically significant interest rate effect does not prove that interest rates enter the marginal rate of substitution, since the interest rate effect could also be interpreted in terms of an arbitrage pricing model (see Roll (1976), Chen, Roll, and Ross (1986)). But a detailed discussion of the APT-model in this context would be beyond the scope of this paper.

[23]As explained in Section 3, c_t differs from y_t by $i_t \cdot c_{2t}$, which is probably a small quantity in most countries. But the analysis suggests how the results might generalize to models where the difference is large, e.g., those with investment or government spending.

[24]In contrast, Svensson's (1985) prices assets in terms of shadow prices (Lagrange multipliers), which do not have obvious empirical counterparts.

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Appendix: Pricing of Cash Goods

This appendix shows that the relative price of money and credit goods is $1 + i_t$. A proof is provided because this feature of the model is distinctive (cf. Lucas (1984), Svennson (1985)) and critical for the simplicity of the money demand and asset pricing equations.

Since a Modigliani-Miller theorem applies, one can assume a specific, simple dividend policy without loss of generality: Assume that firms pay out their present value in every period so that the ex-dividend value of shares is zero. (If the firm followed a different dividend policy, its value might be non-zero, but the following argument would remain unchanged.) Since the firm has no asset except for the cash received for money goods, the policy of paying out all earnings will involve a short position in nominal bonds.

If a firm issues N_t nominal bonds in period t, its dividend in period t is

$$D_t = p_{1t} \cdot c_{1t} + p_{2t-1} \cdot c_{2t-1} - p_t \cdot y_t + N_t - (1 + i_{t-1}) \cdot N_{t-1} ,$$

because it carries $M_{t-1} = p_{2t-1} \cdot c_{2t-1}$ and $M_t = p_{2t} \cdot c_{2t}$ in money between periods. Now suppose the firm issues exactly

$$N_t = \frac{c_{2t} \cdot p_{2t}}{(1 + i_t)} = \frac{M_t}{(1 + i_t)}$$

nominal bonds when it expects to sell c_{2t} goods later in the period. The same policy is pursued every period. Dividends are (using (3) as equality)

$$D_t = (p_{1t} - p_t) \cdot c_{1t} + [p_{2t-1} - p_{1t-1} \cdot (1 + i_{t-1})] \cdot c_{2t-1} + [p_{2t}/(1 + i_t) - p_t] \cdot c_{2t}$$

every period. Thus, competitive prices must satisfy

$$p_{1t} = p_t, \quad \text{and} \quad p_{2t} = (1 + i_t) \cdot p_t , \tag{3}$$

which implies zero dividends. Essentially, firms sell as many nominal bonds as necessary to cancel out their holdings of money. Because of the linear technology and since no uncertainty is realized between the time the bonds are sold and the time the money is received, firms' net worth is always zero.

Table 1: Cross-Sectional Regressions, GLS-Estimates, 1959-87

Regression No.	Coefficient on beta			SEE
	Market	Consumption	Interest Rates	
1	5.40 (9.9)			2.457
2		1.68 (5.4)		2.665
3	5.13 (8.0)	0.27 (0.8)		2.458
4	5.21 (9.3)		-0.19 (1.3)	2.455
5		1.75 (5.7)	-0.58 (3.9)	2.615
6	4.78 (7.1)	0.39 (1.1)	-0.23 (1.5)	2.454

Legend: |t|- statistics are in parentheses, SEE indicates the standard error of the regression.