BIRTH, DEATH AND TAXES

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Abstract

This paper analyzes the effects of lump-sum tax policy in an overlapping generations model in which consumers have uncertain longevity. It extends previous analyses by considering the case in which private insurance arrangements are actuarially unfair. In addition, it considers the polar case of actuarially fair insurance and the polar case of no insurance. A general condition for debt neutrality is derived. This condition depends explicitly on the degree of actuarial unfairness in insurance and on the extent to which parents care about the utility of their children.

One of the motivations underlying Diamond's (1965) classic presentation of the overlapping generations model with capital accumulation was to develop a competitive general equilibrium model with optimizing consumers in which lump-sum tax policy has real effects. Recently, Blanchard (1985) has cast the overlapping generations model in a continuous time framework in order to be able to use familiar continuous time methods to analyze dynamic effects of fiscal behavior in an economy in which lump-sum tax policy has real effects. Blanchard assumed that the instantaneous probability of dying is constant over time and across consumers and, furthermore, that new consumers are born at the same rate at which consumers die. He then demonstrated that changes in the timing of lump-sum taxes have real effects in this model.

In explaining why lump-sum tax policy has real effects, Blanchard emphasized the fact that individual consumers have finite horizons. However, Weil (1985) has argued that the efficacy of tax policy in Blanchard's model is a consequence of the assumption that new consumers will be born rather than the assumption that some currently-living consumers will die before all the extra bonds are redeemed. Weil illustrated this proposition by constructing a model in which consumers live forever and new consumers are born at each instant of time. By breaking the equality between the birth rate and the death rate that was imposed in Blanchard's formulation, Weil was able to disentangle the role of finite horizons from the role of the birth of new consumers.

Weil demonstrated that it is not necessary for individual consumers to have finite lifetimes in order for lump-sum tax policy to have real effects. However, his analysis invites the question of whether the assumption of finite horizons is sufficient for lump-sum fiscal policy to have real effects. This question was addressed by Buiter (1986) who further generalized the Blanchard model by separately specifying a nonnegative birth rate and a nonnegative probability of death. Blanchard's model is a special case of Buiter's model in which the birth rate and the death rate are assumed to be equal. Weil's model is a special case of Buiter's model in which the death rate is assumed to be equal to zero. In his extended model, Buiter showed that debt neutrality (Ricardian Equivalence) holds if and only if the probability of death plus the population growth rate is equal to zero. An alternative statement of Buiter's result is that debt neutrality holds if and only if the birth rate is equal to zero.

In this paper I extend the literature cited above in two directions. First, all of the results derived by Blanchard, Weil and Buiter are based on the assumption that there is a competitive insurance market in which consumers can buy actuarially fair annuities and life insurance. I extend the analysis to examine the effects of tax policy when insurance is not actuarially fair, without explicitly modeling the source of the imperfections in the annuity market. The classic source of imperfection is adverse selection, but I have analyzed the implications of adverse selection on tax policy elsewhere (Abel (1986)) and will not appeal to adverse selection here. Perhaps the simplest assumption is that there are administrative costs of servicing life insurance and annuities and that these costs are proportional to the size of the insurance contract. While this assumption is quite strong, it has the virtue of rigorously justifying actuarially unfair rates of return on annuities and life insurance in a competitive environment, without introducing ancillary assumptions that distract attention from the main point. If administrative costs are sufficiently high, then annuities or life insurance will not be purchased by consumers. This case, without insurance contracts, is particularly interesting because it leads to debt neutrality under Blanchard's demographic assumptions.

The second major extension is to analyze the effects of tax policy in the case in which consumers have altruistic bequest motives toward some, but perhaps not all, of their children. Although this case was examined by Weil, his analysis does not allow for consumers to die and is restricted to the case with actuarially fair insurance markets. By extending the model to include selectively altruistic consumers and imperfect insurance markets, I can derive more general conditions for debt neutrality.

The conditions for debt neutrality depend critically on the nature of insurance markets. In the presence of actuarially fair annuities, debt neutrality holds if and only if the birth rate of "disinherited" consumers is equal to zero. In the absence of annuities, debt neutrality holds if and only if the birth rate of disinherited consumers is equal to the death rate. A more general condition for debt neutrality in the presence of actuarially unfair annuities is presented in Proposition 6.

The analysis begins in Section 1 with a model of saving behavior of consumers who live for either one period or two periods. These consumers are selfish in the sense that they have no bequest motive. Section 2 presents the government's budget constraint, and the effects of changes in the timing of lump-sum taxes are analyzed in Section 3. Section 4 describes the distinction between finite lifetimes and uncertain lifetimes and briefly discusses the implications of this distinction. The model is then generalized in Section 5 to allow consumers to have selectively altruistic bequest motives. More precisely, consumers are assumed to be altruistic toward some, but perhaps not all, of their children. Concluding remarks are presented in Section 6.

1. A Model with Selfish Consumers

Consider an economy that exists for two periods denoted as period 1 and period 2. Consumers who are born at the beginning of period 1 will be called parents, and consumers who are born at the beginning of period 2 will be called children. Each parent gives birth to n children at the beginning period 2. Then each of these parents faces a probability $p > 0$ of dying immediately after the children are born and a probability $1 - p > 0$ of living and consuming during the second period. Let c_1 denote the consumption of the parent in period 1 and c_2 denote the consumption of the parent in period 2 if he survives to period 2. The utility function of a representative parent is

$$u(c_1) + (1 - p)v(c_2) \tag{1}$$

where $u' > 0$, $u'' < 0$, $v' > 0$, $v'' < 0$. The utility function in (1) is based on the Yaari (1965) formulation of the utility function of a consumer with an uncertain length of life. Observe that the utility function in (1) does not include a bequest motive.

Suppose that each parent inelastically supplies one unit of labor in period 1 for which he receives a wage income of w_1. If the parent survives to period 2, he is retired. Let T_1 be a head tax levied on all consumers alive in period 1, and let T_2 be a head tax levied on all consumers alive in period 2. The saving of a parent at the end of period 1 is equal to $w_1 - T_1 - c_1$. Letting Q denote the gross rate of return on saving, the consumption of the parent in the second period is

$$c_2 = (w_1 - T_1 - c_1)Q - T_2 \, . \tag{2}$$

Suppose that there is a (linear) riskless technology that converts one unit of the consumption good in period 1 into R units of the consumption good in period 2. Thus, R is the gross riskless rate of return. The rate of return Q depends on the riskless rate of return and the structure of capital markets. If a perfect annuity market exists, then, as argued by Yaari, consumers will choose to hold their entire portfolios in annuities which pay the actuarially fair gross rate of return $R/(1 - p)$. In this case, $Q = R/(1 - p)$. Alternatively, if, for some reason, there is no market for annuities and the only asset available to consumers is the riskless asset, then $Q = R$.[1] An intermediate case is one in which $R < Q < R/(1 - p)$ so that annuities are actuarially unfair. This case would arise if there were proportional administrative costs for insurance contracts. At this point, I will not restrict attention to a particular case and will simply treat Q as given parametrically to individual consumers.

In period 1 each parent maximizes the utility function in (1) subject to the constraint in (2). The first-order condition for this optimization problem is

$$u'(c_1) = (1 - p)Qv'\big((w_1 - T_1 - c_1)Q - T_2\big) . \tag{3}$$

To interpret (3) consider reducing c_1 by one unit and increasing saving by one unit. The reduction in c_1 decreases first-period utility by $u'(c_1)$. The one unit increase in saving can be used to increase c_2 by Q units which increases utility by $Qv'(c_2)$ if the consumer survives to period 2. However, the consumer has only a $1 - p$ chance of surviving to the second period, so the

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[1] In the absence of annuities, there is a question about what happens to the wealth held by parents who die young. A natural assumption is that their children inherit this wealth. See Abel (1985) for an analysis of fiscal policy in an overlapping generations model in which children receive such accidental bequests. Because the model presented in the text focusses on the contemporaneous effect of a change in the first-period tax on consumption, there is no need to analyze the impact of these accidental inheritances on the behavior of the subsequent generation(s).

increase in expected utility associated with the increased saving is $(1 - p)Qv'(c_2)$. Equation (3) sets the utility loss associated with decreased first-period consumption equal to the expected utility gain from increased second-period consumption.

2. The Government's Budget Constraint

Let G be the present value of the government's purchases of goods in periods 1 and 2. The government pays for these goods by levying lump-sum taxes in periods 1 and 2. It is convenient to normalize the number of parents born in period 1 to be equal to one. With this normalization there are 1-p+n consumers alive in period 2. Recalling that T_i is the tax per person in period i, the government's budget constraint is

$$RT_1 + (1 - p + n)T_2 = RG . \tag{4}$$

I will limit attention to fiscal policy changes that leave G unchanged. In particular, I will allow the taxes T_1 and T_2 to change subject to the government's budget constraint in (4). Letting dT_i denote the change in the head tax in period i, the government's budget constraint implies that

$$RdT_1 + (1 - p + n)dT_2 = 0 . \tag{5}$$

3. The Effects of a Tax Policy Change

In this section, I examine the effect on first-period consumption of a tax change that satisfies the government budget constraint. Recall that equation (3) implicitly gives the optimal value of c_1. Totally differentiating (3) with respect to c_1, T_1 and T_2 yields

$$[u'' + (1 - p)Q^2v'']dc_1 = -(1 - p)Qv''[QdT_1 + dT_2] . \tag{6}$$

Using (5) to eliminate dT_2 from (6) yields

$$dc_1/dT_1 = -H\left[Q/R - \frac{1}{(1 - p + n)}\right] \tag{7}$$

where

$$H \equiv \frac{(1 - p)QRv''}{\left[u'' + (1 - p)Q^2v''\right]} > 0 \ .$$

Several observations follow from equation (7). First, if there is a perfect annuity market, which offers actuarially fair annuities, then $Q = R/(1 - p)$, and (if $Q = R/(1 - p)$)

$$dc_1/dT_1 = \frac{-Hn}{[(1 - p)(1 - p + n)]} \ . \tag{8}$$

Equation (8) can be used to interpret results in the Blanchard model as well as Weil's extension. In the Blanchard model, the birth rate is equal to the death rate, which in the notation above means that $p = n$. It follows immediately from (8) that if $0 < p = n < 1$, then $dc_1/dT_1 = -Hp/(1 - p) < 0$ and, hence, a change in lump-sum tax policy has an affect on consumption. In particular, an increase in the first-period head tax reduces the contemporaneous level of consumption. In Weil's extension of the Blanchard model, the death rate is set equal to zero. Setting p equal to zero in (8) yields $dc_1/dT_1 = -Hn/(1 + n)$ which is negative if and only if n if positive. Thus, as shown by Weil, a positive value of p is not necessary for tax policy to have an effect. To determine whether a positive value of p is sufficient for tax policy to have an effect, set n equal to zero and observe from (8) that dc_1/dT_1 is zero even if p is positive. These results can be summarized as

<u>Proposition 1</u>: In the presence of actuarially fair annuities, a positive value of p is neither necessary nor sufficient for tax policy to have an effect.

Proposition 2: In the presence of actuarially fair annuities, a positive value of n is both necessary and sufficient for tax policy to have an effect.

The expression in (7) for the effect on consumption of a change in tax policy is not restricted to the case of perfect annuity markets. In general, Q is the rate of return on the consumer's saving and could differ from the actuarially fair rate of return if costs associated with administering annuities are present. A particularly interesting example of a departure from perfect annuity markets is the case in which no annuities are available.[2] In this case, the rate of return on saving, Q, is equal to the riskless rate of return R and, hence, the effect of tax policy is given by

$$dc_1/dT_1 = H \frac{p - n}{1 - p + n} \tag{9}$$

(if $Q = R$). Recalling that $H > 0$ and $1 - p + n > 0$ immediately yields

Proposition 3: In the absence of annuities, dc_1/dT_1 has the same sign as $p - n$.

Corollary: If $p = n$ and if there are no annuities, then debt is neutral.

The assumption that $p = n$, which is made in the Corollary, is identical to Blanchard's demographic assumption. The corollary states the remarkable result that in the absence of annuities, and in the absence of a bequest motive, the demographic assumptions of the Blanchard model imply that the Ricardian Equivalence Theorem holds. This result may strike some readers as ironic since the Ricardian Equivalence Theorem is typically derived in the context of perfect capital markets and altruistic consumers. Furthermore, this proposition is typically defended by appealing to a lack of compelling

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[2]Need text for footnote.

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reasons to argue against perfect capital markets. However, if we dispense with the assumption that consumers have altruistic bequest motives, then the Ricardian Equivalence Theorem holds only in the absence of an annuity market; conversely, it fails to hold in the presence of a perfect annuity market.

In the absence of annuities, the Ricardian Equivalence Theorem holds only when the birth rate is equal to the death rate, as Blanchard assumed. If the birth rate exceeds the death rate, as in Weil, then a tax increase reduces contemporaneous consumption. Alternatively, if the birth rate falls short of the death rate, then, in the absence of annuities, a tax increase raises contemporaneous consumption.

Propositions 1, 2, and 3 are all based on equation (7) which indicates that Ricardian Equivalence holds if and only if $Q = R/(1 - p + n)$. This result can be easily interpreted by considering a one dollar reduction in aggregate taxes in period 1. This tax cut must be offset by an R dollar increase in aggregate taxes in period 2. Now consider a representative parent who receives a one dollar tax cut in period 1 (normalizing the population of the parents' cohort to be equal to one). If he saves this dollar, he will have Q additional dollars in period 2. If his second-period taxes are increased by Q dollars, then the consumer can use the Q additional dollars from his additional saving to pay the higher tax bill; thus, the consumer will be able to maintain the original sequence of consumption over his lifetime and will choose to do so. However, if the additional second-period taxes are less than Q dollars, then the parent's lifetime disposable resources are increased, and he will increase his consumption when he is young. By contrast, if the additional second-period taxes are greater than Q, then the consumer will reduce his consumption when he is young. Thus, the parent's response to the tax cut in the first period depends on whether the increase in his second period taxes is greater than, less than, or equal to Q. The increase in his second-period taxes will be equal to the aggregate tax increase, R, divided by the number of consumers alive in the second period, $1 - p + n$. Thus, the parent's response to the first-period tax cut depends on whether Q is greater than, less than, or equal to $R/(1 - p + n)$, just as indicated by equation (7).

4. Finite Lifetimes vs. Uncertain Lifetimes

I have shown above that a positive value of p is neither a necessary nor a sufficient condition for tax policy to have an effect. One may be tempted to interpret this result as implying that finite horizons are irrelevant to the question of Ricardian Equivalence. However, this interpretation is unwarranted for the following reason: If a consumer is certain to die before the final change in taxes, i.e., if $p = 1$ rather than $0 < p < 1$, then lump-sum tax policy will have an effect. Indeed, in the two-period economy in this paper, a parent who knows that he will live for only one period consumes all of his disposable resources in period 1; he reduces his consumption one-for-one with increases in first-period taxes.

5. Altruism and the Effects of Tax Policy

In Barro's (1974) classic presentation of the Ricardian Equivalence Theorem, consumers are assumed to be effectively immortal as a result of operative bequest motives. More precisely, Barro assumed that each consumer obtains utility from the utility of his heirs as well as from his own consumption. This recursive specification of preferences implies that each consumer cares, at least indirectly, about the entire path of current and future consumption of the members of his dynastic family. The analysis in previous sections of this paper has assumed that consumers are entirely selfish and, hence, have no bequest motive. I will now relax this assumption and assume that consumers are selectively altruistic. More precisely, each parent altruistically cares about the utility of m of his n children, where $0 \le m \le n$. This formulation nests both the assumption that consumers are selfish ($m = 0$) and the alternative assumption used by Barro that consumers are altruistic with respect to all of their children ($m = n$). In addition, it allows consideration of the case in which consumers care about some, but not all, of their children ($0 < m < n$).

Consider a selectivity altruistic parent born in period 1. The parent has probability p of dying ($0 < p < 1$) at the beginning of period 2 after giving birth to n children. Now suppose that the parent cares altruistically about m of his children, where $0 \le m \le n$. Since m and n may in general differ, I will use the term "heirs" to designate the m children toward whom the consumer is altruistic; the remaining n - m children will be called "disinherited children." Thus, the consumer has m heirs and n children. The utility function of a representative consumer in period 1 is

$$u(c_1) + (1 - p)v(c_2) + pm\delta z_D(x^D) + (1 - p)m\delta z_S(x^S) \qquad (10)$$

where c_i is the consumption of the parent in period i, i = 1, 2, x^D is the consumption of his representative heir if the parent dies at the beginning of period 2, and x^S is the consumption of his representative heir if the parent survives at the beginning of period 2. Suppose that u(), v(), $z_D()$ and $z_S()$ are each strictly increasing and strictly concave. The functions $z_D()$ and $z_S()$ represent the state-contingent utility functions of the representative heir, and $\delta > 0$ measures the strength of the altruistic bequest motive. In line with the assumption that the economy lasts for only two periods, the heirs of the period 1 consumer are alive only for period 2.

Each parent has to choose a portfolio of riskless bonds, which offer a gross rate of return R and annuities, which offer a gross rate of return $Q \geq R$ contingent on the survival of the parent. The restriction $Q \geq R$ is an implication of market equilibrium in the presence of a bequest motive because selectively altruistic parents would strictly prefer bonds to annuities if Q were less than R. Let A denote the amount of annuities that a parent buys in period 1. The remainder of the parent's wealth, $(w_1 - T_1 - c_1 - A)$, is held in the form of riskless bonds. If the parent dies at the beginning of period 2, then his estate is worth $R(w_1 - T_1 - c_1 - A)$ and is divided equally among his m heirs. Let b^D denote the bequest received at the beginning of period 2 by each of his heirs if the parent dies at the beginning of period 2 so that

$$mb^D = R(w_1 - T_1 - c_1 - A) . \qquad (11)$$

If the parent survives during period 2, then he pays a tax T_2. Because all uncertainty is resolved at the beginning of period 2, I assume that even if the parent survives during period 2, he makes his desired bequest, b^S, to each of his heirs at the beginning of period 2 (after learning that he will survive). Observe that if the consumer survives, then

$$mb^S + c_2 = R(w_1 - T_1 - c_1 - A) + QA - T_2 . \qquad (12)$$

The three terms on the right hand side of (12) are the principal and interest on riskless bonds, the principal and interest on annuities, and the lump-sum tax, respectively. Thus, the right hand side represents the disposable second-period resources of the parent if he survives in the second period. The second-period expenditures on consumption and bequests appear on left hand side of (12).

Because the heirs are alive only for period 2, they have a trivial decision problem. They simply consume their entire resources. Each heir works in period 2 and receives wage income w_2. In addition, each heir pays the lump-sum tax T_2. Taking account of the inheritance received by each heir yields

$$x^D = w_2 - T_2 + b^D \ , \tag{13}$$

$$x^S = w_2 - T_2 + b^S \ . \tag{14}$$

The parent maximizes (10) subject to (11) - (14). The Lagrangean associated with this maximization problem and the first-order conditions are presented in Appendix A. Eliminating the Lagrange multipliers, the first-order conditions can be written as

$$u'(c_1) = Q(1 - p)v'(c_2) \tag{15a}$$

$$p\delta z_D'(w_2 - T_2 + b^D) = [Q/R - 1](1 - p)v'(c_2) \tag{15b}$$

$$\delta z_S'(w_2 - T_2 + b^S) = v'(c_2) \ . \tag{15c}$$

These three equations are easily interpreted by considering various small changes to the optimal consumption and portfolio decisions. First, consider a unit decrease in c_1 that is used to buy an additional unit of annuity. The reduction in c_1 reduces first-period utility by $u'(c_1)$, which appears on the left hand side of (15a). However, the additional annuity can increase second-period consumption by Q units if the consumer survives. The expected increase in second-period utility is $(1-p)Qv'(c_2)$, which appears on the right hand side of (15a).

Second, consider a reduction in the consumer's holding of annuities by 1/R units accompanied by an increase in his holding of riskless bonds by 1/R units. The increase in the holding of riskless bonds allows the consumer to increase his bequest by one unit if he dies at the beginning of the second period. The expected increment to utility from this increased bequest appears on the left hand side of (15b). If the consumer survives, this change in the composition of his portfolio will reduce the second-period value of his portfolio by Q/R - 1 units. If the consumer maintains the bequest b^S unchanged, then c_2 will be reduced by Q/R - 1 units. The expected decrease in second-period utility from this decrease in c_2 appears on the right hand side of (15b).

Third, consider a consumer who survives in the second-period. This consumer can reduce his second-period consumption, c_2, by one unit and increase the bequest to each of his m heirs by 1/m units. The decrease in utility associated with the decrease in c_2 is on the right hand side of (15c) and the increase in utility associated with the increased bequests is on the left hand side of (15c).

The first-order conditions in (15a-c) are three equations in the four variables c_1, c_2, b^D, and b^S. To obtain a fourth equation in these four variables, multiply (11) by (Q - R), multiply (12) by R, and add the resulting equations together to obtain the following lifetime budget constraint

$$Qc_1 + c_2 + mb^S + (Q/R - 1)mb^D = Q(w_1 - T_1) - T_2 \ . \qquad (16)$$

The effect of a change in tax policy can be analyzed by totally differentiating equations (15a - c) and (16) with respect to c_1, c_2, b^D, b^S, T_1 and T_2. After this total differentiation, equation (5), which follows from the government's budget constraint, can be used to eliminate dT_2. The details of the comparative statics derivation are straightforward but tedious and, hence, are relegated to Appendix B. The result of this analysis is

$$dc_1/dT_1 = -M\{(1 - p + n - m)Q - R\} \qquad (17)$$

where $M \equiv -\frac{p(1-p)\delta^2 Qv''z''_D z''_S}{(1-p+n)\Delta} > 0$ and $\Delta > 0$ is the determinant of the 4×4 matrix in Appendix B.

Equation (17) describes the effect of tax policy on the consumption of parents in period 1. It applies to perfect capital markets as well as to imperfect capital markets. In the presence of actuarially fair annuities, $Q = R/(1-p)$ and equation (17) implies

Proposition 4: If annuities are actuarially fair, then $dc_1/dT_1 = -(n-m)QM \leq 0$.

Corollary: Under actuarially fair annuities, debt neutrality will hold if and only if parents are altruistic toward all of their children.

Corollary: If actuarially fair annuities are available, and if there are some children toward whom no one is altruistic, then a tax increase in period 1 will reduce contemporaneous consumption.

Proposition 4, which is based on equation (17), applies to the case with actuarially fair annuities. Equation (17) can also be used to analyze the effects of tax policy in the absence of annuities. If there are no annuities, then $Q = R$ and, hence, equation (17) implies

Proposition 5: In the absence of annuities, $dc_1/dT_1 = -(n-m-p)M$, where $M > 0$.

Corollary: If no annuities exist, then debt neutrality holds if and only if the birth rate of disinherited children, $n - m$, is equal to the death rate p.

A more general condition for debt neutrality, which follows directly from (17), is given in

Proposition 6: Debt is neutral if and only if $Q = R/(1-p+n-m)$.

To understand why the Ricardian Equivalence Theorem would hold in this case, consider a one unit tax cut in period 1. The government's budget constraint implies that the head tax T_2 would have to increase by R/(1 - p + n). Now suppose that when the parent receives the tax cut in period 1 he holds m/(1 - p + n) extra bonds, (1 - p + n - m)/(1 - p + n) extra annuities and maintains c_1 unchanged. Then, in period 2 he can divide the extra riskless bonds equally among his m heirs which gives each heir an additional R/(1 - p + n) units of wealth. This additional wealth is just sufficient for each heir to pay the higher head tax in period 2. If the parent survives, then his extra annuity is worth R/(1 - p + n) which is just sufficient to pay the additional head tax. Thus, if Q = R/(1 - p + n - m), then the parent can afford to maintain the initially planned consumption and bequests and will find it optimal to do so.

In general, the Ricardian Equivalence Theorem does not hold, as is evident from (17). If $Q > R/(1 - p + n - m)$, then a tax increase in period 1 reduces contemporaneous consumption. Alternatively, if $Q < R/(1 - p + n - m)$, then a tax increase in period 1 will increase contemporaneous consumption.

6. Concluding Remarks

The model in this paper was developed to disentangle the roles of the death rate and the birth rate in determining the effect of lump-sum tax policy. It is well-known that the Ricardian Equivalence Theorem holds if (a) there are perfect annuity markets; and (b) parents have operative altruistic bequest motives toward all of their children. However, the Ricardian Equivalence Theorem may hold in other situations as well. For instance, if there are no annuities available to individual consumers, the Ricardian Equivalence Theorem will hold if and only if the birth rate of disinherited consumers is equal to the death rate p. This is precisely the demographic assumption used by Blanchard (1985).

The bequest motive in this paper was specified to have two parameters. The parameter m indicates the number of children toward whom each parent is altruistic and the parameter δ indicates the strength of altruism toward each heir. The existing literature has tended to focus on the parameter δ but has essentially ignored the parameter m, perhaps by implicitly assuming that m is equal to the number of children n. The formal results derived in this paper indicate that the question of whether the Ricardian Equivalence Theorem holds depends crucially on the parameter m, but does not depend at all on the parameter δ. The apparent irrelevance of the parameter δ to the question of Ricardian Equivalence is a consequence of having ignored the non-negativity constraint on bequests. If negative bequests are not possible, then, with perfect capital markets, the Ricardian Equivalence Theorem requires that the bequest motive be operative rather than determined by a corner solution. As shown in Weil (1984) and Abel (1987), the bequest motive will be operative if δ is sufficiently large, but will be at a corner solution if δ is sufficiently small.

Recently, Buiter (1986) has shown that the question of Ricardian Equivalence does not depend on the rate of productivity growth. This result is easily demonstrated using the model developed above. One must simply note that no explicit assumptions were made about the labor income of children as compared to the labor income of parents so that the qualitative results do not depend on the presence or absence of productivity growth. However, it should be noted that the rate of productivity growth may be an important determinant of whether the bequest motive is operative or inoperative (see Drazen (1978)). If the rate of productivity growth is very high, then altruistic consumers may desire to leave a negative because their heirs will be much wealthier than they are. In this case, the non-negativity constraint on bequests is strictly binding and, hence, a tax increase reduces contemporaneous consumption.

Appendix A

The parent maximizes the utility function (10) subject to the constraints implied by (11)-(14). This problem can be solved using the Lagrangean expression

$$L = u(c_1) + (1 - p)v(c_2) + pm\delta z_D(w_2 - T_2 + b^D) + (1 - p)m\delta z_S(w_2 - T_2 + b^S)$$
$$+ \lambda^D[R(w_1 - T_1 - c_1 - A) - mb^D] + \lambda^S[R(w_1 - T_1 - c_1 - A) + QA - T_2 - mb^S - c_2] . \tag{A1}$$

Differentiating the Lagrangean in (15) with respect to c_1, c_2, b^D, b^S, and A yields

$$u'(c_1) = (\lambda^D + \lambda^S)R \tag{A2}$$

$$(1 - p)v'(c_2) = \lambda^S \tag{A3}$$

$$pm\delta z'_D(w_2 - T_2 + b^D) = \lambda^D m \tag{A4}$$

$$(1 - p)m\delta z'_S(w_2 - T_2 + b^S) = \lambda^S m \tag{A5}$$

$$\lambda^D R = \lambda^S(Q - R) . \tag{A6}$$

Eliminating the Lagrange multipliers λ^S and λ^D from (A2) - (A6) yields equations (15a - c) in the text.

Appendix B

In this Appendix, I derive equation (17) in the text. Totally differentiate (15a, b, c) and (16) with respect to c_1, c_2, b^S, b^D, T_1 and T_2 to obtain

$$\begin{vmatrix} u'' & -(1-p)Qv'' & 0 & 0 \\ 0 & -(1-p)[\frac{Q}{R}-1]v'' & 0 & p\delta z''_D \\ 0 & -v'' & \delta z''_S & 0 \\ Q & 1 & m & [\frac{Q}{R}-1]m \end{vmatrix} \begin{vmatrix} dc_1 \\ dc_2 \\ db^S \\ db^D \end{vmatrix} = \begin{vmatrix} 0 \\ p\delta z''_D dT_2 \\ \delta z''_S dT_2 \\ -(QdT_1 + dT_2) \end{vmatrix} \quad (B1)$$

Let Ω denote the 4×4 matrix on the left hand side of (B1). Let m_{ij} be the (i, j) minor of Ω and let Δ be the determinant of Ω. Observe that the first row of Ω^{-1} is equal to $\Delta^{-1}[m_{11} \;\; -m_{21} \;\; m_{31} \;\; -m_{41}]$. It follows from (B1) that dc_1 is equal to the first row of Ω^{-1} post multiplied by the column vector on the right hand side of (B1). Therefore,

$$dc_1 = \Delta^{-1}\{(-m_{21}pz''_D + m_{31}z''_S)\delta dT_2 + m_{41}(QdT_1 + dT_2)\} \; . \quad (B2)$$

Also observe that expanding down the first column of Ω yields

$$\Delta = u''m_{11} - Qm_{41} \; . \quad (B3)$$

In order to calculate the expressions in (B2) and (B3) I will calculate the minors m_{i1}, i = 1, 2, 3, 4. Observe that

$$m_{11} = -\delta\{(1-p)[\frac{Q}{R}-1]^2 mz''_S v'' + pmz''_D v'' + p\delta z''_D z''_S\} < 0 \quad (B4)$$

$$m_{21} = -(1-p)Q[\frac{Q}{R}-1]\delta mz''_S v'' < 0 \quad (B5)$$

$$m_{31} = (1-p)Qp\delta mv''z''_D > 0 \quad (B6)$$

$$m_{41} = (1-p)Qp\delta^2 v''z''_S z''_D < 0 \quad (B7)$$

where the signs of the minus are based on the assumption that $0 < p < 1$ and $Q > R$. It follows immediately from (B3), (B4) and (B7) that $\Delta > 0$.

Substituting (B5), (B6) and (B7) into (B2) yields

$$dc_1 = \Delta^{-1} p(1 - p)\delta^2 Qv'' z_S'' z_D'' \{(m \frac{Q}{R} + 1)dT_2 + QdT_1\} . \tag{B8}$$

Substituting the government's budget constraint (5) into (B8) yields

$$\frac{dc_1}{dT_1} = \frac{p(1 - p)\delta^2 Qv'' z_D'' z_S''}{(1 - p + n)\Delta} \{(1 - p + n - m)Q - R\} . \tag{B9}$$

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