EXPECTATIONS AND VOLATILITY OF LONG-HORIZON STOCK RETURNS

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ABSTRACT

An equilibrium pricing model with time-varying conditional moments of consumption growth is used to analyze the behavior of conditional moments of stock returns for long and short investment horizons. We examine the behavior over time of estimates of the conditional means and variances of consumption growth and returns. Business cycles appear to be associated with all of these estimates. The tendency for estimates of the price of risk to be higher during recessions, when coupled with the business-cycle variation in estimates of the moments of consumption growth, appears to be consistent with the pricing model.

1. Introduction

Much current empirical research on financial assets addresses the volatility and the predictability of rates of return. Two important and related questions are central to this research: (i) Are the estimated properties of changes in investment opportunities consistent with equilibrium models of rational behavior? (ii) What macroeconomic effects, if any, are associated with changes in investment opportunities?

This exploratory study pursues an approach in which changes in the investment opportunity are linked to changes in the conditional moments of real per-capita consumption growth. Implications about the behavior of asset prices are obtained from an equilibrium model in which agents' preferences are combined with exogenously specified processes for the conditional moments of consumption growth. This general approach is applied here in a modeling framework with standard time-additive preferences and constant relative risk aversion. Kandel and Stambaugh (1988) provide an example of such an equilibrium pricing model that is consistent with several empirically observed properties of stock returns, including an annual "equity premium" of about six percent as well as the patterns, with respect to investment holding period or "horizon", of first-order autocorrelations of returns and R-squared values in projections of stock returns on predetermined financial variables. The pricing model includes time-varying conditional moments of the monthly growth rate of consumption along with features similar to those in models developed by Lucas (1978), Mehra and Prescott (1985), and Abel (1988).

In section 2, we use the model in Kandel and Stambaugh (1988), with the same parameters used in the example provided there, to analyze conditional moments of returns across different states of the economy. We examine expected excess returns and conditional variances of returns for investment horizons of both one month and five years. At the end of month t, the states of the economy relevant to these conditional moments of returns can be characterized by the pair $(\mu_t\ \sigma_t^2)$, the conditional mean and the conditional variance of the consumption growth rate for month t+1.

The implied relations between the conditional moments of consumption growth and returns appear to depend on the investment horizon. For a one-month horizon, the expected excess return is increasing in σ_t and is virtually unaffected by μ_t. On the other hand, the expected excess five-year return is increasing in μ_t and is virtually unaffected by σ_t. The standard deviation of the one-month return depends primarily on μ_t, although not monotonically, and it is increasing in σ_t, but only slightly. The standard deviation of the five-year return is increasing in μ_t and decreasing in σ_t, although the latter effect is again weaker than the effect of μ_t.

The price of risk, the ratio of conditional expected excess return to conditional variance, varies across states of the economy, but the implied behavior also differs with respect to investment horizon. For a one-month horizon, the price of risk is increasing in σ_t but U-shaped with respect to μ_t. For a five-year horizon, the price of risk is increasing in σ_t and decreasing in μ_t. For both horizons, the price of risk has its highest value in the state with the highest σ_t and lowest μ_t.

Section 3 presents an exploratory empirical investigation of the behavior over time of conditional moments of consumption growth rates and stock returns. Our investigation focuses on both short and long investment horizons. Examining conditional moments for long horizons is prompted in part by the different relations for short and long horizons implied by the example of the equilibrium model, but another important reason for our investigation of long horizon returns is the nature of existing evidence about the behavior of stock returns for horizons of various lengths.

Returns for short horizons, such as one month, have been used in numerous studies to investigate the relation between expected excess returns and volatility (and the relation is often specified as a constant linear function).[1] The results of these studies do not lead to strong conclusions about the nature of the intertemporal relation between expected returns and volatility. Although many of them detect a positive relation between expected returns and volatility, the relations are often weak [e.g., French, Schwert, and Stambaugh (1987)] and sometimes negative [e.g., Campbell (1987)].

Fama and French (1987, 1988) provide evidence illustrating how patterns in expected returns can become more apparent by examining returns over longer horizons. Although the observed patterns in expected returns may in fact be implied by models for short investment horizons (e.g., the equilibrium model analyzed in this proposal), or even by models estimated using short-horizon returns [e.g., the VAR model for monthly data estimated in Kandel and Stambaugh (1988)], the examination of long-horizon returns may help to provide clues useful in constructing such models. Thus, we use long-horizon returns with a view towards discovering patterns in the conditional expected returns and volatility, and in the relations between them, that might be less apparent in examinations of short-horizon returns.

Section 4 presents a graphical analysis of changes over time in the conditional moments of consumption growth and returns. We first observe that both the conditional mean and the conditional variance of consumption growth display variation that appears to be related to the business cycle. Specifically, recessions appear to accompany peaks in the conditional variance

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[1] "Volatility," in this context, is used to denote either the variance or the standard deviation of the return. A partial list of the studies in this area includes Merton (1980), French, Schwert, and Stambaugh (1987), Poterba and Summers (1986), Gennotte and Marsh (1985), Hasbrouck (1985), Campbell (1987), and Bollerslev, Engle, and Wooldridge (1988).

and troughs in the conditional mean.

The business-cycle-related variation in the conditional moments of consumption, when coupled with the equilibrium pricing model, implies business-cycle-related variation in the conditional moments of returns. We find that, especially when the analysis is conducted using five-year returns, business cycles appear to figure prominently in the behavior through time of expected returns and volatility. Expected stock returns display peaks during recessions, typically in the few months preceding the trough in business activity. Volatility tends to display peaks around the beginnings of recessions, often in the few months preceding the peak in business activity. These patterns are often not as apparent in estimates based on monthly returns.

The estimated price of risk displays a consistent tendency to be higher during recessions than during periods of recovery. In the example of the equilibrium model considered, the price of risk is decreasing in the conditional mean of consumption growth and increasing in the conditional variance of consumption growth. Thus, the business-cycle-related variation in the price of risk appears to be consistent with the joint implications of the equilibrium pricing model and the business-cycle-related variation in the conditional moments of consumption growth.

2. Relations Between Conditional Moments of Consumption Growth and Returns

The equilibrium model analyze is described here, and the relevant implications about conditional moments of returns are stated without proof. A fuller discussion of the model is provided in Kandel and Stambaugh (1988).

The basic framework of the model is that of Lucas (1978), where the physical stock of capital is fixed and aggregate consumption c_t equals aggregate output h_t in each period t. The consumer maximizes expected time-additive utility over an infinite horizon,

$$E\left\{ \sum_{\tau=t}^{\infty} \beta^{\tau-t}\, U(c_\tau) \right\} \quad , \tag{1}$$

where β (>0) is a rate of time preference. To this framework we add a combination of features from models developed by Mehra and Prescott (1985) and Abel (1988):

(i) The utility function exhibits constant relative risk aversion,

$$U(c) = \frac{c^{1-\alpha} - 1}{1 - \alpha} \qquad 0 < \alpha < \infty \quad . \tag{2}$$

(ii) Let λ_{t+1} denote the one-period growth rate in output, i.e.,

$$h_{t+1} = \lambda_{t+1} h_t \tag{3}$$

The quantity $\ln(\lambda_{t+1})$ is, conditional on information at time t, distributed normally with mean μ_t and variance σ_t^2.

(iii) The pair $(\mu_t\ \sigma_t^2)$ follows a joint stationary Markov process with a finite number of states, S. Let s_t denote the state for $(\mu_t\ \sigma_t^2)$ at time t, where s_t can take values 1, ..., S. Let Φ denote the transition matrix with (i,j) element

$$\phi_{ij} = \text{Prob}(s_{t+1} = j \mid s_t = i) \tag{4}$$

(iv) Given s_t, the distribution of s_{t+1} is independent of λ_{t+1}, λ_t, λ_{t-1}, ...

These assumptions imply that the state of the economy follows a Markov process with the state represented as (c, s), $0 < c < \infty$ and $1 \leq s \leq S$, where, at time t, $c_t = c$ and $s_t = s$. Levered equity is defined as the claim on aggregate wealth net of a risky one-period bond that promises to pay, at the end of one period, a fraction θ of current aggregate wealth. Conditional means and variances of returns on levered equity can be shown to depend only on s, and analytic expressions for these moments for various investment horizons are given in the Appendix.

Kandel and Stambaugh (1988) provide an example of this equilibrium pricing model that is consistent with several empirically observed properties of stock returns, including (i) an annual equity premium of about six percent, (ii) a U-shaped pattern of first-order autocorrelations of returns with respect to investment horizon, and (iii) a humped pattern with respect to investment horizon for the R-squared in projections of stock returns on predetermined financial variables. The example is constructed by specifying the preference parameters α and β, the degree of leverage θ, and the Markov process for the conditional moments of consumption.

We use the model in Kandel and Stambaugh (1988), with the same parameters used in the example provided in that study, to analyze conditional expected excess returns and conditional variances across states of the economy. Table 1 summarizes the properties of the consumption process. A nine-state Markov process is specified for $(\mu_t\ \sigma_t^2)$, with each element taking three possible values, and both conditional moments are assumed to be positively autocorrelated. The leverage coefficient θ is set equal to 0.413, so the equity is approximately 60% of aggregate wealth. The rate of time preference is specified as $\beta = 0.99975$ (for monthly periods) and the coefficient of relative risk aversion is specified as $\alpha = 28.55$.

Kandel and Stambaugh (1988) discuss this choice of α that is high by traditional standards, and we will not duplicate that discussion here. We do observe, however, that the price of risk implied by this model, examined below across states of the economy, is between one and two.[2] We also note that the riskless rate implied by the model has a mean of 0.8% and a standard deviation of 4.1% (in annual terms); the real return on nominally riskless securities used as the empirical benchmark by Mehra and Prescott (1985) has the same mean and a somewhat higher standard deviation (5.7%). (As mentioned earlier, our example of this equilibrium model also implies an equity premium equal to the Mehra-Prescott empirical benchmark).

Table 2 reports the implications of the model about conditional moments of returns on levered equity for both one-month and five-year investment horizons. As noted above, the states of the economy relevant to these conditional moments of returns can be characterized by the pair $(\mu_t\ \sigma_t^2)$, the conditional moments of consumption growth. Panel A of table 2 displays, for each of these nine states, the conditional expected excess return, the conditional standard deviation, and the price of risk.

The implied properties of five-year returns differ from those of one-month returns in several important respects. First consider the conditional expected excess return. For a one-month horizon, the expected excess return is increasing in σ_t, the conditional standard deviation of consumption growth, and is virtually unaffected by μ_t, conditional expected consumption growth. For a five-year horizon, however, the expected excess return is increasing in μ_t and is virtually unaffected by σ_t.

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[2] The price of risk, estimated by Friend and Blume (1975) as approximately two, is often used as a measure of relative risk aversion. When consumption growth rates are not identically and independently distributed, the price of risk can deviate significantly from the coefficient of relative risk aversion. See also Kocherlakota (1988).

Conditional standard deviations of returns also display different characteristics depending on the return horizon. For a one-month horizon, the standard deviation of returns depends primarily on μ_t, but not in a monotonic fashion--the standard deviation is highest for the intermediate value of μ_t. The standard deviation is increasing in σ_t, but only slightly.[3] For five-year horizons, the standard deviation is increasing in μ_t and decreasing in σ_t, although the latter effect is again weaker than the effect of μ_t.

The price of risk, the ratio of conditional expected excess return to conditional variance, varies across states, but the behavior again differs with respect to investment horizon. For a one-month horizon, the price of risk is increasing in σ_t but U-shaped with respect to μ_t. For a five-year horizon, the price of risk is increasing in σ_t and decreasing in μ_t. For both horizons, the price of risk has its highest value in the state with the highest σ_t and lowest μ_t.

Panel B reports the mean, the standard deviation, and the first-order (monthly) autocorrelation of the expected excess return, the standard deviation of return, and the price of risk. The one-month-horizon values have lower autocorrelations than the five-year-horizon values, consistent with the overlapping horizons in the latter case. The implied five-year values are also smoother in that, in addition to their higher autocorrelations, they also have lower variances. For example, the implied variance of the one-month price of risk is approximately four times the variance of the five-year price of risk.

[3] This property of the model is suggestive of the results of Schwert (1987), who finds that the volatility of monthly stock returns does not correspond closely to volatilities of other macroeconomic aggregates.

3. Estimates of Conditional Moments of Consumption Growth Rates and Returns

Kandel and Stambaugh (1988) define three financial variables and derive their values across states of the economy. As with the conditional moments of returns, these financial variables depend only on $(\mu_t\ \sigma_t^2)$ and not on realized consumption growth. The three financial variables are (i) a term spread, defined as the difference between the riskless twenty-year rate and the riskless one-month rate, (ii) a default spread, defined as the difference between the yield on the risky one-period bond discussed earlier (used to construct levered equity) and the riskless one-month rate, and (iii) a dividend-price ratio, defined as expected aggregate consumption divided by the price of levered equity,. Although the relations between conditional moments of returns and these variables are nonlinear, Kandel and Stambaugh (1988) find that, in the example of the model considered, a linear projection of conditional moments on these three variables fits well (produces multiple correlation coefficients close to unity).

Conditional means and variances of consumption growth rates and returns are estimated here using simple linear regressions on three predictive variables: a term spread, a default spread, and a dividend price ratio.[4] Although the correspondence between these three variables used in the empirical work and those used in the model is rough, we suggest that the model provides at least some motivation for estimating conditional moments as

[4] These predictive variables are used by Fama and French (1987), and similar variables have been used by other researchers, to predict asset returns. For example, Rozeff (1984) finds that dividend-price ratios predict stock returns, and Keim and Stambaugh (1986) find that (among other variables) the difference in yields between low-grade bonds and Treasury Bills predicts stock and bond returns. Campbell (1987) uses the slope of the (short-term) term structure as a predictive variable. Contemporaneous changes in similar variables have also been used as common risk factors in empirical investigations of multifactor pricing models. In the latter context, Chen, Roll, and Ross (1986) use return spreads between (i) low-grade and high-grade bonds and (ii) long-term high-grade bonds and Treasury Bills

linear projections on such financial variables.

3.1 Conditional Moments of the Consumption Growth Rate

Let c_t denote the consumption growth rate for quarter t. Let x_{t-1} denote a 3×1 vector of state variables observed at the end of quarter t-1:

$(y_{Baa} - y_{Aaa})_{t-1}$: the difference at the end of quarter t-1 between Moody's average yield on bonds rated Baa and bonds rated Aaa.

$(y_{Aaa} - y_{TB})_{t-1}$: the difference at the end of quarter t-1 between the Aaa yield and the yield on a U.S. Treasury Bill with maturity closest to one month.

$(D/P)_{t-1}$: for the equally weighted portfolio of NYSE stocks, the ratio of dividends paid for the twelve months ending at t to the price at the end of quarter t.

The yield-related variables are stated as percent per month.

The expectation of c_t conditional on information at time t-1 is modeled as a linear function of the vector x_{t-1}. We compute ordinary least squares (OLS) estimates of the regression

$$c_t = \alpha_0 + \alpha' x_{t-1} + u_t \tag{5}$$

using the series of quarterly growth rates of real per capita consumption on nondurables and services constructed by Breeden, Gibbons, and Litzenberger (1986).[5] This series covers the period from second quarter 1929 through the fourth quarter of 1982 and consists of a splicing of series constructed by

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[5]We thank Mike Gibbons for providing this data.

different techniques, depending on the subperiod. From 1929 through 1938, the series is estimated by combining annual consumption data and monthly data on personal income, from 1939 through 1958 the series is based on quarterly consumption, and from 1959 through 1982 the series is based on monthly consumption.

Given the above model for the conditional mean, the conditional variance of the consumption growth rate is given by

$$\mathrm{var}_{t-1}\{c_t\} = E_{t-1}\{u_t^2\} \quad , \tag{6}$$

where E_{t-1} and var_{t-1} denote the mean and the variance of the quantity in braces conditional on information at time t-1. We assume that the conditional expectation on the right-hand side of (6) is a function of the same three predictive variables used above in the estimation of the conditional mean. Specifically, we define the regression

$$\ln(u_t^2) = \beta_0 + \beta' x_{t-1} + \varepsilon_t \quad . \tag{7}$$

The log transformation is used in order to avoid negative conditional variances and to reduce the degree of positive skewness that would be more likely in a regression with u_t^2 as the dependent variable.

Since the same predictive variables are used in both (5) and (7), it can be shown that Generalized Method of Moments (GMM) estimation of the parameters in the system of equations given by (5) and (7) is computationally equivalent to a two-stage procedure. OLS estimation of (5) is performed in the first stage. The estimated residual from that regression, $\hat{u}_t$, is substituted for

u_t in (7), and OLS estimates of that regression are then computed.[6]

Table 3 reports estimates of the equations (5) and (7). The standard errors shown are based on the heteroskedasticity-consistent estimator of the covariance matrix analyzed by White (1980) and Hsieh (1983). Both conditional moments appear to vary through time. Tests of the hypotheses that the conditional mean and the conditional variance do not depend on the predictive variables produce Chi-square statistics well above standard significance levels. The conditional mean is estimated in the first equation; the coefficient on $(y_{Baa} - y_{Aaa})_{t-1}$ is reliably positive and the coefficient on $(D/P)_{t-1}$ is reliably negative. In the second equation, used to estimate the conditional variance, the coefficient on $(y_{Baa} - y_{Aaa})_{t-1}$ is reliably positive and the coefficient on $(y_{Aaa} - y_{TB})_{t-1}$ is reliably negative.

We construct the estimated conditional variance of c_t using the relation

$$E_{t-1}\{u_t^2\} = e^{E_{t-1}\{\ln(u_t^2)\} + \frac{1}{2} \text{var}_{t-1}\{\ln(u_t^2)\}} , \tag{8}$$

which holds if ε_t [the residual in (7)] is Normally distributed. Given (8) [and the relation noted earlier in (6)], the estimated value of $\text{var}_{t-1}\{c_t\}$ is constructed by substituting the fitted value of the dependent variable in (7) for $E_{t-1}\{\ln(u_t^2)\}$ and by substituting the sample variance of the estimated ε_t's for $\text{var}_{t-1}\{\ln(u_t^2)\}$.

In order to obtain monthly series of conditional moments of (quarterly) consumption growth rates, we apply the estimates of the coefficients in equations (5) and (7), which are estimated using quarterly consumption growth

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[6] A similar point is discussed by Hasbrouck (1985, 1986) and Campbell (1987).

rates, to the monthly series of the predictive variables. The underlying presumption is that the patterns through time of the conditional moments of the one-quarter-ahead growth rate are similar to those of the one-month-ahead growth rate.

3.2 Conditional Moments of Stock Returns

Let $r_{t,N}$ denote the continuously compounded return on the equally weighted NYSE portfolio for the N-month horizon starting at the beginning of month t. Using monthly data for the period from January 1927 through December 1985, we construct estimates of the conditional moments of returns in essentially the same manner used above to estimate conditional moments of the growth rate of consumption [c.f. equations (5) through (8)]. The predictive variables are observed at the end of month t-1 instead of quarter t-1, and c_t is replaced by the return $r_{t,N}$, for both N=1 and N=60. The 60-month regression uses five-year returns spaced one-month apart, so that adjacent observations contain 59 common months.

Results of these estimations are reported in table 4. The Newey and West (1987) estimator of the covariance matrix is used to construct standard errors of the coefficient estimates. These standard errors are consistent in the presence of heteroskedasticity as well as the residual autocorrelation induced by the overlapping observations. Also reported in table 4 for each regression is a test of the hypothesis that the regression coefficients (excluding the intercept) jointly equal the zero. The test statistic is asymptotically distributed as χ^2 with three degrees of freedom under the null hypothesis.

Note that, in the one-month regression of the returns on the predictive variables (equation I in table 4), the coefficients on the three predictive variables are not reliably different from zero. In the five-year regression (equation III), however, the coefficients on both the Baa-versus-Aaa yield spread and the dividend-price ratio are at least two standard errors above zero. The χ^2 test produces a p-value equal to 0.21 in the one month regression and less than 0.005 in the five-year regression.

Table 4 also displays the results of the estimation of (7), used in estimating the conditional variance, for both one-month returns (equation II) and five-year returns (equation IV). In the one-month regression, the coefficient on the Baa-versus-Aaa yield spread is positive and over six times greater than its standard error; the coefficient on the Aaa-versus-Treasury-bill yield spread is negative and approximately 1.8 standard errors less than zero. The explanatory power of the Baa-versus-Aaa yield spread is consistent with the results of Schwert (1987), who also finds that such a yield spread is positively related to the volatility of one-month returns in the 1927-1986 period.

The Baa-versus-Aaa yield spread also exhibits a reliably positive association with the five-year volatility, given that the estimated coefficient is about 2.5 standard errors above zero. The estimated coefficients for the other two variables are negative in the five-year regression. The coefficient on the Aaa-versus-T-Bill yield spread is more than four standard errors below zero, and the coefficient on the dividend-price ratio is almost two standard errors below zero. These two variables appear to play a more important role here than in the one-month regression. As we will see graphically below, the estimated volatilities for the different holding periods display rather different properties.

The estimates of the conditional variance constructed here should be viewed as preliminary. Other approaches to estimating volatilities could also be considered. Previous studies examining the volatility of monthly returns have used daily data in order to obtain more precise estimates of the volatility within a month [e.g., Merton (1980) and French, Schwert, and Stambaugh (1987)]. The appeal of such approaches is clear when returns are assumed to be identically and independently distributed within the return horizon. When the daily returns within the given horizon are serially dependent, however, then the use of daily returns to estimate volatility becomes more difficult, since the variance of the return over the given horizon depends on the autocovariances of the daily returns. Given the existing evidence about long-term negative serial dependence in returns [e.g., Fama and French (1987a)], the use of daily returns to estimate the conditional volatility of five-year returns would seem to be somewhat complicated.[7] Nevertheless, such approaches should be explored in future research.

4. Business Cycles and the Moments of Consumption Growth and Returns

This section conducts a preliminary analysis, principally using time-series plots, of the behavior of the estimates of the conditional expectation and variance of the consumption growth rate, conditional expected excess returns, conditional standard deviations, and the price of risk (the ratio of expected excess return to variance). We also indicate in the figures the dates for business-cycle peaks and troughs determined by the National Bureau of Economic Research [see Moore and Zarnowitz (1984)]. We focus the

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[7]Note that the most straightforward approach using daily returns would not appear to provide any advantage. French, Schwert, and Stambaugh (1987) consider autocorrelation induced by nonsynchronous trading in their use of daily data to estimate the within-month volatility. They allow returns on adjacent days to be correlated but implicitly assume that returns more than one day apart are uncorrelated. Their variance estimator is of the form

$$\hat{\sigma}^2 = \sum r_t^2 + 2 \sum r_t r_{t+1} \quad ,$$

where the r's denote daily returns and the summations are taken over all days in the holding period (e.g., one month). If the same type of estimator is applied where nonzero autocorrelations for all lags are allowed, the resulting estimator is then simply the square of the return for the holding period.

discussion on the behavior of the series with respect to the business cycles. This business-cycle-related behavior is then linked to the equilibrium pricing model discussed in Section 2.

4.1 A Graphical Analysis

Figure 1 displays plots of the monthly estimates of the conditional mean and the conditional standard deviation of quarterly consumption growth. Both series appear to vary in relation to the business cycle. Peaks in the standard deviation tend to occur during or immediately prior to recessions. Troughs in the conditional mean tend to occur during recessions. World War II is accompanied by a peak in the standard deviation and a trough in the mean.

Figures 2 through 4 display, for one-month holding periods, the conditional expected excess return, the conditional standard deviation, and the price of risk. Figures 5 through 7 display the same quantities for five-year holding periods. Expected excess returns are formed by subtracting from the fitted value of the expected stock return either the yield on a one-month Treasury bill (for the estimates using one-month horizons) or the Aaa bond yield (for the estimates using five-year horizons). Both excess returns and standard deviations are plotted as per-month values.

The graphs of expected excess returns for one-month and five-year horizons are similar in appearance. Both plots exhibit clear business-cycle-related behavior. Note that, with the exception of a peak in each series during World War II, all of the large peaks in the series occur during recessions. Of the twelve recessions included in the period shown, only the 1926-27 and 1945 recessions do not contain a pronounced peak in expected returns. It also appears that the peak in expected excess return tends to occur toward the end of the recession. The latter phenomenon seems particularly evident in the more severe recessions: 1929-33, 1937-38, 1973-75, and 1981-82.

The graphs of conditional standard deviation exhibit different appearances for one-month horizons (figure 3) and five-year horizons (figure 6). The estimated standard deviations for one-month horizons exhibit much larger values during the 1930's than during the rest of the period. Other studies of volatility for one-month horizons observe the same property [e.g., French, Schwert, and Stambaugh (1987)]. The five-year standard deviations, on the other hand, actually achieve their largest values during the latter decades, although large peaks in the series also occur in 1929 and 1932. The monthly standard deviations appear to be higher during recessions. This impression is due primarily to the two recessions of the 1930's, although the somewhat higher values also surround later recessions of the 1970's and 1980's.[8]

Business cycles appear to play a somewhat different role in the behavior of five-year standard deviations (figure 6) than in the behavior of expected returns. Peaks in business activity, especially those preceding the 1929-33 depression and all of the post-1953 recessions, tend to occur within several months of large peaks in the conditional five-year standard deviation. Recall that peaks in the expected excess return tend to occur later in the recession.

Figures 4 and 7 plot the price of risk, the ratio of conditional expected excess return to conditional variance, for one-month and five-year horizons. The differences between the results for the two return horizons are perhaps most dramatic in these plots. Business-cycle effects are clearly evident in the five-year price of risk (figure 7), whereas any business-cycle effects in

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[8] Schwert (1987) also concludes that the volatility of one-month stock returns is higher during recessions, both in the 1927-52 period as well as in the 1953-86 period.

the price of risk for one-month horizons (figure 4) are less apparent.[9] With few exceptions, the large peaks in the estimated five-year price of risk occur during or close to recessions. One of the major exceptions is World War II, which is also accompanied by large values in the series. The plot of the five-year price of risk during periods of expansion tends to be lower and, generally, rather flat. The average estimated five-year price of risk is 2.06 during recessions and 1.13 during non-recession periods (0.97 if the World War II years of 1942-45 are excluded).

4.2 Integrating the Evidence

The business-cycle-related variation in the conditional moments of consumption, when coupled with the equilibrium pricing model, implies business-cycle-related variation in the conditional moments of returns. We now ask whether the variation in the moments of returns is consistent with the variation in the estimated moments of consumption. The example of the model is specified for monthly consumption growth rates, but, as discussed in the previous section, quarterly growth rates are used in the estimation. Given this and other limitations, the analysis must be interpreted cautiously. Nevertheless, we believe that the evidence offers promise for such an approach.

As observed earlier in panel B of table 2, the pricing model implies that the conditional moments of returns for one-month horizons are less smooth than those for five-year horizons, in that the one-month values are both less autocorrelated and more volatile. This implication is consistent in part with the estimates displayed earlier, especially those of the price of risk

[9]Citing an earlier draft of this study, Harvey (1988) provides evidence of apparent business-cycle-related variation in the price of risk for one-month horizons during the 1947-86 period. Harvey also tests and rejects the

(cf. figures 4 and 7). While business-cycle effects appear to be present in the one-month values, these effects are less apparent in the graphs in some cases, perhaps due to the lack of smoothness. We will focus our discussion on whether the business-cycle-related effects in the five year estimates are consistent with the equilibrium model, recognizing that a similar analysis could be made for the one-month estimates.

The expected five-year excess return, graphed in figure 5, experiences peaks late in recessions and during World War II. The pricing model implies that this expected return should be highest when the conditional mean of consumption growth is highest. (Recall that the conditional variance of consumption growth has virtually no effect on this expected return.) Figure 1 suggests that expected consumption growth typically experiences a trough around mid-recession that is followed by a peak at the end of the recession. While the peaks and troughs in the series of estimated expected returns do not coincide precisely with those in estimated expected consumption, both series appear to exhibit a tendency to be high at the end of recessions.

The model implies that the standard deviation of five-year returns should be highest when mean consumption growth is highest and when the variance of consumption growth is lowest. Estimated five-year standard deviations, shown in figure 6, tend to experience peaks at the beginnings of recessions. This behavior does not appear to coincide well with the timing of the peaks and troughs in estimated moments of consumption growth. The standard deviation of consumption growth also appears to peak at the beginnings of recessions, and, while estimated mean consumption growth typically is not low entering recessions, neither does that series experience its largest peaks at those times.

The five-year price of risk, plotted in figure 7, tends to experience peaks during recessions. The model implies that the five-year price of risk is highest when mean consumption growth is low and the variance of consumption growth is high. As observed earlier, figure 1 indicates that peaks in the estimated standard deviation of consumption growth and troughs in estimated mean consumption growth both tend to occur during recessions Thus, the business-cycle-related behavior of the estimated price of risk appears to be consistent with the implications of the pricing model.

World War II provides an interesting test case. The estimated price of risk in figure 7 also exhibits a pronounced peak during the years 1942-44. Note that, in figure 1, these same war years are accompanied by both a peak in the conditional standard deviation of consumption growth and a trough in the conditional mean growth rate. Thus, although this period in history is not classified as a business cycle in the usual sense, the implications of the model concerning the price of risk seem to be supported for this apparently volatile period as well.

6. Conclusions

The equilibrium model considered in this paper is presented in Kandel and Stambaugh (1988) as a model consistent with several observed time-series properties of returns, but that study does not discuss the implications of the model across different states of the economy. We explore such implications in this study and conclude that the same model is capable of providing at least a partial explanation for the observed business-cycle-related behavior in conditional moments of stock returns.

Conditional means and variances of quarterly consumption growth rates and returns for short and long investment horizons are estimated as simple functions of three predetermined financial variables. The estimates of the effects. The estimated conditional mean growth rate is lowest during recessions, and the estimated conditional standard deviation of the growth rate is highest during recessions.

The length of the investment horizon appears to play a role in the behavior of the estimated moments of stock returns, in that business-cycle effects are often more evident in the estimates for five-year horizons. Conditional expected returns appear to reach their highest levels toward ends of recessions, whereas the conditional standard deviation tends to reach its highest levels just prior to peaks in economic activity. The price of risk, the ratio of conditional expected excess return to conditional variance, appears to be higher during recessions than during periods of expansion.

The business-cycle-related behavior of the estimates of the conditional moments of returns appears to be consistent, at least in some respects, with the example of the equilibrium pricing model. In particular, the estimated price of risk appears to be higher during recessions. This result is consistent with the equilibrium model combined with the observed business-cycle-related behavior of the estimated conditional moments of consumption growth.

APPENDIX

Proposition 1. Let $E^{(LN)}$ denote the S-vector of conditional expected N-period (simple) rates of return on levered equity in each of the S states, $1 \leq s \leq S$.

$$E^{(AN)} = \Gamma^N \iota_S - \iota_S \quad , \tag{A1}$$

where Γ is an S×S matrix with (i, j) element

$$\gamma_{ij} = \frac{E\{\max[0,\ \lambda(i)(1 + w_j) - \theta w_i]\}}{w_i - g_i} \quad , \tag{A2}$$

w_i is the ith element of the S-vector $\mathbf{w}$, given by

$$\mathbf{w} = (I - H)^{-1} H \iota_S \quad , \tag{A3}$$

H is the S × S matrix with (i,j) element

$$h_{ij} = \beta\ \phi_{ij}\ E\{\lambda(i)^{1-\alpha}\} \quad , \tag{A4}$$

g_i is the i^{th} element of the S-vector $\mathbf{g} = Y\iota_S$, and Y is an S×S matrix with (i, j) element

$$y_{ij} = \beta\ \phi_{ij} E\{\min[\lambda(i)^{1-\alpha}(1 + w_j),\ \lambda(i)^{-\alpha}\theta w_i]\} \quad . \tag{A5}$$

<u>Proposition 2.</u> Let $V^{(LN)}$ denote the vector of conditional variances of N-period (simple) rates of return on levered equity in each of the S states. Then[10]

$$V^{(AN)} = \Xi^{N}\iota_{S} - [(\Gamma^{N}\iota_{S})*(\Gamma^{N}\iota_{S})] \quad , \tag{A6}$$

where Ξ is an S×S matrix with (i, j) element

$$\xi_{ij} = \phi_{ij}\,\frac{E\{(\max[0,\ \lambda(i)(1 + w_j) - \theta w_i])^2\}}{(w_i - g_i)^2} \quad . \tag{A7}$$

[10]The symbol "*" denotes a Hadamard matrix product. If $[a_{ij}]$ and $[b_{ij}]$ denote the elements of m×n matrices, then $[a_{ij}]*[b_{ij}] = [a_{ij}\cdot b_{ij}]$.

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Table 1

Properties of Monthly Consumption Growth Rates in the Example of the Equilibrium Model

A. States for the Conditional Moments

State	Unconditional Probability	Conditional Mean of the Monthly Growth Rate (%)	Conditional Standard Deviation of the Monthly Growth Rate (%)
1	0.084	0.111	0.915
2	0.088	0.153	0.915
3	0.084	0.194	0.915
4	0.160	0.111	1.023
5	0.168	0.153	1.023
6	0.160	0.194	1.023
7	0.084	0.111	1.144
8	0.088	0.153	1.144
9	0.084	0.194	1.144

B. Probability of moving from state i to state

state i	1	2	3	4	5	6	7	8	9
1	0.459	0.041	0.000	0.397	0.036	0.000	0.062	0.006	0.000
2	0.039	0.422	0.039	0.034	0.365	0.034	0.005	0.057	0.005
3	0.000	0.041	0.459	0.000	0.036	0.397	0.000	0.006	0.062
4	0.209	0.019	0.000	0.500	0.045	0.000	0.209	0.019	0.000
5	0.018	0.192	0.018	0.042	0.460	0.042	0.018	0.192	0.018
6	0.000	0.019	0.209	0.000	0.045	0.500	0.000	0.019	0.209
7	0.062	0.006	0.000	0.397	0.036	0.000	0.459	0.041	0.000
8	0.005	0.057	0.005	0.034	0.365	0.034	0.039	0.422	0.039
9	0.000	0.006	0.062	0.000	0.036	0.397	0.000	0.041	0.459

C. Unconditional Moments[a]

Quantity	Mean	Standard Deviation	First-Order Autocorrelation
Actual Growth Rate	0.1525	1.0301	0.0010
Conditional Expected Growth Rate	0.1525	0.0336	0.9178
Conditional Standard Deviation of the Growth Rate	1.0263	0.0823	0.4320

Table 2

Properties of the Example of the Equilibrium Model

A. Behavior Across States

	Monthly Consumption Growth Rate		One-Month Horizon			Five-Year Horizon[a]		
State	Mean	Std. Dev.	Mean Excess Return on Levered Equity	Std.Dev. of Return on Levered Equity	Price of Risk	Mean Excess Return on Levered Equity	Std.Dev. of Return on Levered Equity	Price of Risk
1	0.0011	0.0091	0.0041	0.0633	1.0151	0.0038	0.0500	1.5141
2	0.0015	0.0091	0.0041	0.0951	0.4517	0.0042	0.0576	1.2755
3	0.0019	0.0091	0.0041	0.0735	0.7589	0.0050	0.0656	1.1553
4	0.0011	0.0102	0.0051	0.0642	1.2312	0.0038	0.0493	1.5439
5	0.0015	0.0102	0.0051	0.0956	0.5579	0.0042	0.0569	1.3004
6	0.0019	0.0102	0.0051	0.0743	0.9280	0.0049	0.0648	1.1781
7	0.0011	0.0114	0.0063	0.0644	1.5287	0.0037	0.0486	1.5794
8	0.0015	0.0114	0.0064	0.0956	0.6964	0.0042	0.0560	1.3301
9	0.0019	0.0114	0.0064	0.0743	1.1552	0.0049	0.0638	1.2053

B. Unconditional Moments

	Mean	Standard Deviation	Auto-correlation
One-Month Horizon			
Conditional Expected Excess Return on Levered Equity	0.0052	0.0008	0.4307
Conditional Standard Deviation of Return on Levered Equity	0.0782	0.0132	0.7764
Price of Risk	0.9141	0.3162	0.7165
Five-Year Horizon[a]			
Conditional Expected Excess Return on Levered Equity	0.0043	0.0005	0.9137
Conditional Standard Deviation of Return on Levered Equity	0.0570	0.0063	0.9136
Price of Risk	1.3414	0.1523	0.9034

Table 3

Estimation of Conditional Means and Variances of Consumption Growth Rates[a]
(4/29 - 12/82)

$$\text{(I)} \quad c_{t+1} = \alpha_0 + \alpha_1 \cdot (y_{Baa} - y_{Aaa})_t + \alpha_2 \cdot (y_{Aaa} - y_{TB})_t + \alpha_3 \cdot (D/P)_t + v_{t+1}$$

$$\text{(II)} \quad \ln(\hat{v}^2_{t+1}) = \beta_0 + \beta_1 \cdot (y_{Baa} - y_{Aaa})_t + \beta_2 \cdot (y_{Aaa} - y_{TB})_t + \beta_3 \cdot (D/P)_t + \eta_{t+1}$$

	Independent variables (lagged one quarter)[b]					
Equation	Intercept	$y_{Baa}-y_{Aaa}$	$y_{Aaa}-y_{TB}$	D/P	adj. R^2	χ^{2}[c]
I.	0.01039 (0.00314)	0.03378 (0.01274)	-0.00706 (0.01685)	-0.23516 (0.06905)	0.079	21.15 (0.00)
II.	-10.41604 (0.51636)	8.29245 (2.90059)	-11.55765 (2.77292)	25.06749 (14.18884)	0.072	19.10 (0.00)

[a]The variables are defined as follows.

c_{t+1} : the growth rate in real per capita consumption (nondurables plus services) for quarter t+1

$(y_{Baa} - y_{Aaa})_t$: the difference at the end of quarter t between Moody's average yield on bonds rated Baa and bonds rated Aaa.

$(y_{Aaa} - y_{TB})_t$: the difference at the end of quarter t between the Aaa yield and the yield on a U.S. Treasury Bill with maturity closest to one month.

$(D/P)_t$: for the equally weighted portfolio of NYSE stocks, the ratio of dividends paid in quarters t-3 through quarter t to the price at the end of quarter t.

[b]The coefficients are estimated using ordinary least squares, and the standard errors are based on the estimator of the covariance matrix proposed by Newey and West (1987).

[c]The statistic reported is asymptotically distributed as χ^2 with three degrees of freedom under the null hypothesis that all of the coefficients on the independent variables (excluding the intercept) are equal to zero. The p-value is shown in parentheses.

Table 4

Estimation of Conditional Means and Variances of Returns[a]
(1/27 - 12/85)

(I) $r_{t+1,1} = \alpha_0 + \alpha_1 \cdot (y_{Baa} - y_{Aaa})_t + \alpha_2 \cdot (y_{Aaa} - y_{TB})_t + \alpha_3 \cdot (D/P)_t + u_{t+1,1}$

(II) $\ln(\hat{u}^2_{t+1,1}) = \beta_0 + \beta_1 \cdot (y_{Baa} - y_{Aaa})_t + \beta_2 \cdot (y_{Aaa} - y_{TB})_t + \beta_3 \cdot (D/P)_t + \varepsilon_{t+1,1}$

(III) $r_{t+1,60} = \alpha_0 + \alpha_1 \cdot (y_{Baa} - y_{Aaa})_t + \alpha_2 \cdot (y_{Aaa} - y_{TB})_t + \alpha_3 \cdot (D/P)_t + u_{t+1,60}$

(IV) $\ln(\hat{u}^2_{t+1,60}) = \beta_0 + \beta_1 \cdot (y_{Baa} - y_{Aaa})_t + \beta_2 \cdot (y_{Aaa} - y_{TB})_t + \beta_3 \cdot (D/P)_t + \varepsilon_{t+1,60}$

	Independent variables (lagged one quarter)[b]					
Equation	Intercept	$y_{Baa}-y_{Aaa}$	$y_{Aaa}-y_{TB}$	D/P	adj. R^2	χ^{2}[c]
I.	-0.0119	0.0758	0.0386	0.1879	0.009	5.90
	(0.0132)	(0.1141)	(0.0297)	(0.2563)		(0.21)
II.	-7.9327	11.1089	-1.6872	2.6372	0.082	48.89
	(0.3133)	(1.7135)	(0.9295)	(6.5496)		(0.00)
III.	-0.4959	1.7780	0.9158	19.3990	0.393	47.82
	(0.2809)	(0.7738)	(0.8675)	(3.1506)		(0.00)
IV.	-1.8599	11.6241	-9.6332	-24.8997	0.100	22.24
	(0.6270)	(4.7215)	(2.2773)	(12.8390)		(0.00)

[a]The variables are defined as follows.

$r_{t,N}$: the continuously compounded return on the equally weighted portfolio of NYSE stocks for the N-month horizon starting at the beginning of month t.

$(y_{Baa} - y_{Aaa})_t$: the difference at the end of month t between Moody's average yield on bonds rated Baa and bonds rated Aaa.

$(y_{Aaa} - y_{TB})_t$: the difference at the end of month t between the Aaa yield and the yield on a U.S. Treasury Bill with maturity closest to one month.

$(D/P)_t$: for the equally weighted portfolio of NYSE stocks, the ratio of dividends paid for the twelve months ending at t to the price at the end of month t.

[b]The coefficients are estimated using ordinary least squares, and the standard errors are based on the estimator of the covariance matrix proposed by Newey and West (1987).

[c]The statistic reported is asymptotically distributed as χ^2 with three degrees of freedom under the null hypothesis that all of the coefficients on the independent variables (excluding the intercept) are equal to zero. The p-value is shown in parentheses.

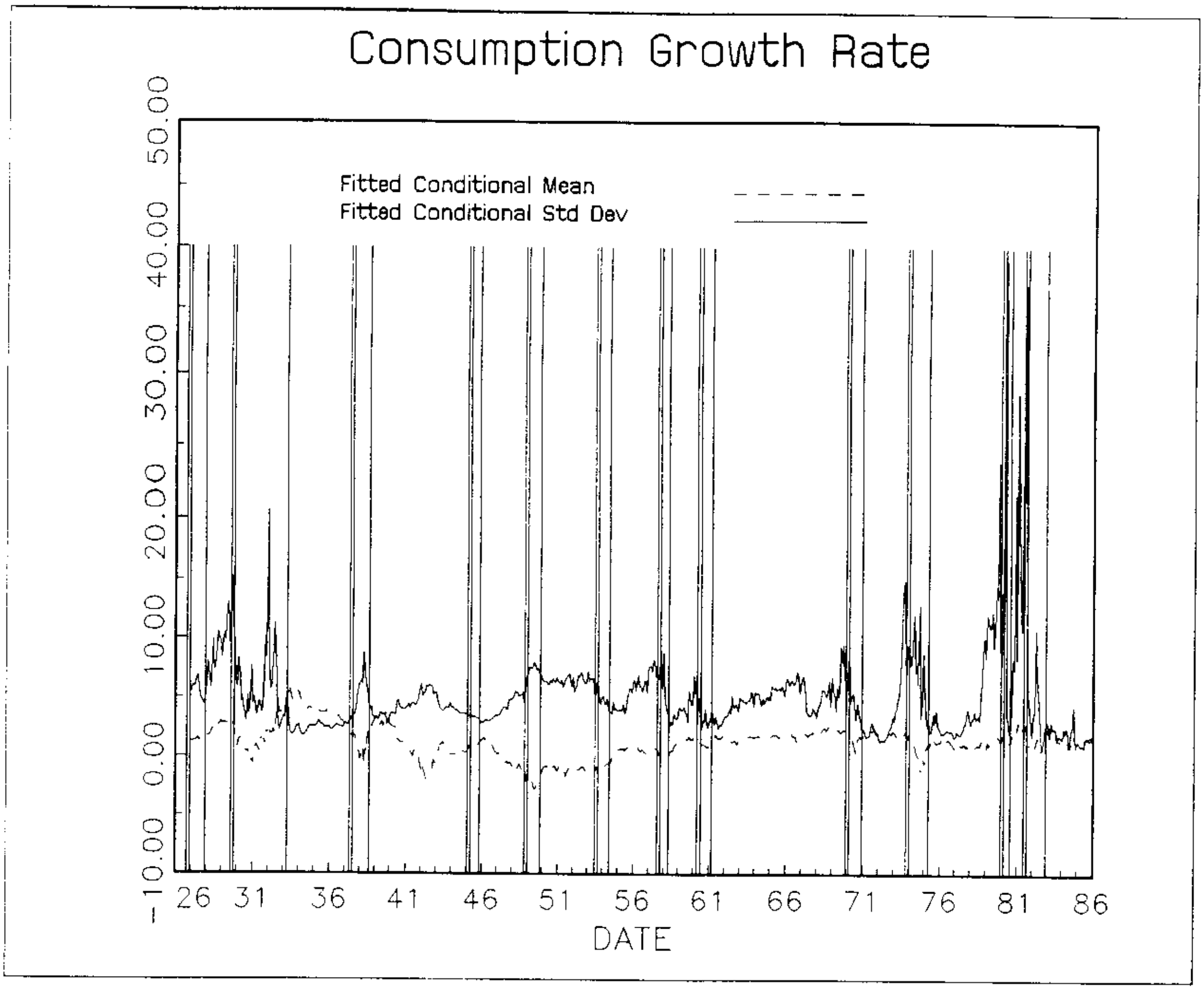

Figure 1. Estimated conditional means and standard deviations of one-quarter-ahead consumption growth (percent, annualized). The monthly values plotted are computed using coefficients obtained in regressions of quarterly growth rates and (the natural log of) squared unexpected growth rates on three predetermined financial variables: the Baa yield minus the Aaa yield, the Aaa yield minus the T-Bill yield, and the dividend-price ratio. Business cycle peaks are indicated by double rules (‖) and troughs are indicated by single rules (|).

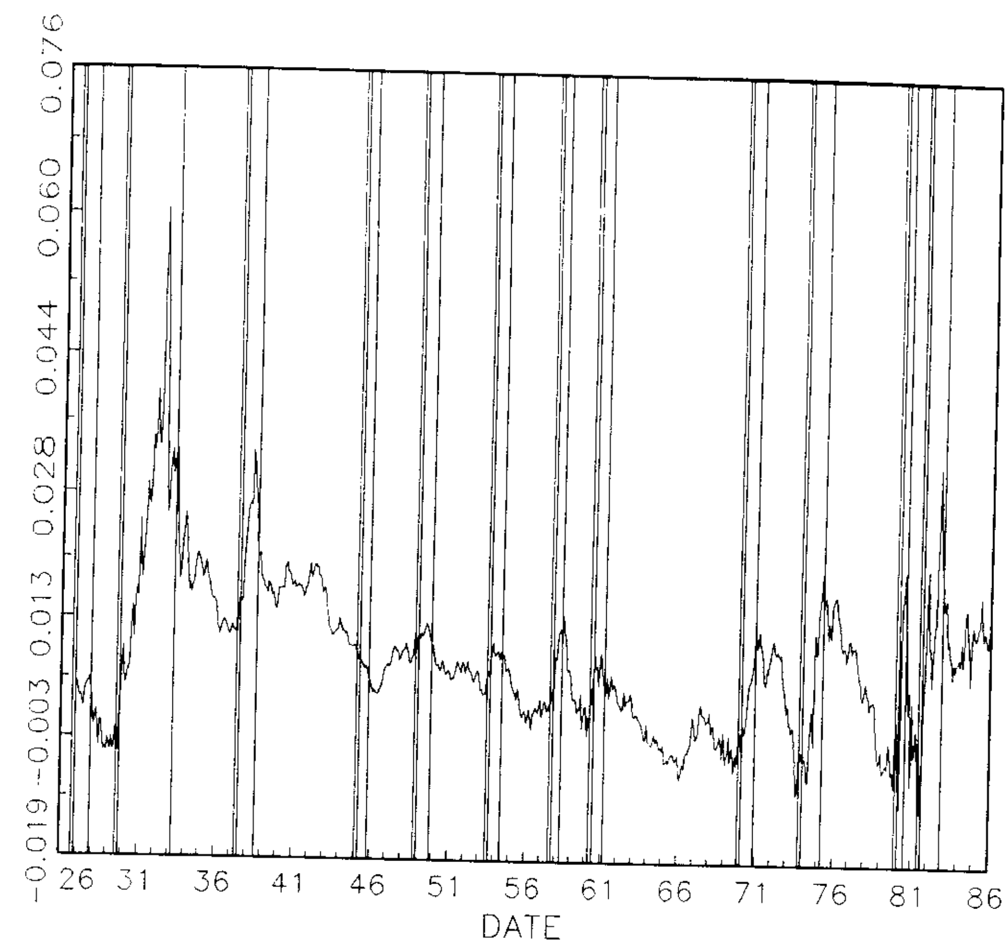

Figure 2. Estimated expected excess returns on the equally weighted NYSE portfolio for one-month horizons. The expected NYSE return is obtained from a regression of one-month returns on three predictive variables (the Baa yield minus the Aaa yield, the Aaa yield minus the T-Bill yield, and the dividend-price ratio). Expected excess returns are computed by subtracting the yield on one-month Treasury bills from this fitted value. Business cycle peaks are indicated by double rules (‖) and troughs are indicated by single rules (|).

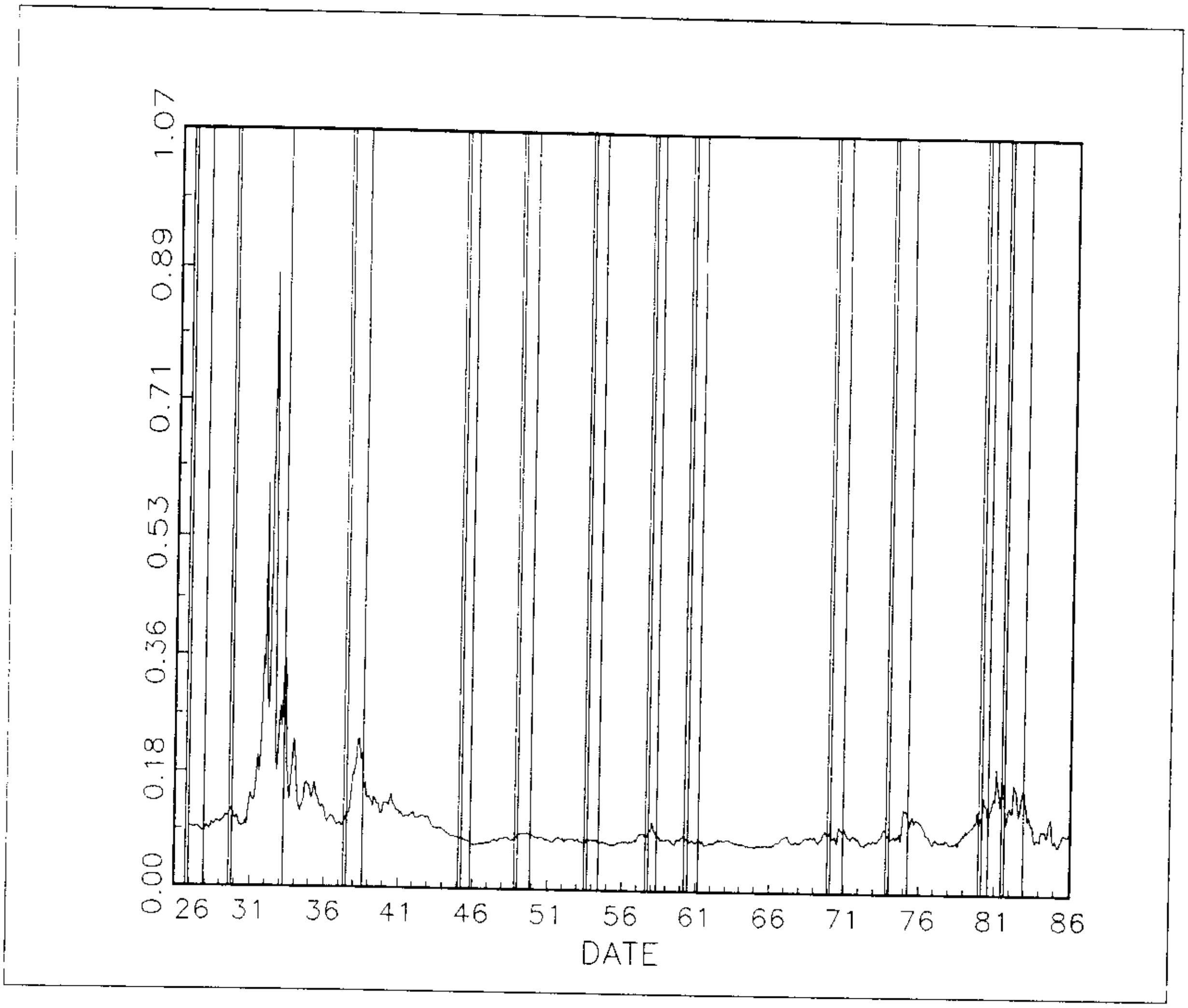

Figure 3. Estimated conditional standard deviations of monthly returns on the equally weighted NYSE portfolio. The value is obtained by regressing (the natural log of) squared unexpected monthly returns on three predictive variables (the Baa yield minus the Aaa yield, the Aaa yield minus the T-Bill yield, and the dividend-price ratio). Business cycle peaks are indicated by double rules (‖) and troughs are indicated by single rules (|).

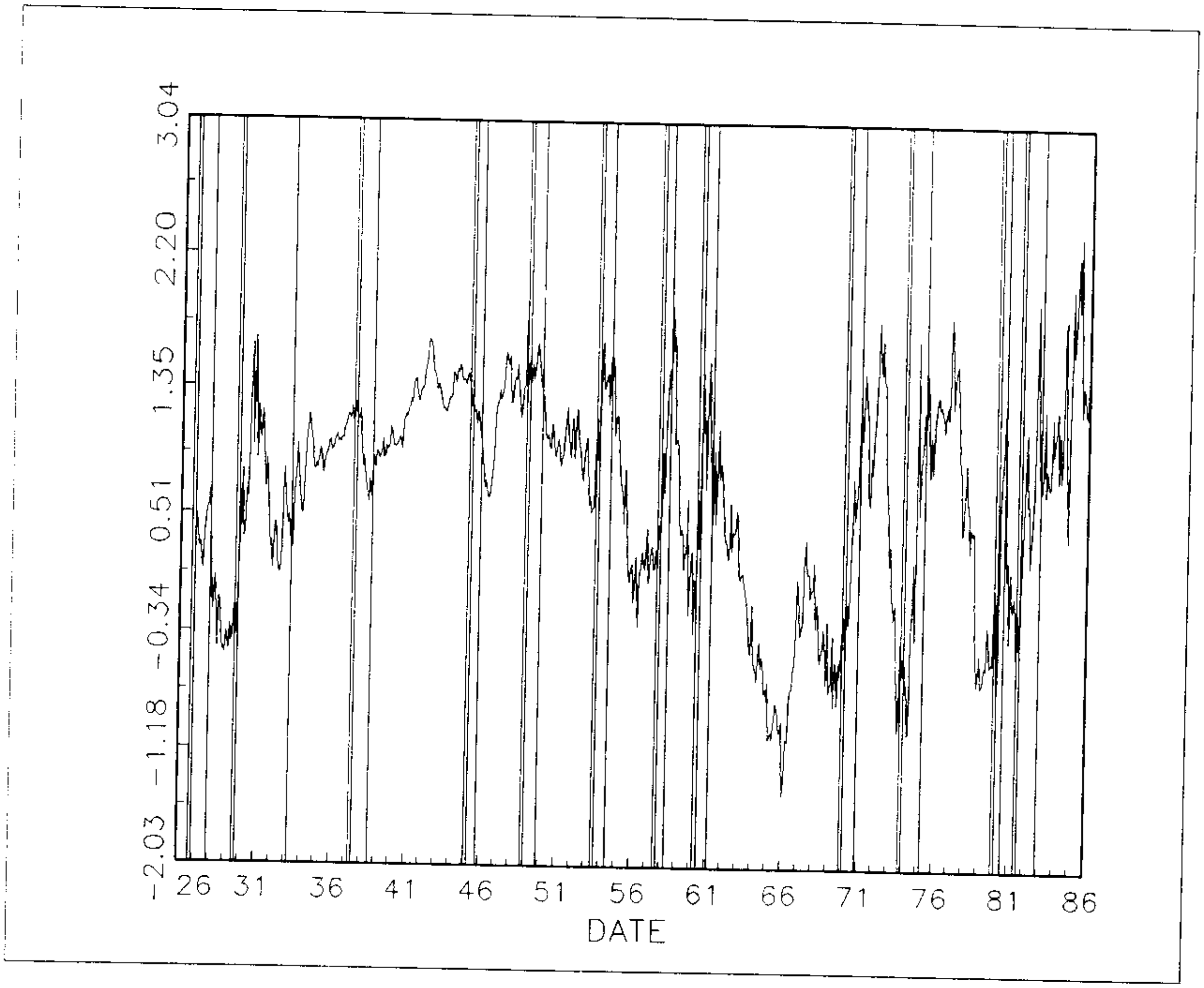

Figure 4. The ratio of estimated expected excess return on the equally weighted NYSE portfolio to the estimated conditional variance of return for one-month return horizons. Business cycle peaks are indicated by double rules (‖) and troughs are indicated by single rules (|).

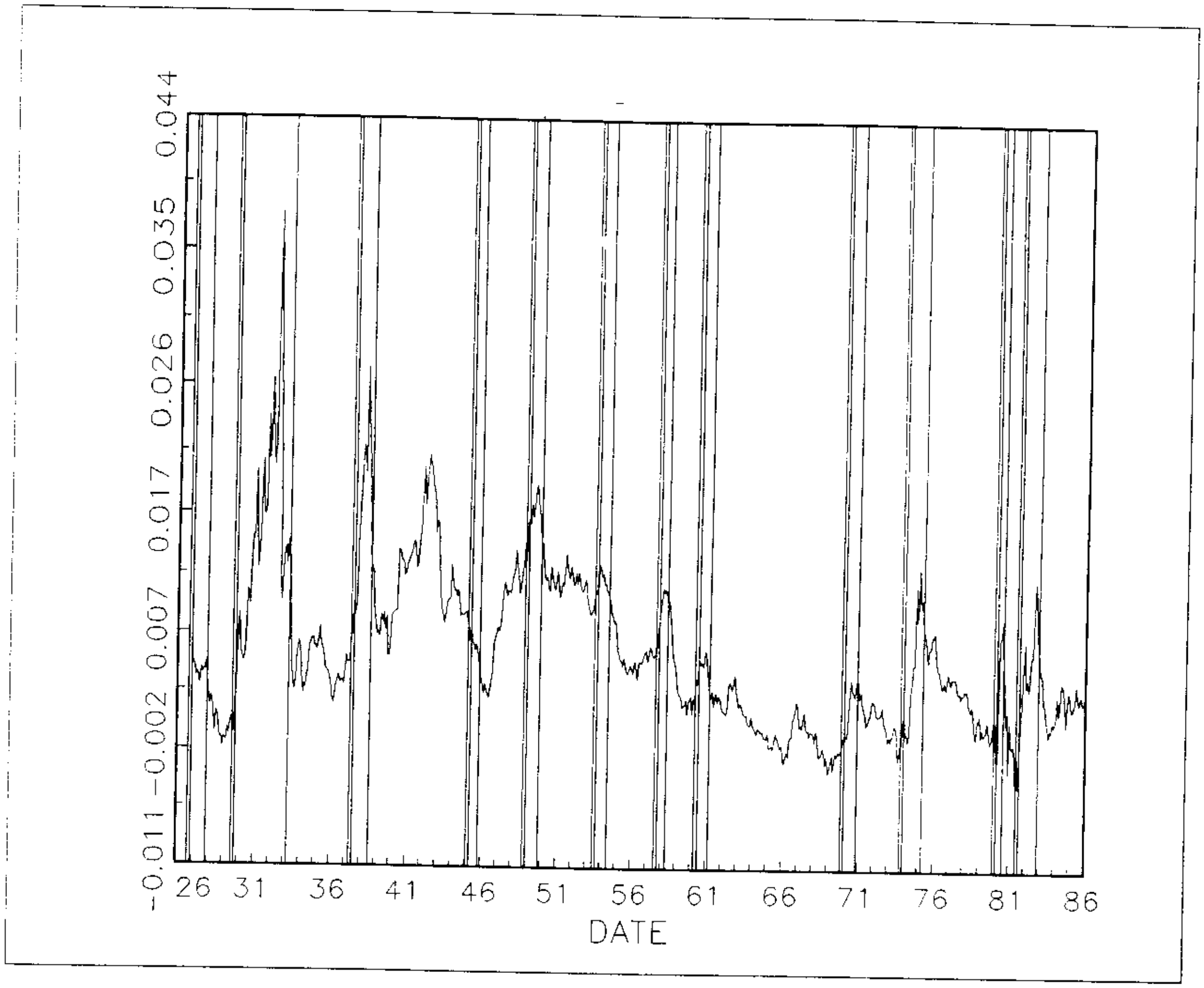

Figure 5. Estimated monthly expected excess returns on the equally weighted NYSE portfolio for five-year horizons. The value plotted corresponds to the expected monthly NYSE return for the five-year horizon beginning on the given date minus the Aaa yield (stated on a per-month basis). The expected NYSE return is obtained from a regression of five-year returns on three predictive variables (the Baa yield minus the Aaa yield, the Aaa yield minus the T-Bill yield, and the dividend-price ratio). Business cycle peaks are indicated by double rules (‖) and troughs are indicated by single rules (|).

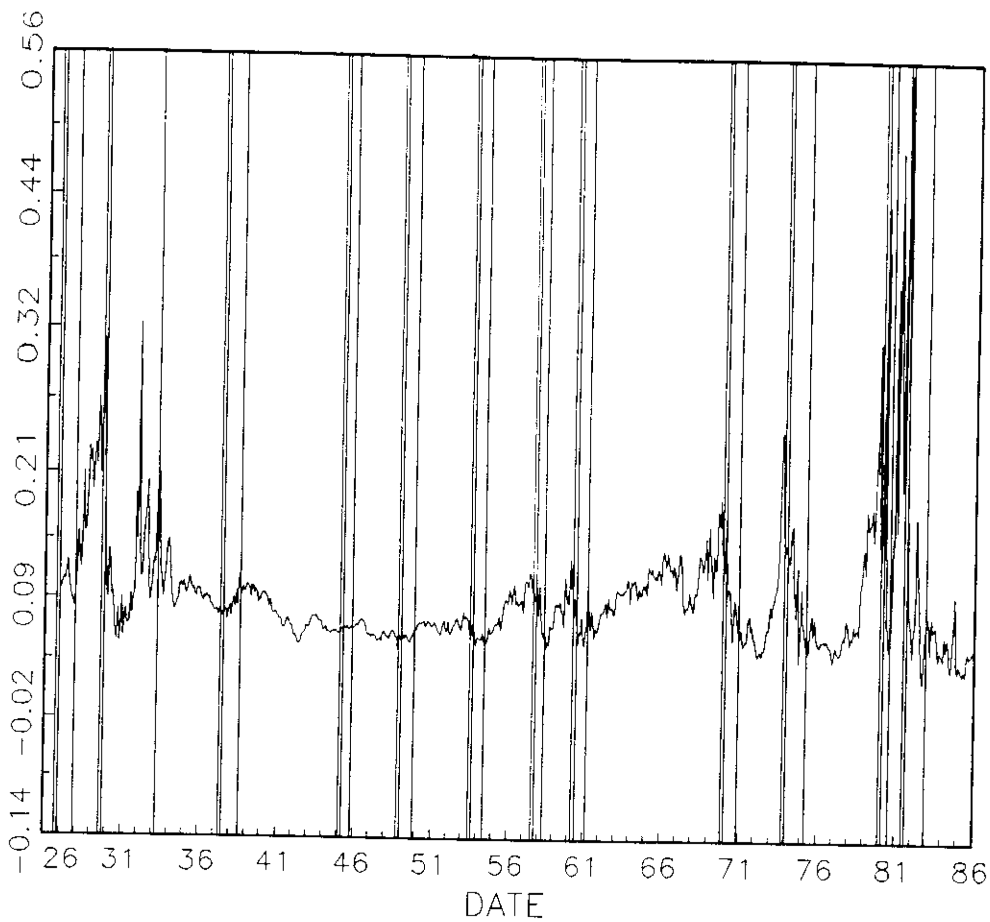

Figure 6. Estimated conditional standard deviations on the equally weighted NYSE portfolio for five-year-horizon returns, stated on a per-month basis. The value plotted corresponds to the standard deviation for the five-year horizon beginning on the given date. The value is obtained by regressing (the log of) squared unexpected five-year returns on three predictive variables (the Baa yield minus the Aaa yield, the Aaa yield minus the T-Bill yield, and the dividend-price ratio). Business cycle peaks are indicated by double rules (‖) and troughs are indicated by single rules (|).

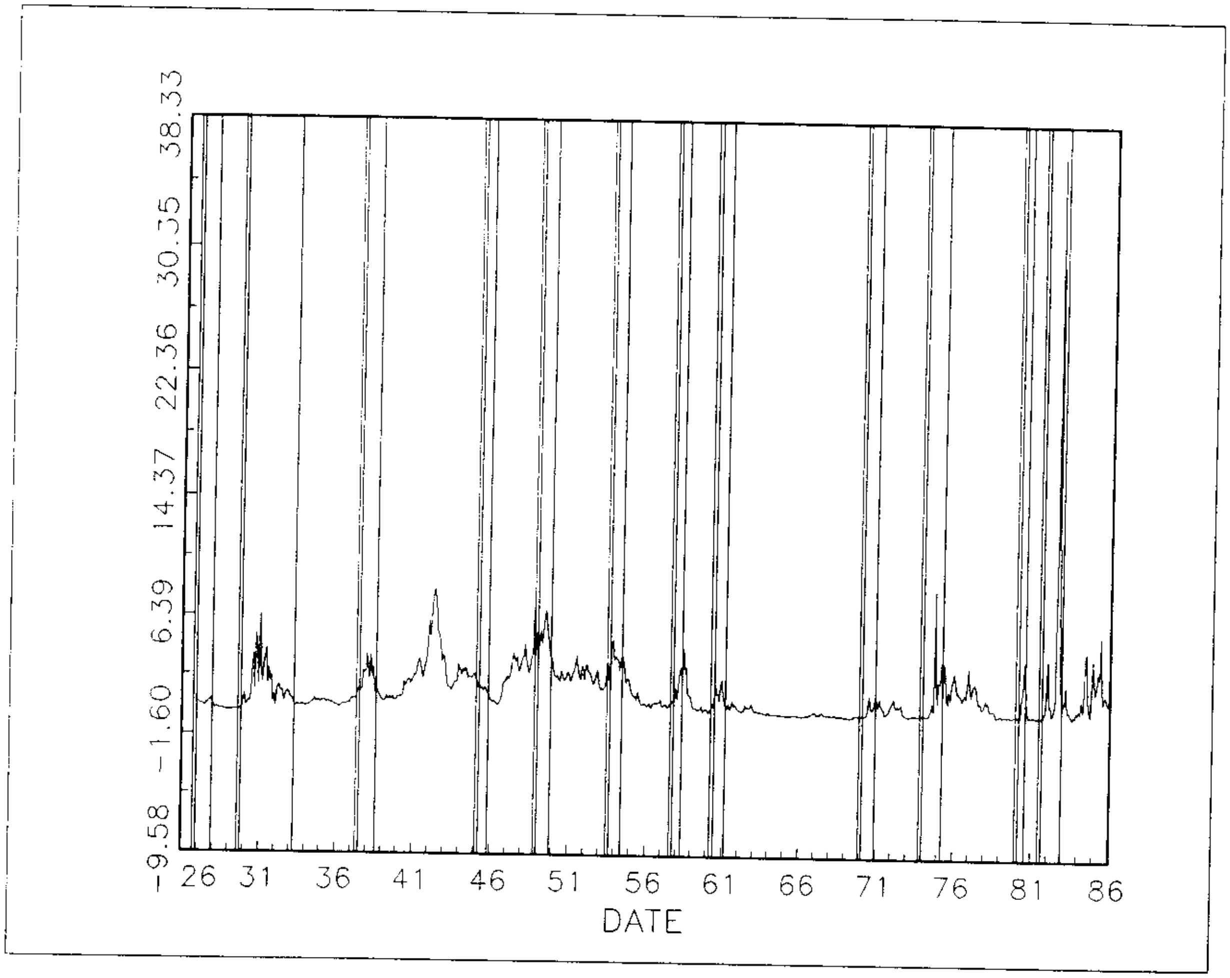

Figure 7. The ratio of estimated expected excess return on the equally weighted NYSE portfolio to the estimated conditional variance of return for five-year return horizons. Business cycle peaks are indicated by double rules (‖) and troughs are indicated by single rules (|).