INCOMPLETE MARKETS AND INCENTIVES TO SET UP AN OPTIONS EXCHANGE

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and an exchange is set up.

Ross (1976), Green and Jarrow (1987) and Green and Spear (1987) have shown that by increasing the number of securities that are traded it is possible to complete markets and hence improve risk sharing in the economy. Their analyses do not take account of the costs of setting up the markets or the incentives of exchange owners. Although opportunities for risk sharing may be improved, it is not clear that the exchange owners have the correct incentives to complete markets and that the benefits outweigh the costs. This is the issue that our analysis focuses on.

These models deal with the limiting case in which markets are approximately complete. When markets are far from complete the introduction of an extra market may not enhance efficiency. Hart (1975) and Newbery and Stiglitz (1984) show that increasing the number of markets can make everybody worse off. The reason is that risk sharing opportunities can be altered by the introduction of a new market. In contrast, in our model the reason that everybody can be worse off with an options market than without one is rather different. It is because there are transaction costs associated with setting up markets. The way in which exchange owners recoup these costs may not provide the correct incentives.

Geanakoplos and Polemarchakis (1987) have studied the efficiency of competitive equilibrium in economies where securities have payoffs denominated in terms of an abstract unit of account. Although the set of securities is given (and incomplete) the real characteristics of the securities can change when the prices of goods (measured in terms of the abstract unit of account) are changed. In this sense securities are endogenous. In a typical equilibrium it would be possible to make everyone better off by manipulating the prices and hence the securities. But price-taking agents do not perceive this possibility. In our analysis, achieving the first best requires a comparison of equilibria, as in the benchmark model. Similarly to Geanakoplos and Polemarchakis (1987), there is an inefficiency when agents take the equilibrium as given, as in the institutional model.

The paper proceeds as follows. Section 2 outlines the benchmark model. Section 3 shows that in this case the equilibrium market structure of the economy is socially efficient. Section 4 considers a model which is closer to institutional practice and demonstrates that the equilibrium market structure of the economy is not necessarily optimal. Section 5 looks at the issue of multiple equilibria. Section 6 considers the interactions between firms' capital structure decisions and the existence of an options exchange. Finally, Section 7 contains concluding remarks.

2. A Benchmark Model

To illustrate these ideas we use an elementary model of asset market equilibrium. The basic ideas are taken from two earlier papers (Allen and Gale, 1988a and 1988b).

There are two dates, indexed by $t = 1, 2$ and a finite set of states of nature, indexed by $s \in S$. Every economic agent has the same information structure: there is no information at the first date and the true state is revealed at the second. At each date there is a single consumption good that can be thought of as "income" or "money."

There is a continuum of identical firms. The set of firms has unit measure except where otherwise stated. A firm's profits at $t = 2$ are represented by a random variable $Z_0: S \to \mathbf{R}_+$. For each $s \in S$, $Z_0(s)$ represents the profits of the firm in state s. To begin with we shall assume that firms are passive: they do not have any decisions to make. Each firm is owned and controlled by a single entrepreneur. The entrepreneur values consumption only at the first date. Consequently, he wants to sell his equity in the firm in order to maximize his consumption at the first date. Since the value of the firm is determined by the market, the firm/entrepreneur is merely a passive consumer.

There are two types of investors, indexed by i = a, b. Each type of investor is a continuum and, for simplicity, we assume that each continuum has unit measure. Investors are risk averse and value consumption at each date. An investor's consumption set is denoted by X, where X is the set of functions from $S \cup \{1\}$ to $\mathbf{R}$. For any $\xi \in X$, $\xi(1)$ denotes consumption at date 1 and $\xi(s)$ denotes consumption at date 2 in state $s \in S$. The preferences of an investor of type i are represented by a utility function $U_i: X \to \mathbf{R}$ for each type i = a, b. If an investor chooses a consumption bundle $\xi \in X$ then his utility is given by $U_i(\xi)$. Each investor has an initial endowment $0 \in X$.

In addition to firms and investors there is an agent, called the exchange owner, who has the right to set up an options exchange for the trading of derivative securities. There is a fixed cost of setting up the exchange which is equal to γ units of first-period consumption. We assume that only a single security can be traded on the exchange. The derivative security is represented by a function $Z_1: S \to \mathbf{R}$. One unit of the security entitles the owner to $Z_1(s)$ units of the consumption good at the second date in state s, for every $s \in S$. The economy's endowment of the derivative security is equal to zero. The exchange owner chooses whether to open the exchange and, if the exchange is opened, what security should be traded. Since he only values consumption at the first date, he makes these decisions with a view to maximizing his first-period consumption.

Equilibrium at date 1 is determined in two stages. At stage 0, it is decided whether the options exchange should open and, if so, what kind of derivative security should be traded and what fees should be levied on the firms and investors. At stage 1, investors trade securities on the available markets. The definition of equilibrium begins with stage 1.

Equilibrium at stage 1

The owner's decision to open the options exchange is indicated by the dummy variable α. The exchange is closed if $\alpha = 0$ and it is open if $\alpha = 1$. The vector of securities that can be traded at stage 1 is denoted by $Z = (Z_0, Z_1)$, where Z_0 is the underlying stock and Z_1 is the derivative security. Finally, let $e = (e_a, e_b, e_f)$ denote the vector of fees (positive or negative) imposed by the owner of the exchange at stage 1. For each $i = a, b$, e_i is the fee imposed on an investor of type i and e_f is the fee levied on a firm.

Suppose that at stage 0 it has been decided to open the exchange ($\alpha = 1$). In that case investors can trade in both securities at stage 1. Let $v = (v_0, v_1)$ denote the vector of security prices, where v_0 (resp. v_1) is the price of the underlying stock (resp. derivative security) measured in terms of consumption at date 1. An investor takes the security prices v as given and chooses a consumption level c and a portfolio $d = (d_0, d_1)$ where d_0 (resp. d_1) is his demand for the equity of the firm (resp. the derivative security). Short sales of equity are not allowed: $d_0 \geq 0$. (This assumption allows the concepts developed in Allen and Gale (1988a) to be used in Section 6 where firms issue both debt and equity. It is discussed further there.) An investor's endowment is normalized to zero at each date. Then the budget constraint of an investor of type i = a, b can be written:

$$c + v \cdot d + e_i = 0, \text{ for } i = a, b .$$

Now suppose that the exchange is not open at stage 1 ($\alpha = 0$). In that case there is no derivative security and there are no fees imposed on the investors and firms. Nonetheless it is convenient to use the same notation for this case as well. An investor is assumed to choose a pair (c, d) subject to the constraints:

$$c + v \cdot d \leq 0 \text{ and } d_1 = 0 \ .$$

The first of these constraints is the budget constraint; the second reflects the fact that he cannot trade the derivative security.

In this notation, $B_i^\alpha(v, e)$ will denote the budget set of an investor i = a, b when the prevailing security prices are v, for any choice of (α, Z, e). Define $B_i^\alpha(v, e)$ by putting

$$B_i^\alpha(v, e) = \begin{cases} \{(c_i, d_i) \in R^3 \mid c + v \cdot d + e_i \leq 0\} \text{ if } \alpha = 1; \\ \{(c_i, d_i) \in R^3 \mid c + v \cdot d \leq 0 \text{ and } d_1 = 0\} \text{ if } \alpha = 0 \ . \end{cases}$$

This allows us to give a concise definition of equilibrium.

<u>Definition</u>: Let (α, Z, e) be given. Equilibrium in stage 1, relative to (α, Z, e), is defined by an array $\{v, (c_i, d_i)\}$, consisting of a price vector v and a choice (c_i, d_i) for each type of investor i = a, b, that satisfies the following conditions:

(i) $(c_i, d_i) \in \arg\max_{B_i^\alpha(v, e)} U_i(c_i, d_i \cdot Z)$ for i = a, b;

(ii) $\Sigma_i \, d_i = (1, 0)$.

Condition (i) simply says that each investor is maximizing his utility subject to the appropriate budget constraint and to the available markets. Condition (ii) is the market-clearing condition. If the security markets clear then

Walras' law ensures the goods market clears as well. Note that equilibrium in stage 1 does not refer explicitly to firms or to the exchange owner. Firms are passive participants in the equilibrium. They simply consume the value of their equity v_0. The exchange owner has made all his decisions at stage 0.

Let $\phi^{\alpha}(Z, e)$ denote the set of stage 1 equilibria relative to (α, Z, e). Let $u_i^{\alpha}(v, e, Z)$ denote the maximum utility attainable when security prices are given by v and the stage 0 choices are given by (α, Z, e). That is,

$$u_i^{\alpha}(v, Z, e) = \sup_{B_i^{\alpha}(v, e)} U_i(c_i, d_i \cdot Z) \text{ for } i = a, b .$$

Also, define the utility of firms to be $u_f^{\alpha}(v, Z, e) = v_0$ if $\alpha = 0$ and $v_0 - e_f$ if $\alpha = 1$.

<u>Equilibrium at stage 0</u>

The exchange owner chooses the derivative security Z_0 that is to be traded on the exchange, the fees e that are to be charged for entry to the exchange and finally decides whether to open the exchange ($\alpha = 1$) or to leave it closed ($\alpha = 0$). Firms and investors are assumed to have a veto on the opening of the exchange. The veto works as follows. The exchange owner announces the derivative security Z_1 he would like to have traded and the fee structure he would like to charge. All agents accept or reject. If one (or more) agent rejects, the exchange cannot be opened ($\alpha = 0$). On the other hand, if they all accept then the exchange owner is given the choice of whether to open the exchange. Notice that the exchange owner is allowed to impose fees on all agents, including the firms, who will not actually use the exchange. Even firms may have an interest in seeing the exchange open, however, because of its effect on the value of their equity. The fees charged can be negative, in which case they constitute a transfer to the agent. These transfers may be necessary in order to obtain the agreement of all the agents to the exchange opening. This mechanism for choosing the market structure is patently artificial. The reason for studying it is simply that it produces an efficient choice of market structure. In subsequent sections we shall look at a model that conforms more closely to the practices of actual institutions.

Suppose that the exchange owner has announced his choice of Z_1 and e. Whether the firms and investors will accept this proposal depends on the equilibrium they anticipate in the second stage following a decision to open the exchange ($\alpha = 1$) or let it remain closed ($\alpha = 0$). In effect, these agents are comparing two equilibria and deciding which they prefer. Suppose the two equilibria are denoted by $\{v^\alpha, (c_i^\alpha, d_i^\alpha)\}$ for $\alpha = 0, 1$. It will be individually rational for an agent of type i = a, b, f to accept the exchange owner's proposal if and only if:

$$u_i^0(v^0, e, Z) \leq u_i^1(v^1, e, Z) .$$

This constraint must be satisfied if the exchange owner wants to open the exchange. On the other hand, he is not compelled to open the exchange even if all the agents accept his proposal, so there is no loss of generality in assuming that his proposal is always individually rational for all agents. We can also assume, again without loss of generality, that agents accept the proposal whenever the individual rationality constraint is satisfied. If not, the exchange owner could always win unanimous approval by decreasing slightly every agent's fee. In a perfect equilibrium agents would always accept when the individual rationality constraint was just satisfied. This leads us to the following definition.

<u>Definition</u>: A pre-equilibrium is defined by an array $\{(v^\alpha, (c_i^\alpha, d_i^\alpha)), \alpha, e, Z\}$ consisting of a second stage equilibrium $(v^\alpha, (c_i^\alpha, d_i^\alpha))$ for each $\alpha = 0, 1$, a

decision α whether to open the exchange, a fee structure e and a choice of securities Z, that satisfies the following conditions:

(i) $u_i^0(v^0, e, Z) \le u_i^1(v^1, e, Z)$ for $i = a, b, f$;

(ii) $\{v^\alpha, (c_i^\alpha, d_i^\alpha)\} \in \Phi^\alpha(e, Z)$ for $\alpha = 0, 1$,

(iii) Z_0 is given.

A pre-equilibrium is called a full equilibrium if there does not consist another pre-equilibrium that yields higher profits $\alpha(\Sigma_{i = a, b, f}\, e_i - \gamma)$ to the exchange owner.

Notice that in the definition of full equilibrium we are not only assuming that the exchange owner maximizes profits with respect to his choice of (α, e, Z). We also implicitly allow him to choose the equilibrium that will follow at the second stage. In Section 5 we consider what would happen if some other selection procedure were followed.

3. Constrained Efficiency

In this section we consider the efficiency of the equilibrium market structure in the benchmark model. The efficiency concept we use is similar to the one used in Allen and Gale (1988a). An equilibrium is said to be constrained efficient if a central planner who is subject to the same transaction costs as individual agents, cannot make some agents better off without making some agents worse off. In this context, the condition "subject to the same transaction costs" is interpreted to the mean that the planner can only change the market structure and reallocate securities and consumption at the first date. We show that, in this sense, the equilibrium market structure in the benchmark model is efficient.

Assumption: U_i is continuous, strictly increasing and quasi-concave for $i = a, b$.

Under this assumption, whenever it is possible to make some agents better off without making any agents worse off, it is possible to make all agents strictly better off. Since firms and the exchange are only interested in first-period consumption, utility is effectively "transferable" between them. An allocation is Pareto-dominated, then, if and only if there exists an alternative allocation that makes investors at least as well off and leaves more consumption for the firms and the exchange owner. This suggests the following definition.

Definition: An equilibrium allocation $\{(c_i, d_i), \alpha, Z\}$ is constrained efficient if there does not exist another allocation $\{(c_i', d_i')\ \alpha', Z'\}$ such that:

(i) $U_i(c_i, d_i \cdot Z) \leq U_i(c_i', d_i' \cdot Z')$ for $i = a, b$;

(ii) $\Sigma_i\ c_i + \alpha\gamma > \Sigma_i\ c_i + \alpha'\gamma$;

(iii) $\Sigma_i\ d_i' = (1, 0)$ and, if $\alpha' = 0$, $d_{i1}' = 0$ for $i = a, b$.

To prove that the equilibrium choice of market structure is constrained efficient, we assume that there exists a better allocation and obtain a contradiction. We need to consider two different cases. Suppose first that there exists a Pareto-preferred allocation that involves opening the exchange. It can be shown that this implies the existence of a Pareto-preferred allocation that can be supported as an equilibrium. That is, there exists a pre-equilibrium that yields higher profits for the exchange owner than the given equilibrium, contradicting the definition of equilibrium.

In the second case, we suppose the equilibrium can be Pareto-improved on by an allocation in which the exchange is closed. When the exchange is closed, the model is isomorphic to an Arrow-Debreu model in which the commodities are consumption and equity. The usual argument suffices to prove constrained efficiency holding the market structure constant. Using this property, we can show that in the original equilibrium there must exist some investor or firm who would be better off in the second stage equilibrium if the exchange were closed. But that agent should have vetoed the exchange, contradicting the definition of equilibrium.

<u>Proposition 1</u>

If $\{(v^\alpha, (c_i^\alpha, d_i^\alpha)), \alpha, e, Z\}$ is a full equilibrium of the benchmark model then the associated allocation $\{(c_i, d_i), \alpha, Z\}$ is constrained efficient.

<u>Proof</u>: Suppose, contrary to what we want to prove, that the allocation is not constrained efficient. Let $\{(c_i', d_i'), \alpha', Z'\}$ be the alternative, Pareto-preferred allocation.

First consider the case where $\alpha' = 1$. Under the maintained assumption there is no loss of generality in taking $\{(c_i', d_i')\}$ to minimize $\Sigma_i\ c_i'$ subject to the individual rationality constraint $U_i(c_i^0, d_i^0 \cdot Z) \leq U_i(c_i', d_i' \cdot Z')$ for $i = a, b$. By the usual supporting hyperplane argument, we can show that the allocation $\{(c_i', d_i')\}$ can be supported as an equilibrium at the second stage. More precisely, under the maintained assumption, there exists a price vector v' and fee structure e' such that $(c_i', d_i') \in \arg\max U_i(c, d \cdot Z')$, where the maximum is taken over the budget set $B_i^1(v', e')$. Define $e_f' = v_0^0 - v_0'$. It is straightforward to check that $\{(v^0, (c_i^0, d_i^0)), (v', (c_i', d_i')), (\alpha', e', Z')\}$ is a pre-equilibrium. By hypothesis, this pre-equilibrium yields higher profits for the exchange owner than the full equilibrium, a contradiction.

Second, consider the case where $\alpha' = 0$. We begin by noting that the second stage equilibrium $\{v^0, (c_i^0, d_i^0)\}$ is constrained efficient when the market structure is taken as given. To show this, suppose to the contrary that there is a Pareto-preferred allocation $\{(c_i', d_i'), \alpha', Z'\}$ with $\alpha' = 0$. Since $\alpha' = 0$ we can assume without loss of generality that $Z' = Z$. From the definition of stage 1 equilibrium, $c_i + v \cdot d_i \le c_i' + v \cdot d_i'$ for $i = a, b$. Since $\Sigma_i d_i = (1, 0)$, this implies that $\Sigma_i c_i + \alpha\gamma \le \Sigma_i c_i' + \alpha'\gamma$, as required. Now suppose that the equilibrium allocation $\{(c_i, d_i), \alpha, Z\}$ is Pareto-dominated by the alternative allocation $\{(c_i', d_i'), \alpha', Z'\}$. Without loss of generality we can assume that the alternative allocation $\{(c_i', d_i'), \alpha', Z'\}$ makes firms and both types of investor strictly better off. Since $\{v^0, (c_i^0, d_i^0)\}$ is constrained efficient, at least one type of investor or firm must be at least as well off at $\{v^0, (c_i^0, d_i^0)\}$ as he is in the alternative allocation $\{(c_i', d_i'), \alpha', Z'\}$. But this means that the individual rationality constraint cannot be satisfied, contradicting the definition of equilibrium.•

4. An Institutional Model

A number of features of the benchmark model differ from actual practice. First, the investors who are going to trade in the exchange do not in practice agree to make payments before the exchange is set up. Second, firms and investors do not in practice have a veto over the opening of the exchange. In fact, the firms whose value forms the basis for the derivative security do not receive or make payments to the owner of the exchange. Finally, we do not observe lump sum payments for the right to trade on the exchange. The fees charged to investors depend in a more or less complex way on the volume of the security traded. The second version of the model we consider is intended to correspond more closely to actual institutional practice. In particular, we assume that the exchange is set up before investors are asked to pay for the right to trade on the exchange. If any investor refuses to pay, he is denied access to the exchange and can only trade shares in the firms. We also assume that there is no payment between the exchange owner and the firms ($e_f = 0$). For simplicity, we retain the assumption that the investors pay lump sum fees. We shall refer to this set of assumptions as the institutional model, to distinguish it from the benchmark model described in the previous section.

If the exchange owner decides not to open the exchange ($\alpha = 0$) then the definition of second stage equilibrium is the same as in the benchmark model. If the owner does open the exchange ($\alpha = 1$) then investors face a more complex problem than before. They must first decide whether to pay the entry fee and then choose an optimal consumption level and portfolio from the appropriate budget set. The description of equilibrium can be considerably simplified, however, if we recognize that in equilibrium all investors will enter the exchange. Since we only consider symmetric equilibria, either all investors of type i will pay the entry fee or none will. However, there can be no trade on the exchange unless both types of investors enter. Thus, in equilibrium, either all investors enter the exchange or none do. The exchange owner will never open the exchange unless he can recoup his costs, so the only equilibria we need to consider are those in which all investors decide to enter the exchange.

With this simplification, an equilibrium at the second stage can be described by the same definition given in Section 2. The only change is that it must now be rational for investors to accept the proposed fees after the exchange has been set up. This requirement can be captured by changing the individual rationality conditions in the definition of equilibrium at stage 0. Suppose that $\{v, (c_i, d_i)\}$ is a stage 1 equilibrium relative to (α, e, Z), where $\alpha = 1$. It is individually rational for an investor of type $i = a, b$ to pay the fee e_i and enter the exchange if and only if the following inequality is satisfied:

$$u_i^0(v, e, Z) \le u_i^1(v, e, Z) .$$

Contrast this individual rationality constraint with the earlier one. In the benchmark model the investor can veto the options exchange. If he refuses to pay the fee he finds himself in a different equilibrium, facing different prices as well as different markets. In the present model, the exchange is open whether he decides to pay the fee or not. If he refuses to pay the fee, he finds himself excluded from the market for the derivative security; but he is in the same equilibrium, facing the same prices.

Definition: A pre-equilibrium of the institutional model is defined by an array $\{(v^\alpha, (c_i^\alpha, d_i^\alpha)), \alpha, e, Z\}$ consisting of a second stage equilibrium $(v^\alpha, (c_i^\alpha, d_i^\alpha))$ for each $\alpha = 0, 1$, a decision α whether to open the exchange, a fee structure $e = (e_a, e_b)$ and a choice of securities Z, that satisfies the following conditions.

(i) $u_i^0(v^1, e, Z) \le u_i^1(v^1, e, Z)$ for $i = a, b$;

(ii) $(v^\alpha, (c_i^\alpha, d_i^\alpha)) \in \Phi^\alpha(e, Z)$ for $\alpha = 0, 1$,

(iii) Z_0 is given.

A pre-equilibrium is called a full equilibrium if there does not exist another pre-equilibrium that yields higher profits $\alpha(\Sigma_{i = a, b}\, e_i - \gamma)$.

Now we can see clearly the difference between the two definitions of equilibrium. In the benchmark model, the individual rationality constraint applies to all firms and investors. Furthermore, it requires that setting up the exchange be individually rational. In deciding whether to accept the exchange owner's proposal, an agent in the benchmark model is comparing his utility levels in two different equilibria, one with an open exchange and one without. He will agree to pay the exchange owner's charge (accept his transfer) only if his utility will be at least as great in the equilibrium with an options exchange as in the equilibrium without it. The maximum charge that can be extracted from him thus reflects the true economic value, to him, of opening the exchange. In this sense, the rents the exchange owner can extract reflect the true economic value of the exchange, so it is perhaps not surprising that he is led to choose an efficient market structure.

In the institutional model, by contrast, the individual rationality constraint applies only to investors. Firms need not be compensated if the opening of the exchange damages them by reducing their value; nor can they be asked for payments if it benefits them by increasing their value. Furthermore, the investors' individual rationality constraints have changed. In the benchmark model, individual rationality for an investor depended on an inter-equilibrium comparison. In the institutional model it depends on an intra-equilibrium comparison. If an investor refuses to pay the exchange owner's charges, the alternative is to remain outside the options exchange trading only in the ordinary equity market. He will pay the charge only if his utility with access to the exchange is at least as great as his utility without access to the exchange, in the same equilibrium. The maximum charge that can be extracted from the investor thus reflects the value of access to the exchange, within a fixed equilibrium.

Once the exchange has been opened, the bargaining power of the exchange owner to extract rents is different from what it was before the exchange opened. It may be greater; it may be less. There is no reason to think that the exchange owner's incentives in the institutional model will lead to an efficient choice of market structure.

We demonstrate next, by means of example, that this is the case: in the institutional model the equilibrium market structure is not necessarily efficient. We start by considering two-state examples; in this case all options are equivalent in the sense that they complete the market so the decision of which type of contract to offer is unimportant. Later we consider three state examples to show how the type of option contract offered matters.

Our second result is the following.

Proposition 2

In the institutional model, the equilibrium market structure is not necessarily constrained efficient: (i) there are parameter values such that an equilibrium exists in which an exchange is not set up but Pareto superior allocations could be reached if an exchange were set up; and (ii) there are also parameter values such that an equilibrium exists in which an exchange is set up but Pareto superior allocations could be reached if an exchange were not set up.

Proof: The proposition is demonstrated by means of examples. Example 1(i) shows the first part of the proposition and Example 1(ii) the second part.

Example 1(i)

There are two equally-probable states of nature. There is one type of producer with outputs (1, 2) in the two states respectively. The measure of firms is 2.

The cost of setting up an options exchange is 0.002. This is independent of the security structure offered by the exchange.

There are two groups of consumers (i = a, b). The measure of both groups of consumer is 1 and their initial endowment is 5. They have von Neumann-Morgenstern utility functions of the form:

$$W_i(x^1, x^2) = x^1 - \exp(-\rho_i x^2) \tag{1}$$

where $\rho_a = 1$ and $\rho_b = 2$. Note that the utility function is linear in first-period consumption. This is an assumption we use in all the examples. It has strong implications for the nature of equilibrium. In particular, since agents have transferable utilities we can identify potential Pareto improvements by looking at the sum of agents' utilities.

First consider the stage 1 equilibrium when no options are available. In this case the only security that is traded is the firms' equity with payoffs (1, 2). It can readily be shown that the equilibrium values of variables of interest are as in Table 1(i)a.

Since there are only two states, any option (except a call with a striking price of 0) leads to complete markets. For the sake of illustration suppose the exchange offers a put with a striking price of 2 which has payoffs (1, 0). In this case the equilibrium values of the variables of interest are given in Table 1(i)b.

Table 1(i)c summarizes the differences between Table 1(i)a and Table 1(i)b. It can be seen that both groups of consumers are better off with complete markets than with incomplete markets: $\Delta W_a = 0.00256$; $\Delta W_b = 0.00256$. However, the value of the firm has fallen so that producers are worse off: ΔFirm Value = -0.00170. Hence in terms of first period consumption the gross social gain is the sum of these three terms which is

0.0034. Since the cost of setting up an exchange is only 0.002, the net social gain is 0.0014. A social planner with the ability to reallocate first-period consumption and securities would be better off to set up the exchange.

What incentives does the exchange owner have? The amount that he can charge depends on the utility levels of the two groups' if they were denied access to the options exchange. In other words, the amount that can be charged in terms of first period consumption is equal to the difference between the utility level with complete markets and the utility level that can be obtained with access to the equity market alone at the complete markets price of 0.25360. Table 1(i)d gives the details of this comparison.

It can be seen that since $0.00168 < 0.002$ the options exchange would not be set up in this case even though it is socially efficient for an exchange to be set up. This demonstrates part (i) of Proposition 1.

Example 1(ii)

The details of this example are identical to those of Example 1(i) with two exceptions. The first is that the measure of firms is 0.4 instead of 2 and the second is that the cost of setting up the options exchange is 0.004 instead of 0.002.

The solutions to this example are given in Tables 1(ii)a-1(ii)d. In this case even though it is undesirable to set up an exchange, nevertheless the exchange owner has an incentive to do so. This demonstrates part (ii) of the proposition.•

The reason Proposition 2 holds is that the amount the exchange owner can charge depends on traders' reservation utilities if they do not participate in the market. In other words, it depends on an intra-equilibrium comparison rather than an inter-equilibrium comparison. The constraint $e_f = 0$ is not crucial for these results to hold; they would also hold if firms could be charged.

One point to notice is that it does not depend on the owner of the exchange being able to discriminate in terms of the fees charged to buyers and sellers. Even if the owner is restricted to charging them the same amount it can be seen that his decision in both of these particular examples is the same as when he can discriminate.

Part (ii) of the proposition is perhaps the more interesting result. It shows that moving to complete markets can lead to an allocation that is Pareto dominated by an allocation that could be reached with incomplete markets. It also shows that leaving the creation of derivative markets to profit-seeking behavior can lead to there being too many markets. Although the result has been derived in terms of a model where an exchange owner has a monopoly right to offer derivative securities, a similar result seems likely to hold in a number of other institutional settings. For example, suppose there is an incumbent exchange and that potential entrants can set up rival exchanges if the incumbent's profits are too high. This will limit the entry fee that the incumbent exchange charges but otherwise the model will be similar. Consider Example 1(ii) but with a cost of setting up the exchange of 0.00739 so that establishing an exchange is just profitable for the incumbent. The profits in this case will be such that the potential entrants do not enter. Competition does not solve the problem.

Another interesting variant of the model is the case where the exchange is a non-profit organization. This is closer to the institutional structure that we actually observe. The problem here is to specify an objective function for the exchange. One possibility is that the exchange can use any surplus that is generated to increase the utilities of its members. For example, the offices provided to members or the salaries paid to employees could be higher than is strictly necessary. In this case the exchange will behave in much the same way as the profit-seeking exchange modelled here. Thus the fact that we observe non-profit exchanges where a large volume of securities is traded does not necessarily mean that such institutions are welfare improving; in fact the reverse can be true.

So far it has been demonstrated that the incentives to set up an exchange are not necessarily the correct ones. However, conditional on the exchange being set up the equilibrium market structure is efficient in the examples considered. We turn next to the question of whether, given the correct decision to set up, the exchange owner has the correct incentives to offer the socially desirable set of contracts.

Proposition 3

It can be more profitable for the options exchange to offer a security which fails to complete the market even though a security is available (at the same cost) which does this.

Proof: The proposition is again demonstrated by means of an example.

Example 2

There are three states of nature, s = 1, 2, 3, with probabilities (0.5, 0.25, 0.25) respectively. The firms have outputs (0.08, 1.8, 2) in the three states respectively. The measure of firms is 1.

The cost of setting up an options exchange which offers one or two options is 0.001. In other words the marginal cost of offering a second option is zero.

There are two groups of investors (i = a, b). Investors of type a have exponential utility functions as in (1) with $\rho_a = 20$. For positive second-period consumption, investors of type b have logarithmic utility functions of the form

$$W_b = x^1 + \ln(x^2). \tag{2}$$

For zero and negative second-period consumption their utility is $-\infty$. The measure of both groups of investors is 1 and their initial endowment is 5.

First consider the stage 1 equilibrium when no options are available. In this case the only security that is traded is the firms' equity with payoffs (0.08, 1.8, 2). It can be shown that the equilibrium values of the variables of interest are as in Table 2a. Now suppose that markets are complete. The equilibrium consumption allocations are given in Table 2b. A derivative security which will support this equilibrium allocation has payoffs (0.01393, 0.17409, 0.17974). Table 2c contains a comparison of the case where there are complete markets with the case where markets are incomplete. It can be seen that the surplus from setting up an exchange is more than enough to cover the cost of doing so. Hence an exchange which issues a security that completes the market is welfare enhancing.

Note that this is the first example in which the derivative security is not an option. The markets could also be made complete by issuing two options. An example is a put with a striking price of 0.18 which will give payoffs of (0.1, 0, 0) and a call with a striking price of 1.8 which will give payoffs of (0, 0, 0.2). This case can be analyzed by extending the model in the obvious way by allowing for two derivative securities rather than one. The results are similar.

Consider next how much the exchange owner can raise by offering a security which completes the market. Table 2d contains the relevant values. It can be seen that it is indeed profitable to do this. However, offering a security which completes the market is not the only possible strategy for the exchange. For example, it could offer a security that leaves markets incomplete. Suppose it just offers a put option with striking price 1.08 so that the option has payoffs (1, 0, 0). The stage 1 equilibrium values of interest are shown in Table 2e. Table 2f compares this with the complete markets equilibrium. It can be seen that the social surplus from the derivative security that completes markets is greater than the social surplus from the derivative security that does not complete markets. Table 2g compares the equilibrium with the derivative security that does not complete markets with the equilibrium with no derivative securities. The latter equilibrium is worse.

Which is the most profitable strategy for the exchange owner? Table 2h shows the amount that can be charged by the exchange if it offers a put with payoffs (1, 0, 0). A comparison with Table 2d shows it is more profitable to do this than to offer the security that completes the market. This demonstrates Proposition 3.•

Hence, the owner may have the wrong incentives to offer the efficient security. He can be better off distorting the security offered even when there is zero marginal cost to completing the market.

It can be seen from Example 1 that introducing an options exchange that completes the market reduces the value of the firm. In other words producers are made worse off by having an option written on the equity of their firm. In Example 2 the reverse is true. The original owners of the firm are made better off by the introduction of an options exchange that completes the market. This gives the following proposition.

Proposition 4

The value of the firm can be increased or decreased by the introduction of an options exchange.

We have assumed above that the owners of firms are a different group from investors and are only concerned with first period consumption. An alternative assumption is that the people that own the firms are also concerned about second period consumption and so are investors as well as owners. In the benchmark model this change would mean that the exchange owner would not need to make separate charges to firms. Instead the fee charged each investor would vary with the amount of shares owned and the change in firm value would be internalised by investors. Apart from this the analysis would be unchanged since utility is transferable. In the institutional version of the model investors take prices as given so their behavior is not altered. The results in Propositions 2-4 are again unchanged because utility is transferable.

5. Equilibrium Selections and Multiple Equilibria

In Section 4 we imposed a restriction that must now be reconsidered. We assumed that the exchange owner could select the equilibrium that would be observed at the second stage. Of course, if the second-stage-equilibrium is unique, this restriction is immaterial. However, when there is more than one equilibrium at stage 1, the selection procedure affects the nature of the analysis in rather subtle ways. In this section we discuss briefly an alternative way of modelling equilibrium, one which does not give the exchange owner the power to choose the second-stage-equilibrium.

We continue to assume, as we did in Section 4, that investors of a given type behave symmetrically. This means, in particular, that when $\alpha = 1$ either all investors of type i enter the exchange or they all stay out. In Section 4 we argued that, since no trade could occur unless both types entered, either all investors enter at the second stage or all stay out. Similarly, we argued that attention could be restricted to equilibria in which investors enter the exchange, since these are the only equilibria in which the owner would find it profitable to open an exchange. These arguments appeal to the properties of a full equilibrium. In the language of game theory, we are assuming that we can restrict attention to the equilibrium path. Once we drop the assumption that the exchange owner can choose the second-stage-equilibrium we have to take account of second-stage-equilibria that cannot be reached along the equilibrium path. Off the equilibrium path, it may well happen that the opening of the exchange is followed by a second-stage-equilibrium in which investors do not enter the exchange. Furthermore, this possibility may be relevant for the analysis of the full equilibrium, even though it cannot occur along the equilibrium path.

Let λ_i be a dummy variable taking the value 0 if no investors of type i = a, b enter the exchange and 1 if they all enter. Suppose that the exchange is opened ($\alpha = 1$) but that no investors enter the exchange ($\lambda_i = 0$ for i = a, b). Since there is no activity on the exchange, the price of the derivative security cannot be determined by supply and demand. Nonetheless, we can define an equilibrium. It is a situation in which, at the prevailing prices, no investor is willing to pay the entry fee. In other respects, a first-stage-equilibrium in which $\alpha = 1$ and $\lambda_i = 0$ for i = a, b is identical to an equilibrium in which $\alpha = 0$.

<u>Definition</u>: Let (α, e, Z) be given. Suppose that $\alpha = 0$. An equilibrium at stage 2 is an array $\{v, (c_i, d_i)\}$, consisting of a price vector v and a choice (c_i, d_i) for each type of investor i = a, b, that satisfies the following

conditions:

(i) $(c_i, d_i) \in \arg\max_{B_i^0(v,e)} U_i(c_i, d_i \cdot Z)$ for $i = a, b$;

(ii) $\Sigma_i d_i = (1, 0)$.

If $\alpha = 1$, on the other hand, equilibrium at stage 2 is defined by an array $\{v, (c_i, d_i), (\lambda_i)\}$, consisting of a price vector v, a choice (c_i, d_i) for each type $i = a, b$ and a number $\lambda_i = 0, 1$ for each type $i = a, b$, that satisfies the following conditions:

(i) $(c_i, d_i) \in \arg\max_{B_i^{\lambda_i}(v,e)} U_i(c_i, d_i \cdot Z)$ for $i = a, b$;

(ii) $\Sigma_i d_i = (1, 0)$;

(iii) $\lambda_i \in \arg\max \lambda u_i^1(v, e, Z) + (1 - \lambda)u_i^0(v, e, Z)$ for $i = a, b$.

Note that in condition (i) the budget set from which the investor chooses depends on the value of λ_i. For example, if $\lambda_i = 1$ then the appropriate budget set is $B_i^1(v, e)$. Condition (iii) ensures that the investors' decision to enter the exchange is optimal at the prevailing prices.

An equilibrium selection is a function $\phi^\alpha(e, Z)$ that associates an equilibrium at stage 1, relative to (α, e, Z), with every choice of (α, e, Z) at stage 0. In the definition of equilibrium given in Section 4, it is implicitly assumed that the exchange owner can choose the equilibrium selection as well as the value of (α, e, Z). Now we assume that the equilibrium selection is taken as given. The earlier definition gives the exchange owner a strong "first mover advantage," allowing him to manipulate the second-stage-equilibrium; the new definition takes away this advantage.

It is much more in the spirit of the non-cooperative, Nash equilibrium. Of course, the old equilibrium will be a special case of the new definition for some choice of $\phi^{\alpha}(e, Z)$.

Definition: Let $\phi^{\alpha}(e, Z)$ be a given equilibrium selection. A pre-equilibrium is defined by an array $\{(v^0, (c_i^0, d_i^0)), (v^1, (c_i^1, d_i^1), (\lambda_i^1)), \alpha, e, Z\}$, consisting of the second-stage-equilibria $(v^0, (c_i^0, d_i^0))$ and $(v^1, (c_i^1, d_i^1), (\lambda_i^1))$ corresponding to $\alpha = 0, 1$, the decision α whether to open the exchange, the fee structure $e = (e_a, e_b)$ and the choice of securities Z, that satisfies the following conditions:

(i) $(v^0, (c_i^0, d_i^0)) = \phi^0(e, Z)$ and $(v^1, (c_i^1, d_i^1), (\lambda_i^1)) = \phi^1(e, Z)$;

(ii) Z_0 is given.

A pre-equilibrium is called a full equilibrium if there does not exist another pre-equilibrium, relative to the given equilibrium selection ϕ, that yields higher profits $\alpha(\Sigma_{i=a,b} \lambda_i e_i - \gamma)$.

The individual rationality condition has disappeared from the definition; condition (iii) of the definition of second-stage-equilibrium has made it redundant. Condition (i) says that the second-stage-equilibrium is determined by the owner's choice of (α, e, Z) and by the selection ϕ, which the owner must take as given. Note also the change in the definition of profits: the owner collects fees only from the investors who enter the exchange.

The main impact of taking the equilibrium selection as a given is that it increases the number of possible equilibria. It is not surprising that the second-stage-equilibria may not be unique. What is less obvious is that the existence of more than one second-stage-equilibrium may lead to a continuum of full equilibria. An analysis of this indeterminacy of equilibrium is out of place here; but we can easily indicate the essential problem. Suppose that in the first stage the exchange owner decides to open the exchange. Let $(\hat{\alpha}, \hat{e}, \hat{Z})$ denote the equilibrium choices of the owner at the first stage and suppose that for any (α, e, Z) sufficiently close to $(\hat{\alpha}, \hat{e}, \hat{Z})$ there exist two second-stage-equilibria, one in which all investors enter the exchange and one in which none of them do. Let $\left(v^1, (c_i^1 d_i^1), (\lambda_i^1)\right) = \phi^1(\hat{e}, \hat{Z})$ be the equilibrium observed at the second stage and suppose that this equilibrium is the one in which all investors enter the exchange ($\lambda_i = 1$ for $i = a, b$). We can suppose without loss of generality that for any first stage choices (α, e, Z) in some sufficiently small neighborhood of $(\hat{\alpha}, \hat{e}, \hat{Z})$, $\phi^1(e, Z)$ is the equilibrium in which no investors enter the exchange. Clearly, any small deviation by the exchange owner from his equilibrium choice will cause a discontinuous drop in profits. Suppose, in fact, that the choice of $(\hat{\alpha}, \hat{e}, \hat{Z})$ uniquely maximizes the exchange owner's profits. Then it is possible to construct a continuum of full equilibria, each one corresponding to a different choice of the point of discontinuity in the equilibrium selection ϕ. Each choice of (α, e, Z) in a small neighborhood will constitute a local and hence a global maximum for profits for an appropriately constructed equilibrium selection. Simply associate (α, e, Z) with the equilibrium in which all investors enter the exchange and all other choices (α', e', Z') in some small neighborhood of (α, e, Z) with the equilibrium in which none of the investors enter the exchange. Then any deviation from (α, e, Z) leads to a discontinuous drop in profits. In this way, we see that any choice of (α, e, Z) in some small neighborhood of $(\hat{\alpha}, \hat{e}, \hat{Z})$ can be supported as a full equilibrium.

The concept of full equilibrium sketched above appears to be the right concept to use in these circumstances. With the appropriate adjustments for the competitive flavor of equilibrium at stage 1, it corresponds to the concept of subgame perfect equilibrium for an extensive form game. But while it gives us the right concept of non-cooperative equilibrium, the multiplicity of equilibria calls for some refinement. In Section 4 we made use of one such refinement by allowing the exchange owner to choose the equilibrium that he liked best in the second stage. Such a refinement is not entirely without justification. Apart from theoretical arguments based on the notion of first mover advantage, there are institutional reasons for thinking the exchange owner may be in a position to choose the equilibrium. In stock exchanges which operate with specialist traders, one of the trader's most important functions--and one from which he is believed to make a great deal of money--is to choose the opening prices at the start of the day's trading. The choice of opening prices is analogous to the choice of an equilibrium and devolves to the specialist, as to our exchange owner, as a matter of institutional design.

6. The Impact of the Options Market on the Financial Decisions of the Firm

So far we have assumed that firms do not make any decisions at all. We have shown that the choice of securities traded on the options exchange affects the value of firms. This suggests that when firms have non-trivial decisions to make there will be some interaction between the options market and the decisions of the firms.

In this section we introduce two extensions to the basic model. The first is to allow for the possibility that firms have issued debt and levered equity as well as unlevered equity in the past. We do this by making use of the equilibrium concepts developed in Allen and Gale (1988a). The second is to allow for asymmetric behavior on the part of firms and investors. Both of these extensions are necessary for an analysis of the interaction between the options market and the financial decisions of the firms.

Equilibrium is determined in three stages. At the first stage (stage 0), firms make financial decisions. At the second stage (stage 1), the exchange owner decides whether to open the exchange and if he does open the exchange, what fees should be charged and what derivative security should be traded. At the third stage (stage 2), investors decide whether to enter the exchange and then trade on the available markets.

Suppose to begin with that firms have made a decision about their financial structure at stage 0. A fraction μ of the firms have decided to issue equity only. The remaining $1 - \mu$ have decided to issue debt and levered equity. Let Z_2 and Z_3 denote the returns to the debt and levered equity respectively, where $Z_2 + Z_3 = Z_0$. Note that we are implicitly assuming that the firms that issue debt all issue the same amount of debt.
Let $Z = (Z_0, Z_1, Z_2, Z_3)$ denote the vector of securities that can be traded, let $v = (v_0, v_1, v_2, v_3)$ denote the vector of security prices and let $d = (d_0, d_1, d_2, d_3)$ denote a generic portfolio.

Suppose that the options exchange does not open ($\alpha = 0$). The budget set of an investor of type $i = a, b$ is denoted by $B_i^0(v, e)$ and defined by putting

$$B_i^0(v, e) = \{(c, d) \in R^5 | c + v \cdot d \leq 0 \ , \quad d_1 = 0 \quad \text{and} \quad d_0, d_2, d_3 \geq 0\} \ .$$

Now suppose that the exchange has been opened ($\alpha = 1$). In the earlier definition, we assumed that either all investors of type i entered the exchange or they all stayed out. Now we allow for the possibility of incomplete participation in the exchange. Let λ_i denote the fraction of investors of type $i = a, b$ who decide to pay the entry fee and enter the exchange. Their budget set is denoted by $B_i^1(v, e)$ and defined by putting

$$B_i^1(v, e) = \{(c, d) \in R^5 | c + v \cdot d + e_i \leq 0 \ , \quad \text{and} \quad d_0, d_2, d_3 \geq 0\} \ .$$

Denote the investors of type $i = a, b$ who remain outside the exchange by $i = A, B$, respectively. The budget set for these investors is the same as when the exchange does not open. For example, $B_A^1(v, e) = B_a^0(v, e)$. With this notation, we can define an equilibrium at stage 2.

Definition: Let (α, μ, Z, e) be given. Suppose that $\alpha = 0$. An equilibrium at stage 2 is an array $\{v, (c_i, d_i)\}$, consisting of a price vector v and a choice (c_i, d_i) for each type of investor $i = a, b$, that satisfies the following conditions:

(i) $(c_i, d_i) \in \underset{B_i^0(v,e)}{\arg\max}\ U_i(c_i, d_i \cdot Z)$ for $i = a, b$;

(ii) $\Sigma_i\, d_i = (\mu, 0, 1 - \mu, 1 - \mu)$.

If $\alpha = 1$, on the other hand, equilibrium at stage 2 is defined by an array $\{v, (c_i, d_i), (\lambda_i)\}$, consisting of a price vector v, a choice (c_i, d_i) for each type $i = a, b, A, B$ and a fraction λ_i for each type $i = a, b$, that satisfies the following conditions:

(i) $(c_i, d_i) \in \underset{B_i^1(v,e)}{\arg\max}\ U_i(c_i, d_i \cdot Z)$ for $i = a, b, A, B$;

(ii) $\Sigma_i\, \lambda_i d_i = (\mu, 0, 1 - \mu, 1 - \mu)$, where $\lambda_A = 1 - \lambda_a$ and $\lambda_B = 1 - \lambda_b$.

Condition (i) simply says that each investor is maximizing his utility subject to the appropriate budget constraint and to the available markets. Condition (ii) is the market-clearing condition. When $\alpha = 1$, the market-clearing condition has been changed to reflect the fact that investors inside and outside the exchange have different demands. In both cases, the market-clearing condition has been changed to reflect the fact that some firms issue equity only and some issue debt and levered equity.

Let $\Phi^{\alpha}(\mu, Z, e)$ denote the set of third-stage-equilibria relative to (α, μ, Z, e). Let $u_i^{\alpha}(v, e, Z)$ denote the maximum utility attainable when security prices are given by v and the stage 0 choices are given by (α, Z, e). That is,

$$u_i^{\alpha}(v, Z, e) = \sup_{B_i^{\alpha}(v,e)} U_i(c_i, d_i \cdot Z) \text{ for } i = a, b .$$

<u>Definition</u>: A pre-equilibrium at stage 1 is defined by an array $\{(v^0, (c_i^0, d_i^0)), (v^1, (c_i^1, d_i^1), (\lambda_i^1)), \alpha, \mu, e, Z\}$, consisting of the third-stage-equilibria $(v^0, (c_i^0, d_i^0))$ and $(v^1, (c_i^1, d_i^1), (\lambda_i^1))$ corresponding to $\alpha = 0, 1$, the decision α whether to open the exchange, a fee structure $e = (e_a, e_b)$ and a choice of securities Z, that satisfies the following conditions:

(i) $\lambda_i \in \arg\max \lambda u_i^1(v^1, e, Z) + (1 - \lambda)u_i^0(v^1, e, Z)$ for $i = a, b$;

(ii) $(v^0, (c_i^0, d_i^0)) \in \Phi^0(\mu, e, Z)$ and $(v^1, (c_i^1, d_i^1), (\lambda_i^1)) \in \Phi^1(\mu, e, Z)$

(iii) (Z_0, Z_2, Z_3) and μ are given .

A pre-equilibrium in the second stage is called an equilibrium if there does not exist another pre-equilibrium that yields higher profits $\alpha(\Sigma_{i=a,b} \lambda_i e_i - \gamma)$.

The financial decisions of firms at stage 0 will be analyzed rather informally. Firms behave non-cooperatively so they take as given the decisions of the other firms. Furthermore, there is a continuum of firms so no one firm perceives that it has any impact on the subsequent equilibrium. As a result firms behave as price-takers. More precisely, they are equilibrium-takers, who take as given all the variables of the subsequent equilibrium. Let v^{α} denote the equilibrium prices when $\alpha = 0, 1$. The market value of the firm will be v_0^{α} if it issues equity only and $v_2^{\alpha} + v_3^{\alpha}$ if it issues debt and equity. Suppose there is no cost of issuing equity and a fixed cost γ_f of issuing debt. Then firms will choose their financial structure to maximize the value of the firm net of issuing costs. This means that μ must satisfy

$$\mu \in \arg\max \mu v_0^{\alpha} + (1 - \mu)(v_2^{\alpha} + v_3^{\alpha} - \gamma_f) ,$$

where v^{α} is taken as given.

What we have not considered yet is the firm's decision about the amount of debt to issue. In order for price-taking firms to decide how much debt to issue, they need to know the prices of securities that are not necessarily issued in equilibrium. In defining an equilibrium we have only quoted prices for the securities actually issued. In Allen and Gale (1988a), we have shown how other securities should be priced in equilibrium. The prices of unissued securities must be such that investors' demand and firms' supply are both zero so that markets clear. Let $p_i : S \to \mathbb{R}$ represent the shadow prices of consumption at date 2, measured in terms of consumption at date 1, for an investor of type $i = a, b, A, B$. In other words, $p_i(s)$ is investor i's marginal rate of substitution of present consumption for future consumption in state $s \in S$, for $i = a, b, A, B$. At the margin an investor of type i is willing to pay $p_i \cdot Z_j$ for a unit of the security Z_j, so the market value must be the $\max_i \{p_i \cdot Z_j\}$ where the maximum is taken over all types $i = a, b, A, B$. The market value of a firm that issues the securities (Z_2, Z_3) is equal to

$$MV(Z_2, Z_3) = \max_i \{p_i \cdot Z_2\} + \max_i \{p_i \cdot Z_3\} .$$

If only debt and equity can be issued, the firm chooses the face value D of the debt that it wishes to issue and then puts $Z_2 = \min\{D, Z_0\}$ and $Z_3 = \max\{Z_0 - D, 0\}$. D is chosen to maximize the market value $MV(Z_2, Z_3)$.

If $\lambda_i = 1$ the measure of investors outside the exchange is zero. Nevertheless, in valuing unissued securities the maximum valuation is taken over all four groups i = a, b, A, B. This is equivalent to taking the limit as $\lambda_i \to 1$ and maintains continuity.

A full equilibrium of the extended institutional model consists of a choice of μ and (Z_2, Z_3), together with an equilibrium in the second stage, such that μ and (Z_2, Z_3) satisfy the two optimality conditions described above, given the equilibrium prices. This rather informal definition should be clarified in the example that follows.

A feature of the model that warrants further discussion is the assumption that investors cannot short sell corporate securities but can sell derivative securities. In Allen and Gale (1988a) we argued that, in practice, short sales of corporate securities are costly and are only rarely undertaken by investors. In contrast, selling options is, in practice, relatively cheap compared to short sales. Hence the assumption that corporate securities cannot be short sold while derivative securities can be does not involve an inconsistency.

In the context of this model it is possible to show the following result.

Proposition 5

An equilibrium exists in which some firms issue debt and levered equity and it is unprofitable for the exchange owner to set up the exchange. There also exists an equilibrium in which the exchange is set up. If the exchange is set up a Pareto superior allocation is feasible.

Proof

The proof is once again by example.

Example 3

The parameters of the example are similar to Example 1(i). The differences are that the cost of setting up the exchange is assumed to be 0.001 and a firm can issue debt for a cost of 0.01.

Two full equilibria exist. In the first, the prices firms anticipate in the equilibrium at stage 2 are consistent with no exchange being set up at stage 1. Thus, at stage 0, a positive measure of firms issue debt and equity. At stage 1, the exchange owner does not find it profitable to set up the options exchange. This is because the debt and equity issued by firms at stage 0 allows investors outside the exchange to smooth consumption across states fairly adequately so that the amounts that can be charged for entry to the exchange are limited. Firms' expectations about the prices they will receive for their securities at stage 2 therefore turn out to be correct.

The prices firms expect to hold at stage 2 correspond to the marginal rates of substitution, or in this context, the marginal utilities of consumption, for groups i = a, b, A, B shown in Table 3a. Suppose a firm were to issue debt with a face value of 1 so the debt has payoffs of (1, 1) and the levered equity has payoffs (0, 1) in the two states respectively. Investors of type b are more risk averse and so value the debt most. Hence the price of debt is (0.5)(1)(0.35930) + (0.5)(1)(0.07473) = 0.21702. Investors of type a value the levered equity the most so that its price is (0.5)(1)(0.09473) = 0.04737. Hence the gross value of a firm that issues debt with a face value of 1 is 0.21702 + 0.04737 = 0.26439. It can easily be seen that this is in fact the optimal capital structure. If the face value on debt is increased above 1 the effect is to transfer consumption in state 2 from group a, who value it most, to group b, who value it least. For example, suppose the face value is 1.1. Then the debt has payoffs of (1, 1.1) and is worth

(0.5)(1)(0.35930) + (0.5)(1.1)(0.07473) = 0.22075 and the levered equity has payoffs of (0, 0.9) and is worth (0.5)(0.9)(0.09473) = 0.04246 so the gross value of the firm is 0.22075 + 0.04246 = 0.26338. Similarly, if the face value on debt is reduced below 1. For example, suppose the face value is 0.9. Then the debt has payoffs of (0.9, 0.9) and is worth (0.5)(0.9)(0.35930) + (0.5)(0.9)(0.07473) = 0.19531 and the levered equity has payoffs of (0.1, 1.1) and is worth (0.5)(0.1)(0.31930) + (0.5)(1.1)(0.09474) = 0.06807 so the gross value of the firm is 0.19531 + 0.06807 = 0.26338.

The highest gross value a firm can obtain if it issues debt and levered equity is 0.26439. In contrast, the gross value of a firm that just issues unlevered equity is 0.25439. Since the cost of issuing debt is 0.01, this means that the firms' net values are the same so they are indifferent between issuing two securities and issuing one. The prices corresponding to the marginal utilities of consumption in Table 3a support a third-stage-equilibrium in which the measure of firms that issue debt and levered equity is 0.07333 and the measure that just issue unlevered equity is 1.92667.

Given that firms issue these securities at stage 0, what does the exchange owner do at stage 1? Taking the supply of firms' securities as given, he considers whether it is worthwhile opening an exchange. If he opens the exchange he must choose a derivative security, a proportion that enters the exchange and entry fees to maximize his profits. Since there are only two states the choice of derivative security is trivial; all options which are linearly independent of the existing securities allow entrants to the exchange to equate their marginal utilities of consumption. Our solution concept allows the exchange owner to choose the third-stage-equilibrium. The owner can choose the entry fees so that all investors are indifferent between being inside or outside the exchange, for any particular proportion of investors that enter the exchange. If he sets the entry fees high, only a small proportion of investors enter the exchange so that his total revenue is small. At the other extreme when all investors enter, the prices of debt and equity issued by the firms are the same as when markets are complete. In this case access to the options exchange does not improve investors' risk sharing possibilities so the amount that can be charged for entry to the exchange is zero and no revenue is raised. Hence the exchange owner's optimal strategy if he opens the exchange is to price entry so that only some portion of investors enter. In Example 3 it can be shown the revenue-maximizing charges for types a and b are 0.00037 and 0.00721 respectively. In this case 47.1% of investors enter the exchange and the total receipts are 0.00051. The exchange owner is therefore unable to cover his costs of 0.001 and so does not find it worthwhile to open the exchange at stage 1. The third-stage-equilibrium is as shown in Table 3a and firms' expectations about prices are fulfilled.

In the second full equilibrium, the prices firms anticipate in the equilibrium at stage 2 are consistent with the options exchange being set up at stage 1 and all investors entering. At stage 0, the measure of firms that issue debt and equity is zero since the anticipated existence of the options exchange means that markets are effectively complete. At stage 1, the exchange owner finds it profitable to set up the options exchange because investors' outside the exchange are unable to smooth consumption across states adequately so that the amounts that can be charged for entry to the exchange are high. Firms' expectations about the third-stage-equilibrium therefore turn out to be correct.

The prices firms expect to hold in the third-stage-equilibrium correspond to the situation where everybody enters the exchange ($\lambda_i = 1$) and the marginal utilities of consumption for groups i = a, b are the same as when markets are complete. These marginal utilities are shown in Table 1(i)b. The marginal utilities of consumption of groups outside the exchange i = A, B as $\lambda_i \to 1$. are used to price the securities. Table 3b shows these. It can be seen that the price of debt with payoffs (1, 1) is (0.5)(1)(0.35853) + (0.5)(1)(0.07434) = 0.21644 and the price of levered equity with payoffs (0, 1) is (0.5)(1)(0.09434) = 0.04717. Similarly for other possible capital structures. Given these prices, the supply of debt and levered equity is zero and the demand is also zero. The amounts that the exchange owner can charge are given by the differences between the utilities investors can obtain with access to the options market and the utilities that they can obtain using unlevered equity, debt and levered equity at the prices within that equilibrium. These charges are shown in Table 3c. Notice that these charges differ from those in Table 1(i)d. This is because in Table 1(i)d the alternative if somebody did not enter the exchange is to use just unlevered equity.

Since the cost of setting up the exchange, 0.001, is less than the amount that can be charged for entry to the exchange, 0.00152, the owner is strictly better off setting up the exchange at stage 1. Moreover, the expectations of the firms that the prices in the third-stage-equilibrium are the complete markets prices are correct.

Thus two full equilibria are possible. Table 3d contains a comparison of these two full equilibria. It can be seen that both types of investor and the exchange owner are better off in the full equilibrium where the exchange is set up. Only the firms are better off in the full equilibrium with no options exchange. However, it is clearly possible to reallocate consumption to make everybody better off in the full equilibrium where the exchange is open. The essential reason such a reallocation is possible is that the cost of setting up the options exchange and sharing risk with options is less than the cost of some firms' issuing two securities and risk being shared with debt and levered equity.•

Example 3 illustrates the interaction between firms' capital structure decisions and the exchange owner's decision. Given that two equilibria exist a question arises as to which is the more "plausible" one. Arguments can be made in favor of both. Firms are better off in the equilibrium where the exchange is not set up; since they move first it can be argued that the one where some firms issue both debt and levered equity and the exchange is not set up is the more plausible one. On the other hand, it can be argued that the equilibrium in which the exchange is opened is more plausible since a reallocation exists which can make everybody better off than in the equilibrium in which the exchange is closed.

For different costs of setting up the exchange there may only exist one equilibrium in Example 3. If the cost is above 0.00152 then the only equilibrium is where some firms issue debt and levered equity and no exchange is set up. If it is below 0.0051 then the only equilibrium is where the exchange is set up and no firms issue two securities.

In demonstrating Proposition 5 we have assumed a particular sequence of events. In particular, we assumed that the firms move first and the exchange owner moves second. Another possibility is that the exchange owner moves first and the firms move second. Consider Example 3 with this sequence. Since the exchange owner moves first he can set up the exchange and preempt firms from issuing debt and levered equity. He prefers to do this since his profits are maximised with the exchange open and no firms issuing two securities. In contrast to the case where firms move first and two equilibria exist, only one equilibrium exists. Another possibility is to model the exchange and firms as moving simultaneously. Both the equilibria described above are again possible.

In situations where the sequencing of moves does affect the equilibrium set, it is not clear which order of moves is more plausible. Firms must exist and therefore must have issued their securities before an exchange can issue derivative securities. On the other hand, firms continually change their capital structure after derivative securities have been issued. Thus it is possible to argue that all the different possibilities are of interest.

Proposition 5 was concerned with the case where an inefficient equilibrium could exist because firms issuance of debt made it unprofitable for the exchange to set up. The reason this is inefficient is that, by issuing derivative securities, the exchange can provide risk sharing opportunities at lower cost than the firms can by issuing debt and levered equity. It is also possible to get an inefficient equilibrium where the opposite is true. In other words, the exchange may be set up even though risk sharing opportunities could be provided more cheaply if firms issued debt and levered equity instead. For example, it can be shown this occurs in Example 1(ii) if the cost of setting up an exchange is 0.002 and the cost of issuing debt is 0.001.

7. Concluding Remarks

In this paper we have considered the incentives of exchange owners to provide a socially efficient market structure. If firms and investors can be charged lump sum fees before the exchange is set up and if the agreement of all agents to pay these fees is required for the exchange to be established then the resulting market structure is socially efficient. These assumptions do not hold in practice. We therefore considered a model which is closer to institutional practice. In particular, it was assumed that the exchange was established before any fees were paid and only investors were charged fees. A number of examples were provided which suggest that the incentives of exchange owners are such that the equilibrium market structure obtained may not be socially efficient.

Markets for derivative securities are extensively regulated by the government. The rationale for this regulation and the form the regulations should take are currently not well understood. The models developed in this paper pose a number of issues related to regulation and suggest that further research along these lines is warranted.

Table 1(i)a

Equilibrium in Example 1(i) with incomplete markets

Group	Equilibrium Demand for Equity	Consumption		Marginal Utility of Consumption		Utility
		State 1	State 2	State 1	State 2	
a	1.15918	1.15918	2.31836	0.31374	0.09843	4.49796
b	0.84082	0.84082	1.68164	0.37214	0.06924	4.67498

The value of a firm = 0.25531

Table 1(i)b

Equilibrium in Example 1(i) with complete markets

Group	Equilibrium Demand for Equity	Consumption		Marginal Utility of Consumption		Utility
		State 1	State 2	State 1	State 2	
a	1.21781	1.102284	2.43562	0.33211	0.08754	4.50052
b	0.78219	0.89772	1.56438	0.33211	0.08754	4.67754

The value of a firm = 0.25360

Table 1(i)c

A comparison between complete and incomplete markets in Example 1(i)

Change in Group a's Utility	0.00256
Change in Group b's Utility	0.00256
Change in Firm Value	- 0.00170
Total Change	0.00340
Surplus from Options Exchange	0.00140

Table 1(i)d

Charges for access to the options exchange in Example 1(i)

Group	Demand for Equity at $v_0 = 0.25360$	Consumption State 1	Consumption State 2	Utility	Charge for Entry to the Options Exchange
a	1.16400	1.16400	2.32800	4.49995	0.00057
b	0.84347	0.84347	1.68694	4.67643	0.00111

The total charge for entry to the options exchange = 0.00168

The total profit from setting up the exchange = - 0.00032

Table 1(ii)a

Equilibrium in Example 1(ii) with incomplete markets

Group	Equilibrium Demand for Equity	Consumption		Marginal Utility of Consumption		Utility
		State 1	State 2	State 1	State 2	
a	0.12110	0.12110	0.24219	0.88595	0.78491	4.01588
b	0.27890	0.27890	0.55781	1.14493	0.65543	4.20745

The value of a firm = 1.22788

Table 1(ii)b

Equilibrium in Example 1(ii) with complete markets

Group	Equilibrium Demand for Equity	Consumption		Marginal Utility of Consumption		Utility
		State 1	State 2	State 1	State 2	
a	0.15114	0.03562	0.30228	0.96501	0.73913	4.01903
b	0.24989	0.36438	0.49772	0.96501	0.73913	4.21421

The value of a firm = 1.22163

Table 1(ii)c

A comparison between complete and incomplete markets in Example 1(ii)

Change in Group a's Utility	0.00315
Change in Group b's Utility	0.00676
Change in Firm Value	- 0.00624
Total Change	0.00367
Surplus from Options Exchange	- 0.00030

Table 1(ii)d

Charges for access to the options exchange in Example 1(ii)

Group	Demand for Equity at v_0 = 1.22163 without access	Consumption State 1	Consumption State 2	Utility	Charge for Entry to the Options Exchange
a	0.12421	0.12421	0.24841	4.01665	0.00238
b	0.28057	0.28057	0.56114	4.20920	0.00501

The total charge for entry to the options exchange = 0.00740

The total profit from setting up the exchange = 0.00340

Table 2a

Equilibrium in Example 2 with incomplete markets

Group	Equilibrium Demand for Equity	Consumption State 1	Consumption State 2	Consumption State 3	Marginal Utility of Consumption 1	Marginal Utility of Consumption 2	Marginal Utility of Consumption 3	Utility
a	0.09998	0.00800	0.17997	0.19996	17.0434	0.54682	0.36657	4.45141
b	0.90002	0.07200	1.62003	1.80004	13.8886	0.61727	0.55554	2.95203

The value of a firm = 1.11109

Table 2b

Equilibrium in Example 2 with complete markets

Group	Consumption State 1	Consumption State 2	Consumption State 3	Marginal Utility of Consumption 1	Marginal Utility of Consumption 2	Marginal Utility of Consumption 3	Utility
a	0.01393	0.17409	0.17974	15.1361	0.61504	0.54937	4.45015
b	0.06607	1.62591	1.82026	15.1361	0.61504	0.54937	2.91272

The value of a firm = 1.15690

Table 2c

A comparison between complete and incomplete markets in Example 2

Change in Group a's Utility	- 0.00126
Change in Group b's Utility	- 0.03931
Change in Firm Value	0.04581
Total Change	0.00524
Surplus from Options Exchange	0.00424

Table 2d

Charges for access to the exchange in Example 2 with complete markets

Group	Demand for Equity at v_0 = 1.15690 without access	Consumption State 1	2	3	Utility	Charge for Entry to the Options Exchange
a	0.09745	0.00780	0.17541	0.19490	4.44689	0.00326
b	0.86438	0.06914	1.55588	1.72876	2.91163	0.00110

The total charge for entry to the options exchange = 0.00436

The total profit from setting up the exchange = 0.00336

Table 2e

Equilibrium in Example 2 with a put option with payoffs (1, 0, 0)

Group	Equilibrium Demand for Equity	Consumption State 1	Consumption State 2	Consumption State 3	Marginal Utility of Consumption 1	Marginal Utility of Consumption 2	Marginal Utility of Consumption 3	Utility
a	0.09335	0.01393	0.16802	0.18669	15.1361	0.69438	0.47801	4.45469
b	0.90665	0.06607	1.63198	1.81331	15.1361	0.61275	0.55148	2.95803

The value of a firm = 1.10692

Table 2f

A comparison between complete markets and partially incomplete markets with a put option with payoffs (1, 0, 0) in Example 2

Change in Group a's Utility	- 0.00454
Change in Group b's Utility	- 0.04530
Change in Firm Value	0.04998
Total Change	0.00013

Table 2g

A comparison between partially incomplete markets with a put option with payoffs (1, 0, 0) and incomplete markets in Example 2

Change in Group a's Utility	0.00328
Change in Group b's Utility	0.00600
Change in Firm Value	- 0.00417
Total Change	0.00511
Surplus from Options Exchange	0.00411

Table 2h

Charges for access to the options exchange in Example 2 with put option (1, 0, 0)

Group	Demand for Equity at v_0 = 1.10692 without access	Consumption State 1	Consumption State 2	Consumption State 3	Utility	Charge for Entry to the Options Exchange
a	0.10022	0.00802	0.18040	0.20044	4.45183	0.00286
b	0.90341	0.07227	1.62614	1.80682	2.95579	0.00224

The total charge for entry to the options exchange = 0.00510

The total profit from setting up the exchange = 0.00410

Table 3a

The third-stage-equilibrium in Example 3 where a positive measure of firms issue debt and equity

Group	Consumption		Marginal Utility of Consumption		Utility
	State 1	State 2	State 1	State 2	
A	1.14163	2.35658	0.31930	0.09474	4.49908
B	0.85837	1.64342	0.35930	0.07474	4.67587

The gross value of a one-security firm = 0.25439

The gross value of a two-security firm with optimal capital structure = 0.26439

Table 3b

The choices of groups A and B as $\lambda_i \to 1$ when an exchange opens in Example 3

Group	Consumption		Marginal Utility of Consumption		Utility
	State 1	State 2	State 1	State 2	
A	1.14403	2.36083	0.31853	0.09434	4.50000
B	0.85944	1.64612	0.35853	0.07434	4.67653

The gross value of a one-security firm = 0.25361

The gross value of a two-security firm with optimal capital structure =0.26361

Table 3c

Charges for access to the options exchange in Example 3 when the exchange is set up

	Utilities Inside the Exchange		Utilities Outside the Exchange	Charge for Entry to the Options Exchange
a	4.50052	A	4.50000	0.00052
b	4.67754	B	4.67653	0.00101

The total charge for entry to the options exchange = 0.00153

The total profit from setting up the exchange = 0.00053

Table 3d

A comparison of the equilibrium where the exchange is set up and the equilibrium where the exchange is not set up in Example 3

	Utilities Net of Charges in the Equilibrium with an Exchange		Utilities in the Equilibrium with no Exchange	Difference in Utilities
a	4.50000	A	4.49908	0.00092
b	4.67653	B	4.67587	0.00066
Firms	0.50720		0.50878	-0.00158
Exchange Owner	0.00053		0.00000	0.00053

The total surplus from the options exchange = 0.00053

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