ASSET PRICES UNDER HETEROGENOUS BELIEFS:
IMPLICATIONS FOR THE EQUITY PREMIUM

by

Andrew B. Abel

(9-89)

Asset Prices under Heterogenous Beliefs:

Implications for the Equity Premium

Andrew B. Abel

Wharton School of the University of Pennsylvania

and

National Bureau of Economic Research

Abstract: An individual investor's demands for risky capital and riskless bonds depend on the investor's subjective beliefs about the payoff to risky capital. This paper determines equilibrium asset prices and returns in a capital market in which investors have heterogeneous subjective expectations of the payoff to capital. Increased heterogeneity increases the riskless rate of return and reduces the stock price. Heterogeneity can also dramatically increase the equilibrium equity premium on stocks relative to bonds. Therefore, calculating the equity premium under the assumption of homogeneous beliefs could dramatically understate the equity premium that would prevail under heterogeneity.

Recent research has exploited the representative consumer abstraction to develop many insights about asset pricing in a general equilibrium framework. Typically it is assumed that all consumers have the same subjective beliefs, the same utility functions and the same opportunity sets. An immediate implication of this set of assumptions is that there is no trade of assets or goods. In general, there may be many risky capital assets and the equilibrium prices of these assets ensure that these assets are not traded. The riskless rate of return may be modeled in a variety of ways. If there is a riskless physical asset with an exogenous rate of return, then, of course, the riskless rate is exogenous. Alternatively, if there is no riskless physical asset, then the price of riskless bonds is determined endogenously; if there are no riskless bonds, then the riskless rate can be calculated as a shadow price.

In order to generate trade among consumers, it is necessary to modify the assumption that all consumers are identical in all respects. Heterogeneity of consumers can be introduced along several dimensions: consumers may have different subjective beliefs, different utility functions and/or different opportunity sets. In this paper I will explore the implications of heterogeneous subjective beliefs for the pricing of bonds and risky physical assets. In general, consumers with different beliefs will choose to hold different portfolios of bonds and risky capital. Even if there are no outside bonds, so that the average holding of bonds is zero, there will be cross-sectional variation in the holding of bonds. Some consumers will borrow from other consumers by selling bonds to them at a market-determined rate of interest. Kraus and Sick (1980) study the effects of heterogeneous beliefs in a model in

which there is no riskless asset, but there is a market for riskless loans (bonds). In their model, as in the model presented below, the equilibrium riskless rate of return is determined endogenously. They focus on the extent to which equilibrium asset prices reveal preferences and beliefs, whereas I focus on the implications of heterogeneity for asset prices, with particular attention to the equity premium on risky stocks relative to riskless bonds.

Recently, Varian (1987) and Cho (1987) have introduced heterogeneity of beliefs to study the impact of differences in beliefs on the prices and volume of trade of risky assets. In Varian's model, the riskless rate of return is specified as the exogenous rate of return on riskless physical capital; for simplicity, Varian sets the net riskless interest rate equal to zero. By contrast, Cho's model has neither riskless capital nor cross-sectional variation in the holding of riskless bonds; private loan markets are ruled out by assumption in his model, and there is no examination of the riskless interest rate.

DeLong, Shleifer, Summers, and Waldmann (1987) study the effects of heterogeneous beliefs in a broader context. They examine the economic consequences of noise traders and apply their model to a variety of questions in corporate finance and asset pricing. In their model, some investors, called rational traders, have subjective distributions equal to the objective distributions of the relevant random variable. Other investors, called noise traders, have subjective distributions that differ from the objective distribution. Because rational traders and noise traders have different subjective beliefs, this model displays heterogeneous beliefs. The model I develop below can be interpreted as a more general model of noise traders. It

includes, as a special case, the DeLong-Shleifer-Summers-Waldmann assumption that there are two classes of investors that are distinguished by their different beliefs. More generally, however, the model developed below allows for heterogeneous beliefs even within the class of noise traders.

Recently, Varian (1987) has stressed the important distinction between heterogeneous *opinions* about random variables and heterogeneous *information* about random variables. If two consumers have different subjective distributions about a particular random variable, then the difference may be a manifestation of different bits of objective information about the distribution of the random variable and/or it may be a manifestation of different opinions that do not reflect information. If the different subjective distributions arise solely as the consequence of different information, then the differences will be eliminated in equilibrium. Essentially, any consumer who possesses some information and who wants to trade an asset at a given price, will, by his willingness to trade, reveal his information. By contrast, consumers with different opinions will, in general, trade in equilibrium, because each consumer views other traders' opinions as uninformative about the distribution of stochastic payoffs. Varian analyzes the case in which consumers have both different opinions and different information. Singleton (1986) analyzes the time-series properties of asset prices in a partial equilibrium model in which consumers have have different information. By contrast, the analysis below will examine asset prices in a general equilibrium framework in which the heterogeneity of subjective distributions reflects only different opinions rather than different information.

This paper critically examines the representative consumer model as a framework for studying the behavior of asset prices in the presence of heterogeneity. Heterogeneity *per se* does not necessarily invalidate the representative consumer framework. For example, Lintner (1969) showed that if investors have different, but constant, degrees of absolute risk aversion, the behavior of asset prices can be described using a single scalar measure of absolute risk aversion. More precisely, equilibrium prices can be calculated assuming that a representative consumer has a coefficient of absolute risk aversion equal to the harmonic mean of the coefficients of absolute risk aversion divided by the number of investors. By contrast, however, heterogeneity *per se* of subjective expected values of the random payoff will affect equilibrium asset prices. The cross-sectional distribution of expectations cannot be completely summarized by a single (scalar) sufficient statistic. In this paper, I derive the impact of heterogeneous beliefs on the riskless interest rate and the price of risky capital.

The effect of heterogeneity on asset prices has important implications for the equity premium puzzle discussed by Mehra and Prescott (1985). They calibrated a representative consumer asset pricing model to specific features of the U.S. economy and then calculated the implied equity premium, which is the excess rate of return on equities relative to riskless bonds. For all reasonable values of the preference parameters for which the riskless rate of return was less than or equal to 4% per year, their model predicted that the equity premium was 35 basis points or lower. However, the observed value of the equity premium in the U.S. over the period 1889-1978 was

slightly more than 600 basis points. This dramatic failure of the representative consumer model to generate an equity premium with an empirically plausible magnitude is presented as strong evidence against the representative consumer model of asset pricing.

Mehra and Prescott conducted their analysis under the assumption of homogeneous beliefs across consumers. However, I will show below that the introduction of heterogeneity of beliefs can substantially increase the equity premium. Moreover, in the context of a particular set of preferences, I derive a simple expression that relates the equity premium to the cross-sectional variance of beliefs and the equity premium that would be obtained under homogeneous beliefs. In general, the riskless interest rate increases monotonically as the cross-sectional variance of beliefs increases. For one of the two alternative formulations of the model, the equity premium increases monotonically without bound as the cross-sectional variance of beliefs increases without bound; for the other formulation of the model, we can identify the conditions that determine whether the equity premium increases or decreases with increased heterogeneity. Although the structure of preferences and technology is not identical to that used by Mehra and Prescott, this finding suggests that the low value of the equity premium calibrated by Mehra and Prescott may be partly due to their assumption of homogeneous beliefs, when, in fact, the economy is characterized by heterogeneous beliefs.

An alternative explanation of the inability of the representative consumer model to explain the large equity premium has been offered by Mankiw (1986). In his model, all consumers are identical ex ante but obtain different realizations of non-diversifiable idiosyncratic risks

ex post. Mankiw shows that if adverse aggregate shocks fall entirely on a random subset of the population, then individual consumers face much larger risks to consumption than would be evident in aggregate consumption data. Therefore, using aggregate data to calibrate the equity premium would understate the equity premium. The model presented below differs from Mankiw's model in at least three important respects. First, Mankiw's results rest upon the existence of a non-traded asset so that aggregate risks cannot be fully diversified. By contrast, the model presented below allows for complete diversification of aggregate shocks. Second, as pointed out by Mankiw (p. 216), the understatement of the equity premium premium calculated under the representative consumer assumption depends crucially on the non-diversifiable shock being an adverse shock; if the non-diversifiable shock were instead a favorable shock, then the representative consumer model would *overstate* the equity premium. By contrast, in the model below, no assumptions are made with respect to the symmetry or skewness of the cross-sectional distribution of the heterogeneous variable. Third, the model below provides a simple quantitative measure of the impact of heterogeneity on the equity premium.

The formal analysis begins with the behavior of an individual consumer in section I. Section II presents equilibrium asset prices in the presence of heterogeneous subjective expectations of the risky payoff. In section III, the equity premium is calculated, and the magnitude of the equity premium is related to the degree of consensus among consumers and to the degree of risk. Section IV extends the analysis to a multi-period framework and develops two alternative formulations. In one formulation, infinite-horizon investors know the

relation among stock price, dividend, and the interest rate, but different investors have different forecasts of the following period's dividend. In the other formulation, investors are myopic and look ahead only one period. In both formulations, the equilibrium interest rate is an increasing function of heterogeneity; however, the impact of heterogeneity on the equity premium is different for the two alternative formulations of the model. Concluding remarks are presented in section V.

I. Individual Consumption and Portfolio Behavior

Consider a two-period version of the Lucas (1978) asset pricing model. Output is produced using only capital. The amount of output produced by each unit of capital is exogenous and stochastic. Output is completely perishable and the capital stock is constant over time. Let K denote the amount of capital per capita.

During the first period, denoted as period 1, each unit of capital yields a dividend equal to y_1 units of the perishable consumption good. Then the consumption good and capital are traded in competitive markets. The consumption good is the numeraire good and p is the ex-dividend price of a unit of capital in period 1. The purchaser of a capital good in period 1 receives the output produced by this capital good, y_2, in period 2.

Now consider an individual consumer who lives for two periods. The consumer is endowed with k_1 units of the risky capital good at the beginning of period 1. Thus, the total resources available to the consumer in period 1 are equal to $(p+y_1)k_1$. These resources are allocated to consumption in period 1 (c_1), capital to be carried into

period 2 (k_2), and riskless bonds to be carried into period 2 (b). The price of riskless bonds in period 1 is equal to unity, and in period 2 each riskless bond is worth R units of the consumption good. Thus, R is the gross rate of interest. The sources and uses of the consumer's resources in period 1 are described by

$$(p+y_1)k_1 = pk_2 + c_1 + b \qquad (1)$$

In the second period, the consumer's total available resources are equal to the capital income, y_2k_2, plus the principal and interest on riskless bonds, Rb. Because this is the last period of life, consumption, c_2, is equal to total available resources

$$c_2 = y_2k_2 + Rb \qquad (2)$$

The utility function of the consumer is specified to be

$$U = u(c_1) + \beta E\{u(c_2)\} \qquad (3)$$

where $u' > 0$, $u'' < 0$, and $0 < \beta \leq 1$ is a positive discount factor that reflects time preference. The expectation operator E{ } denotes the expectation based on the consumer's subjective probability distribution about the stochastic second-period dividend, y_2.

In period 1, the consumer chooses c_1, b, and k_2 to maximize the utility function (3) subject to the budget constraints in (1) and (2). The first-order conditions for this optimization problem can be written as

$$u'(c_1) = \beta R E\{u'(c_2)\} \qquad (4a)$$

$$u'(c_1) = \beta (1/p)E\{y_2u'(c_2)\} \qquad (4b)$$

If the consumer reduces c_1 by one unit and buys an additional riskless bond in period 1, then first-period utility is reduced by $u'(c_1)$. However, the additional riskless bond can be used to increase second-period consumption by R units, thereby increasing expected discounted

utility in the second period by β R E{u'(c$_2$)}. Equation (4a) indicates

that, at the optimum, the consumer's expected lifetime utility will be

unchanged by a marginal change in the holding of riskless bonds.

Alternatively, the consumer can reduce c$_1$ by one unit and hold 1/p

additional units of risky capital. This additional capital will allow

the consumer to increase second-period consumption by (y$_2$/p) units,

thereby increasing the expected discounted value of second-period

utility by βE{(y$_2$/p)u'(c$_2$)}. Equation (4b) indicates that, at the

optimum, a marginal change in the holding of risky capital will not

affect expected discounted lifetime utility.

Now divide equation (4b) by equation (4a) to obtain

$$pR \quad = \quad E\{(y_2 u'(c_2))\}/E\{u'(c_2)\} \qquad (5)$$

In order to explicitly calculate the consumer's demand for risky

capital, I will make specific assumptions about the form of the utility

function and consumer's subjective distribution of the random dividend.

Suppose that the utility function u(c) is a member of the constant

absolute risk aversion family. Under this assumption, the marginal

utility of consumption is u'(c) = exp[-γc] where γ > 0 is the

coefficient of absolute risk aversion. The marginal utility of second-

period consumption, u'(c$_2$), is calculated using the second-period budget

constraint in (2) to obtain

$$u'(c_2) \quad = \quad \exp[-\gamma Rb] \; \exp[-\gamma k_2 y_2] \qquad (6)$$

The consumer believes that y$_2$ is distributed normally with mean m

and variance s^2. Under the normality of y$_2$ with mean m and variance s^2,

it can be easily shown that

$$E\{\exp[\nu y_2]\} \quad = \quad \exp[\nu m + \nu^2 s^2/2] \qquad (7a)$$

and

$$E\{y_2\exp[\nu y_2]\} = (m + \nu s^2)\exp[\nu m + \nu^2 s^2/2]\} \qquad (7b)$$

where ν is a constant.

Using the expressions in (7a,b), and the expression for the marginal utility of second-period consumption (6), we obtain

$$E\{u'(c_2)\} = \exp[-\gamma Rb]\ \exp[-\gamma k_2 m + \gamma^2 k_2{}^2 s^2/2] \qquad (8a)$$

$$E\{y_2 u'(c_2)\} = (m - \gamma k_2 s^2)\ \exp[-\gamma Rb]\ \exp[-\gamma k_2 m + \gamma^2 k_2{}^2 s^2/2] \quad (8b)$$

The expectations in (8a,b) can now be used to calculate the optimal amount of risky capital, k_2, that the consumer wants to carry into the second period. Dividing (8b) by (8a) yields

$$E\{y_2 u'(c_2)\}/E\{u'(c_2)\} = (m - \gamma k_2 s^2) \qquad (9)$$

Substituting the right-hand side of (9) into the right-hand side of (5) yields the optimal value of k_2

$$k_2 = (m - pR)/(\gamma s^2) \qquad (10)$$

Equation (10) shows that the demand for risky capital is a decreasing function of: (a) the price of the risky capital asset, p; (b) the riskless rate of return, R; (c) the variance of the risky payoff, s^2; and (d) the coefficient of absolute risk aversion, γ. However, the demand for risky capital is an increasing function of m, which is the expected value of y_2.

The next step would be to calculate the optimal first-period consumption, c_1, and the optimal holding of riskless bonds, b. However, for the purposes of calculating asset prices, it is not necessary to calculate the optimal values of c_1 and b as functions of variables that are exogenous to the consumer. We need only derive a linear relation between the optimal values of c_1 and b. To obtain this relation, substitute (8a) into (4a) and then take the natural logarithm of each side to obtain

$$c_1 = (-1/\gamma)\ln(\beta R) + Rb + k_2 m - \gamma k_2^2 s^2/2 \quad (11)$$

This expression will be useful in computing equilibrium asset prices in the next section.

II. Equilibrium

A. Heterogeneous Subjective Expectations

In the previous section, I derived the optimal holding of the risky asset for an individual consumer and, in addition, I presented a relation among the individual consumer's optimal values of c_1, b, and k_2. A consumer's optimal decision rules depend, of course, on his subjective probability beliefs about the stochastic payoff y_2. Although all consumers are assumed to have the same utility function, they have different subjective probability beliefs about y_2. In particular, all consumers believe that y_2 is normally distributed with variance s^2, but they have different subjective expected values of y_2; the parameter m, which is the subjective expectation of the risky payoff y_2, varies across consumers. For this particular model, it suffices to specify only the first and second moments of the cross-sectional distribution of m. In particular, let M be the cross-sectional average value of the parameter m and let Var(m) be the variance of the cross-sectional distribution of m.

As a notational convention for variables representing quantities, let upper case letters denote the cross-sectional average values of the corresponding lower case letters. Thus, C_1 is the population average value of c_1, K_2 is the population average value of k_2, etc. Recall that

under the technological assumptions stated above, the consumption good

is perfectly perishable. Therefore, aggregate consumption in the first

period is equal to the aggregate output of the consumption good

$$C_1 = y_1 K \tag{12}$$

The aggregate demand for the risky capital asset, K_2, is

calculated directly by computing the average values of both sides of the

demand for risky capital in equation (10) to obtain

$$K_2 = (M - pR)/\gamma s^2 \tag{13}$$

Capital market equilibrium requires that the per capita demand for

capital in (13) is equal to the fixed per capita supply of capital, K.

Therefore, setting K_2 in (13) equal to K yields the capital market

equilibrium condition

$$pR = M - \gamma s^2 K \tag{14}$$

Equation (14) shows that if risky capital is to have a positive

price in equilibrium, then, on average, consumers must believe that the

expected payoff to risky capital is sufficiently large. More precisely,

in order for the equilibrium price of risky capital to be positive, the

average subjective expectation, M, must exceed $\gamma s^2 K$. Except for this

condition ($M > \gamma s^2 K$), no other restriction is placed on the cross-

sectional distribution of the positive parameter m.[1] Note that this

condition does not require that all consumers will have a positive

demand for risky capital in equilibrium. Indeed, it may well be that

pessimistic consumers (i.e., consumers with low subjective expectations

m) hold negative positions in risky capital by selling short. However,

---

[1]I am ignoring the possibility of bankruptcy. Under the
assumption of normality, it is possible that the disposable second-
period resources of a consumer may be negative. Technically, the
constant absolute risk aversion utility function is defined for negative
values of consumption, and the expectations calculated in this paper
take account of these possible realizations.

on average, there will be a positive demand for risky capital, and the amount of risky capital demanded will be equal to the fixed supply of risky capital.

The equilibrium value of pR depends on the cross-sectional distribution of the parameter m. However, equation (14) displays the striking result that the equilibrium value of pR depends only on the first moment of this cross-sectional distribution. In particular, cross-sectional variation of the parameter m has no effect on the equilibrium value of pR. Therefore, if the riskless rate of return were fixed exogenously, then the equilibrium price of risky capital would depend only on the cross-sectional average of m, but would be independent of the cross-sectional variation in m. Indeed, Varian's (1987) finding that "a mean-preserving spread in these beliefs [m] will leave asset prices unchanged" (p.23) is a direct implication of (14) combined with Varian's assumption that the riskless rate of return is exogenously fixed. However, in the absence of riskless physical capital, the riskless rate of return is determined endogenously, and as shown below, both the riskless rate and the price of risky capital are affected by a mean-preserving spread on the distribution of m.

Using the equilibrium value of pR in equation (14), it is now straightforward to calculate equilibrium optimal holding of the risky asset by an individual consumer. Substituting (14) into (10) yields

$$k_2 \quad - \quad (m - M)/\gamma s^2 \quad + \quad K \qquad (15)$$

An individual's demand for risky capital is an increasing function of that consumer's subjective expectation of y_2. However, under the particular specification of this model, k_2 is independent of the consumer's first-period holding of risky capital.

Consumers can hold riskless bonds or can borrow by issuing riskless bonds. Because all bonds are liabilities of some consumers and assets of some other consumers in this closed economy model, the aggregate stock of riskless bonds, B, is equal to zero.

$$B \quad = \quad 0 \qquad (16)$$

Now calculate the cross-sectional average value of first-period consumption by taking the average of both sides of (11) and using (16) to obtain

$$C_1 \quad = \quad (-1/\gamma)\ln(\beta R) + A\{k_2 m\} - \gamma A\{(s^2/2)k_2^2\} \qquad (17)$$

where $A\{x\}$ denotes the population average value of x. The next step is to calculate the two population averages that appear on the right hand side of (17). Using the demand for risky capital in (15), it is easily shown that

$$A\{k_2 m\} \quad = \quad Var(m)/\gamma s^2 + M\,K \qquad (18)$$

and

$$\gamma A\{(s^2/2)k_2^2\} \quad = \quad Var(m)/(2\gamma s^2) + (\gamma s^2/2)K^2 \qquad (19)$$

Substituting (18) and (19) into (17) yields an expression for the average value of first-period consumption in terms of the riskless rate of return and exogenous parameters

$$C_1 = (-1/\gamma)\ln(\beta R) + Var(m)/(2\gamma s^2) + M\,K - \gamma s^2 K^2/2 \qquad (20)$$

Substituting the equilibrium condition (12) into (20) and rearranging yields an expression for the riskless rate of return in terms of exogenous parameters

$$R = \beta^{-1} \exp[(M-y_1)\gamma\,K - \gamma^2 s^2 K^2/2 + Var(m)/(2s^2)] \qquad (21)$$

Equation (21) can be interpreted by first considering the special case in which there is no uncertainty ($s^2 = 0$) and no disagreement among consumers about the subjective expectation of the random payoff y_2

$(Var(m) = 0)$. If, in addition, all consumers expect y_2 to be equal to y_1 $(M = y_1)$, then equation (21) implies that $R = \beta^{-1}$, which has the interpretation that the riskless rate of interest is equal to the rate of time preference. Equation (21) illustrates three sources of departure of the riskless interest rate from the rate of time preference.

First, to the extent that M exceeds y_1, consumers, on average, expect to consume more in period 2 than in period 1. If the rate of interest were equal to the rate of time preference, consumers would attempt to smooth consumption over time by borrowing in the first period to increase current consumption. The attempt by all consumers to borrow would drive up the interest rate until consumers were satisfied to have second-period consumption exceed first-period consumption.

Second, the riskless rate of return is a decreasing function of $\gamma^2 s^2 K^2/2$, which reflects precautionary saving;[2] in response to increased uncertainty of future income, consumers tend to increase saving in period 1, thereby driving down the equilibrium riskless interest rate.

Third, cross-sectional variation in subjective expectations, which is measured by $Var(m)$, tends to increase the riskless interest rate. This result is the consequence of two opposing effects. The first effect, which is the dominant effect in this case, is that an increase in $Var(m)$ leads to an increase in the average of the subjective expected values of income from risky capital; optimistic consumers hold large amounts of capital and pessimistic consumers hold small amounts of risky capital. This increase in the average subjective expectation of income induces an increase in current consumption. The second effect is that

[2]See Kimball (1987) and Blanchard and Mankiw (1988)

an increase in Var(m) increases the average value of the variance of second period income from risky capital. The increase in the average variance of income tends to reduce current consumption due to the effect of precautionary saving. Under constant absolute risk aversion, the first effect dominates the second effect. Therefore, an increase in Var(m) leads to an increase in the aggregate level of desired first-period consumption at the initial interest rate. However, first-period consumption must be equal to the exogenous level $y_1 K$. In order to equilibrate aggregate desired consumption with the available supply, the interest rate R must rise.

We have derived the effect of an increase in Var(m) under the assumption of constant absolute risk aversion. In this case, aggregate consumption is an increasing linear function of the average subjective expected value of capital income and a decreasing linear function of the average variance of second-period capital income (equation (17)). The effect of increased cross-sectional variation is to unambiguously increase the equilibrium interest rate because the effect of the higher average subjective expectation is greater than the precautionary saving effect. More generally, for any utility function for which the precautionary saving effect is sufficiently small, an increase in Var(m) will increase the equilibrium riskless rate. For instance, under quadratic utility, there is no precautionary saving effect and hence an increase in Var(m) would increase the equilibrium interest rate.

The effects of heterogeneous beliefs on the riskless interest rate can be formally described by defining $R*$ as the gross riskless interest rate that would prevail under homogeneous beliefs. In particular, $R*$ is

the interest rate that would prevail if m = M for all investors. It follows from (21) that

$$R = R^* \exp[\text{Var}(m)/(2s^2)] \qquad (22)$$

Finally, the equilibrium stock price can be easily calculated by substituting (21) into (14) to obtain

$$p = (M-\gamma s^2 K)\beta \exp[-\text{Var}(m)/(2s^2) - (M-y_1)\gamma K + \gamma^2 s^2 K^2/2] \qquad (23)$$

Inspection of (23) reveals that the stock price, p, is a decreasing function of the cross-sectional variation in the subjective expectation m. The decrease in p leads to an increase in the expected rate of return on risky capital. Because an increase in Var(m) increases the expected rates of return on both stocks and riskless bonds, it would appear at first glance that the effect on the equity premium is indeterminate. However, as shown in section III, the equity premium unambiguously increases in response to a mean-preserving spread on the cross-sectional distribution of m.

B. Heterogeneous Subjective Risk Assessments

Now suppose that all investors have identical subjective expectations, m = M, but they have different subjective variances s^2. Equations (10) and (11) still hold, of course, for individual investors, with the modification that m = M. Calculating the cross-sectional average of both sides of (10), and setting the average demand for capital, K_2, equal to the per capita supply K, yields

$$K = (M - pR) A(1/s^2)/\gamma \qquad (10a)$$

Dividing (10) by (10a), and recalling that m = M for all investors, yields

$$k_2 = [(1/s^2)/A(1/s^2)]K \qquad (15a)$$

Equation (15a) shows that an individual's demand for risky capital is a decreasing function of s^2. Under homogeneity, $1/s^2 = A\{1/s^2\}$ for all investors and hence $k_2 = K$ for all investors.

Equation (15a) can be used to calculate the cross-sectional averages $A\{k_2 m\}$ and $\gamma A\{(s^2/2)k_2^2\}$ that appear in (17). Recalling that $m \equiv M$, we have

$$A\{mk_2\} = MK \qquad\qquad (18a)$$

$$\gamma A\{(s^2/2)k_2^2\} = [\gamma K^2/2]\ [1/A\{1/s^2\}] \qquad\qquad (19a)$$

Equations (18a) and (19a) indicate that heterogeneity of s^2 does not invalidate the use of a representative investor model for determining equilibrium asset prices. A representative investor model can be used because $A\{1/s^2\}$ is a sufficient statistic for the cross-sectional distribution of s^2. Asset prices in this economy are exactly the same as in an economy in which all investors have identical values of s^2 equal to $1/A\{1/s^2\}$. This result stands in sharp contrast to the results in subsection II.A with heterogeneity of subjective expectations m. In that case, there was no single scalar sufficient statistic for the cross-sectional distribution of m; equilibrium asset prices depended on M and Var(m) separately.

We have shown that the representative investor model is inappropriate when there is cross-sectional variation in the subjective first moment, m, but it is appropriate when the cross-sectional variation is limited to the subjective second moment s^2. The remainder of the paper will ignore variation in s^2 because such variation can be easily handled in the representative investor framework. The interesting departure from the representative investor model occurs for heterogeneous m.

III. The Equity Premium

The equity premium on stocks is the excess of the rate of return on stocks over the rate of return on bonds. The ex post, or realized, equity premium, q, in this two-period model is $q \equiv y_2/p - R$. Let q^e denote an individual investor's subjective expectation of the equity premium. Recalling that m is the subjective expectation of the dividend y_2, we have

$$q^e = m/p - R \tag{24}$$

Now let Q^e denote the cross-sectional average ex ante equity premium. Recalling that M is the cross-sectional average value of m, we have

$$Q^e = M/p - R \tag{25}$$

Note that if the average investor is rational--more precisely, if M is equal to the objective expectation of y_2--then Q^e is an unbiased forecast of the ex post equity premium.

Now use the equilibrium relation between p and R in (14) to rewrite (24) as

$$Q^e = \gamma s^2 K/p \tag{26}$$

The ex ante equity premium is an increasing function of: (a) investors' aversion to risk; (b) the variance of the dividend; and (c) the amount of capital that must be held in investors' portfolios.

To assess the impact of heterogeneous expectations on the ex post equity premium, let p* and Q^e* denote the stock price and the average ex ante equity premium in the case of homogeneous expectations, i.e., when all consumers have a subjective expectation m equal to M. It follows from (14), (22), and (26) that

$$Q^e/Q^e* = p*/p = R/R* = \exp[\mathrm{Var}(m)/(2s^2)] \tag{27}$$

Equation (27) holds for <u>any</u> cross-sectional distribution of m for which $M > \gamma s^2 K$. Because Var(m) is positive in the case of heterogeneity, equation (27) demonstrates that the equity premium is greater under heterogeneity of subjective expectations than under homogeneity. Thus, if the equity premium were calculated under the assumption of homogeneous expectations, this calculation would understate the equity premium that would emerge in an equilibrium with heterogeneous expectations. Furthermore, the extent of the understatement of the equity premium is a monotonically increasing function of the cross-sectional variance of expectations.

Can plausible degrees of heterogeneity account for the large gap between between the equity premia calibrated by Mehra and Prescott and the actual equity premia observed in the data? Mehra and Prescott could not get the equity premium above 35 basis points for a riskless rate less than 4% and for reasonable values of the preference parameters. The average actual equity premium was somewhat larger than 600 basis points. To attribute the difference to heterogeneous beliefs would require Q^e/Q^{e*} to be around $600/35 \approx 17$. Because Q^e/Q^{e*} is equal to $R/R*$, the introduction of heterogeneous beliefs would have to increase the <u>gross</u> riskless rate by a factor of about 17. The gross riskless rate is equal to one plus the rate of interest. Historically, the average value of the gross riskless rate is about 1.008. Thus, if heterogeneity is to account for a ratio of $Q/Q*$ equal to 17, it would require R* to be approximately 0.059. This value is absurdly low.[3] However, this untenable result is partly a consequence of the assumption that there is only one future period and that capital has an ex-dividend

---

[3]I thank David Romer for this argument.

price of zero in the second period. In the next section, I extend the model to allow for capital to have a positive ex-dividend price in the second period and then re-examine the implications of heterogeneous beliefs for the equity premium.

IV. Asset Pricing in a Multi-period Model

In developing a multi-period model of asset pricing under heterogeneous beliefs, several questions arise. For instance, how does the heterogeneity of beliefs evolve over time? Do different consumers approach consensus, and if so at what rate? How are heterogeneous beliefs about the next period's dividend related to heterogeneous beliefs about next period's stock price? The model presented below will not answer these questions definitively, but it will offer some insights into the behavior of asset prices in a multiperiod economy with heterogeneous beliefs.

I will proceed by extending the simple two-period model in two ways: First, I will assume that the ex-dividend price of a unit of capital in the second period is equal to p_2. In the simple two-period model above, p_2 was assumed to be equal to zero. Letting p_1 denote the ex-dividend price of a unit of capital in the first period, equation (1) is now rewritten as

$$(p_1 + y_1)k_1 = p_1 k_2 + c_1 + b \tag{1'}$$

Second, instead of maximizing the two-period utility function in (3), consumers are assumed to maximize

$$u(c_1) + \beta E\{V(W_2)\} \tag{28}$$

where W_2 is the consumer's wealth in the second period. This wealth is equal to the value of capital, with accrued dividends, plus the value of bonds, with accrued interest,

$$W_2 = (p_2 + y_2)k_2 + Rb \qquad (29)$$

A. A Digression on Rational Expectations in an Infinite-Horizon Model

Before proceeding with the extension of the two-period model, it is useful to present certain results about asset pricing in an infinite-horizon framework with homogeneous beliefs and rational expectations. Suppose that all consumers are identical in all respects, including their subjective probability distributions. Each consumer lives forever and at time t maximizes

$$E_t\{\sum_{j=0}^{\infty} \beta^j u(c_{t+j})\} \qquad (30)$$

where $E_t\{\}$ denotes the expected value conditional on information at time t.

A consumer's maximization problem at time t can be expressed as

$$V(W_t) = \max \{u(c_t) + \beta \, E_t\{V(W_{t+1})\}\} \qquad (31)$$

where the function V(), which is known as the value function, is a solution to the functional equation in (31). The value function depends on the stochastic properties of the rates of return as well as on the specification of the utility function.

The equilibrium price of risky capital is equal to the expected present value of future dividends discounted by the marginal rate of substitution. Therefore,

$$p_t = E_t\{\sum_{j=1}^{\infty} \beta^j u'(c_{t+j})y_{t+j}\} / u'(c_t) \qquad (32)$$

As in previous sections, assume that the consumption good is completely perishable so that aggregate consumption is equal to the aggregate dividend. Because all consumers are identical, they all consume identical amounts in each period; the consumption of the representative consumer in period t is equal to the dividend per capita, $y_t K$. Substituting this equilibrium condition into (32) yields

$$p_t = E_t \{ \sum_{j=1}^{\infty} \beta^j u'(y_{t+j}K)y_{t+j} \} / u'(y_t K) \qquad (33)$$

Now suppose that the dividend, y_t, follows a random walk with normal increments. In particular,

$$y_{t+1} = y_t + \epsilon_{t+1} \qquad (34)$$

where ϵ_{t+1} is $N(0,s^2)$. In addition, assume that the utility function u() displays constant absolute risk aversion

$$u(c_t) = (-1/\gamma)\exp[-\gamma c_t] \qquad (35)$$

Under the stochastic specification in (34) and the utility function in (35), it is straightforward to calculate the equilibrium gross riskless rate of return, R,

$$R = (\beta\phi)^{-1} \qquad (36)$$

where $\phi \equiv \exp[K^2\gamma^2 s^2/2]$. Observe that the riskless rate of return is constant over time. Letting r denote the net riskless rate of return, R - 1, equation (36) implies

$$r = (\beta\phi)^{-1} - 1 \qquad (37)$$

Assume henceforth that the parameters β, γ, s^2, and K are such that $\beta\phi < 1$. Under this assumption, equation (37) shows that the net riskless rate of return, r, is positive.

The equilibrium price of risky capital in (33) can be shown to be

$$p_t = [1/r] [y_t - (R/r)K\gamma s^2] \qquad (38)$$

The ex-dividend price, p_t, is a linear function of the most recent dividend, y_t. Because y_t follows a random walk with normal increments, the ex-dividend price, p_t, also follows a random walk with normal increments. In addition, per capita wealth, which is $W_t \equiv (p_t+y_t)K$, follows a random walk with normal increments; the variance of the increments in W_t is equal to $K^2s^2/(1-\beta\phi)^2 = (R/r)^2K^2s^2$.

Finally, it can be shown that under the stochastic specification in (34) and the utility function in (35), the value function is

$$V(W_t) = -\lambda \exp[-\theta W_t] \tag{39}$$

where $\theta = (1-\beta\phi)\gamma$ and $\lambda = \theta^{-1}\exp[-(Ks\gamma)^2/r]$. The value function displays constant absolute risk aversion, but the degree of risk aversion for the value function, θ, is smaller than for the period utility function, γ. Using (36) and (37), the parameter θ of the value function can be written as

$$\theta = (r/R) \gamma \tag{40}$$

B. Heterogeneous Beliefs in the Extended Two-Period Model

1. Individual Behavior

Now return to the two-period objective function in (28). The optimal portfolio of bonds and risky capital maximizes this two-period objective function and satisfies the following first-order conditions

$$u'(c_1) = \beta R E\{V'(W_2)\} \tag{4a'}$$

$$u'(c_1) = \beta (1/p_1)E\{(p_2+y_2)V'(W_2)\} \tag{4b'}$$

Equation (4a'), which describes the optimal holding of bonds, is identical to equation (4a) except that the marginal utility of second-period consumption, $u'(c_2)$, is replaced by the marginal utility of second-period wealth, $V'(W_2)$. Equation (4b'), which describes the

optimal holding of risky capital, differs from equation (4b) in two respects: (a) the marginal utility of second-period consumption is replaced by the marginal utility of second-period wealth; and (b) the value of a unit of risky capital in the second period is equal to the second-period dividend, y_2, plus the ex-dividend price, p_2. It follows from (4a') and (4b') that

$$p_1 R = E\{(p_2+y_2)V'(W_2)\}/E\{V'(W_2)\} \qquad (5')$$

Now suppose that $V(W_2)$ displays constant absolute risk aversion equal to θ. In particular,

$$V(W_2) = -\lambda \exp[-\theta W_2] \qquad (41)$$

Using the definition of second-period wealth in (29), the marginal utility of second-period wealth is

$$V'(W_2) = \lambda\theta \exp[-\theta Rb] \exp[-\theta(p_2+y_2)k_2] \qquad (6')$$

Suppose that p_2 and y_2 are jointly normally distributed. As before, the subjective mean of y_2 is equal to m; the subjective mean of p_2 is equal to π. Finally, let σ^2 denote the variance of $p_2 + y_2$. Under this stochastic specification, it can be shown that

$$E\{(p_2+y_2)V'(W_2)\}/E\{V'(W_2)\} = (m + \pi - \theta k_2 \sigma^2) \qquad (9')$$

The right-hand side of equation (9') is identical to that of equation (9) except that: (a) the subjective mean dividend, m, is replaced by m + π, the subjective mean of $y_2 + p_2$; (b) the variance of the second-period dividend, s^2, is replaced by σ^2, the variance of the sum of the dividend and the ex-dividend price, and (c) the preference parameter γ is replaced by the preference parameter θ. Substituting (9') into (5') yields the demand for risky capital by an individual

$$k_2 = (m + \pi - p_1 R)/\theta\sigma^2 \qquad (10')$$

Again, the individual demand for capital in (10′) is the same as the individual demand for risky capital in (10) with the three modifications noted above. Finally, we can calculate the relation between optimal consumption in the first period and the optimal holding of bonds to be

$$\gamma c_1 \;=\; -\ln(\beta\; R\; \theta\; \lambda) + \theta Rb + \theta k_2(m+\pi) - \theta^2 k_2^2 \sigma^2/2 \quad (11')$$

2. Characterizing the Equilibrium

Suppose, as before, that consumers differ only in their subjective expectations of the random variables. Recall that M is the population average value of m, and let Π denote the population average value of π. Calculating the population average values of both sides of the demand for risky capital in equation (10′), and setting the average demand equal to the per capita supply K, yields

$$p_1 R \;=\; M + \Pi - \theta\sigma^2 K \quad (14')$$

Substituting (14′) into (10′) yields the individual demand for risky capital

$$k_2 \;=\; (m + \pi - (M + \Pi))/\theta\sigma^2 \;+\; K \quad (15')$$

Equation (15′) can be used to calculate the population average values of $k_2(m+\pi)$ and k_2^2 in a manner to similar to the derivations of (18) and (19). These population average values can be used to calculate the population average values of both sides of (11′) to obtain

$$R = (\beta\theta\lambda)^{-1}\exp[(\theta(M+\Pi)-\gamma y_1)K+Var(m+\pi)/(2\sigma^2)-\theta^2\sigma^2 K^2/2] \quad (21')$$

Recalling that R* is the gross riskless rate of return in an economy with homogeneous beliefs, it follows immediately from (21′) that

$$R \;=\; R^* \exp[Var(m+\pi)/(2\sigma^2)] \quad (22')$$

The relation between R and R* in (22′) is the same as in (22) for the simple two-period economy, except that Var(m) is replaced by Var(m+π) and s^2 is replaced by σ^2.

Now we will calculate the cross-sectional average ex ante equity premium. Because the ex-dividend price of the stock is positive in period 2, we must amend the definition of the ex post equity premium to $q = (y_2 + p_2)/p_1 - R$. The cross-sectional average ex ante equity premium, Q^e, can be calculated using (14') to obtain

$$Q^e = \theta \sigma^2 K/p_1 \qquad (42)$$

Note that (42) is identical to (26) except that the preference parameter γ is replaced by the θ and the variance s^2 is replaced by σ^2.

In analyzing the impact of heterogeneous expectations on the equity premium, I will focus on two cases below. In the first case, investors understand the relation among dividends, stock prices and the rate of interest. Different investors have different subjective expectations of future dividends which induce different subjective expectations of the future price of the stock. In the second case, investors are myopic and do not base their forecasts of future stock prices on the forecasts of future dividends. These investors have heterogeneous beliefs about the next period's dividend and the next period's stock price, but these beliefs are not linked by the interest rate.

3. Case I: Infinite-Horizon Investors

If all investors have infinite horizons, then we must interpret the function V() in the two-period objective function in (28) as the value function of an investor. The value function depends on the equilibrium sequence of probability distributions of asset returns. Recall from the digression on the infinite-horizon model under rational expectations, that the equilibrium price of stock satisfies (38) and that the parameter θ of the value function is given by (40). In

addition, it follows immediately from (38) that σ^2, the variance of (p_2 + y_2), is equal to $(1+1/r)^2 s^2 = (R/r)^2 s^2$.

Now consider the following modification of the rational expectations model: all consumers know the equilibrium relation among the stock price, contemporaneous dividend, and the riskless rate of interest. However, they disagree about the expected value of the dividend in the following period, and, as a consequence, they also disagree about the expected value of p_2. Suppose, however, that the average forecast of y_2 is unbiased; that is, suppose that M is equal to the objective expectation of y_2. Under these assumptions, the relation among stock price, dividend and the riskless interest rate in (38) continues to hold, as shown below. Indeed, the equilibrium is characterized as follows

$$R = \exp[Var(\pi+m)/(2\sigma^2)](\beta\phi)^{-1}$$

$$= \exp[Var(m)/(2s^2)](\beta\phi)^{-1} \qquad (43a)$$

$$p_1 = [1/r] [y_1 - (R/r)K\gamma s^2] \qquad (43b)$$

$$\sigma^2 = (R/r)^2 s^2 \qquad (43c)$$

$$\theta = (r/R)\gamma \qquad (43d)$$

$$\lambda = \theta^{-1}\exp[-(Ks\gamma)^2/r] \qquad (43e)$$

To verify that (43a-e) characterize the equilibrium, note that under the random walk specification of dividends, $M = y_1$. Also equation (43b), together with the random walk assumption on dividends, implies that $\Pi = p_1$. Substituting $M = y_1$ and $\Pi = p_1$ into (21') and simplifying using (43b-e) yields the first equality in (43a). To verify the second equality in (43a), note that if all investors know the relation between dividend and price, then cross-sectional variation in the price forecasts, π, results only from cross-sectional variation in dividend

forecasts. Under the price relation (43b), $\text{Var}(\pi+m) = (1 + 1/r)^2\text{Var}(m)$ $= (R/r)^2 \text{Var}(m)$. Using this expression for $\text{Var}(\pi+m)$ together with (43c) verifies the second equality in (43a).

To verify (43b), substitute $M = y_1$ and $\Pi = p_1$ into (14') to obtain $p_1 = (1/r)[y_1 - \theta\sigma^2 K]$. Equations (43c,d) imply that $\theta\sigma^2 = (R/r)\gamma s^2$ so that (14') now implies that $p_1 = (1/r)[y_1 - (R/r)\gamma s^2 K]$, which is identical to (43b).

Before proceeding, we note an additional quirk of non-rational investors for whom $m \neq M = y_1$. The derivation of the value function in (39) with the parameter θ given by (40) was based on the assumption that investors know that the dividend follows a random walk with normal increments. However, the non-rational investors for whom $m \neq M = y_1$ evidently ignore the random walk assumption when forecasting dividends one period ahead. By assuming that these investors have the same value of θ as given by (40) for rational investors, I am implicitly assuming that as of the next period these consumers believe that the dividend follows a random walk. Nevertheless, in the following period, either these same investors, or other investors, may form one-period forecasts that ignore the random walk nature of dividends. This dynamic formulation of heterogeneous beliefs is admittedly *ad hoc* but, at this stage, any treatment of the dynamic behavior of non-rational investors would be *ad hoc*. An alternative formulation is presented in subsection IV.B.4 below.

In this economy, the riskless rate of interest is constant over time. It is taken as a parameter by individual infinite-horizon investors. Recalling that $V(W)$ is the value function, the parameter θ is given by (40) as $\theta = (r/R)\gamma$. Using this value of the preference

parameter, and observing that $\sigma^2 \equiv \text{Var}(p_2 + y_2) = (R/r)^2 s^2$, the

expression for the average equity premium in (42) becomes

$$Q^e = \gamma s^2 RK/(rp_1) \tag{44}$$

Now substitute (43b) into (44) and use the fact that $M = y_1$ to

obtain

$$Q^e = [\psi/(\gamma R) - 1/(R-1)]^{-1} \text{ where } \psi \equiv M/(s^2 K) \tag{45}$$

Note that ψ/γ must be greater than $R/(R-1)$ in order for the stock price

to be well defined. To interpret ψ, multiply the numerator and

denominator of ψ by K to obtain $\psi = MK/(sK)^2$. Thus, ψ is the ratio of

the average expected level of consumption per capita to the variance of

the level of consumption per capita.

Now consider the effect of heterogeneous beliefs on the equity

premium in (45). It follows from (43a) that R is monotonically

increasing in $\text{Var}(m)$. The effect of an increase in R on the equity

premium can be summarized by the elasticity of Q^e with respect to R,

which can be calculated from (45) as

$$(dQ^e/dR)(R/Q^e) = [\psi/\gamma - (R/r)^2][Q^e/R] \tag{46}$$

The sign of the elasticity of Q^e with respect to R is the same as

the sign of $\psi/\gamma - (R/r)^2$. If $\psi/\gamma > (R/r)^2$, then increased heterogeneity

of beliefs increases R and also increases the equity premium. By

contrast, if $\psi/\gamma < (R/r)^2$, an increase in heterogeneity still increases

R but will decrease the equity premium.

4. Case II: Myopic Investors

Now consider the case of myopic investors who look ahead only one

period. Suppose that the average investor has unbiased forecasts of y_2

and p_2; formally, the assumption is that M is equal to the objective

expectation of y_2 and Π is equal to the objective expectation of p_2. In

addition, suppose that the objective expectation of p_2 is equal to p_1 so that $\Pi = p_1$. Because $M = y_1$ and $\Pi = p_1$, equation (14') implies

$$p_1 r = y_1 - \theta\sigma^2 K \qquad (47)$$

Substituting (47) into (42) yields the following expression for the average ex ante risk premium

$$Q^e = r [\theta\sigma^2 K/(y_1 - \theta\sigma^2 K)] \qquad (48)$$

One distinction between myopic and infinite-horizon investors is that the preference parameter θ is treated as an exogenously specified constant for a myopic investor, but is a component of the value function and depends on the equilibrium (constant) interest rate for an infinite-horizon consumer. The other distinction is that $\sigma^2 = \mathrm{Var}(p_2 + y_2)$ is an exogenous constant in the case of myopia, but it depends on the equilibrium interest rate for infinite horizon-consumers who understand the relation in (38). Because the parameters θ and σ^2 are invariant to the equilibrium interest rate and are treated as constants in the model of myopic investors, equation (48) implies that for any given value of y_1,

$$Q/Q* = r/r* = (R-1)/(R*-1) \qquad (49)$$

Because the gross interest rates, R and R*, are both close to one, the ratio on the right hand side of (49) can be quite large even if R/R* is only slightly greater than one. Thus, the equity premium could be substantially understated by assuming homogeneous expectations. As an example, suppose that a fraction (1-2n) of investors has unbiased forecasts of price and dividend in the following period. For these investors, $m + \pi = M + \Pi$. Half of the remaining investors have a forecast of $p_2 + y_2$ that is one standard deviation below the average forecast, and the other half have a forecast of $p_2 + y_2$ that is one

standard deviation above the forecast. That is, a fraction n of investors have $m + \pi = M + \Pi - \sigma$ and a fraction n have $m + \pi = M + \Pi + \sigma$. Therefore, $Var(m+\pi) = 2n\sigma^2$. Using this expression for $Var(m+\pi)$ in (21') yields $R = exp[n] R*$. For $n = 0.01$, $R/R* = 1.01005$. If, in addition, $R* = 1.001$, then $R = 1.011060$ and $Q^e/Q^e* = 11.06021$. Thus, calculating the equity premium under homogeneous beliefs can dramatically understate the equity premium, even if 98% of investors have unbiased price and dividend forecasts.

V. Concluding Remarks

This paper investigates the impact of heterogeneous beliefs on equilibrium asset prices and rates of return. The model developed in the paper makes two standard modeling assumptions: (a) investors have constant absolute risk aversion; and (b) investors treat the stochastic output produced by capital as normally distributed. This pair of assumptions is convenient for a few reasons. First, it delivers individual demand functions for risky capital that are linear in the subjective mean of the payoff to capital. These linear demand functions are easily aggregated--even under heterogeneous beliefs--to determine equilibrium asset prices. Second, this pair of assumptions has been recently used by Varian (1987) to show that heterogeneity *per se* does not effect on equilibrium asset prices. In particular, he shows that asset prices are invariant to the degree of heterogeneity of subjective expectations. Varian's finding depends crucially on his assumption that the riskless rate of return is exogenous. However, I show in this paper that if the riskless rate of return is determined endogenously in the

capital market, then heterogeneity of subjective means will affect
equilibrium asset prices and rates of return. The equilibrium riskless
rate is a monotonically increasing function of the cross-sectional
variance of subjective means. This result holds true in the two
alternative multiperiod formulations of the model studied in this paper.

This paper also examines the effect of heterogeneity on the
average equity premium. In one formulation of the model, investors are
infinitely-lived and understand the equilibrium relation among the stock
price, the contemporaneous dividend and the interest rate. However,
investors disagree about their subjective forecasts of the dividend in
the following period. In this formulation, the equity premium may
either increase or decrease in response to an increase in heterogeneity,
according to a condition presented in the paper. In the other
formulation, investors are myopic; in this case, the equity premium is
a monotonically increasing function of the cross-sectional variance of
subjective means. The implication of this finding is that if a general
equilibrium model with homogeneous beliefs is used to calculate the
equity premium, this calculation will understate the equity premium that
would prevail in the presence of heterogeneity.

The analysis in the first three sections of this paper was based
on a simple two-period model in which the only random variable was the
exogenous amount of output produced by capital in the second period.
The virtue of this model is its simplicity. Each consumer in this model
makes decisions at only one point in time. There is no scope for
learning in real time or for convergence of expectations as time
proceeds. Thus, the solution of the model was quite straightforward.
One cost of this simplicity is the result that the heterogeneity of

beliefs increases the equity premium and the gross interest rate by the same proportion. This result does not leave much scope for heterogeneous expectations to explain the understatement of the equity premium in empirically calibrated homogeneous expectations models.

Section IV of the paper extends the model to a multiperiod framework in which capital retains some value even after paying the second-period dividend. Two alternative formulations are presented. In one formulation, infinite-horizon investors know the equilibrium relation among the stock price, dividend and the interest rate, but they disagree about the forecast of next period's exogenous dividend. I have solved for the value function of individual investors and then aggregated their behavior to determine equilibrium asset prices and returns. In the other formulation, myopic investors forecast the stock price and dividend one period ahead. These formulations yield useful insights about asset pricing under heterogeneous beliefs in a dynamic framework, but many questions invite further investigation. In particular, how does the heterogeneity of subjective expectations evolve over time? Do the differing subjective expectations of investors approach consensus over time, or is the heterogeneity continually regenerated as a result of new extrinsic shocks impacting differently on different investors? To answer these questions will require substantial progress on the fundamental question of where the heterogeneity comes from in the first place.

References

Blanchard, Olivier J. and N. Gregory Mankiw, "Consumption: Beyond Certainty Equivalence," American Economic Review, 78, 2 (May 1988), 173-177.

Cho, Jaeho, "The Effect of Heterogeneous Beliefs on the Asset Price and the Trading Volume," mimeo, Wharton School of the University of Pennsylvania, 1987.

DeLong, J. Bradford, Andrei Shleifer, Lawrence H. Summers and Robert Waldmann, "The Economic Consequences of Noise Traders," National Bureau of Economic Research Working Paper No. 2395, October 1987.

Kimball, Miles S., Essays on Intertemporal Household Choice, unpublished Ph.D. dissertation, 1987.

Kraus, Alan and Gordon A. Sick, "Distinguishing Beliefs and Preferences in Equilibrium Prices," Journal of Finance, 35, 2 (May 1980), 335-344.

Lintner, John, "The Aggregation of Investor's Diverse Judgments and Preferences in Purely Competitive Security Markets," Journal of Financial and Quantitative Analysis, 4 (December 1969), 347-400.

Lucas, Robert E. Jr., "Asset Prices in an Exchange Economy," Econometrica, 46 (November 1978) 1429-1445.

Mankiw, N. Gregory, "The Equity Premium and the Concentration of Aggregate Shocks," Journal of Financial Economics, 17 (1986), 211-219.

Mehra, Rajnish and Edward C. Prescott, "The Equity Premium: A Puzzle," Journal of Monetary Economics, 15,2 (March 1985), 145-161.

Singleton, Kenneth J. "Asset Prices in a Time Series Model with Disparately Informed Competitive Traders," National Bureau of Economic Research Working Paper No. 1897, April 1986.

Varian, Hal R., "Differences of Opinion in Financial Markets," mimeo, University of Michigan, January 1987.