

SIGNALLING AND THE PRICING OF  
UNSEASONED NEW ISSUES

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## ABSTRACT

This paper develops a signalling model with two signals, two attributes, and a continuum of signal levels and attribute-types to explain a puzzling phenomenon: the underpricing of new issues. Both the fraction of the new issue retained by the issuer and its offering price convey the unobservable "intrinsic" value of the firm and the variance of its cash flows to investors. Many of the model's comparative statics results are quite novel, empirically testable, and consistent with the existing empirical evidence on new issues, as well as the beliefs of many investment professionals. In particular, a closed-form solution for the signalling schedule indicates that the degree of underpricing, which can be inferred from observable variables, is positively related to the firm's post-issue share price.

## 1. INTRODUCTION

One of the most puzzling phenomena in finance is the underpricing of new issues. In perhaps the most widely known study, Ibbotson (1975) found that the average risk-adjusted return from buying a new issue at its offering price and selling it at the end of the same month exceeds 11%. Returns of this magnitude have been confirmed in many other studies. For example, Ritter (1988) documents an average price rise of more than 14% from the offering date to the first date of public trading, which is typically less than one day.<sup>1</sup> In addition, Chalk and Peavy (1987) identify a one-day return of over 21% for 649 issues with a market price on the day after the offering date.

At first glance, large returns for a predictable event seem to contradict the efficient markets paradigm. However, this paper will argue that the underpricing phenomenon can be a consequence of the actions of rational agents. In particular, we present a signalling model in which underpricing is an equilibrium outcome.

To our best knowledge, only two previous papers, by Rock (1986) and Baron (1982), model underpricing as an equilibrium phenomenon in a market with asymmetrically informed agents. In Rock's (1986) model, uninformed investors who subscribe to a new issue face adverse selection because some potential subscribers have superior information. Informed investors do not subscribe to a new issue that they suspect is overpriced, leaving the entire issue to uninformed investors. However, when the issue is expected to earn a high return, they subscribe to a large number of shares and the oversubscribed issue must then be rationed. Thus, the typical share must be offered at a discount (i.e. earn a positive risk-adjusted return) in order for uninformed investors to earn a zero risk-adjusted and ration-adjusted return. Rock's model requires the issuer to have no better information about the value of his

issue than any investor, which is counterintuitive. Moreover, it posits the existence of a large economic rent that can be captured, and thus eliminated, by more efficient contractual arrangements. For example, uninformed clients could contract with investment banks to purchase a guaranteed fraction of every new issue.

Baron (1982) assumes that the value of a new issue is affected by market demand and the investment banker's distribution effort. In his model, the investment banker is better informed about market demand than the issuer, but his distribution effort is unobservable. To address this moral hazard, the optimal contract sets the issue's offering price below its "true value," defined as the equilibrium offering price when the investment banker expends his best effort. However, this result is not the underpricing phenomenon observed. Since the investment banker in Baron's model never expends his best effort, the post-issue price of an equity share is not its value under the best effort scenario.

In our analysis of underpricing, an issuer is assumed to have better information about his firm's future cash flows than outside investors. To overcome the asymmetric information problem, the issuer signals the true value of the firm by offering shares at a discount and by retaining some of the shares of the new issue in his personal portfolio. Such a model can be regarded as a generalization of Leland and Pyle (1977). In their model, the issuer's fractional holding of his firm's equity signals its expected future cash flows--a higher fractional holding signals larger cash flows. Since they analyze only one signal, only one parameter can be unknown, implying that the variance of the cash flows of the firm's projects must be observable. The model examined in this paper assumes that the variance as well as the mean of the project's cash flows are unknown, so that a second signal, the issue's

offering price, is needed to convey the firm's value to the market. In the model's separating equilibrium, a firm's intrinsic value is positively related to the degree its new issue is underpriced.

One appealing aspect of this model is that it is consistent with the rationale for underpricing offered by many investment professionals. They typically state that the investor interest generated by a low-priced new issue tends to subsequently result in higher-priced shares than would have been possible without the underpricing. As we will shortly demonstrate, this belief is perfectly rational. It is also consistent with Ibbotson's (1975, p. 264) conjecture that new issues may be underpriced in order to "leave a good taste in investor's mouths."

Three other contemporaneous signalling models also have this feature. Allen and Faulhaber (1988),<sup>2</sup> Nanda (1988),<sup>3</sup> and Welch (1988)<sup>4</sup> provide conditions for the existence of a separating equilibrium in models with two risk-neutral issuer types and an exogenously specified (binding) constraint on outside financing. Our model, by contrast, has an endogenously determined amount of external financing and a continuum of risk-averse issuer types. Risk aversion, in fact, is critical to our argument. In addition, our model permits the existence of firms with negative net present value projects. These potential issuers are endogenously deterred from floating an unseasoned offering in equilibrium.

The paper is organized as follows: Section 2 outlines the model. Section 3 derives the signalling schedules of the separating equilibria for two special cases. The first case, in which the variance is known and the offering price is not a signal, enables us to relate our model to the work of Leland and Pyle. It also provides a boundary solution for the general case. The second case, in which the issuer's shareholdings are constrained and thus

cannot be a signal, intuitively explains why underpricing a new issue can signal a healthy firm. Section 4 derives an analytic solution for the signalling schedule in the general case, which has two signals and two unknown attributes. This schedule generates a particularly rich set of empirically testable propositions, which are described in Section 5. Section 6 explores the signalling game in a discrete setting in order to investigate the uniqueness of the solution and the possibility of pooling. These issues are analyzed within a discrete version of the model because of technical difficulties in analyzing them with the two-attribute continuum model. The paper is concluded in Section 7.

## 2. THE MODEL

Consider a three-date world in which an entrepreneur owns an investment project that requires a date 0 capital outlay of  $K$ . The project yields a cash flow of  $\mu + \tilde{X}_2$  at date 2 and an independent random cash flow  $\tilde{X}_1$  at date 1;  $\tilde{X}_1$  and  $\tilde{X}_2$  have means of 0 and variances of  $\sigma^2$ .  $\mu$  and  $\sigma^2$  are drawn from a continuous bivariate distribution, with marginal distribution functions that are strictly increasing in  $\mu$  and  $\sigma^2$  and with values of 1 at some finite upper bounds, denoted  $\bar{\mu}$  and  $\bar{\sigma}^2$ . The lower bound on  $\sigma^2$ , denoted  $\sigma_L^2$ , must be positive. There may or may not be a lower bound on  $\mu$ , denoted  $\underline{\mu}$ . If  $\underline{\mu}$  exists,  $\underline{\mu} \leq K$ .<sup>5</sup>

The entrepreneur, henceforth "the issuer," is risk averse, and to achieve a more diversified portfolio, markets the project to the investing public. He has information that leads him to assign a specific value to  $\mu$  and  $\sigma$ , and would like to credibly convey this information directly to investors as soon as possible if  $\mu$  exceeds public expectations. There are two signals that can be employed, each observed by market participants at date 0. The first is the fraction of the new issue retained by the issuer, denoted by  $\alpha$ . The second is

the offering price at which the issue is being sold, denoted as  $S$ .<sup>6</sup> As Section 4.2 later demonstrates, a signalling model where outside investors use  $\alpha$  and  $S$  to perfectly infer  $\mu$  and  $\sigma^2$  is equivalent to a model where they observe  $\alpha$  and the amount by which the issue is underpriced, denoted as  $H$ . We will sometimes take advantage of this equivalence by treating  $H$  as though it is observable and letting  $S$  be endogenously determined by  $H$  and by the beliefs of outside investors about the project mean,  $\mu$ . The innocuous switch in variables allows us to better exposit the role played by underpricing.

In the absence of signalling, outside investors learn  $\mu$  at date 1 with probability  $P$ . Otherwise,  $\mu$  remains unknown until date 2. Date 1 plays an important role in this model for other reasons as well. Since, by law, the issuer must hold his fraction  $\alpha$  of the new issue only for a specified period of time, we would expect him to sell these shares as soon as legally possible in order to hold a more diversified portfolio.<sup>7</sup> To model this, we assume that the issuer sells his remaining fraction of the issue at date 1, after the realization of  $\tilde{X}_1$  becomes public information. Hence, the issuer's objective is to maximize his expected utility of date 1 wealth. For the sake of tractability, expected utility takes the linear form

$$E(U(\tilde{W}_1)) = E(\tilde{W}_1) - \frac{b}{2} \sigma^2(\tilde{W}_1), \quad (1)$$

where  $E(\tilde{W}_1)$  is the expected value of the issuer's wealth at date 1,  $\sigma^2(\tilde{W}_1)$  is the variance of date 1 wealth, and  $b$  is a risk aversion parameter.

Although the objective function used here is the same as the objective function in the Leland and Pyle (1977) model, the two models differ. In Leland and Pyle, the expected value of the project,  $\mu$ , becomes known to outside investors at date 1 with certainty, and the project variance is known at date 0. By contrast, our model assumes that in the absence of signalling,



$\mu$  becomes known to investors with probability  $P$  at date 1, and the variance of the project is unknown.

As in Leland and Pyle, we assume that the issuer can choose any budget feasible combination of three investments--a risk-free asset, the market portfolio, and equity shares in his own firm--and that these investments are priced according to the Capital Asset Pricing Model, using the information available to outside investors.<sup>8</sup> We also assume that the cash flows of the project,  $\bar{X}_1$  and  $\mu + \bar{X}_2$ , are uncorrelated with the returns of the market portfolio in periods 1 and 2. In a perfect markets world, the latter assumption is innocuous if the issuer's knowledge of  $\mu$  does not provide him with additional information about the value of the market portfolio at dates 1 or 2 or about the correlation between the cash flows of the project and the value of the market portfolio.

Under these assumptions, the market value of the project at date 0 is the discounted sum of the project's expected cash flows. Without loss of generality, the risk-free rate can be set to zero. Hence, if uninformed investors assess the second period's expected cash flow to be  $\mu(\alpha, H)$ , they value their portion of the project at  $(1 - \alpha)\mu(\alpha, H)$ . Their actual cash payment to the issuer is  $(1 - \alpha)[\mu(\alpha, H) - H]$ , implying a per share rent to purchasers of the new issue that is proportional to  $H$ . (If the share numeraire is defined so that the firm consists of one share, then  $H$  is the underpricing per share.)

The previous description of the model can be more formally represented as a signalling game. In its extensive form, an informed potential issuer moves first. If he decides not to issue, he achieves the utility associated with investing all wealth in the market portfolio and a risk-free asset. If he chooses to issue, his action is a 2-element vector,  $(\alpha, S)$ , where  $0 \leq \alpha < 1$

and  $S \in \mathbb{R}^1$ . Numerous identical uninformed investors, in perfect competition with each other, move in response. Their actions consist of acceptance or rejection of the initial offering at date 0, and the setting of prices in a secondary market at date 1. They will increase (decrease) their utility if they accept the offering and  $S$  is less (greater) than  $E(\mu|\alpha, S)$ . Although we have not formally modelled flotation costs, we will assume that a rejected offer makes the issuer worse off than an offer that is never made.

Since investor beliefs are rational, the optimal strategy of uninformed investors is to reject the offer only when  $S > E(\mu|\alpha, S)$ . In the secondary market, the price of the firm is set equal to  $E(\mu|\alpha, S)$  if the firm's value is not perfectly revealed, and equal to  $\mu$  if it is.

### 3. SPECIAL CASES OF THE MODEL

#### 3.1 Special Case 1: Known Variance

In this subsection, the variance of the project is known to outside investors at date 0 and the issuer signals the expected value of the project with his fractional holdings. This special case provides a benchmark solution that is used as a boundary condition in the more general case (Section 4). It also illustrates the model's relation to the Leland and Pyle model.

The issuer's date 0 decision problem is to maximize  $E(U(\tilde{W}_1))$  subject to the wealth feasibility constraint

$$\tilde{W}_1 = \alpha(\mu + \tilde{X}_1) + \beta(1 + \tilde{R}_M) + [W_0 + (1 - \alpha)(\mu(\alpha) - H) - \beta - K] , \quad (2)$$

where

$W_0$  is the initial wealth of the issuer, excluding the net worth of the project,

$\beta$  is the number of dollars invested in the market portfolio by the issuer at date 0,

$\tilde{R}_M$  is the excess return of the market portfolio (above the risk-free rate) between dates 0 and 1, which has mean  $\bar{R}_M$  and variance  $\sigma_M^2$ ,

$\mu(\alpha)$  is the market value of the project, as perceived by outside investors at date 0, and

$\tilde{\mu}$  is a random variable.

$\tilde{\mu} = \mu$  with probability  $P$  at date 1; otherwise  $\tilde{\mu} = \mu(\alpha)$ . In the latter case, investors use the signalling schedule  $\mu(\alpha)$  to evaluate the expected value of the project's date 2 cash flows.

Using this distribution of  $\tilde{\mu}$ , one can compute

$$E(\tilde{\mu}) = P\mu + (1 - P)\mu(\alpha) \quad (3)$$

and

$$\text{Var}(\tilde{\mu}) = P(1 - P)(\mu - \mu(\alpha))^2 . \quad (4)$$

Equations (2), (3), and (4) can be used to express the expected value and variance of date 1 wealth,

$$E(\tilde{W}_1) = \alpha[P\mu + (1 - P)\mu(\alpha)] + (W_0 - K) + (1 - \alpha)(\mu(\alpha) - H) + \beta\bar{R}_M \quad (5)$$

and

$$\text{Var}(\tilde{W}_1) = \alpha^2\sigma^2 + \beta^2\sigma_M^2 + \alpha^2P(1 - P)[\mu - \mu(\alpha)]^2 . \quad (6)$$

The expected utility of date 1 wealth can then be computed from equations (1), (5), and (6) as

$$E(U(\tilde{W}_1)) = \alpha[P\mu + (1 - P)\mu(\alpha)] + (W_0 - K) + (1 - \alpha)(\mu(\alpha) - H) + \beta\bar{R}_M - \frac{b}{2} \{ \alpha^2\sigma^2 + \beta^2\sigma_M^2 + \alpha^2P(1 - P)[\mu - \mu(\alpha)]^2 \} . \quad (7)$$

We are now ready to derive the Pareto dominant signalling schedule that separates issuers. These self-fulfilling beliefs will be part of a sequential equilibrium, henceforth "separating signalling equilibrium," if there exist

out-of-equilibrium beliefs that deter out-of-equilibrium actions. It is easily verified that the out-of-equilibrium belief  $\mu = K$  suffices for this purpose, and there is no need to further analyze this aspect of the equilibrium within this section of the paper.

Inspection of equation (7) reveals that since  $0 \leq \alpha < 1$ , the  $H \geq 0$  that maximizes expected utility is  $H = 0$ . The other conditions for the maximization of expected utility are:

$$\frac{\partial E(U(\hat{W}_1))}{\partial \beta} = \frac{\partial E(U(\hat{W}_1))}{\partial \alpha} = 0,$$

which imply

$$\beta = \bar{R}_M / (b\sigma_M^2) \quad (8)$$

and

$$P\mu + (1-\alpha P)\mu_\alpha(\alpha) - P\mu(\alpha) - \alpha b\sigma^2 - \frac{b}{2}P(1-P)\{2\alpha[\mu(\alpha)-\mu]^2 + 2\alpha^2[\mu(\alpha)-\mu]\mu_\alpha(\alpha)\} = 0. \quad (9)$$

A necessary condition for a self-fulfilling belief is  $\mu(\alpha) = \mu$ , which reduces equation (9) to

$$\mu_\alpha(\alpha) = \frac{\alpha}{1-\alpha P} b\sigma^2. \quad (10)$$

Equation (10) is a differential equation, which has the unique solution

$$\mu(\alpha) = -\frac{1}{P^2} b\sigma^2 [\ln(1-\alpha P) + \alpha P] + c.$$

A Taylor series expansion verifies that  $\ln(1-\alpha P) + \alpha P < 0$ .  $c$ , the constant of integration, must be equal to  $K$  in the Pareto optimal schedule. If  $c > K$ , a potential issuer could undertake a negative value project, sell all his holdings to outside investors for  $c$ , and (since  $\mu(0) = c$ ) make a riskless profit of  $c - K$ . On the other hand, if  $c < K$ , the  $\mu(\alpha)$  schedule with  $c < K$  is dominated by the  $\mu(\alpha)$  schedule with  $c = K$ . The latter schedule

signals a given expected value with a smaller  $\alpha$  than the former, and thus offers each issuer a more diversified portfolio and higher expected utility.

The incentive-compatibility of the derived schedule is proved in the following proposition.

**PROPOSITION I:** When the project variance is known to outside investors and only fractional shareholdings are employed as a potential signal, there exists a separating signalling equilibrium if  $\bar{\mu}$ , the upper bound on  $\mu$ , is no greater than  $-\frac{1}{P^2} b\sigma^2[\ln(1 - P) + P] + K$ . The unique Pareto dominant signalling schedule can be expressed as

$$\mu(\alpha) = -\frac{1}{P^2} b\sigma^2[\ln(1 - \alpha P) + \alpha P] + K . \quad (11)$$

**Proof:** The last term in equation (7),

$$-\frac{b}{2} \alpha^2 P(1 - P)(\mu - \mu(\alpha))^2 ,$$

indicates that, if the remaining terms in equation (7) are maximized by  $\mu = \mu(\alpha)$ , the issuer has higher expected utility if he selects an  $\alpha$  that makes  $\mu(\alpha) = \mu$ . After substituting equation (11) and  $H = 0$  into equation (7), the remaining terms have a derivative with respect to  $\alpha$  of

$$P(\mu - \mu(\alpha)) .$$

This expression is negative if  $\alpha$  overstates the issuing firm's value and positive if  $\alpha$  understates the issuing firm's value. Hence, the global maximum occurs at the  $\alpha$  that makes  $\mu = \mu(\alpha)$ .

Pareto optimality follows from the discussion immediately preceding the proposition.

Q.E.D.

Proposition I hinges on the assumption that the issuer signals only with  $\alpha$ , his fractional holding. This assumption is justified by the following corollary:

**Corollary I:** When the project variance is known to outside investors, every incentive-compatible signalling schedule in which  $\mu$  is signalled by underpricing and fractional holdings is Pareto dominated by the schedule (11) in Proposition I.

**Proof:** See Appendix.

It is interesting to compare our signalling schedule with the schedule in Leland and Pyle (1977). Equation (10) indicates that  $\mu_{\alpha} > 0$ , implying that a larger fractional holding signals a higher value firm. Separation occurs because the cost of signalling--a less diversified portfolio--is lower for issuers of high value firms. This conclusion is identical to Leland and Pyle's, although the functional form of the two signalling schedules differs for  $P < 1$ . When  $P = 1$ , equation (11) reduces to the Leland and Pyle schedule.

The derivative of the signalling schedule with respect to the probability  $P$  can be expressed as

$$\frac{\partial \mu(\alpha)}{\partial P} = \frac{2b\alpha^2}{p^3} \left[ (\ln(1 - \alpha P) + \alpha P) + \frac{\alpha^2 P^2}{2(1 - \alpha P)} \right] .$$

A Taylor series expansion proves that the expression above is positive. This is easily explained. When the probability of being detected as a false signaller is higher, the low value issuer perceives the net cost of mimicking the high value issuer to be greater. Therefore, the equilibrium signalling schedule can signal a higher value  $\mu$  for the same  $\alpha$  and still discourage the mimicking of the low value issuer.

### 3.2 Special Case 2: Fixed Issuer's Fractional Holdings.

In the subsection above, issuers use the fraction of equity retained to signal firm value when the variance of the firm's cash flows is known. Here, we examine another special case: the issuer's fractional holdings are fixed and the project's value to outside investors is conveyed by underpricing the offering. Our purpose is to motivate the result that a risk averse issuer can signal his firm's value by giving money away (i.e. selling shares at a discount).

Given a group of issuers with the same  $\alpha$ , denote the LM issuer as the issuer with the lowest project mean,  $\mu^L$ , and the HM issuer as any issuer with a higher project mean,  $\mu^h$ . We initially assume that investors perceive only issuers with underpriced offerings to be HM issuers. We will then show that this system of beliefs can be self-fulfilling, i.e. a separating signalling equilibrium. The subsequent analysis begins by focussing on a single HM issuer and his attempts to distinguish himself from the LM issuer. It will be shortly apparent, however, that the derived schedule for signalling one's type applies to a continuum of HM issuers.

Table 1 lists the date 1 payoffs of different strategies to the LM issuer in two different states: State 1, in which issuer type is revealed, occurs with probability  $P$ . State 2, in which truth is not revealed and investors use a signalling schedule to draw inferences about the issuer type, occurs with probability  $1 - P$ .  $\bar{X}_{1L}$  and  $\bar{X}_{1h}$  denote the date 1 cash flows from the respective projects of the issuers, which have means of zero.

Column 1 in Table 1 is the payoff to the LM issuer from holding shares in his firm when he chooses to be honest, and not mimic the HM issuer by selling his shares at a discount. At date 0, the honest LM issuer receives  $(1 - \alpha)\mu^L$  for selling the fraction  $1 - \alpha$  of his shares. At date 1, irrespective of

Table 1: The Payoff of Different Strategies Taken by the LM (Low Mean) Issuer

	The LM Issuer Does Not Signal (1)	The LM Issuer Falsely Signals (2)	The Cost and Benefit of Falsely Signalling (3)
Date 0	$(1 - \alpha)\mu^L$	$(1 - \alpha)(\mu^h - H)$ $= (1 - \alpha)S^h$	$(1 - \alpha)(\mu^h - \mu^L - H)$ $= (1 - \alpha)(S^h - \mu^L)$
State at Date 1	<u>Probability</u>		
State 1	$\alpha(\mu^L + \bar{X}_{1L})$	$\alpha(\mu^L + \bar{X}_{1L})$	0
State 2	$\alpha(\mu^L + \bar{X}_{1L})$	$\alpha(\mu^h + \bar{X}_{1L})$	$\alpha(\mu^h - \mu^L)$

Table 2: The Payoff of Different Strategies Taken by the HM (High Mean) Issuer

	The HM Issuer Does Not Signal (1)	The HM Issuer Truthfully Signals (2)	The Cost and Benefit of Truthfully Signalling (3)
Date 0	$(1 - \alpha)\mu^L$	$(1 - \alpha)(\mu^h - H)$ $= (1 - \alpha)S^h$	$(1 - \alpha)(\mu^h - \mu^L - H)$ $= (1 - \alpha)(S^h - \mu^L)$
State at Date 1	<u>Probability</u>		
State 1	$\alpha(\mu^h + \bar{X}_{1h})$	$\alpha(\mu^h + \bar{X}_{1h})$	0
State 2	$\alpha(\mu^L + \bar{X}_{1h})$	$\alpha(\mu^h + \bar{X}_{1h})$	$\alpha(\mu^h - \mu^L)$



which state occurs, investors purchase the LM issuer's remaining shares for  $\alpha(\mu^L + \bar{X}_{1L})$ . In column 2, the LM issuer, being dishonest, mimics the HM issuer, and is perceived as a high mean issuer unless truth is revealed, which occurs with probability  $P$ . At date 0, he receives  $(1 - \alpha)(\mu^h - H)$  for selling  $1 - \alpha$  of his shares, because he signals and investors erroneously believe that he is a high mean issuer. If state 1 occurs at date 1, the true value of the firm is revealed, and he can only sell his remaining shares for what the honest LM issuer would have received. However, if state 2 occurs, the LM issuer benefits from signalling, because investors pay  $\alpha(\mu^h + \bar{X}_{1L})$  for his remaining shares, the amount dictated by the signalling schedule.

Column 3, obtained by subtracting the entries in column 1 from those in column 2, illustrates the marginal benefits and costs of pretending to be an HM issuer. From Table 1, column 3, we can see that by deceptively signalling, the LM issuer acquires a lottery at a date 0 cost of  $(1 - \alpha)(H - \mu^h + \mu^L)$ . At date 1, the lottery pays  $\alpha(\mu^h - \mu^L)$  with probability  $1 - P$  and zero with probability  $P$ . Table 2 illustrates that the HM issuer acquires the identical lottery at the margin by truthfully signalling.

How much would the LM issuer and the HM issuer be willing to pay for the date 1 outcome of the lottery? Or to ask the question differently, what is the maximum amount of underpricing that each issuer can profitably incur? The answer depends on the lottery's marginal effects on the issuers' wealth. A comparison of the payoffs in columns 1 and 3 for Tables 1 and 2 indicates that the lottery is uncorrelated with the LM issuer's wealth, but is negatively correlated with the wealth of the HM issuer. Alternatively, the HM (LM) issuer eliminates (increases) the variability in his wealth across the two states by underpricing. Since both issuer-types are risk averse, the lottery is more valuable to the HM issuer. The maximum amount that the HM issuer is

willing to pay for the lottery is larger than the lottery's expected payoff, which is  $\alpha(1 - P)(\mu^h - \mu^L)$ , but paying  $\alpha(1 - P)(\mu^h - \mu^L)$  is sufficient for the HM issuer to bid the lottery away from the LM issuer. Thus, if

$H = \frac{1 - \alpha P}{1 - \alpha} (\mu^h - \mu^L)$  for the HM issuer, implying an offering price  $S^h$  below  $\mu^L$ , the LM issuer is deterred from mimicking and will optimally choose

$H = 0$ . Rearranging terms in the expression yields the corresponding

signalling schedule:

$$\mu^h = \mu^L + \frac{(1 - \alpha)}{1 - \alpha P} H.$$

This schedule is a risk neutral pricing of the column 3 lottery in Tables 1 and 2. Hence, for all  $\mu^h$ , the expected wealth of each issuer is independent of his degree of underpricing. It follows that each issuer would choose to minimize the variance of his wealth,

$$\alpha^2 \sigma^2 + \beta^2 \sigma_M^2 + \text{Var}(\mu^{\tilde{x}}).$$

This can only be achieved by truthfully signalling, so that  $\text{Var}(\mu^{\tilde{x}})$  is zero. Thus, not only does the schedule deter LM issuers from mimicking HM issuers, it also deters HM issuers from mimicking each other, (even when the variances of the projects and the risk aversion of the issuers are unknown).

The equation for  $\mu^h$  above is a straight line with a slope less than one. This implies that the offering price declines as the firm's intrinsic value increases. The offering price,  $S^h$ , is selected by the HM issuer so that the date 0 cash flow of the column 3 lottery,  $(1 - \alpha)(S^h - \mu^L)$ , plus the date 1 expected cash flow of the lottery,  $\alpha(1 - P)(\mu^h - \mu^L)$ , is zero. Thus,

$$S^h = \mu^L - \frac{\alpha(1 - P)}{1 - \alpha} (\mu^h - \mu^L),$$

which yields the signalling schedule

$$\mu^h = \mu^L \frac{(1 - \alpha P)}{\alpha(1 - P)} - S^h \frac{(1 - \alpha)}{\alpha(1 - P)}.$$

4. THE GENERAL MODEL: PROJECT VARIANCE UNKNOWN, FRACTIONAL HOLDINGS NOT FIXED

4.1 Derivation of the Signalling Schedule

When two issuers with different project variances retain the same fractional holdings in equilibrium, equation (11) indicates that the higher variance issuer has higher expected value. Consequently, if the variances are unknown, a single parameter separating equilibrium in  $\alpha$  does not exist. This suggests that issuers with high value firms may be willing to pay the additional cost of a second signal, assumed here to be the degree of underpricing.

In this section, issuers with the lowest project variance are designated as LV issuers. Before deriving  $\mu(\alpha, H)$ , the signalling schedule for an arbitrary issuer, we recognize that in a separating equilibrium, the portion of the signalling schedule for the LV issuers must be a separating signalling schedule for the case where the variance is known. If outside investors conjecture that an LV issuer signals only with  $\alpha$ , then the LV issuers' signalling schedule and its derivative with respect to  $\alpha$  are given by a parameter specific solution of special case 1. The derivative is

$$\mu_{\alpha}^L(\alpha) = \frac{\alpha}{1 - \alpha P} b\sigma_L^2, \quad (12)$$

and the relevant portion of the signalling schedule is

$$\mu^L(\alpha) = -\frac{1}{P^2} b\sigma_L^2 [\ln(1 - \alpha P) + \alpha P] + K, \quad (13)$$

where  $\sigma_L^2$  is the project variance of the lowest variance issuers. These equations are identical to equations (10) and (11) with  $\sigma_L^2$  as the variance parameter. We will later demonstrate that this equation is part of a more general incentive-compatible schedule.

The date 1 wealth of an arbitrary issuer is expressed as

$$\tilde{W}_1 = \alpha(\tilde{X}_1 + \mu) + \beta\tilde{R}_M + (W_0 - K) + (1 - \alpha)[\mu(\alpha, H) - H], \quad (14)$$

where  $\overset{x}{\mu} = \mu$  with probability  $P$  and  
 $= \mu(\alpha, H)$  with probability  $1 - P$ .

Equation (14) is identical to equation (2), except that  $\mu(\alpha, H)$  has replaced  $\mu(\alpha)$ . The issuer's optimization problem is to choose  $\alpha$ ,  $H$ , and  $\beta$ , given parameters  $\sigma^2$  and  $\mu$ , in order to maximize  $E(U(\tilde{W}_1))$ . Combining equations (14) and the expressions for the expected value and variance of  $\overset{x}{\mu}$ , (equations (3) and (4) with  $\mu(\alpha, H)$  replacing  $\mu(\alpha)$ ), we can write  $E(U(\tilde{W}_1))$  as follows:

$$\begin{aligned} E(U(\tilde{W}_1)) &= E(\tilde{W}_1) - \frac{b}{2} \text{Var}(\tilde{W}_1) \\ &= [\alpha P \mu + \alpha(1 - P)\mu(\alpha, H)] + \beta \bar{R}_M + (W_0 - K) + (1 - \alpha)[\mu(\alpha, H) - H] \\ &\quad - \frac{b}{2} \{ \alpha^2 \sigma^2 + \beta^2 \sigma_M^2 + \alpha^2 P(1 - P)[\mu - \mu(\alpha, H)]^2 \}. \end{aligned} \quad (15)$$

We are now ready to prove the paper's central result.

**PROPOSITION II:** When both the expected value ( $\mu$ ) and the variance ( $\sigma^2$ ) of the firm's cash flows are unknown to investors, there exists a two-parameter separating signalling equilibrium if  $\bar{\mu}$ , the upper bound on  $\mu$ , is smaller than  $\mu^L(1) = -\frac{1}{P^2} b \sigma_L^2 [\ln(1 - P) + P] + K$ .

$$\mu(\alpha, H) = \mu^L(\alpha) + \frac{1 - \alpha}{1 - \alpha P} H \quad (16)$$

and

$$\sigma^2(\alpha, H) = \sigma_L^2 + \frac{(1 - \alpha)P}{(1 - \alpha P)ab} H \quad (17)$$

are the signalling schedules for the firm's intrinsic value and variance, respectively, with  $\mu^L(\alpha)$ , equation (13), the signalling schedule for the lowest variance issuer.

**Proof:** See Appendix.

Note that  $\sigma^2$ , being diversifiable, does not affect the utility of investors. However, because it does affect the utility of issuers with

$\alpha > 0$ , knowledge of it is needed to unravel the  $\mu$ 's of their firms. Thus, while equation (17) represents irrelevant information to investors once equation (16) is known, any inference of  $\mu$  with equation (16) implies the simultaneous inference of  $\sigma^2$  with equation (17).

It is trivial to verify that the out-of-equilibrium beliefs ( $\mu = K$ ) used to support the sequential equilibrium in Proposition II satisfy the Cho and Kreps (1987) intuitive criterion when  $b(\bar{\mu} - K) \leq \frac{1}{p}$ . The latter inequality is implied by positive marginal utility of wealth. We can also prove that the equilibrium has desirable efficiency properties.

**Corollary II:** The equilibrium in Proposition II is Pareto efficient with respect to all schedules that separate  $\mu$  (including those that pool  $\sigma^2$ ).

**Proof:** The lowest variance types cannot separate from each other and achieve higher utility than in an equilibrium in which their variance is known to outside investors without signalling. Corollary I, however, indicates that the portion of the schedule for the issuer types with  $\sigma^2 = \sigma_L^2$  is the unique Pareto dominant schedule when the variance is known.

Given this, higher variance issuers, irrespective of their choice of  $\alpha$ , minimize signalling cost (i.e., maximize utility) and reveal their  $\mu$ , only when they select H that is consistent with the first order condition

$$\mu_H = \frac{1 - \alpha}{1 - \alpha p} .$$

Equation (16) is the only solution to this equation that is consistent with efficient signalling by the  $\sigma_L^2$  issuers. As shown in the proof of Proposition II, equation (17) is the only equation that is consistent both with (16) and the choice of  $\alpha$  that maximizes utility.

Q.E.D.

#### 4.2 The Observability of the Underpricing Signal

Since investors do not observe the true expected value of the project, it is not obvious that they can determine the degree of underpricing from the offering price alone. For instance, if investors observe an issuer who retains  $\alpha$  of his issue and sells the remaining fraction for \$30 per share, how do they determine whether the issuer has offered a \$10 discount on a \$40 share or a \$5 discount on a \$35 share? In this section, we demonstrate that a one-to-one correspondence exists between the discount  $H$  and the offering price  $S$ , given  $\alpha$ , the fractional shareholding. Without this correspondence, the separating equilibrium could not be achieved with observable variables.

The offering price can be written as

$$\begin{aligned} S &= \mu(\alpha, H) - H \\ &= \mu^L(\alpha) + H\left(\frac{1 - \alpha}{1 - \alpha P} - 1\right), \end{aligned} \quad (18)$$

with the latter equality derived from the signalling schedule for firm value, equation (16). Since  $P$ ,  $\alpha$ , and  $\mu^L(\alpha)$  are known, investors can infer the exact degree of underpricing by observing  $S$ . This verifies that there exists a mathematically equivalent signalling schedule in  $\alpha$  and  $S$ , which is obtained by substituting the function  $\hat{\mu}(\alpha, S)$  and  $\hat{\sigma}(\alpha, S)$  for  $\mu(\alpha, H)$  and  $\sigma(\alpha, H)$  in equations (16) and (17) and rearranging terms. The signalling schedules are

$$\hat{\mu}(\alpha, S) = \mu^L(\alpha) \frac{1 - \alpha P}{\alpha(1 - P)} - S \frac{1 - \alpha}{\alpha(1 - P)} \quad (19)$$

and

$$[\hat{\sigma}(\alpha, S)]^2 = \sigma_L^2 + \frac{(1 - \alpha)(\mu^L(\alpha) - S)P}{\alpha^2(1 - P)b}. \quad (20)$$

As currently written, equation (19) is undefined at the feasible point  $\alpha = 0$ . However, when  $\alpha = 0$ , equation (18) indicates that the only equilibrium offering price is  $S = K$ . Hence, only fairly-priced zero NPV projects can be

offered when  $\alpha = 0$ . This implies that  $\hat{\mu}(0, K)$  in equation (19) takes on its limiting value, i.e.  $\hat{\mu}(0, K) = K$ , even though the variance cannot be inferred, as indicated by equations (17) and (20). The inability to infer the variance makes economic sense here, since an issuer who markets his entire firm at date 0 is unaffected by the variance of the firm's cash flows at date 1.

The one-to-one correspondence between the degree of underpricing and the offering price is broken for pooling equilibria (if they exist) and signal levels that are off-the-equilibrium path. Thus, the analysis of these issues in Section 6 cannot employ the earlier technique of assuming that the degree of underpricing is observed.

## 5. ANALYSIS OF THE SIGNALLING SCHEDULE

### 5.1 Comparative Statics and Empirical Implications

The signalling schedule, represented by equations (16) and (17), has several interesting empirical implications. Inspection of equation (16), for instance, reveals that holding  $\alpha$  constant,  $H$  and  $\mu$ , the degree of underpricing and the intrinsic value of the firm, respectively, are positively related. In testing this and all subsequent relations cross-sectionally, one would also have to control for  $K$ , the capital outlay of the project;  $P$ , the probability of early revelation; and  $b$ , the risk aversion of the issuer. Note, however, that one does not control for the variance of the firm in this test.

Equation (17) implies that there is a positive relation between  $\sigma^2$  and  $H$ , holding  $\alpha$  constant and a negative relation between  $\sigma^2$  and  $\alpha$ , holding  $H$  constant. It immediately follows that  $\alpha$  and  $H$  are positively related, holding  $\sigma^2$  constant.

If equation (17) is solved for  $H$  and substituted into equation (16), the resulting equation is

$$\mu = m(\alpha, \sigma^2), \quad \text{where}$$

$$m(\alpha, \sigma^2) = \mu^L(\alpha) + (\sigma^2 - \sigma_L^2)ab/P . \quad (21)$$

Equation (21) implies that there is a positive relation between  $\mu$  and  $\alpha$ , holding  $\sigma^2$  constant, and a positive relation between  $\mu$  and  $\sigma^2$ , holding  $\alpha$  constant. It follows that  $\alpha$  and  $\sigma^2$  are negatively related, holding  $\mu$  constant. This, of course, is the sign of

$$\frac{\partial \alpha(\mu, \sigma^2)}{\partial \sigma^2} ,$$

where  $\alpha(\mu, \sigma^2)$  represents one member of the pair of inverse equations to (16) and (17), the other being  $H(\mu, \sigma^2)$ .

The final "testable" implication is the sign of a partial derivative in the complementary inverse function. It is easy to verify that

$$\frac{\partial H(\mu, \sigma^2)}{\partial \mu} > 0 ,$$

which demonstrates a positive relation between  $H$  and  $\mu$ , holding  $\sigma^2$  constant.

This is done by rewriting equations (16) and (17) as  $F(\alpha, H, \mu, \sigma^2) = 0$  and  $G(\alpha, H, \mu, \sigma^2) = 0$ , where

$$F(\alpha, H, \mu, \sigma^2) = \mu^L(\alpha) + HQ(\alpha) - \mu ,$$

$$G(\alpha, H, \mu, \sigma^2) = \frac{PH}{ab} Q(\alpha) + \sigma_L^2 - \sigma^2 ,$$

and where  $Q(\alpha)$ , a function between 0 and 1, is defined by

$$Q(\alpha) = (1 - \alpha)/(1 - \alpha P) .$$

$\frac{\partial H(\mu, \sigma^2)}{\partial \mu}$  must satisfy the system of equations

$$F_\alpha \frac{\partial \alpha(\mu, \sigma^2)}{\partial \mu} + F_H \frac{\partial H(\mu, \sigma^2)}{\partial \mu} + F_\mu = 0 \quad \text{and}$$

$$G_\alpha \frac{\partial \alpha(\mu, \sigma^2)}{\partial \mu} + G_H \frac{\partial H(\mu, \sigma^2)}{\partial \mu} + G_\mu = 0 ,$$



which, using Cramer's rule, is solved by

$$\frac{\partial H(\mu, \sigma^2)}{\partial \mu} = \frac{-F_{\alpha \mu} G_{\alpha \mu} + G_{\alpha \mu} F_{\alpha \mu}}{F_{\alpha H} G_{\alpha H} - G_{\alpha H} F_{\alpha H}} = \frac{-G_{\alpha}}{F_{\alpha H} G_{\alpha H} - G_{\alpha H} F_{\alpha H}} .$$

The numerator,  $\frac{PH}{\alpha^2 b} Q - \frac{P}{\alpha b} HQ_{\alpha}(\alpha)$ , is positive since  $Q(\alpha)$  has a negative derivative for  $0 < \alpha < 1$ , and the denominator,  $\frac{QP}{1 - \alpha P} \sigma_L^2 + \frac{PH}{\alpha^2 b} Q^2$ , is also positive, completing the proof.

The four remaining comparative statics results are of ambiguous sign, since they depend on the sign of

$$F_{\alpha} = \frac{\alpha}{1 - \alpha P} b \sigma_L^2 + HQ_{\alpha}(\alpha) ,$$

which is of the same sign as

$$\frac{\alpha(1 - \alpha P)b\sigma_L^2}{1 - P} - H .$$

There is a negative (positive) relation for large (small)  $\sigma^2$  between  $\mu$  and either  $\alpha$  or  $\sigma^2$ , holding  $H$  constant, and between  $H$  and  $\sigma^2$ , holding  $\mu$  constant. There is also a positive (negative) relation for large (small)  $\sigma^2$ , between  $\alpha$  and  $H$ , holding  $\mu$  constant. Note that holding either  $H$  or  $\mu$  constant, small (large)  $\sigma^2$  is equivalent to large (small)  $\alpha$ . Hence, these results could easily be rewritten so that  $\alpha$ 's magnitude, rather than  $\sigma$ 's, determines the sign of the relation. These comparative statics results can be easily seen in Figure 1, which is discussed in the next section.

It is also easy to verify  $\mu$  and  $\sigma^2$  are increasing functions of  $P$ , holding  $H$  and  $\alpha$  constant. The intuition for this is similar to that presented at the end of Section 1.

We can contrast our empirical implications with those of Welch (1988). In general, these comparisons are a bit strained because Welch considers only two issuer-types. Hence, all of his comparative statics require changing the

parameters of the economy, while some of our cross-sectional comparisons analyze different issuing firms within the same economy. With this in mind, we note that both models predict that

- (i) There can be underpricing;
- (ii) Entrepreneurs offering an unseasoned issue will later sell additional shares to the investing public, and
- (iii) Fixing the lowest value firm, the perceived  $\mu$  of higher value firms with a given degree of underpricing and fractional ownership is an increasing function of  $P$ , the probability of revelation.

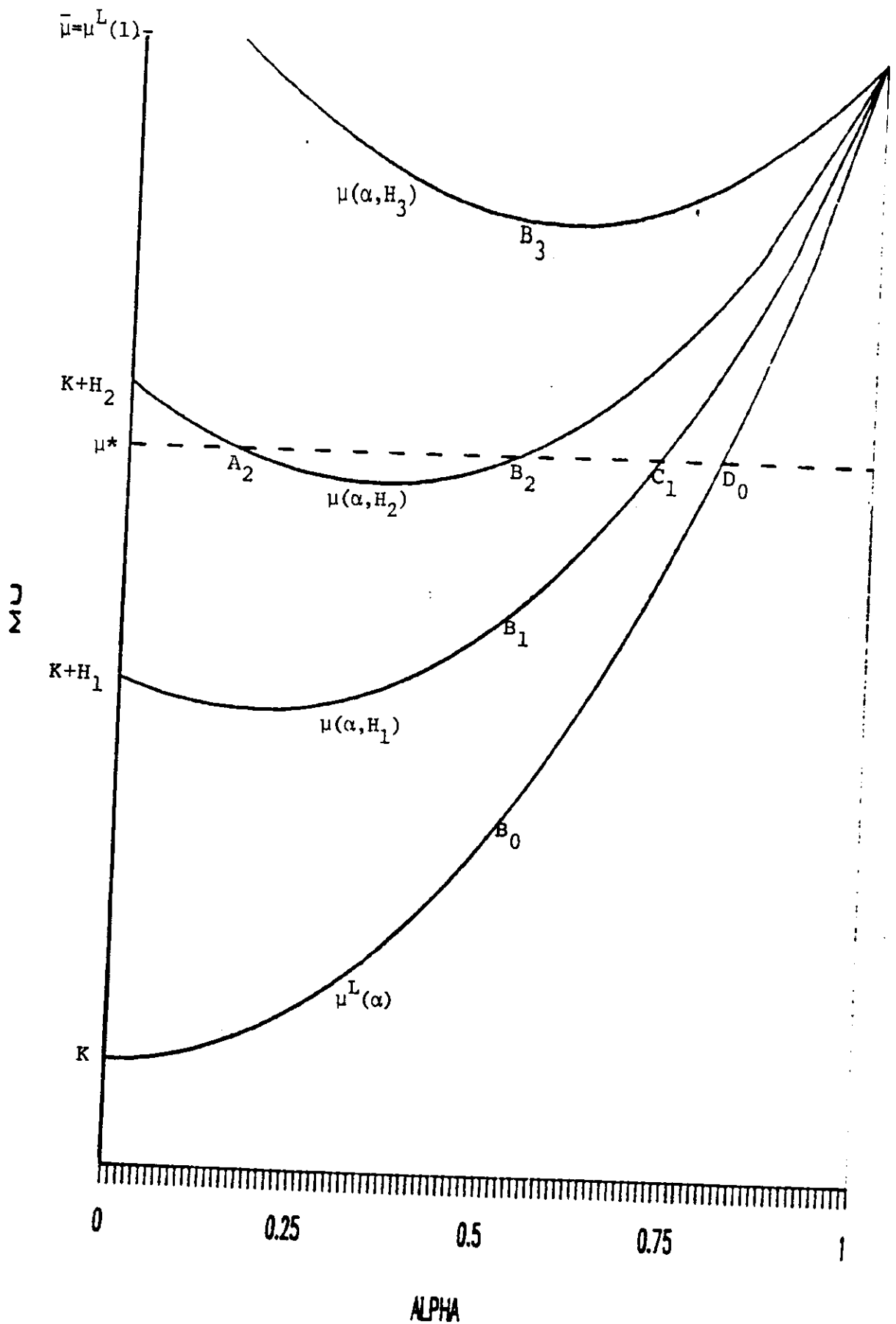
Welch also predicts that the perceived  $\mu$  and the fractional ownership of the higher value firm are positively related, whereas in our model, the relation between  $\alpha$  and  $\mu$  depends on the variability of the firm's operating cash flows.

Finally, Welch predicts that the issuers of underpriced offerings believe that there will be low aftermarket stock price volatility whereas the opposite is true in our model. Only the latter is supported by the empirical evidence. Every cross-sectional regression in Hwang (1988), for instance, displays a positive relation between the degree of underpricing and aftermarket volatility. In addition, Beatty and Ritter (1986) and Chalk and Peavy (1987) find a positive relation between various proxies for ex ante uncertainty and the degree of underpricing.<sup>10</sup>

## 5.2 Graphic Representation of the Schedule

The signalling schedule for the intrinsic value of the firm, equation (16), is graphed in Figure 1, where  $H_3 > H_2 > H_1$ , etc.,  $\bar{\mu} = \mu^L(1)$ , and  $P' < \frac{1}{2}$ . (For lower  $\bar{\mu}$ , truncate the top part of the figure. For  $P \geq \frac{1}{2}$ , some schedules with large  $H$  may have inflection points.) Each of the lines in the figure corresponds to a signalling schedule for a fixed  $H$ . We can, of course, translate this into a diagram with offering prices,  $S$ , instead of share price discounts,  $H$ . Towards the left of Figure 1, the offering prices for different

FIGURE 1: THE SIGNALLING SCHEDULE



values of  $H$  are similar, though the  $\mu$ 's differ substantially; whereas to the right of the diagram, the  $\mu$ 's are similar, but the offering prices differ.

Figure 1 suggests that increases in  $\alpha$ , (holding  $H > 0$  constant), decrease the perceived intrinsic value of the firm when  $\alpha$  is sufficiently close to zero. This results from the interaction of two opposing effects. The first effect, associated with the  $\mu^L(\alpha)$  term in equation (16), is the undiversified variance effect, discussed in Leland and Pyle and Section 3.1 of this paper. This represents a cost to the issuer that is monotonically increasing in  $\alpha$ . A small increase in  $\alpha$  has little effect on this cost when  $\alpha$  is close to zero, since the covariance of the firm's date 1 cash flow with the issuer's wealth is close to zero in this case. However, the cost associated with the second term on the right side of equation (16) is monotonically decreasing in  $\alpha$  and is never trivial, even near  $\alpha = 0$ . This effect is the value of the firm that is given away by underpricing the issue. An issuer with a smaller  $\alpha$  sells more shares at a discount. Since the latter effect must dominate when  $\alpha$  is small, the perceived mean must rise as  $\alpha$  falls to compensate issuers who reduce  $\alpha$ . Also, since the second effect is stronger for larger  $H$ , the  $\mu$ -minimizing  $\alpha$  for a given  $H$  must be an increasing function of  $H$ .

The proof of Proposition II in the Appendix can be sketched from Figure 1. Consider, for instance, an issuer who would be at point  $C_1$  if truthfully signalling his mean and variance. If  $C_1$  maximizes the expected utility of the issuer, then the schedule is incentive compatible for him.

This is proved in two parts. The first part of the proof demonstrates that choices of  $\alpha$  and  $H$  on the horizontal line through  $C_1$  dominate choices that are vertically above or below them. Hence, for this issuer,  $B_2$  offers higher expected utility than  $B_0$ ,  $B_1$ , or  $B_3$ , and for any  $\alpha$ , the choice of  $H$  that maximizes expected utility is the one that truthfully signals  $\mu$ . The

intuition for this is straightforward. Each of the points along a vertical line, such as the line through  $B_0, B_1, B_2, B_3$ , correspond to the same expected wealth for the issuer. This finding, discussed in Section 3.2, suggests that the market does not reward an issuer who, by merely selecting a larger  $H$ , claims to have a higher variance portfolio. Given  $\alpha$ , it thus pays the issuer to select the  $H$  that minimizes the variance of his portfolio. Examining the final braced term in equation (15), there are three components to the variance of the issuer's wealth. The first two, which correspond to the unsystematic and systematic risk of the issuer's portfolio, are identical for points on a vertical line, such as  $B_0, B_1$ , and  $B_2$ . The last component, which is related to the variability in his wealth because of false signalling, was discussed in Section 3.2, and is zero only when the issuer truthfully signals his mean.

The second half of the proof in the Appendix demonstrates that, along the horizontal dotted line, expected utility is a strictly increasing function of  $\alpha$  to the left of  $C_1$  and a strictly decreasing function to the right of  $C_1$ . The optimal  $\alpha$ , at  $C_1$ , truthfully signals the variance, demonstrating incentive-compatibility.

The sign of the derivative of expected utility with respect to  $\alpha$  along the horizontal dotted line is a function of the relative cost of signalling with  $\alpha$  and  $H$ . Because the nondiversification cost is an increasing function of  $\alpha$ , signalling with  $\alpha$  becomes more expensive relative to  $H$  as we move to the right along the dotted line. Moreover, the marginal cost of signalling with  $\alpha$  is an increasing function of the cash flow variance,  $\sigma^2$ . Hence, the larger  $\sigma^2$ , the smaller the  $\alpha$  at which the marginal costs of the two signals will be equated, implying that  $\sigma^2$  decreases as we move to the right along the dotted line. For similar reasons, the convex lines where  $H$ , rather than  $\mu$ , is fixed also separate variance. As one increases  $\alpha$ , holding  $H > 0$  constant, the perceived variance decreases.

## 6. EQUILIBRIUM UNIQUENESS AND POOLING IN A DISCRETE MODEL

It is generally rare to find unique equilibria in signalling models unless the models are augmented either with assumptions about the distribution of attribute-types or with more stringent definitions about what constitutes a plausible equilibrium. To date, neither the distributional approach nor the equilibrium refinement approach have provided a tractable analysis of uniqueness in models with two attributes and a continuum of attribute-types. We are thus forced to adopt a "second-best" approach, where we study uniqueness by developing a discrete model with two issuer-types.<sup>11</sup>

Type 0 firms in the discrete model are assumed to have intrinsic value  $\mu_0$  and variance  $\sigma_0^2$ . Type 1 firms have value  $\mu_1$ , where  $\mu_1 > \max(0, K, \mu_0)$ , and variance  $\sigma_1^2$ . The prior probability of a high value firm (type 1) is  $q$ .

If the issuer of a type  $i$  firm perceives investor posteriors to be summarized by  $E(\mu|\alpha, S)$ , he acts to maximize

$$u(\alpha, S; E(\mu|\alpha, S); \mu, \sigma^2) = \mu_i - K + (1 - \alpha P)(E(\mu|\alpha, S) - \mu_i) + (1 - \alpha)(S - E(\mu|\alpha, S)) - \frac{b}{2} \alpha^2 \{ \sigma_i^2 + P(1 - P)[\mu_i - E(\mu|\alpha, S)]^2 \}, \quad (22)$$

which is expected utility, equation (15), shifted by a constant.

If the equilibrium involves pooling where both types issue, the efficient signal levels set the offering price equal to  $(1 - q)\mu_0 + q\mu_1$ , implying no underpricing on average (i.e.,  $S = E(\mu|\alpha, S)$ ), and issuers will retain no shares. From equation (22), the utility of both issuers is then

$$-K + \mu_0 + q(\mu_1 - \mu_0),$$

which is lower than the utility of zero achieved when not issuing if

$$q < (K - \mu_0)/(\mu_1 - \mu_0) .$$

Hence, pooling equilibria do not exist if low value firms are sufficiently abundant and their projects have negative net present values. Under this distributional restriction, the price paid for the pooled issue is less than  $K$ . The intuition behind this result can be generalized to the continuous-attribute case to yield conditions that prevent global pooling.

In any efficient separating equilibrium, issuers of low value firms do not underprice and retain no shares (thus bearing zero signalling cost), or they do not issue. The issuers of high value firms make low value firms indifferent about mimicking high value firms. From equation (22), incentive-compatible beliefs that separate exist if issuer 1's signals,  $(\alpha, S)$ , with  $S \leq \mu_1$  and  $0 \leq \alpha < 1$  can satisfy

$$(1 - \alpha P)(\mu_1 - \mu_0) + (1 - \alpha)(S - \mu_1) - \frac{b}{2} \alpha^2 [\sigma_0^2 + P(1 - P)(\mu_1 - \mu_0)^2] \leq \max(0, K - \mu_0) . \quad (23)$$

To be an efficient separating equilibrium,  $\alpha$  and  $S$  must also minimize the issuer 1's signalling cost

$$C = (1 - \alpha)(\mu - S) + \frac{b}{2} \alpha^2 \sigma_1^2 .$$

Starting from values of  $S$  that induce separation, high offering prices reduce the marginal loss of a low value issuer who mimicks a high value issuer and reduce the signalling cost of the high value issuer. When there is no underpricing, lower fractional holdings do the same. Thus, (23) will be an equality in an efficient separating equilibrium, implying that the high value issuer bears lower signalling costs in the discrete model than in the continuous model. This verifies that issuer 1 prefers his own more costly signal levels to the signals of the low value issuer when he "just deters" mimicking. It also implies that his perceived  $\mu$  will be higher at his optimal signal level than indicated by equations (16) or (21) for the continuous case. Inspection of (23) also yields:

**PROPOSITION III:** Neither issuer will underprice in the most efficient separating equilibrium if  $\sigma_1^2 \leq \sigma_0^2$  and it is possible to deter low value issuers from mimicking high value issuers without underpricing, (i.e., (23) holds for signals  $\alpha = 1$  and  $S = \mu_1$ ).

**Proof:** The efficient separating equilibrium must minimize  $C$ , subject to (23) as an equality, which is rewritten as

$$-C + (1 - \alpha P)(\mu_1 - \mu_0) - \frac{b}{2}\alpha^2 P(1 - P)(\mu_1 - \mu_0)^2 - \frac{b}{2}\alpha^2(\sigma_0^2 - \sigma_1^2) = \max(0, K - \mu_0) . \quad (24)$$

If the efficient separating equilibrium has  $S < \mu_1$ , it must have lower  $\alpha$  and lower  $C$  than the schedule in which  $S = \mu_1$  and  $\alpha$  equals the lowest  $\alpha$  that makes (23) an equality. But then (23) is violated since lowering  $C$  and lowering  $\alpha$  raises the left side of (23).

Q.E.D.

This result is consistent with the results about the  $\sigma_L^2$  issuers in the continuous case. Underpricing can arise when  $\sigma_1^2 > \sigma_0^2$  because it offers a more efficient signal vector than a pure signal of fractional holdings.

We have now characterized both the efficient pooling equilibrium in the class of pooling "equilibria" and the efficient separating equilibrium. We have also proposed a distributional condition that prevents pooling. In the absence of this condition, we are forced to employ equilibrium refinements to justify the uniqueness of the separating equilibrium. The weakest of these refinements, the Cho and Kreps (1987) intuitive criterion, rules out the viability of the pooling equilibrium.

**PROPOSITION IV:** The efficient separating equilibrium, but not the efficient pooling sequential equilibrium satisfies the intuitive criterion.

**Proof:** See Appendix.



Unappealing dominated pooling and separating are eliminated with similar lines of reasoning. Hence, the efficient separating equilibrium appears to be the unique plausible equilibrium. While there is no reason to expect this uniqueness result to apply to the continuous-attribute model, there are more stringent equilibrium refinements that often rule out pooling sequential equilibria in more complex models, including continuous-attribute models. These include Cho's (1987) "forward induction equilibrium," the "D1 criterion" in Cho and Kreps (1987), and the "perfect sequential equilibrium" proposed by Grossman and Perry (1986).<sup>12</sup> We are unaware of applications of these criteria to continuous models with two attributes and have been unsuccessful in applying them here for technical reasons. Hence, the uniqueness of the equilibrium in the general signalling game remains an open question.

An alternative approach is to reverse the ordering of the players in the games. In screening games, investors (perhaps through investment banking intermediaries), simultaneously announce schedules of acceptable combinations of  $\alpha$  and  $S$ , along with the signal-contingent prices for shares that are likely to prevail in the secondary market. Issuers then respond. The Nash equilibria in this game, if they exist, can only have Pareto efficient separating schedules. The non-existence problem can often be handled by adopting the concept of a "reactive equilibrium," developed in Riley (1979). Engers (1984) has shown that a reactive equilibrium necessarily has a Pareto efficient separating schedule as the strategy of the first mover, even in the case of multiple signals.<sup>13</sup>

## 7. CONCLUSION

A two-parameter signalling model was developed here to analyze and explain the underpricing of new issues. In the model, there is information asymmetry about both the expected value and the variance of a project that is

about to be capitalized. In contrast to the Leland and Pyle model, the issuer's fractional holding alone is not sufficient to signal the expected value of the project. Since the equilibrium signalling schedule is a function of both the project variance and the issuer's fractional holding, a second signal, the degree of underpricing per share, is also needed to infer the variance. This signal is indirectly observed when the offering price of the issue is announced.

The model yields eight testable implications. Four of them are consistent with the Leland-Pyle model:

- (1) The variance of the firm's cash flows and the issuer's fractional holding are negatively related, given the degree of underpricing (although the Leland and Pyle model does not predict underpriced offerings);
- (2) The value of the firm is positively related to the variance of its cash flows, keeping the issuer's fractional holding constant;
- (3) The value of the firm is positively related to the fractional holding of the issuer, holding the variance constant; and
- (4) The fractional holding of the issuer is negatively related to the variance, holding the value of the firm constant.

One of them is consistent with Rock's model (see Beatty and Ritter (1986)):

- (5) The degree of underpricing is an increasing function of the variance, given the issuer's fractional holding.

The remaining three are unique to the model presented here:

- (6) Given the variance of the firm, the degree of underpricing is positively related to the issuer's fractional holdings;
- (7) Given the issuer's fractional holdings, firm value is positively related to the degree of underpricing; and
- (8) Given the variance of the firm, firm value and the degree of underpricing are positively related.

The existing empirical evidence on new issues, found in Ibbotson (1975), Ibbotson and Jaffe (1975), Downes and Heinkel (1982), Ritter (1984), Beatty and Ritter (1986), Chalk and Peavy (1986), Ritter (1988), Hwang (1988), and a number of other papers, are consistent with these results.

The concept of signalling firm value by issuing shares at a discount (i.e. giving money away) can also be applied to different situations. Two applications come to mind: one where managers signal high firm value through the retainment of high-priced investment bankers, auditors, and advertising; the other where high firm value is signalled through high dividends, since dividends essentially give money away to the government in the form of higher taxes. A simple change in variables allows us to directly apply these alternative interpretations to the model's solution.

$P$ , the probability that the project value is revealed at date 1, plays an important role in this model. If  $P = 1$  or  $P = 0$ , then investors are unable to infer the degree of underpricing and project variance from the offering price. In this case, underpricing cannot be a signal. However, a value of  $P$  between 0 and 1 imposes restrictions on the prior distributions of the firms' intrinsic values. These restrictions can be relaxed to a certain degree by allowing the upper bound on the mean to be an increasing function of the variance. However, such a model permits unbounded negative offering prices when means are bounded, which we find implausible.

We have also assumed that the risk aversion of each issuer,  $b$ , is known to investors. One is tempted to infer that this assumption is unnecessary because  $\beta$ , the issuer's investment in the market portfolio, could be used as a signal of the issuer's unknown risk aversion. In this case, however, the issuer's investment in the market portfolio is a function not only of risk aversion, but also of its effect on the signalled mean and variance of the new issue. This makes the problem virtually intractable. Hence, the possibility of a three-parameter signalling model remains a subject for future research.

## Appendix

**Proof of Corollary I:** The continuous model's Pareto efficient signalling schedules are incentive-compatible when there are discrete issuer types. This implies that, at worst, these schedules can only be approximately Pareto dominated in large elements of any sequence of discrete models that converges to the continuous model. No schedule with underpricing as a signal has this property, as we will demonstrate shortly. Moreover, the unique Pareto dominant schedule in a sequence of discrete models converges to equation (11).

The Nth discrete model in the sequence, where N is large, has N + 1 issuer types, indexed by i. Issuer i,  $i = 0, 1, \dots, N$ , has a firm with intrinsic value  $\mu_i$ , where

$$\mu_i = K + i \left( \frac{\bar{\mu} - K}{N} \right),$$

and where a selection of  $\alpha_i$ ,  $H_i$  makes

$$\mu_i = \mu(\alpha_i, H_i) \tag{A1}$$

if  $\mu(\alpha, H)$  is the signalling schedule,  $0 \leq \alpha < 1$ ,  $H \geq 0$ .

The expected utility of issuer i is identical to equation (7) with  $\mu(\alpha, H)$  replacing  $\bar{\mu}(\alpha)$ . Letting  $Q_i$  denote the terms in this equation that are unaffected by the signal levels,  $\alpha$  and  $H$ , issuer i's expected utility can be rewritten as

$$\begin{aligned} E(U_i(\bar{W}_{1i})) &= Q_i - \left[ (1 - \alpha)H + \frac{b}{2} \alpha^2 \sigma^2 \right] + (1 - \alpha P)(\mu(\alpha, H) - \mu_i) \\ &\quad - \frac{b}{2} \alpha^2 P(1 - P)[\mu_i - \mu(\alpha, H)]^2. \end{aligned} \tag{A2}$$

From equation (A2), issuer i's signalling cost, incurred when selecting signal levels  $\alpha_i$  and  $H_i$ , is

$$C_i = (1 - \alpha_i)H_i + \frac{b}{2} \alpha_i^2 \sigma^2 \tag{A3}$$

and issuer j,  $j \neq i$ , is deterred from mimicking investor i when

$$C_i \geq C_j + (1 - \alpha_i P)(\mu_i - \mu_j) - \frac{b}{2} \alpha_i^2 P(1 - P)(\mu_j - \mu_i)^2. \quad (A4)$$

If a signalling schedule,  $\mu(\alpha, H)$ , is incentive-compatible, equations (A1) and (A4) hold for all  $i, j$ . If it also makes (A3) weakly smaller for all  $i$  and smaller for some  $i$  than any other incentive-compatible schedule, it is Pareto dominant (and necessarily unique).

An incentive-compatible schedule cannot be Pareto efficient (and hence, not dominant) if for some  $i \geq 1$ , there is slack in inequality (A4) for all  $j \neq i$  or if  $C_0 \neq 0$ . Hence, we restrict our search to all schedules with

$$C_i = \sup_{\{j:j \neq i\}} C_j + (1 - \alpha_i P)(\mu_i - \mu_j) - \frac{b}{2} \alpha_i^2 P(1 - P)(\mu_j - \mu_i)^2 \quad (A5)$$

for  $i \geq 1$ . If issuer  $j$  satisfies (A5), he is the binding issuer for  $i$ .

For sufficiently large  $N$ , the following algorithm generates the unique Pareto dominant schedule: Set  $C_0 = 0$  and, proceeding sequentially from  $i = 1$  to  $i = N$ , select  $H_i = 0$  and set  $\alpha_i$  equal to the  $\alpha$  between 0 and 1 that makes

$$C_i = C_{i-1} + (1 - \alpha P)(\mu_i - \mu_{i-1}) - \frac{b}{2} \alpha^2 P(1 - P)(\mu_i - \mu_{i-1})^2, \quad (A6)$$

implying

$$C_i = \frac{b}{2} \alpha_i^2 \sigma^2. \quad (A7)$$

The analysis of the constraint (A6) is facilitated by ignoring the last term on its right side, which is trivial for large  $N$ . This constraint makes issuer  $i - 1$  binding for issuer  $i$  and implies  $\alpha_i > \alpha_j$  if  $i > j$ . Since  $\mu_i - \mu_{i-1}$  is constant for all  $i$  and the right side of (A6) is decreasing in  $\alpha$ ,  $C_i$  is necessarily an increasing and concave function of  $i$ . Hence, if issuer  $i - 1$  is deterred from mimicking issuer  $i$ , issuer  $i + 1$ , as well as all issuers with larger  $\mu$ , are also deterred from mimicking issuer  $i$ . (It is also easily verified from the continuous case that issuers with  $i \geq 1$  will not

defect to  $\alpha = 0$  and  $H = 0$  since these higher mean issuers bear lower signalling costs in the discrete case than in the continuous case.) This proves the incentive-compatibility of the schedule obtained with our algorithm.

Now, consider another schedule that is not Pareto dominated by our schedule. Trivially, this cannot have  $H_i = 0$  for all  $i$ . Find the lowest value of  $i$  at which  $H_i > 0$ . For  $j < i$ , this schedule is identical to ours. By equation (A3), if  $H_i > 0$  and the signalling cost of issuer  $i$  is lower than in our schedule,  $\alpha_i$  must be smaller. But (A6), (and hence (A4)), is then violated, since the right side of (A6) is decreasing in  $\alpha$ . This disproves the incentive-compatibility of the alternative schedule.

On the other hand, if  $C_i$  is larger in the alternative schedule, deterring issuer  $i - 1$  from mimicking issuer  $i$ , an algorithm similar to ours, but which starts at issuer  $i$  instead of issuer 0, produces the lowest incentive-compatible signalling costs for issuers  $j$  with  $j > i$ . Hence, issuers  $j$  with  $j > i$  will also have larger signalling costs with the alternative schedule than with ours, proving the former's Pareto inferiority.

Finally, we demonstrate convergence of the sequence of the Pareto dominant discrete schedules to the schedule given by equation (11). From (A6), letting  $\Delta X_i$  denote  $X_i - X_{i-1}$ , we rewrite (A6) as

$$\Delta C_i = \Delta \mu_i (1 - \alpha_i P) + o(\Delta \mu_i)^2 \quad (A8)$$

and after substituting in (A7) obtain

$$\frac{b}{2} \sigma^2 [2\alpha_i \Delta \alpha_i - (\Delta \alpha_i)^2] = \Delta \mu_i (1 - \alpha_i P) + o(\Delta \mu_i)^2 .$$

Letting  $N \rightarrow \infty$ , this converges to the differential equation  $b\alpha\sigma^2 d\alpha = (1-\alpha P)d\mu$ , which is identical to the first order condition, equation (10), that has (11) as its unique solution when the boundary condition  $\mu_0 = K$  is imposed.

Q.E.D.

**Proof of Proposition II:** It is easily verified that the solution in equation (8) maximizes expected utility for the choice of  $\beta$ .

Substituting equations (8) and (16) into equation (15) yields

$$E(U(\tilde{W}_1)) = \alpha P\mu + (1-\alpha P)\mu^L(\alpha) + \frac{\bar{R}_M^2}{2b\sigma_M^2} + W_0 - K - \frac{b}{2}[\alpha^2\sigma^2 + \alpha^2P(1-P)[\mu - \mu(\alpha, H)]^2]. \quad (A9)$$

Since H appears nowhere but in the last term of this expression, the optimal H makes

$$\mu = \mu(\alpha, H), \quad (A10)$$

and, from equation (16), is easily shown to be

$$H(\alpha, \mu) = \frac{1 - \alpha P}{1 - \alpha} [\mu - \mu^L(\alpha)]. \quad (A11)$$

Substituting equation (A10) into (A9) yields

$$E[U(\tilde{W}_1)] = \alpha P\mu + (1 - \alpha P)\mu^L(\alpha) - \frac{b}{2} \alpha^2 \sigma^2 + \frac{\bar{R}_M^2}{2b\sigma_M^2} + W_0 - K. \quad (A12)$$

Equation (A12) has the first order condition

$$P\mu + ab(\sigma_L^2 - \sigma^2) - P\mu^L(\alpha) = 0.$$

This is satisfied when

$$\sigma^2 = \sigma_L^2 + \frac{P}{ab} (\mu - \mu^L(\alpha)),$$

which, after substituting equation (A11) into equation (17), only holds when

$$\sigma^2 = \sigma^2(\alpha, H).$$

This proves incentive compatibility if the second order conditions for a global maximum apply. The second derivative of equation (A12),

$$b(\sigma_L^2 - \sigma^2) - \frac{Pab\sigma_L^2}{1 - \alpha P}$$

is negative for  $0 < \alpha < 1$ , which confirms that the interior solution is a maximum and that it is unique. It is also easily verified that the first derivative is nonnegative at  $\alpha = 0$  and nonpositive at  $\alpha = 1$  for  $\mu \leq \bar{\mu}$  and  $\sigma^2 \geq \sigma_L^2$ , with strict inequalities at interior values of  $\mu$  and  $\sigma$ . This confirms that the interior maximum dominates any exotic boundary solutions where division by zero may have occurred.

It remains to show that actions off the equilibrium path are deterred by plausible beliefs. One type of off-the-equilibrium path behavior, excessive perceived underpricing of the issue, occurs when the offering price is so low as to make  $\mu(\alpha, H) > \bar{\mu}$  in equation (19). This is deterred by the belief  $\mu(\alpha, H) = \bar{\mu}$ . The other type has  $S > \mu^L(\alpha)$ . Any belief that firm value is less than  $S$  results in rejection of the issue, which deters the behavior.

Q.E.D.

**Proof of Proposition IV:** Equation (22) indicates that the type 0 issuer will not defect from his signal levels in the pooling equilibrium to points  $(\alpha, S)$  off the equilibrium path if

$$\begin{aligned} q(\mu_1 - \mu_0) &> (1 - \alpha P)[E(\mu|\alpha, S) - \mu_0] + (1 - \alpha)[S - E(\mu|\alpha, S)] \\ &\quad - \frac{b}{2}\alpha^2[\sigma_0^2 + P(1 - P)(E(\mu|\alpha, S) - \mu_0)^2] \quad \text{or} \\ (1 - \alpha)[S - E(\mu|\alpha, S)] + (1 - q)(\mu_1 - \mu_0) &< \frac{b}{2}\alpha^2[\sigma_0^2 + P(1 - P)(E(\mu|\alpha, S) - \mu_0)^2] \\ &\quad + \alpha P(\mu_1 - \mu_0) + (1 - \alpha P)(\mu_1 - E(\mu|\alpha, S)) . \end{aligned}$$



For small  $\alpha$ , the posterior belief associated with maximum utility is

$E(\mu|\alpha, S) = \mu_1$ , which allows us to rewrite the equation as

$$(1-\alpha)(S-\mu_1) + (1-q)(\mu_1-\mu_0) < \frac{b}{2}\alpha^2[\sigma_0^2 + P(1-P)(\mu_1-\mu_0)^2] + \alpha P(\mu_1-\mu_0) . \quad (A13)$$

On the other hand, equation (22) indicates that the type 1 issuer will defect to the same signal level,  $(\alpha, S)$ , if perceived as a type 1 issuer, provided that  $\alpha$  and  $S$  satisfy

$$(1 - \alpha)(S - \mu_1) + (1 - q)(\mu_1 - \mu_0) > \frac{b}{2}\alpha^2\sigma_1^2 . \quad (A14)$$

Combining (A13) and (A14), it follows that the pooling equilibrium fails the intuitive criterion if there exists an  $\alpha$  sufficiently close to zero and an  $S \leq \mu_1$  satisfying

$$\frac{b}{2}\alpha^2\sigma_1^2 < (1-\alpha)(S-\mu_1) + (1-q)(\mu_1-\mu_0) < \frac{b}{2}\alpha^2[\sigma_0^2 + P(1-P)(\mu_1-\mu_0)^2] + \alpha P(\mu_1-\mu_0) .$$

For small  $\alpha$ , the far right expression exceeds the far left expression since  $\alpha^2$  converges to zero faster than  $\alpha$ . The middle expression can be "squeezed" between the outer two because it is linear in  $S$ . Moreover,  $S$  must be smaller than  $\mu_1$  with  $(1 - q)(\mu_1 - \mu_0) > 0$ .

The efficient separating equilibrium, by construction, cannot fail the intuitive criterion. Any  $(\alpha, S)$  that would induce defection of the type 1 issuer alone must impose a lower signalling cost on him. However, the efficient separating equilibrium minimized signalling cost over all signal levels that would not induce defection of the low value issuer.

Q.E.D.

## FOOTNOTES

<sup>1</sup>The 14% return is in excess of the return of the CRSP value-weighted index, and was computed for a sample of over 1500 firms. For 78.5% of Ritter's sample, this is a return of 1 trading day or less; for 92.7% of the sample, it is a 3-day return or less; for less than .9% of the sample, the return represents a one month return or more, but it is always less than half of a year.

<sup>2</sup>In Allen and Faulhaber's (1988) model, bad managers, who are more likely to run bad firms, are deterred from mimicking good managers who underprice because subsequent cash flows partially reveal the firm type. The Bayesian revisions about firm type that occur upon observation of a high cash flow are dramatic for a manager who underprices. High cash flows rarely occur, however, for the bad manager who mimics the underpricing of good managers. In addition, the good manager who does not incur the cost of underpricing will not experience a strong upward revision in beliefs about his firm type when his firm has a high cash flow.

<sup>3</sup>In Nanda's (1988) model, firms with high mean returns also have low variances. Since this model has high-variance low-mean firms issuing debt, high-mean firms are penalized by issuing debt that is perceived as being riskier than it really is. They separate themselves from the low-mean high-variance debt issuers by underpricing their equity offerings.

<sup>4</sup>In Welch's (1988) model, risk-neutral entrepreneurs can sell a fraction of their firm in an unseasoned offering. If they do so, they are exogenously assumed to sell the remainder of their firm in a subsequent seasoned offering. Low quality firms who mimic high quality firms must pay an exogenously specified operation cost that the high quality firms do not incur. In some cases, this cost may be insufficient to deter mimicking, and underpricing becomes the additional wedge that deters low quality firms in the separating equilibrium.

<sup>5</sup>If  $\underline{\mu} > K$ , all signalling schedules replace  $K$  with  $\underline{\mu}$ . Nothing fundamental changes.

<sup>6</sup>The other parameters of the model, such as the capital outlay,  $K$ , the utility function of the issuer, and the probability of learning  $\mu$  over time in the absence of signalling, are also known by market participants.

<sup>7</sup>Rule 144 under the Security Act of 1933 requires a two-year holding period. This rule applies to some, but not a majority of new issues, since equity that was privately placed with the entrepreneur more than two years prior to the public offering could be sold as part of the offering, or sold separately in the public market. Shares retained at the offering, however, cannot be sold by insiders within 90 days of the offering. Moreover, it is very common for inside shareholders to precommit to legally binding holding periods in excess of 90 days by stating so in the prospectus. In addition to securities regulations, much of the law in this area is administrative case law, the implications of which will vary from situation to situation.

<sup>8</sup>In our model, as well as the Leland and Pyle model, the entrepreneur would not optimally hold the market portfolio, but would hold more (less) of

individual securities that are negatively (positively) correlated with the project. Here, the word "market" represents this alternative optimal portfolio.

<sup>9</sup>Risk-free borrowing (lending) by the newly incorporated firm will have no effect on the results in this model. Since personal risk-free investment or borrowing is a perfect substitute for corporate lending and borrowing, the Modigliani-Miller theorem applies in this setting. For this reason, we can assume without loss of generality that any difference between  $(1 - \alpha)S$  and  $K$  is kept by the entrepreneur, rather than invested (borrowed) by the firm at a risk-free rate. This allows us to interpret  $\mu$  as the value of the firm, as well as the value of the firm's risky project.

<sup>10</sup>This comparison can be shown to be somewhat misleading, however, when stock price volatility is broken into its two components: cash flow volatility conditional on knowing the mean and resolution of ex ante uncertainty about the mean. Our model describes a positive cross-sectional relation between the former and the degree of underpricing, whereas Welch's model does not contain a variable for conditional cash flow. His opposite relation thus follows entirely from the resolution of uncertainty about the mean of the future cash flows of underpriced firms prior to the aftermarket.

<sup>11</sup>We note that general distributional conditions have eluded other work in the area, notably Riley's (1979, 1985) seminal work on one-attribute models with a continuum of attribute types.

<sup>12</sup>Application of the criteria might require minor parameter restrictions. For example, a lower bound  $\underline{\mu}$  on  $\mu$  might have to be imposed. To make utility monotonic in the perceived  $\mu$ , it might be necessary to require

$$b(\bar{\mu} - \underline{\mu}) \leq \frac{1}{p} \text{ or } b(\bar{\mu} - K) \leq \frac{1}{p} .$$

This would ensure the "single-crossing property" on which the non-existence of pooling equilibria with the criteria is based.

<sup>13</sup>Engers (1987) has also shown that for screening models satisfying mild regularity conditions (the multiple-signal attribute analogy of the single-crossing condition), Pareto-dominating schedules, which necessarily are the only candidates for reactive equilibria, exist.

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