PERISHABLE INVESTMENT AND HYSTERESIS IN CAPITAL FORMATION

by

Bernard Dumas

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RODNEY L. WHITE CENTER FOR FINANCIAL RESEARCH
The Wharton School
University of Pennsylvania
Philadelphia, PA 19104-6367

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ABSTRACT

Entry into a market seems to necessitate some investment into "marketing capital" (or distribution capital: advertising, dealerships etc...). This form of investment has the property that, if it is unused for some time, it quickly becomes worthless. When entry into a market requires marketing investment, firms which are currently out of this market tend to delay entry until price vs cost conditions have become extremely favorable. Conversely, firms which are in the market tend to delay exit until they can no longer bear large operating losses. This is because they know that, if they do exit, and if price vs cost conditions later become favorable again, they will have to incur afresh the investment in marketing capital.

The purpose of the present paper is to produce a general-equilibrium model of capital formation in an economy subject to random shocks, when marketing capital (with the above properties) is used in distribution, in addition to the "normal" capital used in production.

We exhibit an analytical solution to the dynamic program representing the welfare optimum problem, along with the shadow prices corresponding to this program. These are also the prices which would support the general equilibrium of a decentralized market economy.

Our results pertain to the effect of entry costs, risk, risk aversion and productivity on the balance between marketing and productive capital, to the nature of growth paths in this economy and to the level of prices (such as the price of shares in the stock market, or the price of final goods) as well as the extent to which productivity shocks are passed through into these prices.

1. Introduction

Some forms of capital have the property that they quickly become useless if they are left unused for some time. We describe capital which has this property as "perishable." The limiting case of perishable capital or investment is encountered when that capital depreciates immediately if it is out of use for any time at all. In what follows, we consider only that limiting case.[1]

One example of perishable investment or capital is "marketing capital".[2] We call marketing capital the (cumulative) expenditure which a firm must incur in order to penetrate a market and sell a product. This expenditure is in the nature of a setup cost: as a firm enters a market it must incur some cost which allows it thereafter permanently to distribute a given flow of product to consumers. If the firm later exits from the market it can often recoup those costs by selling the brand or the dealerships to another firm which enters. But if firms in the aggregate withdraw, the setup cost is lost.

When entry into a market requires marketing investmemt, firms tend to delay entry until price vs cost conditions have become extremely favorable. Conversely, after they have entered, firms tend to delay exit until they can no longer bear large operating losses. This is because they know that, if they do exit, and if price vs cost conditions later become favorable again, they will have to incur afresh the investment in marketing capital. As a result of this behavior pattern, the amount of goods distributed to final consumption in any given market tends to adjust sluggishly.[3]

Entry costs have been invoked to explain the persistence of the U.S. trade deficit during the years 1986-1988, after the dollar has dropped so markedly. Because of them, it has been argued, foreign firms selling in the

U.S. accept the losses they currently incur, while concurrently U.S. firms have not as yet decided to enter foreign markets. In other words, foreign and domestic firms do not behave in a symmetric way when the dollar randomly rises and when it randomly falls. The phenomenon has been labelled "hysteresis" by Baldwin and Krugman (1986). The same phenomenon explains that foreign firms selling in the U.S. have not "passed through" to customers the change in their costs brought about by the change in the exchange rate.

The entry cost can usefully be interpreted as an investment to discover and establish distribution or marketing channels, or as an advertising expenditure to make potential customers aware of a product. After the investment is made, the rate at which goods can be delivered to final consumers is bounded by the capacity of the marketing channels which have been put in place,[4] or by the size of the market segment which has been generated by the advertising effort.[5,6]

In a contribution which is the closest antecedent to, and the major impetus for the present study, Dixit (1987a, b) has implemented the Baldwin-Krugman idea in a partial-equilibrium model of firm decisions. The source of randomness is again the exchange rate but that rate exogenously follows a Brownian motion (instead of being identically independently distributed over time as it was in Baldwin-Krugman). Foreign firms decide to enter a home market where they sell their imported goods at a given home price, while the costs of production have been incurred in the foreign country. The exchange rate is really a cost shock for these firms.

The purpose of the present paper is to produce a general-equilibrium model of capital formation in an economy subject to random productivity shocks, when marketing capital (with the above properties) is used in distribution, along with the "normal" capital used in production. This means that, in addition to optimizing the behavior of firms, as Dixit did, we intend simultaneously to optimize the consumption-investment choices of individual agents, thereby obtaining the process for the various price variables. We can expect to derive three kinds of benefits from this analysis.

First, the general equilibrium approach will force us to revise some of Dixit's (1987b) results. These are the results pertaining to the degree to which cost shocks are passed-through to final consumers.

Second, rational expectations cause some prices (particularly financial prices) to anticipate events which take place in the market for goods. In turn, the anticipatory adjustment of prices somewhat reduces the need for goods markets to effectively adjust.[7] The challenge is to explain the observed pass-through in an equilibrium situation.

Finally, this being a general equilibrium model, we can expect to draw from it some implications regarding the growth path of the economy. Hysteresis implies that small deviations occurring in the present can have long-term repercussions.[8] We will show that in the present model there can exist a bifurcating equilibrium in which the economy can equally well embark on a path of sustained (although random) positive growth or on a path of sustained decline. It all depends on the capital mix (i.e., the mix of perishable vs regular productive capital) that the economy has been able to achieve.

The outline of the paper is as follows. Section 2 lays out the modelling choices which are made. It leads to a certain mathematical programming problem which must be solved in order to derive the aggregate behavior of the economy. Section 3 provides the mathematical solution of the programming problem and the optimal decisions of agents while section 4 outlines the aggregate dynamics which they generate. Section 5 presents some comparative analysis of these decisions as parameter values are changed. Section 6 derives the price of shares in the stock market and the price of final consumption and examines their behavior over time. The conclusion in section 7 contains some suggestions for extensions (especially to a two-market economy).

2. Modelling perishable capital in general equilibrium

For the sake of simplicity, the model to be studied here does not aim to encompass the international setting or the multiplicity of markets found in a typical economy. There is only one product sold in one market. This is in fact as in Dixit's (1987a, b) partial-equilibrium analysis. As we just pointed out, the exchange rate in his work really acts as a cost or productivity shock. We model the shock explicitly as a productivity shock.

The exchange rate, being the source of randomness in Dixit's work, introduces an asymmetry between home and foreign firms. This is probably very realistic. Dixit, however, allows foreign firms to compete on the home market but not home firms to compete on the foreign market and it would appear that home firms do not compete on their own market under the same conditions as foreign firms do.[9] These added asymmetries seem artificial. In the present analysis, for better or for worse, all firms are in the same situation.

Finally, in order to facilitate interpretation, one more correspondence between Dixit's (1987b) model and this one must be pointed out.[10] In Dixit's work the total number n of foreign (e.g., Japanese) firms which have decided to enter the home (e.g., American) market is also a measure of the cumulative amount of setup cost which has been expended by the economy at large as far as the home market is concerned. We choose to deal with this state variable directly and it is referred to as the stock m of "marketing capital" in place. The value of m indicates the size of the market which has been developed so far: when m increases firms "enter" the market; when it decreases they "leave" it.[11]

In the single-market economy considered here there are, therefore, two types of capital:

- productive (or durable or tangible) capital generates a random output (according to a random constant-return-to-scale production function). In addition, one consumes out of the stock of capital. This is represented by the following stochastic differential equation:

$$dk = (\alpha k - c)dt + \sigma k dz ;$$

where: k = the current stock of productive capital,

α = the expected rate of output per unit of capital,

c = the rate of consumption,

σ = the volatility of output per unit of capital,

dz = a "white noise" reflecting productivity shocks;

- marketing capital (or perishable or intangible capital) m has the following properties:

  - it is not possible to consume unless the corresponding marketing capacity is in place:

$$c \leq m$$

  - m decays immediately if unused:

$$m \downarrow c$$

(hence m = c at all times);

  - m is formed (this is called entry) by giving up 1/s units of capital k for each unit of m;

  - m is abandoned and the abandoned amount becomes worthless (this is called exit), if and when m = c, in the aggregate, is brought down.

The economy is populated by one (or several identical) consumer(s) with time-additive von-Neumann Morgenstern isoelastic utility (power γ), infinite horizon and a rate of impatience δ which is constant. Irrespective of any assumption regarding the financial market, it is therefore allowable to examine a welfare optimum problem as a computational shortcut towards the determination of a market equilibrium. This shortcut assumes, however, that expenditures in marketing capital are not firm specific, i.e., that the stock of marketing capital in place can be traded freely when one firm leaves the market, and is replaced by another one.[12] Otherwise one may have some difficulty in arguing that the market equilibrium one is looking for is also a Pareto optimum.

A correct mathematical model representing this economy must now be stated. We are guided in so doing by Harrison (1985) (pages 102 and following) and Constantinides (1986). Considering the proportional nature of the setup costs incurred, it stands to reason that there exists an area[13] of the state space (the (m, k) plane) within which it is optimal to do nothing, i.e., let k evolve of its own accord following the stochastic differential equation above (with $c = m$) and let m be constant. (This is in contrast to the situation of increasing average setup cost where it would have been optimal to perform some action at all times.) The frontier of the region of no action is made up of two parts: when one part of the frontier is reached a decision is made to transform some productive capital into marketing capital; when the other part of the frontier is reached a decision is made to abandon some marketing capital.

A <u>policy</u> is defined as a pair of adapted processes U and L (parametrized by the initial conditions (m, k)) which are right-continuous, non-decreasing and non-negative and which represent the <u>cumulative</u> amounts of capital formed abandoned since time 0. These functions will serve to regulate the joint process for k and m. To be precise the problem statement is:

(1) $$J(m, k) \equiv \underset{\substack{U_{m,k} \\ L_{m,k}}}{\text{Max}} \quad E^{L,U}_{m,k} \int_0^\infty e^{-\delta t} \frac{1}{\gamma} (m_t)^\gamma dt \ ;$$

where the superscripts on the expectation operator E refer to the policy and the subscripts indicate that the conditional expectation is to be calculated with respect to the new processes for k and m given by the following differential equations:[14]

(2) $$dk = (\alpha k - m)\ dt + \sigma k dz - dU$$

(3) $$dm = s dU - dL\ .$$

Considering the linear nature of the constraints and the isoelasticity of the instantaneous utility function, it is clear that the solution for J(m, k), if it exists, must be homogeneous of degree γ and the solutions for L and U, if they exist, must have the property that if policy (U, L) is optimal for initial conditions (m, k), then policy (2U, 2L) is optimal for initial conditions (2m, 2k). Furthermore, since the setup costs considered here are strictly of a proportional nature, one could prove[15] that it is optimal to choose the L and U processes to be continuous, an example of what Harrison (1985) (page 105) calls a "barrier policy." This means that the problem and its solution can be formulated on the basis of a new variable x = k/m, a new function:

(4) $$F(x) \equiv J(m, k)/m^\gamma\ ,$$

and two numbers $\ell < u$ which operate as follows:[16]

(a) U increases only when $x = k/m = u$;

(b) L increases only when $x = k/m = \ell$.

The interpretation of this policy is as follows. In the (m, k) space, there exists a cone delimited by the two rays of slopes ℓ and u with the following meanings (see figure 1). The upper ray (of slope u) is the "entry ray": when the existing combination of capital stocks (m, k) places the economy on that ray, a decision is made instantaneoulsy to form some marketing capital by cutting back on productive capital ($dU > 0$). The amount of marketing-capital formation is very small and is such as to push the economy back inside the cone (along a line of slope $- 1/s$). Immediately thereafter the volume of sales in the market is slightly larger.

FIGURE 1 GOES HERE

The lower ray of slope ℓ is the "exit ray": when the existing combination of capital stocks (m, k) places the economy on that ray, a decision is made instantaneoulsy to abandon some marketing capital ($dL > 0$). The exact amount of marketing capital abandoned is very small and is such as to push the economy back inside the cone (along a horizontal line). Immediately thereafter the volume of sales in the market is slightly lower.

Inside the cone, $dU = dL = 0$ so that consumption and the stock of marketing capital are constant: entry costs prevent the firms from acting immediately. They prefer to "wait and see"; the future succession of output shocks may cause them to act later but they do not act for the time being. In the meanwhile, as is indicated by equation (2), output shocks are buffered by the stock of productive capital: any output beyond the fixed current consumption goes into the formation of productive capital k and any output lower than the fixed current consumption is made up for by reducing the stock of productive capital.

In what follows, we seek to optimize the two numbers ℓ and u and we obtain an analytical expression for the functions J and F. Before we proceed any further, we establish that the function J exists, in the sense that there are conditions under which the expected integral in (4) converges for finite values of k (and for any value of m) and the maximum in (1) exists. Imagine for a minute an economy which would be identical to the one described above, except for the fact that there would be no entry costs ($1/s = 0$). In that case, the solution $J^*(k)$ of the maximization problem is known exactly. Provided that:

$$(5) \qquad \delta > \gamma\left[\alpha - \tfrac{1}{2}(1 - \gamma)\sigma^2\right] ,$$

we know from Merton (1971) that:

$$(6) \qquad J^*(k) = \frac{1}{\gamma}\, k^\gamma \left[\frac{\delta - \gamma\left(\alpha - \frac{1}{2}(1 - \gamma)\sigma^2\right)}{1 - \gamma}\right]^{1-\gamma}$$

Now, imposing entry costs cannot possibly increase the achievable indirect utility level: $J(m, k) \le J^*(k)$. The above restriction (5) on parameter values is therefore sufficient to ensure existence of J^* and J.[17]

3. The solution of the programming problem

The expected discounted utility function J is defined in (1) as the expected value of an integral computed from the current time to an infinite horizon. For this reason, as time goes by and the regulated process (m, k) evolves, the trajectory of J cannot have jumps for as long as the kernel m^γ/γ of the integral does not take on an infinite value at the current time.[18] An essential property of the expected discounted utility is that the trajectory of its values over time must be continuous with probability one.[19]

The conditionally expected rate at which J evolves during a small interval of time dt is dictated by discounting at the rate δ and by consumption which procures a rate of utility equal to m^γ/γ. When the regulator is applied, however, the movement to the target position is instantaneous: if the trajectory of the values of F is to remain continuous with probability one, the value of expected discounted utility at the arrival point must be equal to the value at the departure point (with probability one).

The first statement of the previous paragraph translates mathematically into the following linear ordinary differential equations to be satisfied by the functions J and F inside the cone:

(7) $$0 \equiv \frac{1}{\gamma} m^\gamma - \delta J + J_k(\alpha k - m) + \tfrac{1}{2} J_{kk}\sigma^2 k^2 .$$

(8) $$0 \equiv \frac{1}{\gamma} - \delta F + F'(\alpha x - 1) + \tfrac{1}{2}F''\sigma^2 x^2 .$$

As for the second statement, it translates into the following two equalities:

-when $k/m = u$: $J(m, k) = J(m + sdU, k - dU)$;

(9) or: $$0 = sJ_m(m, k) - J_k(m, k) ;$$

which transforms into:

(10) $$\frac{F'(u)}{F(u)} = \gamma \frac{s}{1 + su} .$$

-when $k/m = \ell$: $J(m, k) = J(m - dL, k)$

(11) or: $$0 = J_m(m, k)$$

which transforms into:

(12) $$\frac{F'(\ell)}{F(\ell)} = \gamma .$$

These properties would actually suffice to fully calculate the functions J and F prior to any optimization (i.e., for <u>given</u> values of u and ℓ). The general solution of the O.D.E. (8) is:

(13) $$F(X; C_1, C_2) \equiv \frac{1}{\delta\gamma} + C_1 N_1\left[\frac{2}{\sigma^2 X}\right] + C_2 N_2\left[\frac{2}{\sigma^2 X}\right]$$

where: C_1 and C_2 are two integration constants,

(14) $$N_i(Y) \equiv e^{-y}\, y^{\pi_i}\, M(a_i, b_i, y) \; ; \qquad i = 1, 2$$

M is the confluent hypergeometric function,[20]

π_1 and π_2

are the two real solutions of the following second-degree algebraic equation:[21]

(15) $$-\delta = \pi(\alpha - \sigma^2) + \tfrac{1}{2}\, \pi(\pi - 1)\sigma^2 = 0 ,$$

and, for

i = 1, 2:

(16a,b) $$a_i = \frac{\alpha - \sigma^2 - \tfrac{1}{2}\pi_i\sigma^2}{\tfrac{1}{2}\sigma^2} \qquad b_i = \frac{\alpha - \sigma^2 - \pi_i\sigma^2}{\tfrac{1}{2}\sigma^2} .$$

Since the integration constants C_1 and C_2 intervene linearly in this solution it is then simply a matter of solving a linear 2 x 2 system to determine their values C_1 (ℓ, u) and C_2 (ℓ, u) such that the two equations (10) and (12), serving as boundary conditions, are satisfied:

(17) $$\frac{F'(u; C_1, C_2)}{F(u; C_1, C_2)} = \gamma\, \frac{s}{1 + su} \; ;$$

(12) $$\frac{F'(\ell)}{F(\ell)} = \gamma \; .$$

These properties would actually suffice to fully calculate the functions J and F prior to any optimization (i.e., for given values of u and ℓ). The general solution of the O.D.E. (8) is:

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Since the integration constants C_1 and C_2 intervene linearly in this solution it is then simply a matter of solving a linear 2 x 2 system to determine their values $C_1\,(\ell,\, u)$ and $C_2\,(\ell,\, u)$ such that the two equations (10) and (12), serving as boundary conditions, are satisfied:

(17) $$\frac{F'(u;\, C_1,\, C_2)}{F(u;\, C_1,\, C_2)} = \gamma\,\frac{s}{1 + su} \; ;$$

$$\frac{F'(\ell;\ C_1,\ C_2)}{F(\ell;\ C_1,\ C_2)} = \gamma \tag{18}$$

We now look for the choice of the boundary parameters ℓ, u which maximizes the expected discounted utility. Even though we have not been able to establish this by analytical methods on the basis of equations (13) to (16), it is intuitive from the economic nature of the problem that improving the choice of boundary parameters is beneficial uniformly, i.e., irrespective of the current state (m, k) of the economy. This means that the functions J, F and the two integration constants C_1 and C_2 reach an optimum simultaneously for the same set of values of ℓ, u.

Differentiating (17) and (18) with respect to u and ℓ respectively, using the fact that, at the optimum, the derivatives of the integration constants are zero, one obtains:

$$\frac{F''(u;\ C_1,\ C_2)}{F'(u;\ C_1,\ C_2)} = (\gamma - 1)\ \frac{s}{1 + su}\ ; \tag{19}$$

$$\frac{F''(\ell;\ C_1,\ C_2)}{F'(\ell;\ C_2,\ C_2)} = \gamma \tag{20}$$

which will serve as first-order conditions for the optimal choice of u and ℓ.[22]

The system (17, 18, 19, 20) is linear in two of the unknowns viz. C_1 and C_2 so that these can easily be eliminated. In the end, the slopes ℓ and u are the only two remaining unknowns and they are provided by a system of two non-linear algebraic equations which can be solved numerically by the Newton-Raphson technique, for instance.

In what follows, frequent reference will be made to a "base case" combination of parameter values which are as follows:

(21) $\alpha = 0.1 \quad \sigma = 0.5 \quad \delta = 0.2 \quad s = 1 \quad \gamma = -1$.

For these parameter values, but in the absence of setup costs ($1/s = 0$), the optimal consumption decision c^* would have been to maintain a rate of consumption per unit of capital c^*/k equal to 2.5%; or a ratio of capital to consumption k/c^* equal to $x^* = 40$.[23]

In the presence of setup costs such that productive capital can be turned into marketing capital unit for unit ($s = 1$), the solution is found to be:

(22) $u = 72.38 \quad \ell = 31.59$.

It is optimal for the ratio of productive to marketing capital to fluctuate around $x^* = 40$ within these two control limits.

The rate of consumption per unit of productive capital must fall -- as a result of a sequence of favorable output shocks -- as low as 1/72.38 (compared to 1/40) before the decision is made to expand the market and to form some additional marketing capital. The reason for this cautiousness on the part of decision makers is that they know that the sequence of favorable output shocks has no reason to persist. There is no sense in forming marketing capital now, only to have to abandon it soon thereafter. Only when the stock of productive capital is high enough to guarantee a high level of output do they start building the marketing channels needed to distribute an increased flow of consumption.

Similarly the rate of consumption per unit of productive capital must rise as high as 1/31.59 -- as a result of a severely eroded stock of productive capital -- before a decision is made reluctuantly to pull back from the market and to abandon some marketing capital.

In both cases, the size of the market is not allowed to respond immediately to fluctuations in the exogenous source of risk (in this case productivity shocks). A "safety zone" is created on both sides of what would otherwise be the optimal capital/consumption ratio. This is the essence of the hysteresis phenomenon.

4. Expanding vs contracting economies

The behavior of the economy over time can be characterized on the basis of the following criterion. Since $c = m$ at all times, equation (2) above implies that, within the cone:

$$\text{(23)} \qquad d\ln k = (\alpha - \frac{m}{k} - \frac{1}{2}\sigma^2)\, dt + \sigma\, dz \ .$$

Since the coefficient of the white noise in this stochastic differential equation is a constant, the dynamics are entirely provided by the drift term.[24]

When $m/k < \alpha - \frac{1}{2}\sigma^2$, the passage of time implies a displacement of the entire conditional probability distribution of $\ln k$ towards the right of the current point. When that is so, we say that the economy is expanding. When the opposite is true, the displacement is to the left and we describe the economy as contracting.

Furthermore, it has been established that at all times: $\ell \le k/m \le u$, and that within the cone ($\ell < k/m < u$) m is a positive constant[25] whereas on the edges of the cone, m/k is a constant. One can therefore compute the following two quantities:

$$\text{(24)} \qquad \alpha - 1/\ell - \frac{1}{2}\sigma^2 \quad \text{and} \quad \alpha - 1/u - \frac{1}{2}\sigma^2$$

and use them to draw the following conclusions.

If they are both negative, the economy is at all times contracting. If they are both positive, the economy is at all times expanding. For instance, for the base case corresponding to parameter values (21) the quantities (24) are equal to -0.056 and -0.03881 so that the economy is unambiguously a contracting one, as it would have been in the absence of setup costs (c^*/k = 0.025 is larger than $\alpha - \frac{1}{2}\sigma^2 = -0.025$).

If the two quantities (24) are of opposite signs, the critical value of m/k equal to $\alpha - \frac{1}{2}\sigma^2$ falls within the [1/u, 1/ℓ] range. To this critical value of m/k corresponds, for any given m, critical values of k and lnk such that, when lnk is above that value the economy is expanding and when lnk is below the economy is contracting. The critical value of k/m materializes into a new ray (which is in this case situated within the allowable range of fluctuations) where a <u>knife-edge bifurcation</u> appears: if the economy is placed above that ray or reaches there by chance, it goes on upward in a probabilitic sense (eventually reaching the entry ray); if it placed below, it shoots down probabilistically (eventually reaching the exit ray).[26]

This knife-edge bifurcation -- of which we shall exhibit an example in section 5.4. below -- is an interesting new phenomenon because it could not have arisen in the absence of hysteresis. An economy which is not subject to setup costs is unambiguously either expanding or contracting[27] (in the probabilitic sense given above), irrespective of the current or initial situation.

The mechanism which generates this phenomenon can be described as follows. In a country with a shortage of marketing capital, i.e., one where $m/k < \alpha - \frac{1}{2}\sigma^2$, consumption is frequently curtailed by the insufficient capacity of marketing channels. This curtailment of consumption increases the speed at which productive capital can be accumulated and later production can be increased. This increased production is to some extent channeled into new marketing capital. But risk and the hysteresis phenomenon imply that when marketing capital is formed, the amount which is formed is just sufficient to maintain the capital mix close to its preexisting level. The shortage of marketing capital is not allowed to become exceedingly severe but it remains a shortage and growth continues to be stimulated by the curtailment of consumption. In a country with an abundance of marketing capital ($m/k > \alpha - \frac{1}{2}\sigma^2$), however, consumption is artificially stimulated for fear of leaving marketing capital unused. This enhanced consumption thwarts growth.

In his survey on long-run growth in the presence of externalities, Romer (1988) identified only two models capable of generating bifurcating behavior. The first model is the Marshall (1961) - Romer (1987) model in which aggregate output depends on the total number of varieties of goods produced in the economy at large. The degree of development is then defined by the number of existing varieties. If two economies of this type are allowed to trade with each other and

> "if there is a significant range of goods that are too expensive to transport and trade outside of a limited area, developed areas will tend to have a built in advantage over the less developed areas. Under these circumstances, convergence will fail. Starting from symmetric positions for two different countries, the country that can first take the lead may have a permanent advantage over the other."[28]

The second category of models identified by Romer (1988) as having this property are those which endogenize population growth (e.g., Nerlove (1974), Barro and Becker (1986)). In such models,

> "the larger is the human capital of the parents, the higher is the cost of a child.... Thus, depending on the initial conditions, one family or country might be stuck in a permanent state of low per capita income growth and high population growth, while another might be in an equilibrium with low population growth and high per capita income growth."[29]

In our model, countries with a larger amount of productive capital relative to perishable capital have an inherited or acquired growth advantage. But this result is achieved without any external effect being present.

5. Comparative dynamic analysis

The intensity of the hysteresis phenomenon as well as the dynamic behavior of the economy depend, of course, on parameter values. We examine successively the influence of the size of setup costs (s), the influence the volatility of the source of risk (σ), the influence of the degree of risk aversion (γ) and that of the expected productivity of the production process (α).

5.1. Varying the size of the entry or setup cost

The curves showing the evolution of the two control limits (1/u and 1/l) as functions of the size of entry costs are given here as figure 2. Not surprisingly, the optimal range of allowed fluctuations of m/k widens as setup costs increase. The extreme case is the one where 1/s = 0 (expansion of the market requires no entry cost); in that case, the two control limits become equal to each other as they must evidently both be equal to the inverse of the optimal rate of costless consumption ($1/u = 1/\ell = 1/x^* = 0.025$).

FIGURE 2 GOES HERE

This graph displays also one property already obtained by Dixit (1987a). The slope of the curves is infinite in a neighborhood of 1/s = 0. Hence <u>even very small setup costs can produce a sizable hysteresis effect</u>. The reason for this phenomenon is the fact that a diffusion process drives output in this model.[30] It is well known, since the Black-Scholes (1973) theory of option pricing, that a policy of continuously adjusting a dynamic hedge, when one or both of the assets under consideration follows such a process, would produce infinite transactions costs in finite time, for any finite rate at which transactions costs are levied. The only way to avoid infinite transactions costs is to adjust a hedge (in our case the capital mix k/m) at irregular random times. To achieve this goal the trigger points applying to the capital mix must be separated from each other.

A rapid check on the quantities (24) would reveal that, for all values of setup costs, the economy defined by parameter values (22) remains a contracting one. The allowable range $[1/u, 1/\ell]$ of variations of m/k is uniformly above the critical value $\alpha - \frac{1}{2}\sigma^2$ (which happens to be negative).

5.2. Varying the volatility of output

As one gradually reduces the value of σ down from the base case value of 0.5 (leaving other parameters as in the base case), the optimal value x^* decreases. Equivalently, as risk increases people consume less, as a hedge against future production shocks.[31] Concurrently, as σ drops, the optimal range between the two control limits ($1/\ell$ and $1/u$) first expands and then narrows again (see figure 3). But this range does not vanish entirely as σ goes to zero and one reaches the certainty situation. Setup costs still play a role under certainty but a smaller one than under uncertainty, as the diagram of figure 3 indicates.[32]

FIGURE 3 GOES HERE

As σ is changed, it is easy to verify, based on criteria (24), that the particular economy of the numerical illustration is a contracting one unambiguously for all values of the volatility. This is reflected in figure 3 by the fact that the domain of allowed variations of m/k is uniformly above the line $\alpha - \frac{1}{2}\sigma^2$. [33]

The fact that the economy is contracting is the reason for one phenomenon which is observable on figure 3: as σ approaches zero the curve for $1/\ell$ meets the curve for $1/x^*$. Under certainty a contracting economy does contract (i.e., k goes down over time) with probability one. Furthermore consumption also contracts as soon as the exit ray is reached. But, if consumption is declining with certainty, the amount of marketing capital in place is not a constraint (marketing capital is simply disposed of as the rate of consumption declines). Consequently the effective consumption rate per unit of productive capital $1/\ell$ is set at the cost-free optimal level. The spread which gradually develops between $1/\ell$ and $1/x^*$ ($1/\ell > 1/x^*$) as σ rises above zero is perhaps what most narrowly characterizes the hysteresis phenomenon: even though the economy, as far as transition probabilities are concerned, remains a contracting one, decision makers know that there is a positive probability of a temporary expansion at some future point.[34] Their desire to postpone market exit on the way down is the direct result of this random prospect. In the language of Finance, they prefer to postpone exercise of their option to exit, because the probability is large enough that future events may render this option kept alive very valuable.

5.3. Varying risk aversion

A reduction of the consumer-investor's risk aversion ($1 - \gamma$), down from the base case value of 2, increases the value of[35] $1/x^* = c^*/k$ as well as the range which separates $1/\ell$ from $1/u$. This range becomes infinite as one approaches risk neutrality. Here again, because the economy is a contracting one, the exit ray of slope $1/\ell$ and the ray of slope $1/x^*$ which would be followed in the absence of setup cost, become very close to each other as one approaches risk neutrality. Hysteresis (but only in the restricted sense defined at the end of paragraph 5.2.) disappears under risk neutrality just as it did under certainty. These effects are summarized in figure 4.

FIGURE 4 GOES HERE

As is indicated on the diagram by the value $\alpha - \frac{1}{2}\sigma^2$, the economy of our numerical illustration remains a contracting one for all values of risk aversion.

5.4. Varying the expected productivity of the production process

Figure 5 displays the effect of increasing the expected physical rate of return of the production process α from the base case of 0.1 to 0.6 without varying other parameter values. Naturally, as productivity rises, the optimal rate at which one can consume per unit of capital rises. Correspondingly the mix of marketing vs productive capital in place must evolve in favor of marketing capital. Also, the optimal range $[1/u, 1/\ell]$ of fluctuations of the m/k capital mix increases as indicated by the non shaded area of figure 5.[36]

FIGURE 5 GOES HERE

As α rises, it becomes possible for the economy to turn from a contracting into an expanding one. This is visible on figure 5 from the fact that the $\alpha - \frac{1}{2}\sigma^2$ line cuts across the non shaded zone. Loosely speaking, the determining factor is whether the physical rate of return is sufficiently lower or higher than the rate of discount of utilities δ (set here equal to 0.2). Indeed, when α is lower than approximately[37] 0.154, the economy, on the basis of criteria (24), is unambiguously contracting. Similarly when α is higher than approximately 0.261, the economy is unambiguously expanding.

When α is between these two values, the two quantities defined in (24) are of opposite signs and the economy's behavior depends on the current capital mix. If and when the ratio m/k is lower than the critical value $\alpha - \frac{1}{2}\sigma^2$, the probability distribution of lnk is moving downward. Otherwise it is moving upward. This is an <u>example of a bifurcating equilibrium</u> of the kind described in section 4 above.

6. The stock market, the price of consumption and other prices in the decentralized economy

Three concepts of marginal utility are present in the model:

-the marginal indirect utility of one more unit of productive capital which is equal to the derivative J_k of the J(m, k) function with respect to k;

-the marginal indirect utility of one more unit of marketing capital which is equal to the derivative J_m of the J(m, k) function with respect to m;

-the marginal utility of consumption $c^{\gamma-1}$. This quantity has not played a role explicitly in the solution of the welfare optimum problem. But, in the decentralized version of this economy, it would guide the choices of individuals in their decisions to consume and/or to invest into financial securities and would therefore also be equal to their marginal utility for financial wealth.

All three quantities generally take on different values and their various ratios are indicative of the equilibrium prices which would prevail in the corresponding purely competitive market economy.[38] We examine now three such prices as functions of the prevailing capital mix k/m:[39] the price of marketing capital, the average value of shares in the stock market and the price of final consumption. We choose to express all prices in units of productive capital on the grounds that productive capital is the common fungible good from which other goods and services (marketing capital and final consumption) are produced.

6.1. The prices of assets

The marginal price of marketing capital, i.e., the price of one more unit of marketing capital relative to one more unit of physical capital, is equal to J_m/J_k. Since the function J(m, k) is fully known, there is no difficulty in obtaining this price whose graph is shown in figure 6 for the base-case parameter values (21). Given the boundary conditions which have been imposed (see section 3), there is no suprise in finding that the price of marketing capital is equal to zero for $k/m \leq \ell$ when firms are exiting and equal to $1/s$ (= 1 in the numerical illustration) for $k/m \geq u$ when firms are entering. In between, marketing capital everywhere has a nonnegative price, since it could be disposed of at no cost. The price varies between 0 and $1/s$ and its graph connects smoothly to the two boundary levels. This last property is due to the super-contact conditions (32) and (33). For $\ell < k/m \leq x^*$ one could say,[40] in a sense, that there is an excess of marketing capital since consumers are lead to consume as much as they do, only in order to keep the marketing capital[41] alive. Even then the price is positive, simply because of the prospect that marketing capital may again become scarce and would then have to be re-built at some cost. The asset (or stock) character of marketing capital explains that it may retain a positive value even when its immediate social worth is negative.

FIGURE 6 GOES HERE

The value of total capital in the stock market (also equal to aggregate financial wealth) is given by $k + (J_m/J_k)m$, which is the stock of productive capital plus the stock of marketing capital valued at its (marginal) price. It will probably be more telling to scale down this quantity by dividing it by its replacement cost or "book value" $k + m/s$. Figure 7 shows the stock market capitalization per unit of total capital measured at cost which can be regarded as "the price of a share." This is in essence an average Tobin q, but one which arises from costs of adjustment on the consumption side rather than on the investment side. This ratio is always less than or equal to 1.

Its variation reflects the evolution of the price of marketing capital J_m/J_k given above: it rises from a value of $k/(k + m/s)$ left of ℓ, when marketing capital is extremely abundant to the point of being worthless, to a value of 1 for $k/m \geq u$ when marketing capital has to be formed by new entries. The effective value of this Tobin q at any time does not signal the need to invest more or less intensively, as it does, for instance, in the installation-cost approach to the investment schedule. Only when q reaches the extreme values which we just indicated does that event signal the formation or abandonment of marketing capital.

FIGURE 7 GOES HERE

If we regard q as the price of an average share, we may be interested in its volatility. In the absence of entry costs q would always have been equal to 1, with a volatility of zero therefore. In the presence of entry costs, the depreciation of marketing capital relative to productive capital, when the former is abundant, introduces some movement in the price of a share. When computing the elasticity of the q function numerically, one finds that it is very close to 0 for values of k/m close to u, then rises to a maximum of 0.048 as k/m decreases, to finally come down to 0.032 if k/m is close to ℓ.[42] Since the volatility of k, in our example, is equal to 50% per unit of time, we find that the quantity stickiness associated with setup costs on the consumption side, and the resulting hysteresis phenomenon, raise the volatility of the price of a share from 0 to approximately 2.4% per unit of time. We have seen above that setup costs -- even small ones -- have a sizable impact on physical quantities such as the flow of consumption but their impact on the price of equity measured in units of productive capital is not nearly as large. The reason is that marketing capital remains a comparatively small component of total capital.[43]

5.6.4

6.2. The price of goods

The price of final consumption is given by the marginal utility of consumption $m^{\gamma-1}$ divided by the marginal utility of productive capital J_k. This ratio is represented in figure 8 as a function of the capital mix k/m. One sees that the price of final consumption is an increasing function of k/m (or a decreasing function of m/k): the more the economy consumes, relative to its size, the less dear that consumption is per unit.

FIGURE 8 GOES HERE

It should be remembered that consumption takes place out of productive capital (see equation (2)),[44] so that the "factory cost" of each unit of consumption is equal to 1. Figure 8 shows[45] that the price of final consumption takes on the value of 1 for some value of k/m which we denote $\hat{x}$, equal to 41.43. Not surprisingly, this number is close to the capital/consumption ratio $x^* = 40$ which would be chosen in the economy without setup costs. When $k/m > \hat{x}$, the price is larger than 1: there is a scarcity of marketing capital and the value of marketing (or distribution) services is factored into the consumer price which is therefore larger than pure factory cost. There is a (variable) markup.

When $k/m < \hat{x}$, however, there is an excess of marketing capital and the price of consumption is less than 1: firms maintain consumption at the current level only in order to preserve the market at its current size and to keep marketing capital alive. To entice individuals to consume more than their current wealth alone would warrant, firms must charge a consumer price below factory cost. Legally, this is a case of dumping but it is not one which arises from the desire to preserve a monopoly position. Rather, the reason for it is that, when the stock of marketing capital is excessive, the services of marketing capital take on a negative market value.[46] Although our setup is different, this rationale is strongly reminiscent of the one proposed by Ethier (1982), who introduced dumping as a phenomenon which occurs in response to demand uncertainty when constraints created by labor contracts eliminate a channel of adjustment through layoffs.

Figure 8 contains also a comparison of the price of goods for two levels of underlying risk σ. On the left-hand side of the figure is the price curve corresponding to $\sigma = 0.4$ (while the curve on the right-hand side is valid for $\sigma = 0.5$). This comparison leads to the following observations:

-as σ rises (from 0.4 to 0.5), the entry price rises (from 2.68 to 3.07) and the exit price falls (from 0.62 to 0.58) so that the range of allowable price fluctuations widens as volatility increases;

-in figure 8, the k/m ranges corresponding to $\sigma = 0.4$ and $\sigma = 0.5$ do not overlap.[47] But the $\sigma = 0.5$ curve is unambiguously to the right of the $\sigma = 0.4$ curve. Hence, had the two values of σ been sufficiently close to permit an overlap of the k/m ranges, it would also have been true unambiguously that, for any given (m, k) state of the economy, an increase in underlying volatility lowers the price which is charged for consumption.[48]

-the lower price charged for any given state of the economy does not automatically translate into a lower price to be observed most of the time: the economy we have been examining is not stationary and one could not define an average (i.e., unconditionally expected) price. But, in the numerical example, we have established that the economy is a contracting one. It follows from this that the economy should spend "most of the time" in the neighborhood of the exit points. As we saw above, an increase in σ lowers the exit price. We can therefore expect that a higher σ will indeed translate into a lower price of consumption to be observed most of the time.

6.3. Pass-through and consumer price volatility

We now examine the impact of a given productivity shock on consumption prices in order to address the questions of pass through and commodity price volatility. The answers to these questions depend on where the economy currently is: it can be sufficiently inside the cone of no action for the given productivity shock to leave unchanged the amount of marketing capital or or it can be sufficiently close to one of the edges for the shock to cause firms to enter or leave.

Inside the cone, a positive productivity shock ($dz > 0$) increases k relative to a fixed m, creating a shortage of marketing capital, so that the price of consumption goods rises, as figure 8 demonstrates. A negative productivity shock has the exact symmetric effect. This is a case of negative passthrough since a positive productivity shock results in a higher price for consumers. The result is in sharp contrast with Dixit (1987b) who found a passthrough equal to zero when the number of foreign firms currently in the market does not change. The difference between the two results is a difference between partial-equilibrium and general-equilibrium frameworks. Dixit postulated that consumption was driven by a (linear) purely price sensitive excess demand function. If the number of foreign firms does not change, the supply is fixed so that the price cannot move. In our setting the entire industry is explicitly subject to setup costs so that consumption inside the cone is constrained to be equal to the existing stock of marketing capital: supply therefore is also fixed, as in Dixit. But demand in a general equilibrium setting is not only price sensitive, it is also related to wealth and, in our case, to its composition. In equilibrium, following a productivity shock, consumption is unchanged while the price of consumption is the result of the new composition of wealth as between productive and marketing capital.

The elasticity of the price-of-consumption schedule is related to "the" wealth elasticity of demand and is always close to the value $1 - \gamma$ (equal to 2 in our numerical example).[49] The volatility of the price of consumption is therefore approximately equal to $(1 - \gamma)\sigma$, or, in our numerical example, 100% per unit of time. This very large volatility is entirely attributable to setup costs since, in their absence, there would have been no price volatility whatsoever. Setup costs which were found to impart only a very small volatility on the price of equity do impart a large volatility on the price of the flow of consumption.[50]

Close to the edges of the cone,[51] productivity shocks have an asymmetric effect on the price of consumption. When k/m is close to u, a positive productivity shock ($dz > 0$) triggers firm entries and leaves the price unchanged: zero passthrough materializes. A negative productivity shock, however, has the same effect as it does inside the cone: it causes a fall in the price. When k/m is close to ℓ, the opposite is true: a negative productivity shock causes firms to exit and leaves the price unchanged (while a positive shock causes it to rise). By way of contrast, Dixit found close to 100% passthrough in circumstances where firms enter or leave.[52]

7. Conclusion

This general-equilibrium model of an economy with setup costs which are present on the consumption side has produced a number of interesting dynamic features. Consumption behaves sluggishly and remains constant for random periods of time, because firms are reluctant to incur the marketing investment needed to enter the market and are then also reluctant to exit from the market, as this would mean abandoning the marketing capital which may later be useful again. This is the interpretation given here and in Dixit (1987a, b) of the phenomenon of hysteresis identified by Baldwin and Krugman (1986). Technically, marketing capital provides an option to entrenched firms. This option is not easily abandoned.

The existence of setup costs profoundly affects the behavior of the aggregate economy. It has been found here, as in Dixit (1987a), that an economy with even very small setup costs can exhibit a sizable hysteresis phenomenon. The presence of risk modelled by a Brownian motion accounts for this result.

Furthermore, an economy with well-specified parameter values, which would otherwise be unambiguously an expanding or a contracting one at all times, assumes a new bifurcating behavior pattern. If its capital mix happens to fall above some critical level, the economy is expanding (and has a better than even chance of remaining so in the future); otherwise, it is contracting.

Setup costs also impart interesting features on the prices which would prevail in a decentralized version of this economy. The price of final consumption in particular is shown to be sometimes above and sometimes below the factory cost of producing the goods. The reason is that the services of marketing capital are factored into the final consumer price. When the stock of marketing capital is excessive, relative to that of productive capital, those services take on a negative value and the final price is below factory cost. This gives the appearance of dumping.

Generally, setup costs seem to generate on the part of firms behavior patterns which have so far been ascribed to a noncompetitive attitude. These include reluctance to enter or leave a market, markups above factory cost and, in some instances, pricing below cost. It is conceivable that setup costs might provide a simpler model of these behavior patterns than do theories of oligopoly.

In the international context -- which was after all the original motivation of the present line of attack (Baldwin (1986), Baldwin-Krugman (1986)) -- researchers have often noted, more or less casually, that deviations from the Law of One Price may arise from the local content of the goods delivered to the final consumer, e.g., point-of-sale services, information services etc. Dealerships, and the difficulty of setting them up, have been mentioned as explaining that the price of a BMW may not be the same in Germany and in the United States. To our knowledge, these ideas have not been formalized in a general-equilibrium way.

The present framework can provide the basis for such a formalization. What is needed is an extension of the present model to two markets, with investment into marketing capital taking place in both. One could also distinguish two categories of investors, based on the market to which they have access and two categories of firms, based on their locus of production (which could alternatively be an object of choice).[53] The purpose would be to obtain the dynamic behavior of the difference in the price of final consumption between the two markets and also to examine the impact of the relative size of marketing capital in place in the two markets, on the trade balance between them.[54] In addition, when two types of firms are introduced, the reluctance to abandon a market, which we have exhibited, should generate interesting dynamics in their relative market shares.

The possibility that hysteresis on the trade side might feed back into a more volatile exchange rate has been suggested by Krugman (1988)) and examined (in the context of a macroeconomic model) by Baldwin and Lyons (1988). Formally we have no exchange rate in the present model so that we cannot address the issue directly. But we can offer the following conjecture. If the exchange rate in any given model plays mostly the role of a relative price of goods, hysteresis in trade may very well cause it to be very volatile. If, however, the exchange rate has all the characteristics of a relative price between long-lived assets, then our model augurs poorly of the possibility that setup costs imposed on physical flows would account for a high degree of volatility.

FOOTNOTES

[1]A contrast must be drawn between perishable investment, in the sense used here, and irreversible investment which has been the subject of much attention in the recent literature (McDonald and Siegel (1986), Pindyck (1988), Bertola (1987)). The investment we consider here is indeed irreversible, in that its salvage value upon abandonment is zero. But perishable investment has an added feature: it goes up in smoke if unused. This second aspect has not so far been incorporated in the literature dealing with irreversible investment. For this reason, there has been no need in this literature to model a decision to abandon the investment: since its salvage value was zero, it never was optimal to abandon it, after it had been made. McDonald and Siegel (1985), in a separate article considered in isolation the decision to shut down a project. Brennan and Schwartz (1985) modelled a gold mining venture which could be opened and closed at will. While the investment was shut down, however, it was still standing and able to be reused. The difference between this and an investment which goes up in smoke is only a matter of degree: in Brennan and Schwartz' analysis a cost had to be incurred afresh to re-open the mine. Similarly, in Bentolila and Bertola (1988), a firm makes decisions to hire and fire workers, each time incurring a setup cost. None of the models mentioned is a general equilibrium model but the Brennan-Schwartz model being an arbitrage model is, by construction, compatible with general equilibrium provided only that the process for the underlying price variable (the price of gold) is itself so compatible.

[2]Another example is human capital. The model of this paper is cast in terms of a marketing capital interpretation: the capital in question will be considered in use if and when a flow of consumption is distributed to final consumers. But the model could be re-cast in terms of a human capital interpretation, keeping in mind that skills learned by a worker often go wasted if and when he does not engage in production.

[3]A model with properties similar to the present one but based on consumer habit formation has been developed by Sundaresan (1988). In the forties, the macroeconomic model of Duesenberry (1949) incorporated a "ratchet effect" on the consumption side. This effect, however, was operative in one direction only: consumers were reluctant to reduce consumption but felt no hindrance to increase it.

[4]The marketing capacity interpretation of accumulated entry costs has already been proposed by Foster and Baldwin (1986).

[5]In other words, every good at the customer level has a "local content" viz. the value of the distribution services which are built into the product. I make use of one alleged property of the technology which produces those services, to generate a new form of aggregate dynamics in the economy. The "local-content" idea has already been exploited in the international trade literature, but in a static setting, by Sanyal and Jones (1982). In their model, only middle products are traded and some transformation process is needed to generate final goods which are not traded internationally.

[6]There is no claim made where that all marketing investment has the perishability property, only that some does. For instance, it must be the case frequently that connections, made with middlemen to sell one product, are lost once the flow of business between the firm and the middlemen has been interrupted for some time. I.e., the same connections cannot simply be reactivated later in order to start selling the product again. Costs will have to be incurred to re-establish the old connections and to create new ones.

In the realm of advertising, a similar phenomenon occurs as a result of customers' imperfect information and imperfect memory. A promotion campaign is effective for the purpose of selling a product made available immediately thereafter. Once the customers know the product, sales, to some extent, can be maintained, even if the level of advertising is reduced. But, if the product is withdrawn and customers no longer get to purchase and use the product, the initial promotion effort has no lasting effect; it must be exerted anew, if and when the firm later (because costs or prices have changed) wishes to re-introduce the product. (In fact, it is more likely that upon re-entry the firm would introduce another product. In that case, it is obvious that a new promotion must be launched. It is an excessive simplification to consider entry and exit decisions in isolation. One should concurrently examine decisions to switch from one product to another.)

[7]To illustrate this (perhaps obvious) point, consider the closely related model of Froot and Klemperer (1988). It is a two-period model where, because of switching costs on the part of customers, second-period demand for a firm's product depends on first-period market share. Froot and Klemperer show that exchange rate changes which businessmen expect to be temporary produce a lower pass-through of the exchange rate into goods prices than permanent changes do.

In fact exchange rates, being asset prices, follow a quasi-martingale process, which means that for the larger part exchange rate changes must be assumed to be permanent. Furthermore, the quasi-martingale process they follow must exactly reflect what businessmen around the world do.

[8]A recent revival of growth theory, pioneered by Romer (1986) and others (see the survey by Romer 1988)), has lead to the introduction of increasing returns and externalities between firms. The purpose of that specification is to explain a number of stylized facts of secular growth which have been observed over time and comparatively across developed and less developed countries. One of these facts is that different countries seem to grow persistently at different rates. There appear to exist several possible equilibrium growth paths for an economy.

[9]That issue is not clearly settled by Dixit. E.g., do local firms also incur a set-up cost when they enter? The only thing which is specified about the agents on the home side is that they enforce the exogenously set selling price. This implies an infinite elasticity of their reaction.

[10]The reader will remember the difference between the two Dixit articles: in (1987a) the optimal behavior of one firm is studied whereas in (1987b) the number of foreign firms which decide to enter the home market is endogenized.

PRICE OF CONSUMPTION REL. TO PROD. CAP.

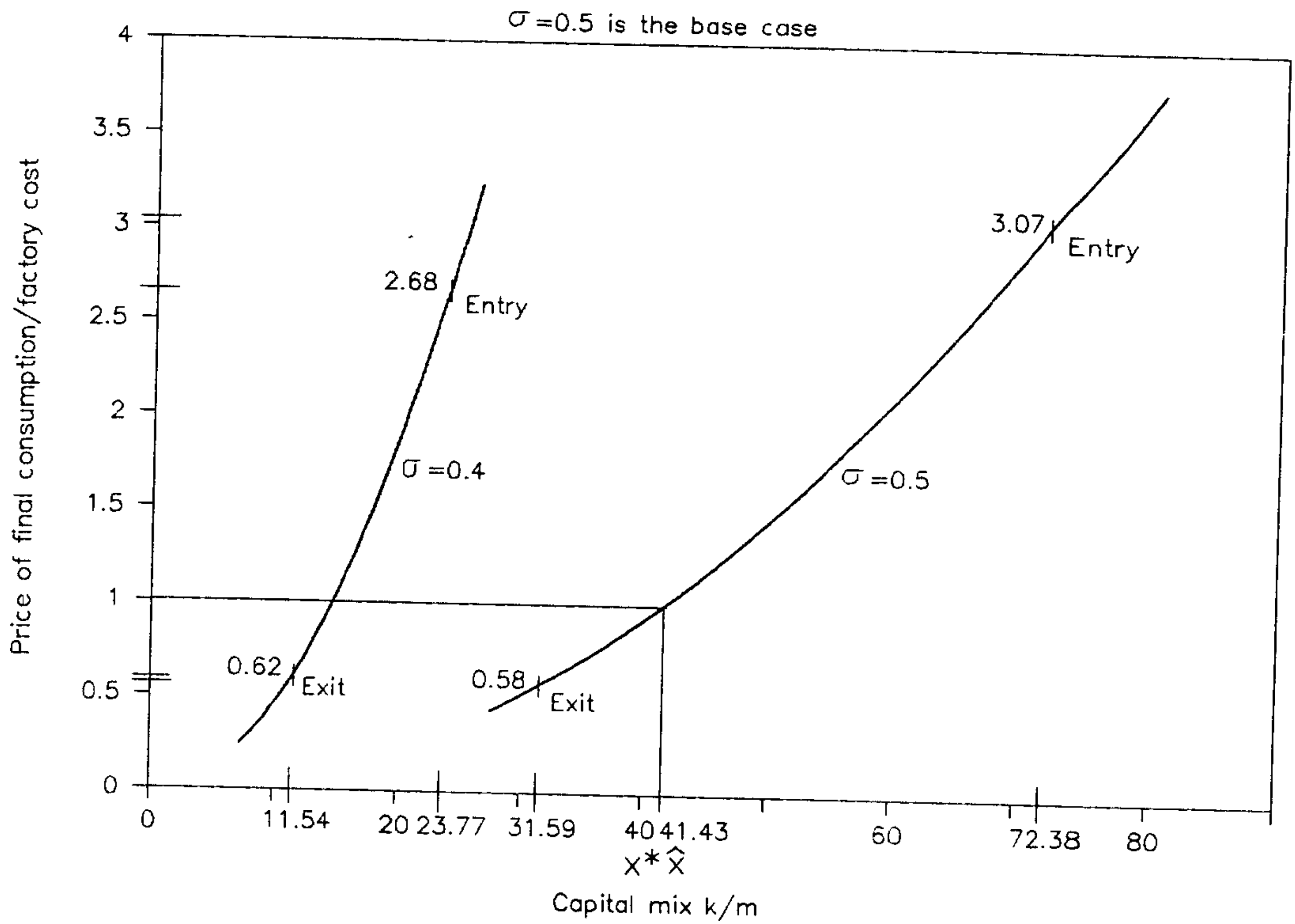

Figure 8

[11]In addition, m is treated here as a real variable, whereas Dixit's number of firms n was restricted to being an integer.

[12]If we had several markets and several products, we could still allow marketing expenditures to be tied to a specific product.

[13]We assume that there is only one such connected area.

[14]Stochastic differential equations are truly a notational convenience. Only their integral counterparts are well-defined. The "impulses" dL and dU must be interpreted as potentially taking finite values (when a discrete jump occurs).

[15]The proof is available form the author on request. See also Dumas (1988b).

[16]The following formulation assumes that the initial conditions m, k satisfy:

$$\ell < x = k/m < u$$

Otherwise, it is optimal instantaneously and immediately at time 0 to give a discrete "impulse" dL or dU.

[17]This line of reasoning is borrowed from Bertola (1987). We, of course, systematically verify that the restriction on parameters is satisfied every time we investigate new parameter values, as we do in section 5 below.

[18]Reminder: under parameter restriction (5) above, the integral converges.

[19]Actually this property obtains before as well as after any optimization.

[20]For the definition and derivative properties of confluent hypergeometric functions see Abramowitz and Stegun (1965), chapter 13, or Erdelyi (1953), chapter VI, or Slater (1960).

[21]Both roots are real. One is positive and the other is negative.

[22]These second-degree conditions have been labelled "super-contact" conditions in Dumas (1988a).

[23]Reminder (from Merton (1971)):

$$\frac{c^*}{k} = \frac{\delta - \gamma(\alpha - \frac{1}{2}(1 - \gamma)\sigma^2)}{1 - \gamma} .$$

[24]This was of course the purpose of characterizing the growth behavior of the economy in terms of the time change in lnk rather than the relative change dk/k.

[25]so that $\alpha - m/k - \frac{1}{2}\sigma^2$ is a monotonically increasing function of k.

[26]The possibility of cycling is remote as a cyclical pattern can only result from a chance succession of random shocks working against the displacement of the probability masses.

[27] If:

$$\frac{c^*}{k} = \frac{\delta - \gamma(\alpha - \frac{1}{2}(1 - \gamma)\sigma^2)}{1 - \gamma} > \alpha - \frac{1}{2}\sigma^2 ,$$

an economy without setup cost is contracting. In the opposite case it is expanding. When equality prevails, it is stationary. In no case is the nature of the economy dependent on the current state.

[28] Romer (1988), page 76.

[29] Romer (1988), page 81.

[30] I am grateful to K. Froot for bring this explanation to my attention.

[31] Refer to footnote #23 which gives the value of c^*/k and note that hedging is "normal" in this case because our base parameter values include a value of γ which is negative (risk aversion larger than 1). "Reverse hedging" would have obtained for $\gamma > 0$. We do not examine that possibility.

[32] The calculations for the case $\sigma = 0$ cannot be carried out on the basis of the equations given so far which become degenerate. A separate analytical development is needed; it will be made available by the author on request. The numerical values of u and ℓ given in figure 3 for $\sigma = 0$ are based on that separate analytical treatment.

[33] This is largely due in this particular economy to the fact that the line $c^*/k = [\delta - \gamma(\alpha - \frac{1}{2}(1 - \gamma)\sigma^2]/(1 - \gamma)$ happens to be parallel to the line $\alpha - \frac{1}{2}\sigma^2$. The choice of parameter value $\gamma = -1$ is responsible for this occurrence. For other choices of parameter values but keeping $\gamma < 0$, it is quite conceivable for increased risk to turn a contracting economy into an expanding one because increased risk discourages consumption. When $\gamma > 0$ the opposite is true.

[34] resulting from a succession of favorable output shocks arising by chance.

[35] Please refer again to footnote #23 which gives the formula for c^*/k. The reader is reminded that the rate of relative risk aversion, which is chosen equal to the base case value of 2 in these calculations, plays a double role as the rate of intertemporal substitution.

[36] Contrary to appearances, the curves for the upper and the lower boundaries in figure 5 are curves, not straight lines.

[37] This number is supplied by the intersection of the $\alpha - \frac{1}{2}\sigma a^2$ line with the $1/\ell$ curve. See figure 5.

[38] Naturally there would be no effective trading in those markets since all individuals are identical. The prices we are about to examine are shadow or virtual prices. This first line of attack on the problem of asset price determination is in the tradition of Lucas (1978).

[39]All prices are homogeneous functions (of degree 0) in m and k.

[40]Reminder: $x^* = 40$ for the base-case parameter values.

[41]which is an option to expand;

[42]When $k/m = \ell$ and a negative productivity shock ($dz < 0$) occurs, firms abandon marketing capital. When they do, the price q is stabilized at its existing level. A positive productivity shock, however, would have reduced the price.

[43]If, however, the price of equity had been measured in units of consumption -- which may seem more natural from a portfolio investment point of view -- the conclusion would have been markedly different, as will be apparent below.

[44]I.e., firms which distribute dividends have this much less productive capital to produce from.

[45]At this point we refer to the graph on the right-hand side of figure 8 which corresponds to the base case $\sigma = 0.5$.

[46]As we saw above, the stock of marketing capital itself would still be valued positively: the value of a flow can only derive from its immediate social worth but the value of a stock may also derive from its future social worth.

[47]These ranges are (11.54 to 23.77) in the case $\sigma = 0.4$ and, as we know, (31.59 to 72.38) in the case of $\sigma = 0.5$.

[48]This result is similar in spirit to the one obtained by Froot and Klemperer (1988) in a partial-equilibrium setting reflecting switching costs incurred by customers.

[49]The sensitivity of price to shocks (the degree of passthrough) is not very much dependent on the amount of risk present in the economy.

[50]The implication is that, if we had measured the price of equity in units of consumption, rather than in units of productive capital, a fairly high volatility would have been obtained for that price.

[51]i.e., when the distance to the boundary is of the same order of magnitude as the size of the shock.

[52]On the supply side, Dixit introduced a collection of firms, each with a different variable cost of production. In that setting, when new firms enter they are relatively high cost firms. This explains that the passthrough is not exactly 100%.

[53]as in Dumas (1988a).

[54]One will then be in a position to address the Japan-U.S. current trade issue.

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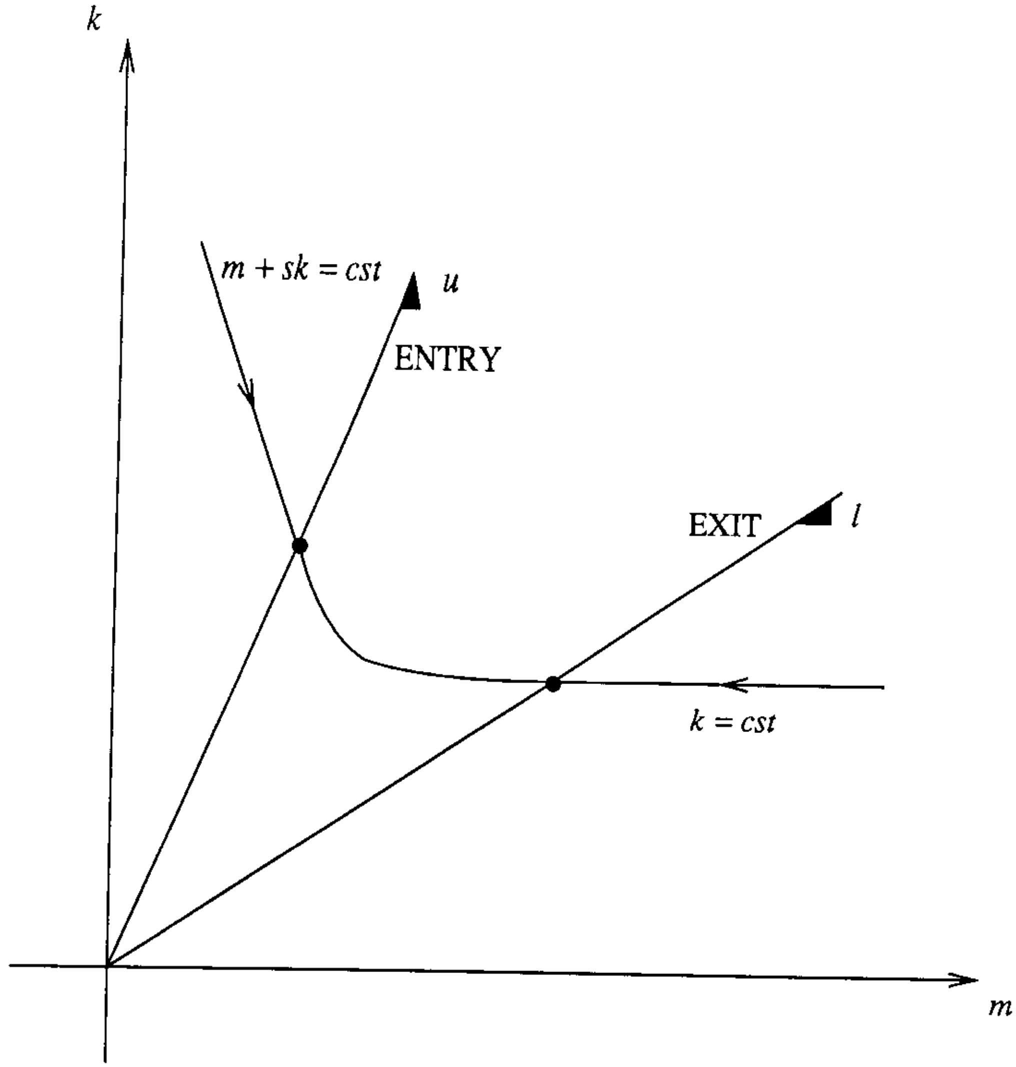


$J(m, k; l, u) = cst$ INDIFFERENCE CURVE

Figure 1

THE EFFECT OF ENTRY COSTS

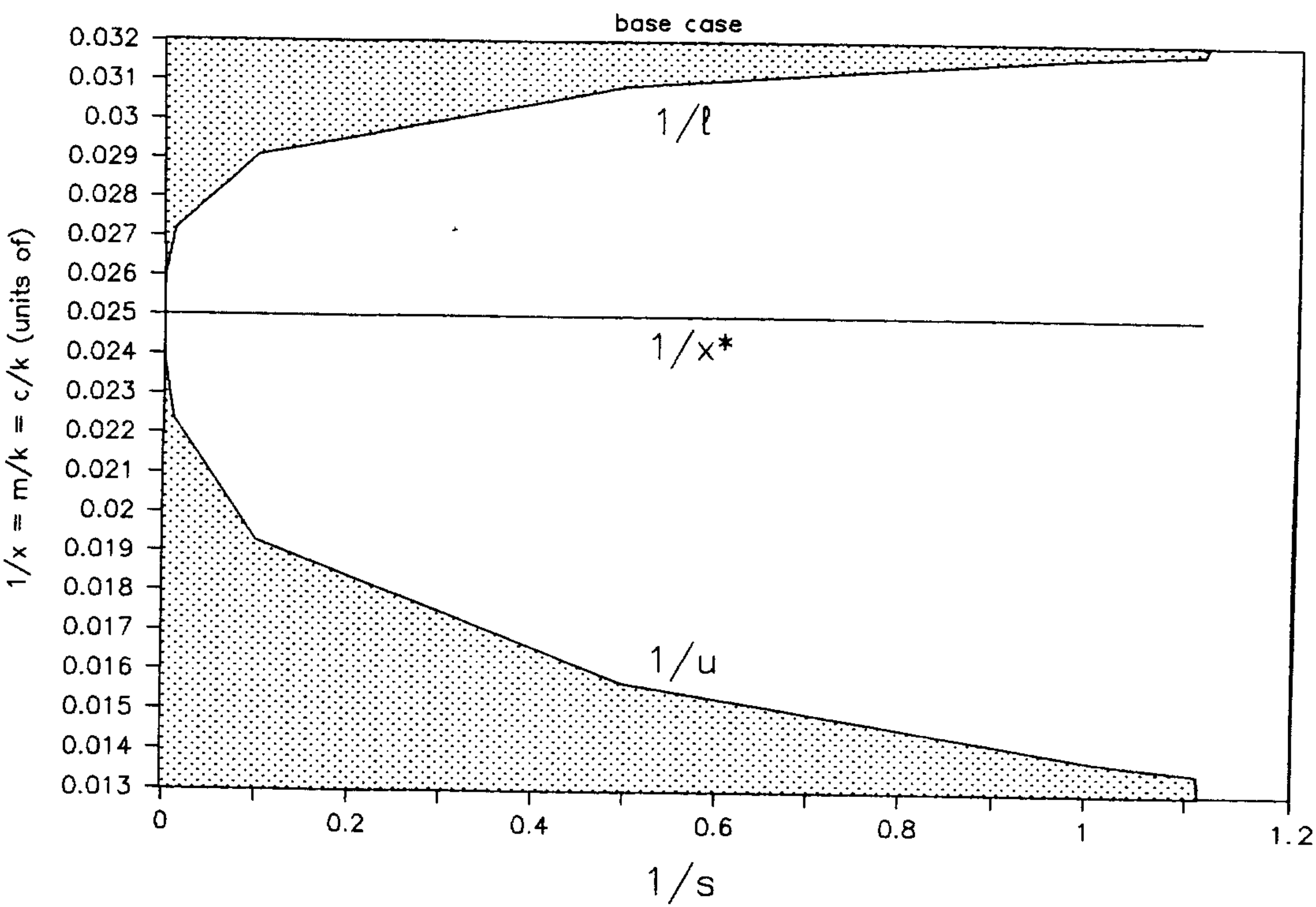
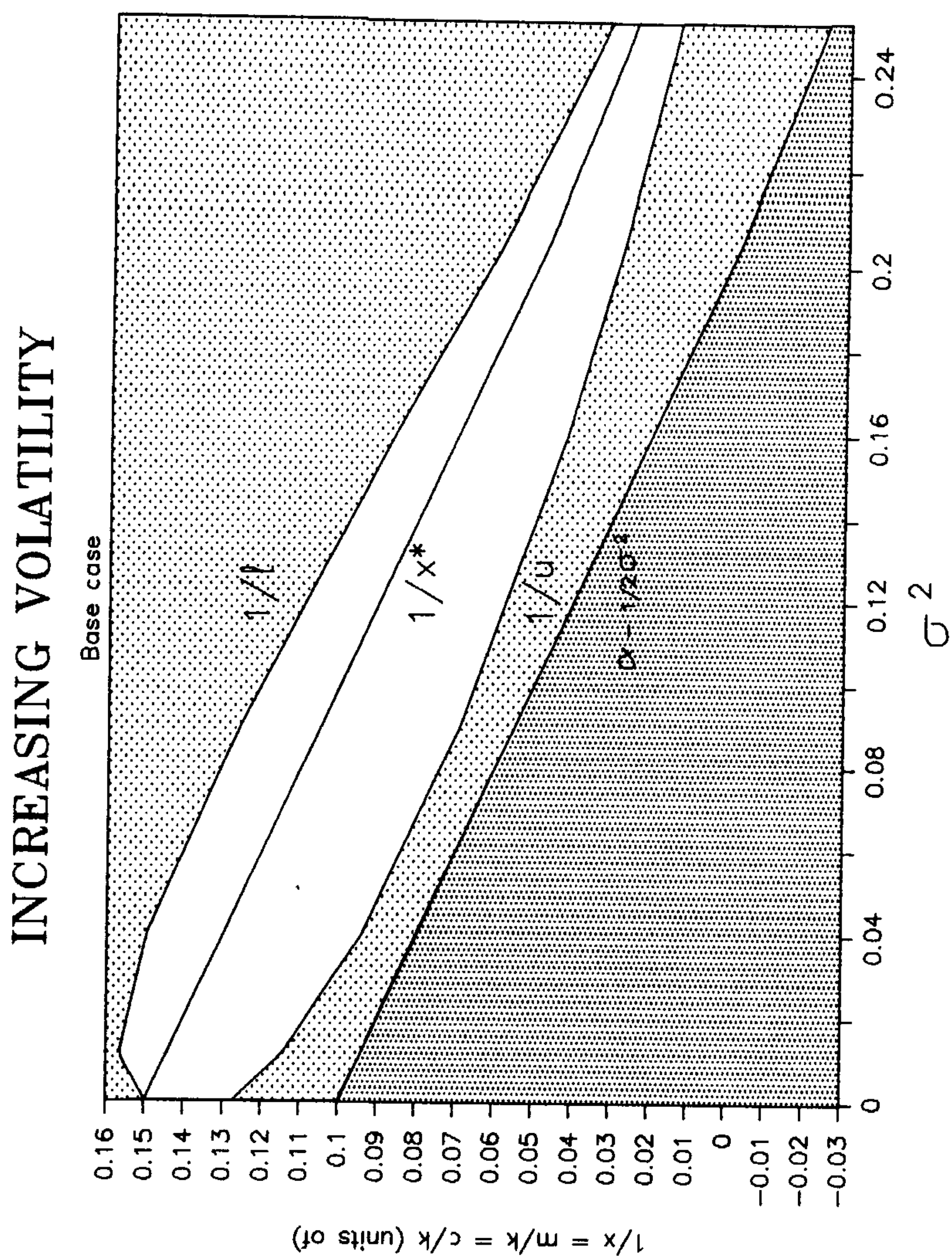


Figure 2



INCREASING VOLATILITY
Base case
1/l
1/x*
1/u
α − 1/2σ²
1/x = m/k = c/k (units of)
σ²
0.16
0.15
0.14
0.13
0.12
0.11
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
−0.01
−0.02
−0.03
0
0.04
0.08
0.12
0.16
0.2
0.24

Figure 3

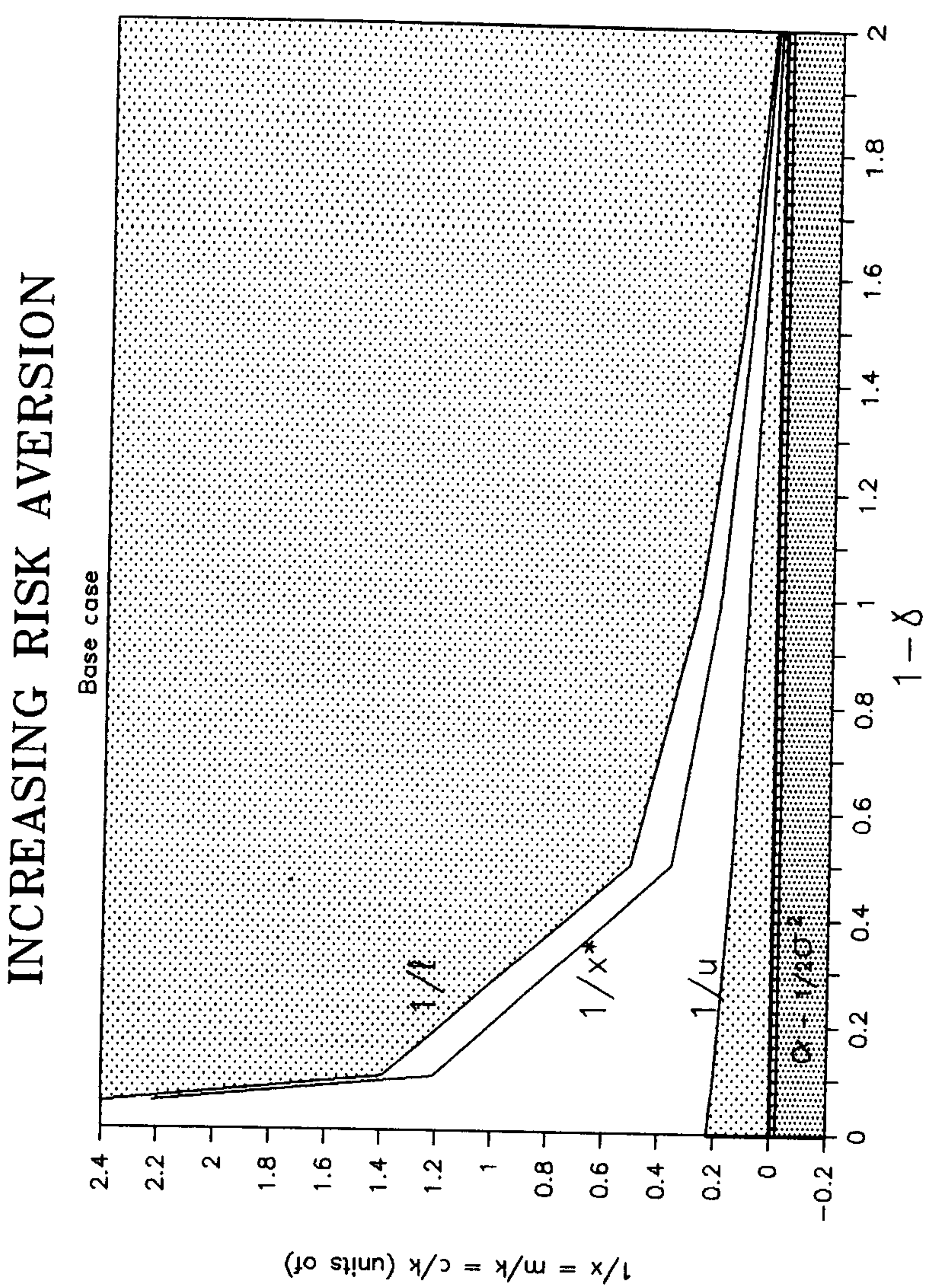

INCREASING RISK AVERSION
Base case
1/ℓ
1/x*
1/u
α − 1/2σ²
1 − δ
1/x = m/k = c/k (units of)
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
−0.2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

Figure 4

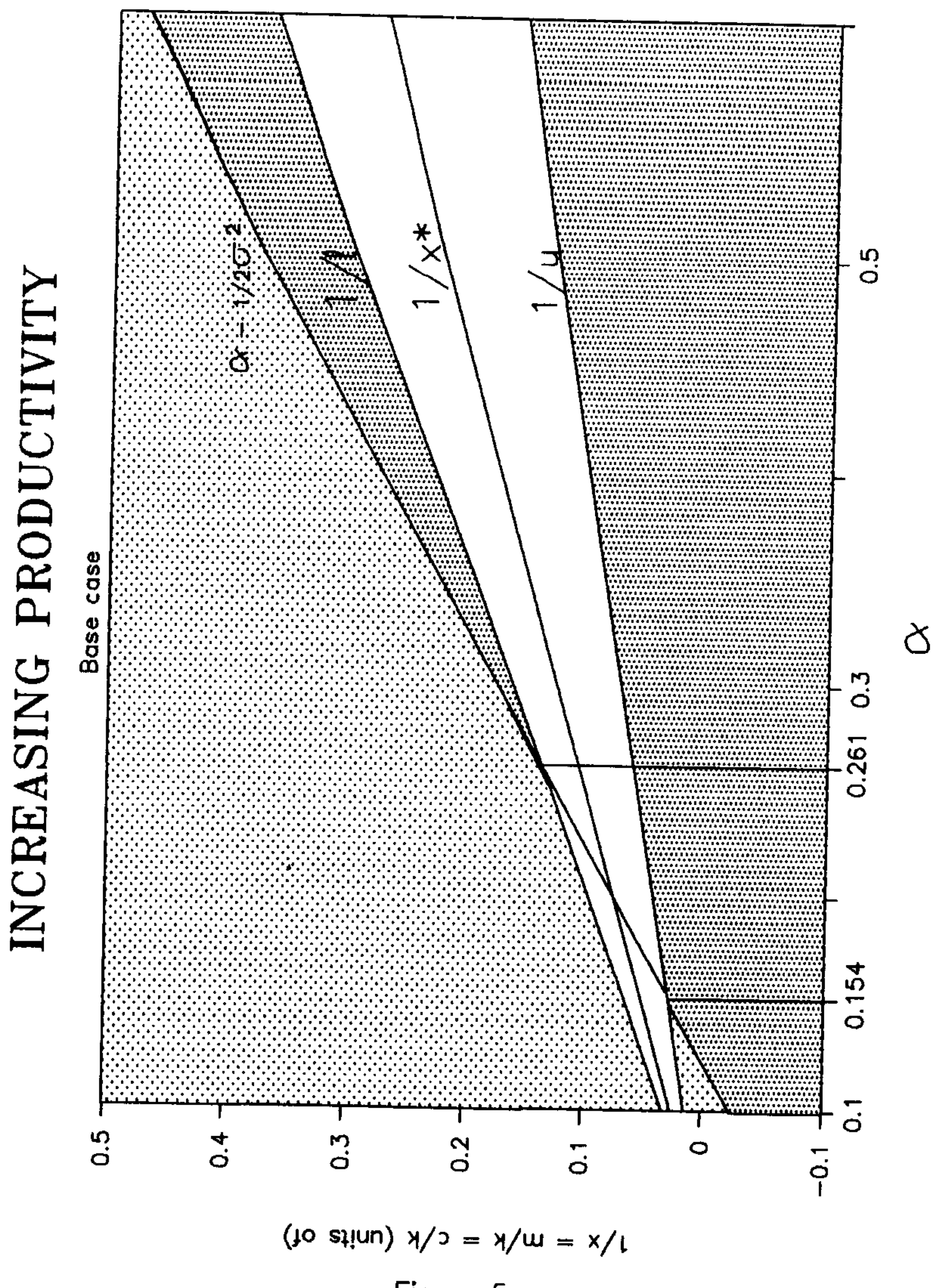
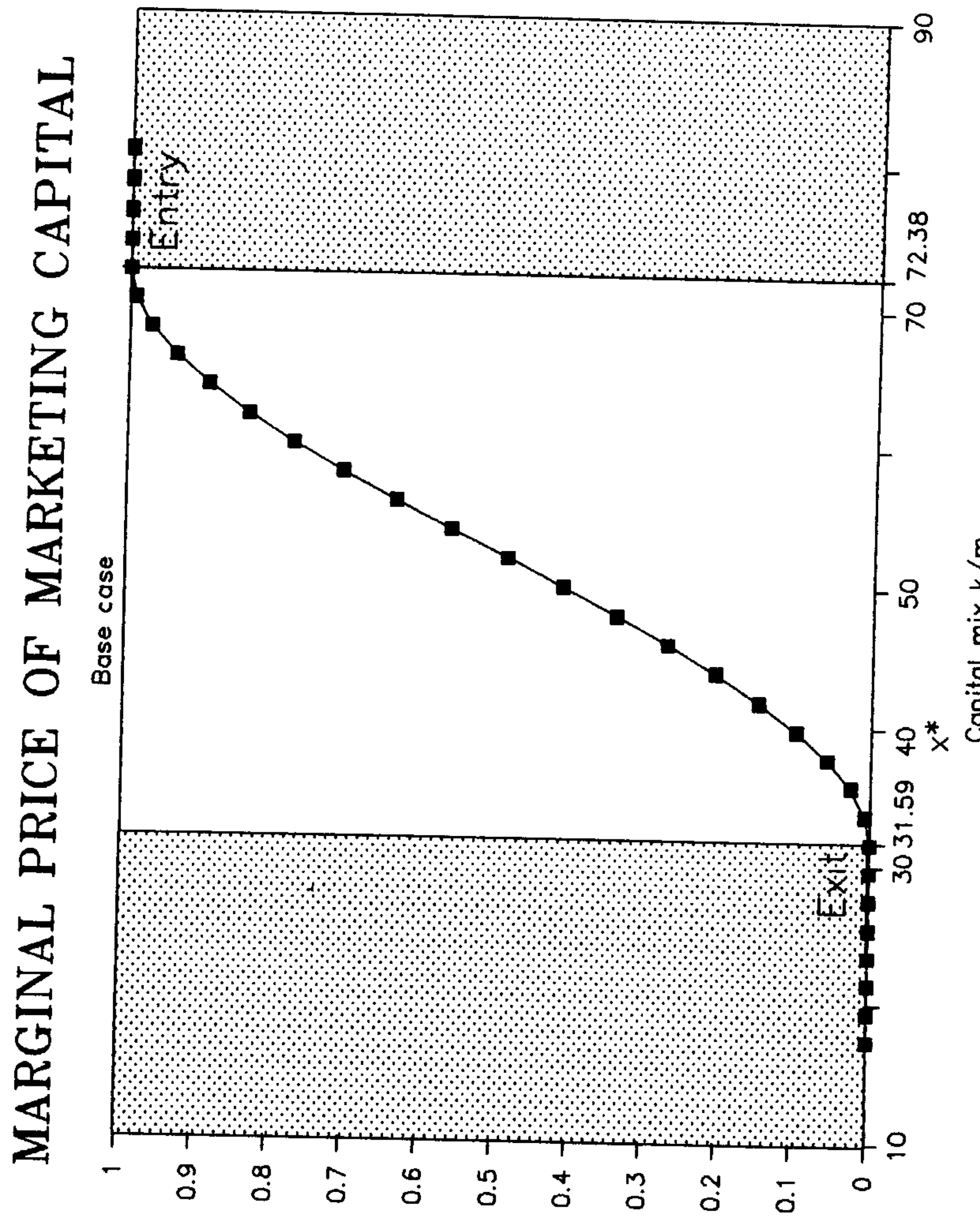

INCREASING PRODUCTIVITY
Base case
α − 1/2σ²
1/x*
1/x = m/k = c/k (units of)
α
0.5
0.4
0.3
0.2
0.1
0
−0.1
0.1
0.154
0.261
0.3
0.5

Figure 5



MARGINAL PRICE OF MARKETING CAPITAL
Base case
Entry
Exit
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
10
30
31.59
40
50
70
72.38
90
x*
Capital mix k/m

Figure 6

VALUE OF TOTAL CAPITAL/REPLACEMENT COST

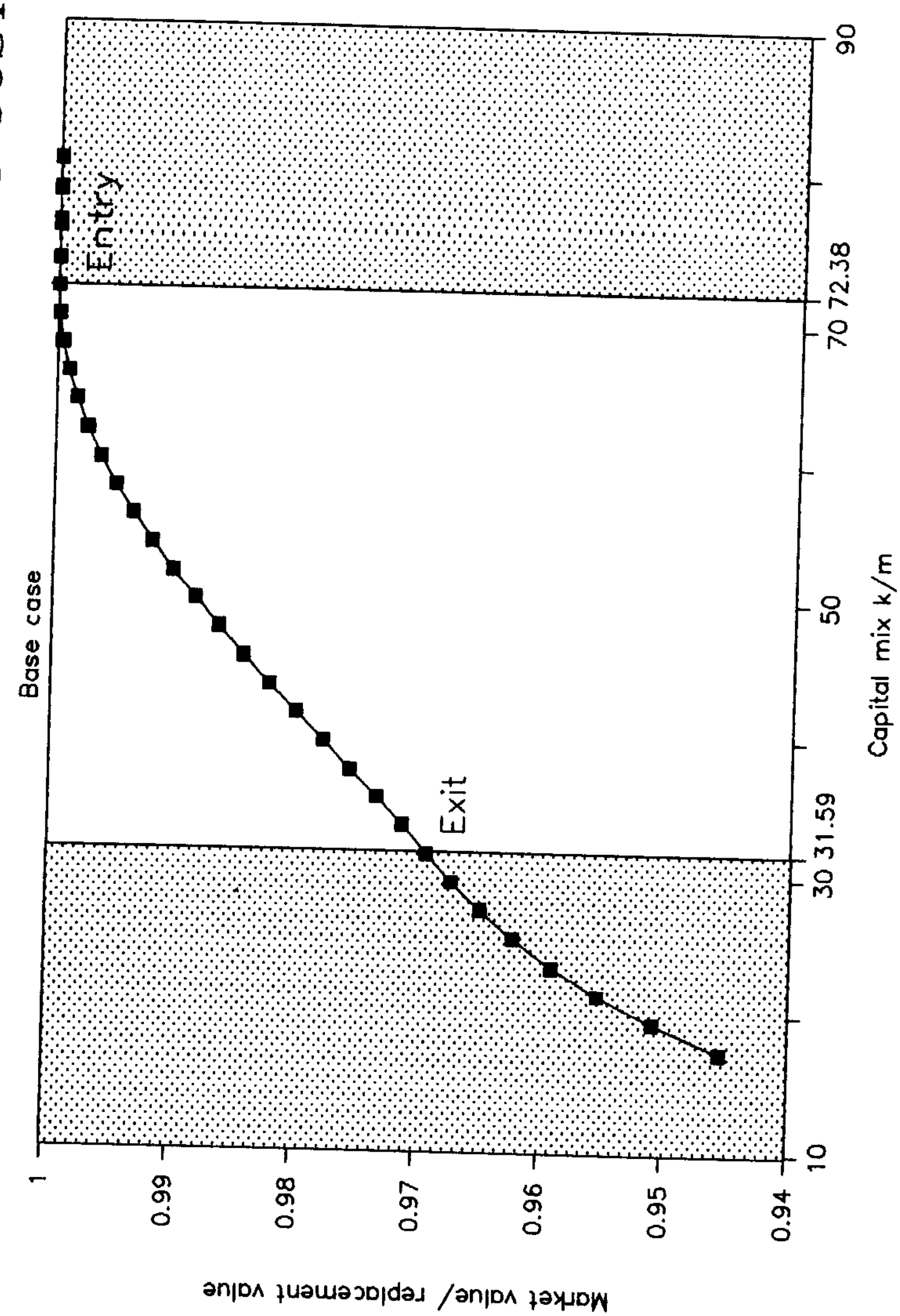


Figure 7