LINEAR TRANSFORMATION OF ASSET RETURNS AND THE APT

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Linear Transformation of Asset Returns and the APT

Abstract

The capital market is abound of mergers, spin-offs, sell-offs, and construction of mutual funds. All these activities impose linear or nonlinear transformations on the return generating process. The validity of the APT under linear transformations of asset returns has been discussed but not fully explored in the literature. The purpose of this paper is to examine the robustness of the APT with respect to arbitrary linear transformations. We show that the APT holds under any linear transformation as long as the product of the transformation matrix and its transpose is uniformly bounded.

I. Introduction

The capital restructuring process in business, such as merger and spin-off, and the construction of mutual funds, gives rise to linear or nonlinear transformation of asset returns in the economy. The validity of the APT with respect to arbitrary linear transformations of asset returns caused many concerns but has not been fully explored in the literature. For example, Dybvig and Ross (1985) made the following observation:

> Many people we have talked with have thought that the APT "should" be robust to linear transformations or else its validity would be affected by mergers or spinoffs. However, mergers and spin-offs are not arbitrary transformations; they represent the adding together of returns of two stocks or the separation of a single security into two parts. Neither case is likely to affect the validity of the APT. This is in contrast to the type of extreme transformation required for Shanken's argument, which might create a new security which is long 1000 shares of the security one and short one share each of 999 other securities. There is no reason to require or expect the APT's distributional assumptions to be robust to such transformations. (p. 1179)

In this paper, we will demonstrate that the validity of the APT should not be impaired by arbitrary linear transformations as long as the product of the transformation matrix and its transpose is uniformly bounded. Our proof is first based on a simple but rather restrictive linear factor structure adopted in Ross (1975, 1976) and Huberman (1982). We then demonstrate the robustness of the APT in a much more general framework which includes the circumstances where the asset returns are weakly dependent and/or their second moments do not exist. Specifically, we will either assume the sequence of the idiosyncratic risks is a lacunary system of order p for some $p \geq 1$ or assume it is a convergence system in order to suit for different needs for alternative definitions of asymptotic arbitrage.[1]

Note that this paper considers a market in which a countable number of assets are traded. We normalize the price system by assuming each asset costs one dollar. The assets are arranged in a sequence. We often look at what happens to various objects as n increases to infinity.

The rest of the paper is organized as follows. The robustness of the APT in a basic model is demonstrated in Section II. In Section III, the robustness of the APT with respect to arbitrary linear transformations is examined in a generalized model where the sequence of the idiosyncratic risks is a lacunary system of order p for some $p \geq 1$. The conclusion in Section III can be used to prove several interesting results in the theory of arbitrage pricing. In Section IV, we derive results similar to those in Section III with the assumption that the sequence of the idiosyncratic risks is a convergence system. The last section concludes the paper.

II. The Robustness of the APT in the Ross-Huberman Model

II.1. The Ross-Huberman Model

The basic model of the linear K-factor structure upon which Ross (1975, 1976) and Huberman (1982) demonstrated an approximate linear pricing relation as the result of market arbitrage activities is summarized in equation (1).

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[1]. The APT is composed of three basic elements: the stochastic structure of asset returns, the no-arbitrage condition, and the approximate linear pricing relation. There is a trade-off between the linear factor structure and the no-asymptotic-arbitrage condition that keeps the approximate linear pricing relation intact. Wang and Lee (1988) and Lee and Wang (1988) establish the validity of the APT when the sequence of the idiosyncratic risks is a lacunary system and convergence system respectively.

$$\mathbf{x} = \mathbf{a} + \mathbf{B}\ \mathbf{f} + \mathbf{e} \qquad \text{(1a)}$$ [2]

$$E(\mathbf{e}) = \mathbf{0} \qquad \text{(1b)}$$

$$E(\mathbf{f}) = \mathbf{0} \qquad \text{(1c)}$$

$$E(\mathbf{ff}') = \mathbf{I}_K \quad \text{(Identity matrix of rank K)} \qquad \text{(1d)}$$

$$E(\mathbf{ef}') = \mathbf{0} \quad \text{(n x K matrix)} \qquad \text{(1e)}$$

$$E(\mathbf{ee}') = \mathbf{D} \quad (\mathbf{D} \text{ is a diagonal matrix}) \qquad \text{(1f)}$$

$$1_i'\mathbf{D}1_i < \sigma^2 < \infty \quad \text{for all i, where } 1_i \text{ is the } i^{th} \text{ column of an identity matrix with rank n.} \qquad \text{(1g)}$$

The vectors **x**, **a**, and **e**, each of which is n x in dimension, respectively represent the realized returns, expected returns and (nonobservable) "residual" portions of the returns. The residual e_i measures the uncertainty unexplained by the common factors and is known as the idiosyncratic risk in the literature. The expected return of each individual asset, a_i, is assumed to be bounded. **f** is a K x vector of nonobservable values of the common factors; the second moment of f_k is assumed to exist for all k. **B** is the n x K matrix of bounded factor loadings, i.e., $|b_{ik}| < \infty$ for all i and k. The elements in the n x n diagonal matrix of variance-covariance, **D**, are assumed to be bounded.

Conditions (1b), (1c), (1e), and (1f) are rather innocuous. They are merely normalization conditions and do not impose any real restriction on the structure of asset returns. Condition (1f) implies uncorrelated idiosyncratic risks and condition (1g) imposes a bound on variances of asset returns.[3] In

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[2] The variables, **x**, **f**, and **e**, are random. The values of **x**, **a**, **B**, **f**, and **e** all depend on n. To avoid messy presentations, we omit the notation for randomness and the sequential indices.

[3] Wang and Lee (1988) relaxed these two conditions in their generalization of the APT.

this section, we will first show the robustness of the APT in the basic Ross-Huberman model. More general results concerning top the relaxation of (1f) and (1g) will be given in the following two sections.

II.2. Robustness of the APT

Consider an arbitrary linear transformation of asset returns in a linear factor structure:

$$\mathbf{x}^T = \mathbf{a}^T + \mathbf{B}^T\mathbf{f} + \mathbf{e}^T \qquad (2)$$

where $\mathbf{x}^T = \mathbf{Hx}$, $\mathbf{a}^T = \mathbf{H}\mathrm{a}$, $B^T = \mathbf{HB}$, $\mathbf{e}^T = \mathbf{He}$, and $\mathbf{H}$ is an m x n matrix. Proposition 1 shows that the APT holds for the transformed asset returns $\mathbf{x}^T$ with sufficient restriction on $\mathbf{H}$.

Proposition 1: Suppose that the original linear factor structure (1) holds. Consider the above transformed linear structure (2) with the product of the transformation matrix and its transpose, $\mathbf{HH}'$, being uniformly bounded ($\mathbf{H}$ is m x n in dimension). If, for some portfolio $\mathbf{w}$ (where $\mathbf{w}' = (w_1, w_2, \ldots, w_n)$, w_i is the proportion of the wealth invested in i^{th} asset; $\mathbf{1}$ is a column vector of n 1s) of the transformed assets, $\mathbf{w}'1 \to 0$ and $\mathbf{w}'\mathbf{x}^T - \mathbf{w}'\mathbf{a}^T \xrightarrow{L_p} 0$ imply $\mathbf{w}'\mathbf{a}^T \to 0$ (no asymptotic arbitrage condition), then there exists a vector $\mathbf{c} = (c_0, c_1, \ldots, c_k)$ such that

$$\lim_{m\to\infty}\|\mathbf{a}^T - \mathbf{B}^{T*}\mathbf{c}\|^2 = \lim_{m\to\infty}(\mathbf{a}^T - \mathbf{B}^{T*}\mathbf{c})'(\mathbf{a}^T - \mathbf{B}^{T*}\mathbf{c}) < \infty$$

where $\mathbf{B}^{T*} = (\mathbf{1} \;\; \mathbf{B}^T)$.

Proof: It is easy to find out that

$$\mathbf{X}^T = \mathbf{a}^T + \mathbf{B}^T\mathbf{f} + \mathbf{e}^T$$

$$E(\mathrm{e}^T) = 0,\ E(f) = 0,\ E(ff') = I,\ E(\mathbf{e}^T\mathbf{f}) = 0.$$

$$E|\mathrm{d}'\mathbf{e}^T|^2 \equiv E|\mathbf{d}'\mathbf{He}|^2 \le \sigma^2|\mathbf{d}'\mathbf{HH}'\mathbf{d}| \le \sigma^2|g_1\mathbf{d}'\mathbf{d}| \le \delta|\mathbf{d}'\mathbf{d}|, \text{ for all d,}$$

where $\delta \equiv g_1\sigma^2$ and g_1 is the largest eigenvalue of $\mathbf{HH'}$ and is bounded since $\mathbf{HH'}$ is uniformly bounded. The rest of the proof is similar to Theorem 1 of Ingersoll (1984). Q.E.D.

III. The Case of Lacunary System

III.1. Nonexistence of Second Momement in Asset Returns and The Cross-sectional Dependence of Idiosyncratic Risks

Fama (1965) tested the normality hypothesis on the daily returns of the Dow Jones Industrial stocks. The result revealed more kurtosis (fatter tails) than that predicted from a sample of independent and identically distributed normal variates. This signifies that the second moments of asset return random variables may not exist. The same result can be found in numerous empirical researches. Moreover, there is no compelling reason to assert that the idiosyncratic risks in the asset market are uncorrelated. Thus, conditions (1f) and (1g) in the original Ross-Huberman model may be too stringent.

Wang and Lee (1988) establish the validity of the APT under a more general setting which includes the circumstances where the idiosyncratic risks are weakly dependent and/or the second moments of asset returns do not exist. In this section, we demonstrate that the robustness of the APT assuming a linear factor structure where the sequence of idiosyncratic risks is a lacunary system of order p for some $p \geq 1$. This also covers "approximate factor structure" in Chamberlain and Rothschild (1983) and "uniform boundedness" of variance-covariance matrix of idiosyncratic risks of Ingersoll (1984) as special cases.

III.2. The Lacunary System

Before showing the robustness of the APT with respect to arbitrary transformations in a general setting, we summarize some of the important results in Wang and Lee (1988) which are needed in showing our general result concerning the robustness of the APT to arbitrary linear transformation.

Definition: Given $p > 0$, a sequence of real-valued random variables $\{e_i\}$ is called a **lacunary** system of order p, or an **S_p system**, if there exists a positive constant K_p such that for any sequence of real constants d_i,

$E|\Sigma_{i=1}^{n} d_i e_i|^p \leq K_p(\Sigma_{i=1}^{n} d_i^2)^{p/2}$ for all $n \geq 1$.

If the system $\{e_i\}$ is an S_p system for every $p > 2$, then it is called an **S_∞ system**. ∎

The lacunary system has properties that are, in some sense, similar to properties of systems of independent variables. Probably we may call it a weakly dependent system. If $\{e_i\}$ is a lacunary system, then $\Sigma_{i=1}^{\infty} d_i e_i$ is called a lacunary series. This concept is discussed in detail in Gaposhkin (1966) and Lai and Wei (1983). The lacunary system is fairly general. For example, let $\{e_i\}$ be sequence of i.i.d. standard normal random variables, then $\{e_i\}$ is an S_p system. As another example, the sequence of idiosyncratic risks in the approximate factor structure in Chamberlain and Rothschild (1983) is an S_2 system. The random elements in a lacunary system are neither necessarily uncorrelated, nor do they necessarily possess second moments. Please see Wang and Lee (1988) for more examples.

The following definition is a generalization of the usual no arbitrage condition in terms of convergence in quadratic mean to one in terms of convergence in p^{th} mean.

Definition: There are no **asymptotic-arbitrage opportunities in terms of convergence in p^{th} mean** if the following condition holds: for all $\mathbf{w} \in R^n$,

$$w'1 \dashrightarrow 0, \text{ and } w'x - w'a \ {-L_p}{\rightarrow} \ 0 \implies \mathbf{w}'\mathbf{a} \dashrightarrow 0,$$

where $-L_p\!\!\rightarrow 0$ indicates the convergence to zero in p^{th} mean.[4] ∎

By relaxing the restriction on the linear factor structure and using the concept of asymptotic arbitrage in terms of convergence in pth mean, Wang and Lee (1988) proved a generalized version of the APT. Their result is reiterated in Lemma 1.

Lemma 1: Suppose the linear K-factor structure holds and the sequence of idiosyncratic risks is a lacunary system of order p for some $p \geq 1$. If there are no asymptotic arbitrage opportunities in terms of convergence in p^{th} mean, then there exist a column vector $\mathbf{c}' = (\ c_0\ c_1\ c_2\ \ldots\ c_K\ )$ such that $\lim_{n \to \infty} \|\mathbf{a} - \mathbf{B}^{*}\mathbf{c}\| < \infty$, where $\|\ \|$ denotes the Euclidean norm.

Proof: See Wang and Lee (1988). Q.E.D.

It can be shown that the results about the linear pricing relations in Ingersoll (1984) and Chamberlain and Rothschild (1983) are special cases of the above lemma.[5]

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[4] The condition is similar to Condition (Ai) of Chamberlain and Rothschild (1983):

$$\mathbf{w}'1 \dashrightarrow 0, \text{ and } Var(z) = \mathbf{w}'\Sigma\mathbf{w} \dashrightarrow 0 \implies E(z) = \mathbf{w}'\mathbf{a} \dashrightarrow 0,$$

where z is the random return of the portfolio $\mathbf{w}$, and Σ is the variance-covariance matrix of $\mathbf{e}$. When $p = 2$, they are exactly the same.

[5] Ingersoll (1984) assumes that the variance-covariance matrix of the residual risks is uniformly bounded. Chamberlain and Rothschild (1983) and Chamberlain (1983) assume that only K eigenvalues of the returns variance-cova-

Lemma 2: Given that **e** is an S_p ($p \geq 1$) system, e^T = **He** is also an S_p ($p \geq 1$) system if **H** is m x n in dimension and **HH'** is uniformly bounded.

Proof: Since **e** is an S_p system, there exists a positive constant K_o such that for any sequence of $\{d_i\}$, $E|\mathbf{d'e}|^p \leq K_o|d'd|^{p/2}$.

$$E|d'He|^p \leq K_o|\mathbf{d'HH'd}|^{p/2} \leq K_o|g_1\mathbf{d'd}|^{p/2} \leq K_1|\mathbf{d'd}|^{p/2},$$

where $K_1 = K_o g_1{}^{p/2}$ and g_1 is the largest eigenvalue of **HH'** and is bounded since HH' is uniformly bounded. Therefore, e^T is an S_p system. Q.E.D.

III.3. The Robustness of the APT

Proposition 2 shows that the APT remains valid with respect to the transformed asset returns $\mathbf{x}^T$.

Proposition 2: Suppose that the original linear factor structure holds with the exception that $\{e_i\}$ is an S_p system for some $p \geq 1$. Consider the transformed linear model (2) with the product of the transformation matrix and its transpose, **HH'**, being uniformly bounded (**H** is m x n in dimension). If, for some portfolio **w** of the transformed assets,

$$\mathbf{w'1} \to 0 \quad \text{and} \quad \mathbf{w'x}^T - \mathbf{w'a}^T \xrightarrow{L_p} 0 \quad \text{imply} \quad \mathbf{w'a}^T \to 0,$$

then there exists a vector $\mathbf{c} = (c_0, c_1, \ldots, c_k)$ such that

$$\lim_{m \to \infty}\|\mathbf{a}^T - \mathbf{B}^{T*}\mathbf{c}\|^2 = \lim_{m \to \infty}(\mathbf{a}^T - \mathbf{B}^{T*}\mathbf{c})'(\mathbf{a}^T - \mathbf{B}^{T*}\mathbf{c}) < \infty$$

where $\mathbf{B}^{T*} = (\mathbf{1} \;\; \mathbf{B}^T)$.

Proof: e^T is a S_p system with $p \geq 1$ by Lemma 2. Then apply Lemma 1. Q.E.D.

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riance matrix become unbounded. These are equivalent to the assumption of lacunary system if p = 2. See Wang and Lee (1988).

Dybvig and Ross (1985) worried about the effect of transformation on the variance-covariance matrix of asset returns. They argued that:

> The Appendix verifies that either Shanken's transformed variances of the inverse of the transformation blows up. As we have seen in the above example, if variances blow up, then the assumptions used to motivate the APT (such as small idiosyncratic variance) will not be preserved when moving from the original assets to the transformed assets (pp. 1179-1180).

What "variance" means in the above discussion is the norm of the transformed variance-covariance matrix of the returns on the assets (not idiosyncratic risks). In fact, it is perfectly all right to have blown-up-variances (it should be). The assumptions used to motivate the APT will generally remain valid when moving from the original factor structure to the transformed factor structure as long as the product of the transformation matrix and its inverse is uniformly bounded.

III.4. An Example of Application of Proposition 2

Proposition 2 can be invoked to prove other results in the theory of arbitrage pricing. The next corollary is an example.

Corollary 1: Given the K-factor linear model with $E(\mathbf{ee}') = \Omega$ where Ω is the n x n positive definite variance-covariance matrix of the idiosyncratic risks. **(i)** If no arbitrage opportunities in terms of convergence in quadratic mean are available, then there is a sequence of vectors of factor premium **c** and there exists a positive number α such that the weighted sum of the squared pricing errors is uniformly bounded, i.e,

$$(\mathbf{a} - \mathbf{B}^*\mathbf{c})'\Omega^{-1}(\mathbf{a} - \mathbf{B}^*\mathbf{c}) < \infty \; . \qquad (3)$$

(ii) If assets variances are uniformly bounded, then $\mathbf{\Omega}$ may be replaced by the correlation matrix of idiosyncratic risks, **R**, in (3).

Proof: (i) Since $E(d'e^T)^2 = E(\mathbf{d'He})^2 = \mathbf{d'd}$, $\mathbf{e}^T$ is an S_2 system. When $\mathbf{HH'} = \Omega^{-1}$ ($\Omega = E(\mathbf{ee'})$), $\|\mathbf{a}^T - \mathbf{B}^{T*}\mathbf{c}\|^2 = (\mathbf{a} - \mathbf{B}^*\mathbf{c})'\, \mathbf{\Omega}^{-1}\, (\mathbf{a} - \mathbf{B}^*\mathbf{c})$. The result follows immediately.

(ii) Let $\mathbf{HH'} = \mathbf{R}^{-1}$. $(\mathbf{a} - \mathbf{B}^*\mathbf{c})'R^{-1}(\mathbf{a} - \mathbf{B}^*\mathbf{c}) \le m < \infty$ if R is positive definite, since $\mathbf{HH'}$ has to be uniformly bounded.

Note that $R = D_\Omega^{-1/2}\Omega D_\Omega^{-1/2}$ where D_Ω is the diagonal matrix of idiosyncratic-risk variances constructed from the main diagonal of $\mathbf{\Omega}$. **R** is positive definite due to the fact that Ω is positive definite and that $\mathbf{D}_\Omega$ is uniformly bounded and positive definite. Q.E.D.

Corollary 1 is exactly the same as Theorem 1 of Ingersoll (1984). Since Ω^{-1} can have negative entries, this is a useless bound. Note that Ingersoll does not assume $\mathbf{\Omega}$ to be uniformly bounded. But if $\mathbf{\Omega}$ is not uniformly bounded, **a** can be "arbitrarily" far away from $\mathbf{B}^*$c, i.e., $(\mathbf{a} - \mathbf{B}^*\mathbf{c})'(\mathbf{a} - \mathbf{B}^*\mathbf{c})$ can be arbitrarily large without violating (3).[6] Thus we should perhaps write $\mathbf{G}'^{-1}\mathbf{a} \approx \mathbf{G}'^{-1}\mathbf{B}^*\mathbf{c}$ instead of $\mathbf{a} \approx \mathbf{B}^*\mathbf{c}$ as in Ingersoll (1984, p.1024), where $\Omega = \mathbf{GG'}$. Furthermore, c is not unique in this case.[7] Theorem 2 of Ingersoll (1984) can be proved by using Proposition 2 in a similar fashion.

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[6] This argument does not apply to Proposition 2, since in Proposition 2 we are concerning about the pricing of the transformed assets. However, here, we are dealing with the original assets.

[7]

Suppose $\mathbf{\Omega}$ is not uniformly bounded and let $\mathbf{\Omega} = \mathbf{GG'}$. Since $\|\Omega\|$ is unbounded, $\|G\|$ is also unbounded and G^{-1} is a singular matrix. The rank of $\mathbf{B}^*$ is K+1, however the rank of $G^{-1}\mathbf{B}^*$ will probably be less than K+1. c is then not unique.

IV. The Case of Convergence System

The above result can be extended to the case where the sequence of the idiosyncratic risks is a **convergence system**. This is a suitable structure for the idiosyncratic risks especially if we define asymptotic-arbitrage in terms of **almost sure convergence**. The idea of convergence system may be traced to Gaposhkin (1966). The reason to consider this type of no-arbitrage condition and the application of the convergence system to the generalization of the APT can be found in Lee and Wang (1988).

Definition: Suppose that $\{e_i\}$ is a sequence of random variables satisfying the following condition:

$\Sigma_{i=1}^{\infty} d_i e_i$ converges almost surely for all real sequences $\{d_i\}$ such that $\Sigma_{i=1}^{\infty} d_i^2 < \infty$ (i.e., $\{d_i\} \in l_2$).

Then it is called a **convergence system**. If any rearrangement of the system $\{e_i\}$ is a convergence system, then $\{e_i\}$ is called an **unconditional convergence system**. ∎

Definition: There are no **asymptotic-arbitrage in terms of almost sure convergence** if the following condition holds: for all $\mathbf{w} \in R^n$,

$\mathbf{w}'\mathbf{1} \to 0$, and $\mathbf{w}'\mathbf{x} - \mathbf{w}'\mathbf{a} \to 0$ a.s. (i.e., $\mathbf{w}'\mathbf{x} - \mathbf{w}'\mathbf{a}$ converges almost surely to 0) $\Longrightarrow$ $\mathbf{w}'\mathbf{a} \to 0$. ∎

We now show that the APT is still correct after arbitrary linear transformation as long as the product of the transformation matrix and its transpose is uniformly bounded. First, we need the following result.

Lemma 3: If $\{e_i\}$ is a convergence system and if $\mathbf{HH}'$ is uniformly bounded, then e^T = $\mathbf{H}e$ is also a convergence system.

Proof: Similar to Lemma 2. Q.E.D.

Lemma 4: Suppose the linear K-factor structure holds and the sequence of idiosyncratic risks is a convergence system. If there are no asymptotic arbitrage opportunities in terms of almost sure convergence, then there exist a column vector $\mathbf{c}' = (c_0\ c_1\ c_2\ \ldots\ c_K)$ such that $\lim_{n \to \infty} \|\mathbf{a} - \mathbf{B}^{*}\mathbf{c}\| < \infty$, where $\| \ \|$ is the Euclidean norm.

Proof: See Lee and Wang (1988). Q.E.D.

Corollary 2: Suppose the sequence of idiosyncratic risk, $\{e_i\}$, is a convergence system. Consider a transformed linear factor structure where the product of the transformation matrix and its transpose is uniformly bounded. If for all portfolio $\mathbf{w} \in \mathbf{R}^n$, $\mathbf{w}'1 \to 0$ and $\mathbf{w}'\mathbf{x}^T - \mathbf{w}'\mathbf{a}^T \to 0$ a.s. imply $\mathbf{w}'\mathbf{a}^T \to 0$, then the approximate linear pricing relation holds, i.e., $\lim_{m \to \infty} \|\mathbf{a}^T - \mathbf{B}^{T*}\mathbf{c}\| < \infty$.

Proof: Use Lemmas 3 and 4. Q.E.D.

V. Conclusion

The Arbitrage Pricing Theory has generated wide interests in scholarly pursuit since its introduction in mid 70's. It represents one of the major attempts to surmount the problems with testability and the anomalous empirical findings that have plagued other theories. Understanding the generality of this theory would help researchers to formulate empirical studies and to apply this theory to solve practical problems. This paper resolve one of the major concerns about the generality of the APT: would the APT be robust to arbitrary linear transformations in asset returns? We show that the APT holds under any linear transformation as long as the product of the transformation matrix and its transpose is uniformly bounded.

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