# PORTFOLIO PERFORMANCE EVALUATION: OLD ISSUES AND NEW INSIGHTS

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Revised June 1988

Comments Welcome
215-898-4184

*We are especially grateful to Michael Brennan for helpful discussions and comments on earlier drafts. We also wish to thank Anat Admati, Thomas Copeland, Brad Cornell, Phillip Dybvig, David Hirshleifer, Jonathan Ingersoll, Ronald Masulis, Krishna Ramaswamy, Richard Roll, Stephen Ross, Walter Torous, Brett Trueman, Robert Verrecchia, Arthur Warga, an anonymous referee, and seminar participants at Chicago, Michigan, Northwestern, Wharton, UCLA/USC, and the European Finance Association for their comments. Sheridan Titman gratefully ackowledges financial support from the Batterymarch Fellowship program.

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# PORTFOLIO PERFORMANCE EVALUATION: OLD ISSUES AND NEW INSIGHTS


## Abstract

This paper presents a simple model that provides insights about various measures of portfolio performance. The model explores three criticisms of these measures: (i) the inability to identify an appropriate benchmark portfolio; (ii) the possibility of overestimating risk because of market timing ability; and (iii) the failure of informed investors to earn positive risk-adjusted returns because of increasing risk aversion. The paper argues that these are not serious impediments to performance evaluation. In particular, it shows (i) that the appropriate benchmark portfolio is the unconditional mean-variance efficient portfolio of the evaluated investor's tradable assets, even when the investor does not optimally hold the mean-variance efficient portfolio; (ii) that the market timing risk-adjustment problem can be overcome with new measures; and (iii) that informed investors display negative risk-adjusted returns only for pathological preferences that treat risky assets as Giffen goods.

## 1. Introduction

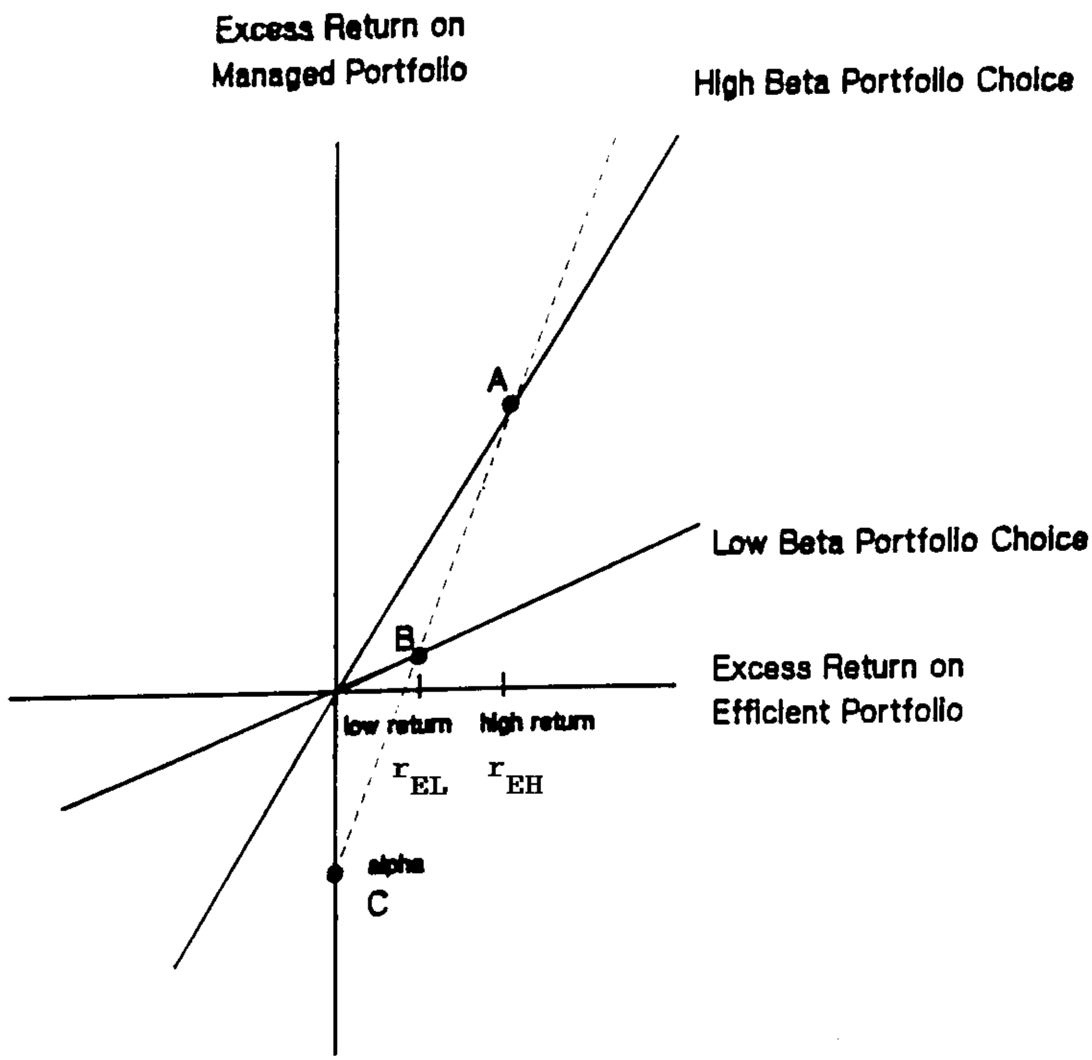
One of the widely held "folk theorems" in finance is that informed investors can achieve a better risk-return tradeoff than uninformed investors. For this reason, all measures of portfolio performance in the academic literature have sought to adjust returns for risk. Risk, however, is difficult to define and measure in markets with asymmetric information, especially when one considers that it must be computed by an uninformed observer.

As a result of this, there has been a great deal of controversy over whether the performance measures proposed by Treynor (1965), Sharpe (1966), and Jensen (1968, 1969) can identify investors with superior information. Jensen's alpha, which measures the deviation of a portfolio from the securities market line, has been the focus of most of the controversy because it is the most widely used in academic empirical studies. We, too, will focus on it, although many of our results could be extended to the measures suggested by Treynor and Sharpe.

One criticism of the Jensen Measure is that it provides an upwardly-biased estimate of the risk of a market-timing investment strategy. That is, the estimated beta of an investor who increases his portfolio beta when he (correctly) forecasts an abnormally high market return and decreases it when he forecasts an abnormally low return will be an inconsistent overestimate of his average risk. Examples provided by Jensen (1972), Admati and Ross (1985), and Dybvig and Ross (1985) demonstrate that an informed investor can display a negative Jensen Measure in large samples because of this.

Diagram 1, which graphs the excess return of the evaluated portfolio (above a risk-free rate) against the excess return of the benchmark portfolio, illustrates the tendency to overestimate beta. The portfolio manager is constrained here to select a high or low beta portfolio, designated by the steeper and gentler-sloped solid lines, respectively. If the benchmark is mean-variance efficient, both of these lines will pass through the origin.

DIAGRAM 1


Excess Return on
Managed Portfolio
High Beta Portfolio Choice
A
Low Beta Portfolio Choice
B
Excess Return on
Efficient Portfolio
low return
high return
$r_{EL}$
$r_{EH}$
alpha
C

The manager is presumed to be able to forecast the benchmark return. He receives one of two signals: that the benchmark excess return will be $r_{EH}$, which is above its unconditional mean, or it will be $r_{EL}$, which is below its mean. If he acts as a market timer, he will select a high beta portfolio, and be at point A upon receipt of the high return signal, and at point B if he receives the low return signal.

An observer, unaware of the market timing of the manager, would estimate the risk of this investment strategy as the slope of the dotted line connecting points A and B. This slope exceeds the risk of the portfolio in either information state, which is represented by the slopes of the solid lines. Moreover, the Jensen Measure, which is the intercept of the dotted line at C, is negative for this example, erroneously indicating that the informed investor is an inferior performer.

Obviously, this result is not robust for all market timers. It is possible to draw the graph differently to generate a positive Jensen Measure. The issue, however, is not the robustness of the negative Jensen Measure. It is that there are plausible scenarios where it can be negative.

The most important contribution of this paper is the development of an alternative measure that has the same data requirements as the Jensen Measure, but which correctly identifies informed investors as positive performers. We call this more reliable measure, "The Positive Period Weighting Measure." In addition to demonstrating that the measure circumvents the timing-related problem discussed above, the paper addresses two other criticisms of the Jensen Measure that may also apply to our new measure.

The most well known of these criticisms was put forth by Roll (1978, 1979), who argued that the Jensen Measure is sensitive to the choice of the benchmark portfolio that is used to compute beta. The CAPM-related empirical anomalies documented in the past decade serve to motivate this criticism. For example, from 1975-84, the smallest 8% of AMEX and NYSE-listed securities outperformed the largest 8% by a statistically significant 12.5% per year after adjusting for risk with a monthly-rebalanced, equally-weighted index. Similar anomalies have been documented for other indexes, other securities characteristics, and other time periods. Since it is undesirable to classify uninformed investors who engage in passive strategies (e.g., buy and hold small firms) as superior performers, index portfolios that yield such "anomalies" are inappropriate as performance benchmarks.

The portfolio of tradable assets that is mean-variance efficient from the perspective of an uninformed observer, by contrast, correctly classifies uninformed investors as zero performers. We demonstrate that under certain conditions, it is the appropriate benchmark portfolio for both Jensen's alpha and our new measure. In contrast to the CAPM benchmark, which requires the observability of all assets, our benchmark may consist of a relatively small set of assets, since it can be limited to those assets that the evaluated investor considers tradable. Moreover, since our analysis allows for non-traded assets, this benchmark is appropriate even in cases where the evaluated investor does not optimally select a portfolio on the mean-variance efficient frontier. In Section 7, we argue that a model with non-traded assets may be particularly appropriate for evaluating the behavior of professional fund managers.

The third line of criticism was raised by Verrecchia (1980), who presented an example where an informed investor realizes average returns below that expected by uninformed observers who know the risk of the portfolio. Since this means that informed investors can realize negative risk-adjusted returns, even when returns are properly adjusted for risk, the example challenges the validity of all measures of portfolio performance. Indeed, Dybvig and Ross (1985) first pointed out that Verrecchia's example applies to the measure of performance proposed by Cornell (1979). The Cornell Measure, which examines the difference between each holding period return of an investor's portfolio and the return realized with the same portfolio in a time period outside of the holding period--and then averages, is also studied here.

In Section 6, we demonstrate that counterexamples in the class described by Verrecchia can only occur when the informed investor has preferences with extreme increasing absolute risk aversion. Because of these extreme preferences, good news about the market makes the informed investor wealthier, and consequently so risk averse that he decreases his beta and hence, times perversely. Since this behavior is probably pathological, we contend that it does not affect the viability of performance measures.

These three criticisms are analyzed within a framework that follows from a decomposition of the Jensen Measure. The decomposition, presented in Section 2, illustrates that the Jensen Measure is the sum of three terms. The "bias-in-beta term" arises only in cases where the estimated beta is an inconsistent measure of the true average beta. The other two terms relate to true performance: The "selectivity term" reflects the ability to select investments that will do well relative to the benchmark portfolio; the "timing term" represents the contribution to performance of the ability to forecast the return of the benchmark portfolio.

In Section 2, we propose two other new performance measures, the Selectivity Measure and the Timing Measure, which are specifically designed to capture the selectivity and timing terms, and which have desirable properties for evaluating performance. These measures require the observation of portfolio holdings, like the Cornell Measure, which captures the sum of the timing and selectivity terms.

Much of the prior research on performance, as well as the research in this paper, can be neatly categorized with this decomposition. For instance, the example at the beginning of the paper, where the Jensen Measure of a market timer is negative, illustrates that the "bias-in-beta" term can be negative and dominate the other two terms. This is the motivation for the Positive Period Weighting Measure, described in Section 4.

The asymmetric information models of Mayers and Rice (1979) and Dybvig and Ross (1985), where informed investors display positive Jensen Measures (when appropriate benchmarks are used), draw conclusions about the sign of the selectivity term when the timing term is zero. Our extensions of their work to models with non-traded assets, in Section 5, also analyze the selectivity term.

The analysis in Section 6 of Verrecchia's example presents results about the sign of the timing term. This section also contains a proof of Cornell's implicit assertion that the measure proposed by him is positive for an informed investor.

The organization of the paper, its main results, and the assumptions needed to prove each result are given in Table One. These assumptions come from the following global set:

1. A risk-free asset exists.
2. Unconditional asset returns are distributed i.i.d. over time.
3. The benchmark portfolio, from the perspective of an uninformed observer, is ex ante mean-variance efficient with respect to the evaluated investor's set of tradable assets.
4. The evaluated investor maximizes the expected utility of his end-of-period tradable plus non-tradable wealth.
5. Returns and information are multivariate normally distributed.
6. The evaluated investor's timing and selectivity signals are independently distributed.
7. The evaluated investor's portfolio beta is a monotonic increasing function of his forecast of the return of the benchmark portfolio.
8. The evaluated investor has non-increasing absolute risk aversion.
9. The evaluated investor has constant absolute risk aversion.
10. All assets that contribute to the evaluated investor's wealth are tradable.

These assumption numbers are referred to in the table.

## 2. The Measures of Performance and Their Decomposition

To measure performance, we observe the excess returns (above the risk-free rate) of an investor's portfolio of N tradable risky assets in each period t,

**TABLE 1**

ORGANIZATION OF PAPER, RESULTS, AND ASSUMPTIONS

| Section and Description | | Main Results for Large Samples | Assumptions |
|---|---|---|---|
| 2 The Decomposition and the Performance of an Uninformed Investor | i) | Various measures of performance can be decomposed into 3 terms. | 1-3 |
| | ii) | (Lemma 1) The bias-in-beta component is zero in the absence of timing ability. | 1-3 |
| | iii) | (Prop. 1) Uninformed investors have zero measures of performance in large samples with either of four measures. | 1-3 |
| 3 The Model of Information | | Not applicable. | |
| 4 The Bias-in-Beta Component and the Positive Period Weighting Measure | i) | The Jensen Measure is a Period Weighting Measure but not a Positive Period Weighting Measure. | 1 & 3 |
| | ii) | (Prop. 2) The Positive Period Weighting Measures of Informed Investors are Positive. | 1-6, & 7 or 8 |
| | iii) | (Prop. A1) Same as (ii). | 1-5 & 9 |
| 5 The Selectivity Component | i) | (Cor. 2) The selectivity component of an informed investor with selectivity information and possibly timing information is positive. | 1-6 |
| | ii) | (Prop. A2) Same as (i). | 1-5 & 9 |
| 6 The Timing Component | i) | (Prop. 3 or 4) The timing component of an informed investor with timing information and possibly selectivity information is positive. | 1-6, & 7 or 8 |
| | ii) | (Prop. A2) Same as (i). | 1-5 & 9 |
| | iii) | (Prop. 5) The Cornell Measure of an informed investor, as well as the sum of his timing and selectivity components, is positive. | 1,2,4,5,8&10 |
| 7 The Appropriate Benchmark Portfolio | | Not applicable. | |

$t = 1,\ldots,T$. A tradable risk-free asset is also assumed to exist. The investor's portfolio weights are assumed to be fixed within each time period, but change from period to period. Let

$\tilde{R}_{jt}$ = excess return of tradable asset j in period t,

$\tilde{x}_{jt}$ = the investor's period t portfolio weight on asset j, and

$\tilde{r}_{pt} = \sum_{j=1}^{N} \tilde{x}_{jt}\tilde{R}_{jt}$ = period t excess return of the investor's portfolio of traded assets;

where $\tilde{x}_{jt}$ is random, since the investor may alter his portfolio in response to (real or imagined) information. Similarly, let $\tilde{r}_{Et}$ denote the period t excess return of the portfolio of tradable risky assets that is mean-variance efficient from the perspective of an uninformed investor. It has population mean $\bar{r}_E$ and variance $\sigma_E^2$.

The excess returns of each asset can then be expressed as

$$\tilde{R}_{jt} = \beta_j\tilde{r}_{Et} + \tilde{\varepsilon}_{jt}, \quad \text{where} \quad \beta_j = \frac{\text{cov}(\tilde{R}_{jt}, \tilde{r}_{Et})}{\sigma_E^2},$$

and where the mean of $\tilde{\varepsilon}_{jt}$ is zero because the benchmark portfolio is mean-variance efficient. This, in turn, implies that the excess return of the investor's portfolio can be expressed as

$$\tilde{r}_{pt} = \tilde{\beta}_{pt}\tilde{r}_{Et} + \tilde{\varepsilon}_{pt}, \quad \text{where} \tag{1}$$

$$\tilde{\beta}_{pt} = \sum_{j=1}^{N} \tilde{x}_{jt}\beta_j \quad \text{and} \quad \tilde{\varepsilon}_{pt} = \sum_{j=1}^{N} \tilde{x}_{jt}\tilde{\varepsilon}_{jt}.$$

The means, variances, and covariances expressed above and throughout the paper are calculated from the perspective of an uninformed observer who, by assumption, views the excess return vector as being drawn from an i.i.d. distribution.[1] These will henceforth be called the unconditional distributions. Although the distribution of asset returns, conditioned on the information signals of informed investors, is nonstationary, the model is consistent with a general equilibrium as long as the effect of informed traders on market clearing prices is insignificant.

The i.i.d. assumption is required because it is otherwise impossible for an uninformed observer to distinguish between performance and changes in the parameters of the return-generating process. Indeed, any taxonomy that distinguishes between these possibilities is necessarily based on the number of investors who are assumed to observe the nonstationarities. In a market with traders who have special information, only a few investors observe the nonstationarities, whereas in a market with nonstationary parameters, virtually all investors observe the nonstationarities (and only the evaluator is naive).

I.i.d. unconditional returns imply that the mean-variance efficient portfolio has constant portfolio weights. They also imply that the beliefs of an uninformed investor, and hence his portfolio weights and portfolio beta, must be independent of the realizations of $\tilde{r}_{Et}$ and each $\tilde{R}_{jt}$. In contrast, an informed investor may change his portfolio weights in response to new information, inducing a nonzero covariance between his portfolio weights and asset returns. In particular, he may increase (decrease) his holdings in assets with increased (decreased) expected returns in order to realize superior investment performance.

In later sections, we demonstrate that the ability of various measures to capture superior performance depends on whether the informed investor's information relates to $\tilde{r}_{Et}$ or to the $\tilde{\varepsilon}_{jt}$'s. For this reason, it is convenient to distinguish between these types of information.

Definition: An investor is said to have timing information if the expected value of $\tilde{r}_{Et}$, conditioned on his information, changes from period to period.

Definition: An investor is said to have selectivity information if the expected value of $\tilde{\varepsilon}_{jt}$, conditioned on his information, changes from period to period for at least one asset.

These definitions of timing and selectivity are closely related to their common usage by investment professionals. The two types of information will be explicitly modelled in Section 3.

### a. The Decomposition of the Jensen Measure and its Relation to Other Measures of Abnormal Performance

This subsection examines the large sample value (or probability limit) of the Jensen Measure in order to derive a "decomposition" that simplifies and synthesizes our analysis. We do this because it is easier to first obtain results about the elements of the decomposition and then piece the results together than it is to obtain results about the Jensen Measure directly. Moreover, since other measures capture portions of the decomposition, these results have implications for other performance measures.

The Jensen Measure is

$$J = \hat{r}_p - b_p \bar{r}_E \,, \quad \text{where} \tag{2}$$

$b_p$ = the probability limit of the least squares slope coefficient obtained by regressing the time series of excess returns of the evaluated portfolio against the time series of excess returns of the efficient benchmark portfolio, and

$\hat{r}_p$ = the probability limit of the sample mean of $\tilde{r}_{p1}, \tilde{r}_{p2}, \ldots, \tilde{r}_{pT}$.

In general, we denote

$$\hat{q} = \text{plim}\left(\frac{1}{T} \sum_{t=1}^{T} \tilde{q}_t\right)$$

as the limiting sample mean of a sequence of random variables $\tilde{q}_1, \ldots, \tilde{q}_T$. Wherever necessary, this probability limit is assumed to exist.

The Jensen Measure can be decomposed into a (large sample) bias-in-beta term, a timing term, and a selectivity term. Using equation (1), the limiting large sample mean of the excess return of the portfolio can be expressed as

$$\hat{r}_p = \text{plim}\left(\frac{1}{T} \sum_{t=1}^{T} (\tilde{\beta}_{pt} \tilde{r}_{Et} + \tilde{\varepsilon}_{pt})\right) ,$$

$$= \hat{\beta}_p \bar{r}_E + \text{plim}\left(\frac{1}{T}\sum_{t=1}^{T} \tilde{\beta}_{pt}(\tilde{r}_{Et} - \bar{r}_E)\right) + \hat{\varepsilon}_p . \qquad (3)$$

Substituting equation (3) into equation (2) yields the decomposition:

$$J = (\hat{\beta}_p - b_p)\bar{r}_E + \text{plim}\left(\frac{1}{T}\sum_{t=1}^{T} \tilde{\beta}_{pt}(\tilde{r}_{Et} - \bar{r}_E)\right) + \hat{\varepsilon}_p . \qquad (4)$$

The three terms in equation (4) will be referred to as follows:

$(\hat{\beta}_p - b_p)\bar{r}_E$ = component of abnormal performance due to large sample biases in estimated beta as a measure of average risk,

$\text{plim}\left(\frac{1}{T}\sum_{t=1}^{T} \tilde{\beta}_{pt}(\tilde{r}_{Et} - \bar{r}_E)\right)$ = component of abnormal performance due to timing, and

$\hat{\varepsilon}_p$ = component of abnormal performance due to selectivity.

If the weights of the evaluated investor's portfolio are observable, the elements of the decomposition can be separately identified. For instance, Cornell (1979) proposed a measure that averages the difference between the holding period return of an investor's portfolio and the return realized with the same portfolio weights in a benchmark time period outside of the holding period.[2] If asset returns in the benchmark period are distributed independently of their respective portfolio weights in the holding period (which is assumed throughout the paper), then the asymptotic value of this measure can be expressed as

$$C = \hat{r}_p - \hat{\beta}_p \bar{r}_E ,$$

which, upon substitution of equation (3), yields

$$C = \text{plim}\left(\frac{1}{T}\sum_{t=1}^{T} \tilde{\beta}_{pt}(\tilde{r}_{Et} - \bar{r}_E)\right) + \hat{\varepsilon}_p . \qquad (5)$$

Thus, the Cornell Measure captures the sum of the timing and selectivity components.

Observation of the investor's portfolio weights also permits the development of measures that separately estimate the timing and selectivity components. The Selectivity Measure is constructed by estimating the period t beta of a portfolio as period t's portfolio-weighted average of individual asset betas. Multiplying this beta by $r_{Et}$, subtracting from $r_{pt}$, and averaging yields a measure with an asymptotic value of

$$S = \hat{r}_p - \text{plim}\left(\frac{1}{T}\sum_{t=1}^{T} \tilde{\beta}_{pt}\tilde{r}_{Et}\right) .$$

After substitution of equation (3), this can be expressed as

$$S = \hat{\varepsilon}_p = \text{the selectivity component.} \qquad (6)$$

The counterpart to the Selectivity Measure, the Timing Measure, is defined as the sample covariance between the portfolio beta and the excess return of the benchmark portfolio. Its asymptotic value is

$$TI = \text{plim}\left(\frac{1}{T}\sum_{t=1}^{T} \tilde{\beta}_{pt}(\tilde{r}_{Et} - \bar{r}_E)\right) = \text{the timing component.} \qquad (7)$$

The above components are analyzed separately in sections 4, 5, and 6. Section 4 analyzes the bias-in-beta component; Section 5 analyzes the selectivity component; and Section 6 examines conditions under which the timing component of performance is positive for informed investors.

**b. The Measured Abnormal Performance of an Uninformed Investor**

A minimum requirement of an "appropriate" performance measure is that it generates, in large samples, no abnormal performance for the portfolios of uninformed investors. The analysis in this section demonstrates that, with a mean-variance efficient benchmark, all of the above measures satisfy this criterion. This requires a preliminary result, which holds even when the uninformed investor chooses suboptimal or random portfolio weights. The result implies that the bias-in-beta component can only be non-zero if there is timing information.

<u>Lemma 1</u>: $\hat{\beta}_p = b_p$ for an investor who lacks market timing information.

<u>Proof</u>: Using equation (1), the portfolio's excess return can be written as the sum of several terms,

$$\tilde{r}_{pt} = \hat{\varepsilon}_p + \hat{\beta}_p \tilde{r}_{Et} + \left[(\tilde{\beta}_{pt} - \hat{\beta}_p)\tilde{r}_{Et} + (\tilde{\varepsilon}_{pt} - \hat{\varepsilon}_p)\right] ,$$

where $\hat{\varepsilon}_p$ and $\hat{\beta}_p$ respectively represent the theoretical intercept and slope coefficient of a regression. Without timing information, the bracketed expression, which can be regarded as a regression residual, is asymptotically uncorrelated with $\tilde{r}_{Et}$, so that the least squares procedure yields a consistent estimator of the slope coefficient, $\hat{\beta}_p$. Q.E.D.

For most utility functions, the risk of the investor's portfolio, $\beta_{pt}$, is affected by selectivity signals because they affect his expected future wealth, which, in turn, determines his aversion to risk and his choice of beta. However, without timing information, these wealth induced changes in risk aversion cannot correlate with the return on the benchmark portfolio. Hence, the large sample least squares estimate, $b_p$, equals the large sample average beta of the portfolio, $\hat{\beta}_p$.

With this lemma, the next result is straightforward.

Proposition 1: The portfolio of an investor who lacks superior information exhibits zero performance with either the Jensen Measure, the Cornell Measure, the Selectivity Measure, or the Timing Measure in large samples. Moreover, if the investor lacks timing (selectivity) information, his portfolio has a zero Timing (Selectivity) Measure in large samples.

Proof: Examine the three elements in the decomposition of abnormal performance. Lemma 1 implies that the bias-in-beta component is zero if the investor lacks timing information. The absence of timing information also implies that $\tilde{\beta}_{pt}$ is uncorrelated with $\tilde{r}_{Et}$; so the timing component of abnormal performance is zero in this case. Finally, if the investor lacks selectivity information, $\tilde{x}_{jt}$ and $\tilde{\varepsilon}_{jt}$ are uncorrelated, which (along with the zero mean of $\tilde{\varepsilon}_{jt}$) implies that the selectivity component of performance is zero. The result follows immediately from equations (4)-(7). Q.E.D.

## 3. The Informed Investor

Observe that Proposition 1 requires only a minimal set of assumptions and hence is a very general result. If one is willing to accept negative as well as positive deviations from zero abnormal performance as an indication of superior information, the analysis can end here. The ability to measure performance is then an empirical issue, which hinges on benchmark observability, the stationarity of returns, data availability, and the small sample properties of the various techniques.

However, since negative measured performance can arise from high transaction costs or embezzlement, as well as from superior information, the efficacy of the different performance measures cannot be based on this result alone. Thus, if the measures are to be useful, it is necessary to demonstrate that they generally are positive for investment strategies that utilize superior information.[3] Although others have pointed out that this more stringent criterion is not always met, we will argue that it can be met under plausible conditions.

To demonstrate this, a model of superior information is developed. The model imposes additional assumptions--most notably, that returns and signals about returns are multivariate normally distributed. The normality assumption is not required for all of the results in the paper,[4] nor is it necessary if the utility function of the evaluated investor is known.[5] The advantage of multivariate normality is that it yields the same benchmark portfolio, (the unconditional mean-variance efficient portfolio), for all investors, and hence allows us to obtain results without specifying the utility function. It also implies that the conditional covariance matrix of asset returns does not vary as the information signals of an informed investor take on different realizations.

The model is used to analyze the optimal portfolio of an informed investor. This portfolio determines the signs of the ex ante means of the random variables $\tilde{\varepsilon}_{pt}$ and $\tilde{\beta}_{pt}(\tilde{r}_{Et} - \bar{r}_E)$, and hence the signs of the selectivity and timing components, $\hat{\varepsilon}_p$ and $\mathrm{plim}(\frac{1}{T}\sum_{t=1}^{T}(\tilde{\beta}_{pt} - \hat{\beta}_p)\tilde{r}_{Et})$, as well.[6] Because the analysis of the ex ante means does not depend on the time period, time subscripts can be omitted to simplify notation, and we will conveniently refer to $E(\tilde{\varepsilon}_p)$ and $\text{cov}(\tilde{\beta}_p, \tilde{r}_E)$ as the selectivity and timing components, respectively.

Our model of information and portfolio selection abstracts from issues that can arise in the multiperiod consumption-investment optimization problem of an informed investor. Each period's investment decision is assumed to be determined by maximizing the expected utility of end-of-period wealth,

$$E\left(U(\tilde{W} + \tilde{W}_H)\right),$$

conditioned on information available just prior to trading in that period. In the expression above,

$W_H$ = end-of-period wealth from non-tradable assets and

$W$ = end-of-period wealth from tradable assets = $W_0(R_F + r_p)$, where

$W_0$ = wealth available at the beginning of the period for investment in tradable assets and

$R_F$ = one plus the risk-free rate.

Let the return of the mean-variance efficient portfolio be expressed as

$$\tilde{r}_E = \bar{r}_E + \tilde{m} + \tilde{y} ,$$

where m is a signal observed by the informed investor and y is the realization of uncorrelated random noise. Similarly, the unconditional return of the mean zero residual, $\tilde{\varepsilon}_j$, in the regression,

$$\tilde{R}_j = \beta_j \tilde{r}_E + \tilde{\varepsilon}_j ,$$

can be expressed as

$$\tilde{\varepsilon}_j = \tilde{s}_j + \tilde{z}_j , \qquad j = 1,\ldots,N ,$$

where $s_j$ is a signal observed by the informed investor and $z_j$ is the realization of uncorrelated noise. The private information signals, m and $s_j$, are drawn from distributions with unconditional means of zero by definition and are assumed to be observed prior to trading in the period.

The information structure is summarized by the equations

$$\tilde{R}_j = \beta_j(\bar{r}_E + \tilde{m} + \tilde{y}) + \tilde{s}_j + \tilde{z}_j$$

for individual assets and for the evaluated portfolio by

$$\tilde{r}_p = \tilde{\beta}_p(\bar{r}_E + \tilde{m} + \tilde{y}) + \tilde{s}_p + \tilde{z}_p \text{ , where}$$

$$\tilde{s}_p = \sum_{j=1}^{N} \tilde{x}_j\tilde{s}_j \quad \text{and} \quad \tilde{z}_p = \sum_{j=1}^{N} \tilde{x}_j\tilde{z}_j \text{ , which implies } \tilde{\varepsilon}_p = \tilde{s}_p + \tilde{z}_p \text{ .}$$

Note that the unconditional efficient portfolio's weighted average of the $\tilde{\varepsilon}_j$'s is identically zero, which implies, by the independence of $\tilde{s}_j$ and $\tilde{z}_j$, that its portfolio-weighted averages of the $\tilde{s}_j$'s and $\tilde{z}_j$'s are also identically zero. Other constant-weight portfolios do not have this property, although their portfolio-weighted averages of the $\tilde{z}_j$'s and $\tilde{s}_j$'s have unconditional expectations of zero. The portfolios of informed investors, by contrast, dynamically change in response to information. We represent this in a single period as a random vector of portfolio weights. Such portfolios necessarily have $\tilde{z}_p$'s with means of zero, but may have $\tilde{s}_p$'s with non-zero expected values.

While superior information about non-tradable wealth cannot be used to increase the investor's trading profits, the mean of this wealth component may vary with the signals, $m, s_1, \ldots, s_N$. For the sake of generality, we model this nonstationarity as follows: The ratio $W_H/W_0$ is first separated into a market and non-market component,

$$\tilde{W}_H/W_0 = \alpha_H + \beta_H\tilde{r}_E + \tilde{\varepsilon}_H \text{ ,}$$

where $\beta_H$ is the population regression coefficient of $\tilde{W}_H/W_0$ on $\tilde{r}_E$. As with the tradable assets, we assume that $\tilde{\varepsilon}_H$ has an observable and an unobservable component, expressed as $\tilde{\varepsilon}_H = \tilde{s}_H + \tilde{z}_H$.

The realizations of the mean zero random variable $\tilde{m}$ and the mean zero random vector $\underline{\tilde{s}} = (\tilde{s}_1,\ldots,\tilde{s}_N,\tilde{s}_H)$ will be referred to as the timing and selectivity signals, respectively. All of the $\tilde{s}_j$'s, $\tilde{z}_j$'s, and $\tilde{y}$ are assumed to be jointly normal. Hence, the random vector $(\tilde{\varepsilon}_1,\ldots,\tilde{\varepsilon}_N,\tilde{\varepsilon}_H,\tilde{r}_E)$ is multivariate normally distributed, both unconditionally and conditionally, with unconditional mean $(0,0,\ldots,0,\bar{r}_E)$ and mean $(s_1,\ldots,s_N,s_H,\bar{r}_E + m)$ conditional on private information.

The standard first order conditions for portfolio optimality are

$$E(U'(\tilde{W} + \tilde{W}_H)\underline{\tilde{R}}|m, \underline{s}) = \underline{0} .$$

Using Stein's Lemma,[7] this can be rewritten as[8]

$$\begin{aligned} E(\underline{\tilde{R}}|m, \underline{s}) &= a(m, \underline{s})\mathrm{cov}(\tilde{W} + \tilde{W}_H, \underline{\tilde{R}}|m, \underline{s}) \\ &= aW_0\mathrm{cov}(\tilde{R}_p + \tilde{R}_H, \underline{\tilde{R}}|m, \underline{s}) , \qquad (8) \end{aligned}$$

where the positive parameter $a = a(m, \underline{s})$, termed "the Rubinstein Measure of Risk Aversion," is defined by

$$a(m, \underline{s}) = -\frac{E(U''(\tilde{W} + \tilde{W}_H)|m, \underline{s})}{E(U'(\tilde{W} + \tilde{W}_H)|m, \underline{s})} .$$

Equation (8) defines the optimal portfolio of an informed investor.

Many of the results in the next three sections assume that the timing signal is uncorrelated with each of the selectivity signals. This has been a fairly standard assumption in the performance literature.[9] An example of a selectivity signal that provides no information about "the market" would be inside information about which of two defense contractors will win a large government contract. Good news to one firm is offset by bad news to the other, so that the overall effect on the economy is unchanged. While information of this type certainly exists, it is unreasonable to assume that all selectivity signals are independent of information about the broader economy. For instance, inside information that the United Auto Workers will strike General Motors certainly affects $\varepsilon_{GM}$, but it also has repercussions throughout the economy that can affect the "market return." Correlated timing and selectivity information, however, presents technical difficulties in generalizing some of the subsequent results. The technical problems, which only arise when there are wealth effects in addition to correlated timing and selectivity information, are discussed in more detail in later sections.

## 4. Timing-Related Estimation Problems and Their Solutions

### a. The Bias-in-Beta Component

Consider an investor with timing information but no selectivity information, as defined by the model in the previous section. We now formally demonstrate that this superior investor may display a negative Jensen Measure. To simplify the example, the beta response function is assumed to be monotonically increasing in his timing signal and symmetric about the "long run target beta," denoted $\hat{\beta}_p$. That is,

$$\tilde{\beta}_p = \hat{\beta}_p + f(\tilde{m}) \ , \qquad \text{where}$$

$$f(m) = -f(-m) \ , \quad f(0) = 0 \ , \quad \text{and} \quad f'(m) > 0 \ .$$

This model of beta adjustment implies (after substitution into equation (1)) that

$$\tilde{r}_p = \hat{\beta}_p \tilde{r}_E + f(\tilde{m})(\bar{r}_E + \tilde{m} + \tilde{y}) + \tilde{\varepsilon}_p \ .$$

In this case, the large sample least squares estimate of the Jensen Measure beta is

$$b_p = \frac{\text{cov}(\tilde{r}_p, \tilde{r}_E)}{\sigma_E^2} = \frac{\hat{\beta}_p \sigma_E^2}{\sigma_E^2} + \frac{\text{cov}\big((\tilde{\beta}_p - \hat{\beta}_p)(\bar{r}_E + \tilde{m} + \tilde{y}),\ \tilde{r}_E\big)}{\sigma_E^2} + \frac{\text{cov}(\tilde{\varepsilon}_p, \tilde{r}_E)}{\sigma_E^2}$$

$$= \hat{\beta}_p + \Big\{\frac{E\big[(\tilde{\beta}_p - \hat{\beta}_p)\bar{r}_E \tilde{r}_E\big]}{\sigma_E^2} + \frac{E\big[f(\tilde{m})(\tilde{m} + \tilde{y})^2\big]}{\sigma_E^2}\Big\} + 0$$

$$= \hat{\beta}_p + \frac{\bar{r}_E}{\sigma_E^2} \text{cov}(\tilde{\beta}_p, \tilde{r}_E) \ ,$$

which tends to overestimate the average risk of the portfolio by a factor proportional to the timing component.

Substituting the above expression for $b_p$ into equation (4) yields the large sample estimate of the Jensen Measure,

$$\left(1 - \frac{\bar{r}_E^2}{\sigma_E^2}\right)\text{cov}(\tilde{\beta}_p, \tilde{r}_E) .$$

The positive derivative for $f(\cdot)$ implies a positive timing component. Hence, this expression is negative if the absolute value of the Sharpe Ratio of the benchmark, $\bar{r}_E/\sigma_E$, exceeds one. In terms of our decomposition, we have shown that the bias-in-beta component can be negative and of larger magnitude than the timing component.

If $f'(m) = \frac{\partial \beta_p}{\partial m} < 0$, so that the investor times perversely, $\hat{\beta}_p$ will be underestimated in large samples. If this is the case, the analysis above indicates that the positive bias-in-beta component would dominate the negative timing component if $\bar{r}_E > \sigma_E$, resulting in a positive Jensen Measure. The Jensen Measure would thus correctly identify the investor as having superior information, but it would fail to indicate that he was using the information in a contrary manner--to lower returns rather than to increase them. We will return to this possibility in Section 6. For now, however, we are not terribly concerned about this shortcoming of the Jensen Measure, since it seems more plausible that an investor would increase (decrease) his portfolio beta when he forecasts a high (low) market return. Hence, for the remainder of this section, we will assume that $\frac{\partial \beta_p}{\partial m} > 0$ for investors with timing information.

**b. Period Weighting Measures**

The example above suggests that the Jensen Measure may indicate that positive market timers are inferior performers. Although the Cornell, Selectivity, and Timing Measures exclude the bias-in-beta component and hence, are not subject to this problem, they require knowledge of the holdings of the evaluated portfolio.

This subsection introduces a new measure that does not require the observation of portfolio holdings and is not subject to this timing-related problem.

We begin by studying a general class of performance measures,

$$\alpha^* = \sum_{t=1}^{T} w_t r_{pt} , \tag{9}$$

where $w_t = w(r_{Et}; r_{E\tau}, \tau \neq t)$ satisfies

$$\sum_{t=1}^{T} w_t r_{Et} = 0 , \tag{10}$$

of which the Jensen Measure is a special case.[10] To ensure that the measure's variance converges to zero as T approaches infinity, the weights, $w_t$, are scaled to sum to one and each weight is assumed to approach zero as the time series gets large. To prevent uninformed investors from achieving positive measured performance with an investment strategy based on past benchmark returns, we also assume that the function $w(r_{Et}; r_{E\tau}, \tau \neq t)$ is invariant to rearrangements of $r_{E\tau}$, $\tau \neq t$ and asymptotically depends only on $r_{Et}$. The statement that $T\tilde{w}_t$ is asymptotically independent of $\tilde{r}_{E\tau}$, $\tau \neq t$, formally expresses the latter notion.

Measures that satisfy these conditions are called "Period Weighting Measures." To show that the Jensen Measure is a Period Weighting Measure, let

$$w_t = \frac{v_E - (r_{Et} - r_E^*)r_E^*}{Tv_E} , \tag{11}$$

where $v_E$ denotes the (maximum likelihood) sample variance of $r_{E1},\ldots,r_{ET}$; and $r_E^*$ the sample mean. Using the weights from equation (11), it is easy to verify that equation (10) holds. Substituting the weights from equation (11) into equation (9) yields

$$\sum_{t=1}^{T} w_t r_{pt} = r_p^* - b_p^* r_E^* = \text{the (small sample) Jensen Measure,}$$

where $r_p^*$ and $b_p^*$ are respectively the sample mean and the sample beta of the portfolio returns $r_{p1},\ldots,r_{pT}$.

Viewing the Jensen Measure from this perspective yields further insights into its failure to identify some market timers as superior investors. It also suggests a modification to the class of Period Weighting Measures that prevents erroneous inferences from being induced by a negative bias-in-beta component. The condition imposed is that $w_t$ be nonnegative for all t. Measures in the class that satisfy this additional restriction will be called "Positive Period Weighting Measures."

The intuition for this additional restriction is simple. For large $r_{Et}$, the weights implicit in the Jensen Measure, equation (11), are negative. Thus, the Jensen Measure weights do not satisfy the criteria of a Positive Period Weighting Measure. If the investor "times the market," the large positive portfolio returns that tend to occur when the benchmark's return is extremely high are multiplied by negative weights, which reduces the Jensen Measure. Consequently, this measure can be negative for the portfolios of investors who time. Replacing the negative weights with positive weights, and adjusting the other weights accordingly, eliminates this possibility, since it always rewards market timing. This intuition is confirmed in the following proposition.

<u>Proposition 2</u>: Let $\tilde{w}_t = \tilde{w}(\tilde{r}_{Et}; \tilde{r}_{E\tau}, \tau \neq t)$, a function that is symmetric (or invariant) to rearrangements of the latter T - 1 arguments, satisfy

$$\sum_{t=1}^{T} \tilde{w}_t\tilde{r}_{Et} = 0, \quad \sum_{t=1}^{T} \tilde{w}_t = 1, \text{ and let } \tilde{\alpha}^* = \sum_{t=1}^{T} \tilde{w}_t\tilde{r}_{pt}$$

define a class of performance measures called "Period Weighting Measures." If, for all t, the asymptotic distribution of $T\tilde{w}_t$ is independent of $\tilde{r}_{E\tau}$ for each $\tau \neq t$, then

(i) The large sample Period Weighting Measure of an uninformed investor's portfolio, $\text{plim}(\tilde{\alpha}^*) = 0$.

(ii) If the period weights additionally satisfy $\tilde{w}_t > 0$, t = 1,...,T, and if

$\frac{\partial \beta_{pt}}{\partial m_t} > 0$ for all realizations of the signals of an informed investor with selectivity and/or independent timing information, then the investor's portfolio has a large sample performance measure of $\text{plim}(\tilde{\alpha}^*) > 0$.

(iii) The portfolio of an informed investor with selectivity ability but no timing ability has a large sample Period Weighting Measure, $\text{plim}(\tilde{\alpha}^*) > 0$.

Proof: See Appendix.

The Positive Period Weighting Measures, which mitigate econometric biases due to timing ability, require nonnegative weights that make the weighted-average excess return of the efficient portfolio zero. Proposition 2, which demonstrates that this class of measures can be used to identify superior investors, has implications for a number of other measures of performance. These implications are discussed in Section 5.

Because of technical problems associated with wealth effects, (mentioned earlier), Proposition 2 requires that timing and selectivity information be independent. However, when the investor possesses constant absolute risk aversion, and hence, exhibits no wealth effects, this assumption can be relaxed. For these preferences, Proposition A1 in the Appendix extends Proposition 2 to correlated timing and selectivity.

### c. Marginal Utility Interpretations

An interesting interpretation of the Period Weighting Measures (suggested to us by Michael Brennan), can be made if we substitute expectations for the summations and interpret the weights, $w_t$, as marginal utilities. Equation (10) then becomes the first order condition for maximizing the expected utility of an uninformed investor who holds the benchmark portfolio,

$$E\left[U'\left(W_0(R_F + \tilde{r}_E)\right)\tilde{r}_E\right] = 0 ,$$

and equation (9) measures this investor's marginal change in utility from adding a small amount of the evaluated portfolio's excess return to his existing portfolio,

$$E\left[U'\left(W_0(R_F + \tilde{r}_E)\right)\tilde{r}_p\right] .$$

If this quantity is positive, it indicates that an uninformed investor, with marginal utilities equal to the weights used to evaluate performance, wishes to add some of the evaluated portfolio to his unconditionally optimal portfolio.

For instance, the Jensen Measure weights expressed in equation (11) are linear in $r_{Et}$ and are thus the marginal utilities of a quadratic utility investor. Under this interpretation, the Jensen Measure is the marginal gain in utility, obtained by adding a small amount of the evaluated portfolio to the unconditionally optimal portfolio of an uninformed quadratic utility investor. With normal distributions, as in subsection (a)'s example, there are always large realizations of $\tilde{r}_{Et}$ that make the period weight negative. These correspond to wealth levels that exceed the satiation point of the quadratic utility investor. It is not surprising that a measure, based on preferences that consider additional wealth bad in some states of nature, sometimes provides erroneous inferences. The Positive Period Weighting Measures, by contrast, always have weights that can be interpreted as positive marginal utilities.

It is not always the case, however, that large $r_{Et}$'s imply negative Jensen Measures for portfolios that "time the market," even if $\bar{r}_E/\sigma_E$ exceeds one. The sign of the Jensen Measure is also determined by the $\tilde{r}_{pt}$'s that are generated when the period weights are negative. These depend on the model of beta adjustment, which in turn depends on the market timer's preferences. The assumptions for the example in subsection (a), for instance, are satisfied if $\beta_p$ is a linear function of m, which would imply exponential utility. The negative Jensen Measure for this example suggests that an uninformed quadratic utility investor may not wish to marginally add the returns of an informed exponential utility investor to his existing portfolio. On the other hand, the large sample Jensen Measure of a portfolio that is managed by an informed quadratic utility investor is positive, even with timing ability and arbitrary asset return distributions. This is because an uninformed investor with quadratic utility will prefer to add a portion of the portfolio of his more informed counterpart, even if his risk aversion parameter or initial endowment differs.[11]

## 5. The Component of Abnormal Performance That is Due to Selectivity

Mayers and Rice (1979) developed a model with complete markets where an informed quadratic utility investor receives selectivity information about state probabilities. They proved that this investor exhibits a positive Jensen Measure in large samples if the CAPM holds and the benchmark is the market portfolio. Dybvig and Ross (1985) extended this result to more general preferences, state spaces, and information signals. They showed that one only needs to assume that the informed investor is a mean-variance optimizer over the set of tradable assets, that a riskless asset exists, and that the information does not alter the investor's estimate of the mean and variance of the benchmark.

Since neither paper allows timing information, the timing component is zero, and by Lemma 1, the bias-in-beta component is zero. One interpretation of these results is that the selectivity component, represented by the expected value of $\tilde{\varepsilon}_p$, is positive in the absence of timing information. Proposition 2, in the previous section, generalizes this finding in two ways. In this section, we use Proposition 2 to show that the selectivity component is still positive even when (i) the investor does not select a conditional mean-variance efficient portfolio (since we allow for non-traded assets) and (ii) there is independent timing information.

These implications of Proposition 2 are developed in two corollaries. The first is

Corollary 1: In the absence of timing information, the asymptotic Jensen, Cornell, and Selectivity Measures (and hence, the selectivity component) of an informed investor's portfolio are positive.

Proof: The result immediately follows from part (iii) of Proposition 2 for the Jensen Measure, since, by equation (10), the Jensen Measure is a Period Weighting

Measure. In addition, when there is no timing information, the timing component is zero by Proposition 1, and by Lemma 1, the bias-in-beta component is zero. In conjunction with equations (4) - (6), this implies that the asymptotic values of all three measures are identical, since each equals the selectivity component.

Q.E.D.

In contrast to the Dybvig and Ross and Mayers and Rice models, which do not allow for non-traded assets, the expected Jensen Measure in our model, conditioned on the informed investor's selectivity signal, $\underline{\tilde{s}}$, is not necessarily positive, even when the investor lacks timing information. However, the unconditional Jensen Measure, (which in this case is the selectivity component of performance, $E(\tilde{\varepsilon}_p)$), is positive for large samples. An example illustrates why this is the case. Consider an investor who holds a large amount of IBM stock in his portfolio to hedge the risk associated with his non-traded assets. If his private information indicates that IBM is likely to have a negative return this month, he will hold less IBM stock than he typically would, but still might hold a positive amount of it for its hedging properties. Conditioned on this information signal, the selectivity component, $E(\tilde{\varepsilon}_p|\underline{s})$, may be negative. However, since his holdings of IBM are higher when he has favorable information, deviations from his average portfolio holdings are always of the same sign as his information signal. This implies that the selectivity component, $E(\tilde{\varepsilon}_p)$, is positive unconditionally, since the covariance of the portfolio weight with the conditional excess return, $E(\tilde{\varepsilon}_{IBM}|\underline{s})$, is positive and the unconditional excess return has a mean of zero.

The scenario in the previous paragraph also suggests that the portfolio of an informed investor with non-traded assets could plot well inside the efficient frontier of an uninformed evaluator, but the Jensen Measure would still identify him as having superior information. Hence, the mean-variance diagram may not be a suitable alternative to the Jensen Measure for the measurement of performance, as some earlier research has conjectured.

Of course, the Jensen Measure may not identify superior investors who possess timing information in addition to selectivity information. This is because the asymptotic Jensen Measure, in this case, is no longer equal to the selectivity component, but has a timing and bias-in-beta component as well. According to Proposition 2, this problem can be overcome by employing a Positive Period Weighting Measure. Alternatively, we can identify the investor as having selectivity information by applying the Selectivity Measure, as suggested in the next corollary.

Corollary 2: The selectivity component of performance of an informed investor's portfolio (and hence, the asymptotic Selectivity Measure), is positive if the investor's selectivity signal for each asset is distributed independently of the timing signal.

Proof: Identical to the proof of part (iii) of Proposition 2 in the Appendix, which in turn, follows from the proof of part (ii).

If timing and selectivity signals are correlated, pure selectivity information no longer exists, in that realizations of $\tilde{s}_j$ provide information about realizations of $\tilde{m}$. In this case, the selectivity component of performance is affected by changes in risk aversion that arise from information signals and may be negative for some utility functions.[12] Wealth effects of this type have a more direct impact on the timing component of performance and are discussed in detail in the next section.

## 6. The Timing Component of Abnormal Performance

### a. Verrecchia's Example: The Giffen Good Effect

Section 5 simplified and extended the Mayers and Rice result that an investor with only selectivity information would, on average, realize a positive Jensen Measure. Verrecchia (1980), responding to some of the restrictive assumptions in the Mayers and Rice model, presented a counterexample to what he called the "broader hypothesis" that "the superior investor will on average achieve a greater return than the uninformed investors expect."[13]

The analysis in the last section indicates that Verrecchia's counterexample must be due to timing information since his assumption of quadratic utility has the same implications for portfolio choice as our assumption that returns are normally distributed. The assumption of quadratic utility also implies that increases in wealth always increase risk aversion. Consequently, information that increases expected wealth, such as information that the "market" return will be high (i.e. m is large), can make an investor with quadratic utility so much more averse to risk that he holds less of that portfolio, rather than more. This wealth effect, which, ceteris paribus, induces a negative correlation between $\tilde{\beta}_p$ and $\tilde{r}_E$, dominates substitution effects for investors who, in most states of nature, are close to their satiation point, where risk aversion becomes arbitrarily large.

**b. The Sign of the Timing Component of Abnormal Performance**

The discussion in the previous subsection suggests that the violations of the broader hypothesis considered by Verrecchia are linked to perverse income or wealth effects (in the sense of the income/substitution effects and the superior/inferior goods classification in Hicks (1939)). In Verrecchia's example, the preferences of the informed investor make the efficient portfolio of risky assets a Giffen good (i.e. less is purchased as its expected return increases). This type of behavior is probably pathological. One expects the portfolio beta of most investors to be an increasing, not a decreasing function of m. If this is the case, the portfolio of an investor with timing information exhibits a positive timing component of abnormal performance, as shown in the next proposition.

<u>Proposition 3</u>: If $\beta_p = \beta_p(m, \underline{s})$ satisfies $\frac{\partial \beta_p}{\partial m} > 0$ for all m, $\underline{s}$, and if the selectivity signal for each asset is distributed independently of the timing signal, then the timing component of performance is positive.

<u>Proof</u>: It suffices to show that $E(\tilde{\beta}_p \tilde{m}) > 0$. Using the multivariate version of Stein's Lemma,[14]

$$E(\tilde{\beta}_p\tilde{m}) = E(\frac{\partial\beta_p}{\partial m})\mathrm{var}(\tilde{m}) + \sum_{j=1}^{N} E(\frac{\partial\beta_p}{\partial s_j})E(\tilde{m}\tilde{s}_j) = E(\frac{\partial\beta_p}{\partial m})\mathrm{var}(\tilde{m}) > 0 \ .$$

Q.E.D.

Corollary 3: Under the conditions specified in Proposition 3, the large sample Timing and Cornell Measures of an informed investor will be positive.

Proof: Follows trivially from equations (5) and (7), Corollary 2, and Proposition 3.

Q.E.D.

One can examine the class of utility functions that make $\frac{\partial\beta_p}{\partial m} > 0$ and ask whether this class is sufficiently broad to represent realistic behavior. The proposition below demonstrates that if the timing and selectivity signals are independent, a sufficient (but not necessary) condition is that the investor have non-increasing Rubinstein absolute risk aversion.

Proposition 4: If an informed investor with independent timing and selectivity information has nonincreasing Rubinstein absolute risk aversion for $t = 1,\ldots,T$, his timing component of abnormal performance (and hence the large sample Timing and Cornell Measures) will be positive.

Proof: See Appendix.

The Rubinstein (1973) measure of absolute risk aversion,

$$a(m, \underline{s}) = - \frac{E(U''(\tilde{W} + \tilde{W}_H)|m, \underline{s})}{E(U'(\tilde{W} + \tilde{W}_H)|m, \underline{s})} ,$$

depends not only on preferences, but on the investment opportunity set and the probability distribution of the states of the world. Although it is different from the Arrow-Pratt measure of risk aversion, its preference properties are similar. For instance, an investor is said to have decreasing (increasing) Rubinstein risk aversion if this measure is a monotonically decreasing (increasing) function of expected utility. This means that with decreasing Rubinstein risk aversion, changes in m and $\underline{s}$ that increase (decrease) $E(U(\tilde{W} + \tilde{W}_H)|m, \underline{s})$ will decrease (increase) $a(m, \underline{s})$.

Increasing or decreasing absolute risk aversion cannot be defined in terms of wealth or expected wealth with the Rubinstein measure because the former is a random variable and the latter is only one of the two return components that affect the Rubinstein measure, the other being the variance of wealth. However, intuition about wealth effects with the Arrow-Pratt measure is generally applicable here. For instance, holding variability constant, an increase in expected wealth increases expected utility, which decreases the risk aversion of an investor with decreasing absolute risk aversion, as well as the fraction of his wealth held in the risk-free asset. In addition, an investor with constant Arrow-Pratt absolute risk aversion also has constant Rubinstein absolute risk aversion.

We have not been able to prove that Proposition 4 generalizes to cases where timing and selectivity information are correlated. However, the following proposition proves in this more general information setting that the sum of the timing and selectivity components of performance (and by extension, the large sample Cornell Measure) is positive if an informed investor has nonincreasing Rubinstein absolute risk aversion.[15,16]

Proposition 5: If an informed investor has nonincreasing Rubinstein absolute risk aversion for $t = 1,\dots,T$ and if all assets that affect his wealth are tradable, then the sum of his portfolio's timing and selectivity components (and hence its large sample Cornell Measure), is positive.

Proof: See Appendix.

One caveat deserves discussion here: Proposition 5 requires that all assets be tradable. This assumption is dictated by the same technical difficulties that prevent a generalization of Propositions 2-4 to a model where timing and selectivity signals are correlated.[17] In spite of this technical difficulty, it is unlikely that the measurability of performance critically depends on whether Propositions 2-5 generalize. Counterexamples to such generalizations are necessarily founded on the possibility that changes in risk aversion will dominate substitution effects. Since this is unlikely, the behavior described by Verrecchia is probably unrealistic.

### c. Why the Timing and Selectivity Components are Fundamentally Different

Note that both timing and selectivity signals can affect expected wealth, and hence the risk aversion of an informed investor. However, with independent timing and selectivity signals, only the timing information can potentially lead to pathological behavior. Consider, for example, the case of an investor who, with neutral information (i.e. $\underline{s} = \underline{0}$), holds the mean-variance efficient portfolio along with IBM stock to hedge against variability in the value of his human capital. If this investor receives favorable private information about IBM that is independent of the "market," his expected wealth increases. However, regardless of how his risk-aversion increases or decreases in response to this change in expected wealth, he will always increase his holdings of IBM when he receives favorable private information. To illustrate this, consider the self-financing portfolio consisting of one dollar long in IBM stock, $\beta_{IBM}$ dollars short in the efficient portfolio, and $1 - \beta_{IBM}$ dollars short in the risk-free asset, which has end-of-period value $\tilde{\varepsilon}_{IBM}$. With neutral information, this self-financing portfolio has an expected end-of-period value of zero (i.e. no risk premium) and the investor will hold the amount of this portfolio that minimizes the unsystematic variance of his total wealth (tradable plus nontradable "nonmarket" risk). Note that the portfolio weights of this minimum unsystematic variance portfolio are fixed and do not change with information signals. Hence, deviations from this portfolio position in response to information always increase risk and will only be taken to increase expected return. This implies that increases in the expected return of IBM that stem from selectivity information always increase the investor's holdings of IBM's unsystematic disturbance, while decreases always decrease the investor's holdings.

The same cannot be said for the unconditional efficient portfolio, since it carries a risk premium, $\bar{r}_E$. Thus, the different effects of timing and selectivity ability on measured abnormal performance stem from timing information being related to priced risk and selectivity information being related to non-priced risk.

## 7. The Appropriate Benchmark Portfolio

The previous sections present conditions under which the unconditional mean-variance efficient portfolio of tradable assets can be used to evaluate portfolio performance. This benchmark portfolio is used with all of the measures analyzed in Sections 2-6, except for the Cornell Measure. The intuition for the appropriateness of this portfolio is quite simple. The efficiency of any particular index portfolio will be rejected if a strategy with constant portfolio weights realizes a significant "Jensen Measure" with respect to it. Moreover, for a strategy with constant portfolio weights, the Jensen Measure is asymptotically equivalent to all of the measures. Hence, if a managed portfolio realizes a significant positive performance measure and if the efficiency of the index used to compute that measure cannot be rejected, the positive measure is probably due to portfolio weights that change in response to superior information.

In his reply to the Mayers and Rice paper, Roll (1979) suggested that choosing an appropriate benchmark portfolio may be particularly difficult if uninformed investors need to include real estate and other non-equity assets in their optimal portfolio, as the CAPM suggests. However, our results indicate that the appropriate benchmark portfolio consists only of those assets that can be included in the portfolio being evaluated. For example, portfolio managers who specialize in oil stocks can be evaluated with a mean-variance efficient benchmark portfolio consisting only of oil stocks.[18] This is because, from the perspective of these managers, non-oil investments may be regarded as non-traded assets. A related example is the case of partially delegated portfolio management, where an investment manager, aware that his clients invest a portion of their assets on their own or with other professionals, regards those investments as non-traded assets. A rational manager should select a portfolio that partly hedges the unmanaged wealth of a representative client. Our results suggest that portfolio performance can be evaluated in these circumstances.[19]

## 8. Summary and Conclusion

This paper examines the problem of evaluating portfolio performance. A decomposition of the Jensen Measure into three components--bias-in-beta, selectivity, and timing--offers insights into its ability to identify truly superior performance and allows us to contrast its properties with those of other measures of portfolio performance. The paper also presents new performance measures which, under certain circumstances, are more appropriate than those previously employed.

We demonstrate that the bias-in-beta component, which arises because timing ability, does not affect measures that require the observation of portfolio weights, (e.g., the Cornell, Timing, and Selectivity Measures), but it can erroneously generate a negative Jensen Measure. However, under plausible conditions, the Positive Period Weighting Measure, introduced here, provides correct inferences about abnormal performance arising from either timing and/or selectivity information in cases where the portfolio composition is not observable.

Our analysis also shows that the selectivity component of performance is always positive when timing and selectivity information are independently distributed. This implies that the Jensen Measure detects the abnormal performance of informed investors without timing ability. While earlier research has also demonstrated this result, we extend it to cases where investors do not desire to hold mean-variance efficient portfolios of tradable assets.

The analysis of the timing component of performance implies that it, too, is positive for investors with timing ability except for cases that are based on (what we consider) pathological preferences. Counterexamples to what Verrecchia (1980) referred to as "the Mayers/Rice conjecture" are shown to be based on unrealistic wealth effects and are only of esoteric interest.

This paper also addresses the issue of the appropriate benchmark portfolio. Because of its mathematical (and not its equilibrium) properties, the unconditional mean-variance efficient portfolio of assets that are considered tradable by the evaluated investor provides correct inferences about the investor's performance. This indicates that the missing asset problem, which is important in tests of the CAPM, does not apply to the evaluation of managed portfolios that consist of traded stocks or bonds.

Roll's (1977) critique of CAPM tests had led some authors to question the appropriateness of the Jensen methodology.[20] However, our analysis illustrates that links between performance measures and particular equilibrium models are not necessary. Despite the recent work (by Connor and Korajczyk (1986) and Lehmann and Modest (1987)) on performance evaluation with APT-based measures, equilibrium models do nothing more than suggest candidates for mean-variance efficient benchmarks.

This does not necessarily imply that a one-factor model is superior to the multifactor approach employed by Lehmann and Modest (1987) and Connor and Korajczyk (1986). Indeed, one may prefer multiple index benchmarks because they generally yield more powerful test statistics and intuition suggests that they are less likely to be inefficient than a single index. The propositions in this paper apply directly to multiple indexes if the index portfolios are locally mean-variance efficient, as defined in Grinblatt and Titman (1987).

An i.i.d. return-generating process is a critical assumption in our analysis. If unconditional returns are nonstationary, it may be impossible to test for the mean-variance efficiency of a particular benchmark portfolio. Indeed, a portfolio with constant weights that is ex ante efficient for each time period may not exist. There could also be a more fundamental problem in distinguishing between shifts in mean returns known by just a few informed investors from changes in mean returns due to parameter shifts known by most investors, but not the observer.

There is more robustness here, however, than is apparent from the formal modelling. In order to synthesize and simplify the analysis, the different performance measures were analyzed under identical stationarity assumptions. However, various types of nonstationarities affect the different performance measures in different ways. Consider, for example, a portfolio that specializes in the purchase of the stock of bankrupt firms.[21] These stocks would tend to have higher betas at the time they are in this portfolio than at any other time in their existence. The Cornell Measure, which effectively measures the betas of securities during the sample period when they are not held in the portfolio, would be upwardly biased for such a portfolio strategy. However, the Jensen Measure, which obtains its risk adjustment from a contemporaneous period, would not be biased by this nonstationarity (assuming that the bankrupt stocks comprise an infinitesimally small component of the efficient portfolio). On the other hand, nonstationarities in the composition of the efficient portfolio alone would affect the Jensen Measure much more than the Cornell Measure.

At a more fundamental level, we may be less concerned that the Jensen Measure is sensitive to changes in the composition of the efficient portfolio. After all, many investors would like to know if there are strategies with changing portfolio weights that dominate simple buy-and-hold strategies or rebalancing strategies with constant portfolio weights. If some evaluated portfolios exhibit positive Jensen, Period Weighting, or Selectivity Measures, it indicates that either the managers of these funds have special information or that the composition of the mean-variance efficient portfolio is changing over time. In either case, the evidence indicates that simple passive strategies can be beaten.

## Appendix

The Appendix is organized in the following order:

1. Preliminaries for all but Proposition 5
2. Proof of Proposition 2
3. Proposition A1
4. Proposition A2
5. Proof of Proposition 4
6. Proof of Proposition 5

### Preliminaries

The proofs presented here, except for the proof of Proposition 5, are simplified if we consider an equivalent economy formed from N portfolios of the tradable risky assets. This repackaging of the economy's assets does not limit the generality of the proofs. In the repackaged securities market, the N excess returns of the redefined assets satisfy the index model

$$\underline{\tilde{R}} = \begin{bmatrix} \tilde{R}_1 \\ \tilde{R}_2 \\ \tilde{R}_3 \\ \cdot \\ \cdot \\ \tilde{R}_N \end{bmatrix} = \begin{bmatrix} \tilde{r}_E \\ \tilde{\varepsilon}_2 \\ \tilde{\varepsilon}_3 \\ \cdot \\ \cdot \\ \tilde{\varepsilon}_N \end{bmatrix} = \begin{bmatrix} \bar{r}_E + \tilde{m} \\ \tilde{s}_2 \\ \tilde{s}_3 \\ \cdot \\ \cdot \\ \tilde{s}_N \end{bmatrix} + \begin{bmatrix} \tilde{y} \\ \tilde{z}_2 \\ \tilde{z}_3 \\ \cdot \\ \cdot \\ \tilde{z}_N \end{bmatrix} .$$

Note that there are only N - 1 non-redundant assets with zero unconditional risk premia. Prior to repackaging, the efficient portfolio's weighting of the N ("market model") residuals was zero, implying that they were linearly dependent. This dependence has been eliminated in the repackaged market. We also assume, without loss of generality, that the covariance matrix of asset returns, conditioned on the information signals m and $\underline{s}$, is positive definite. We denote this N x N matrix as $\underline{\underline{V}} = \mathrm{var}(\tilde{y}, \underline{\tilde{z}}^T)$, where the superscript T denotes "transpose." Entry i, j of this matrix is denoted as $V_{ij}$.

If $\tilde{m}$ and $\tilde{s}_j$ are uncorrelated, $\tilde{y}$ and $\tilde{z}_j$ are uncorrelated, implying $V_{1j}$ and $V_{j1}$ are zero for $j \neq 1$. Thus, in the case where timing and selectivity information are uncorrelated, both $\underline{\underline{V}}$ and $\underline{\underline{V}}^{-1}$ have off-diagonal elements of zero in the first row and column.

Letting

$\underline{e}_1$ = the first column of the identity matrix

$\underline{\sigma}_H$ = the conditional covariance between $\tilde{\varepsilon}_H$, (the component of $\tilde{W}_H/W_0$ that is uncorrelated with $\tilde{r}_E$), and the excess return vector $\underline{\tilde{R}}$,

$\underline{z}_0^T = (0, \underline{z}^T)$,

$$\underline{s}_0 = \begin{bmatrix} 0 \\ s_2 \\ \cdot \\ \cdot \\ s_N \end{bmatrix}, \text{ and the vector of optimal portfolio weights, } \underline{x}_\beta = \begin{bmatrix} \beta_p \\ x_2 \\ \cdot \\ \cdot \\ x_N \end{bmatrix},$$

the first order condition for selecting $\underline{x}_\beta$, equation (8), can be rewritten as

$\underline{e}_1(\bar{r}_E+m) + \underline{s}_0 = aW_0\left(\underline{\underline{V}}\underline{x}_\beta + (\beta_H V_{11}\underline{e}_1 + \underline{\sigma}_H)\right)$, which yields portfolio weights

$$\underline{x}_\beta = \underline{\underline{V}}^{-1}\left[\left(\underline{e}_1(\bar{r}_E + m) + \underline{s}_0\right)/(aW_0) - (\beta_H V_{11}\underline{e}_1 + \underline{\sigma}_H)\right] . \qquad \text{(A1)}$$

Note that "a," the risk aversion measure, is unconditionally a random variable because it depends on information. An exception to this, in Propositions A1 and A2, occurs when the investor has constant absolute risk aversion. The Arrow-Pratt measure of absolute risk aversion is then equal to "a."

In addition to equation (A1), Propositions (A1) and (A2) make use of

Lemma A1: If a symmetric matrix $\underline{\underline{M}}$ is positive definite, then for all i, the product of the ith diagonal element of $\underline{\underline{M}}$ and that in $\underline{\underline{M}}^{-1}$ equals or exceeds one.

Proof: Available on request.

Proof of Proposition 2: The large sample expectations encountered here easily translate into a more notationally convenient "one period framework." Because $\tilde{r}_{Et}$ has a stationary distribution, we can view the infinite sequence of time series draws of $r_{Et}$ and $Tw_t$ as random drawings from a population distribution. In this framework, we use the realizations of $\tilde{w}$ and $\tilde{r}_E$ to represent some time period t's outcome of $Tw_t$ and $r_{Et}$. Thus, $E(\tilde{w}) = 1$, $E(\tilde{\alpha}^*) = E(\tilde{w}\tilde{r}_p)$, and $E(\tilde{w}\tilde{r}_E) = 0$, where $\tilde{w}$ is a function of $\tilde{r}_E$ by the assumption of asymptotic independence between $T\tilde{w}_t$ and $\tilde{r}_{E\tau}$, $\tau \neq t$.

For parts (i) and (iii), consider an investor who lacks timing ability. In this case,

$$E(\tilde{w}\tilde{r}_p) = E\left(\tilde{w}(\tilde{\beta}_p\tilde{r}_E + \tilde{\varepsilon}_p)\right) = E(\tilde{w}\tilde{r}_E)E(\tilde{\beta}_p) + E(\tilde{w})E(\tilde{\varepsilon}_p) = E(\tilde{w})E(\tilde{\varepsilon}_p) = E(\tilde{\varepsilon}_p) \quad \text{(A2)}$$

where the first equality follows from equation (1) and, in the absence of timing ability, the second follows from the independence of $\tilde{\varepsilon}_p$ and $\tilde{w}$, and of $\tilde{w}\tilde{r}_E$ and $\tilde{\beta}_p$. Thus, in the absence of timing, the Period Weighting Measure equals the selectivity component of performance.

(i) Equation (A2) is zero since $E(\tilde{\varepsilon}_p) = 0$ by the proof of Proposition 1.

(ii) $E(\tilde{w}\tilde{r}_p) = E(\tilde{w}\tilde{r}_E\tilde{\beta}_p) + E(\tilde{w}\tilde{\varepsilon}_p)$ by equation (1).

We first demonstrate that $E(\tilde{w}\tilde{\varepsilon}_p) = E(\tilde{w}\tilde{s}_p) = E(\tilde{w}\underline{\tilde{s}}_0^T\underline{\tilde{x}}_\beta) > 0$. Using equation (A1),

$$E(\tilde{w}\tilde{s}_p) = E\left[\tilde{w}\underline{\tilde{s}}_0^T\underline{\underline{V}}^{-1}\underline{e}_1(\bar{r}_E+\tilde{m})/(\tilde{a}W_0)\right] + E\left[\tilde{w}\underline{\tilde{s}}_0^T\underline{\underline{V}}^{-1}\underline{\tilde{s}}_0/(\tilde{a}W_0)\right] - E\left[\tilde{w}\underline{\tilde{s}}_0^T\underline{\underline{V}}^{-1}(\beta_H V_{11}\underline{e}_1+\underline{\sigma}_H)\right] .$$

With selectivity signals that are independent of the timing signal, the off-diagonal elements in the first row and column of $\underline{\underline{V}}^{-1}$ are zero. Hence, the term inside the first expectation on the right side of the equation is zero. The last expectation is zero because $\underline{\tilde{s}}_0$ has a mean of zero and is independent of $\tilde{w}$. Hence,

$$E(\tilde{w}\tilde{s}_p) = E\left[\tilde{w}\underline{\tilde{s}}_0^T\underline{\underline{V}}^{-1}\underline{\tilde{s}}_0/(\tilde{a}W_0)\right] > 0$$

because $\tilde{a} > 0$, $\tilde{w} > 0$, and $\underline{\underline{V}}^{-1}$ is a positive definite matrix.

The proof is completed by demonstrating that $E(\tilde{w}\tilde{r}_E\tilde{\beta}_p) \geq 0$. By the law of iterated expectations,

$$E(\tilde{w}\tilde{r}_E\tilde{\beta}_p) = E_{r_E}(\tilde{w}\tilde{r}_E\tilde{\beta}_p | r_E) ,$$

and since $\tilde{w}\tilde{r}_E$ is non-stochastic conditional on $r_E$,

$$E(\tilde{w}\tilde{r}_E\tilde{\beta}_p) = E_{r_E}[E(\tilde{w}\tilde{r}_E|r_E)E(\tilde{\beta}_p|r_E)]$$

$$= E_{\{r_E:r_E\geq 0\}}[E(\tilde{w}\tilde{r}_E|r_E)E(\tilde{\beta}_p|r_E)]pr(\tilde{r}_E \geq 0)$$

$$+ E_{\{r_E:r_E<0\}}[E(\tilde{w}\tilde{r}_E|r_E)]E(\tilde{\beta}_p|r_E)pr(\tilde{r}_E < 0)$$

$$> E_{\{r_E:r_E\geq 0\}}[E(\tilde{w}\tilde{r}_E|r_E)\beta_p^*]pr(\tilde{r}_E \geq 0) + E_{\{r_E:r_E<0\}}[E(\tilde{w}\tilde{r}_E|r_E)\beta_p^*]pr(\tilde{r}_E < 0) ,$$

where $\beta_p^* = E(\tilde{\beta}_p|\tilde{r}_E = 0)$. The inequality follows from $\tilde{w} > 0$ and $\frac{\partial\beta_p}{\partial m} > 0$. The latter assumption makes $E(\tilde{\beta}_p|r_E)$ an increasing function of $r_E$.[22]

Note that $\beta_p^*$ is independent of $\tilde{r}_E$. Thus,

$$E(\tilde{w}\tilde{r}_E\tilde{\beta}_p) > \beta_p^* E_{\{r_E:r_E\geq 0\}}[E(\tilde{w}\tilde{r}_E|r_E)]pr(\tilde{r}_E \geq 0) + \beta_p^* E_{\{r_E:r_E<0\}}[E(\tilde{w}\tilde{r}_E|r_E)]pr(\tilde{r}_E < 0)$$

$$= \beta_p^* E(\tilde{w}\tilde{r}_E) = 0 \quad \text{since} \quad E(\tilde{w}\tilde{r}_E) = 0.$$

Q.E.D.

(iii) Equation (A2) is positive, since with selectivity ability, $E(\tilde{\varepsilon}_p)$ is shown to be positive by substituting "1" for "$\tilde{w}$" in part (ii) and following the steps in part (ii) that demonstrate $E(\tilde{w}\tilde{\varepsilon}_p) > 0$.

<u>Proposition A1</u>: If the Rubinstein measure of absolute risk aversion, a, is constant over all the information realizations of an informed investor, then the expectation of the investor's Positive Period Weighting Measure, $E(\tilde{w}\tilde{r}_p)$, is positive.

<u>Proof</u>: Using equation (1) and the asset packaging described at the beginning of the Appendix,

$$E(\tilde{w}\tilde{r}_p) = E(\tilde{w}\tilde{r}_E\tilde{\beta}_p) + E(\tilde{w}\underline{\tilde{\varepsilon}}_0^T\underline{\tilde{x}}_\beta) ,$$

where the first element of $\underline{\tilde{\varepsilon}}_0$ is 0 and the ith element is $\tilde{\varepsilon}_i$. Using equation (A1) and noting that $\tilde{w}\tilde{r}_E$ is non-stochastic conditional on $r_E$, the timing term

$$E(\tilde{w}\tilde{r}_E\tilde{\beta}_p) = E[\tilde{w}\tilde{r}_E(\underline{e}_1^T\underline{\underline{V}}^{-1}\underline{e}_1\bar{r}_E/(aW_0) - \underline{e}_1^T\underline{\underline{V}}^{-1}(\beta_H V_{11}\underline{e}_1 + \underline{\sigma}_H))]$$

$$+ \frac{1}{aW_0} E_{r_E}[wr_E(\underline{e}_1^T\underline{\underline{V}}^{-1}\underline{e}_1 E(\tilde{m}|\tilde{r}_E = r_E) + \underline{e}_1^T\underline{\underline{V}}^{-1}E(\underline{\tilde{s}}_0|\tilde{r}_E = r_E))]$$

$$= \frac{1}{aW_0} E_{r_E}[wr_E^2((\underline{e}_1^T\underline{\underline{V}}^{-1}\underline{e}_1) \frac{var(\tilde{m})}{\sigma_E^2} + \underline{e}_1^T\underline{\underline{V}}^{-1} \frac{E(\underline{\tilde{s}}_0\tilde{m})}{\sigma_E^2})]$$

$$> E\left(\frac{\tilde{w}\tilde{r}_E^2}{\sigma_E^2 aW_0}\right)\left[\underline{e}_1^T \underline{\underline{V}}^{-1} E(\tilde{\underline{s}}_0 \tilde{m})\right] = E\left(\frac{\tilde{w}\tilde{r}_E^2}{\sigma_E^2 aW_0}\right)\left[\underline{e}_1^T \underline{\underline{V}}^{-1}(-\underline{\underline{V}}\underline{e}_1 + V_{11}\underline{e}_1)\right] .$$

This is positive because $(wr_E^2)/(\sigma_E^2 aW_0)$ is positive and Lemma A1 implies that the latter factor is positive. The last equality stems from $\tilde{\varepsilon}_j$ and $\tilde{r}_E$ being uncorrelated, $j = 2,\dots,N$, which implies

$$E(\tilde{\underline{s}}_0 \tilde{m}) = -E(\tilde{\underline{z}}_0 \tilde{y}) = -(\underline{\underline{V}}\underline{e}_1 - V_{11}\underline{e}_1) .$$

Using equation (A1), the selectivity term,

$$\begin{aligned} E(\tilde{w}\tilde{\underline{\varepsilon}}_0^T \tilde{\underline{x}}_\beta) &= E\left[\tilde{w}\left(\tilde{\underline{\varepsilon}}_0^T \underline{\underline{V}}^{-1}\underline{e}_1 \bar{r}_E/(aW_0) - \tilde{\underline{\varepsilon}}_0^T \underline{\underline{V}}^{-1}(\beta_H V_{11}\underline{e}_1 + \underline{\sigma}_H)\right)\right] \\ &\quad + \frac{1}{aW_0} E\left[\tilde{w}\left(\tilde{\underline{\varepsilon}}_0^T \underline{\underline{V}}^{-1}\tilde{\underline{s}}_0 + \tilde{m}\tilde{\underline{\varepsilon}}_0^T \underline{\underline{V}}^{-1}\underline{e}_1\right)\right] \\ &= \frac{1}{aW_0} E_{r_E}\left[wE\left(\tilde{\underline{\varepsilon}}_0^T \underline{\underline{V}}^{-1}\tilde{\underline{s}}_0 + \tilde{m}\tilde{\underline{\varepsilon}}_0^T \underline{\underline{V}}^{-1}\underline{e}_1 \mid r_E\right)\right] \\ &= \frac{1}{aW_0} E\left[\tilde{w}\left(E(\tilde{\underline{\varepsilon}}_0^T \underline{\underline{V}}^{-1}\tilde{\underline{s}}_0) + E(\tilde{m}\tilde{\underline{\varepsilon}}_0^T \underline{\underline{V}}^{-1}\underline{e}_1)\right)\right] \\ &= \frac{1}{aW_0} E\left[\tilde{w}\left(E(\tilde{\underline{s}}_0^T \underline{\underline{V}}^{-1}\tilde{\underline{s}}_0) + E(\tilde{m}\tilde{\underline{s}}_0^T \underline{\underline{V}}^{-1}\underline{e}_1)\right)\right] > 0 . \qquad \text{(A3)} \end{aligned}$$

The second equality stems from the zero mean of $\tilde{\underline{\varepsilon}}_0$ and $\tilde{w}$ being non-stochastic conditional on $r_E$. In the line below, the independence of $\tilde{\underline{\varepsilon}}_0$ and $\tilde{r}_E$ implies the equality of the unconditional and conditional expectations. To see this, regress $\tilde{m}$ and $\tilde{\underline{s}}_0$ onto $\tilde{r}_E$ and note that the covariances of the regression residuals with $\tilde{\underline{\varepsilon}}_0$ are identical to the covariances of $\tilde{m}$ and $\tilde{\underline{s}}_0$ with $\tilde{\underline{\varepsilon}}_0$. Finally, the first interior expectation in equation (A3) is positive because its argument is a quadratic form in a positive definite matrix. The second interior expectation is positive by Lemma A1, as demonstrated in the first part of the proof. Since $w > 0$, the exterior expectation is positive as well.

Q.E.D.

<u>Proof of Proposition 4</u>: By Corollary 2 and Proposition 3, it suffices to show that $\frac{\partial \beta_p}{\partial m} > 0$ for all realizations of $\tilde{m}$ and $\tilde{\underline{s}}$. Since, by equation (A1),

$$\beta_p = \left[(\underline{e}_1^T \underline{\underline{V}}^{-1}\underline{e}_1)(\bar{r}_E + m) + \underline{e}_1^T \underline{\underline{V}}^{-1}\underline{s}_0\right]/(aW_0) - \underline{e}_1^T \underline{\underline{V}}^{-1}(\beta_H V_{11}\underline{e}_1 + \underline{\sigma}_H) ,$$

$$\frac{\partial \beta_p}{\partial m} = -\frac{1}{a^2 W_0}\frac{\partial a}{\partial m}\, aW_0\left(\beta_p + \underline{e}_1^T \underline{\underline{V}}^{-1}(\beta_H V_{11}\underline{e}_1 + \underline{\sigma}_H)\right) + \frac{1}{aW_0}\underline{e}_1^T \underline{\underline{V}}^{-1}\underline{e}_1$$

$$= -\frac{1}{a}\frac{\partial a}{\partial m}(\beta_p + \beta_H) + \frac{1}{aW_0}\underline{e}_1^T \underline{\underline{V}}^{-1}\underline{e}_1 ,$$

with the last equality following from the independence of the timing and selectivity signals, which makes the (1,1) element of $\underline{\underline{V}}^{-1}$ equal to $1/V_{11}$ and the first entry of $\underline{\sigma}_H$ zero. This expression is positive because the quadratic form on the right side is $1/\mathrm{var}(\tilde{y})$ and $\frac{\partial a(m, \underline{s})}{\partial m}$ is zero or of the opposite sign of $\beta_p + \beta_H$. To see the latter, note that if $\beta_p + \beta_H$ is positive (negative), an increase (decrease) in m, ceteris paribus, results in an expected utility increase even if the investor's portfolio weights do not change. Consequently, expected utility increases (and hence risk aversion decreases) even more after portfolio weights shift in response to these information changes.

Q.E.D.

Proposition A2: If the Rubinstein measure of absolute risk aversion, a, is constant over all the information realizations of an informed investor, then both the investor's selectivity component of abnormal performance (and hence the asymptotic Selectivity Measure) and the investor's timing component of abnormal performance (and hence the asymptotic Timing Measure) are positive.

Proof: (i) By equation (A1), the selectivity component of performance is

$$E(\tilde{s}_p)=E(\tilde{\underline{s}}_0\tilde{\underline{x}}_\beta)=E[\tilde{\underline{s}}_0^T\underline{\underline{V}}^{-1}\underline{e}_1\bar{r}_E]/(aW_0)-E[\tilde{\underline{s}}_0^T\underline{\underline{V}}^{-1}(\beta_H V_{11}\underline{e}_1+\underline{\sigma}_H)]+E[\tilde{\underline{s}}_0\underline{\underline{V}}^{-1}\tilde{\underline{s}}_0+\tilde{\underline{s}}_0^T\underline{\underline{V}}^{-1}\underline{e}_1\tilde{m}]/(aW_0)$$

$$= \frac{1}{aW_0}\left[E(\tilde{\underline{s}}_0^T\underline{\underline{V}}^{-1}\tilde{\underline{s}}_0)+E(\tilde{m}\tilde{\underline{s}}_0^T\underline{\underline{V}}^{-1}\underline{e}_1)\right] > \frac{1}{aW_0}E(\tilde{m}\tilde{\underline{s}}_0^T\underline{\underline{V}}^{-1}\underline{e}_1) = (-\underline{e}_1^T\underline{\underline{V}}+V_{11}\underline{e}_1^T)\underline{\underline{V}}^{-1}\underline{e}_1 ,$$

which is positive by Lemma A1. The last equality stems from $\tilde{\varepsilon}_j$ and $\tilde{r}_E$ being uncorrelated, $j = 2,\ldots,N$, which implies

$$E(\tilde{m}\tilde{\underline{s}}_0^T) = -E(\tilde{y}\tilde{\underline{z}}_0^T) = -(\underline{e}_1^T\underline{\underline{V}} - V_{11}\underline{e}_1^T) .$$

(ii) From equation (A1), the timing component,

$$E(\tilde{\beta}_p\tilde{m}) = E\left[\frac{1}{aW_0}\, \underline{e}_1^T \underline{\underline{V}}_1^{-1}(\underline{e}_1\tilde{m} + \underline{\tilde{s}}_0)\tilde{m}\right] + E\left[\underline{e}_1^T \underline{\underline{V}}_1^{-1}\underline{e}_1 \bar{r}_E\tilde{m}/(aW_0) + \underline{e}_1^T \underline{\underline{V}}_1^{-1}(\beta_H V_{11}\underline{e}_1 + \underline{\sigma}_H)\tilde{m}\right]$$

$$= \frac{1}{aW_0}\left[\underline{e}_1^T \underline{\underline{V}}_1^{-1}\underline{e}_1 \mathrm{var}(\tilde{m}) + \underline{e}_1^T \underline{\underline{V}}_1^{-1} E(\underline{\tilde{s}}_0\tilde{m})\right] > \frac{1}{aW_0}\, \underline{e}_1^T \underline{\underline{V}}_1^{-1} E(\underline{\tilde{s}}_0\tilde{m}) ,$$

which is positive, as shown in part (i).

Q.E.D.

Proof of Proposition 5: To simplify the proof, without loss of generality redefine the tradable primitive assets in the economy so that

$$\mathrm{cov}(\beta_j\tilde{y} + \tilde{z}_j, \beta_i\tilde{y} + \tilde{z}_i) = 0 \quad \text{for} \quad i \neq j .$$

By forming portfolios, one can always repackage the primitive assets to have this covariance structure, although this will not permit the repackaging used for the other proofs. Investors are indifferent to such repackagings since they can be undone in their personal portfolios. Moreover, if the sum of the timing and selectivity components of performance is positive in the repackaged economy, it is positive in the original economy because the two sums are identical.

This repackaging allows us to rewrite the standard first order condition, equation (8), as

$$\bar{R}_j + \beta_j m + s_j = aW_0 x_j \mathrm{var}(\beta_j\tilde{y} + \tilde{z}_j) . \tag{A4}$$

Without loss of generality, one can express the deviations of conditional mean returns from unconditional mean returns with the factor model

$$\beta_j\tilde{m} + \tilde{s}_j = \sum_{i=1}^{N} \gamma_{ji}\tilde{f}_i \quad \text{for} \quad j = 1,\ldots,N ,$$

where the normally distributed factors $\tilde{f}_1,\ldots,\tilde{f}_N$ are normalized to have zero mean and zero covariance with each other. There is no loss of generality here because the number of factors equals the number of assets, merely implying a change to an orthogonal basis.

Taking the partial derivative of equation (A4) with respect to $f_k$, holding $f_i$ constant for $i \neq k$, implies

$$\gamma_{jk} = aW_0 \operatorname{var}(\beta_j \tilde{y} + \tilde{z}_j)\left[\frac{x_j}{a}\frac{\partial a}{\partial f_k} + \frac{\partial x_j}{\partial f_k}\right] , \quad \text{or}$$

$$\gamma_{jk}\frac{\partial x_j}{\partial f_k} = \frac{\gamma_{jk}^2}{aW_0 \operatorname{var}(\beta_j \tilde{y} + \tilde{z}_j)} - \frac{x_j \gamma_{jk}}{a}\frac{\partial a}{\partial f_k} .$$

Summing over j and noting that $\gamma_{jk}$ is a constant yields

$$\sum_{j=1}^{N} \frac{\partial(x_j \gamma_{jk})}{\partial f_k} = \frac{1}{a}\left[\sum_{j=1}^{N} \frac{\gamma_{jk}^2}{W_0 \operatorname{var}(\beta_j \tilde{y} + \tilde{z}_j)} - \frac{\partial a}{\partial f_k}\sum_{j=1}^{N} x_j \gamma_{jk}\right] .$$

This is positive because with nonincreasing Rubinstein absolute risk aversion, $\frac{\partial a}{\partial f_k}$ is zero or of the opposite sign of $\sum_{j=1}^{N} x_j \gamma_{jk}$. Thus, the sum of the timing and selectivity components of performance,

$$E\left(\sum_{j=1}^{N} \tilde{x}_j(\tilde{R}_j - \bar{R}_j)\right) = E\left(\sum_{k=1}^{N}\sum_{j=1}^{N} \tilde{x}_j \gamma_{jk} \tilde{f}_k\right) ,$$

is positive because

$$E\left(\sum_{j=1}^{N} \tilde{x}_j \gamma_{jk} \tilde{f}_k\right) = E_{f_i, i\neq k} \operatorname{cov}\left(\sum_{j=1}^{N} \tilde{x}_j \gamma_{jk}, \tilde{f}_k \mid f_i, i\neq k\right),$$

and the conditional covariance is positive because

$$\frac{\partial\left(\sum_{j=1}^{N} x_j \gamma_{jk}\right)}{\partial f_k} = \sum_{j=1}^{N} \frac{\partial(x_j \gamma_{jk})}{\partial f_k} > 0 , \qquad \text{as shown above .}$$

Q.E.D.

## ENDNOTES

[1]For the analysis of some of the performance measures, this strong stationarity assumption can be relaxed somewhat. This is discussed in more detail in the conclusion.

[2]Copeland and Mayers (1982) pointed out that because trading strategies may be based on past returns, the Cornell approach should be modified by measuring securities' benchmark returns in periods after they were held by investors, rather than before. This, however, introduces survivorship bias, which the former authors argue is negligible in their study. Their study implements a more powerful version of the Cornell Measure by employing the market model. Since the properties of this measure are virtually the same as those of the Cornell Measure, it is not directly analyzed in the paper.

[3]It would be better if abnormal performance measures could also select the more informed of two informed investors. Unfortunately, risk aversion and preferences for higher order moments also affect these measures, making it impossible, except for special cases, to extract the information related component of performance. See, for example, Henriksson and Merton (1981), Admati and Ross (1985), Admati, Bhattacharya, Pfleiderer, and Ross (1986), and Connor and Korajczyk (1986).

[4]Dybvig and Ross prove a special case of Corollary 1 without the normality assumption.

[5]In the latter case, the benchmark portfolio that will satisfy the criteria discussed above is the optimal portfolio of the investor if he does not receive superior information. A marginal utility condition, based on this portfolio, can be used to develop a measure that has the desirable properties discussed above. This is discussed further in footnote 11.

[6]The distinction between one period mean and asymptotic sample mean does not exist when the random variables under consideration are i.i.d. Here, however, variables like $\beta_{pt}$ and $\varepsilon_{pt}$ depend on the derived utility of wealth function $U_t(\ )$, which may vary from period to period.

[7]$\text{cov}(\tilde{u}, g(\tilde{v})) = E(g'(\tilde{v}))\text{cov}(\tilde{u}, \tilde{v})$ if $g(\cdot)$ is continuously differentiable, all expectations are finite, and $(\tilde{u}, \tilde{v})$ is bivariate normally distributed. See Stein (1973) or Rubinstein (1973) for a proof.

[8]The intermediate algebraic steps are straightforward. For more detail, see Grinblatt and Titman (1983, p. 501).

[9]See, for example, Jensen (1972) and Admati, Bhattacharya, Pfleiderer, and Ross (1986).

[10]The intercept in the Treynor and Mazuy (1966) regression also falls in this class and is subject to the same bias-in-beta problem as the Jensen Measure. For brevity's sake, we will forego the analysis that demonstrates this.

[11]This is because $E(U'(\tilde{W}^u)\tilde{r}_p) > 0$ for concave utility functions, where $r_p$ represents the excess return of the portfolio of an informed investor with utility function U and $W^u$ represents the wealth from the optimal investment of an uninformed investor with the same utility function. The proof is available on request. In addition (irrespective of risk aversion or endowments), the ratio of the marginal utilities of two optimizing quadratic utility investors with the same opportunity set is constant across states of nature. This proportionality implies that if one uninformed quadratic investor desires a bit of some portfolio, all uninformed quadratic utility investors desire a bit of it.

One can infer from this that Verrecchia's "counterexample," in which an informed quadratic utility investor displays a negative risk-adjusted return, does not apply to the Jensen Measure. It is informative to confirm this numerically. For Verrecchia's parametization of the example, we have computed a positive bias-in-beta component of performance, which more than outweighs the negative component of performance from timing plus selectivity. We computed the Jensen Measure to be .0005 in Verrecchia's counterexample, which has correlated timing and selectivity information. In this example, the selectivity component is negative and larger in absolute magnitude than the timing component, which is positive.

[12]Proposition A2 in the Appendix demonstrates that with constant absolute risk aversion, the selectivity component of performance is positive, even with correlated timing and selectivity information.

[13]Verrecchia's example is widely regarded as an example where the Jensen Measure is negative. It is, however, an example where the Cornell Measure (and not the Jensen Measure) is negative. This is because Verrecchia implicitly assumes that the bias-in-beta component is zero.

[14]Let $\underline{\tilde{v}} = (\tilde{v}_1,\ldots,\tilde{v}_n)$. $\mathrm{cov}(\tilde{u}, g(\underline{\tilde{v}})) = \sum_{j=1}^{N} E(\frac{\partial g}{\partial v_j})\mathrm{cov}(\tilde{u}, \tilde{v}_j)$ if $g(\cdot)$ is a continuously differentiable function, all expectations are finite, and $(\tilde{u}, \underline{\tilde{v}})$ is multivariate normally distributed. See Losq and Chateau (1982) for a proof.

[15]In a complete markets framework, Verrecchia (1980) implicitly demonstrated that the sum of the timing and selectivity components of the portfolio of an informed investor with constant absolute or relative risk aversion is positive, even if the timing and selectivity signals are correlated.

[16]Furthermore, Proposition A2 in the Appendix demonstrates that with constant absolute risk aversion, the timing component of abnormal performance is positive, even with correlated timing and selectivity information.

[17]In both cases, there are occasional realizations of information for which the investor is shorting an asset (but shorting less than for the average information realization) when he knows its expected return will be slightly larger than average. This is because the short position hedges unforecastable changes in the value of other assets in his portfolio--in the case of Proposition 5's generalization, the non-traded assets. For these realizations, an increase in the expected return of the asset decreases expected utility and makes him more risk averse. With hedging now relatively

more valuable, he may increase his short position in response to the information rather than decrease it.

[18]This assumes that the covariance between the personally managed portion of the client's wealth and that portion under the jurisdiction of the manager is independent of the manager's information.

[19]Performance may also be detectable with an efficient benchmark portfolio that includes additional assets not contained in the investor's choice set. For instance, if the investor's information provides no information about either the unconditional mean-variance efficient portfolio of assets within his choice set or the larger portfolio used as the benchmark, our propositions apply to measures that use the larger benchmark. In such cases, the efficient portfolio of NYSE stocks is also an appropriate benchmark for evaluating a portfolio of NYSE oil stocks. We have not, however, been able to determine the extent to which this can be generalized.

[20]See, for example, Cornell (1979, p. 390) and Wallace (1980).

[21]Two of the best performing mutual funds (in terms of total return) over the last ten years, Fidelity Magellan and Mutual Shares, followed trading rules that were very similar to this. Other mutual funds make trades based on P/E ratios, (i.e., they buy particular stocks when their P/E ratios are below some historical average), which also has a systematic tendency to purchase securities when they are riskier than average.

[22]Let $g(m|r_E)$ represent the conditional density function of $\tilde{m}$ given $\tilde{r}_E = r_E$. $\tilde{m}$ and $\tilde{r}_E$ are normally distributed. Thus, for any constant $c > 0$, there exists a unique critical value $m^* = m^*(r_E, c)$ where the conditional density functions $g(m|r_E)$ and $g(m|r_E + c)$ are equal (i.e., where they cross). For all $m > m^*$, $g(m|r_E + c) > g(m|r_E)$. For $m < m^*$, $g(m|r_E + c) < g(m|r_E)$. This implies that the conditional expectation of m given $\tilde{r}_E = r_E + c$ exceeds the conditional expectation of m given $\tilde{r}_E = r_E$. It also implies that any monotonic increasing function of m has the same property. Since $\beta_p(m, \underline{s})$ is a monotonic increasing function of m, and since $\tilde{\underline{s}}$ is independent of $\tilde{r}_E$, $E(\tilde{\beta}_p|r_E + c) > E(\tilde{\beta}_p|r_E)$.

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