A POSITIVE THEORY OF FOREIGN CURRENCY DEBT

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Abstract

The paper makes a case for foreign currency debt as a hedging device in an open economy subject to stochastic shocks to output. A government can reduce uncertainty in net wealth and in consumption by issuing foreign or domestic currency debt, if unexpected domestic and foreign inflation are negatively correlated with domestic output.

Foreign currency debt is desirable in comparison to domestic currency debt, if growth rates of output of both countries are closely related and if domestic inflation is relatively uncertain. In addition, foreign currency debt replaces domestic currency debt as hedge, whenever issuing domestic debt is prevented or discouraged by incentive problems.

It also shows that time-consistency problems may motivate capital controls or taxes on international borrowing.

1. Introduction

Many countries are in debt to the rest of the world. A large body of basic economic literature addresses the question of which countries are debtors and which are creditors. Much less has been said about the contractual terms of the debt, which is the focus of this paper.

Most debt contracts express the repayment of obligations in terms of fiat money, domestic or foreign. Hence, debt is state contingent in the sense that it depends on the real value of the currency at maturity. Why do countries show a preference for such types of state-contingencies?

The case for foreign currency debt is based on the interaction of hedging and incentive considerations. The hedging argument is an application of basic portfolio analysis in a general equilibrium setting (see Arrow (1971), Fischer (1975), Lucas (1978)). State-contingent debt is preferred to indexed debt (the risk free security in real terms), if fluctuations in the real value of state-contingent debt provide a hedge against other shocks to the economy.[1] Here the uncertainty is about output.[2] Nominal debt, foreign currency debt, or some combination of both is useful, if domestic output is negatively correlated with domestic and foreign inflation.

If domestic and foreign inflation are positively correlated, the two types of debt are substitutes in the government's liability portfolio. It depends on the structure of macroeconomic shocks, which of them is preferred. Domestic currency debt has an advantage over foreign currency debt as a hedging device, if domestic shocks are "more closely" correlated with domestic than foreign inflation. Foreign currency debt has an advantage, if the domestic monetary sector is more "noisy" than the foreign one while output is correlated internationally. The optimal amount of foreign currency debt issued for hedging purposes can be positive or negative, depending on the exact structure of worldwide macroeconomic disturbances.

The second argument for foreign currency debt is based on the fact that a government's ability to manipulate the domestic price level can be a significant obstacle against issuing nominal liabilities (Kydland and Prescott (1977), Calvo (1978)). If investors must fear that the government will inflate the domestic currency as soon as it is a net debtor, foreigners may refuse to buy its nominal debt or they may demand high nominal interest rates.[3] When such a time consistency problem exists, foreign currency debt has an additional advantage over nominal debt and can take over more of the hedging role.

Optimal debt policy in an environment with uncertainty and incentive problems has been analyzed previously by Lucas and Stokey (1983) and Bohn (1988). Lucas and Stokey focus on the incentive aspect.[4] They show that the optimal debt policy can solve all incentive problems and achieve the first-best allocation in a barter economy, but not in a monetary economy. The important implications for this paper are that, in our monetary economy, debt policy is linked to incentives and that nominal debt leads to a second-best outcome. Bohn (1988) shows in a closed economy model that hedging considerations make some nominal liabilities desirable even in the presence of time-inconsistency problems. However, the choice in that paper is between two alternatives that may both be relatively "bad": the government gives up hedging by issuing indexed debt or incurs incentive problems by issuing nominal debt.

The point of this paper is to identify foreign currency debt as a simple state-contingent contract that combines the benefits of both alternative choices. Foreign currency debt has desirable hedging properties and does not create incentive problems. The paper generalizes the main messages of Bohn (1988) that both hedging and incentive arguments must be considered in finding an optimal government policy on financial markets and that nominal liabilities (in some currency) have a role in the optimal portfolio.

Several issues arise in modeling. First, we have to specify the game. We have a (domestic) government of a small country, its domestic individuals, and foreign investors. Infinitely lived, risk averse domestic individuals maximize an intertemporal utility function over a single, perishable consumption good. The government maximizes their welfare. It may issue indexed debt, debt denominated in the domestic currency (referred to as nominal debt) and debt denominated in a foreign currency.[5] Demand for money is generated through a cash-in-advance constraint. To abstract from issues of asset pricing, we assume that foreign investors are risk neutral and that they buy any quantity of bonds, provided their expected rate of return is sufficient.[6] The government also raises lump sum taxes and it can print money.

For most of the discussion, we assume that domestic individuals do not have direct access to foreign capital markets. This may approximate the situation in countries with capital controls. Assuming taxation is lump-sum, private and government borrowing are perfect substitutes (see Barro (1974)) so that individuals can rely on optimal government borrowing.[7] The key question is how the government should structure its debt so that risk neutral foreign investors insure its residents against economic uncertainty.

A critical assumption is that the government cannot commit itself to limit the future supply of money. This raises a problem because, without further assumptions, there may not be an optimal supply (short of infinity).[8] In the main text, we therefore adopt Barro and Gordon's (1983a) ad-hoc assumption that there is some welfare-cost of inflation. But we show in an appendix that inflation is indeed costly in a slightly generalized model with heterogeneity in individual money holdings. This formal model of why even anticipated inflation is costly may be interesting in itself. In any case, a time-consistent equilibrium must have a path of money supply and inflation that balances the cost of inflation with the benefit of devaluing external nominal debt.

The paper is organized as follows. Section 2 sets up the model. Section 3 shows under what conditions nominal and foreign currency debt can serve to hedge against macroeconomic shocks. Incentive problems are excluded in this section. We derive the first-best policy that achieves optimal hedging. The general game between an opportunistic government and the investor's problem is formulated in Section 4. We characterize the effect of incentives on the optimal structure of debt and compute the optimal policy for the class of quadratic utility functions. Section 5 presents a brief discussion of the strategic implications of private borrowing from foreigners. We show that capital controls or taxes on international borrowing can be justified by time-consistency problems. In Section 6, we explore some generalizations of the macroeconomic model. Section 7 summarizes the conclusions and the appendix presents our model of costly inflation.

2. A Framework for Analysis

We consider a small economy with identical, infinitely lived individuals. Each individual is endowed with a stochastic stream of endowments, Y_t, and has an intertemporal utility function over a consumption good, c_t,

$$U_t = \sum_{i=0}^{\infty} \beta^i \cdot u(c_{t+i}), \quad u' > 0,\ u'' < 0 . \tag{1}$$

Notice that individuals are risk averse and that their expected utility in period t is $E_t[U_t]$. The government maximizes social welfare, i.e., the expected utility of the representative individual.

To introduce money, we assume that individuals cannot consume or store their own endowments but must sell them to others (individuals, foreigners, or the government) for immediate consumption. All purchases are made with checks denominated in the nominal unit of account drawn on the government's clearing bank. Check-writers must have money, meaning account balances, in the bank. Depositors of checks either obtain immediate credit on their account (fraction $1 - v_t$ of all payments) or experience a technical delay in clearing (fraction v_t). In case of delay, the check is not credited until goods markets have closed for that period. Thus, individuals are stuck with money balances

$$M_t = v_t \cdot p_t \cdot Y_t \, , \tag{2}$$

in their accounts, where p_t is the price of the consumption good and v_t can be interpreted as the inverse of velocity.[9] Since the government determines the nominal amount of money M_t, equation (2) determines the price level p_t.

To complete the individuals' problem, we denote taxes by T_t and assume that they are lump-sum.[10] Since the distinction between private and government debt is then irrelevant (see Barro (1974)), one can assume without loss of generality that there is no internal debt (except for money) and that only the government borrows from foreigners. Thus, individual consumption is

$$c_t = (1 - v_t) \cdot Y_t + M_{t-1}/p_t - T_t \, . \tag{3}$$

The government can issue 3 types of debt claims to foreigners: indexed bonds, domestic nominal bonds, and foreign currency bonds.[11] All bonds are and foreign currency bonds as bonds that pay interest in real terms so that only the real value of the principal is stochastic.[12] Indexed (safe) bonds, S_t, require a payment of $(1 + r)$ units of consumption in period $t + 1$. Nominal bonds, N_t, promise a payment of $1 + i_t - \pi_{t+1}$ in real terms, where i_t is the nominal interest rate on debt issued in period t and π_{t+1} is the rate of inflation between periods t and t+1. Foreign currency bonds, F_t, promise an interest payment of i^*_t and are repaid in the foreign currency. Throughout the paper, an asterisk (*) will denote foreign variables. The government budget constraint requires that old government debt (to foreigners) is financed by new debt, taxes, or money creation. In real terms, this is

$$T_t + S_t + N_t + F_t + M_t/p_t = (1 + r) \cdot S_{t-1} + (1 + i_{t-1} - \pi_t) \cdot N_{t-1} + (1 + i^*_{t-1} - \pi^*_t) \cdot F_{t-1} + M_{t-1}/p_t \, . \tag{4}$$

Output and velocity are stochastic. Denote the growth rates of money, prices, output, and velocity between periods t-1 and t by m_t, π_t, y_t, and α_t respectively. We assume that y_t and α_t are independent white-noise processes with means and standard deviations (Ey, σ_y) and $(0, \sigma_\alpha)$.[13] From equation (2), inflation is determined by money growth and these two random determinants, $\pi_t = m_t - y_t - \alpha_t$. We assume that foreign inflation is determined in a similar way, $\pi^*_t = m^*_t - y^*_t - \alpha^*_t$, where y^*_t and α^*_t are independent and white-noise.[14] It is important that domestic and foreign shocks may be correlated. Most of the analysis holds for general correlation patterns; but sometimes we will assume that monetary and real shocks have both country specific (denoted by ν and ψ) and common worldwide components (denoted η and μ), so that one can write

Assumption:

$$y_t = \psi_t + \mu_t + Ey , \qquad \alpha_t = \nu_t + \eta_y ,$$
$$y^*_t = \psi^*_t + \mu_t + Ey^*, \qquad \alpha_t = \nu^*_t + \eta_t . \tag{5}$$

The random variables ν, ψ, ν*, ψ, η and μ are independent and white-noise with mean zero and finite second moments. Whenever these specific assumptions on shocks are needed, we will refer to assumption (5). Foreign money supply is considered exogenous by the domestic government (running a small country) so that m^*_t can be treated as a parameter.

We further assume that we are in a one-good world, i.e., one unit of the foreign good is equivalent to one unit of the domestic good. This is a key simplification. It implies purchasing power parity and allows us to sidestep issues related to changes in relative prices of tradeable versus non-tradable or import versus export goods. Simple randomness in real exchange rates is discussed in Section 6.

Purchasing power parity implies that the nominal exchange rate must change by the difference in inflation rates. Then a foreign currency bond promises a real payment of $1 + i^*_t - \pi^*_{t+1}$. Foreign investors are risk neutral and they are willing to absorb any amount and type of debt, if and only if interest rates are such that

$$i_t = r + E_t\pi_{t+1} , \quad \text{and} \quad i^*_t = r + E_t\pi^*_{t+1} . \tag{6}$$

All international payments are settled with money (clearing account balances). The government repays debt by crediting foreigners with positive balances of domestic money, which they use immediately to buy domestic goods. In case of new loans, the government buys goods abroad with foreign money obtained from creditors, which the government will immediately pass on to consumers (without additional use of money).[15]

Since all bonds pay the same expected return, the budget constraints simplify. If we define $D_t = S_t + N_t + F_t$ as the total amount of bonds issued, the government budget constraint (4) becomes

$$T_t = (1 + r) \cdot D_{t-1} - D_t + M_{t-1}/p_t - M_t/p_t + \hat{\pi}_t \cdot N_{t-1} + \hat{\pi}^*_t \cdot F_{t-1}\ , \tag{7}$$

where $\hat{\pi}_t = \pi_t - E_{t-1}\pi_t$ and $\hat{\pi}^*_t = \pi^*_t - E_{t-1}\pi^*_t$. Nominal and foreign currency debt then enter only through unexpected changes in inflation rates. Only total bonds D_t and D_{t-1} matter in expectation. Using equation (3) for consumption, we obtain a national budget constraint

$$\begin{aligned} c_t &= Y_t + M_{t-1}/p_t - M_t/p_t - T_t \\ &= Y_t + (1 + r) \cdot D_{t-1} - D_t + \hat{\pi}_t \cdot N_{t-1} + \hat{\pi}^*_t \cdot F_{t-1}\ . \end{aligned} \tag{8}$$

The equation shows directly how consumption depends on policy. In particular, the uncertainty in consumption created by stochastic output can be reduced by an appropriate choice of nominal and foreign currency debt, provided innovations in output and rates of inflation are correlated. This is the hedging problem.

3. Hedging against Macroeconomic Shocks

The government derives its optimal policy under the influence of two sets of determinants. The first consideration is that its residents are risk averse and exposed to changes in output, while foreign investors are ready to take risk and insure domestic residents. Second, the government has to consider the potential incentive problems caused by its power to print money.

The hedging aspect is valid whether or not the government can pre-commit monetary policy. We will therefore abstract from incentive issues in this section and concentrate on the government policy that uses financial assets to hedge against macroeconomic risks in an optimal way.

In this section only, assume that money growth follows some preannounced path that the government can commit to. This may even be realistic in some countries with independent central banks or strong reputation for not breaking commitments. For those countries, the analysis of this section is directly applicable. For countries with discretionary monetary policy, we will see that the arguments become a part of the general analysis in Section 4.

Optimal debt policy is characterized by the first order conditions of maximizing (1) subject to the budget constraint (8). Past values of debt and the realization of period t shocks are given. The first order conditions for D_t, F_t, and N_t, are

$$U_D = u'(c_t) - \beta \cdot E_t[u'(c_{t+1}) \cdot (1 + r)] = 0 \qquad (9d)$$

$$U_F = \beta \cdot E_t[u'(c_{t+1}) \cdot \hat{\pi}^*_{t+1}] = 0 \qquad (9f)$$

$$U_N = \beta \cdot E_t[u'(c_{t+1}) \cdot \hat{\pi}_{t+1}] = 0 \qquad (7n)$$

where derivatives of u are denoted by u'.

The path of total debt and consumption (or marginal utility) is determined in (9d) under the influence of the real rate r and the rate of time preference in the usual way (see, e.g., Lucas (1978)). We assume that a unique solution for consumption and debt exists.

The net supply of the "risky assets" N_t and F_t is given in (9f) and (9n). Given risk neutral investors and risk averse taxpayers, optimal risk sharing requires that bonds are issued (or purchased) until the covariance of taxpayer's marginal utility with unexpected returns is zero.

Our goal is to determine the optimal levels of nominal and foreign currency debt. In particular, will a country issue bonds in terms domestic and/or foreign currency as opposed to holding them as assets, and under what conditions?

We can address these questions, because we know how future consumption is determined in the equilibrium model as a function of debt structure and macroeconomic shocks (cf. equation (8)). However, it is difficult to evaluate the first order conditions in general. Following standard asset pricing theory, we have to make some additional assumptions on u(c) and/or on the distributions of shocks to show precise results. To save space, we concentrate on one set of assumptions. Alternatives are discussed in Appendix 2. Define

Assumption: u(c) is quadratic, $Ey < r$, and $\beta \cdot (1 + r)^2 > 1$. (10)

This will be referenced as assumption (10) below. Then the main result of this section can be stated as

Proposition 1: Optimal Hedging

a. Under assumption (10), we have

$$N_t = N_{hedge} = \kappa \cdot Y_t \cdot a_n/a_0 \quad \text{and}$$

$$F_t = F_{hedge} = \kappa \cdot Y_t \cdot a_f/a_0 \text{ , where}$$

$$a_0 = var(\pi^*) \cdot var(\pi) - cov(\pi, \pi^*)^2 \text{ ,}$$

$$a_n = -cov(y, \pi) \cdot var(\pi^*) + cov(y, \pi^*) \cdot cov(\pi, \pi^*) \text{ ,}$$

$$a_f = -cov(y, \pi^*) \cdot var(\pi) + cov(y, \pi) \cdot cov(\pi, \pi^*) \text{ , and}$$

$$\kappa = (1 + r)/(r - Ey) \text{ .}$$

b. Under the additional assumption (5), we have moreover

$$a_0 = \mathrm{var}(\pi^*) \cdot \mathrm{var}(\psi + \nu) + \mathrm{var}(\mu + \eta) \cdot \mathrm{var}(\psi^* + \nu^*) > 0 ,$$

$$a_n = \mathrm{var}(\pi^*) \cdot \mathrm{var}(\psi) + \mathrm{var}(\mu) \cdot \mathrm{var}(\psi^*) > 0 ,$$

$$a_f = \mathrm{var}(\mu) \cdot \mathrm{var}(\nu) - \mathrm{var}(\psi) \cdot \mathrm{var}(\eta) .$$

Proof: see Appendix 1. ‖

Part (a) is a characterization of optimal policy that is fairly independent of the stochastic structure of the macro-model (see Section 6). Notice that assumption (10) is sufficient, but far from necessary (see Appendix 2). Part (b) is clearly specific to the macro-model, but very strong. Given assumption (5), a government should always issue nominal debt.

Intuitively, some "risky" nominal or foreign currency debt should be issued, if the marginal benefits are positive at $N_t = F_t = 0$ ($U_N > 0$ or $U_F > 0$). If $N_t = F_t = 0$, innovations in consumption are perfectly correlated with innovations in output. The desire to stabilize consumption therefore provides an incentive to issue nominal debt, if domestic inflation is negatively correlated with domestic output. This is the case in most cash-in-advance models; $N_t>0$ is not too surprising. More interestingly, the same argument justifies foreign currency debt, if foreign inflation is negatively correlated with domestic output. This correlation should generally be expected in an interdependent world economy; it is guaranteed by assumption (5).

The amount of foreign currency F_t depends on the structure of shocks. It is large, if output of different countries is closely related (σ_μ large, and σ_ψ small)--giving foreign currency debt a role as hedge--and if monetary disturbances have a large domestic component (ν) relative to the international one (η)--giving foreign currency debt an advantage over nominal debt as hedge. Thus, foreign currency debt is most useful for a country with an unstable monetary system that is exposed to world business cycles. The foreign country should be one with a stable monetary system and with output closely related to domestic output.

Incentive problems will force us to add other determinants of nominal debt later. In preparation, it is useful to know how a change in nominal debt affects the optimal level of foreign currency debt. Portfolio theory suggests that nominal and foreign currency bonds are substitutes in the portfolio, if and only if their returns--rates of inflation--are positively correlated. Positively correlated inflation rates across countries seem empirically plausible and they are definitely satisfied under assumption (5), since then $\mathrm{cov}(\pi, \pi^*) = \sigma_\mu^2 + \sigma_\eta^2 > 0$. The formal result is as follows.

Proposition 2: Substitution

a. Nominal and foreign currency debt are substitutes in the government's portfolio, if

$$E_t[u''(c_{t+1}) \cdot c_D \cdot \hat{\pi}_{t+1} \cdot \hat{\pi}^*_{t+1}] > 0 ,$$

where c_D is the derivative of optimal consumption with respect to initial debt.

b. Given assumption (10), the optimal value of F_t conditional on N_t is

$$F_t = F_{hedge} - \mathrm{cov}(\pi, \pi^*)/\mathrm{var}(\pi^*) \cdot (N_t - N_{hedge}) , \qquad (11)$$

where F_{hedge} and N_{hedge} are defined in Proposition 1.

Proof: By taking the total differential of (9n) or (9f). ‖

The expression in part (a) can be interpreted as a "weighted" covariance of inflation rates, where the inflation terms are weighted by the derivative of marginal utility with respect to initial debt. A positive sign can be expected if the covariance of inflation rates is positive and if u" and c_D do not vary "too much." The exact solution in part (b) is obtained when the weights are constant; alternative sufficient conditions for (11) are stated in Appendix 2.

4. Incentive Problems

In this section, we derive the optimal monetary and debt policy in an environment in which the government cannot pre-commit future policy. Everybody expects the government to optimize subject to the constraints at the time of decision.

4.1. The Game

The government plays a repeated game against foreign investors. Every period, the government has to determine optimal values of total debt D_t, the amount of nominal and foreign currency debt N_t and F_t in this total, and the growth rate of money supply m_t. We assume that money supply is determined early in the period, before the shocks are observed. After shocks are observed, the government chooses taxes and the level and structure of debt.[16]

Foreign investors set interest rates. Given current and past debt policy, they form rational expectations about future policy and future stochastic shocks. This determines expected inflation for period t+1, and therefore interest rates required in period t. As we will see, the current level and structure of debt affects optimal money growth in period t+1. Therefore, actual and expected inflation depend on current debt policy. The government has to keep this in mind when making its period-t choice.

The structure of the game remains unchanged over time and investors' decisions are completely characterized by the pricing equations (6). Thus, we can focus on the government's decision in some period t.[17]

Endogenous monetary policy creates an existence problem whenever nominal liabilities are positive. Without further assumptions, optimal money supply is infinite, i.e., nominal liabilities cannot exist in equilibrium (see Calvo (1978), Lucas and Stokey (1983)). Most economists would probably agree that inflation has some cost, though there is disagreement precisely what these costs are. For the main discussion, we will therefore follow Barro and Gordon (1983a) and assume that there is some utility cost of inflation, say, because individuals simply have an aversion against tracking rapidly changing prices. We denote this cost by a function $k(\pi)$, where $k' > 0$ and $k'' > 0$ for $\pi > 0$ and rewrite preferences as

$$U_t = \sum_{i=0}^{\infty} \beta^i \cdot \left[u(c_{t+i}) - k(\pi_{t+i})\right] , \quad u' > 0 , \quad u'' < 0 \qquad (12)$$

which replaces equation (1).

In Appendix 3, we present a slightly generalized model in which the cost of inflation is derived from basic assumptions on trading opportunities. Since the generalized model has a much more complicated notation and yields exactly the same policy implications as the $k(\pi)$- function, we use the simple ad-hoc version here.

To determine period-t outcomes, we have to look forward. Let V_{t+1} be the value of the game for the government in period t+1, i.e.,

$$V_{t+1} = \max E_{t+1} \sum_{i=0}^{\infty} \beta^i \cdot \left[u(c_{t+1+i}) - k(\pi_{t+1+i})\right] .$$

The value is limited by the budget constraint (8). Define current wealth

$$W_{t+1} = Y_{t+1} - (1 + r) \cdot D_t + \hat{\pi}_{t+1} \cdot N_t + \hat{\pi}^*_{t+1} \cdot F_t ,$$

then the budget constraint (8) reduces to

Given white-noise disturbances, the variable W_{t+1} and the level of output Y_{t+1} are the only state variables at that time.[18] Hence, the value is a (time independent) function $V_{t+1} = V(W_{t+1}, Y_{t+1})$.

Money supply in period t+1 maximizes V_{t+1} at the start of period t+1, i.e., it solves max $E_t[V(W_{t+1}, Y_{t+1})]$. The state variables at that time are level and structure of initial debt, D_t, N_t, and F_t, output Y_t, and interest rates i_t and i_t^* (the latter does not play a significant role). Instead of interest rates, we can use the level of investors' inflationary expectations $\pi_t^e = E_t\pi_{t+1} = i_t - r$ as state variable. Thus, the value of the game at the start of period t+1 is a function V^m,

$$V_{t+1}^m = \max E_t[V(W_{t+1}, Y_{t+1})] = V^m(D_t, N_t, F_t, Y_t, \pi_t^e) .$$

Optimal money supply is a function $m_{t+1} = m(D_t, N_t, F_t, Y_t, \pi_t^e)$. We will characterize the function below.

Investors determine inflationary expectations rationally, i.e.,

$$\pi_t^e = E_t\pi_{t+1} = E_t(m_{t+1} - y_{t+1} - \alpha_{t+1}) = m(D_t, N_t, F_t, Y_t, \pi_t^e) - Ey .$$

If the derivative of $m(\cdot)$ with respect to π^e is less than one in absolute value, π_t^e is the unique fixed point of this equation. It is clearly a function of initial debt, i.e., $\pi_t^e = \pi^e(D_t, N_t, F_t, Y_t)$.

Optimal debt policy in period t must then maximize

$$u(c_t) - k(\pi_t) + \beta \cdot V^m(D_t , N_t, F_t, Y_t, \pi^e(D_t, N_t, F_t, Y_t)) \qquad (14)$$

subject to the period-t equivalent of (13). The results are optimal values of total, nominal, and foreign currency debt.

4.2. Optimal Policy

We are interested in characterizing optimal policy rather than studying the existence of solutions. Therefore, we assume that value functions V and V^m exist and that they are differentiable and strictly concave.[19] Optimal policy is then characterized by the first order conditions of maximizing (14) subject to the budget constraint, taking into account investors' expectational reactions. Past values of debt and endowments, money supply, and the realizations of period t shocks are given as initial conditions.

The first order conditions for D_t, F_t, and N_t, are (after using the envelope theorem to eliminate derivatives of the value functions)

$$U_D = u'(c_t) - \beta \cdot E_t[u'(c_{t+1}) \cdot (1+r + \pi_D \cdot N_t)] = 0 \qquad (15d)$$

$$U_F = \beta \cdot E_t[u'(c_{t+1}) \cdot (\hat{\pi}^*_{t+1} - \pi_F \cdot N_t)] = 0 \qquad (15f)$$

$$U_N = \beta \cdot E_t[u'(c_{t+1}) \cdot (\hat{\pi}_{t+1} - \pi_N \cdot N_t)] = 0 \qquad (15n)$$

where π_D, π_F, and π_N denote the marginal changes in expected inflation caused by increases D_t, F_t, and N_t, respectively.

Comparing these equations to the first order conditions (9) shows that the problem has the same structure as before, except for the expected inflation effects. A country with discretionary monetary policy faces the same considerations of intertemporal allocation and hedging as a country that can commit itself not to manipulate monetary policy in response to changes in government debt. Discretionary monetary policy complicates the hedging problem because it is more difficult to make debt contingent on variables that the government can manipulate. By construction, this temptation exists only for domestic inflation. The government has no influence over real rates and

The values of the derivatives π_D, π_F, and π_N are crucial in determining how much and in which direction the incentive problem modifies the previous results. A positive derivative of expected inflation indicates a marginal change of welfare (a loss, if $N_t > 0$) due to higher expected inflation. A financial instrument is less desirable, if its existence encourages attempts to generate surprise inflation, which is counter-productive in a game against rational investors.

To interpret (15), we need the derivatives π_D, π_F, and π_N. They are related to the derivatives of optimal money supply growth by $\pi_D = m_D/(1 - m_\pi)$, $\pi_N = m_N/(1 - m_\pi)$, and $\pi_F = m_F/(1 - m_\pi)$. Money supply m_{t+1} is determined by the first order condition

$$U_m = E_t[u'(c_{t+1}) \cdot N_t - k'(\pi_t)] = 0 \; . \tag{16}$$

A time consistency problem arises, because the government takes expected inflation and interest rates as given at this point, while investors set interest rates based on expected monetary policy. Since $u' > 0$, high nominal debt clearly provides incentives to inflate. Less obviously, high debt D_t, foreign currency debt, and high expected inflation may also create incentives to inflate. The reason is that all three variables may reduce consumption, increase $u'(c)$, and therefore change the marginal benefit of money creation.

Unfortunately, the mapping from debt policy to money supply and inflation is not easy to compute.[20] Additional assumptions will be made in Section 4.3, but the general intuition is as follows. Provided $\pi_N > 0$, the incentive problem reduces the marginal benefit of nominal debt, U_N, by the amount $\pi_N \cdot N_t$ that increases with N_t (compare (9n) and (15n)). The higher π_N, the lower is the optimal value of N_t. Optimal nominal debt is

Then the key implication follows from Proposition 2: If F_t and N_t are substitutes, the reduction in nominal debt increases the optimal amount of foreign currency debt, i.e., $F_t > F_{hedge}$. This provides the second justification for foreign currency debt. Foreign currency debt may have to be substantially larger than the amount suggested by unrestricted hedging, if incentive problems make nominal debt undesirable.

Notice that the incentive effect $\pi_N \cdot N_t$ is zero, if $N_t = 0$. Therefore, if there is a hedging argument for nominal debt ($N_{hedge} > 0$), optimal nominal debt is positive (though smaller) even with the incentive problems present; this is consistent with Bohn (1988). However, if the cost of inflation is small (as k' and k" converge to zero), N is small and the level of foreign currency debt converges to $-\kappa \cdot Y_0 \cdot \mathrm{cov}(y, \pi^*)/\mathrm{var}(\pi^*)$. Thus, we obtain a simple condition for optimal foreign currency debt in countries with severe incentive problems: Foreign currency debt should be issued whenever domestic output is negatively correlated with foreign inflation. Under assumption (5), this is always the case.

4.3. Solution for the Quadratic Case

We can determine how money supply changes with state variables, if we assume quadratic utility and if shocks are zero after period 1 (the current period being t = 0). Moreover, we assume that $k(\pi)$ is quadratic with second derivative $k = k''(\pi) > 0$ and that $\beta = 1/(1 + r)$.[21]

With the simplifying assumptions, consumption for periods $t \geq 1$ is[22]

$$c_1 = r/(1 + r) \cdot \left[(1 + r)/(r - Ey) \cdot Y_1 + W_1\right] ,$$

expected inflation is a function of $N_0 = N$ and D_0,[23] and the first order conditions (15) reduce to[24]

$$c_0 = E_0c_1 \cdot (1 + N^2/K) \ , \tag{17d}$$

$$-\kappa \cdot Y_0 \cdot cov(y, \pi^*) - cov(\pi, \pi^*) \cdot N - var(\pi^*) \cdot F = 0 \ , \tag{17f}$$

$$-\kappa \cdot Y_0 \cdot cov(y, \pi) - var(\pi) \cdot N - cov(\pi, \pi^*) \cdot F - N \cdot G(N) = 0 \ , \tag{17n}$$

where $K = k/(u \cdot c_W) > 0$, $u = -u'' > 0$, $c_W = r/(1 + r)$, and

$$G(N) = \frac{u'(0)/(u \cdot \beta)}{K} - \frac{r/\beta \cdot \left(W_0 + (1 + Ey)/(r - Ey) \cdot Y_0\right)}{(1 + r) \cdot K + r \cdot N^2} \ .$$

Equation (17d) describes the intertemporal allocation of consumption. If we had pre-commitment ($\pi_D = 0$), we would get $c_0 = E_0c_1$. Given discretionary policy, however, high debt creates a temptation to inflate whenever $N > 0$ (or to deflate, if $N < 0$) Since higher debt in period zero increases rationally expected inflation, $\pi_D > 0$, optimal government debt is lower than it would be with pre-commitment. Period-0 consumption remains below the level needed to smooth consumption, $E_0c_1 < c_0$.

Foreign currency debt is determined in (17f), based purely on considerations of optimal hedging. Equation (17n) for nominal debt combines incentive and hedging arguments. Substituting (17f) into (17n), we get a reduced form equation for nominal debt N and an equation for foreign currency debt conditional on N,

$$\kappa \cdot Y_0 \cdot a_n/var(\pi^*) - a_0/var(\pi^*) \cdot N - N \cdot G(N) = 0 \ , \tag{18n}$$

$$F(N) = -\kappa \cdot Y_0 \cdot cov(y, \pi^*)/var(\pi^*) - cov(\pi, \pi^*)/var(\pi^*) \cdot N \tag{18f}$$

where a_n and a_0 are the constants involving second moments of shocks defined in Section 3 and $\kappa = (1 + r)/(r - Ey) > 0$.

The incentive effect on optimal nominal debt depends on the shape of the "marginal cost function" $G(N)$. The requirement of positive marginal utility guarantees $G(N) > 0$ for all N. Also, $G'(N) > 0$ for all $N > 0$ and $G'(0) = 0$. Several results are immediate:

Proposition 3: Incentives

a. Incentive effects reduce the amount of optimal nominal debt, N_{opt}, solving (18n), in absolute value below the value suggested by hedging, but do not reduce it to zero. That is, if $N_{hedge} = \kappa \cdot Y_0 \cdot a_n / a_0 > 0$, then $0 < N_{opt} < N_{hedge}$; if $N_{hedge} < 0$, then, $0 > N_{opt} > N_{hedge}$.

b. If $cov(\pi, \pi^*) > 0$, incentive effects increase the amount of optimal foreign currency debt, $F_{opt} = F(N_{opt})$, solving (18f), in absolute value above the value suggested by hedging. That is, if it would be desirable to issue nominal debt for hedging purposes, i.e., $N_{hedge} > 0$, then $F_{opt} > F_{hedge}$. If $N_{hedge} < 0$, then $F_{opt} < F_{hedge}$.

Proof: The properties of $G(N)$ guarantee that the left hand side of (18n) is strictly decreasing in N. At $N = 0$, it has the sign of N_{hedge}, at $N = N_{hedge}$, its value is $-N_{hedge} \cdot G(N_{hedge})$, which has the opposite sign of N_{hedge}. Thus, the unique solution must be between 0 and N_{hedge}. Part (b) follows directly from (18f). ‖

The result shows that incentive problems do not only reduce the role of nominal debt in the government's portfolio of liabilities but also increase the role of foreign currency debt, its substitute. In view of Proposition 1(b), we are most interested in the case of $N_{hedge} > 0$. However, our ability to sign N_{hedge} depends critically on our macroeconomic model, while

Proposition 3 may be applied even if correlations between inflation and output variables arise from different macroeconomic assumptions.

The next issue is the optimal amount of nominal and foreign currency debt. If the marginal cost $G(N) = Eu'(c_1) \cdot \pi_N$ were linear, one could easily obtain the solutions. However, the effect of nominal debt on inflation is weighted by expected marginal utility of period-1 consumption, which is increasing in N, because nominal debt distorts the intertemporal allocation (equation (17d)). Still, a linearization provides a useful benchmark. Since $G'(0) = 0$, we approximate $G(N) = G(0) = g$ and obtain linearized solutions

$$N_{lin} = \kappa \cdot Y_0 \cdot a_n/(a_0 + g \cdot var(\pi^*)) = a_0/(a_0 + g \cdot var(\pi^*)) \cdot N_{hedge} , \quad (19n)$$

$$F_{lin} = \frac{\kappa \cdot Y_0 \cdot a_f + g \cdot (-cov(y, \pi^*))}{a_0 + g \cdot var(\pi^*)} = F_{hedge} + \frac{g \cdot cov(\pi, \pi^*)}{a_0 + g \cdot var(\pi^*)} \cdot N_{hedge} . \quad (19f)$$

For $N \geq 0$, the linearized solution understates the marginal cost of nominal debt and therefore overstates the optimal level of nominal debt. It is an upper bound on N, hence a lower bound for F. Thus, incentive effects increase optimal foreign currency debt by at least a factor proportional to the amount that should "ideally" be issued as nominal debt. The factor is large, if inflation is closely correlated across countries, and if foreign inflation is not too volatile.

The linearized formulae for optimal policy may also be attractive under a somewhat different perspective. The function G(N) involves the ad-hoc parameter $K = k/(u \cdot c_W)$, about which we do not know much. Although one may try to find a structural interpretation for K (see the appendix), a wide range of values for K would have to be considered, if one actually wanted to compute optimal debt policies.[25] In such a calculation, the error caused by error of linearization. Thus, the model may be most easily applicable, if we set $G(N) = g$, as in the linearization and consider the parameter g as a free parameter, which indicates the severity of incentive problems.

In this general interpretation, optimal nominal debt can take any value between zero and N_{hedge}, depending on g. Optimal foreign currency debt will take the corresponding value between $F(0) = -\kappa \cdot Y_0 \cdot cov(y, \pi^*)/var(\pi^*)$ and F_{hedge}. Since $F(0) > 0$, any government that faces severe incentive problems should issue foreign currency debt. It is not surprising that its use is widespread.

5. Private Foreign Debt and Capital Controls

Our analysis assumed that the government has a monopoly in dealing with foreigners. If private (domestic) individuals can interact with foreign investors themselves, the strategic interaction of government and residents becomes important. By Ricardian equivalence, individuals care about total public and private debt, i.e., the national debt. Since they know the government's debt structure and the implied tax liabilities, their optimal decisions will in effect "undo" any government action.

A problem arises because atomistic individuals take interest rates as given. They do not take into account that the rate of money growth that their welfare maximizing government will later set is a function of their borrowing decisions. Therefore, they issue debt until the amount and structure of national debt satisfy equations (9).[26] Incentive problems are effectively ignored.

As a result, individuals issue too much nominal debt to foreigners and, at least in the quadratic case, too much total debt. The outcome yields even lower welfare than the second-best solution derived in Section 4.

This overissue problem provides a strong motivation for government restrictions. Capital controls are an obvious response that can support the second-best optimal policy. An example is a complete ban on private foreign borrowing; this is the case analyzed in Section 4. But because of Ricardian equivalence, the specific type of controls is largely irrelevant, provided they set a finite limit on all types of private foreign borrowing.

Alternatively, the government may consider taxes on private borrowing on international markets. To obtain the second-best optimum, taxes would have to bridge the gap between marginal private benefits (equations (9)) and marginal social benefits (equations (15)) of debt. Expressed in real terms payable at repayment, taxes have to be $\pi_D \cdot N_{opt}$ on indexed debt, $(\pi_D + \pi_N) \cdot N_{opt}$ on nominal debt, and $(\pi_D + \pi_F) \cdot N_{opt}$ on foreign currency debt. Notice that this optimal structure of taxes is unique: taxes must be differentiated by type of security and they should be levied on all private debt to foreigners. Even foreign investment in the domestic-currency bond market should be taxed!

6. Extensions

The optimal debt structure arising from hedging considerations depends critically on correlation properties of macroeconomic shocks with rates of inflation. These correlations depend on the type of shocks and the macroeconomic transmission mechanism. In this section, we consider generalizations of our macroeconomic structure. Non-neutrality of money is the focus of Section 6.1, while Section 6.2 concentrates on real exchange rates and real interest rate uncertainty.

6.1. Real Effects of Monetary Policy

In Section 3, we demonstrated that correlations between output and inflation are critical in determining optimal debt structure. However, all links between real and monetary disturbances (y_t and α_t) are potentially important. The results of Sections 3 and 4 are valid if money is neutral. As an alternative, now consider an environment with nominal price or wage contracts that generate monetary non-neutrality.

Suppose that there are nominal contracts in the economy so that the government can increase output temporarily by generating unexpected inflation. Moreover, assume that contract prices contain monopolistic elements so that increased output improves welfare.[27] Since the effect on the level of output is temporary, growth is reduced when output reverts to the "natural-rate" level after surprise inflation ends. Let $\theta > 0$ be the marginal effect of inflation on current growth and let $y_t^n = Ey + \mu_t + \psi_t$ be natural rate growth. Then output growth is

$$y_t = y_t^n + \theta \cdot \hat{\pi}_t - \theta \cdot \hat{\pi}_{t-1} .$$

Such a modified equation for output affects our results in two places. First, it adds a positive element to the covariance of the present value of income and unexpected inflation. The covariance expression determining nominal debt, $\text{cov}(\kappa \cdot Y_{t+1}, \pi_{t+1})$ (see the proof of Proposition 1), is replaced by $Y_t \cdot [\kappa \cdot \text{cov}(y_{t+1}^n, \pi_{t+1}) - \theta \cdot \text{var}(\pi_{t+1})]$. If shocks are primarily real (μ or ψ), this term is still negative. However, if most shocks are monetary and θ is large, it may become positive. The amount of nominal debt is a declining function of θ; it is reduced by $\theta \cdot Y_t$ relative to the amount without monetary non-neutrality.

Concerning foreign currency debt, the term $\text{cov}(\kappa \cdot Y_{t+1}, \pi_{t+1}^*)$ in the proof of Proposition 1 is replaced by $\kappa \cdot T_t \cdot \text{cov}(y_{t+1}^n, \pi_{t+1}^*)$, which depends on shocks in a similar way as the term a_f of Proposition 1(b). Optimal foreign currency debt is unaffected by monetary non-neutrality.

The second place where monetary non-neutrality matters is in determining the government's incentives to inflate and consequently in determining expected inflation. Taking the quadratic case of Section 4.3, it is straightforward to show that in equations (17n) and (17f) N will be replaced by $N + \theta \cdot Y_0$ and y by y^n. Again, nominal debt is reduced by $\theta \cdot Y_0$ while foreign currency debt is unaffected. Intuitively, the government has an incentive to create inflation even if N = 0. The strength of this incentive is related to the effect of surprise inflation on output, θ. It reduces optimal nominal debt even further below N_{hedge} and may make it negative, even if $N_{hedge} > 0$. In effect, the government may want to post a bond to convince investors that it will not print money.

Overall, non-neutral money may weaken the case for nominal debt. But as far as foreign currency debt is concerned, it will still be issued as a hedge, if F_{hedge} was positive before.

6.2. Real Interest Rate and Real Exchange Rate Uncertainty

We noted that the strong results on hedging in Section 3 depend on the sources of macroeconomic shocks. Shocks to economic output are only one, though important, risk for individuals. Two other shocks may be worth mentioning.

First, consider uncertainty in real interest rates. Formally, the real rate would enter as a state variable in the value function of the government. If the country is a debtor country, higher interest rates tighten the intertemporal budget constraint and reduce consumption through intertemporal substitution. Thus, the country should hedge against them. Risky debt is desirable, if its value is low in states of nature in which real rates are high.

Notice that long term bonds have this property. Thus, real interest rate risk may provide arguments for an optimal maturity choice. I have tried to steer clear of this issue by using one period bonds and discrete time. This makes the most sense, if we interpret the basic period as a long one (say, several years). Notice that a long period also justifies the neglect of short-run monetary non-neutralities in the basic model.

As a second extension, consider changes in real exchange rates. A general analysis would require a model with preferences over domestic and imported goods in both countries and is therefore complicated.[28] We will only discuss a case that can be analyzed in the one-good framework.

Suppose all agents consider one unit of the foreign good to be the equivalent of e_t units of the domestic good. Suppose this real exchange rate moves randomly at an exogenous rate ε_t. This covers the case when foreign and domestic goods are still perfect substitutes at any point in time, but when the rate of substitution (e.g., quality) changes over time.[29] Then the real cost of foreign currency debt is $1 + i_t^* + \varepsilon_{t+1} - \pi_{t+1}^*$. Our analysis remains unchanged, except that π_{t+1}^* must be replaced by $\pi_{t+1}^* - \varepsilon_{t+1}$ in all equations for optimal debt policy.

The resulting values of optimal debt depend on the stochastic properties of ε_t. If ε_t is an i.i.d. random variable that is uncorrelated with all other variables, the amount of F_{opt} is simply "scaled down" to a fraction $var(\pi^*)/var(\pi^* + \varepsilon)$ of the original amount. The reason is that foreign currency debt exposes the country to an additional risk.

Modeling correlation of real exchange rates and other variables would be straightforward. An interesting case may be one in which the nominal exchange rate reacts to and "overshoots" in response to monetary policy. Then money growth reduces the real exchange rates, increases the cost of foreign currency debt, and hence discourages excessive inflation. This would provide an additional motivation for foreign currency debt.

On the other hand, if real disturbances to domestic output (the shock ψ) are negatively correlated with the real exchange rate, foreign currency debt will be weaker. This may be relevant for the case for countries that are significantly affected by shifts in demand for their natural resources. Again, these extensions can only be analyzed properly in a multi-good model so that they should be considered as conjectures.

Many other shocks can be handled in a similar way. For example, all shocks that enter only through the budget constraint (e.g., exogenous government spending for disaster relief or investment spending) are analogous to endowment shocks. To protect consumption, one would have to compute how endowment net of these other spending items varies with inflation rates.

7. Summary and an Application

The paper makes a case for foreign currency debt based on a hedging motive in an open economy subject to stochastic shocks to output. The derivation can be summarized in three main steps.

First, if inflation is positively correlated across countries, debt denominated in the domestic currency and debt denominated in a foreign currency are substitutes in the liability portfolio of a government.

Second, if unexpected domestic (foreign) inflation and domestic output are negatively correlated, nominal (foreign currency) debt has a role as a device to hedge the domestic economy against shocks to output. Foreign currency debt is desirable, if growth rates of output of both countries are closely related and if domestic inflation is relatively uncertain.

Third, if the government has the ability to manipulate domestic money and nominal interest rates. This effect reduces welfare and discourages the issue of domestic nominal debt.

Given these three arguments, the level of foreign currency debt has two determinants. Foreign currency debt may serve as a hedge against output fluctuations even if domestic currency debt can be issued easily to foreigners, depending on the structure of stochastic shocks. In addition, foreign currency debt replaces domestic currency debt as hedge, whenever issuing domestic debt is prevented or discouraged by incentive problems. If the optimal amount of nominal debt is small, the foreign currency debt should be issued, if domestic output and foreign inflation are negatively correlated.

Our results have some direct implications for policy. For example, since the "debt crisis" of 1982, troubled debtors have been criticized for issuing US-dollar denominated debt. With hindsight, most debtors probably regretted this policy. But was it a mistake ex ante? The paper suggests a defense of LDC-policy: first, it seems plausible that incentive problems prevented issuing significant amounts of domestic nominal debt to foreigners. In terms of the model, investors would worry about monetary policy so much that the cost-of-inflation parameter g is infinite. Second, large economic disturbances in the 1970's originated on the supply side and were worldwide. For example, in the 1974-75 and the 1980 recessions, US inflation accelerated while output in the US and in many less developed countries declined. Thus, it may have been reasonable to gamble that debt denominated in US dollars would protect the domestic economy against worldwide business cycles; dollar-denominated debt appeared to have a role as hedge. The downside risk of this strategy was the exposure to the effects of idiosyncratic changes in US monetary policy. Unfortunately, the disinflation in the United States beginning in October 1979 revealed this exposure.

Footnotes

[1]This is also an example of the general notion that state-contingent contracts or rules can improve welfare in an uncertain world, as recognized, e.g., in the literature on wage indexation, on monetary rules, and on optimal exchange rate regimes (see Aizenman and Frenkel (1985)).

[2]In general, one might worry about alternative ways of purchasing insurance. The maintained assumption here is that the government has to rely on simple contracts such as nominal debt that can be manipulated only at a cost.

[3]We assume that there are no defaults in the sense that the government will always honor its commitments. That is, "repudiation" as defined by Grossman and Van Huyck (1987) is excluded a priori. A prohibitive cost of repudiation could easily be added as a formal justification. A fall in the real value of nominal debt is not considered a default, because the government still pays what it promised; it is the investor's task to figure out what the real value of such a promise is. Alternatively, one could interpret inflation as a partial, "excusable default," as Grossman and Van Huyck (1987) do. In contrast to their reputational arguments, we use an endogenous default-penalty, the welfare cost of inflation, which is continuous in the degree of default. Notice that we have various degrees of "excusable fault" as opposed to the discrete choice in many other theories of sovereign debt (see, e.g., Eaton et al. (1986)).

[4]As Persson and Swensson (1984) point out, uncertainty is not essential in Lucas and Stokey's results.

[5]A generalization to many foreign currencies or other securities is straight forward. It is important that the government cannot sell off all claims to future domestic output. Otherwise the trivial solution would be to sell all claims to output to foreigners against risk free assets. In reality, such a strategy may be prevented by information problems, which we do not want to model here. Sales of some output-contingent assets (e.g., through debt-equity swaps) could be allowed, as long as individuals are subject to some residual output-related risk (e.g., through future labor income) that cannot be sold.

[6]In reality, investors may be somewhat risk averse. However, one should expect that international investors are more diversified and less risk averse than the typical taxpayer. This is approximated by our assumption. Notice that these foreigners will not hold fiat money because it does not pay interest.

[7]A companion paper will discuss the case when the budget constraints of government and individuals are separated by distortionary taxation and when individuals borrow from foreigners directly.

[8]Unless inflation causes some loss in welfare, the time inconsistency problem precludes nominal debt (Lucas and Stokey (1983), Calvo (1978)). This

seems unrealistic. Still, our results for foreign currency debt apply (by setting N = 0 in all equations below).

[9]The important feature of this story is that it generates a simple money demand function with stochastic velocity. Nothing important would change if the government would pay a fixed rate of interest on money balances. But one has to be careful that sellers do not find a reason to price-discriminate against buyers using money. This is why we use a story based on exogenous delays in check clearing (where all checks look identical) instead of one based on cash-in-advance (where sellers see whether a buyer uses cash or credit).

[10]If negative, T_t denotes transfers. Taxes are paid in (immediately consumable) goods or in money.

[11]We refer to all government debt as "bonds" without distinguishing securitized claims, bank loans, or other claims. Indexed debt only exists in a few countries. However, the combination of indexed bonds and nominal bonds with fixed maturity may be interpreted as a choice of duration. For example, suppose we add a continuous time dimension to the model in the following way: actual bonds have any maturity τ in the interval [0,1]. Period t + 1 shocks are revealed "soon" (in continuous time) after the bond is issued. Maturing bonds must be refinanced at time $t + \tau$ for the remaining interval $(1 - \tau)$ at interest rate r. Then any portfolio with nominal bonds of different maturities τ can be replicated by a linear combination of indexed and nominal bonds. The same holds for the foreign currency bond.

[12]Interest payments in nominal terms would add products of the type $i_{t+1} \cdot \pi_{t+1}$ and $i^*_{t+1} \cdot \pi^*_{t+1}$, which would introduce technical complications. As long as inflation is not too large in absolute value, the distinction is quantitatively small. The definition is also exact, if the bonds are coupon bonds and we use the continuous time interpretation described in the previous footnote.

[13]The white-noise assumption is adopted to simplify the computation of optimal policy, though a generalization is straightforward. Adding other securities, longer term bonds in particular, would also be easy. However, I feel that this would distract from the main issues related to incentives and choice of currency. For these issues, it is sufficient to focus on contemporaneous correlations and on one "typical" bond of each currency. It may be best to interpret the basic period as an interval of several years, which is long enough so that autocorrelation of inflation and growth are small. This interpretation is consistent with the notion that decisions about debt structure are fundamental policy decisions that are not revised frequently. It also fits our assumption of discretionary policy (e.g., governments cannot bind future administrations) and the interpretation of indexed bonds in the previous footnotes.

[14]To be precise, we may assume that the foreign economy has the same structure as the domestic one, except that the foreign country is also populated by some risk neutral agents. Notice that foreign money supply is only exogenous, if the foreign currency debt issued by the domestic government is a negligible quantity in determining foreign monetary poli-

Alternatively, we may assume that the foreign government has some way of precommitting its monetary policy to m_t^*.

[15]In terms of Helpman and Razin (1984), this corresponds to payments in the seller's currency.

[16]If money supply were chosen later, the government could offset all shocks to inflation, which would make the problem rather uninteresting. For tax and debt policy, shocks must be known because they affect the real value of liabilities in the budget constraint.

[17]We assume that policy is determined period-by-period and do not consider issues of reputation that may link policy across periods (cf. Barro and Gordon (1983b)). Reputational equilibria may be impossible to sustain, if investors are unable to coordinate a "punishment" for deviations.

[18]Notice that this could be generalized easily. If Y_t itself were white noise (and growth negatively autocorrelated), only W would remain as a state variable. Also notice that one could factor-out Y_t in all budget constraints. Since Y_t is known in period t, the government essentially decides about the ratio of D, N, and F to current endowments.

[19]If we end the game at some final period T, the value function in period T would be $u(c_T) - k(\pi_T)$, which is strictly concave. As T becomes large, concavity of V^T and V^m can be shown easily. The only issue is whether the concavity is strict.

[20]We obtain the determinants of optimal money supply by taking the total differential of (16), which yields

$$m_D = (1 + r) \cdot E_t z_{t+1} \;, \quad m_\pi = N_t \cdot E_t z_{t+1} \;, \quad m_F = -E_t[z_{t+1} \cdot \hat{\pi}^*_{t+1}] \;, \text{ and}$$

$$m_N = \frac{E_t u'(c_{t+1})}{E_t[k''(\pi_{t+1}) - u''(c_{t+1}) \cdot c_W \cdot N_t^2]} - E_t[z_{t+1} \cdot \hat{\pi}_{t+1}] \;,$$

where

$$z_{t+1} = \frac{-u''(c_{t+1}) \cdot c_W \cdot N_t}{E_t[k''(\pi_{t+1}) - u''(c_{t+1}) \cdot c_W \cdot N_t^2]}$$

is a function of all state variables and c_W is the derivative of optimal consumption c_{t+1} with respect to W_{t+1}. Since $k'' > 0$, $u' < 0$, and $c_W \geq 0$, the denominator is always positive. Hence, money growth is finite and the derivatives of money growth with respect to bond supplies are well defined. At this step--and only here--a cost of inflation is essential. We just assume $k'' > 0$, but the appendix provides a more rigorous motivation. If $|u''(c) \cdot c_W|$ does not vary too much, the covariance of z_{t+1} with inflation rates is small, implying $m_F \simeq \pi_F \simeq 0$. Under the same assumption, the sign of m_N depends primarily on $u'(c_{t+1})$, which is positive, i.e., $m_N > 0$ and $\pi_N > 0$.

[21] Quadratic utility implies that $u''(c)$ is constant and can be taken out of the expression for z_1 (defined in a previous footnote). The assumption of no shocks after period 1 allows us to solve explicitly for period 1 consumption, i.e., to obtain c_W in the equation for z_1. Notice that these are not necessary assumptions. But since the same decision problem is essentially repeated over and over (except for starting with different levels of current wealth and endowments), it is difficult to see how adding or omitting noise in the continuation game (from t = 1 on) should affect the qualitative results. Quadratic utility is certainly a strong assumption. It may perhaps be justified as an approximation of other utility functions. The results would only change if higher order derivatives of $u(c)$ were critical. This seems unlikely, though we should be cautious in interpreting the final equations for extreme values of the arguments.

[22] Under certainty, we have

$$c_t = c_1 = \frac{\beta \cdot (1+r)^2 - 1}{\beta \cdot (1+r)^2} \cdot \left[\sum_{\tau=1}^{\infty} \frac{1}{(1+r)^\tau} \cdot E_1 Y_{\tau+1} + W_1 \right]$$

where the sum reduces to $(1 + r)/(r - Ey) \cdot Y_1$ because of known output growth, Ey. In addition the derivative c_W is a positive constant, which can be simplified as

$$c_W = \left(\beta \cdot (1+r)^2 - 1\right)/\left(\beta \cdot (1+r)^2\right) = r/(1+r) .$$

Positive marginal utility requires that consumption in any period is less than $u'(0)/u$. This can be guaranteed by starting with a low enough value of W_0 (negative, if the country is in debt).

[23] Optimal money growth m_1 has a derivative $m_\pi = N^2/(K + N^2)$, which satisfies $0 \le m_\pi < 1$ for all N. The variable z_1 reduces to $z_1 = u \cdot c_W \cdot N/(k + u \cdot c_W \cdot N^2) = N/(K + N^2)$, where $K = k/(u \cdot c_W)$. Hence, expected inflation has derivatives $\pi_D = (1 + r) \cdot N/K$, $\pi_F = 0$, and $\pi_N = E_0 u'(c_1)/K > 0$.

[24] They are

$$U_D = \left(u'(0) - u \cdot c_0\right) - E_t\left[\left(u'(0) - u \cdot c_1\right) \cdot (1 + N^2/K)\right] = 0$$

$$U_F = -\beta \cdot u \cdot \mathrm{cov}(c_1, \pi_1) = 0$$

$$U_N = -\beta \cdot u \cdot \mathrm{cov}(c_1, \pi_1) - N \cdot E_t\left(u'(0) - u \cdot c_1\right)/K = 0 .$$

Notice that $N_t = F_t = 0$ for $t \ge 1$, because all covariances are zero for $t > 1$.

[25] It seems very difficult to defend a high value of K on the basis of tangible economic costs or benefits. But if K is small, inflation is high, welfare losses are high, and nominal debt is close to zero. This may be realistic for some developing countries with inflation bordering on hyperinflation. Other economies appear to work as if individuals had a much stronger aversion against inflation. Perhaps they have been able to build a

appointing an inflation-averse central banker (Rogoff (1985)). The case of a country that has found a device to reduce the incentive cost of nominal debt below Eu'(c)/K can be modeled by a low value of g.

[26]This can be shown easily in the quadratic model of Section 4.3. Let the government's debt be denoted S^G, N^G, and F^G, respectively. Then individuals choose private debt S^P, N^P, and F^P to satisfy the first order conditions $c_0 = Ec_1$, $\text{cov}(c_1, \pi) = 0$, and $\text{cov}(c_1, \pi^*) = 0$. Hence $N^P = -N^G + \kappa \cdot Y_0 \cdot a_n/a_0$ and $F^P = -F^G + \kappa \cdot Y_0 \cdot a_f/a_0$.

[27]To be specific, suppose there is a technology that transforms endowments of agent j to $h(L^j)$ units of consumption agent k, if L^j units of j's consumption good are given up, $h' > 0$, $h'' < 0$. Suppose they must get together in period t to negotiate payment w_t for inputs to be delivered in period t. Following the contract literature (see Fischer (1977)), we assume that w_t is a nominal quantity and that the quantity L^j is determined by demand (agent k) in period t+1. Similar delivery relations are assumed between other agents. Denote total income of agent j by Y_t^j (which enters the budget constraint) and denote endowments by X_t. Total income then consists of three physical components, the endowment X_t, production $h(L^k)$, $k \neq j$ and two monetary flows, $+w_{t-1}/p_t \cdot K^j$ for deliveries and $-w_{t-1}/p_t \cdot L^j$ as payment for inputs. Agent j will set L^k so that $h'(L^k) = w_{t-1}/p_t$, which means that L^k is clearly increasing in inflation. On aggregate, we have output $Y_t = X_t - \int h(L^j)dj - \int L^k dk$, where $L^k = L^j = L$ in a symmetric equilibrium. Competitive suppliers would set w_{t-1} so that $E_{t-1}[w_{t-1}/P_t] = 1$, which maximizes Y_t. To generate a policy problem, suppose the market for inputs has monopolistic elements on the supply side, which lead to $E_{t-1}[w_{t-1}/p_t] = \theta \gg 1$. Then w_t is too high and Y_t lower than the optimal level.

[28]In particular, changes in relative prices of imported consumption goods, imported production inputs and/or export goods may enter into budget constraints and/or preferences. Hedging against such changes may effect the optimal structure of debt.

[29]Even if the government manipulates the price at which individuals can buy foreign exchange from the central bank, the country faces some real opportunity cost of acquiring foreign goods. If we interpret that real rate as this true opportunity cost, exogeneity may not be a bad assumption, no matter how the central bank sets the nominal exchange rate.

References

Aizenman, Joshua and Jacob Frenkel, 1985, "Optimal Wage Indexation, Foreign Exchange Intervention, and Monetary Policy," American Economics Review 75, 402-423.

Arrow, Kenneth, 1971, "Essays in the Theory of Risk Bearing," Markham Pub. Co.

Barro, Robert J., 1974, "Are Government Bonds New Wealth?" Journal of Political Economy 82, 1095-1117.

Barro, Robert J. and David B. Gordon, 1983a, "A Positive Theory of Monetary Policy in a Natural Rate Model," Journal of Political Economy 91, 589-610.

Barro, Robert J. and David B. Gordon, 1983b, "Rules, Discretion, and Reputation in a Model of Monetary Policy," Journal of Monetary Economics 12, 101-121.

Bohn, Henning, 1988, "Why Do We Have Nominal Government Debt?" Journal of Monetary Economics 21, 127-140.

Calvo, Guillermo, 1978, "On the Time Consistency of Optimal Policy in a Monetary Economy, Econometrica 46, 1411-1428.

Eaton, Jonathan, Mark Gersovitz, and Joseph Stiglitz, 1986, "The Pure Theory of Country Risk," European Economic Review 30, 481-513.

Fischer, Stanley, 1975, "The Demand for Index Bonds," Journal of Political Economy 88, 509-534.

Fischer, Stanley, 1977, "Long Term Contracts, Rational Expectations, and the Optimal Money Supply Rule," Journal of Political Economy 85, 191-206.

Grossman, Herschel, and John Van Huyck, 1987, "Sovereign Debt as a Contingent Claim: Excusable Default, Repudiation, and Reputation," mimeo, Brown University and Texas A&M University.

Helpman, Elhanan and Assaf Razin, 1984, "The Role of Savings and Investment in Exchange Rate Determination under Alternative Monetary Mechanisms." Journal of Monetary Economics 13, 307-325.

Kydland, Finn and Edward Prescott, 1977, "Rules Rather Than Discretion: The Inconsistency of Optimal Plans," Journal of Political Economy 85, 473-491.

Lucas, Robert E., 1978, "Asset Prices in an Exchange Economy," Econometrica 46, 1429-1445.

Lucas, Robert E. and Nancy Stokey, 1983, "Optimal Fiscal and Monetary Policy in an Economy without Capital," Journal of Monetary Economics 12, 55-93.

Persson, Torsten, and Lars Swensson, 1984, Time-Consistent Fiscal Policy and Government Cash-Flow, Journal of Monetary Economics 14, 365-374.

Rogoff, Kenneth, 1985, "The Optimal Degree of Precommitment to an Intermediate Monetary Target, Quarterly Journal of Economics 100, 1169-1189.

Appendix

A.1. Proof of Proposition 1

Define wealth

$$W_t = \Sigma_{i\geq 0}(1 + r)^{-i}\cdot Y_{t+i} - (1 + r)\cdot D_t + \hat{\pi}_t \cdot N_{t-1} + \hat{\pi}^*_t \cdot F_{t-1} . \quad \text{(A-1)}$$

Then the budget constraint in period t + 1 is $\Sigma_{i\geq 0}(1 + r)^{-i}\cdot c_{t+i+1} = W_{t+1}$. Denote marginal utility by $u_1 - u_2 \cdot c_{t+1}$, where $u_1, u_2 > 0$. The first order conditions in period t+1 are

$$u_1 - u_2 \cdot c_{t+1} = \beta(1 + r)\cdot(u_1 - u_2 \cdot E_{t+i}c_{t+i})$$

for all $i \geq 1$. Then

$$\Sigma_{i\geq 0}(1 + r)^{-i}\cdot E_{t+1}c_{t+i+1} = \Sigma_{i\geq 0}(1 + r)^{-i}\cdot c_{t+1} + C = E_{t+1}W_{t+1} ,$$

where C is a constant that depends on the parameters of the model. Notice that $\Sigma_{i\geq 0}(1 + r)^{-i} = (1 + r)/r$ and $\Sigma_{i\geq 0}(1 + r)^{-i}\cdot E_{t+1}Y_{t+1+i} = (1 + r)/(r - Ey)\cdot Y_{t+1} = \kappa \cdot Y_{t+1}$. Hence,

$$\text{cov}\big(u'(c_{t+1}), \pi_{t+1}\big) = -u_2 \cdot \text{cov}(\kappa \cdot Y_{t+1}, \pi_{t+1}) - u_2 \cdot \text{cov}\big(-(1 + r)\cdot D_t + \hat{\pi}_{t+1}\cdot N_t + \hat{\pi}^*_{t+1}\cdot F_t, \pi_{t+1}\big) ,$$

and an analogous formula holds for $\text{cov}\big(u'(c_{t+1}), \pi^*_{t+1}\big)$. Solving (9n) and (9f) with these covariances proves part (a). Part (b) is straightforward algebra.

A.2. Alternative Assumptions

Restrictions on utility and/or distributions of shocks are needed for Propositions 1 and 2, because incomplete markets (in the sense that future endowments are not traded) complicate the analysis. The complication is not tractable in the subsequent section, when incentive problems are added. Alternatively, the problem can be solved with normal distributions and only two relevant periods. By assuming that all uncertainty is resolved in period 1, after which the game is just mechanically played out, we essentially consider a two-period model. Since the model does not have any significant multi-period dynamics, we do not loose much generality. The problem would be modeled easily in a two-period setting, except that money is awkward in a finite model. Formally, define

Assumption (A-2): All macroeconomic shocks are jointly normally distributed, the current period is $t = 0$, and all macroeconomic shocks are zero for periods $t \geq 2$. Moreover the utility function and parameters of the model satisfy $Ey < u$ and

$$\lambda_W = \Sigma_{i\geq 0}\left[(\beta \cdot (1 + r)^2)^{-i} \cdot 1/u''((\beta \cdot (1 + r))^{-i} \cdot \lambda)\right] > 0$$

for all $\lambda > 0$. ‖

Then the following is true:

Proposition A: Proposition 1 and 2(b) also hold, if assumption (A2) is imposed instead of assumption (10).

Proof:

Under assumption (A-2), W_1 (defined in equation (A-1)) is known in period 1 and it has a normal distribution as seen from period 0. The optimal solution of the government's problem in period 1 is a set of functions $c_t = c_t(W_1)$ for $t \geq 1$. Therefore

$$\mathrm{cov}(u'(c_1), \pi_1) = E_0 u''(c_1) \cdot c_W \cdot \mathrm{cov}(W_1, \pi_1) ,$$

with respect to wealth. Hence (9n) and (9f) are equivalent to $\mathrm{cov}(W_1, \pi_1) = 0$ and $\mathrm{cov}(W_1, \pi_1^*) = 0$, which implies the stated results. ‖

Weaker results can be obtained without assumptions (10) or (A-2). For example, one can follow the intuitive argument to show that we have at least $N > 0$ or $F > 0$ for any utility function. The factor $\kappa = (1 + r)/(r - Ey)$ is due to the assumption that endowments follow a random walk. It indicates how much the current growth in endowments affects the present value of the stream endowments. It would be lower, if shocks to endowment were less persistent. For example, if Y_t were i.i.d., the factor would be 1. (Then the results with normality would hold even with uncertainty in periods $t \geq 2$.)

A.3. A Model of Costly Inflation

Here we will show why inflation may have a welfare-cost in an equilibrium model. The key assumption is that individuals (a continuum indexed by j, j in [0,1]) are heterogeneous in their money holdings instead of being identical. Otherwise, all assumptions of Section 2 apply.

Heterogeneity in money holdings can be generated by assuming that the fraction of non-cleared checks (which is v_t on aggregate) has an idiosyncratic component x^j. We assume that it averages to zero, $\int x^j dj = 0$, and has a variance, $\sigma_x^2 = \int (x^j)^2 dj > 0$. Individual j keeps $(v_t + x^j) \cdot p_t \cdot Y_t$ on account, which is available in the subsequent period.

Two additional assumptions are crucial to prevent the heterogeneity from being eliminated: first, the government cannot distinguish agents with different values of x^j, i.e., cannot differentiate in taxation. Second, domestic individuals must not be able to "trade away" the differences. Since individuals only differ in money holdings, we can still assume without loss of generality that only the government transacts with foreigners. This leaves differences in wealth due to differences in money holdings as only motivation for trading among individuals. A small transactions cost would be sufficient to prevent such trading. Alternatively, it is sufficient to assume that there is a domestic bond market that excludes a small subset of individuals (who face prohibitively high transactions cost), provided money holdings of the subset differ from the average.

Then individual j's consumption, c_t^j , deviates from the average, c_t, by

$$c_t^j - c_t = -x^j \cdot Y_t + x^j \cdot p_{t-1} \cdot Y_{t-1}/p_t \ .$$

To simplify the algebra in this last expression, we will use the approximations $\pi_t = \log(p_t/p_{t-1}) = \log(-p_{t-1}/p_t) \approx -p_{t-1}/p_t$ to compute the decline in purchasing power of money; a similar approximation is used for output growth. Then

$$c_t^j - c_t = -x^j \cdot Y_{t-1} \cdot (y_t + \pi_t) = -x^j \cdot \phi_t \qquad \text{(A-3)}$$

We see that the cross-sectional distribution of this difference depends on inflation, which will be important for optimal monetary policy.

Each individual has an intertemporal utility function U_t^j over own consumption c^j, as in equation (1). The government maximizes social welfare

$$U_t = E_t \int_0^1 U_t^j dj = \sum_{i>0} \beta^i \cdot \int u(c_{t+i}^j) dj \ . \qquad \text{(A-4)}$$

Notice that we do <u>not</u> assume a cost function $k(\pi)$. We almost have a representative agent model, except for differences in money holdings. As far as debt policy is concerned (Sections 2 and 3), the model gives the same results as the representative agent model, provided that marginal utilities are integrated over individuals. An exception is the proof of Proposition 1 based on Assumption (A-2), which requires identical individuals.

In the game of Section 4, the variable ϕ_t defined in (A-3) enters as additional state variable in $V(\cdot)$. Since ϕ_t depends on monetary policy through π_t, the first order condition (16) for monetary policy must be replaced by

$$U_m = E_t\left[\int u'(c^j_{t+1}) \cdot (N_t - x^j Y_t) dj\right] = 0 . \tag{A-5}$$

In the total differential, we now have

$$z_{t+1} = \frac{-\int u''(c^j_{t+1}) \cdot c_w \cdot (N_t - x^j Y_t) dj}{E_t\left[-\int u''(c^j_{t+1}) \cdot c_W \cdot (N_t - x^j Y_t)^2 dj\right]} .$$

Since x^j has positive variance, the denominator is always positive. Hence, money growth is finite and the derivatives of money growth with respect to bond supplies are well defined.

Intuitively, money growth has a welfare cost because it hurts individuals with above average money holdings ($x^j > 0$). Since these individuals typically have below average consumption and high marginal utility (assuming $y + \pi > 0$), the cross sectional covariance $\int u'(c^j_{t+1}) x^j dj$ in (A-5) is positive even though $\int x^j dj = 0$. Inflation increases heterogeneity in consumption, which lowers welfare. This is the only place in the paper where heterogeneity of individuals makes a difference.

In the quadratic case of Section 4.3, we have

$$z_1 = u \cdot c_W \cdot N / \left(u \cdot c_W \cdot (N^2 + Y_0^2 \cdot \sigma_x^2)\right) .$$

If we define $K = Y_0^2 \cdot \sigma_x^2$ and $k = u \cdot c_W \cdot Y_0^2 \cdot \sigma_x^2$, the first order conditions in Section 4.3 remain unchanged, but the parameters K and k have a structural interpretation: the cost of inflation k depends on the curvature of the utility function (u) and the heterogeneity in money holdings (σ_x) normalized by the size of the economy (Y_0).