AN EMPIRICAL INVESTIGATION OF BOND PRICES AND INFLATION

by

George G. Pennacchi

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RODNEY L. WHITE CENTER FOR FINANCIAL RESEARCH
The Wharton School
University of Pennsylvania
Philadelphia, PA 19104-6367

July 1988

Comments Appreciated

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*Department of Finance, The Wharton School, University of Pennsylvania, Philadelphia, PA, 19104, (215) 898-6298. I am indebted to the members of the Penn Macro Lunch Group, the Penn Finance Workshop, Lars Hansen, Bob Inman, Ravi Jagannathan, Krishna Ramaswamy, and Mark Watson for valuable comments. Andrew Lo deserves special thanks for many helpful discussions. I am grateful to Geewax-Terker & Company for providing research support, to Nicholas Valerio for excellent research assistance, and to Thomas Juster, Krishna Ramaswamy, and Tim Schiller for supplying data. All errors are my responsibility.

Abstract

This paper investigates the dynamics of real interest rates and inflation in the context of an equilibrium asset pricing model. Formulas for bond prices and optimal forecasts of inflation are shown to form a state space system. The model's parameters are estimated by maximum likelihood, using a Kalman filter to compute the likelihood function. The estimation uses time series data on Treasury bill prices of various maturities and survey forecasts of inflation. The results suggest that the stochastic processes for real interest rates and expected inflation are mutually dependent; innovations in the processes display significant negative correlation while expected changes in each variable are significantly positively related to the level of the other variable. There is evidence that over the past decade inflation and real interest rates have displayed somewhat less mean reversion than previously. Distinguishing real rates from expected inflation is likely to lead to gains in interest rate modelling.

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I. Introduction

There is a long history to modeling the link between interest rates and inflation, with research dating back to the work of Irving Fisher (1896). More recently, Cox, Ingersoll, and Ross (1985), Richard (1978), and others have made significant progress by developing equilibrium, "arbitrage-free" bond pricing models that explicitly recognize the effect of inflation on the term structure of interest rates. However, with the notable exception of the recent empirical work by Gibbons and Ramaswamy (1986), little effort has been devoted to estimating equilibrium models where separate dynamics for real interest rates and inflation are distinguished.

Economic theory suggests that an underlying "state" variable might have an effect on the equilibrium real interest rate that is different from its effect on the equilibrium inflation rate. For example, a state variable such as technological change can directly influence investment opportunities and real rates of return. However, if monetary policy is accommodating, changes in technology need not exert much effect on inflation. Alternatively, a government's monetary policy may itself be viewed as a changing state variable. In cases where theoretical models display "monetary (super-) neutrality," an anticipated change in money growth has a proportional effect on inflation but little influence on real interest rates. An implication of these theoretical examples is that if the underlying state variables affecting real rates follow processes that are dissimilar to those primarily affecting inflation, a model of the term structure should reflect this dissimilarity.

This paper investigates the dynamics of real interest rates and inflation within an equilibrium term structure framework. Starting with a model which specifies consumer preferences, technological change, and a nominal price level process, we derive an equilibrium term structure of interest rates with characteristics similar to models by Vasicek (1977), Langetieg (1980), and Marsh (1980). This bond pricing model can be transformed into a state space system. The unobserved state variables are the instantaneous real interest rate and expected inflation, which follow a bivariate stochastic process that allows for mutual dependence between the variables.

The observed variables consist of bond prices of various maturities and survey forecasts of inflation.

The model's parameters are estimated by maximum likelihood, using a Kalman filter to simplify computation of the likelihood function. The results provide evidence that real interest rates and expected inflation follow significantly different, though correlated, stochastic processes. This is empirical support for the importance of making a distinction between real and nominal variables when formulating equilibrium bond pricing models.

The plan of the paper is as follows. Section II motivates our empirical work by showing its consistency with a simple intertemporal capital asset pricing model. In section III, we derive the equilibrium term structure of interest rates that results from this model. Section IV discusses the technique used to estimate this model, while section V reports on the data chosen for the empirical work. Section VI follows with an analysis of the empirical results.

II. An Intertemporal Model

Let us consider an economy similar to that of Merton (1971, 1973) and Cox, Ingersoll, and Ross (1985a, b) hereafter referred to as CIR. The following three assumptions are made:

Assumption 1 Infinitely lived individuals maximize utility of the form;

(1) $$\max E_t \int_t^{\infty} e^{-\rho s} \ln(C(s))\, ds$$

where $(C(t))$ is the time t consumption of a representative individual.

Assumption 2 There exists a single capital-consumption good. A single technology is available which transforms capital, $K(t)$, into output according to the process;[1]

(2) $$dK/K = \alpha_k(t)\,dt + \sigma_k\,dz_k$$

where the expected rate of return on capital, $\alpha_k(t)$, also varies stochastically, following the process;

(3) $$d\alpha_k = (a_1 + b_{11}\,\alpha_k(t) + b_{12}\,\pi(t))\,dt + \sigma_\alpha\,dz_\alpha$$

and where $\pi(t)$ is another state variable, following the process;

(4) $$d\pi = (a_2 + b_{21}\alpha_k(t) + b_{22}\pi(t))\,dt + \sigma_\pi\, dz_\pi \,.$$

Assumption 3 The monetary policy of the government is assumed to result in a process for the nominal price level, $P(t)$, of the form;

(5) $$dP/P = \pi(t)\,dt + \sigma_p\, dz_p\,, \quad dz_k dz_p = \rho_{kp}\, dt$$

where $\pi(t)$ is the expected rate of inflation, which follows the process previously specified in equation (4).[2]

Similar to CIR (1985b), we assume an exogenous nominal price level process without explicitly modelling the nature of money supply and demand that would give rise to it. However, unlike CIR, the specifications in equations (2) to (5) allow for the possibility of interaction between the real return on capital and the rate of inflation. This could be important since certain monetary models predict that technological change, in affecting the efficiency (return) of capital, influences the economy's rate of inflation, e.g., see Siegel's (1983) extension of the Sidrauski (1967) model. Equation (4) captures this effect by allowing the expected rate of inflation to depend on the current expected return on capital.

In addition to the return on capital affecting inflation, there may well be a causal relationship working in the opposite direction. Fischer and Modigliani (1978) outline a number of ways that inflation can affect the real economy. Many of these influences are a result of taxation of nominal asset returns or non-indexed private contracts. Research, such as Feldstein (1980), has been able to demonstrate a link between higher expected inflation with lower asset valuation. Equation (3) allows for this result by making the expected real return on capital depend on the level of expected inflation.[3]

The solution to the consumption and portfolio choice problem described by assumptions A.1 to A.3 is similar to others found in the literature, and the derivation is outlined in Appendix A. Without loss of generality, let $i(t)$ be the (instantaneous) nominal yield on a currently maturing, riskless in terms of default, discount bond. In addition, let $r(t)$ be the (instantaneous) real yield on a currently maturing, riskless in terms of default, real discount bond. These bonds are assumed to be in zero net supply, and since individuals have identical preferences, their individual holdings equal zero.

Individuals' optimal consumption behavior implies that aggregate consumption is proportional to aggregate real capital;

$$C(t) = \rho K(t)\ . \tag{6}$$

In equilibrium, the nominal interest rate equals;

$$i(t) = \alpha_k(t) - \sigma_k^2 + \pi(t) - \sigma_p^2 - \rho_{kp}\sigma_k\sigma_p \tag{7}$$

and the equilibrium real interest rate is;

$$r(t) = \alpha_k(t) - \sigma_k^2\ . \tag{8}$$

Thus the instantaneous riskless real interest rate has dynamics similar to the dynamics of the expected rate of return on risky capital;

$$\begin{aligned} dr = d\alpha_k &= \left(a_1 + b_{11}\sigma_k^2 + b_{11}r(t) + b_{12}\pi(t)\right)dt + \sigma_\alpha dz_\alpha\ . \\ &\equiv (a_r + b_{11}r(t) + b_{12}\pi(t))\,dt + \sigma_r dz_r\ . \end{aligned} \tag{9}$$

The state of the economy is determined by the level of real wealth, $K(t)$, along with the bivariate system consisting of the real interest rate and expected inflation. Letting $s(t)' = (r(t)\ \pi(t))$, equations (9) and (4) can be re-written in the form;

$$ds(t) = (A + Bs(t))\,dt + \sigma dZ \tag{10}$$

where $A' = (a_r\ a_\pi)$, $B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$, and $(\sigma dZ)' = (\sigma_r dz_r\ \sigma_\pi dz_\pi)$, with $\sigma dZ(\sigma dZ)' = \Sigma dt$, where $\Sigma = \begin{pmatrix} \sigma_r^2 & \sigma_{\pi r} \\ \sigma_{r\pi} & \sigma_\pi^2 \end{pmatrix}$.

The process for $r(t)$ and $\pi(t)$ will be stationary if the real parts of the eigenvalues of B are negative. Also note that, unlike CIR (1985b), the process given by equation (10) will not restrict either $r(t)$ or $\pi(t)$ from becoming temporarily negative. In one sense, this is an advantage of the present model, since without a riskless real storage technology, $r(t)$ could be negative while expected deflation implies $\pi(t)$ could be negative. There appear to be historical episodes that are consistent with $r(t) < 0$, such as in the U.S. during the mid 1970's, and other times where $\pi(t) < 0$, such as in the U.K. during the early 1920's when attempting to return to the gold standard at pre-war parity. Nevertheless, we would ideally like to impose the restriction that the nominal interest rate be non-negative, i.e. $i(t) = r(t) + \pi(t) - \sigma_p^2 - \rho_{kp}\sigma_k\sigma_p \geq 0$. Unfortunately, this leads to significant difficulties in deriving solutions for equilibrium bond prices, so that this restriction will not be imposed in the analysis that follows.[4] But we can guess that the effect from $i(t) > 0$ will lead to estimates of b_{12}, b_{21}, and/or $\sigma_{r\pi}$ which reflect negative correlation between $r(t)$ and $\pi(t)$.

III. Derivation of the Term Structure

We can now derive the term structure of interest rates for (zero net supply, default-free) nominal bonds. Following the work of Richard (1978) and Langetieg (1980), let $N(\tau, s(t))$ be the nominal price at date t of a discount bond which pays \$1 with certainty at date $t+\tau$. Since the bond price depends on the vector of state variables, $s(t)$, Ito's lemma allows us to write its dynamics as;

(11) $$dN(\tau, s(t))/N(\tau, s(t)) = \mu_n(s(t))\,dt + (N_r\sigma_r/N)\,dz_r + (N_\pi\sigma_\pi/N)\,dz_\pi$$

where

(12) $$\mu_n = (1/N)\left[N_r\mu_r + N_\pi\mu_\pi + \tfrac{1}{2}N_{rr}\sigma_r^2 + \tfrac{1}{2}N_{\pi\pi}\sigma_\pi^2 + N_{r\pi}\sigma_{r\pi} - N_\tau\right]$$

where μ_r and μ_π are the instantaneous expected changes in $r(t)$ and $\pi(t)$, respectively, given in equation (10).

A currently maturing bond is instantaneously riskless in nominal terms, so that we can define the yield on this bond to be the nominal interest rate, $i(t, s(t))$;

(13) $$dN(\tau = 0, s(t))/N(\tau = 0, s(t)) = -N_\tau \equiv i(t, s(t))$$

since $N_r = N_\pi = N_{rr} = N_{\pi\pi} = N_{r\pi} = 0$ as the bond price approaches its maturity value of \$1. If bonds are priced such that no arbitrage opportunities exist in equilibrium, then a standard hedging argument can be employed to show that the expected rate of return on a τ maturity bond must be of the form;

(14) $$\mu_n = i(t) + \chi_r N_r \sigma_r / N + \chi_\pi N_\pi \sigma_\pi / N$$

where χ_r and χ_π are the "factor risk premia" or "market prices of risk" of a unit of standard deviation from the real interest rate and inflation, respectively. CIR (1985a) shows that, in equilibrium, the factor risk premia equal;

(15) $$\chi_s = \left(\frac{-U_{CC}(C*)}{U_C}\right)\text{cov}(C*, s)$$

where $U(C*)$ is the utility function evaluated at individuals' optimal level of consumption, and $\text{cov}(C*, s)$ is the covariance between changes in optimal consumption and changes in the state variable, s. Given that utility is logarithmic, $C* = \rho K(t)$, and using equation (10), we have $\chi_r = \sigma_k \sigma_r \rho_{kr}$ and $\chi_\pi = \sigma_k \sigma_\pi \rho_{k\pi}$.

Equating the right hand sides of equations (12) and (14) we obtain a partial differential equation that the equilibrium bond price, $N(\tau, s(t))$, must satisfy. Richard (1978) shows that the solution to this equation can be written in the form;

(16) $$N(\tau, s(t)) = E_t \exp\left[-\int_t^{t+\tau}\left(i(v) + \tfrac{1}{2}\Psi'\Sigma^{-1}\Psi\right)dv - \int_t^{t+\tau}\Psi'\Sigma^{-1}\sigma dZ(v)\right]$$

where $\Psi' = (\sigma_r \chi_r \;\; \sigma_\pi \chi_\pi)$, and where $i(t) = r(t) + \pi(t) - \sigma_p^2 - \rho_{kp}\sigma_k \sigma_p$.

Some insight may be gained by comparing this formula to that of a forecast of inflation over the life of the bond, i.e., the period from t to $t+\tau$. From the stochastic process for inflation, equation (5), we obtain;

(17) $$E_t[P(t+\tau)/P(t)] = E_t \exp\left[\int_t^{t+\tau}\left(\pi(v) - \sigma_p^2\right)dv + \int_t^{t+\tau}\sigma_p dz(v)\right].$$

Note the similarity in equations (16) and (17), especially the first terms under the integral signs on the right hand side of each equation. One component of the bond price is the expectation of an exponential function of the integral of the nominal interest rate, of which one component is the instantaneous mean rate of inflation, $\pi(t)$. Similarly, the expected rate of inflation over the bond's life, equation (17), has this same component.

The results of Langetieg (1980) allow us to solve for the equilibrium price of a bond of any maturity. Denote the log of the bond price as $n(\tau, s(t)) = \ln N(\tau, s(t))$. Using equations (16) and (10), the formula for $n(\tau, s)$ can be shown to take the form;

$$\begin{aligned} (18) \qquad n(\tau, s) &= K_1(t) \\ &+ \left[c_1 + g_1(b_{11} + b_{21} - \lambda_2)e^{\lambda_1 \tau} + g_2(b_{11} + b_{21} - \lambda_1)e^{\lambda_2 \tau}\right] r(t) \\ &+ \left[c_2 + g_1(b_{22} + b_{12} - \lambda_2)e^{\lambda_1 \tau} + g_2(b_{22} + b_{12} - \lambda_1)e^{\lambda_2 \tau}\right] \pi(t) \\ &\equiv K_1(\tau) + \alpha_1(\tau) r(t) + \alpha_2(\tau)\pi(t) \end{aligned}$$

where $\lambda_{1,2} = \frac{1}{2}\left(b_{11} + b_{22} \pm \left[(b_{11} - b_{22})^2 + 4b_{12}b_{21}\right]^{\frac{1}{2}}\right)$,

$$c_1 = (b_{22} - b_{21})/(b_{11}b_{22} - b_{21}b_{12}) \; ,$$

$$c_2 = (b_{11} - b_{12})/(b_{11}b_{22} - b_{21}b_{12}) \; ,$$

$$g_1 = [(b_{22} + b_{12} - \lambda_1)(b_{21} - b_{22}) + (b_{11} + b_{21} - \lambda_1)(b_{11} - b_{12})]/d \; ,$$

$$g_2 = [(b_{22} + b_{12} - \lambda_2)(b_{22} - b_{21}) + (b_{11} + b_{21} - \lambda_2)(b_{12} - b_{11})]/d \; ,$$

with $d = [(b_{11} + b_{21} - \lambda_2)(b_{22} - b_{12} - \lambda_1) - (b_{11} + b_{21} - \lambda_1)(b_{22} + b_{12} - \lambda_2)](b_{11}b_{22} - b_{12}b_{21})$ and $K_1(\tau)$ is a constant, given the bond's maturity τ. Note that the coefficients multiplying $r(t)$ and $\pi(t)$ in equation (18) are only functions of τ and the elements of the matrix B.

Similarly using equations (17) and (10), we can solve for the optimal forecast of the price level, given the current price level, $P(t)$, real interest rate, $r(t)$, and instantaneous rate of inflation, $\pi(t)$. Denoting the time t optimal forecast of the price level at $t + T$ as $E_t[P(t + T)]$, we find it satisfies;

(19)
$$\ln(E_t[P(t+T)/P(t)]) = K_2(T)$$
$$+ \left[c_3 + g_3(b_{11} + b_{21} - \lambda_2)e^{\lambda_1 T} + g_4(b_{11} + b_{21} - \lambda_1)e^{\lambda_2 T}\right] r(t)$$
$$+ \left[c_4 + g_3(b_{22} + b_{12} - \lambda_2)e^{\lambda_1 T} + g_4(b_{22} + b_{12} - \lambda_1)e^{\lambda_2 T}\right] \pi(t)$$
$$\equiv K_2(T) + \alpha_3(T)r(t) + \alpha_4(T)\pi(T)$$

where $c_3 = b_{21}/(b_{11}b_{22} - b_{21}b_{12})$,

$c_4 = -b_{11}/(b_{11}b_{22} - b_{21}b_{12})$,

$g_3 = -b_{21}(b_{22} + b_{12} - \lambda_1) - b_{11}(b_{11} + b_{21} - \lambda_1)/d$

$g_4 = b_{21}(b_{22} + b_{12} - \lambda_2) + b_{11}(b_{11} + b_{21} - \lambda_2)/d$

and $K_2(T)$ is a constant, independent of $r(t)$ and $\pi(t)$. Again, the coefficients multiplying $r(t)$ and $\pi(t)$ are functions only of T and the elements of the matrix B.

IV. Estimation of the Model

In this section, we outline a method of estimating the parameters of the previous model. This bond pricing model was formulated in continuous time so that analytic solutions for bond prices could be obtained. However, since the data used to estimate the model are available only at discrete observation intervals, it will be useful to specify the discrete time analog of this model. We show that this model can be put in the form of a discrete time state space system.[5]

The process for the state variables, $s(t)$, given by equation (10) above, can be re-written in a discrete time form as;

(20)
$$s(t+\delta) = \gamma(\delta) + \Phi(\delta)s(t) + v_t(\delta) ,$$

where $\Phi(\delta) = e^{B\delta} = \begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix}$ is the fundamental solution matrix of the state variable process. Its elements take the form;

$$\phi_{11} = -(b_{22} + b_{12} - \lambda_1)(b_{11} + b_{21} - \lambda_2)e^{\lambda_1\delta} + (b_{11} + b_{21} - \lambda_1)(b_{22} + b_{12} - \lambda_2)e^{\lambda_2\delta}/\theta$$
$$\phi_{21} = (b_{11} + b_{21} - \lambda_1)(b_{11} + b_{21} - \lambda_2)e^{\lambda_1\delta} - (b_{11} + b_{21} - \lambda_1)(b_{11} + b_{21} - \lambda_2)e^{\lambda_2\delta}/\theta$$
$$\phi_{12} = -(b_{22} + b_{12} - \lambda_1)(b_{22} + b_{12} - \lambda_2)e^{\lambda_1\delta} + (b_{22} + b_{12} - \lambda_1)(b_{22} + b_{12} - \lambda_2)e^{\lambda_2\delta}/\theta$$

$$\phi_{22} = (b_{11} + b_{21} - \lambda_1)(b_{22} + b_{12} - \lambda_2)e^{\lambda_1 \delta} - (b_{22} + b_{12} - \lambda_1)(b_{11} + b_{21} - \lambda_2)e^{\lambda_2 \delta}/\theta$$

and $\theta = (b_{11} + b_{21} - \lambda_1)(b_{22} + b_{21} - \lambda_2) - (b_{22} + b_{12} - \lambda_1)(b_{11} + b_{21} - \lambda_2)$.

Also $\gamma(\delta) = \int_0^\delta \Phi(s) A ds$ and $v_t(\delta)$ is a normally distributed random variable with zero mean and covariance matrix $Q \equiv \int_0^\delta \Phi(\omega) \Sigma \Phi'(\omega) d\omega$.

Equation (20) can be viewed as the "state transition equation" in a discrete time state space model where these states, the instantaneous real interest rate and expected inflation, are unobservable to the econometrician. Now if data on bid and ask prices of discount bonds are available, in addition to survey data on individuals' inflation forecasts, a set of measurement equations that correspond to this state transition equation can be constructed. As will be discussed below, our use of inflation forecasts, in addition to data on bond prices, arises from a need for identification restrictions on the parameters of the real interest rate-inflation process.

Suppose that we are not able to observe actual traded prices of nominal bonds, but these traded prices are hypothesized to be of the form given in equation (18). Instead, if we have only bid and ask prices on bonds, then these observed prices will measure the "true" traded bond prices with error. For simplicity, we assume that the log of the average of the bid and ask prices of a bond maturing in τ periods, $n*(\tau, s)$, differs from the hypothetical log of the trading price by a normally distributed error term which is independent across time, but whose variance may depend on the maturity of the given bond. This assumption gives us a measurement equation of the form;

(21) $$n*(\tau, s(t)) = n(\tau, s(t)) + e_{1t}(t) \text{ where } e_{1t}(\tau) \text{ is } N(\mu_\tau, \sigma_\tau) .$$

Another type of measurement equation can be constructed by the use of survey data on inflation forecasts that are assumed to be noisy measures of the "true" market forecast of inflation. We make the assumption that the mean survey inflation forecast deviates from the market forecast by an error term that is distributed independently across time and whose variance may depend on the time into the future over which the forecast is made. Denoting this survey inflation forecast as $E_t^* [P(t+T)/P(t)]$, we have;

(22) $\ln E_t^*[P(t+T)/P(t)] = \ln E_t[P(t+T)/P(t)] + e_{2t}(T)$ where $e_{2t}(T)$ is $N(\mu_T, \sigma_T)$.

In practice, the error terms in equations (21) and (22) will also be due to slight data misalignment, as the survey responses may not correspond to the exact time that the bond price quotations are reported.[6]

Now suppose that at date t we can observe (with error) the prices of discount bonds at M maturities, $\tau_1, ..., \tau_M$, and survey forecasts of inflation for N periods into the future, $T_1, ..., T_N$. Then we can write these observations in matrix form as;

(23) $$y(t) = K + \alpha s(t) + e_t$$

where

$$y'(t) = \left[n*(\tau_1, s) \; ... \; n*(\tau_M, s) \; \ln\left(E_t^*[P(t+T_1)/P(t)]\right) \; ... \; \ln\left(E_t^*[P(t+T_N)/P(t)]\right)\right] ,$$

K is an $M+N$ vector of constants, and α is an $(M+N) \times 2$ matrix whose first M rows are of the form $(\alpha_1(\tau_i) \; \alpha_2(\tau_i))$, $i = 1, ... M$, and whose last N rows are of the form $(\alpha_3(T_j) \; \alpha_4(T_j))$, $j = 1, ..., N$. The $M+N$ vector of error terms is distributed as $N(0, R)$ where the covariance matrix R has diagonal elements $\left(\sigma^2_{\tau_1} \; ... \; \sigma^2_{\tau_M} \; \sigma^2_{T_1} \; ... \; \sigma^2_{T_N}\right)$.

Equations (20) and (23) now comprise a state space model. However, another slight simplification can be obtained by defining the state variables not as $s(t)' = (r(t) \; \pi(t))$, but as the deviations of the real interest rate and expected inflation from their unconditional (long run) means. We can define a new vector of state variables, $x(t)' = (x_1(t) \; x_2(t))$ with zero unconditional mean;

$$x_1(t) = r(t) - (\gamma_1(1-\phi_{22}) + \gamma_2\phi_{12})/[(1-\phi_{11})(1-\phi_{22}) - \phi_{12}\phi_{21}]$$

$$x_2(t) = \pi(t) - (\gamma_2(1-\phi_{11}) + \gamma_1\phi_{21})/[(1-\phi_{11})(1-\phi_{22}) - \phi_{12}\phi_{21}] .$$

With this state variable transformation, the state space model becomes;

(24a) $$x_t = \Phi x_{t-1} + v_t$$

(24b) $$y_t = \alpha x_t + \beta + \varepsilon_t$$

with $\begin{pmatrix} v_t \\ e_t \end{pmatrix}$ distributed normal with covariance matrix $\begin{pmatrix} Q & 0 \\ 0 & R \end{pmatrix}$ and where β is an $M + N$ dimensional vector of constants and, without loss of generality, δ in the state transition equation (20) has been set to 1.

Our goal is now to estimate the parameters of the instantaneous real interest rate and expected inflation dynamics given by equation (10), and test the validity of the hypothesized bond pricing formula, equation (18). Specifically, we wish to estimate the elements of the matrix B and the elements of the covariance matrix Σ. Note that in the state space system (24), both Φ and α are uniquely determined by the parameters b_{11}, b_{12}, b_{21}, and b_{22} (i.e., the matrix B). In addition, the covariance matrix Q is uniquely determined by the covariance matrix Σ and the matrix B. Thus we want to fit the model (24) subject to the cross equation restrictions on the parameters of Φ, α, and Q, and thus obtain estimates of B and Σ.

The question arises whether these cross equation restrictions will be sufficient to identify the parameters of this state space system. Note that had we used only bond price data, one could not be sure whether the real interest rate or expected inflation was the first or second element of the state vector x_t, since each log bond price is just a linear combination of the underlying state variables and these coefficients cannot be identified, a priori.[7] However, the use of data on inflation forecasts provides us with the requirement that the shorter the inflation forecast horizon, the greater the dependence of this forecast on the current instantaneous expected rate of inflation. In the limit, as the forecast horizon, T, goes to zero, the inflation forecast simply equals the current expected rate of inflation, implying an exclusion (zero) restriction on the coefficient of the real interest rate, $r(t)$.

Following the work of Engle and Watson (1981, 1983), we now describe a maximum likelihood technique to estimate the parameters of the system in (24). Define the vector of "innovations," η_t, as;

(25)
$$\begin{aligned} \eta_t &= y_t - \hat{y}_t \\ &= y_t - \alpha\Phi E_{t-1}[x_{t-1} | y_{t-1}, y_{t-2}, ..., y_1] - \beta . \end{aligned}$$

Let $\hat{x}_{t-1} \equiv E_{t-1}[x_{t-1} | y_{t-1}, y_{t-2}, ..., y_1]$ be the optimal forecast of the unobservable state vector given the current and past values of the measurement equation observations. Also let H_t denote the time t covariance matrix of the innovations vector, η_t. Both $\hat{x}_{t-1}$ and H_t can be computed from a Kalman Filter recursion, given starting estimates for $\hat{x}_0$ and the covariance of x_0, denoted G_0, and given values of the parameters Φ, α, β, Q, and R (or equivalently given values for the parameters B, Σ, β, and R). $\hat{x}_t$ and H_t are computed as follows. Define $\hat{G}_t \equiv \Phi G_{t-1} \Phi' + Q$. Then we have;

(26) $$\hat{x}_t = \Phi \hat{x}_{t-1} + \hat{G}_t \alpha' H_t^{-1} (y_t - \alpha \Phi \hat{x}_{t-1} - \beta)$$

(27) $$G_t = \hat{G}_t - \hat{G}_t \alpha' H_t^{-1} \alpha \hat{G}_t$$

(28) $$H_t = \alpha \hat{G}_t \alpha' + R \ .$$

Once n_t and H_t are computed using the Kalman Filter, we can calculate the log-likelihood of the observations, y_t. Schweppe (1965) has shown that the log-likelihood can be written in terms of the innovations as;

(29) $$L = \sum_t L_t = \sum_t -\tfrac{1}{2}\left(\log|H_t| + \eta_t' H_t^{-1} \eta_t\right) \ .$$

The remaining step is to find those parameter values B, Σ, β, and R which maximize the above function by using a numerical iteration technique.

V. Data

A monthly time series of Treasury bill prices was obtained from the Center for Research in Security Prices (CRSP) tape. Similar to Gibbons and Ramaswamy (1986), we constructed time series of (hypothetical) 30, 90, 180, and 345 day Treasury bill prices by linearly interpolating the yields derived from the average bid and ask prices of Treasury bills whose maturities immediately surround the desired maturity. This time series of Treasury bill prices is available over the period 1964 through 1986.

Two different time series of survey inflation forecasts were used in carrying out estimations of the model The Institute for Social Research (ISR) at the University of Michigan produces a monthly survey of households' forecasts of inflation over the next 12 months.[8] These monthly surveys covered the period 1978 through 1986. The second source of inflation expectations data was obtained from the Livingston survey, which covers the period 1947-1986. The data comprises responses from semi-annual questionnaires sent to participating economists. They are asked to forecast the level of the Consumer Price Index (CPI) for 8 and 14 months into the future.[9] These forecasts are made around the end of May and November of each year. Although the Livingston survey is only available on a semi-annual, not a monthly, basis, we can still use this data along with monthly bond price data to estimate our model of the term structure. Appendix B outlines how the state space model can be re-formulated and estimated when the inflation forecasts are only observed at the end of every sixth month.

VI. Estimation Results

This section reports maximum likelihood estimates for the parameters B, Σ, β, and R. First, the model was estimated using the monthly Treasury bill prices along with the semi-annual Livingston survey 8 and 14 month mean inflation forecasts over the period 1964 through 1986. The elements of the observation vector, y_t, were ordered to equal the log of 30, 90, 180, and 345 day bond prices, and 8 and 14 month mean inflation forecasts, respectively. The covariance matrix R was restricted to be diagonal with elements r_i^2, where the standard deviations of the measurement errors were assumed to be the same for each of the log bond prices ($= r_1$), and the same for the log 8 and 14 month CPI forecasts ($= r_2$). The results are given in Table A.

The parameter estimates of the covariance matrix, Σ, indicate significant negative correlation ($\rho_{r\pi} = -.394$) between the innovations of the instantaneous real rate and expected inflation.[10] In addition, the standard deviation of the instantaneous real rate, σ_r, is almost twice as large as that of expected inflation, σ_π.

Turning to the estimates of the elements of matrix B, i.e., b_{ij}, $i, j = 1, 2$, it can be shown that these point estimates in Table A imply negative eigenvalues, consistent with the state variables following a stationary process. These estimates are generally significantly different from zero, including the off-diagonal elements, b_{12} and b_{21}. This implies that the expected change in real rates is positively related to the level of expected inflation while inflation is expected to rise as the current level of the real interest rate increases. A likelihood ratio test of the hypothesis of zero overall correlation between real rates and inflation, $b_{12} = b_{21} = \rho_{r\pi} = 0$, results in rejection at a 1% significance level.

In Figure 1, parameter estimates from Table A were used to simulate the expected paths of the instantaneous real interest rate and expected inflation resulting from a one standard deviation jump (innovation) in the real interest rate from its unconditional (long run) mean.[11] Note that the real interest rate returns half way to its long run value in approximately 5 years, implying rather weak mean reversion and leading to the "overshooting" of expected inflation from its long run value. Figure 2 repeats this simulation exercise for a one standard deviation innovation in the rate of expected inflation. Here, the half-life of expected inflation's mean reversion is approximately 2.5 years, one half that of the real interest rate which was simulated in Figure 1.

The model was re-estimated over the 1978-86 period, using the ISR's 12 month household mean inflation forecasts instead of the 8 and 14 month Livingston survey forecasts. These results are reported in Table B. Unlike the estimates using the Livingston survey data, the estimation using the ISR's forecast produces parameter estimates of the B matrix that are all insignificantly different from zero.[12] In addition, one cannot reject the hypothesis of zero correlation ($b_{12} = b_{21} = \rho_{r\pi} = 0$) between the processes for real interest rates and expected inflation.

There are a number of possible explanations for the differences in Table A and Table B. One obvious explanation is that the inflation forecasts reported in the Livingston survey are those of professional economists while the ISR survey reports forecasts of households. Also, in the Livingston survey, the participating economists are likely to have filled out their forecast questionnaires near the end of the months of May and November of each year, and thus their forecasts would coincide fairly closely with the reported end of month Treasury bill prices from the CRSP tape used in the estimation. In contrast, the ISR telephone surveys of households are spread fairly evenly throughout the month,[13] possibly causing significant data misalignment. This would imply a non-zero correlation between the error vector in the state transition equation and that of the measurement equation, invalidating this assumption of our estimation technique

Another potential explanation for the difference in the estimation results would be the shorter recent time period used in the ISR estimation. To investigate this possibility, the Livingston data sample was split in half and the model was re-estimated over the separate 1964-75 and 1975-86 periods. Table C gives the results of this estimation for the earlier period while Table D gives the results for the latter period.

It is rather striking that in both Tables C and D, the estimates for the parameters of the covariance matrix, Σ, imply high standard deviations and correlation for the innovations in real interest rates and expected inflation. In each case σ_r and σ_π are close to .1 and $\rho_{r\pi}$ is close to -1. However, while the estimates of the covariance matrix are similar, the estimates of the matrix B are quite different. The estimates obtained in the earlier 1964-75 period imply relatively strong mean reversion in both the real interest rate and expected inflation. This can be seen from Figures 3 and 4 which give the expected paths of real interest rates and inflation from a one standard deviation increase. For this earlier period, the half life for mean reversion is less than one year for both the real rate and expected inflation. Mean reversion is weaker in the latter 1975-86 period. Figures 5 and 6 show that the half life for mean reversion from one standard deviation innovations is almost two years for both the real interest rate and expected inflation. A possible interpretation of this result is the earlier period coincided largely with a fixed exchange rate regime, while the latter period was one of flexible exchange rates. Fixed exchange rates may have resulted in greater monetary discipline with more predictable inflation and real interest rates.

VII. Conclusion

This paper formulated and estimated an equilibrium bond pricing model that explicitly accounts for the difference between real and nominal variables. An important feature of the model and the estimation technique is its allowance for interdependence between real interest rates and expected inflation. Macroeconomic theory gives justification for making a distinction between the process for the real interest rate and that for inflation. This paper produces some evidence indicating this distinction to be worthwhile.

Our empirical results show that unexpected changes in real interest rates are significantly negatively correlated with those of expected inflation, while a higher level of one variable implies a greater expected increase in the other. Evidence for significant mutual dependence between real interest rates and expected inflation has implications for a number of areas in macroeconomics and finance, such as the plausibility of real business cycle models or the general hypothesis of monetary neutrality.

Appendix A

The derivation of the solution to the consumers' consumption and portfolio choice problem, given by equations (6) to (8) in section II, is outlined here.

Let w_b, w_r, and w_k (= 1 - w_b - w_r) be the proportions of an individual's portfolio invested in nominal bonds, real (indexed) bonds, and real capital, respectively. Denoting an individual's total wealth as W, and $J(W, \alpha_k, \pi)$ as the indirect utility function, Merton (1971) shows that the optimality equation is;

$$\text{(A)} \quad \begin{aligned} \phi(C, w_b, w_r) &= \ln(C) + J_W\left\{W\left[(1 - w_b - w_r)\,\alpha_k + \left(i - \pi + \sigma_p^2\right) w_b + r\,w_r\right] - C\right\} \\ &+ \tfrac{1}{2} J_{WW} W^2 \left[(1 - w_b - w_r)^2 \sigma_k^2 + w_b^2 \sigma_p^2 - 2\rho_{kp} (1 - w_b - w_r)\, w_b\, \sigma_k \sigma_p\right] - \rho J \\ &+ J_{\alpha_k}(a_1 + b_{11}\alpha_k + b_{12}\pi) + J_\pi (a_2 + b_{21}\alpha_k + b_{22}\pi) \\ &+ \tfrac{1}{2} J_{\alpha_k \alpha_k} \sigma_\alpha^2 + \tfrac{1}{2} J_{\pi\pi} \sigma_\pi^2 + J_{\alpha_k \pi}\, \rho_{\alpha_k \pi}\, \sigma_\alpha \sigma_\pi \\ &+ J_{W\alpha_k} W \left(\rho_{\alpha_k k} (1 - w_b - w_r)\, \sigma_k \sigma_\alpha - \rho_{\alpha_k p}\, w_b\, \sigma_\alpha \sigma_p\right) \\ &+ J_{W\pi} W \left(\rho_{\pi k} (1 - w_b - w_r)\, \sigma_k \sigma_\pi - \rho_{\pi p}\, w_b\, \sigma_\pi \sigma_p\right) . \end{aligned}$$

Differentiating with respect to C, w_b, and w_r, one obtains;

$$\text{(B)} \qquad C = 1/J_W$$

$$\text{(C)} \quad \begin{aligned} & J_W W\left(-\alpha_k + i - \pi + \sigma_p^2\right) + J_{WW} W^2 \left(-(1 - w_b - w_r)\,\sigma_k^2 + w_b \sigma_p^2 - \rho_{kp}(1 - 2w_b - w_r)\,\sigma_k \sigma_p\right) \\ & - J_{W\alpha_k} W (\rho_{\alpha_k k} \sigma_k \sigma_\alpha + \rho_{\alpha_k p} \sigma_\alpha \sigma_p) - J_{W\pi} W (\rho_{\pi k} \sigma_k \sigma_\pi + \rho_{\pi p} \sigma_\pi \sigma_p) = 0 \end{aligned}$$

$$\text{(D)} \quad \begin{aligned} & J_W W(-\alpha_k + r) + J_{WW} W^2 \left(-(1 - w_b - w_r)\,\sigma_k^2 + \rho_{kp}\, w_b\, \sigma_k \sigma_p\right) \\ & - J_{W\alpha_k} W \rho_{\alpha_k k} \sigma_k \sigma_\alpha - J_{W\pi} W \rho_{\pi k} \sigma_k \sigma_\pi = 0 . \end{aligned}$$

Using equations (C) and (D), we can solve for the optimal portfolio proportions, w_b^* and w_r^*. Substituting these proportions back into equation (A), we then have a partial differential equation for $J(W, \alpha_k, \pi)$. Merton (1971) shows that the solution to this differential equation is of the form;

$$\text{(E)} \qquad J = \frac{1}{\rho} \ln W + H(\alpha_k, \pi) .$$

Since nominal and real bonds are assumed to be in zero net supply, we have $W(t) = K(t)$. Using conditions (B) and (E), we arrive at equation (6) in the text. Using condition (E) along with conditions (C) and (D), setting $w_b = w_r = 0$, we arrive at equations (7) and (8) in the text.

Appendix B

This appendix gives the state space representation used to estimate the bond pricing model when bond prices, y_{1t}, are observed at more frequent intervals than are survey inflation forecasts, y_{2t}.

The measurement equation (24b) can be re-written as:

(A) $$y_t = \begin{bmatrix} y_{2t} \\ y_{2t} \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} x_t + \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} + \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix}$$

Now define a new vector of observations, Y_t, where

(B) $$Y_t = \begin{bmatrix} y_{1t} \\ y_{2t\text{-}1} \end{bmatrix} \quad \text{when } y_{2t\text{-}1} \text{ is observed at time } t\text{-}1$$
$$= \begin{bmatrix} y_{1t} \\ 0 \end{bmatrix} \quad \text{when } y_{2t\text{-}1} \text{ is not observed at time } t\text{-}1 \,.$$

In addition, define the vector w_t such that

(C) $$w_t = y_{2t\text{-}1} - \beta_2 \,, \quad \text{when } y_{2t\text{-}1} \text{ is observed at } t\text{-}1$$
$$= 0 \,, \qquad \text{otherwise} \,.$$

A new vector of state variables, X_t, can be defined where

(D) $$X_t = \begin{bmatrix} x_t \\ w_t \end{bmatrix} = \begin{bmatrix} \Phi & & 0 \\ \alpha_{21} & \alpha_{22} & 0 \end{bmatrix} \begin{bmatrix} x_{t\text{-}1} \\ w_{t\text{-}1} \end{bmatrix} + \begin{bmatrix} v_t \\ e_{2t\text{-}1} \end{bmatrix}$$

is the form of the new state transition equation.

The measurement equation corresponding to this state equation is;

(E) $$Y_t = a_t X_t + b_t + \begin{bmatrix} e_{1t} \\ 0 \end{bmatrix}$$

where

$$a_t = \begin{bmatrix} \alpha_{11} & \alpha_{12} & 0 \\ 0 & & I \end{bmatrix} \quad \text{when } y_{2t-1} \text{ is observed at } t\text{-}1$$

$$= \begin{bmatrix} \alpha_{11} & \alpha_{12} & 0 \\ 0 & & 0 \end{bmatrix} \quad \text{otherwise}.$$

$$b_t = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} \quad \text{when } y_{2t-1} \text{ is observed at } t\text{-}1$$

$$= \begin{bmatrix} \beta_1 \\ 0 \end{bmatrix} \quad \text{otherwise}.$$

Calculation of the likelihood function when a_t and b_t are deterministic functions of time is similar to the method given in section IV of the text. See Harvey (1981, p. 110) for details.

Footnotes

[1]For simplicity, we assume a single technology. However, this assumption could be weakened to allow for multiple technologies, without changing the form of the process followed by the equilibrium real interest rate, e.g. see CIR (1985a).

[2]The formulation in equation (5) allows the government's monetary policy to be modelled as one in which real capital shocks to the price level may or may not be accommodated. $\rho_{kp} = -1$ could be regarded as a non-accommodating monetary policy, while if $\rho_{kp} = 0$, capital shocks are completely offset.

[3]The stochastic components of equations (3) and (4), $\sigma_\alpha dz_\alpha$ and $\sigma_\pi dz_\pi$, may also be correlated.

[4]As with the Vasicek (1977) and Langetieg (1980) models, leaving out this restriction still leads to sensible bond prices when the nominal interest rate is currently positive.

[5]A state space representation is also used by Hansen and Singleton (1987) to test asset pricing models by a general method of moments (GMM) technique. Their procedure allows for greater generality in consumers' utility but relies on the use of consumption data.

[6]Technically, error due to slight data misalignment in the measurement equations will imply a non-zero correlation between the error terms in the state transition equation and the measurement equations. Our estimation procedure assumes this correlation to be zero. However, it is clear that if the data misalignment is small relative to the observation interval, then this correlation between the measurement equation error terms and the state equation errors will be small.

[7]A similar problem arises in identifying the unobservable "sources of risk" in empirical work based on the Arbitrage Pricing Model of Ross (1976). Identification of these risk factors is unique only up to a nonsingular linear mapping.

[8]Juster and Comment (1980) give a detailed description of this data series.

[9]See Carlson (1977) for a description of the Livingston survey and the logic of picking 8 and 14 month forecast horizons when using this data.

[10]This is consistent with the results of other research on interest rates, such as Summers (1982) and Mishkin (1987).

[11]Given the non-zero correlation between the innovations in the real rate and expected inflation, a rise of σ_r in the real rate would be expected to produce a change of $\rho_{r\pi}\sigma_\pi$ in the level of expected inflation. For the current set of parameter estimates, this implies a fall in expected inflation equal to .0053.

[12]However, these point estimates imply negative eigenvalues.

[13]Reported in personal correspondence from Professor Thomas Juster.

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Table A

Parameter estimates using monthly 30, 90, 180, and 345 day Treasury bills and semi-annual 8 and 14 month Livingston CPI forecasts, 1964-1986.

278 monthly observations

(Standard errors in parentheses.)

b_{11}	b_{12}	b_{21}	b_{22}
-.15635	.11039	.16398	-.16598
(.08497)	(.05469)	(.07938)	(.05285)

σ_r	σ_π	$\rho_{r\pi}$
.02440	.01337	-.39419
(.00189)	(.00147)	(.14934)

β_1	β_2	β_3	β_4	β_5	β_6
-.00020	-.00176	-.00497	-.01144	.00838	.01687
(.00801)	(.02390)	(.04763)	(.09071)	(.03606)	(.06274)

r_1	r_2
.00089	.00129
(.00001)	(.00018)

Likelihood ratio test of H_0: $b_{12} = b_{21} = \rho_{r\pi} = 0$.
$-2\ln(L(\theta_0)/L(\theta)) = 12.74$
$\chi^2(3)|1\% = 11.34$

Table B

Parameter estimates using monthly 30, 90, 180, and 345 day Treasury bills and monthly University of Michigan Institute for Social Research 12 month inflation forecasts, 1978-1986.

108 monthly observations

(Standard errors in parentheses.)

b_{11}	b_{12}	b_{21}	b_{22}
-.03713	-.12929	-.01051	-.17837
(.18566)	(.14926)	(.18507)	(.14938)

σ_r	σ_π	$\rho_{r\pi}$
.03137	.01572	-.16470
(.00396)	(.00354)	(.31839)

β_1	β_2	β_3	β_4	β_5
-.00018	-.00717	-.01639	-.03539	.03495
(.00754)	(.02210)	(.04367)	(.08175)	(.02457)

r_1	r_2
.00103	.00567
(.00003)	(.00063)

Likelihood ratio test of H_0: $b_{12} = b_{21} = \rho_{r\pi} = 0$.
$-2\ln(L(\theta_0)/L(\theta)) = 0.71$
$\chi^2(3)\,|10\% = 6.25$

Table C

Parameter estimates using monthly 30, 90, 180, and 345 day Treasury bills and semi-annual 8 and 14 month Livingston CPI forecasts, 1964-1975.

140 monthly observations

(Standard errors in parentheses.)

b_{11}	b_{12}	b_{21}	b_{22}
-.69459	.06249	.21089	-.20358
(.19651)	(.10657)	(.20547)	(.10443)

σ_r	σ_π	$\rho_{r\pi}$
.11626	.10682	-.98925
(.01110)	(.01077)	(.21829)

β_1	β_2	β_3	β_4	β_5	β_6
.00004	-.00071	-.00276	-.00711	-.01952	-.02876
(.00445)	(.01333)	(.02654)	(.05029)	(.04690)	(.07739)

r_1	r_2
.00033	.00084
(.00001)	(.00027)

Likelihood ratio test of H_0: $b_{12} = b_{21} = \rho_{r\pi} = 0$.
$-2\ln(L(\theta_0)/L(\theta)) = 269.44$
$\chi^2(3)|1\% = 11.34$

Table D

Parameter estimates using monthly 30, 90, 180, and 345 day Treasury bills and semi-annual 8 and 14 month Livingston CPI forecasts, 1975-1986.

140 monthly observations

(Standard errors in parentheses.)

b_{11}	b_{12}	b_{21}	b_{22}
-.25149	.17228	.21089	-.61310
(.09477)	(.15836)	(.08837)	(.14615)

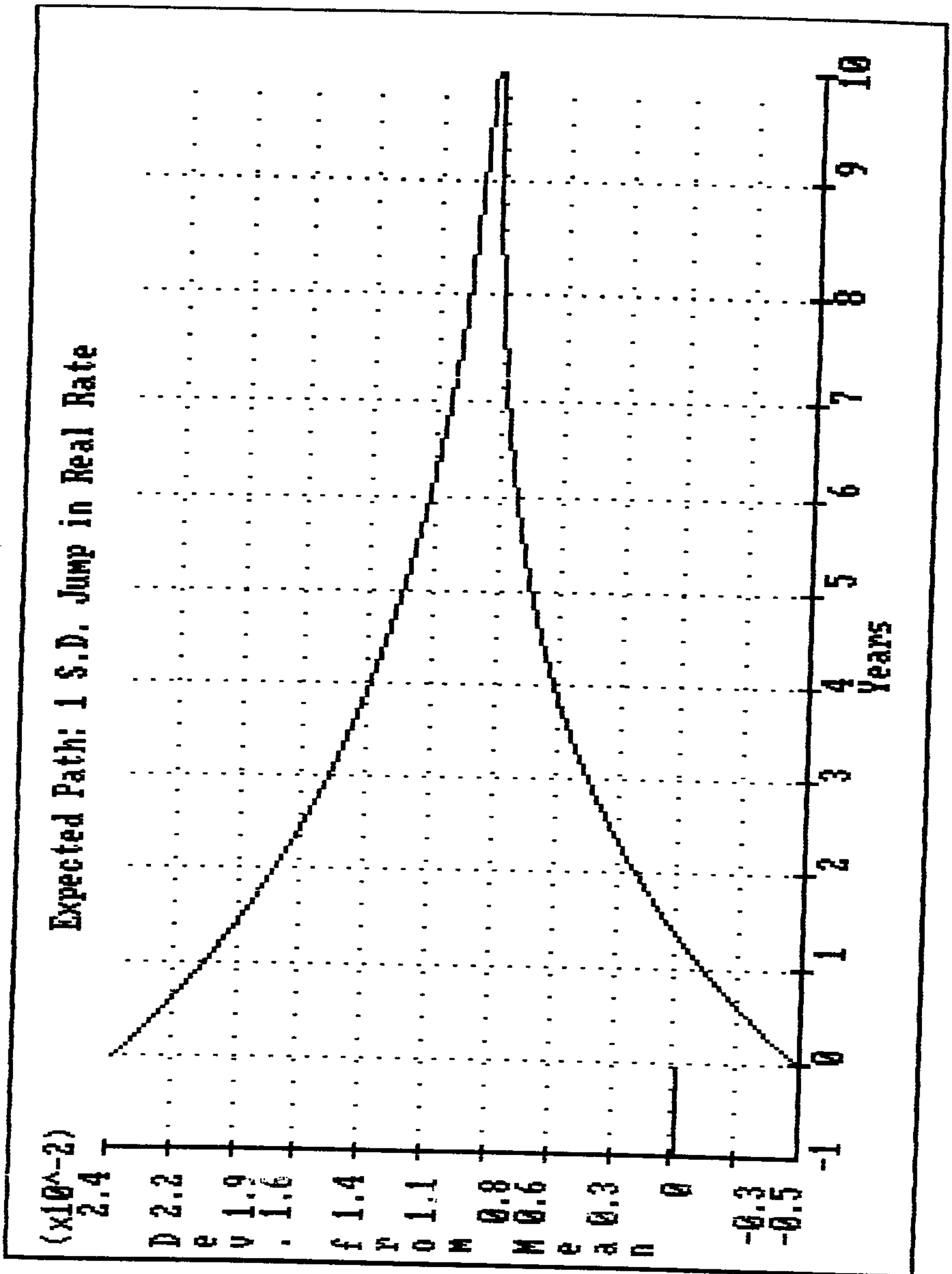
σ_r	σ_π	$\rho_{r\pi}$
.09384	.09837	-.94882
(.01048)	(.01204)	(.22412)

β_1	β_2	β_3	β_4	β_5	β_6
-.00041	-.00282	-.00767	-.01861	.02819	.05161
(.00868)	(.02586)	(.05125)	(.09641)	(.01226)	(.02102)

r_1	r_2
.00061	.00108
(.00001)	(.00019)

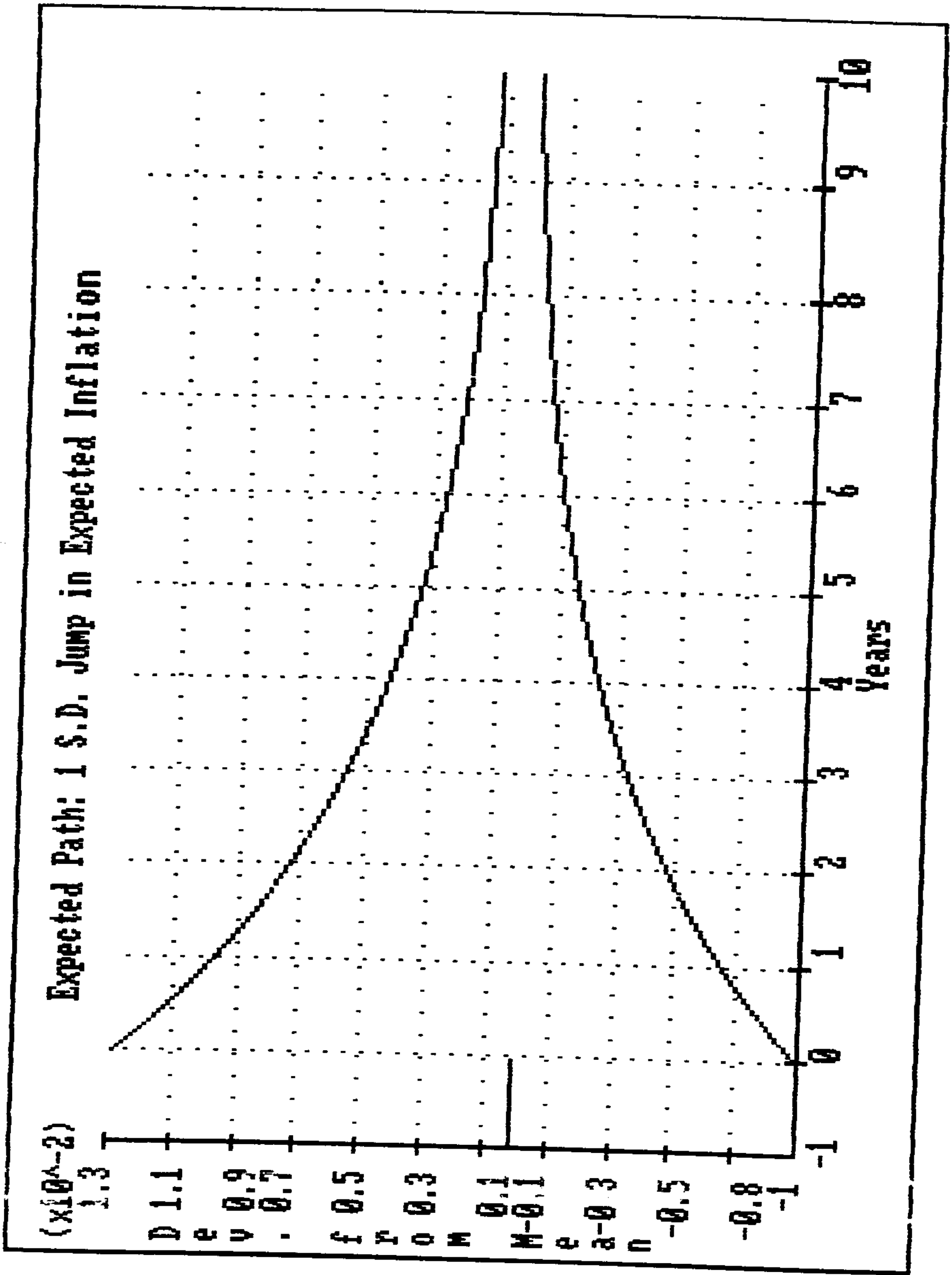
Likelihood ratio test of H_0: $b_{12} = b_{21} = \rho_{r\pi} = 0$.
$-2\ln(L(\theta_0)/L(\theta)) = 137.93$
$\chi^2(3)|1\% = 11.34$

Figure 1



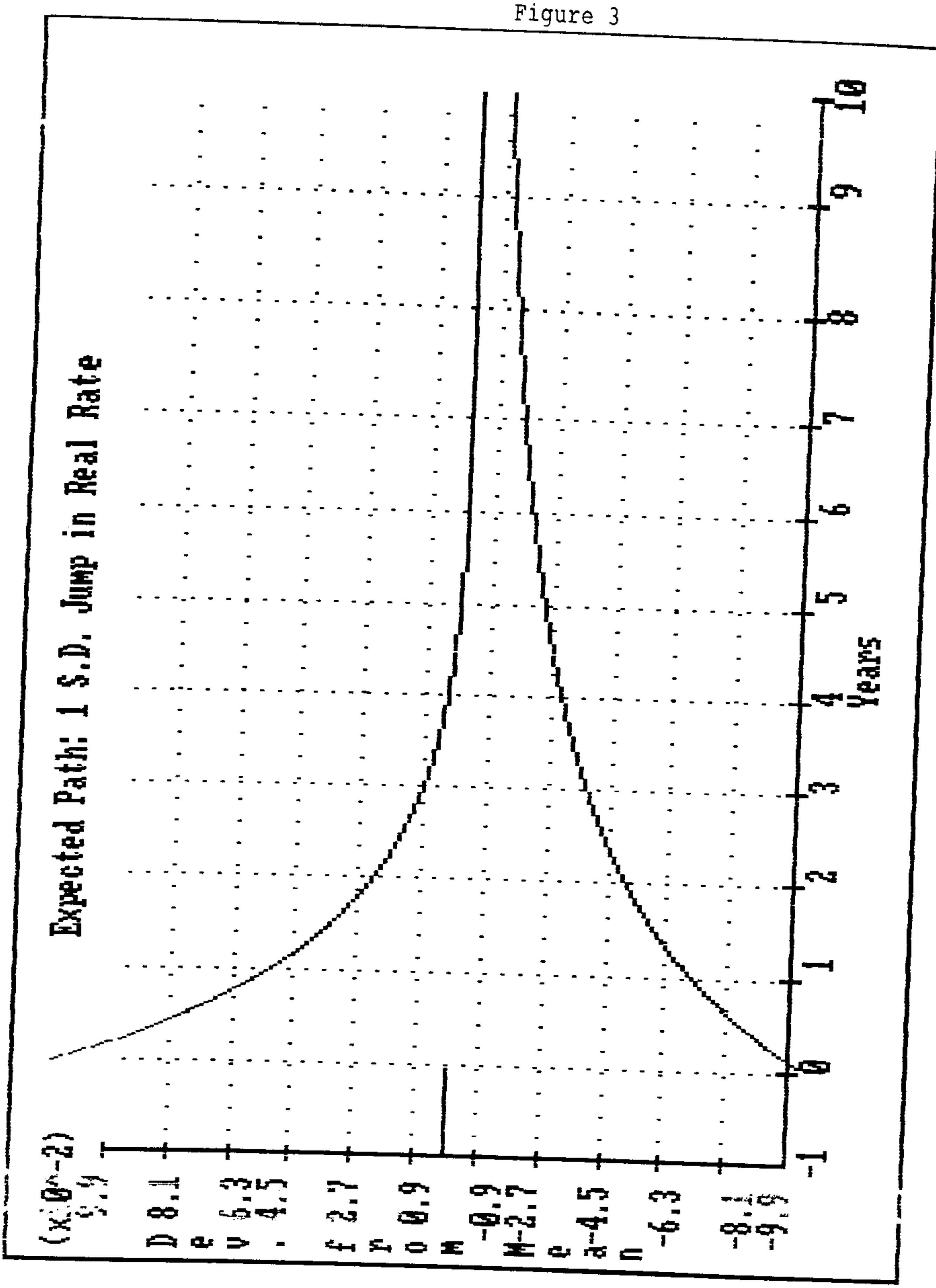
Data: 30, 90, 180, and 345 day Treasury bills. 1963.11 to 1986.12
8 and 14 month Livingston survey inflation forecasts.

Figure 2



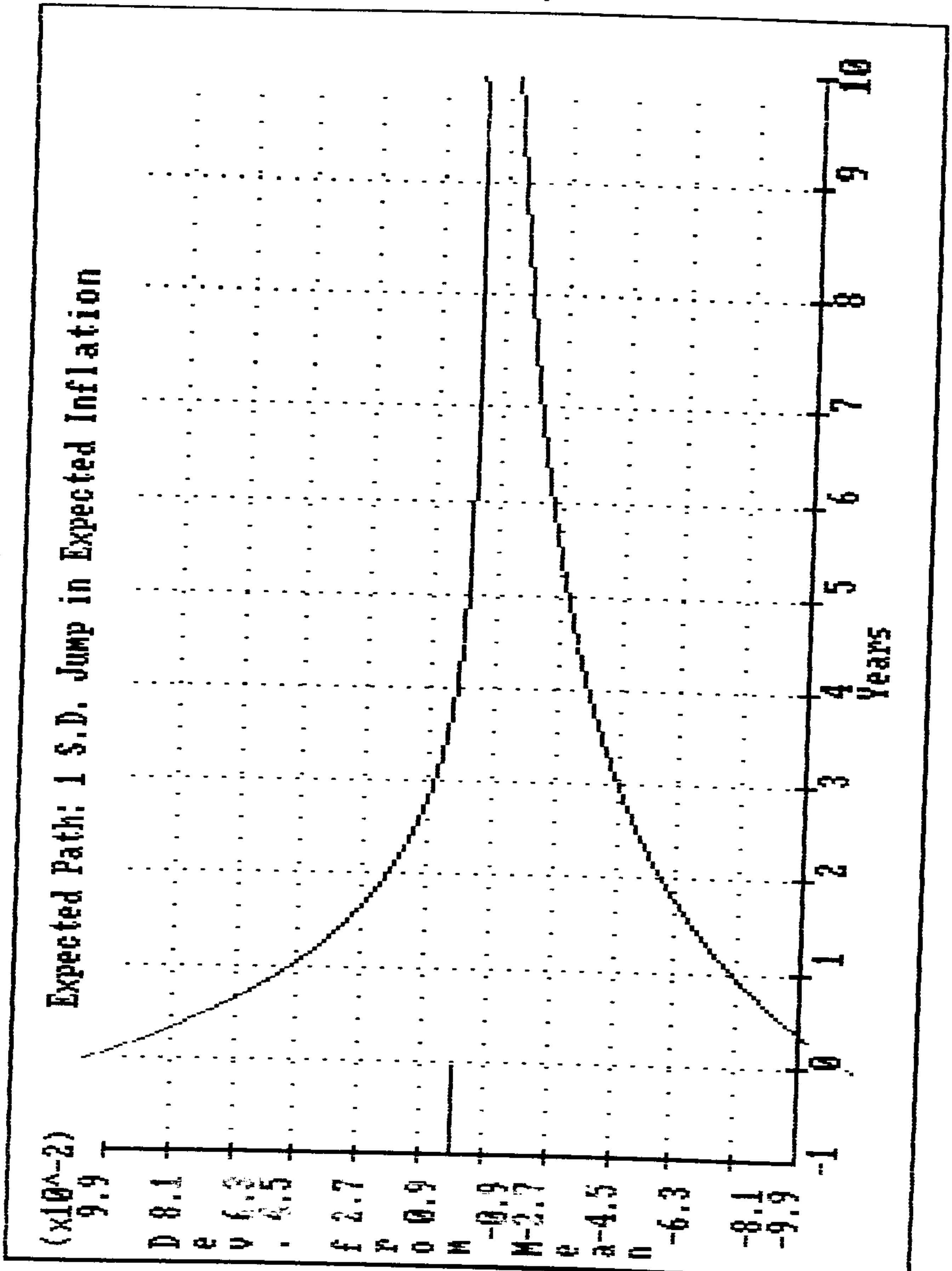
Data: 30, 90, 180, and 345 day Treasury bills. 1963.11 to 1986.12
8 and 14 month Livingston survey inflation forecasts.

Figure 3



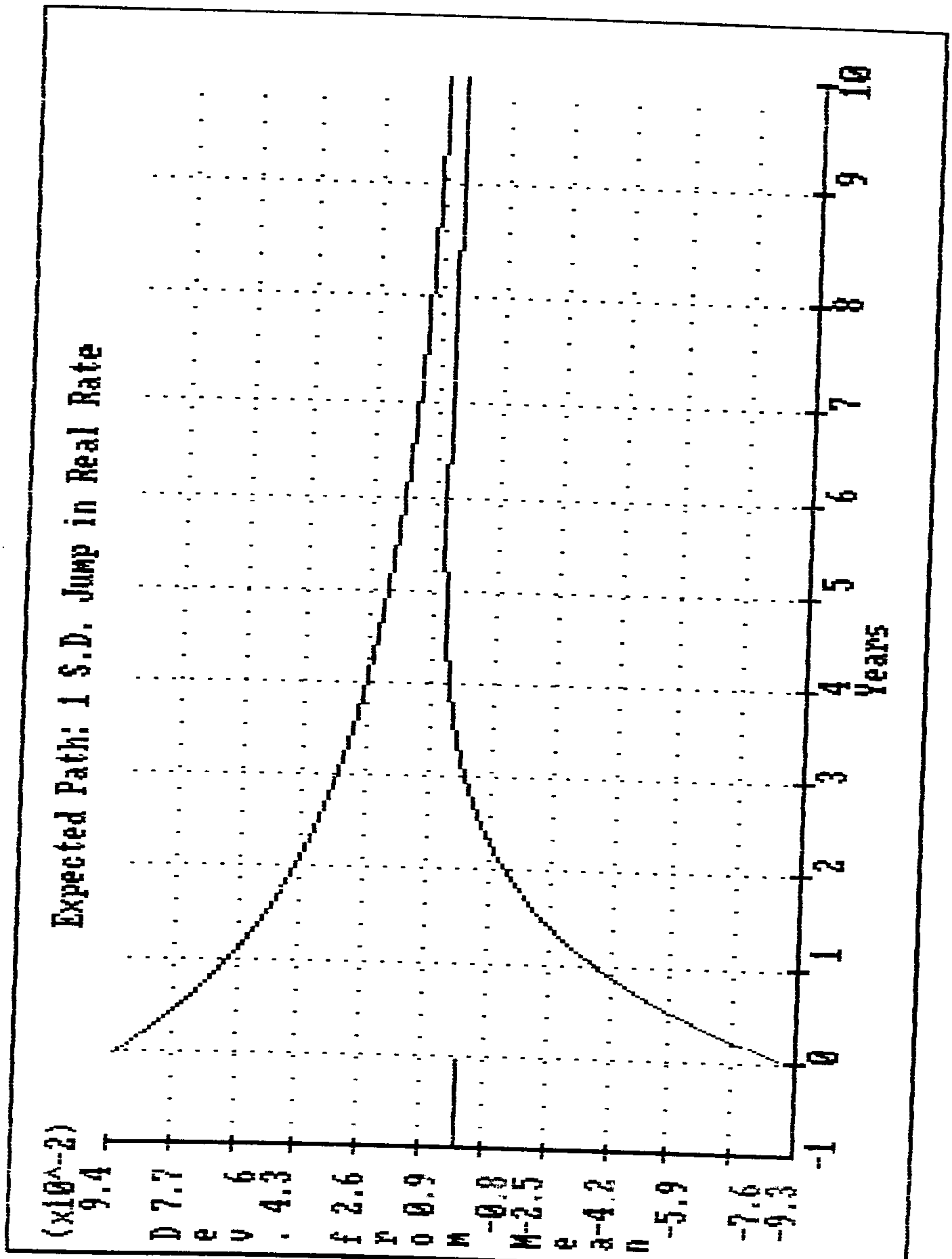
Data: 30, 90, 180, and 345 day Treasury bills. 1963.11 to 1975.6
8 and 14 month Livingston survey inflation forecasts.

Figure 4



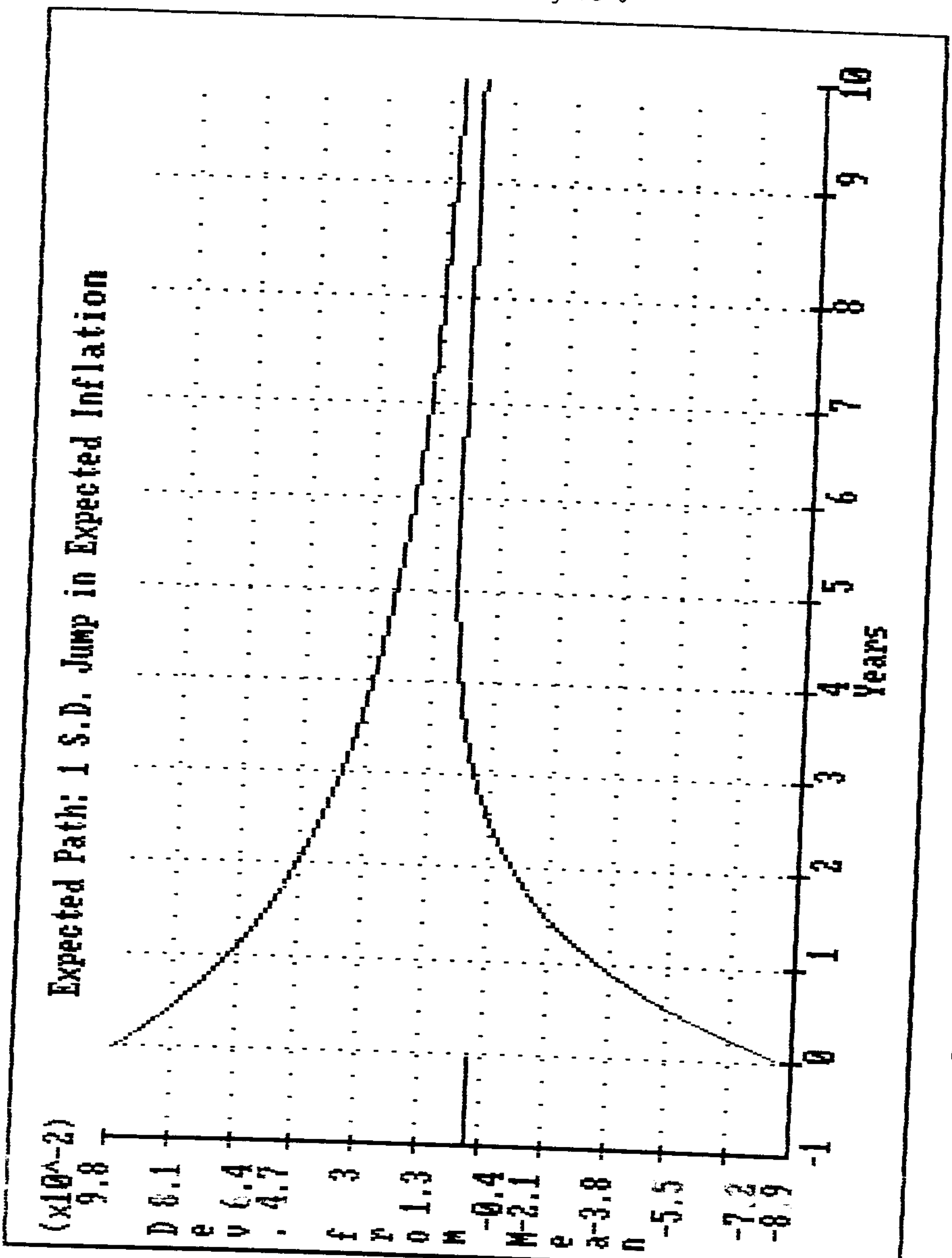
Data: 30, 90, 180, 345 day Treasury bills. 1963.11 to 1975.6
8 and 14 month Livingston survey inflation forecasts.

Figure 5



Data: 30, 90, 180, and 345 day Treasury bills. 1975.5 to 1986.12
8 and 14 month Livingston survey inflation forecasts.

Figure 6



Data: 30, 90, 180, and 345 day Treasury bills. 1975.5 to 1986.12
8 and 14 month Livingston survey inflation forecasts.