

**MANAGERIAL INCENTIVES IN AN
ENTREPRENEURIAL STOCK MARKET MODEL**

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1. INTRODUCTION

A moral hazard problem arises when the shareholders of a firm are unable to observe the conduct of its manager. If the firm's shares are publicly traded, it is natural to ask whether market forces effectively deal with the problem. This paper provides a formal answer. We define a competitive rational expectations equilibrium, referred to as a "Competitive Expectations Equilibrium" (CEE), in which the contract relating the manager's compensation to the firm's profit is determined by the manager's own portfolio choices. We then ask whether the market-determined contract is efficient in the second best sense of the principal-agent literature as represented, for example, by Holmstrom [1979] and Grossman and Hart [1983].

The principals in our model are the outside investors in a firm; their agent is an entrepreneur who is both the firm's initial owner and its manager. The moral hazard problem arises because the entrepreneur's labor supply ("effort") decision affects the firm's profit and is unobserved by other investors. The firm's equity is traded in a competitive market. Although the investors cannot observe the effort exerted by the entrepreneur, they can observe the fraction of the firm's equity that he retains. They also know that the entrepreneur's incentive to supply labor increases with his fractional share of profit. Thus, in a CEE, the equilibrium value of the firm's equity is an increasing function of the stake in the firm retained by the entrepreneur. The entrepreneur knows that by retaining a larger fraction of the firm's equity, he can realize a higher price for the equity that he sells. But increasing his stake in the firm not only induces the entrepreneur to exert

more effort, it also forces him to bear more risk. Thus, in making his portfolio decision, the entrepreneur balances his reluctance to bear risk and exert effort against his desire to raise his firm's market value.

Although the entrepreneur can, in a CEE, raise his firm's market value by raising his fractional share in the firm, this is not a consequence of his having monopoly power in the market for his firm's equity. In fact, a CEE is defined so that the entrepreneur has no monopoly power. The increase in the firm's market value caused by an increase in his fractional share occurs solely because investors use the entrepreneur's observable stake in the firm to infer his unobservable level of effort and, therefore, expect the entrepreneur's increased stake in the firm to provide him with an incentive to exert more effort.

We consider equilibria that are competitive in this sense because our aim is to establish an analog, applicable when there is moral hazard, of the first fundamental welfare theorem. It is, of course, also of interest to describe the conditions under which entrepreneurs can reasonably be assumed to act competitively in the equity market. Such conditions will be discussed in the body of the paper. At this point we merely observe that the entrepreneur must be a competitor in our sense if there is a risk neutral investor.

In studying the efficiency of an equilibrium contract, we ask whether it is Pareto optimal in an appropriately chosen set of contracts. This set of contracts is restricted in two ways. First, since effort is not observable,

each contract in this set must specify an effort choice that is optimal for the entrepreneur given the way in which the contract relates his compensation to profit. Second, the terms of the contract must result in a linear relationship between the entrepreneur's compensation and profit. This linearity restriction, imposed originally in Diamond [1967], is appropriate here because the only assets traded are the firm's equity and a riskless asset. Any equilibrium contract must, therefore, be linear, and the best that can be achieved by a CEE is efficiency in the set of linear contracts. We refer to contracts that are Pareto optimal in this restricted set as "restricted efficient".

By means of an example we show that a given economy can have many CEE that are not restricted efficient and are even Pareto-ranked. However, our main result, Theorem 3.1, gives sufficient conditions for a CEE to be restricted efficient. The theorem implies that every economy with multiple equilibria will have at least one that is restricted efficient. When an economy has multiple equilibria, each one is associated with a different function (consistent with rational expectations) relating the firm's market value to the entrepreneur's stake in the firm. An equilibrium is efficient if the associated function assigns to each fractional share (less than one) that might be retained by the entrepreneur the maximum value of the firm consistent with rational expectations on the part of investors.

The construction of the example with multiple restricted inefficient equilibria relies on the presence of income effects. Corollary 3.1 asserts

that if income effects are absent, then a CEE is restricted efficient. When uncertainty enters the firm's profit function "additively", and the firm's profit is a strictly concave function of effort, the income effects are, indeed, absent. The income effects are also absent if the investors have CARA utility functions.

A CEE is a generalization of the equilibrium concept employed in Marshall [1976] and Grossman and Hart [1982]. These authors considered the case of a risk-neutral principal. Marshall is concerned with an insurance market, but the equilibrium in his formal model can be reinterpreted as a CEE with risk-neutral investors. In Grossman and Hart, the unobserved action is an investment decision and the capital structure chosen by the firm's manager plays a role analogous to that of the entrepreneur's stake in the firm in a CEE.

A CEE is also similar to the signalling equilibrium of Leland and Pyle [1977]. In their model, however, the informational asymmetry arises because of adverse selection rather than moral hazard. Their manager doesn't take an unobserved action that determines the firm's profit; rather, he simply knows more about the firm's prospects than do the investors. In their equilibrium, the share of the firm retained by the manager is a signal of his private information rather than a determinant of his incentives to exert effort.

A CEE also has features in common with the rational expectations equilibrium introduced in Kihlstrom and Laffont [1982], which is essentially the same as

that defined by Diamond [1967]. The Kihlstrom-Laffont equilibrium is appropriate when there are no informational asymmetries; it would arise in the model of this paper if the entrepreneur's labor supply could be observed. In a Kihlstrom-Laffont equilibrium, investor's expectations would link the firm's market value to the entrepreneur's labor supply choice rather than to his fractional share in the firm. Kihlstrom and Laffont establish that their equilibria are always efficient in a restricted sense analogous to that used in this paper, i.e., efficient when attention is restricted to linear risk-sharing contracts.

The efficiency of market equilibria in the presence of agency problems has been discussed earlier by Stiglitz [1974] and Jensen and Meckling [1976]. Stiglitz considered this problem in a sharecropping setting, but his analysis is of obvious relevance for the study of financial market equilibrium. Jensen and Meckling's paper is the seminal contribution on this topic in the finance literature. Our analysis can be interpreted as a formal elaboration of the discussion in their Sections 2.2 and 2.5. In particular, a CEE is a specific equilibrium concept incorporating their rational expectations hypotheses, and our efficiency results are an indication of the extent to which their optimality discussion can be formalized. (An earlier attempt at formalizing the Jensen-Meckling efficiency discussion is contained in Beck and Zorn [1982]. Our explicit inclusion of effort costs is the most significant difference between our treatment and theirs.)

Diamond and Verrechia [1982] also integrate the theory of moral hazard with a

theory of financial market equilibrium. In their model, the manager's compensation, rather than being determined as it is in a CEE by the manager's own portfolio decisions, is instead entirely determined by an incentive contract imposed by the firm's owners. If the Diamond-Verrechia manager did choose a portfolio, its composition would, of course, influence his incentives in a way that would have to be considered by the investors in choosing a contract. Similar remarks apply to the model of Ramakrishnan and Thakor [1984]. One model that does incorporate the effect of the manager's portfolio decisions on his incentives and on the investor's choice of a contract is Campbell and Kracow [1985]. Although their equilibrium is similar to a CEE, they do not study its efficiency properties. In the context of a financial market equilibrium resembling our CEE, Hughes [1985] discusses moral hazard's effects on the CAPM, paralleling our discussion in Section 2.3.1. An earlier contribution, Marcus [1982], studied portfolio and investment decisions of managers whose fractional share of the firm's equity is constrained by the terms of an incentive contract. The terms of the incentive contract are, however, exogenous to Marcus' analysis.

The outline of the paper is as follows. In Section 2, we use a simple CAPM to introduce the equilibrium and efficiency concepts and to discuss the effects of moral hazard. The CAPM we consider incorporates the assumptions of additive uncertainty, normal probability distributions and CARA preferences. As noted earlier, these assumptions imply the absence of the income effects that can result in restricted inefficient equilibria. Thus, Corollary 3.1 of Theorem 3.1 implies that the CEE in the CAPM is restricted efficient. Section

2 contains a direct proof of this result. The general model is described and the main results established in Section 3. The example with multiple, Pareto-ranked CEE follows the formal definition of a CEE. The example is followed by the definition of restricted efficiency and by Theorem 3.1 and its corollaries. The section and the paper conclude with a discussion of the case of a risk neutral entrepreneur.

2. MORAL HAZARD IN A SIMPLE CAPM

Subsection 2.1 introduces moral hazard into the CAPM framework and defines the concepts of competitive expectations equilibrium (CEE) and restricted efficiency. The main result is that, in a CAPM setting, any CEE is restricted efficient. Subsection 2.2 briefly describes a noncompetitive equilibrium that is not restricted efficient. Subsection 2.3 describes the effects of moral hazard in the CAPM. Although this final subsection is a digression from our major focus on the restricted efficiency of CEE, this discussion is included because the influence of moral hazard can be clearly described under the special assumptions of the CAPM.

2.1 COMPETITIVE EXPECTATIONS EQUILIBRIUM AND RESTRICTED EFFICIENCY

We let L denote the amount of labor the manager supplies to the firm. The firm's profit is $f(L) + \tilde{x}$, where \tilde{x} is a random variable with mean $E\tilde{x} = 0$. We assume that $f'(L) > 0$ and $f''(L) < 0$, so that the firm's expected profit increases at a diminishing rate with the manager's labor supply. Initially we make no assumptions about the distribution of \tilde{x} . (Later in this section, we will assume that \tilde{x} is normally distributed.) The manager is assumed to have a fixed stock, \bar{L} , of labor that can either be supplied to the firm or to a competitive labor market in which the wage is w . Thus, the cost of the managerial labor supplied to the firm is the forgone wage that could be earned in alternative employment. A moral hazard problem arises because L cannot be observed by the firm's outside investors.

There are n outside investors, and it is assumed that they and the manager

maximize their expected utility of wealth. The manager's utility function is u_0 , and the utility function of the i^{th} investor is u_i . At this point, it is not necessary to be more explicit about the preferences of the investors and the manager; later in this section we will assume that their utility functions exhibit constant absolute risk aversion.

The manager is an entrepreneur who creates the firm. Hence, he is initially the sole owner of the firm, and it is he who can sell equity in the firm to the investors. The fractional share of the firm's equity retained by the entrepreneur is denoted by s , and the market value of the firm is denoted by V . Because the investors cannot observe L , the firm's market value cannot depend explicitly on L . The investors can, however, observe s .

The equilibrium can be loosely described as follows. The investors have rational expectations and therefore assume correctly that, in equilibrium, the manager's labor supply is an increasing function $L(s)$ of s . They, therefore, believe that the firm's expected profits will be $f(L(s))$ when the manager is observed to retain the fractional share s of the firm's equity. Thus, as will be shown, the firm's market value is an increasing function, $V(s)$, of s . The entrepreneur is assumed to know, when he chooses s , that his firm's market value is related to his s choice by the function $V(s)$. He, therefore, knows that by raising s , he can raise the price of the fraction, $(1 - s)$, of the firm's equity that he sells. The cost to the entrepreneur of raising s is borne in three forms: (i) he sells less equity, (ii) he is exposed to a larger fraction of the risks associated with the firm and (iii) he is induced to work

longer hours. The entrepreneur chooses s by comparing these costs to the benefit of increasing $V(s)$.

We can now be explicit about the effect of s on the entrepreneur's incentive to supply labor. Specifically, we can define the function $L(s)$. When the entrepreneur works L hours for the firm and retains the fractional share s , he receives $(1 - s)V(s)$ for the equity he sells. His total wealth is, thus,

$$s[f(L) + \tilde{x}] + (1 - s)V(s) + w[\bar{L} - L].$$

$L(s)$ is the L level that maximizes the entrepreneur's expected utility

$$Eu_0(s[f(L) + \tilde{x}] + [1 - s]V(s) + w[\bar{L} - L]). \quad (2.1)$$

Note that $L(s)$ maximizes

$$sf(L) - wL, \quad (2.2)$$

the net benefit to the entrepreneur of working L hours for the firm. Thus, $L(s)$, satisfies the first order condition

$$sf'(L(s)) - w = 0. \quad (2.3)$$

When the entrepreneur increases his share of the firm's equity from s to $s + \Delta s$, he has an incentive to work $L'(s)\Delta s$ additional hours for the firm and the firm's expected profit increases by $f'(L(s))L'(s)\Delta s$. Both of these terms are positive as we observe by implicitly differentiating (2.3) to obtain

$$L'(s) = - \frac{f'(L(s))}{sf''(L(s))} = - \frac{w}{sf''(L(s))} > 0 \quad (2.4)$$

and

$$f'(L(s))L'(s) = - \frac{[f'(L(s))]^2}{sf''(L(s))} > 0. \quad (2.5)$$

We now derive an expression for $V(s)$, the value of the firm when the

entrepreneur retains the fractional share s of the firm's equity. At this point we assume that \tilde{x} is normally distributed with variance σ^2 , and that the investors have CARA utility functions.

We let t_i represent the fraction of the firm's equity acquired by investor i , and \bar{y}_i represent his income endowment which he uses to purchase shares. The expected utility of investor i is

$$Eu_j(\bar{y}_i + t_i[f(L) + \tilde{x}] - t_i V(s)), \quad (2.6)$$

where

$$u_i(I_i) = -e^{-c_i I_i} \quad (2.7)$$

and c_i is the investor's Arrow-Pratt measure of absolute risk aversion.

Under the hypotheses that \tilde{x} is normally distributed and that u_i is given by (2.7), the expected utility (2.6) is a monotonically increasing function of

$$t_i f(L(s)) - t_i V(s) - c_i t_i^2 \sigma^2 / 2. \quad (2.8)$$

The first order condition satisfied by the t_i that maximizes (2.8) can be solved for $V(s)$ to obtain

$$V(s) = f(L(s)) - c_i t_i \sigma^2. \quad (2.9)$$

In the equilibrium, the expression for $V(s)$ must be independent of i . Thus,

in equilibrium, t_i and t_1 must be related by the equation

$$c_i t_i = c_1 t_1. \quad (2.10)$$

The equity market will be in equilibrium when supply equals demand, i.e., when

$$\sum_{i=1}^n t_i = (1 - s). \quad (2.11)$$

Together, (2.10) and (2.11) imply that, for each i ,

$$c_i t_i = c_m (1 - s), \quad (2.12)$$

where

$$c_m = \left[\sum_{i=1}^n (1/c_i) \right]^{-1}. \quad (2.13)$$

The parameter c_m is a measure of the market's aggregate risk aversion. When (2.12) is substituted in (2.9) the result is

$$V(s) = f(L(s)) - c_m (1-s)\sigma^2. \quad (2.14)$$

In expression (2.14), s appears twice. The term

$$\pi((1-s)\sigma^2) \equiv c_m (1-s)\sigma^2 \quad (2.15)$$

can be interpreted as an inverse demand curve relating the price of risk to the supply of risk. When the entrepreneur retains the fraction s of the firm's equity, he supplies the profits $(1-s)[f(L) + \tilde{x}]$ to the equity market. In the present mean-variance setting, the variance, $(1-s)\sigma^2$, of these profits measures the supply of risk. The term, $\pi((1-s)\sigma^2)$ defined in (2.15) is the mean return that makes the risk $(1-s)\sigma^2$ acceptable to investors. It is the price investors require for bearing this risk.

The entrepreneur can lower the price of risk, $\pi((1-s)\sigma^2)$, thereby raising $V(s)$, by using an s increase to reduce the supply of risk. If, however, the entrepreneur does this he is effectively behaving as a monopolist in the equity market. In a *competitive* expectations equilibrium, the entrepreneur should be a competitor in the firm's equity market. We, therefore, assume that when the entrepreneur chooses s , he does not recognize the effect that changes in $(1-s)\sigma^2$, the supply of risk, have on $\pi((1-s)\sigma^2)$, the price of risk. He effectively takes the price of risk as fixed at its equilibrium

level $\pi((1 - s^*)\sigma^2)$, where s^* is the equilibrium s level. This implies that the entrepreneur believes that the value of his firm is given by

$$V(s) = f(L(s)) - c_m(1 - s^*)\sigma^2 \quad (2.16)$$

where $L(s)$ is given by (2.3). The entrepreneur, therefore, expects an s increase to raise the market price of the firm by only

$$V'(s) = f'(L(s))L'(s). \quad (2.17)$$

REMARK 2.1: An entrepreneur who has the beliefs just described will be said to have *competitive expectations*. It should be emphasized that competitive expectations are not, in general, completely rational. Observe, however, that, if one of the investors, say investor i , is risk neutral, then $c_i = c_m = 0$. When $c_m = 0$, (2.15) implies that the price of risk is zero independent of the supply. In this case, the entrepreneur must necessarily be a competitor in the firm's equity market, and competitive expectations are, therefore, fully rational.

REMARK 2.2: Assuming that the entrepreneur takes the price of risk as given is consistent with the approaches of Diamond [1967] and Kihlstrom and Laffont [1982]. But it is in contrast to the approach taken by Stiglitz [1972]. The inefficiency in the stock market allocation of risk observed by Stiglitz is, in fact, attributable to the monopoly power exercised by the firm managers. In Stiglitz's model, managers exploit the fact that the amount of risk they supply determines the price of risk.

At this point, we introduce the hypothesis that the entrepreneur's utility

function is in the CARA class with an Arrow-Pratt absolute risk aversion measure equal to c_0 . The investor's expected utility, as shown in (2.1), is now an increasing function of

$$sf(L(s)) + (1-s)V(s) - wL(s) - c_0 s^2 \sigma^2 / 2. \quad (2.18)$$

Using the expression (2.16) for $V(s)$, (2.18) becomes

$$\begin{aligned} & sf(L(s)) + (1-s) \left\{ f(L(s)) - c_m (1-s^*) \sigma^2 \right\} - wL(s) - c_0 s^2 \sigma^2 / 2 \\ & = f(L(s)) - wL(s) - c_m (1-s)(1-s^*) \sigma^2 - c_0 s^2 \sigma^2 / 2. \end{aligned} \quad (2.19)$$

The first-order condition satisfied by the s that maximizes (2.19) is

$$\begin{aligned} & [sf'(L(s)) - w]L'(s) - \sigma^2 [c_0 s - c_m (1-s^*)] \\ & + (1-s)f'(L(s))L'(s) = 0. \end{aligned} \quad (2.20)$$

Because of (2.3), i.e., because, when he chooses $L = L(s)$, the entrepreneur equates his marginal benefit, $sf'(L(s))$, to his marginal cost, w , (2.20) reduces to

$$(1-s)f'(L(s))L'(s) - \sigma^2 [c_0 s - c_m (1-s^*)] = 0. \quad (2.21)$$

(If $f(L(s)) - wL(s)$ were concave in s , then, when $c_0 > 0$ (2.19) would be strictly concave and would be maximized by the unique s that satisfied the first order-condition (2.21). We will not analyze general conditions under which $f(L(s)) - wL(s)$ is concave. We observe, however, that if

$$f(L) = AL^\gamma,$$

where $A > 0$, then the first order condition (2.3) implies that

$$f(L(s)) - wL(s) = \left[(1/\gamma) s^{\gamma/(1-\gamma)} - s^{1/(1-\gamma)} \right] \left\{ [\gamma A/w^\gamma]^{1/(1-\gamma)} \right\},$$

which is a concave function of s when $0 < \gamma \leq 1/2$.)

The equilibrium s level, s^* , maximizes (2.19). Substituting $s = s^*$ in (2.21),

yields

$$(1 - s^*)f'(L(s^*))L'(s^*) - \sigma^2[c_0 s^* - c_m(1 - s^*)] = 0, \quad (2.22)$$

which implicitly defines s^* . It is easily verified that $s^* < 1$, when $c_0 > 0$.

Once s^* is obtained as the solution of (2.22), equations (2.3) and (2.12) can be used to solve for $L^* = L(s^*)$ and t_i^* , the equilibrium levels of L and t_i . The vector $\langle L^*, V^*(\cdot), s^*, t_1^*, \dots, t_n^* \rangle$ is the CEE for this example.

We can now define the set of restricted efficient contracts and demonstrate that the CEE is restricted efficient. A restricted efficient contract is, by definition, Pareto optimal in the set of available contracts. As noted in the introduction, it is appropriate to define the set of available contracts by imposing two restrictions. In general, the terms of a contract specify the amount of labor to be supplied by the entrepreneur as well as the compensation scheme that determines the relationship between the entrepreneur's compensation and the firm's profit. The first restriction we impose is on the form of the compensation scheme and requiring that it relate the entrepreneur's compensation to the firm's profit linearly. The second restriction, imposed because entrepreneurial labor is unobserved and cannot, therefore, be explicitly specified by the contract, is that the contract can explicitly specify only the compensation scheme; the labor input choice associated with the contract will be that which is preferred by the entrepreneur when he faces the specified compensation scheme.

The entrepreneur's compensation will vary linearly with the firm's profit if

we assume that the contract provides compensation in two forms: equity in the firm and a fixed salary. The fixed salary will be denoted by y , and the fraction of the firm's equity received by the entrepreneur will be denoted by s . The vector (s,y) will be referred to as the compensation package specified by the contract. The investors' incomes will also be linear functions of the firm's profit if, under the terms of the contract, each investor, i , receives a fraction t_i of the firm's equity and is obligated to pay a fraction q_i of the fixed wage y . The fractions t_i must, of course, be related to s by (2.11) and the fractions q_i must sum to one.

Because a contract can explicitly specify only the compensation package (s,y) , the labor input associated with the contract will be that which the entrepreneur has an incentive to supply when his compensation is determined by (s,y) . Specifically, when faced with the compensation package (s,y) , the entrepreneur will work $\lambda(s,y)$ hours, where

$$\lambda(s,y) \equiv \underset{L}{\operatorname{argmax}} E u_0(s[f(L) + \tilde{x}] + y + w[\bar{L} - L]).$$

Note that $\lambda(s,y) = L(s)$, where $L(s)$ is as defined above in (2.3). Thus, the fraction of the firm's equity retained by the entrepreneur is the only aspect of the compensation package that affects the entrepreneur's incentive to supply labor.

When, for all i , the utility function of investor i is CARA and given by (2.7), it is easily verified that the restricted efficient shares t_i will be related to s by (2.12). In this case, the restricted efficient (s,y) can be

defined by proceeding as though there were a single investor with utility function

$$u_m(I_m) = -e^{-c_m I_m}, \quad (2.23)$$

where c_m and c_1, \dots, c_n are related by (2.13). Thus, a contract is restricted efficient if the compensation package (s, y) specified by the contract solves

PROBLEM 1:

$$\max_{(s, y)} Eu_m((1-s)[f(L(s)) + \tilde{x}] - y) \quad (2.24)$$

subject to the constraint

$$Eu_0(s[f(L(s)) + \tilde{x}] + y + w[\bar{L} - L(s)]) = \bar{U}, \quad (2.25)$$

where \bar{U} is some arbitrarily fixed level of utility, and $L(s)$ is the function defined by (2.3).

If we had not restricted attention to linear contracts, this would be exactly the problem studied in the principal-agent literature. In that literature, \bar{U} is interpreted as the utility level the entrepreneur can obtain in some other employment if he chooses not to work under the contract. Under the hypothesis of CARA utility functions, the optimal s is independent of the alternative utility level \bar{U} and y is simply chosen to guarantee the satisfaction of the constraint. We can, therefore, solve the above maximization problem for an arbitrary \bar{U} . We will use the utility level

$$\bar{U} = -e^{-c_0 w \bar{L}}. \quad (2.26)$$

which would be the entrepreneur's utility if he simply worked all of the time for the competitive wage w .

Problem 1 can be restated as

$$\max_{(s,y)} (1-s)f(L(s)) - c_m(1-s)^2\sigma^2/2 - y \quad (2.27)$$

subject to the constraint

$$sf(L(s)) - c_0s^2\sigma^2/2 + y - wL(s) = 0. \quad (2.28)$$

Solving the constraint for y and substituting the result in the maximand, we obtain the efficient s as the solution of the problem

$$\max_s f(L(s)) - [c_0s^2 + c_m(1-s)^2]\sigma^2/2 - wL(s). \quad (2.29)$$

Using the fact that $L(s)$ satisfies (2.3), the first order condition satisfied by the solution of the maximization problem (2.29) is

$$(1-s)f'(L(s))L'(s) - \sigma^2[c_0s - c_m(1-s)] = 0.$$

This is exactly the condition (2.22) satisfied by s^* , the equilibrium s level. Observe that, like the entrepreneur's maximand (2.19), the maximand (2.29) is a concave function of s when $f(L(s)) - wL(s)$ is concave. Thus, when $f(L(s)) - wL(s)$ is concave, the s^* that solves (2.22) maximizes (2.29). The CEE is, therefore, restricted efficient when $f(L(s)) - wL(s)$ is concave.

2.2 A MONOPOLISTIC EQUILIBRIUM

As pointed out earlier, the restricted efficiency of the CEE is attributable to the hypothesis of competitive behavior embodied in the definition of the CEE. If this hypothesis were dropped and we assumed that the entrepreneur exerted monopoly power, the resulting equilibrium would not be restricted efficient and the inefficiency would take a familiar form. The entrepreneur would retain an inefficiently large share of the firm and thereby restrict the

supply of the firm's equity as a way of raising the market value of the firm. The formal demonstration of this result can be accomplished by assuming that the entrepreneur realizes that he can lower the price of risk, $\pi((1-s)\sigma^2)$, and raise $V(s)$, the firm's market value, by raising s and thereby reducing the supply of risk. Thus, we now assume that the entrepreneur realizes that

$$V(s) = f(L(s)) - c_m(1-s)\sigma^2, \quad (2.14)$$

so that he expects the market price of the firm to rise by

$$V'(s) = f'(L(s))L'(s) + c_m\sigma^2 \quad (2.30)$$

when he raises s . By way of contrast, in the definition of a CEE the entrepreneur is assumed to believe that he can't affect the price of risk and that $V(s)$ and $V'(s)$ are, therefore, given by (2.16) and (2.17), respectively.

Using the expression (2.14) for $V(s)$ in (2.18), the monopolistic entrepreneur's maximand is

$$\begin{aligned} & sf(L(s)) - c_0s^2\sigma^2/2 + (1-s)\left\{f(L(s)) - c_m(1-s)\sigma^2\right\} - wL(s) \\ & = f(L(s)) - \left\{c_0s^2/2 + c_m(1-s)^2\right\}\sigma^2 - wL(s). \end{aligned} \quad (2.31)$$

Using (2.3), the first-order condition satisfied by the s that maximizes (2.31) is

$$(1-s)\left[f'(L(s))L'(s) + c_m\sigma^2\right] - \sigma^2[c_0s - c_m(1-s)] = 0. \quad (2.32)$$

It is easily verified that, s^M , the s value that satisfies (2.32), is larger than s^* , the equilibrium s in the CEE. The monopolistic equilibrium is, thus, not restricted efficient; as noted earlier, the inefficiency is analogous to that observed by Stiglitz [1972].

2.3 THE COSTS OF MORAL HAZARD

We can now discuss the effects of moral hazard by comparing the CEE to the CAPM equilibrium that would arise if investors could observe L . The discussion begins in Subsection 2.3.1 with a description of the fully efficient CAPM contract, which is then compared to the restricted efficient CEE contract derived in Subsection 2.1. Subsection 2.3.2 briefly describes a specific example in which both contracts can be explicitly computed and compared.

2.3.1 A COMPARISON OF THE RESTRICTED EFFICIENT AND FULLY EFFICIENT CONTRACTS

We could proceed with a direct comparison of the two equilibria. We choose instead an equivalent approach. Specifically, we compare the restricted efficient contracts to the contracts that are Pareto optimal in the set of linear contracts explicitly specifying the entrepreneur's labor supply (as is possible when there is no moral hazard). These two approaches are equivalent because, as shown in Section 2.1, the CEE are restricted efficient and, when there is no moral hazard, the CAPM equilibria are well known to be Pareto optimal in the set of linear contracts.

We begin by describing the first-order conditions satisfied by the s associated with a Pareto optimal contract in the set of linear contracts that specify L . This is a problem of describing the first-order conditions satisfied by the linear contract (s, y, L) that solves

PROBLEM 2:

$$\max_{(s, y, L)} E u_m((1 - s)[f(L) + \tilde{x}] - y) \quad (2.33)$$

subject to the constraint

$$Eu_0(s[f(L) + \tilde{x}] + y + w[\bar{L} - L]) = \bar{U}. \quad (2.34)$$

REMARK 2.3: It is well known that, when the utility functions are CARA, the solution to this problem is, in fact, Pareto optimal in the class of all contracts. Thus, the restriction to linear contracts is not a restriction at all when moral hazard is absent and the utility functions are CARA. It is, therefore, appropriate in this setting to refer to the contract that solves Problem 2 as the fully optimal contract.

As in the case of Problem 1, the hypothesis of CARA utility functions implies that the optimal s is independent of \bar{U} . We can, therefore, again substitute

$$\bar{U} = -e^{-c_0 w \bar{L}} \quad (2.26)$$

for \bar{U} in the constraint (2.34). When the expected utilities in (2.33) and (2.34) are computed using the CARA utility functions, Problem 2 can be restated as

$$\max_{(s,y,L)} (1-s)f(L) - c_m(1-s)^2\sigma^2/2 - y \quad (2.35)$$

subject to the constraint

$$sf(L) - c_0s^2\sigma^2/2 + y - wL = 0. \quad (2.36)$$

Solving the constraint for y and substituting the result in the maximand, we obtain the efficient (s,L) as the solution of the problem

$$\max_{(s,L)} f(L) - [c_0s^2 + c_m(1-s)^2]\sigma^2/2 - wL. \quad (2.37)$$

Since the maximand in (2.37) is concave in (s,L) , the first order conditions

$$f'(L) - w = 0 \quad (2.38)$$

and

$$[c_0 s - c_m (1 - s)] \sigma^2 = 0 \quad (2.39)$$

are necessary and sufficient for a maximum. Recalling the definition of $L(s)$ as the L that maximizes (2.2), we observe that the efficient L which solves (2.38) is $L(1)$, the amount of labor the entrepreneur would supply if he were the sole owner of the firm. The efficient s which solves (2.39) is

$$\hat{s} = \frac{c_0^{-1}}{[c_0^{-1} + c_m^{-1}]}, \quad (2.40)$$

a standard result in the CAPM when utility functions are CARA.

The effect of moral hazard can be observed by simply comparing s^* with \hat{s} and $L(s^*)$ with $L(1)$. We will show that s^* exceeds \hat{s} . This means that, because of moral hazard, the entrepreneur retains too large a fractional share of the firm's equity in order to convince other investors that he has an incentive to work long hours for the firm. This incentive is effective in the sense that the entrepreneur does supply more labor when s equals s^* than he would if his fractional share of the firm's equity were \hat{s} . The effectiveness of this incentive is limited, however, by the fact, noted earlier, that s^* is less than one. Thus, although the entrepreneur retains an excessive share of the firm's equity, he is not the sole owner of the firm, and the presence of moral hazard reduces the amount of time the entrepreneur works from $L(1)$ to $L(s^*)$.

For the purpose of comparing s^* and \hat{s} , we will discuss the role of s when moral hazard is and is not present. When moral hazard is absent and L can be

chosen explicitly, the s choice can be used for a single purpose: to allocate risk efficiently, i.e., to minimize the total risk premium

$$[c_0 s^2 + c_m (1 - s)^2] \sigma^2 / 2. \quad (2.41)$$

At \hat{s} , risk is efficiently allocated and the first order condition (2.39) satisfied at \hat{s} simply asserts that the marginal risk premium is zero. Note that because the risk premium (2.41) is a convex function of s , the marginal risk premium is negative below \hat{s} and positive above \hat{s} .

When moral hazard is present, the choice of s must play two roles: it allocates risk and it determines the entrepreneur's incentive to work. These goals may or may not be in conflict depending on whether s is larger or smaller than \hat{s} . When $s < 1$, increases in s always improve the entrepreneur's incentives because the incentive benefits of an s increase,

$$(1 - s) f'(L(s)) L'(s), \quad (2.42)$$

(which is the first term in (2.21)) are always positive. When s is smaller than \hat{s} , there is no conflict in the roles assigned to s . Specifically, when $s < \hat{s}$, the marginal risk premium

$$[c_0 s - c_m (1 - s)] \sigma^2 \quad (2.43)$$

(which is the second term in (2.21)) is negative and increases in s improve the allocation of risk as well as the entrepreneur's incentives. The conflict between the roles played by s arises if $s > \hat{s}$. Then, the marginal risk premium (2.43) is positive and further increases in s worsen the allocation of risk while improving incentives. Condition (2.22) asserts that, at s^* , the marginal risk premium (2.43) equals the positive marginal incentive benefit (2.42). This means that, at s^* , the goals of allocating risk and providing

incentives are in conflict, with s^* exceeding \hat{s} . At s^* , the benefit of a marginal improvement in the entrepreneur's incentives just equals the cost of a marginal deterioration in the allocation of risk.

The tradeoff between efficient risk-sharing and the provision of incentives is a familiar feature of principal-agent models. The assumption of linear contracts and CARA preferences has made it possible to clearly illustrate how this tradeoff arises when s is chosen.

Inefficiency due to moral hazard can be interpreted as occurring because of an externality created by the unobservability of the entrepreneur's labor supply. In this interpretation, the entrepreneur works an inefficiently low number of hours because of the difference between the private and social benefits of his labor. The social benefits are $f'(L)$, the increase in the expected profit achieved when L is increased. It is the social benefits that should be equated to the marginal cost of labor when L is chosen. The social benefits of the L increase are, of course, the sum of $sf'(L)$, the entrepreneur's private benefits, and $(1 - s)f'(L)$, the benefits that accrue to other investors. When L cannot be observed by other investors, the entrepreneur cannot appropriate the gains of an L increase that accrue to other investors. Thus, when the entrepreneur decides how many hours to work, his choice is made by equating only his private gains, $sf'(L)$, to the marginal cost of labor.

In addition to the undersupply of labor, the externality results in an inefficient s choice. This happens because by raising s , the entrepreneur

can raise the market value of the firm's equity by $f'(L(s))L'(s)$. This increase in the firm's value permits the entrepreneur to capture $(1 - s)f'(L(s))L'(s)$, the benefits to other shareholders of the additional labor the entrepreneur is induced to supply when s increases. It is these benefits which are equated, in (2.22), to the marginal risk premium when s is chosen, and it is the fact that these benefits are included in the calculation of the gain from an s increase that causes the entrepreneur to overinvest in the firm.

The difference between the private and social benefit of an L increase would not arise if L were observable to the investors. Were L to be observable and investors to have rational expectations, an L increase would raise the value of the firm by $f'(L)$. The higher price would permit the entrepreneur to capture $(1 - s)f'(L)$, the benefits of the L increase that accrue to the other investors who buy the fraction $(1 - s)$ of the firm's equity. As a shareholder he receives the benefit $sf'(L)$. Thus, when he increases L , the total private benefit captured by the entrepreneur is the social benefit,

$$f'(L) = (1 - s)f'(L) + sf'(L).$$

When L is observable, increases in s do not raise the firm's market value and it is not necessary for the entrepreneur to overinvest in the firm in order to capture the benefits other investors obtain when he works longer hours.

There is one important and well known case in which the unobservability of L does not give rise to an externality and the CEE is efficient in the strongest sense. This case arises when the entrepreneur is risk neutral; i.e., $c_0 = 0$.

In this case, $\hat{s} = 1$, i.e., it is efficient for the entrepreneur to be the sole owner of the firm and bear all risk. This implies that when the entrepreneur works longer hours for the firm he captures all of the benefits of his increased labor and no externality arises. (A more complete discussion of this case is at the end of Section 3).

2.3.3 AN EXPLICIT EXAMPLE:

One interesting special case in which the equilibria with and without moral hazard can be computed and explicitly compared is that in which

$$f(L) = A\sqrt{L}.$$

In this case,

$$L(s) = [sA/2w]^2,$$

$$f(L(s)) = s[A^2/2w],$$

(2.22) becomes

$$(1 - s^*)[A^2/2w] - \sigma^2[c_0 s^* - c_m(1 - s^*)] = 0,$$

and

$$s^* = \frac{c_0^{-1}}{\left[c_0^{-1} + \left[c_m + [A^2/2w] \right]^{-1} \right]}.$$

Note that in this example, s^* does exceed \hat{s} and

$$L(1) = [A/2w]^2 > [s^*A/2w]^2 = L(s^*).$$

3. THE GENERAL EFFICIENCY THEOREM

In this section we study the restricted efficiency of CEE under more general assumptions about technology and preferences. We assume that either (i) investors are risk neutral or (ii) the production function satisfies a spanning condition. These are sufficient conditions for an analog of the "first fundamental welfare theorem" to hold in stock market economies; see, e.g., Diamond [1967] and Radner [1974]. The fact that CEE are restricted efficient in the CAPM framework of Section 2 suggests that a similar analog of the first welfare theorem holds in other cases in which moral hazard is present. This is not true, however, as we demonstrate below by presenting an example in which there exists a continuum of Pareto-ranked CEE. In this example, there are restricted inefficient CEE because the proceeds from the sale of equity create income effects that influence the entrepreneur's incentives. These income effects are absent if the entrepreneur is assumed to have a CARA utility function, or if the uncertainty is assumed to enter the profit function additively. Both of these assumptions are satisfied in the CAPM model of Section 2.

In the Pareto-dominated CEE of the example presented below, the entrepreneur's incentive to exert effort is low as a result of the income effect that arises because the firm's value is low and the proceeds from the sale of equity are, therefore, low. The low valuation of the firm's equity is consistent with rational expectations because the firm's expected profit actually is low when the entrepreneur exerts a low level of effort. Although, the welfare loss that results from the income effect could be recaptured by an income transfer

to the entrepreneur, a CEE incorporates no market mechanism for making this transfer.

In the example, the CEE with the highest valuation function for the firm is not Pareto-dominated by any of the others. Theorem 3.1, which follows the example, states that a CEE in which the entrepreneur is not the sole owner of the firm and the firm's value is as large as it can be when expectations are rational is restricted efficient. One corollary of this theorem is that, as in the CAPM case of Section 2, CEE are restricted efficient whenever the uncertainty in the firm's profit enters additively or the entrepreneur's utility function is CARA.

In the general model considered in this section, the entrepreneur begins as the sole owner of a firm whose profit is a function, $g(L, \tilde{x})$, of the entrepreneur's labor supply, $L \geq 0$, and of a random variable, \tilde{x} . The profit function is strictly increasing in L , and L is unobserved by investors. The opportunity cost to the entrepreneur of an hour of labor supplied to the firm is the wage, w , that could have been earned in alternative employment. The entrepreneur's utility is a concave function, $u_0(I_0)$, of his income, I_0 . For $i = 1, \dots, n$, investor i 's utility is also a concave function, $u_i(I_i)$, of his income, I_i .

The entrepreneur sells a fraction, $(1 - s)$, of the firm's equity to the other investors. The investors observe s and use it to infer the level of L . Thus, as in the previous section, the firm's market value, V , is a function, $V(s)$,

of s . The entrepreneur chooses the vector (s, L) to maximize his expected utility,

$$Eu_0(\tilde{sg}(L, \tilde{x}) - wL + (1 - s)V(s)). \quad (3.1)$$

The fraction of the firm's equity purchased by investor i is t_i . He chooses t_i to maximize his expected utility,

$$Eu_i(t_i \tilde{g}(L, \tilde{x}) - t_i V(s)). \quad (3.2)$$

(The entrepreneur's and the investors' constant endowments, $w\bar{L}$ and \bar{y}_i , respectively, have been suppressed in (3.1) and (3.2) in order to simplify notation — compare (2.1) and (2.6).)

DEFINITION 3.1: An Expectations Equilibrium (EE) is an $(n + 3)$ -tuple $\langle L^*, V^*(\cdot), s^*, t_1^*, \dots, t_n^* \rangle$ such that

- (i) (s^*, L^*) maximizes (3.1) when $V(\cdot) = V^*(\cdot)$,
- (ii) for each i , t_i^* maximizes (3.2) when $V(s) = V^*(s^*)$ and $L = L^*$;

and

$$(iii) \sum_{i=1}^n t_i^* = (1 - s^*). \quad (3.3)$$

In this definition of an EE, no restrictions are placed on the function $V^*(\cdot)$.

In a Competitive Expectations Equilibrium, expectations are both "rational" and "competitive" in ways that we shall define. First define a correspondence

$\lambda(\cdot, \cdot)$ by

$$\lambda(s, y) \equiv \operatorname{argmax}_{L \geq 0} Eu(\tilde{sg}(L, \tilde{x}) - wL + y). \quad (3.4)$$

Note that in an EE,

$$L^* \in \lambda(s^*, (1 - s^*)V^*(s^*)). \quad (3.5)$$

Investors with rational expectations who observe the entrepreneur retain the fractional share s^* of the firm's equity will infer that the entrepreneur has an incentive to choose an L^* that satisfies (3.5). Rational investors who observe s choices other than s^* should (arguably) still believe that L will be chosen optimally, i.e., they should expect the entrepreneur to choose an L in $\lambda(s, (1-s)V^*(s))$. The assumption of such rational expectations, particularly when combined with the assumption that all traders act competitively, restricts the form of the valuation function $V(\cdot)$.

It remains to restrict the expectations of investors to be "competitive". We do this first for the case in which some investor is risk neutral. In this case, competition among rational investors will cause the firm to be valued at the expected profit level consistent with optimizing behavior by the entrepreneur. That is, for each s , risk neutral rational investors will expect the entrepreneur to choose an L in $\lambda(s, (1-s)V^*(s))$, and on the basis of this expectation they will bid the price of the firm up to $Eg(L, \tilde{x})$. Formally, if some investor is risk neutral, we have:

DEFINITION 3.2: A Competitive Expectations Equilibrium (CEE) when at least one investor is risk neutral is a vector $\langle L^*, V^*(\cdot), s^*, t_1^*, \dots, t_n^* \rangle$ that is an EE and satisfies

$$(iv) V^*(s^*) = Eg(L^*, \tilde{x}), \text{ and}$$

$$V^*(s) \in Eg(\lambda(s, (1-s)V^*(s)), \tilde{x}) \text{ for all } s.$$

REMARK 3.1: The first part of (iv) requires the investors to accurately infer

that the entrepreneur has chosen L^* when he has been observed to retain s^* of the firm's equity. This is implicit in their maximization condition (ii). For out-of-equilibrium s values, (iv) does not require the investors to correctly infer the entrepreneur's "true" labor choice (however that is defined!), unless $\lambda(s, (1-s)V^*(s))$ is single-valued.

Another assumption under which we can define a Competitive Expectations Equilibrium is that the production technology satisfies *spanning* (SP):

$g(L,x)$ satisfies spanning if there exist functions $f(\cdot)$ and $h(\cdot)$ such that

$$g(L,x) = f(L) + h(L)x \text{ for all } (L,x). \quad (\text{SP})$$

This condition was introduced in the shareholder unanimity literature; see, e.g., Eckern and Wilson [1974], Leland [1974] or Radner [1974]. A special case of spanning, one that is satisfied by the CAPM in Section 2, is that of *additive uncertainty* (AU):

$g(L,x)$ satisfies additive uncertainty if $g(L,x)$ satisfies

$$\text{spanning with } h(L) \equiv 1. \quad (\text{AU})$$

The following is a motivation for our definition of a CEE in the case of spanning. With spanning, owning the firm is equivalent to holding a portfolio composed of $f(L)$ shares of a safe asset and $h(L)$ shares of a risky asset. If profits were measured in dollars, then a dollar invested in the safe asset would pay a return of one dollar with certainty, and a dollar invested in the

risky asset would pay a return of \tilde{x} dollars. Traders who are competitors would take the prices of these assets as given. If the price of the safe asset equalled one and the price of the risky asset that pays \tilde{x} equalled p , then a standard arbitrage argument would imply that, when rational investors inferred that $L = L^*$ when they observed $s = s^*$, the firm would be valued at

$$V^*(s^*) = f(L^*) + ph(L^*). \quad (3.6)$$

Rational expectations require that if investors were to observe an s different from s^* , they should believe that the L supplied by the entrepreneur is in the set $\lambda(s, (1-s)V^*(s))$. Thus, for $s \neq s^*$, *competitive rational expectations* require that

$$V^*(s) \in f(\lambda(s, (1-s)V^*(s))) + ph(\lambda(s, (1-s)V^*(s))). \quad (3.7)$$

Formally, when the production function satisfies the spanning condition (SP), we have:

DEFINITION 3.3: *When the spanning condition (SP) is satisfied, a Competitive Expectations Equilibrium (CEE) is a vector $\langle L^*, V^*(\cdot), s^*, t_1^*, \dots, t_n^* \rangle$ that is an EE and for which a constant p exists such that*

- (iv') $V^*(s^*)$ satisfies (3.6) and
 $V^*(s)$ satisfies (3.7) for $s \neq s^*$.

REMARK 3.2: Definitions 3.2 and 3.3 are consistent. When both the spanning and risk neutrality conditions hold, (iv) implies (iv') directly, with $p = \tilde{Ex}$. If there actually were to be a market for an asset with return \tilde{x} that the risk neutral investors could enter, then the price of the risky asset would, in any

equilibrium, equal its expected value, $p = E\tilde{x}$.

The motivation for (iv') requires that traders take the price of a risky asset paying a return \tilde{x} as given. This is a standard price-taking assumption in securities market models in which spanning holds, see, e.g., Diamond [1967] or Radner [1974]. Although the assumption of competitive trading in a risky asset yielding a return perfectly correlated with the firm's profit would certainly provide a solid rationale for the price-taking assumption, we do not make this assumption. Unfortunately, in the present model, the assumption of a competitive market in such an asset would create two conceptual difficulties. First, if the risky asset paying \tilde{x} could be sold short, the entrepreneur could then avoid the conflict between incentives and risk bearing. He would achieve this by taking a short position in the risky asset while retaining all of the shares in his firm. When he follows this strategy, he is the sole owner of the firm, and therefore has the incentive to employ the first-best level of labor. By optimally choosing a short position in the risky asset whose return is perfectly correlated with his firm's profit, the entrepreneur would avoid bearing excess risk. We do not want the moral hazard to be so circumvented in our model. Thus, if we were to explicitly assume the existence of a competitive market for a risky asset with return \tilde{x} , we would also have to assume that the entrepreneur could not take a short position in this asset. (Similarly, the entrepreneur must also be unable to take a long position in an asset that pays a return of $-\tilde{x}$, i.e., an asset whose return is negatively correlated with the return on his firm.)

The second difficulty created by assuming actual competitive trading in a risky asset yielding a return perfectly correlated with the firm's profit is that then the firm's investors would like to offer the entrepreneur a contract that relates his compensation to the observable return paid by this asset. In fact, first-best optimality could be achieved with a contract of this type. We could avoid this problem by simply assuming that investors cannot directly impose a contract on the entrepreneur that relates his compensation to the return on any other asset. Such an approach would be in accord with our maintained assumption that investors cannot impose an explicit incentive contract on the entrepreneur that relates his compensation to his firm's profits.

There is an alternative to assuming a competitive market in an asset yielding a return perfectly correlated with the firm's profit. The alternative is to simply view (iv') as the appropriate notion of price-taking for a model of *competitive* expectations. It is certainly a plausible notion of price-taking behavior, and it is basically the same price-taking assumption as that used in the shareholder unanimity literature (see, e.g., Eckern and Wilson [1974], Leland [1974] or Radner [1974]) to obtain restricted efficiency results. In this connection, it is quite interesting that not all CEE defined by (i)-(iv') need be restricted efficient.

EXAMPLE (A CONTINUUM OF PARETO-RANKED CEE): The production function is

$$g(L, \tilde{x}) = f(L)\tilde{x},$$

where

$$f(L) = \begin{cases} L, & \text{if } 0 \leq L \leq 1, \\ 1, & \text{if } 1 \leq L. \end{cases}$$

The random variable \tilde{x} is uniformly distributed on $[0,2]$ and therefore has mean $E\tilde{x} = 1$. The (exogenous) wage rate is $w = 1/4$. The investors are risk neutral. The entrepreneur is risk averse with the utility function

$$u_0(I_0) = \begin{cases} \alpha I_0, & \text{if } I_0 < 0, \\ I_0, & \text{if } 0 \leq I_0, \end{cases}$$

where α , the entrepreneur's marginal utility at negative income levels, is much greater than one.¹

Expected profit, $Eg(L, \tilde{x}) - L/4$, is maximized at $L = 1$. Since the investors are risk neutral, $L = 1$ is the first-best L level. For all $y \geq 0$ and $s \leq 1$, the entrepreneur's optimal L choice is, however,

$$\lambda(s, y) = \begin{cases} 0, & \text{if } s < 1/4, \\ \min(1, 4y), & \text{if } s > 1/4. \end{cases} \quad (3.8)$$

To see that (3.8) is true, first note that the entrepreneur never sets $L > 1$, since then his marginal product of labor would surely be zero and his marginal cost of labor would be $1/4$. If the entrepreneur chooses $L \in [0,1]$, his income is $\tilde{I}_0 = y + L(s\tilde{x} - 1/4)$, which is uniformly distributed on $[y - L/4, y + L(2s - 1/4)]$. His expected income is $E\tilde{I}_0 = y + L(s - 1/4)$. If $s < 1/4$, setting $L = 0$ maximizes the entrepreneur's expected income without subjecting him to risk. Hence, $L = 0$ does, as (3.8) asserts, maximize the

¹We have chosen a piecewise linear utility function for convenience only. In particular, for $I_0 < 0$ we could have let $u(\cdot)$ be any concave function with $u'(\cdot)$ sufficiently large; such a $u(\cdot)$ could exhibit decreasing absolute risk aversion.

entrepreneur's expected utility if $s < 1/4$. If now $s > 1/4$, then the entrepreneur's expected income increases in L for $L < 1$. Because all income levels in the support of \tilde{I}_0 are positive if $L \in [0, 4y)$, the entrepreneur is risk neutral with respect to L choices in this range. He, therefore, raises L to at least $\min(1, 4y)$. Raising L above $4y$ lowers the left endpoint of the support of \tilde{I}_0 to a level that is below zero. This must decrease expected utility because the marginal utility of income is very large at negative income levels. Thus, when $s > 1/4$, the labor supply $\lambda(s, y) = \min(1, 4y)$ indicated in (3.8) does maximize the entrepreneur's expected utility.

We do not consider the case of $s > 1$; allowing the entrepreneur to sell shares in his firm short, as he would be doing if s were to exceed one, would introduce tedious complications without yielding essentially different results. Hence, the valuation function $V^*(\cdot)$ associated with any CEE will only be defined on $[0, 1]$, in this example.

Let $L_0 \in [0, 1]$ be chosen arbitrarily. We now demonstrate that $\langle L^*, s^*, V^*(\cdot), t_1^*, \dots, t_n^* \rangle$ is a CEE if $L^* = L_0$, $s^* = 3/4$,

$$V^*(s) = \begin{cases} L_0, & \text{if } s = 3/4, \\ 0, & \text{otherwise,} \end{cases}$$

and t_1^*, \dots, t_n^* sum to $1/4$. To verify this we observe that:

- (a) Because the t_i^* 's sum to $1/4$, condition (iii) of Definition 3.1 is satisfied.

- (b) The investor maximization condition, (ii), is satisfied because risk neutrality and the fact that $Eg(L_0, \tilde{x}) = L_0 = V^*(s^*)$ imply that all investors are indifferent as to the number of fractional shares they hold.
- (c) If the entrepreneur retains any fraction $s \neq 3/4$ of his firm's equity, his proceeds from the sale of equity will be $y = (1 - s)V^*(s) = 0$. From (3.8), he, therefore, will not work, and both expected output and $V^*(s)$ will equal zero. The competitive expectations condition (iv) is, therefore, satisfied at any $s \neq 3/4$. If the entrepreneur retains $s^* = 3/4$ of his firm's equity, the proceeds from his sale of equity is $y = (1 - s^*)V^*(s^*) = L_0/4$. From (3.8) we see that he will then work $L = 4y = L_0$. Expected output is $L_0 = V^*(3/4)$, so that (iv) is satisfied at $s = s^* = 3/4$.
- (d) As noted in (c), if $s \neq 3/4$, then $\lambda(s, y)$ and y both equal zero. As a consequence, \tilde{I}_0 is zero with probability one and the entrepreneur's expected utility is $u_0(0) = 0$. If, however, the entrepreneur sets $s = 3/4$, then $y = L_0/4$ and $L = L_0$. In this case, the entrepreneur's expected utility is $Eu_0(s^* \tilde{x} L_0 - L_0/4 + (1 - s^*)L_0) = 3L_0/4 \geq 0$. Thus, $(L^*, s^*) = (L_0, 3/4)$ maximizes the entrepreneur's expected utility and (i) is satisfied.

In the CEE of this example, the entrepreneur's expected utility is (positively) indexed by his labor supply L_0 . The investors have zero expected utility in all equilibria and are, therefore, indifferent among them. Thus, high- L_0 CEE Pareto-dominate low- L_0 CEE. The CEE in which $L_0 = 1$ is not only

restricted efficient, it is fully efficient. In this equilibrium, the entrepreneur maximizes expected profits because he is effectively risk neutral; he is risk neutral in this equilibrium because his income is sure to be positive.

There are multiple equilibria in this example because many valuation functions $V^*(\cdot)$ satisfy the competitive expectations condition (iv). In particular, any $V^*(\cdot)$ with $V^*(s) = 0$ when $s \neq 3/4$, and with $V^*(3/4) \in [0,1]$, is an equilibrium valuation function. If $V^*(3/4) < 1$, the entrepreneur does not receive enough from the sale of shares to induce him to work as hard as he should, i.e., to set $L = 1$. His income from selling shares is only $y = (1/4)V^*(3/4) < 1/4$, so that he prefers to set $L = 4y < 1$.

In the fully efficient CEE that has $L_0 = 1$, the value of the firm is $V^*(s^*) = 1$, which is larger than the value of the firm at s^* in any of the Pareto dominated CEE's. As Theorem 3.1 shows, this feature of the example is a general property.

REMARK 3.3: In the example, when $V^*(3/4) < 1$ and the entrepreneur's labor supply is less than one, he would be induced to work $\Delta L = 4\Delta y$ additional hours if the investors paid him a transfer of Δy . The investors would be indifferent about making this transfer since $(1 - s^*)(4\Delta y)$, the investors' share of the resulting increase in the firm's expected profit, is equal to the transfer, Δy , because $s^* = 3/4$. But the entrepreneur's expected utility would increase by $3\Delta y$. In our model, such transfers are impossible; all exchanges

between the entrepreneur and investors must take place through the stock market.

REMARK 3.4: In the CAPM case discussed in Section 2, the production function exhibits additive uncertainty (AU). Whenever (AU) holds,

$$\lambda(s,y) = \operatorname{argmax} [sf(L) - wL], \quad (3.9)$$

so that $\lambda(s,y)$ does not vary with y and does not depend on the entrepreneur's utility function. Thus, there would be no income effect in the CAPM case even if the entrepreneur's utility function did not satisfy CARA.

As a preliminary to the statement of Theorem 3.1 we must define precisely the set of feasible allocations and the concept of restricted efficiency.

DEFINITION 3.4: A linear allocation is a vector $\langle L, s, t_1, \dots, t_n, y, y_1, \dots, y_n \rangle$.

The interpretation of the variables L , s and t_i is as before. The variables y and y_i represent fixed payments to the entrepreneur and the i^{th} investor, respectively. Thus, for the entrepreneur and the i^{th} investor, the incomes associated with a linear allocation $\langle L, s, t_1, \dots, t_n, y, y_1, \dots, y_n \rangle$ are, respectively,

$$sg(L, \tilde{x}) - wL + y \quad (3.10)$$

and

$$t_i g(L, \tilde{x}) + y_i. \quad (3.11)$$

DEFINITION 3.5: A linear allocation $\langle L, s, t_1, \dots, t_n, y, y_1, \dots, y_n \rangle$ is feasible if

it satisfies:

$$(i) \sum_{i=1}^n t_i = (1 - s), \quad (3.12)$$

$$(ii) y + \sum_{i=1}^n y_i = 0 \quad (3.13)$$

and

$$(iii) L \in \lambda(s, y). \quad (3.14)$$

Conditions (i) and (ii) guarantee that the income distribution satisfies the appropriate resource availability constraints. Condition (iii) is a consequence of the moral hazard caused by the informational asymmetry; it asserts that the L specified by a feasible allocation is optimal for the entrepreneur, given the relationship (3.10) between his income and the firm's profit.

DEFINITION 3.6: A feasible linear allocation is restricted efficient if it is Pareto optimal in the class of all feasible linear allocations.

THEOREM 3.1: (i) Suppose that (SP) holds and $\langle L^*, V^*(\cdot), s^*, t_1^*, \dots, t_n^* \rangle$ is a CEE satisfying Definition 3.2. If, for all s ,

$$(1-s)V^*(s) = \sup\{y \mid y \leq (1-s)[f(L)+ph(L)] \text{ for some } L \in \lambda(s, y)\}, \quad (3.15)$$

then $\langle L^*, V^*(\cdot), s^*, t_1^*, \dots, t_n^* \rangle$ is restricted efficient.

(ii) Suppose that some investor is risk neutral and $\langle L^*, V^*(\cdot), s^*, t_1^*, \dots, t_n^* \rangle$ is a CEE satisfying Definition 3.1. If, for all s ,

$$(1-s)V^*(s) = \sup\{y \mid y \leq (1-s)Eg(L, \tilde{x}) \text{ for some } L \in \lambda(s, y)\}, \quad (3.16)$$

then $\langle L^*, V^*(\cdot), s^*, t_1^*, \dots, t_n^* \rangle$ is restricted efficient.

REMARK 3.5: Under standard regularity conditions, (3.15) and (3.16) are strengthenings of conditions (3.6) of (iv') and (iv), respectively. Take the case of (3.16). Assume that g and u are continuous and concave and that L must be chosen from a compact interval. Then $\lambda(s,y)$ is bounded, convex-valued and upper hemicontinuous. This can be used to show that (3.16) implies that, for each s , $V^*(s) = \text{Eg}(L, \tilde{x})$ for some $L \in \lambda(s, (1-s)V^*(s))$, which is the competitive expectations condition (iv).

REMARK 3.6: Our analysis does not rule out the possibility that s might exceed one. If $s > 1$, the entrepreneur is the sole owner of the firm and also sells short some shares in his firm. In the empirically relevant case in which $s < 1$, (3.15) and (3.16) reduce to

$$V^*(s) = \sup\{y \mid y \leq [f(L) + ph(L)] \text{ for some } L \in \lambda(s,y)\} \quad (3.15')$$

and

$$V^*(s) = \sup\{y \mid y \leq \text{Eg}(L, \tilde{x}) \text{ for some } L \in \lambda(s,y)\}, \quad (3.16')$$

respectively. Theorem 3.1, therefore, asserts that when the entrepreneur is not the sole owner of the firm, the CEE in which the firm's value is as large as it can be when expectations are rational is restricted efficient.

PROOF: (i) Suppose that $\langle L, s, t_1, \dots, t_n, y, y_1, \dots, y_n \rangle$ is a feasible linear allocation that Pareto dominates the CEE satisfying (3.15). Then because of (i) of Definition 3.1,

$$\begin{aligned} \text{Eu}_0(\text{sg}(L, \tilde{x}) - wL + y) &\geq \text{Eu}_0(s^* g(L^*, \tilde{x}) - wL^* + (1 - s^*)V^*(s^*)) \\ &\geq \text{Eu}_0(\text{sg}(L, \tilde{x}) - wL + (1 - s)V^*(s)). \end{aligned}$$

Consequently,

$$y \geq (1 - s)V^*(s). \quad (3.17)$$

Similarly, (ii) of Definition 3.1 implies that

$$\begin{aligned} Eu_i(t_i g(L, \tilde{x}) + y_i) &\geq Eu_i(t_i^* g(L^*, \tilde{x}) - t_i^* V^*(s^*)) \\ &\geq Eu_i(\bar{t}_i g(L^*, \tilde{x}) - \bar{t}_i V^*(s^*)) \end{aligned} \quad (3.18)$$

for any \bar{t}_i . Using (SP) and setting $\bar{t}_i = t_i h(L)/h(L^*)$ in (3.18),

$$Eu_i(t_i [f(L) + h(L)\tilde{x}] + y_i) \geq Eu_i(\{t_i h(L)/h(L^*)\} [f(L^*) + h(L^*)\tilde{x} - V^*(s^*)]). \quad (3.19)$$

Hence,

$$t_i f(L) + y_i \geq \{t_i h(L)/h(L^*)\} [f(L^*) - V^*(s^*)]. \quad (3.20)$$

Summing over i and using (3.12) and (3.13) yields

$$(1 - s)f(L) - y \geq (1 - s)\{h(L)/h(L^*)\} [f(L^*) - V^*(s^*)]. \quad (3.20)$$

Now, the competitive expectations condition (3.6) is that

$$f(L^*) - V^*(s^*) = -ph(L^*),$$

which, when substituted in (3.20), implies

$$y \leq (1 - s)[f(L) + ph(L)]. \quad (3.21)$$

Condition (iii) in Definition 3.5 of feasibility implies that $L \in \lambda(s, y)$.

Thus, from (3.21) and the hypothesis (3.15),

$$y \leq (1 - s)V^*(s). \quad (3.22)$$

Since either the entrepreneur or at least one of the investors strictly prefers $\langle L, s, t_1, \dots, t_n, y, y_1, \dots, y_n \rangle$ to the CEE, one of the inequalities (3.17) or (3.22) must be strict, a contradiction.

(ii) The arguments establishing (3.17) and (3.18) are the same as in case

(i). Setting $\bar{t}_i = 0$ in (3.18) yields

$$Eu_i(t_i g(L, \tilde{x}) + y_i) \geq u_i(0). \quad (3.23)$$

Because $u_i(\cdot)$ is concave, Jensen's inequality can be applied to obtain

$$u_i(t_i \text{Eg}(L, \tilde{x}) + y_i) \geq \text{Eu}_i(t_i g(L, \tilde{x}) + y_i), \quad (3.24)$$

which when combined with (3.23) implies

$$y_i \geq -t_i \text{Eg}(L, \tilde{x}). \quad (3.25)$$

Summing over i and using (3.12), (3.13) and the hypothesis (3.16) yields (3.22). Again, since either the entrepreneur or at least one of the investors strictly prefers $\langle L, s, t_1, \dots, t_n, y, y_1, \dots, y_n \rangle$ to the CEE, one of the inequalities (3.17) or (3.22) must be strict, a contradiction. \square

COROLLARY 3.1: *Assume that either some investor is risk neutral or the profit function satisfies (SP). If $\lambda(s, y)$ is a single-valued function independent of y , then any CEE $\langle L^*, V^*(\cdot), s^*, t_1^*, \dots, t_n^* \rangle$ is restricted efficient.*

PROOF: We consider only the case in which an investor is risk neutral; the proof for the case in which (SP) holds is similar. We must show that, under the hypotheses, (iv) implies (3.16). As λ is independent of y , we can write $\lambda(s, y)$ as $L(s)$ for all (s, y) . For any s , condition (3.16), therefore, becomes,

$$(1 - s)V^*(s) = \sup \{ (1 - s)\text{Eg}(L, \tilde{x}) \mid L \in L(s) \}.$$

Since $L(s)$ is single valued, this, in turn, becomes

$$(1 - s)V^*(s) = (1 - s)\text{Eg}(L(s), \tilde{x}) \quad (3.26)$$

Similarly, (iv) becomes

$$V^*(s) = \text{Eg}(L(s), \tilde{x}),$$

and this implies (3.26). \square

COROLLARY 3.2: (i) If the profit function satisfies (AU) and f is strictly concave, then any CEE $\langle L^*, V^*(\cdot), s^*, t_1^*, \dots, t_n^* \rangle$ is restricted efficient.

(ii) If the entrepreneur's utility function is CARA and, for each possible x value, $g(L, x)$ is strictly concave in L , then any CEE $\langle L^*, V^*(\cdot), s^*, t_1^*, \dots, t_n^* \rangle$ is restricted efficient.

PROOF: In view of Remark 3.5, (i) follows from Corollary 3.1. (ii) follows from the well known fact that with CARA utility functions, there are no income effects. \square

Corollary 3.2 demonstrates that the restricted efficiency of the CAPM case studied in Section 2 can be attributed to either the assumption that the profit function exhibits additive uncertainty or the assumption that the entrepreneur's utility function is CARA.

We conclude by considering the case in which the entrepreneur is risk neutral. A well known result in the principal-agent literature is that if the principal is risk neutral, then full efficiency can be achieved by having the principal bear all of the risks created by his action. In this case, there is no conflict between incentives and an efficient allocation of risk. It is efficient to have the agent bear all risk, and when he does, he has an incentive to choose an efficient action.

We now show that the market also achieves full efficiency when the entrepreneur is risk neutral. If, furthermore, rational expectations require

that investors believe the entrepreneur will work harder when he retains more equity, then the entrepreneur will choose to retain all of the firm's equity.

When the entrepreneur is risk neutral, it is possible to adapt Definition 3.2 to define a CEE even when the investors are all risk averse and spanning fails to hold. Note first that, when the entrepreneur is risk neutral,

$$\lambda(s, y) = \operatorname{argmax}_{L \geq 0} \{sEg(L, \tilde{x}) - wL + y\},$$

which is independent of y . In fact, $\lambda(s, y) = L(s)$, where

$$L(s) \equiv \operatorname{argmax}_{L \geq 0} \{sEg(L, \tilde{x}) - wL\}. \quad (3.27)$$

Rational expectations should, therefore, imply that

$$V^*(s) \in Eg(L(s), \tilde{x}) \text{ for all } s.$$

Formally, we have:

DEFINITION 3.7: A Competitive Expectations Equilibrium (CEE) when the entrepreneur is risk neutral is a vector $\langle L^*, V^*(\cdot), s^*, t_1^*, \dots, t_n^* \rangle$ that is an EE and satisfies

$$\begin{aligned} \text{(iv')} \quad & V^*(s^*) \in Eg(L^*, \tilde{x}), \text{ and} \\ & V^*(s) \in Eg(L(s), \tilde{x}) \text{ for all } s, \text{ where } L(s) \text{ is given by (3.27)}. \end{aligned}$$

We do not give a formal definition of full efficiency. We instead recall the previously mentioned result in the principal-agent literature which, using the terminology of this paper, asserts that in a fully efficient allocation, the entrepreneur should bear all risk when he is risk neutral. Thus, a linear allocation, $\langle L, s, t_1, \dots, t_n, y, y_1, \dots, y_n \rangle$, will be fully efficient if $s = 1$ and

$L \in L(1)$. If some investors are also risk neutral, efficiency is also consistent with $s < 1$ and $t_i > 0$ for risk neutral investors, but efficiency still requires that $L \in L(1)$. The following theorem demonstrates that CEE are fully efficient when the entrepreneur is risk neutral. It also establishes that $s^* = 1$ if $L(\cdot)$ is *monotonically strictly increasing*, where monotonicity is defined for correspondences by the following:

DEFINITION 3.8: *The correspondence $L(\cdot)$ is strictly (weakly) increasing if, for all $s < \bar{s}$, $L \in L(s)$ and $\bar{L} \in L(\bar{s})$ $L < (\leq) \bar{L}$.*

A standard revealed preference argument can be used in combination with the assumption that $g(L, \tilde{x})$ is strictly increasing in L to prove that $L(s)$ is weakly increasing in s . (It may help to note that s can be reinterpreted as the output price and $sEg(L, \tilde{x}) - wL$ can be reinterpreted as profit when $Eg(L, \tilde{x})$ is interpreted as a production function.)

THEOREM 3.2: *If the entrepreneur is risk neutral, a CEE $\langle L^*, V^*(\cdot), s^*, t_1^*, \dots, t_n^* \rangle$ satisfying Definition 3.6 is fully efficient. If, furthermore, $L(\cdot)$ is strictly increasing, then $s^* = 1$.*

PROOF: Because of (iv'), the entrepreneur's equilibrium expected earnings are

$$\begin{aligned} & s^* Eg(L^*, \tilde{x}) - wL^* + (1 - s^*)V^*(s^*) \\ &= s^* Eg(L^*, \tilde{x}) - wL^* + (1 - s^*)Eg(L^*, \tilde{x}) = Eg(L^*, \tilde{x}) - wL^* \\ & \leq \max_{L \geq 0} \{Eg(L, \tilde{x}) - wL\}. \end{aligned} \quad (3.28)$$

Hence, the entrepreneur's earnings do not exceed the maximum of $Eg(L, \tilde{x}) - wL$.

He can achieve this maximum by setting $s = 1$ and choosing

$L \in L(1)$, for then his income is

$$\begin{aligned} sEg(L, \tilde{x}) - wL + (1 - s)V^*(s) &= Eg(L, \tilde{x}) - wL \\ &= \max_{L \geq 0} \{Eg(L, \tilde{x}) - wL\}. \end{aligned}$$

The inequality in (3.28) must, therefore, be an equality, and L^* must be in $L(1)$, i.e., L^* must maximize $\{Eg(L, \tilde{x}) - wL\}$, the expected profit of the firm. In general, we cannot prove that $s^* = 1$. However, since $V^*(s^*) = Eg(L^*, \tilde{x})$, risk averse investors will choose $t_1^* = 0$ and only risk neutral investors will choose $t_1^* > 0$. Together $L^* \in L(1)$ and $t_1^* = 0$ for risk averse investors imply full efficiency.

We have shown that L^* must be in $L(1)$. The definition of equilibrium requires that $L^* \in L(s^*)$. Thus, L^* must be in both $L(s^*)$ and $L(1)$. When $L(\cdot)$ is monotonically strictly increasing, $L(s^*)$ and $L(1)$ do not intersect unless $s^* = 1$. Thus, when $L(\cdot)$ is monotonically strictly increasing, $s^* = 1$. \square

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