

**THE IMPLICATIONS OF INSURANCE
FOR THE
EFFICACY OF FISCAL POLICY**

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(6-88)

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February, 1988

This paper was prepared for presentation to the American Risk and Insurance Association in Montreal, August 16, 1987. I thank J. David Cummins for discussion and Stanley Fischer, S. Travis Pritchett, and Stephen Zeldes for comments on an earlier draft. Financial support from the National Science Foundation, the Sloan Foundation, and the Amoco Foundation Term Professorship in Finance is gratefully acknowledged.

Abstract

The Implications of Insurance for the Efficacy of Fiscal Policy

Various tax policies provide consumers with forms of insurance. Social security has the payoff characteristics of an annuity. Income tax provides consumers with a degree of income insurance because the government shares part of the individual's income risk. Redistributive taxes can be used to spread aggregate income risks across different generations. The effects of these and other tax policies are shown to depend crucially on the nature of existing private insurance arrangements.

Introduction

What does insurance have to do with the macroeconomic effects of fiscal policy? To an economist schooled in the traditional Keynesian multiplier analysis, the answer would be a resounding "nothing!" In the simplest Keynesian paradigm, the effects of fiscal policy are analyzed using the multiplier, which is based on a simple marginal propensity to consume. The role of uncertainty--not to mention insurance--is not at all apparent in this analysis. However, macroeconomics and the analysis of fiscal policy have progressed well beyond this simple framework. Consumption behavior continues to be stressed in analyzing the effects of fiscal policy, but the mechanisms that are currently emphasized are quite different from those in the early Keynesian framework.

Most recent theoretical research on the effects of fiscal policy proceeds by examining the effects of fiscal policy on the consumption and portfolio decisions of individual consumers. The macroeconomic effects of fiscal policy are then determined by aggregating the behavior of individual consumers. If all consumers are identical, then, of course, aggregation is particularly simple. Alternatively, if, as in much of the research presented below, there is heterogeneity among consumers, then the aggregation of individual behavior must explicitly take account of general equilibrium considerations and market-clearing relations.

The preferred frameworks for analyzing individual consumption behavior are the life-cycle model of Modigliani and Brumberg [22] and the permanent income model of Friedman [12]. Each of these approaches is based on explicit utility maximization by an individual consumer subject to the constraints that the consumer faces. The important insight shared by these theories is that consumers form their consumption decisions on

the basis of their lifetime income rather than simply their on current income as assumed in the Keynesian consumption function. Optimal consumption behavior requires consumers to forecast their future after-tax incomes. Therefore, in responding to a tax change, for example, consumers must forecast the future course of taxes as well as the current tax. Because future incomes and taxes are not perfectly predictable, there is a demand by risk-averse consumers for insurance. The savings and consumption decisions of individual consumers will be greatly affected by whether insurance of various types is available and, if so, at what price. In particular, the responses of individual consumers to various changes in taxes depend on the nature of available insurance arrangements.

In discussing the importance of insurance arrangements, a broad definition of insurance will be used. For the purposes of this paper, insurance will be defined as any contingent arrangement that allows individual consumers to mitigate random fluctuations in marginal utility. This definition is deliberately general in order to convey the view that insights about insurance can be applied to questions that at first glance do not appear to have anything to do with insurance.

The majority of this paper is devoted to situations in which individuals face idiosyncratic risks. More precisely, much of the analysis examines situations in which all individuals in a group face the same ex ante probability distribution for a random variable; but, ex post, different members of the group obtain different realizations of the random variable. If each individual's realization of this random variable were publicly observable, there would evidently be scope for private insurance markets to pool these idiosyncratic risks. By

contrast, the last part of this paper will ignore idiosyncratic risks and will focus instead on aggregate risks, in which all members of a cohort experience the same realization of the random variable. In this situation, the scope for private insurance is less evident, but a fiscal authority could provide insurance.

The particular risks analyzed in this paper are of three sorts. The first risk is associated with the fact that individuals do not know in advance exactly when they will die. After analyzing the implications of this individual longevity risk for individual saving and the distribution of wealth, this framework is used to analyze the effects of social security in the presence of alternative private insurance arrangements. The second risk is associated with the unpredictability of future income, and it gives rise to precautionary saving. An income tax provides a form of insurance against fluctuations in income and thus mitigates the need for precautionary saving. This interaction of the insurance aspects of the income tax and saving behavior has important implications for the effects of fiscal policy. The third risk is a cohort-wide income risk that cannot be shared in private insurance arrangements. However, a fiscal system of taxes and transfers can be established to share this risk optimally across generations. After presentation of the features of an optimal system, the viability of such a system is discussed.

Longevity Risk

Before analyzing the saving behavior of consumers in the presence of longevity risk, it is useful to summarize briefly the implications of the life-cycle model under the assumption that each consumer knows in

advance how long he or she will live. The life-cycle model has two fundamental components. First, each individual cares about lifetime utility and, consequently, attempts to have a smooth profile of consumption over his or her lifetime. Second, there is a typical life-cycle pattern of income in which individuals earn labor income during early and middle adulthood and are retired in late adulthood. In order to achieve the same level of consumption during retirement as during working years, it is necessary for individuals to save some of their labor income and accumulate wealth during their working years. Then this wealth is gradually decumulated to provide for consumption during retirement.

In a particularly restrictive form of the life-cycle model, consumers are assumed not to have bequest motives. In this formulation then, it is optimal for a consumer to end life with precisely zero wealth. However, this implication is simply not borne out by the data.¹ While the implication that consumers die holding zero wealth is perhaps too strong to be true, many studies have indicated that consumers decumulate wealth too slowly, or not at all. Does this failure of elderly consumers to decumulate their wealth indicate the importance of a bequest motive, does it indicate an imperfection in life and/or health insurance markets², or does it indicate some more basic flaw in the model? Although this question is still waiting for a definitive answer, recent research, which has focussed on the role of insurance markets and bequest motives, has produced a rich array of insights.

1 See, for example, Kotlikoff and Summers [21].

2 Davies [8] calibrated a theoretical model to actual mortality probabilities and concluded that the uncertainty about one's date of death could potentially explain the failure of elderly consumers to dissave.

Absence of Private Annuities

To begin the study of saving and bequests under uncertain longevity, it is convenient to start with as simple a model as possible. This model is taken from Abel [1]. A similar model of individual behavior, which does not include a capital stock, is analyzed in Eckstein, Eichenbaum and Peled [10].

Suppose that each consumer can live for at most two periods. For the moment, assume that a consumer does not have a bequest motive and cannot buy life insurance or annuities. In the first period, the consumer receives an inheritance, I , and inelastically supplies one unit of labor thereby earning labor income Y . Also in the first period, the consumer pays a tax T and consumes an amount c . Therefore, the consumer's total wealth at the end of the first period of life, which is denoted by W , is

$$W = I + Y - T - c \quad (1)$$

Suppose that this wealth is held in the form of riskless capital and let R denote the gross rate of return on wealth between the first and second period. Thus, at the beginning of the second potential period of life, the consumer's wealth, including accrued interest, is RW . At the beginning of the second potential period of life, the consumer gives birth to G heirs. After the G heirs are born, the uncertainty about the consumer's longevity is resolved. With probability p , the consumer dies at the beginning of the second period and the consumer's estate is divided equally among the G heirs; thus each heir receives RW/G . Alternatively, with probability $1-p$, the consumer survives. Each surviving consumer receives a social security benefit S . Knowing that this is the last period of his or her life, the consumer consumes all of

his or her resources. Letting x denote the consumption in the second period of life, it follows that

$$x = RW + S = R[I + Y - T - c] + S \quad (2)$$

Equation (2) is the consumer's lifetime budget constraint. The next step is to specify the consumer's utility function. It is convenient to use the following special case of the Yaari [26] utility function

$$U = \ln c + (1-p) D \ln x \quad (3)$$

where $0 < D < 1$ is a discount factor representing the pure rate of time preference. The utility of old age consumption is discounted both because of time preference and because of uncertainty. The weight $(1-p)$ is the probability of survival.³

To derive the optimal consumption in the first period, substitute the budget constraint (2) into the utility function (3) so that the level of expected lifetime utility depends only on consumption when young. Differentiating this expression with respect to c and setting the derivative equal to zero yields the optimal level of consumption

$$c = a [I + Y - T + S/R] \quad (4)$$

where $a = 1/[1+(1-p)D]$. The coefficient a , which is between zero and one, is the marginal propensity to consume out of lifetime resources. Note that, in calculating the present of value of lifetime resources, the social security benefit S is discounted by the riskless rate of interest R , even though S will be received only with probability $1-p$. Because the

³ The utility function in (3) can be interpreted as the expected present value of lifetime utility. Under this interpretation, it is implicitly assumed in (3) that if the consumer dies at the beginning of the second period, then second-period utility is equal to zero. More generally, one could write (3) as $U = \ln c + (1-p) D \ln x + p D \phi$ where ϕ is the level of second-period utility if the consumer dies young. For the purpose of deriving the optimal behavior of the consumer, the value of ϕ is irrelevant.

consumer is assumed not to have a bequest motive, he or she does not care about anything that occurs in the second period if death occurs. That is, in valuing assets that pay off in the second period, the selfish consumer cares only about the payoffs in the state in which he or she survives and discounts these payoffs by the riskless rate of return.

The consumption function (4) can be substituted into the definition of wealth (1) to obtain an equation for the optimal saving of a young consumer

$$W = (1-a) [I + Y - T] - a S/R \quad (5)$$

The Distribution of Wealth: The analysis to this point has been partial equilibrium in nature. The consumption function in (4) presents the optimal consumption of a young consumer given labor income, Y , given the tax structure as parameterized by T and S , and given the inheritance received at birth, I . However, in a dynamic general equilibrium model, the inheritance, I , is an endogenous variable that depends on the mortality history of the consumer's ancestors. In particular, the relevant mortality history can be summarized completely by the non-negative integer j which is the number of consecutive previous generations which died young, thereby leaving (albeit accidental) bequests. For example, if $j = 0$, then the consumer's parent died old and thus left no bequest. Alternatively, if $j = 1$, then the consumer's parent died young leaving a bequest, but the consumer's grandparent died old and left no bequest.

The behavior of type 0 consumers (i.e., consumers for whom $j = 0$) is particularly important in characterizing the cross-sectional distribution of wealth and consumption. Letting $I^{(0)}$ denote the inheritance received by a type 0 consumer, it is clear that $I^{(0)} = 0$,

because this consumer's parent died old and left no bequest.

Substituting $I^{(0)} = 0$ into the consumption function (4) yields the consumption of a young type 0 consumer, $c^{(0)}$,

$$c^{(0)} = a [Y - T + S/R] \quad (6)$$

Also, substituting $I^{(0)} = 0$ into (5) yields the saving of a young type 0 consumer

$$W^{(0)} = (1-a) [Y - T] - a S/R \quad (7)$$

The consumption of type j ($j > 0$) consumers differs from the consumption of type 0 consumers only because these consumers receive inheritances at birth. Because the consumption function is linear, the consumption of a type j consumer exceeds the consumption of a type 0 consumer by an amount equal to the inheritance, $I^{(j)}$, multiplied by the marginal propensity to consume, a . Therefore,

$$c^{(j)} = c^{(0)} + a I^{(j)} \quad (8)$$

Similarly, the wealth of a type j consumer, $W^{(j)}$, exceeds the wealth of a type 0 consumer, $W^{(0)}$, by the inheritance, $I^{(j)}$, multiplied by the marginal propensity to save, $1-a$. Therefore,

$$W^{(j)} = W^{(0)} + (1-a) I^{(j)} \quad (9)$$

The expressions for the consumption and wealth of type j consumers in (8) and (9), respectively, depend on the inheritance received by a type j consumer. Of course, the type j consumer treats the value of the inheritance, $I^{(j)}$, as given, but the value of this inheritance is generated endogenously within the model. To calculate the value of this inheritance, observe that a type j consumer is a consumer whose parent was a type $j-1$ consumer that died young. In this case the inheritance received by a type j consumer is

$$I^{(j)} = R W^{(j-1)}/G \quad (10)$$

Equation (10) relates the inheritance of a type j consumer to the wealth of a type $j-1$ consumer. Using this relation to substitute for the inheritance in (9) yields

$$W^{(j)} = W^{(0)} + (1-a) R W^{(j-1)}/G \quad (11)$$

Equation (11) can be used to solve for the wealth of all consumers. Technically, it is a first-order linear constant coefficient difference equation with the boundary condition given by (7). This equation can be easily solved. It can be shown that the wealth of a young type j consumer, $W^{(j)}$, increases monotonically in j and, if $(1-a)R/G < 1$, it approaches a finite limit as j approaches infinity. Rather than present the complete solution here⁴, it is convenient to focus on the average value of $W^{(j)}$ in the steady state, which is denoted as W^* . It can be shown that

$$W^* = W^{(0)}/[1-(1-a)pR/G] \quad (12)$$

The variable W^* is an interesting macroeconomic quantity; in particular, it is the per capita value of the private capital stock.⁵ To see that W^* is the per capita stock of private capital, recall that surviving old consumers consume all of their resources. Thus, all private saving in the economy is done by young consumers. Since capital is the only asset in this economy, the saving of young consumers, which averages W^* per capita, is equal to the private capital stock.

This simple model endogenously generates a cross-sectional distribution of wealth. The mechanism generating the cross-sectional variation is that a fraction p of each cohort of consumers dies young and thus leaves accidental bequests to their heirs. In this model, all of

⁴ See Abel [1] for a complete solution.

⁵ Strictly speaking, W^* is equal to the total private capital stock of the economy divided by the number of young consumers, rather than divided by the total number of consumers.

the cross-sectional variation in wealth results from cross-sectional variation in bequests. An additional feature of this model is that it predicts a potentially substantial ratio of bequests to total private wealth. Indeed, since a fraction p of each type of consumer dies young, the ratio of bequests to total private wealth is equal to p .

Fully Funded Social Security: Although the model presented above is quite simple, it provides some important insights into the effects of a social insurance program. In particular, this model can be used to examine the effects on consumption, capital accumulation and the distribution of wealth of either a fully funded or a pay-as-you-go social security system.

First, consider the effects of a fully funded social security system. In such a system, the government collects a tax T from each young consumer and invests the proceeds in capital. In the next period, the social security fund is worth RT and is distributed evenly to the surviving members of the cohort of elderly consumers. Thus, each surviving consumer receives a social security payment S such that

$$RT = (1-p)S \quad (13)$$

The effects of the introduction of social security can be evaluated by comparing the equilibrium values of variables under the social security system with the values that these variables would attain in the absence of social security (with $T = S = 0$). The consumption of type 0 consumers can be calculated by substituting the relation between the social security parameters S and T in (13) into (6) to obtain

$$c^{(0)} = a [Y + pT/(1-p)] \quad (14)$$

Inspection of (14) reveals that the introduction of fully funded social security increases the consumption of young type 0 consumers by

$apT/(1-p)$. This increase in consumption reflects the intra-cohort risk pooling of the social security system. Each consumer contributes T to the social security system, but a fraction p of each cohort dies young and thereby surrenders its claim to social security benefits to the remaining fraction $1-p$ of the cohort. Thus, risk pooling increases the present value of lifetime resources of each survivor by $pT/(1-p)$. Multiplying this increase in lifetime resources by the marginal propensity to consume, a , yields the increase in consumption of young type 0 consumers.

The wealth held by young type 0 consumers can be calculated by substituting the relation between the social security parameters S and T from (13) into equation (7) to obtain

$$w^{(0)} = (1-a)Y - T - a p T/(1-p) \quad (15)$$

The introduction of fully funded social security reduces the wealth held by young type 0 consumers, and this reduction in wealth is decomposed into two parts in (15). First, even if a young type 0 consumer maintained consumption unchanged with the introduction of social security, the consumer's wealth would decline by the amount of the social security tax, because first-period disposable income is reduced by T . Furthermore, as explained above, a young type 0 consumer increases consumption by $apT/(1-p)$, which reduces saving by an additional $apT/(1-p)$.

Because the saving of young type 0 consumers is reduced by the introduction of fully funded social security, those type 0 consumers who die young leave smaller bequests in the presence of social security than in its absence. Therefore, the introduction of social security reduces the inheritances received by type 1 consumers. These consumers in turn

leave smaller bequests in the presence of social security than in its absence. Indeed, in the new steady state the accidental bequest left by each type j consumer is reduced by the introduction of social security. Therefore, the introduction of social security reduces the inheritances received by all consumers (except for type 0 consumers who receive no inheritance in either case). Because the only source of cross-sectional variation is the intra-cohort variation in inheritances, it follows that fully funded social security narrows the steady state distributions of consumption and wealth.⁶

In addition to reducing the intra-cohort variation in wealth, the introduction of fully funded social security affects the average level of wealth in the economy and the size of the national capital stock. The national capital stock (per young person), K^* , is equal to the sum of the private capital stock, W^* , and the capital held by the social security system, T . Recalling from (12) that the private capital stock is proportional to $W^{(0)}$, and recalling that the introduction of social security reduces the value of $W^{(0)}$, it follows that the introduction of social security reduces the private capital stock. Moreover, it can be shown that the reduction in the private capital stock, W^* , is greater than T . Therefore, the national capital stock, $K^* = W^* + T$, declines in response to the introduction of social security.

The effect of social security on the average level of consumption can be calculated using the national income identity. To derive this identity, let N_t denote the number of young consumers born in period t .

⁶ Chu [7] extends this model to make the rate of return on capital and labor income endogenous. He further modifies the model to make social security taxes proportional to labor income and shows that linking the social security tax to income leads to different results concerning the distribution of wealth.

The assumption that each consumer has G children implies that $N_t = GN_{t-1}$. Gross national product in period t is equal to the labor income of young consumers, $N_t Y$, plus gross capital income, $R N_{t-1}(W^*+T)$.⁷ Gross national product is allocated to consumption and saving. Total consumption is equal to the consumption of the young consumers, $N_t c^*$, plus the consumption of the surviving old consumers, $(1-p)N_{t-1}x^*$. Gross national saving is equal to the saving of the young consumers, $N_t W^*$, plus the gross saving of the social security system, $N_t T$. Equating the sources and uses of gross national product yields

$$N_t Y + R N_{t-1}(W^*+T) = N_t c^* + (1-p)N_{t-1}x^* + N_t(W^*+T) \quad (16)$$

Equation (16) simply states that gross national product is equal to consumption plus gross investment. Dividing both sides of (16) by N_t and recalling that $N_t/N_{t-1} = G$ and $K^* = W^* + T$ yields

$$c^* + [(1-p)/G] x^* = Y + [R/G - 1] K^* \quad (17)$$

The left hand side of (17) is aggregate consumption per capita.

Recall that the introduction of fully funded social security leads to a reduction in the national capital stock, K^* , on the right hand side of

⁷ The definitions of gross national product, gross capital income, gross national saving and gross investment used here differ somewhat from those used in the national income accounts. Recall that one unit of capital in period t yields R units of the consumption good in period $t+1$. Using more standard terminology, R is equal to $1 - d + r$ where d is the rate of depreciation and r is the rate of return on capital before subtracting depreciation. With this notation, gross national product in period t is $N_t Y + rN_{t-1}(W^* + T)$; gross capital income is $rN_{t-1}(W^*+T)$; gross investment, which is net investment plus depreciation, is $N_t(W^*+T) - N_{t-1}(W^*+T) + dN_{t-1}(W^*+T)$; gross saving of the young generation is $N_t W^*$; and gross saving of the old generation, which is net saving plus depreciation, is $-(1-p)N_{t-1}x^* + d(1-p)N_{t-1}W^*$. In the special case of complete depreciation, $d = 1$ and therefore $R = r$. In this case, gross national product is $N_t Y + RN_{t-1}(W^* + T)$; gross capital income is $RN_{t-1}(W^*+T)$; gross investment is $N_t(W^*+T)$; gross saving of the young generation is $N_t W^*$; and gross saving of the old generation is zero. Thus in the case of complete depreciation, the concepts of the gross national product, gross capital income, gross national saving and gross investment used in the text correspond to the standard national income accounting definitions.

(17). If the rate of interest exceeds the population growth rate, then $R/G - 1$ is positive and the reduction in K^* implies a reduction in consumption per capita. Alternatively, if the interest rate is less than the population growth rate, then $R/G - 1$ is negative and the reduction in the national capital stock K^* implies an increase in consumption per capita.

The relation between aggregate consumption and the aggregate capital stock in (17) is related to Phelps's [23] famous Golden Rule result. In order to maintain a constant level of capital per capita, it is necessary that the capital stock grow at the same rate that population grows. Thus, if the level of capital per capita is to be increased permanently by one unit, then the level of gross investment in each period must be increased by G units per capita. The benefit of increasing the capital stock by one unit per capita is that gross capital income is increased by R units per capita. If R is greater than G , then the increased capital stock will increase steady state consumption; whether the economy should temporarily decrease consumption in order to accumulate capital and increase long-run consumption depends on society's preferences for present consumption relative to future consumption. Alternatively, if R is less than G , then steady state consumption can be increased by a decrease in the capital stock. There is no tradeoff between current and future consumption in this case. An increase in current consumption will reduce the capital stock and increase future consumption. Clearly, if an economy is ever in the case with $R < G$, it should decrease its capital stock; this would increase consumption at all dates, which would be Pareto-improving.⁸

The case in which R is equal to G receives special attention. In a model with a neoclassical production function, the rate of return on capital, R , is a strictly decreasing function of the capital stock; hence, there is a unique value of the capital stock for which $R = G$. This value of the capital stock is called the Golden Rule capital stock. The Golden Rule capital stock is the capital stock that maximizes the permanently sustainable level of consumption. Any capital stock greater than the Golden Rule capital stock is too large in the sense described above, because R would be less than G .

Pay-as-you-go Social Security: A pay-as-you-go social security system differs from a fully funded system in that the social security system does not hold any capital under a pay-as-you-go system; the taxes collected from the young consumers are used to pay the benefits to the old consumers in the same period. In each period, the young cohort is $G/(1-p)$ times as large as the surviving old cohort. Therefore, setting total tax collections from young consumers equal to the total benefits paid to the old consumers yields the following relation between the social security parameters S and T

$$GT = (1-p)S \quad (18)$$

The consumption of young type 0 consumers is calculated by substituting (18) into (6) to obtain

$$c^{(0)} = aY + a[1 - (1-p)R/G] (S/R) \quad (19)$$

Equation (19) indicates that the consumption of young type 0 consumers may either increase or decrease with the introduction of pay-as-you-go social security, depending on whether G exceeds or falls short of $(1-p)R$. This result is to be contrasted with the finding that fully

⁸ See Diamond [9] for a demonstration that a competitive economy may end up with an inefficient overaccumulation of capital.

funded social security unambiguously increases $c^{(0)}$. To understand this difference, one can view the social security tax T as the price paid for a contingent claim that pays S if the consumer lives for two periods. The gross rate of return on this claim is S/T . If this gross rate of return, S/T , exceeds the rate of return available on the consumer's portfolio, R , then the introduction of social security will effectively make the consumer richer and will increase consumption. However, if the rate of return on social security falls short of the rate of return on the consumer's portfolio, then the introduction of social security will reduce the consumption of young type 0 consumers. Now observe from (13) that the rate of return on fully funded social security, S/T , is equal to $R/(1-p)$ which exceeds R if $p > 0$. Thus, fully funded social security leads to an increase in $c^{(0)}$. Alternatively, (18) implies that the rate of return on pay-as-you-go social security, S/T , is equal to $G/(1-p)$ which may be greater than, less than, or equal to R . Therefore, pay-as-you-go social security may lead to an increase, decrease, or no change in $c^{(0)}$.

Although the introduction of pay-as-you-go social security may either raise or lower $c^{(0)}$, inspection of (7) reveals immediately that it unambiguously reduces the saving of young type 0 consumers, $W^{(0)}$. As with fully funded social security, the reduction in $W^{(0)}$ implies that the inheritance received by each type j consumer ($j > 0$) is reduced. Again, the reduction in all non-zero inheritances implies that the cross-sectional variation in wealth is reduced. Also, the reduction in $W^{(0)}$ again implies that the per capita value of private wealth, W^* , is reduced by the introduction of social security. Under pay-as-you-go social security, the government does not hold any capital; the national capital

stock is equal to the private capital stock. Because pay-as-you-go social security reduces the private capital stock, W^* , it also reduces the national capital stock. Again (17) indicates that consumption per capita will fall if the interest rate, R , exceeds the population growth rate, G , but will increase if R is less than G .

Annuities

The model presented above has yielded some important insights about the behavior of individual and aggregate saving, and about the effects of social insurance, in the presence of uncertain individual longevity. However, the model has at least two unsatisfactory implications. First, because consumers are assumed to be selfish, they would choose to hold all of their wealth in annuities, even if the annuities were not actuarially fair. Provided that an annuity pays a greater return than riskless capital in the event the consumer survives, the consumer would choose to fully annuitize his or her wealth. Thus, there is an incipient demand for annuities, and a satisfactory treatment would either include annuities or would provide an economic reason why there are no annuities in equilibrium. The model presented above simply rules out annuities by assumption. However, annuities will be introduced into the analysis below.

The second unsatisfactory feature of the model is that the children of the richest consumers are among the poorest members of the economy if their rich, but selfish, parents live for two periods and thus leave no bequest. In this model, the only channel for the preservation of a family's wealth across generations is through accidental bequests which occur with early death. This feature of the model can be eliminated by introducing a bequest motive. In order to be able to focus first on the

implications of an annuity market, the introduction of the bequest motive will be delayed until the next section.

Now suppose that there is a competitive market for annuities.⁹ Each dollar invested by a young consumer in an annuity yields Q dollars in the following period if the consumer survives; the consumer's estate receives nothing if the consumer dies after only one period of life. Insurers sell these annuities and invest the proceeds in riskless capital which pays a gross rate of return R . In the following period, insurers distribute the premiums with accrued interest to the surviving annuitants in proportion to their contributions when young. The gross rate of return earned by survivors is equal to $R/(1-p)$.

The introduction of a competitive annuity market into the model dramatically alters the nature of the equilibrium and the effects of social security. Because $R/(1-p)$ exceeds R , annuities dominate riskless capital and all consumers would choose to hold all of their wealth in the form of annuities. Therefore there would be no bequests--accidental or otherwise. Hence, there would be no cross-sectional variation in wealth. In this situation, it is appropriate to use a "representative consumer" model.

In the presence of an annuity market offering a gross rate of return Q , the consumption of a representative old consumer, x , is

$$x = QW + S = Q[Y - T - c] + S \quad (20)$$

The optimal level of consumption when young can be calculated by substituting the lifetime budget constraint (20) into the utility

⁹ Kotlikoff and Spivak [20] discuss the role of the family in helping to provide annuities. The annuity protection offered by family members is not, of course, as complete as the insurance available in competitive annuity markets. Nevertheless, this annuity protection does affect consumer behavior.

function (3) and then differentiating the resulting expression with respect to c . Setting this derivative equal to zero yields

$$c = a [Y - T + S/Q] \quad (21)$$

where, as earlier, the marginal propensity to consume, a , is equal to $1/[1+(1-p)D]$. Equation (21) states that the consumption of a young consumer is proportional to the present value of lifetime resources, where the future social security benefit, S , is discounted by the actuarial rate of return Q . Alternatively, recalling that in a competitive annuity market $Q = R/(1-p)$, the consumption function in (21) can be written as

$$c = a [Y - T + (1-p)S/R] \quad (22)$$

The consumption function in (22) indicates that in the presence of a competitive annuity market, the appropriate concept of lifetime income is the expected present value of income.

Fully Funded Social Security: Now consider the effects of introducing a fully funded social security system as characterized by (13). Substituting the social security parameters from (13) into the consumption function (22) yields

$$c = a Y \quad (23)$$

Equation (23), which presents the optimal level of consumption of a young consumer in the presence of a fully funded social security system, displays a remarkable result. This equation indicates that the optimal level of c is independent of the values of the social security parameters T and S . Thus, the optimal level of c is invariant to the introduction of fully funded social security.

The reason for the irrelevance of social security in the presence of a competitive annuity market is that fully funded social security

simply provides consumers with a redundant asset. As stated earlier, a consumer's claim to the social security benefit S can be viewed as an asset with a gross rate of return equal to S/T . Under fully funded social security, this rate of return is equal to $R/(1-p)$, which is precisely equal to the rate of return on privately available annuities. Thus, while the social security system essentially forces young consumers to purchase an annuity, the consumers can exactly offset this effect by reducing their holdings of private annuities by an equal amount. Because the payoff characteristics of the private annuity are identical to those of social security, the consumer can obtain exactly the same state-contingent stream of consumption after the introduction of social security that could be obtained before its introduction. Furthermore, it will be optimal for the consumer to offset the effect of social security, because the initial state-contingent consumption plan was optimal. Since the introduction of fully funded social security does not change any relative price and does not change the consumer's opportunity set in any way, the original optimal plan remains optimal.

The irrelevance of fully funded social security in the presence of a competitive annuity market is an example of a more general phenomenon that is known as the Ricardian Equivalence Theorem. Briefly, the Ricardian Equivalence Theorem states that changes in the timing of lump-sum taxes, holding constant the path of government spending, have no effect on the allocation of consumption. The reason is essentially that consumers can, and will, offset the effects of such changes by adjusting their savings and/or bequest behavior.

As shown above, the Ricardian Equivalence Theorem applies to fully funded social security in the presence of an actuarially fair private

annuity market. However, there are at least two sources of departure from the Ricardian Equivalence Theorem in the case of fully funded social security. First, it should be noted the invariance of consumption to the introduction of fully funded social security is a direct consequence of the fact that the rate of return on social security is exactly the same as the rate of return on private annuities. If, for some reason, the rates of return on private annuities and social security were not identical, then the Ricardian Equivalence Theorem would not hold. For example, if the rate of return on social security exceeded the rate of return on private annuities, then the introduction of social security would increase the expected present value of lifetime income and would lead to an increase in consumption by young consumers.¹⁰ Of course, the question that then arises is why the government would be able to offer a higher rate of return on annuities than the private sector could. It would appear to be difficult to make the case that the government is more efficient in providing annuities, and it would also appear to be difficult to make the opposite case, a priori. An alternative explanation, which is discussed below, is that if consumers face different mortality risks, and have private information about these risks, then the private annuity market would be subject to adverse selection. However, the government could, by requiring a compulsory level of social security coverage, be immune to adverse selection and thus offer a higher rate of return.

A second reason why the Ricardian Equivalence Theorem may not hold is that if the social security tax is large enough, the consumer may not

¹⁰ Karni and Zilcha [17] examine the effect of social security in the presence of unfair annuity and life insurance markets in a model that includes a bequest motive.

be able to offset it completely. Equivalently, a large social security tax may force the consumer to hold more of the publicly provided annuity than he or she would have held of the private annuity. Therefore, the consumer would not be able to offset completely the effect of the publicly provided annuity by reducing private annuity holdings; the consumer was simply not planning to hold that much wealth in private annuities. Formally, this argument can be stated by observing that the private saving of a young consumer is $W = Y - T - c$. In view of the optimal level of consumption in (23), the optimal level of private saving of a young consumer is

$$W = (1-a) Y - T \quad (24)$$

Provided that T is less than $(1-a)Y$, the consumer could offset the effects of social security by reducing the holding of private annuities by T . However, if T is greater than $(1-a)Y$, then in order to maintain the originally planned level of consumption, the consumer would have to hold a negative amount of annuities. A negative holding of annuities could be achieved if the consumer could borrow resources and repay the debt if he or she survives but have the debt cancelled if he or she dies. The actuarially fair rate of return on such loans would be $Q = R/(1-p)$. The Ricardian Equivalence Theorem would hold if the consumer could borrow at the actuarially fair rate Q . However, if the consumer were unable to borrow (perhaps because the social security benefit is legally prohibited from serving as collateral), then the Ricardian Equivalence Theorem would fail to hold if T is greater than $(1-a)Y$. In this case, social security would reduce consumption by young consumers.¹¹

Pay-as-you-go Social Security: Although a fully funded social

¹¹ See Hubbard and Judd [16] for a more complete discussion of the effects of social security in the presence of borrowing constraints.

security system is ineffectual in the presence of a competitive annuity market, this is not true, in general, for a pay-as-you-go system. To determine the optimal level of consumption under pay-as-you-go social security, substitute the social security parameters from (18) into the consumption function (22) to obtain

$$c = a Y + a [G/R - 1] T \quad (25)$$

If the rate of interest exceeds the population growth rate, then $G/R - 1$ is negative and the introduction of social security leads to a reduction in consumption by young consumers. The reason for this reduction is quite clear. The rate of return on pay-as-you-go social security, S/T , is equal to $G/(1-p)$ and the rate of return on private annuities is $R/(1-p)$. Thus, if $R > G$, then the introduction of social security forces young consumers to hold annuities with a lower rate of return than is available in the private market. Therefore, consumers are made poorer by the introduction of social security and they reduce their consumption.

If the rate of interest is less than the population growth rate, then $G/R - 1$ is positive and the introduction of social security increases consumption. Furthermore, the introduction of social security is Pareto-improving in this case. In the period in which pay-as-you-go social security is introduced, the members of the old generation are clearly better off because they receive the social security benefit S without ever having had to pay social security taxes. In addition, each subsequent generation is made better off by the introduction of social security because it provides them with an annuity that dominates the annuity available in the private market. The fact that the introduction of social security is Pareto-improving indicates that the initial

equilibrium was not Pareto-efficient. Indeed, the Golden Rule results discussed above indicate that if $R < G$, and if the capital stock is positive, then the economy suffers from an inefficient overaccumulation of capital. As applied to pay-as-you-go social security when $R < G$, the Golden Rule result indicates that any increase in T and S is Pareto-improving until private saving by young consumers is driven to zero (or, with a neoclassical production function, until R is equal to G).

Bequest Motive

The preceding analysis introduced annuities but has the shortcoming that there are no bequests in the model. As a consequence, the distribution of wealth is degenerate. In order to generate bequests in a model with a competitive annuity market, a bequest motive is introduced in this section. The two most common formulations of the bequest motive are altruism and the joy-of-giving. Although these specifications are similar in many respects, they have different implications for the validity of the Ricardian Equivalence Theorem and the efficacy of fiscal policy.

Altruism: The altruistic specification of the bequest motive became popular in macroeconomics after Barro's [4] presentation of the Ricardian Equivalence Theorem. Under an altruistic specification of the bequest motive, a consumer obtains utility from the utility of his heirs as well as from his own consumption. Consider a consumer born in period t who consumes c_t in period t and, contingent on survival, consumes x_{t+1} in period $t+1$. Let V_t denote the total utility of this consumer and let V_{t+1} denote the total utility of his or her representative heir, who is born in period $t+1$. Suppose that the consumer's utility function is

$$V_t = \ln c_t + (1-p)D \ln x_{t+1} + \beta E_t(V_{t+1}) \quad (26)$$

where $E_t(\)$ denotes the expectation conditional on information in period t . The parameter β , which is assumed to lie between zero and one, measures the strength of the altruistic bequest motive.

The utility of an altruistic consumer depends on his or her children's utility, which in turn depends on their children's utility, and so on. Thus, the utility of a consumer depends on the entire stream of consumption over his or her own lifetime and over the lifetimes of all of his or her descendants. Formally, the recursive specification of altruistic preferences in (26) is a linear difference equation that is satisfied by the following infinite-horizon utility function¹²

$$V_t = \sum_{j=0}^{\infty} E_t(\beta^j [\ln c_{t+j} + (1-p)D \ln x_{t+1+j}]) \quad (27)$$

Now consider the effects of social security in the presence of an altruistic bequest motive. It is easiest to begin with the case in which all consumers live for two periods with certainty (formally $p = 0$). First, consider a fully funded social security system in which $RT = S$. As in the case without a bequest motive, fully funded social security will have absolutely no effect. In response to the introduction of fully funded social security, young consumers will maintain their original levels of consumption and will simply reduce their private saving by T . In the following period, when they are old, their portfolios of private assets will be worth RT less than in the absence of social security, but they will receive a social security benefit of $S = RT$ that allows them to maintain the original level of old-age consumption.

Now consider the introduction of pay-as-you-go social security in

¹² Douglas Gale [13, pp. 55-61] has emphasized that equation (27) is only one of an infinity of solutions to the difference equation in (26). However, (27) is the specification that is generally used in the literature.

which $S = GT$. Under altruistic preferences, it turns out that this policy also has no effect. The old consumers who are alive in the period in which social security is introduced will receive a payment of S but they will not increase their consumption at all. Instead, they will choose to increase their bequest to each of their heirs by S/G . This increased inheritance exactly offsets the tax burden T levied on each of the young consumers. Thus, all young and old consumers are able to maintain the original levels of consumption. Because all relative prices and marginal rates of substitution remain unchanged, there is no incentive for anyone to change consumption.

The discussion above demonstrates that in the absence of longevity risk and with altruistic preferences, the Ricardian Equivalence Theorem holds both for fully funded social security and for pay-as-you-go social security. That is, private consumption is invariant to the introduction of social security whether it is fully funded or pay-as-you-go. When longevity risk is re-introduced into the model, the Ricardian Equivalence Theorem does not hold quite so generally. It does continue to hold for fully funded social security. Because the rate of return on fully funded social security, S/T , is equal to the rate of return on private annuities, $R/(1-p)$, young consumers respond to the introduction of social security by reducing their holding of private annuities by T ; they maintain their consumption unchanged.

The effects of pay-as-you-go social security in the presence of longevity risk and altruistic preferences are more interesting. Consider the period in which the pay-as-you-go social security system is introduced. All surviving old consumers receive a social security benefit $S = GT/(1-p)$. However, because a fraction $1-p$ of families have a

surviving old consumer and the remaining fraction p of the families do not have a surviving old consumer, this payment to surviving old consumers induces a redistribution of wealth across families. In particular, there is a redistribution from families without a surviving old consumer to families with a surviving old consumer. Each surviving old consumer will see that the present value of his or her family's resources is increased by the introduction of social security. Therefore, surviving old consumers will increase their own consumption somewhat and will also increase their bequests somewhat in order to share the increase in wealth with subsequent generations. By contrast, the young consumers in families without survivors will see a decrease in their families' wealth and they will reduce their own consumption and their bequests.

The argument above indicates that the Ricardian Equivalence Theorem fails to apply to pay-as-you-go social security under longevity risk, even if consumers have altruistic bequest motives. However, a staunch defender of the Ricardian Equivalence Theorem would not concede the case so quickly. A defense of the Ricardian Equivalence Theorem would argue that the analysis in the paragraph above has ignored a relevant insurance market. More precisely, if the introduction of social security was at least conceivable in the prior period, then each young consumer would have taken steps to guard against the risk of having social security introduced during a period in which there were no surviving old consumers in his or her family. Each young consumer in the previous period would have agreed to give up any social security benefit, S , received in the subsequent period in exchange for $(1-p)S$ to be paid to the consumer, or his estate, in the following period. If this sort of tax liability

insurance arrangement were in force, then the introduction of pay-as-you-go social security would have no effect on the allocation of private consumption. Although this argument is theoretically sound, it will undoubtedly strike many readers as far-fetched. This type of insurance arrangement is not typically observed in practice, either because of legal prohibitions on trading future social security benefits or because of the limited ability and/or willingness of consumers to anticipate and write contracts for all conceivable contingencies.

Joy of Giving: The altruistic specification of the bequest motive often implies that individual consumers will take actions to completely nullify the effects of various lump-sum tax and transfer policies. An alternative specification of the bequest motive is the joy of giving. Under the joy of giving, consumers obtain utility from their own consumption and from leaving a bequest. The utility from leaving the bequest depends only on the size of the bequest; it does not depend on the utility or consumption of the recipients of the bequest. An example of a utility function that displays a joy-of-giving bequest motive is

$$V_t = \ln c_t + (1-p) D \ln x_{t+1} + pH(B_t^D) + (1-p)H(B_t^S) \quad (28)$$

where B_t^D is the bequest if the consumer dies after one period, and B_t^S is the bequest if the consumer survives for two periods. Under the specification in (28), the utility from leaving a bequest of size b is $H(b)$; it is assumed that $H'(b) > 0$ and $H''(b) < 0$.

To analyze optimal consumption and portfolio behavior under the utility function in (28), let A_t denote the amount of wealth that the consumer holds in the form of annuities at the end of period t ; the remainder of the portfolio, $I + Y - T - c_t - A_t$, is held in the form of riskless capital. If the consumer dies young, the bequest, B_t^D , is equal

to the value of riskless capital with accrued interest

$$B_t^D = [I + Y - T - c_t - A_t] R \quad (29a)$$

Alternatively, if the consumer survives for two periods, wealth in the second period consists of the principal and interest on annuities as well as on riskless capital; in addition, the consumer receives a social security benefit, S . Total available resources are allocated to consumption, x_{t+1} , and to the bequest, B_t^S , so that

$$B_t^S = [I + Y - T - c_t - A_t]R + A_t Q + S - x_{t+1} \quad (29b)$$

The young consumer's consumption and portfolio decisions can be solved by substituting (29a,b) into (28) and then differentiating with respect to c_t , x_{t+1} , and A_t . The solution to this problem is presented in Abel [2]. The discussion below focusses on a few interesting implications of optimal behavior.

Optimal behavior implies that the consumer would be indifferent between investing an additional dollar in riskless capital or in annuities.¹³ An additional dollar invested in annuities would be worth Q dollars in the following period if the consumer survives. This additional wealth could be used to increase the bequest B_t^S by Q units, thereby increasing expected utility by $(1-p)Q H'(B_t^S)$. Alternatively, an extra dollar invested in riskless capital would be worth R dollars in the following period regardless of whether the consumer dies or survives. In

13 If the consumer faces a binding constraint on the holding of riskless capital or annuities (such as a nonnegativity constraint), then he would not in general be indifferent about whether to invest an additional dollar in riskless bonds or in annuities. For this particular optimization problem, the consumer will choose to hold positive amounts of both riskless bonds and annuities provided that $H'(b)$ approaches infinity as b approaches zero; thus, any nonnegativity constraints on the holdings of capital or annuities would not be binding.

either case, the consumer could increase the bequest by R dollars, thereby increasing expected utility by $(1-p)RH'(B_t^S) + pRH'(B_t^D)$ dollars. Therefore, the consumer will be indifferent between investing the dollar in riskless capital and annuities if

$$(1-p)Q H'(B_t^S) = (1-p)R H'(B_t^S) + pR H'(B_t^D) \quad (30)$$

Recall that if the annuities are actuarially fair, then $(1-p)Q = R$. In this case, it follows directly from (30) that $B_t^S = B_t^D$. That is, in the presence of actuarially fair annuities, the consumer plans to leave the same bequest whether he or she lives for one or two periods. This result depends on the fact that the marginal utility of a bequest does not depend on whether the consumer lives for one period or for two periods. In particular, the joy-of-giving function $H(b)$ does not depend on whether the consumer lives for one period or two periods. (The same result also holds under an altruistic bequest motive.) The intuition behind this result is that actuarially fair annuities permit the consumer to completely insure the consumption basket which consists of c_t , x_{t+1} , B_t^D , and B_t^S . The strategy to achieve full insurance is implemented by holding just enough riskless capital to provide for the desired bequest and just enough annuities (including the contingent claim on the future social security benefit S) to provide for second-period consumption.¹⁴

The introduction of a fully funded social security system has no effect under a joy-of-giving bequest motive. The reason, as in the absence of a bequest motive, and as in the presence of an altruistic bequest motive, is that the annuity provided by the social security system offers exactly the same payoffs as the privately available

14 See Sheshinski and Weiss [24].

annuity. Therefore, consumers can, and will, choose to fully offset the effects of social security.

The effects of pay-as-you-go social security under the joy-of-giving bequest motive differ quite dramatically from the effects under altruism. The difference is most clear in the case in which all consumers live for two periods with certainty ($p=0$). Recall that under altruism, when the pay-as-you-go social security system is introduced, old consumers simply bequeath the payment, S , to their children in order to compensate them for their increased tax of S/G per capita. However, under the joy-of-giving bequest motive, it would not be optimal for old consumers to maintain their consumption unchanged while increasing their bequests by S . The reason is that the utility from leaving a bequest depends only on the size of the bequest and not on the utility or consumption of the heirs. Thus, in response to receiving the payment S , old consumers would increase both their consumption and the bequest they leave. Essentially, consumption and bequests are both goods that enter the consumer's utility function, and, furthermore, these are the only arguments of the utility function. Consumption and bequests are each specified to be normal goods in (28) so that in response to an increase in income, the consumer optimally increases consumption of both of these goods.

The analysis in the paragraph above indicates that for the purpose of analyzing the Ricardian Equivalence Theorem, it is extremely important whether the bequest motive is of the altruistic or joy-of-giving variety. The reason is that under altruistic preferences the consumer cares about the entire stream of his or her family's consumption. Because the

consumer does not care about the size of bequests per se, he or she is indifferent among changes in bequest patterns that maintain the initial allocation of consumption. Thus, in response to certain lump-sum tax and transfer policies, the consumer maintains the original path of consumption simply by rearranging bequests. However, under the joy-of-giving bequest motive, the consumer cares directly about the level of bequests, and therefore does not find it optimal to rearrange bequests while keeping consumption unchanged.

Heterogeneous Mortality Probabilities

Up to this point it has been assumed that all consumers face identical mortality probabilities ex ante. However, if consumers have different probabilities of survival, then there are additional channels through which fiscal policy may operate. In addition, heterogeneity of ex ante mortality probabilities raises the possibility of adverse selection in the private annuity market, which has important consequences for the pricing of annuities and for the efficacy of fiscal policy.

The implications of heterogeneous ex ante mortality probabilities are clearest in the absence of a bequest motive so the discussion below will be confined to this case.¹⁵ In the absence of bequest motive, and in the presence of private annuities, all consumers will choose to hold their wealth entirely in annuities and hence there will be no bequests or inheritances. The major strategic decision in developing a model with heterogeneous mortality probabilities is whether to assume that an individual consumer's probability of dying, p , is known only by that

¹⁵ The effects of fiscal policy under heterogeneous mortality probabilities and a joy-of giving bequest motive are examined in Abel [2] and [3].

consumer or whether it is a publicly available bit of information. The discussion below begins with the assumption that each consumer's value of p is known by insurance companies. Next, the discussion will turn to the case in which the value of an individual's mortality probability is private information. These two cases are based on Abel [3] and [2], respectively.

Public Knowledge of Mortality Probabilities: Suppose that the ex ante mortality probability of each consumer is known to everyone, including insurers. Under this assumption, of course, competitive insurers will offer annuities with different rates of return to consumers with different values of p . Annuities will be priced to be actuarially fair to each consumer so that a consumer with a probability p_j of dying young can buy annuities that offer a rate of return

$$Q_j = R/(1-p_j) \quad (31)$$

It follows immediately from (31) that consumers with a high probability of dying young will be able to purchase annuities with a high rate of return. Equivalently, these consumers can buy a given contingent payoff in the second period more cheaply than could healthier consumers who have a lower value of p . However, the expected rate of return on annuities, $(1-p_j)Q_j$, is identical for all consumers and is equal to R .

Suppose that all consumers have logarithmic preferences as specified in (3). Let $c(p)$ denote the consumption of a young consumer whose probability of dying young is equal to p . It follows immediately from the consumption function in (21) that

$$c(p) = a(p) [Y - T + (1-p)S/R] \quad (32)$$

where $a(p) = 1/[1 + (1-p)D]$. Observe that the marginal propensity to consume, $a(p)$, is an increasing function of p . Thus, if $S = T = 0$, then consumption by young consumers would be an increasing function of p . The reason is that with logarithmic preferences and no second-period income, consumption when young is independent of the rate of return on savings. Thus, the fact that consumers with a higher value of p can obtain annuities with a higher rate of return is irrelevant for the consumption decision. However, because consumers with a high p have a small chance of enjoying consumption in the second period, they will consume more when they are young. This result, that $c(p)$ is an increasing function of p , holds also for positive S and T , provided that the values of these tax parameters are small.

Fully Funded Social Security: Now consider the effects of a fully funded social security system that ignores differences in ex ante mortality probabilities. The assumption is not that the government is unable to observe ex ante mortality probabilities that are observable by insurance companies; rather, the assumption is that, for some reason, the social security system does not discriminate according to mortality probabilities. Under a fully funded social security system, the benefits and taxes satisfy

$$(1-p^*)S = RT \quad (33)$$

where p^* is the average of the ex ante mortality probabilities of young consumers. Substituting the social security parameters from (33) into the consumption function (32) yields

$$c(p) = a(p) [Y + (p^*-p)T/(1-p^*)] \quad (34)$$

Observe from (34) that the consumption of young consumers with a higher than average probability of dying ($p > p^*$) is reduced by the introduction of social security; the consumption of young consumers with a lower than average probability of dying is increased by the introduction of social security. These effects on consumption reflect the fact that non-discriminatory social security redistributes income from consumers with a high value of p to consumers with a low value of p . The social security system forces consumers to hold an annuity with gross rate of return $S/T = R/(1-p^*)$. For consumers with $p > p^*$, this rate of return is less than the rate of return available on private annuities and thus these consumers are made poorer by the introduction of social security. By contrast, for consumers with $p < p^*$, the annuity provided by social security offers a higher rate of return than is otherwise available to them. Hence these consumers are made wealthier and increase their consumption.

The introduction of social security shifts resources, and hence consumption, away from consumers with a high value of p toward consumers with a low value of p . Because consumers with a high value of p initially had high consumption relative to consumers with a low value of p , this redistribution of resources reduces the cross-sectional variation in consumption. Note that the mechanism for reducing cross-sectional variation differs from the mechanism in the case without annuities and with accidental bequests. In that case, the source of cross-sectional variation was accidental bequests; by reducing savings, social security reduced accidental bequests and cross-sectional variation. In the present model, there are no bequests. The source of variation, in the

absence of social security, is the difference in marginal propensities to consume which results from different mortality probabilities.

In addition to reducing cross-sectional variation in consumption, the introduction of fully funded social security reduces the average level of consumption by young consumers. This reduction in average consumption arises because the social security system transfers resources from consumers with a high marginal propensity to consume (high p consumers) to those with a low marginal propensity to consume (low p consumers). This result can be derived formally by defining $E\{Z(p)\}$ as the population average value of some arbitrary function $Z(p)$.¹⁶ With this definition, the average consumption of young consumers is $E\{c(p)\}$ where

$$E\{c(p)\} = E\{a(p)\}Y + E\{(p^*-p)a(p)\}T/(1-p^*) \quad (35)$$

It can be shown formally that $E\{(p^*-p)a(p)\}$ is negative.

Intuitively, the reason is that $E\{(p^*-p)a(p)\}$ is equal to $E\{p^*-p\}E\{a(p)\} + \text{Cov}\{p^*-p, a(p)\}$. Because $E\{p^*-p\}$ is, by definition, equal to zero, it follows that $E\{(p^*-p)a(p)\}$ is equal to $\text{Cov}\{p^*-p, a(p)\}$. Since p^*-p is decreasing in p and $a(p)$ is increasing in p , this covariance is negative. Therefore,

$$E\{(p^*-p)a(p)\} < 0 \quad (36)$$

¹⁶ At a formal level, let $f(p)$ denote the density function of the ex ante mortality probability p . With this definition, the average ex ante probability p^* is equal to

$\int_0^1 pf(p)dp$. Now define $E\{Z(p)\} = \int_0^1 Z(p)f(p)dp$ so that $E\{Z(p)\}$ is the average value of $Z(p)$.

The inequality in (36) implies that the coefficient of T in (35) is negative. Therefore, as argued above, an increase in T leads to a reduction in average consumption by young consumers.

Private Information and Adverse Selection: Now suppose that there is heterogeneity of ex ante mortality probabilities, and that individual mortality probabilities are private information. More specifically, suppose that each individual consumer knows his or her own ex ante mortality probability, but that no one else knows that person's value of p . However, the distribution of ex ante mortality probabilities in the population is public knowledge. This information structure gives rise to a classic adverse selection problem. In the case of annuities, the high risk consumers from the viewpoint of insurance companies are those consumers with a low mortality probability p . These consumers will demand more annuities than the consumers with high mortality probabilities and they will be more likely to survive and receive annuity payments.

In general, the equilibrium in the presence of adverse selection is either a pooling equilibrium, in which consumers do not reveal their private information, or a separating equilibrium in which the optimal behavior of consumers reveals their private information. To simplify the determination of the market equilibrium, an additional assumption will be made. In particular, assume that an insurer cannot determine whether any given consumer has purchased annuities from other insurance companies. The force of this assumption is to rule out separating equilibria in which consumers with different mortality risks face different rates of return on annuities. If insurers tried to charge higher prices (i.e.,

offer lower rates of return) to consumers with low p , then these consumers would masquerade as high p consumers and would buy only a small amount of annuities from a given insurer. Then these consumers would satisfy their relatively large demand for annuities by purchasing additional annuities from one or more other insurers. Therefore, an insurer's attempt to separate its customers by offering different quantities of annuities at different prices would fail. Instead, the market would be characterized by one rate of return that is offered on all annuities. Because of adverse selection, this rate of return would have to be lower than $R/(1-p^*)$, which is the actuarially fair rate based on population average mortality.

In the absence of a bequest motive the demand for annuities by a consumer will be equal to the consumer's savings, which is equal to first-period income, $Y - T$, minus consumption in the first period. Let $A(p;Q)$ denote the amount of annuity demanded by a young consumer with a mortality probability p when the rate of return on annuities is Q . Using the consumption function in (21), it follows that

$$A(p;Q) = [1-a(p)] [Y - T] - a(p)S/Q \quad (37)$$

Equation (37) implies that in the absence of social security ($S = T = 0$), the demand for annuities is invariant to the rate of return they offer. This invariance is a consequence of the offsetting income and substitution effects associated with logarithmic utility. Now recall that $a'(p) > 0$ which implies that $1 - a(p)$ is a decreasing function of p . Thus, if $Y - T - S/Q > 0$, then the demand for annuities is a decreasing function of p .

The equilibrium rate of return on annuities is such that the expected profit of insurers is equal to zero. Insurers will, on average, earn positive profits on annuities sold to consumers with high values of p but will, on average, suffer losses on annuities sold to consumers with low values of p . More precisely, in the absence of social security, the expected profit on annuities sold to a consumer with mortality probability p is $[R - (1-p)Q]A(p;Q)$. Let π denote the expected profit of the annuity industry, averaging over all consumers, and observe that

$$\pi = E\{[R - (1-p)Q]A(p;Q)\} \quad (38)$$

This expression can be rewritten using the fact that $1-p = (1-p^*) + (p^*-p)$ to obtain

$$\pi = [R - (1-p^*)Q]E\{A(p;Q)\} - E\{(p^*-p)A(p;Q)\}Q \quad (39)$$

Observe that $E\{p^*-p\} = 0$ and recall that, in the absence of social security, $A(p;Q) = [1 - a(p)]Y$. Thus, in the absence of social security, the expected profit of the annuity industry can be rewritten as

$$\pi/Y = [R - (1-p^*)Q]E\{1-a(p)\} + E\{(p^*-p)a(p)\}Q \quad (40)$$

The two terms on the right hand side of (40) have a simple interpretation. The first term is the expected profit of the annuity industry that would prevail if all consumers purchased the same amount of annuities regardless of their ex ante mortalities probabilities. The second term, which is negative according to equation (36), represents the expected losses inflicted on insurance companies due to adverse selection. Observe that each of the two terms on the right hand side of (40) is a decreasing function of Q . Therefore, the expected profit of the annuity industry is strictly decreasing in the rate of return offered on annuities. In addition, note that if the rate of return on annuities

is actuarially fair based on the population average probability p^* , i.e., if $Q = R/(1-p^*)$, then the first term on the right hand side of (40) would be equal to zero. In this case, π/Y would equal $E\{(p^*-p)a(p)\}Q$ which is negative. Therefore, any rate of return on annuities greater than or equal to $R/(1-p^*)$ would lead to expected losses for the annuity industry and could not be an equilibrium rate of return.

The annuities' equilibrium rate of return must yield zero expected profits. In this case, with logarithmic utility, and in the absence of social security, this rate of return is unique. Setting the expected profit, π , equal to zero in (38) yields

$$Q = R E\{1-a(p)\} / E\{(1-p)(1-a(p))\} \quad (41)$$

Fully Funded Social Security: Now consider the effects of introducing fully funded social security. Although social security has the payoff characteristics of an annuity, there is an important difference between social security and the annuities available in the private market. Because the social security system forces all young consumers to "purchase" equal amounts of the annuity it provides, the social security system is immune to adverse selection. The rate of return implicit in social security, S/T , is equal to $R/(1-p^*)$ as in (33).

To calculate the effect of social security on the national capital stock, substitute the relation between the social security parameters S and T in (33) into the annuity demand function (37) to obtain

$$A(p;Q) = [1-a(p)]Y - T - [R - (1-p^*)Q][a(p)/(1-p^*)Q]T \quad (42)$$

Recall that as a consequence of adverse selection, $R - (1-p^*)Q > 0$. Therefore, equation (42) indicates that the introduction of fully funded social security reduces the demand for annuities by more than T ; hence,

the private capital stock falls by more than T . The demand for annuities would decrease by precisely T if Q were equal to $R/(1-p^*)$. However, because social security provides an annuity with a larger payoff than is available on private annuities, the introduction of social security expands the opportunity set of all consumers and hence induces all consumers to increase their consumption when young. This increase in consumption means that the private capital stock falls by more than T . Therefore the national capital stock, which is equal to the private capital stock plus T , also declines.¹⁷

In order to determine effects of social security on the equilibrium rate of return on private annuities, first calculate the change in the structure of the demand for private annuities. Differentiating the annuity demand function in (42) with respect to T , and evaluating the derivative at $S = T = 0$, yields

$$dA(p;Q)/dT|_{S=T=0} = - \{1 + [R - (1-p^*)Q][a(p)/(1-p^*)Q]\} \quad (43)$$

The term in curly brackets on the right hand side of (43) is an increasing function of p . Therefore, consumers with a high value of p reduce their demand for annuities by a greater amount than consumers with a low value of p . Since consumers with a high value of p began with a lower annuity demand than low p consumers, it is clear that the high p consumers reduce their demand for annuities by a larger fraction than do the low p consumers. Hence the share of annuities bought by high p consumers is reduced, which reduces the expected profits of the annuity

¹⁷ This reduction in the private capital stock depends on the absence of a bequest motive. If there is a sufficiently strong joy-of-giving bequest motive, then the national capital stock may increase in response to the introduction of fully funded social security. See Abel [2].

industry. In order to restore zero expected profits, it is necessary for the rate of return on annuities to decline.

In concluding the discussion of annuity markets, it is useful to examine the quantitative effect of adverse selection in private annuity markets. Friedman and Warshawsky [11] and Warshawsky [25] have analyzed annuity prices and the mortality experiences of annuity purchasers in the United States. They found that annuity purchasers tend to live longer than the average life expectancy shown for Americans in the U.S. Life Tables. To get a measure of how much longer annuity purchasers live, they calculate load factors. The gross load factor is defined as the ratio of the price paid for an annuity to the expected present value of the payments accruing to an annuity purchaser; the net load factor is equal to the gross load factor minus one. These load factors are calculated under two different assumptions about mortality: in one calculation, the mortality probabilities are taken from the U.S. Life Tables and in the other calculation, the mortality probabilities are calibrated to match the mortality experience of annuity purchasers. Not surprisingly, they found that the expected present value of payments using annuity purchasers' mortality is greater than the expected present value of payments using the U.S. Life Table. Therefore, the load factor based on annuity purchasers' mortality is less than the load factor based on the U.S. Life Table. Very roughly, the average net load factor based on the U.S. Life Table was around 30 cents on the dollar; the average net load factor based on the mortality of annuity purchasers was about 15 cents on the dollar. The difference in load factors, 15 cents on the

dollar, was attributed to adverse selection.¹⁸ However, the extent to which the difference in mortality probabilities was private information or public information could not be determined from these studies.

Although the load factors reported in these studies are substantial, they do not appear to be large enough to explain the widespread shunning of annuity markets by private consumers. Friedman and Warshawsky [11] attribute to a bequest motive at least part of consumers' reluctance to buy annuities, but unanswered questions remain. For example, to what extent do consumers hold bequeathable wealth rather than level-payment annuities as a precaution against the need to make very large medical expenditures? Perhaps this risk of catastrophic medical expenditure explains the fact that retired consumers decumulate their wealth much more slowly than predicted by the life-cycle model. Clearly more research into these risks and consumers' reactions to these risks is needed.

Summary: The discussion of the effects of fiscal policy in the presence of longevity risk has examined several different sets of assumptions about bequest motives, the type of fiscal policy and the availability and pricing of annuities. Rather than summarize all of these cases, a few of the major themes will be highlighted. The insight of the Ricardian Equivalence Theorem is that fiscal policy will affect private economic activity only if it changes the opportunities available

¹⁸ More precisely, Friedman and Warshawsky [11] examine data for the period 1968-1983 and find load factors of 32-48 cents on the dollar using the U.S. Life Tables; they find load factors of 18-33 cents on the dollar after allowing for adverse selection. Warshawsky [25] examines data from the period 1919-1984 and finds load factors of 10-29 cents on the dollar using the U.S. Life Tables. He attributes 8 to 16 cents on the dollar of these load factors to adverse selection.

to individuals. Such changes in opportunities could take the form of changes in relative prices or changes in the present value of resources.

In applying the insight of the Ricardian Equivalence Theorem to social security, it is useful to think of social security as an annuity because consumers pay something when they are young in exchange for something that they will receive only if they survive. Clearly, if there is no market in which consumers can buy annuities, then the introduction of social security changes consumers' opportunity sets by providing an annuity. Not only does the introduction of social security affect the saving decisions of consumers who receive no inheritances, it also reduces the inheritances of those people who do receive them. Alternatively, if there is a private market for annuities, then the introduction of social security will have an effect only if the annuity provided by social security offers different terms than those offered by privately traded annuities. If consumers have identical mortality probabilities and if the rate of return on private annuities is actuarially fair, then fully funded social security offers the same rate of return as private annuities and thus has no effect. This invariance of economic behavior to the introduction of fully funded social security holds true regardless of whether or not consumers have a bequest motive and regardless of the form of the bequest motive.

There are several reasons why the rate of return on social security may differ from the rate of return on private annuities. First, pay-as-you-go social security offers an expected rate of return equal to the population growth rate rather than the rate of return on capital. Thus, in general, pay-as-you-go social security offers a different rate of

return from the rate of return in competitive annuity markets. Second, if consumers have different mortality probabilities and if individual consumers possess private information about their own mortality probabilities, then the private annuity market will be subject to adverse selection which drives down the rate of return on annuities. In this case, fully funded social security would offer a higher rate of return than private annuities. Third, even if each individual consumer's mortality probability is publicly known, then social security will have an effect if the government decides not to discriminate on the basis of mortality probabilities. In this case, the government offers the same taxes and benefits to all consumers, but in the private market consumers face different prices for a given level of second-period benefits. Therefore, for at least some consumers, social security will offer a different rate of return than private annuities.

It might seem that if the rate of return on social security is different from the rate of return on privately available annuities, then the introduction of social security would have an effect on private saving decisions. This presumption is indeed true if consumers do not have bequest motives or if they have joy-of-giving bequest motives. However, if consumers have altruistic bequest motives, then it may be that social security has no effect even though it offers a rate of return that differs from the rate on any privately traded asset. For instance, in the absence of longevity risk, fully-funded social security would have no effect even though the rate of return on social security differs from the rate of return on capital. However, in the presence of longevity

risk, the Ricardian Equivalence Theorem could fail to hold even under altruism.

Income Risk

In the previous sections of this paper, the risks have been confined to uncertainty about the length of an individual's lifetime. The market for life insurance and annuities allows consumers to reduce the effects of these risks and, as discussed above, the functioning of these markets has important implications for the effects of fiscal policy. This section will ignore longevity risk and focus instead on the risk associated with an individual's future labor income. Future labor income is risky for two reasons. First, there is a chance that a consumer will not be able to work as a result of an accident or illness. Second, even if the consumer is able to work, future income will fluctuate as a result of fluctuations in productivity or in the demand for his or her services. Because disability insurance is available to reduce, or even eliminate, the first of these sources of income risk, the discussion will ignore this source of risk, focusing instead on the second source of income risk.

At first glance, it appears that there is no insurance available to reduce the risk associated with fluctuations in productivity or in demand. Although there is no active insurance market to reduce the riskiness of a future income stream, the income tax system provides a form of income insurance. If the income tax rate is constant, say at 27 percent, then the government essentially shares 27 percent of the risks associated with fluctuations in labor income. Not only does the income

tax provide risk reduction as would more conventional types of insurance, it is also subject to the problem of moral hazard, as discussions of the Laffer Curve have made clear. More precisely, while the income tax provides some insurance against fluctuations in labor income, it also provides a disincentive to work; the Laffer Curve is based on the possibility that a tax rate increase will reduce work effort to such an extent that income tax revenue would decline. In order to isolate the risk-reducing effects of the income tax, and to focus on precautionary saving, the analysis will be based on the assumption that labor supply is perfectly inelastic. Therefore, future labor income will be treated as an exogenous stochastic variable from the viewpoint of the individual consumer.

Consider a consumer who lives for two periods and receives exogenous income y_1 and y_2 in periods 1 and 2, respectively. The consumer pays total taxes t_1 and t_2 in periods 1 and 2, respectively. The determination of the consumer's tax bill will be discussed in more detail below. Let c_1 and c_2 be consumption in periods 1 and 2, respectively. The saving of a young consumer is $y_1 - t_1 - c_1$. For simplicity, suppose that the net rate of return on saving is equal to zero. In this case, the consumer's second-period consumption is equal to saving plus second-period income net of taxes, $y_2 - t_2$,

$$c_2 = y_1 - t_1 - c_1 + y_2 - t_2 \quad (44)$$

Suppose that the consumer's utility function is

$$u(c_1) + u(c_2) \quad (45)$$

Now consider a young consumer's saving decision. When making this decision, the consumer knows the values of y_1 and t_1 but knows only the

probability distributions of y_2 and t_2 . At the optimal level of consumption, the consumer is indifferent between consuming an additional unit in period 1 and increasing savings by one unit. If c_1 is increased by one unit, then the consumer's utility increases by $u'(c_1)$.

Alternatively, if the consumer saves an additional unit, then second-period consumption, c_2 , increases by one unit, which increases expected utility by $E(u'(c_2))$ where $E(\cdot)$ denotes the expectation conditional on first-period information. The optimal consumption decision is characterized by

$$u'(c_1) = E(u'(c_2)) \quad (46)$$

Certainty Equivalence

Suppose, for the moment, that the utility function $u(c)$ is quadratic: $u(c) = -c^2/2 + bc$, where the parameter b is positive. In this case, the marginal utility is linear in consumption

$$u'(c) = -c + b \quad (47)$$

Substituting the marginal utility function (47) into the first-order condition (46) yields

$$E(c_2) = c_1 \quad (48)$$

Equation (48), which displays Robert Hall's [15] famous random walk theory of consumption, indicates that the expectation of future consumption is equal to current consumption. Equivalently, consumption follows a random walk. The reason is that, with concave utility, consumers attempt to mitigate fluctuations in consumption over time. In response to an increase in income, a consumer increases both current consumption and planned future consumption. In the case with quadratic utility and equal rates of interest and time preference (both are zero in

this particular case), it turns out the increases in c_1 and the expected value of c_2 are exactly equal. Under a more general utility function, consumption does not follow a random walk exactly, but the marginal utility of consumption does follow a random walk as in (46).

The optimal level of c_1 under quadratic utility can be determined by substituting the budget constraint (44) into the first-order condition (48) to obtain

$$c_1 = (1/2)[y_1 - t_1 + E\{y_2 - t_2\}] \quad (49)$$

The consumption function in (49) displays the permanent income/life cycle theory of consumption. It states that consumption is a function of the expected present value of lifetime income, net of taxes. In this particular example, it is optimal to consume one half of expected lifetime income in the first period. The consumption function in (49) also illustrates the certainty equivalence principle. More generally, the certainty equivalence principle applies to optimization problems with a quadratic objective function and linear constraints with additive uncertainty. It states that optimal decision rules depend on the expected values of random variables, but do not depend on any other moments of the distributions of the random variables. In particular, the variance of the random variables is irrelevant and may as well be assumed to be zero. Equivalently, the optimal decision rule is identical to the rule that would prevail if all random variables were equal to their expected values with certainty.

Precautionary saving is defined as the additional saving induced by the introduction of uncertainty about future income. Because the consumption function in (49) is independent of the variance of future

income, it is not useful for examining precautionary saving. Although the quadratic utility function on which (49) is based displays risk aversion, optimal behavior does not display precautionary saving. An increase in the variance of $y_2 - t_2$ reduces the expected utility of the consumer; however, it does not change the consumer's behavior at all. If there were actuarially fair insurance against the risks associated with second-period income, the consumer would buy it. However, the consumer would choose the same level of c_1 regardless of whether or not such insurance is available. In terms of the specification of the utility function, risk aversion requires a positive second derivative, but precautionary saving requires a positive third derivative.¹⁹ The quadratic utility function, of course, has a positive second derivative but a zero third derivative.

Precautionary Saving

Now consider a utility function with a positive third derivative so that the optimal consumption function will display precautionary saving. For simplicity, suppose that the utility function is $u(c) = -\exp[-kc]$ where $k > 0$ is the coefficient of absolute risk aversion.²⁰ The marginal utility function is

$$u'(c) = k \exp[-kc] \quad (50)$$

Substituting the utility function from (50) into the first-order condition (46), and using the budget constraint (44) to eliminate c_2 , yields

¹⁹ For an excellent discussion of the relation between risk aversion and precautionary saving, see Kimball [18].

²⁰ Kimball and Mankiw [19] examine the precautionary saving of an infinite-horizon consumer with a constant absolute risk aversion utility function. They use their model to examine the interaction of tax policy and precautionary saving in a richer dynamic framework.

$$\exp[-kc_1] = E\{\exp[-k(y_1 - t_1 + y_2 - t_2 - c_1)]\} \quad (51)$$

To calculate the expectation on the right hand side of (51), the distribution of the random variable $y_2 - t_2$ must be specified. Suppose that $y_2 - t_2$ is normally distributed with mean $E\{y_2 - t_2\}$ and variance $\text{var}\{y_2 - t_2\}$. Under this distributional assumption, the expectation of $\exp[-k(y_2 - t_2)]$ is equal to $\exp[-kE\{y_2 - t_2\} + (1/2)k^2\text{var}\{y_2 - t_2\}]$ and equation (51) can be rearranged to yield

$$c_1 = [y_1 - t_1 + E\{y_2 - t_2\} - (1/2)k \text{var}\{y_2 - t_2\}]/2 \quad (52)$$

The consumption function in (52) displays precautionary saving. Consumption in the first period is a linear and decreasing function of the variance of second-period after-tax income. Therefore, saving is a linear and increasing function of the variance of future after-tax income.

Fiscal Policy

The simple consumption function in (52), which displays precautionary saving, can be used to examine the interaction of precautionary saving and various tax policies. In particular, this framework can be easily used to examine the impact of both lump-sum taxes and income taxes. Many of the results presented below were derived for a more general utility function by Louis Chan [6]. In this particular model, as in Chan's model, income is exogenous so that the incentive effects of taxes on labor effort will be ignored. By treating income as exogenous, this model focuses on the insurance aspects of income tax.

Suppose that the second-period tax consists of a head tax, t^* , plus a proportional income tax, at rate τ ($0 \leq \tau < 1$), so that

$$t_2 = t^* + \tau y_2 \quad (53)$$

and after-tax income is given by

$$y_2 - t_2 = (1 - \tau)y_2 - t^* \quad (54)$$

Now consider an increase in the income tax rate τ accompanied by a reduction in the head tax t^* that leaves the expected second-period tax payment, $E\{t_2\}$, unchanged. Because the expected tax payment is held fixed, this change in tax structure leaves $E\{y_2 - t_2\}$ unchanged.

However, the increase in τ reduces the variance of after-tax income, $\text{var}\{y_2 - t_2\}$, which is equal to $(1-\tau)^2\text{var}\{y_2\}$. It follows immediately from (52) that this reduction in the variance of after-tax income induces an increase in first-period consumption. Thus, when the income insurance associated with the income tax is increased, there is a decline in precautionary saving. Note, in addition, that this increase in the income tax rate, compensated by a decrease in the head tax, leads to an increase in expected utility.

The next step in the analysis of fiscal policy is to examine aggregate income and to specify the relation between individual income and aggregate income. Let Y_2 denote the level of aggregate income per capita in the second-period. Suppose that individual income, y_2 , is

$$y_2 = Y_2 + e \quad (55)$$

where e represents the idiosyncratic random component of income and $E\{e\} = E\{e Y_2\} = 0$. These assumptions imply that the idiosyncratic component, e , is uncorrelated with aggregate income Y_2 .

In examining various tax and transfer policies, one must make sure that the policy changes satisfy the government's budget constraint. For simplicity, suppose that all consumers pay the same tax, t_1 , in period 1 and that all consumers pay the same head tax, t^* , in period 2. Second-

period tax bills will differ across consumers to the extent that their second-period incomes differ. The government budget constraint states that total tax revenues over the two periods must equal total government spending over the two periods. Letting g denote the total value of government spending, the government's budget constraint is

$$t_1 + t^* + \tau Y_2 = g \quad (56)$$

The lifetime tax liability of an individual may be calculated by adding τe to both sides of (56), and using the fact that $y_2 = Y_2 + e$, to obtain

$$t_1 + t^* + \tau y_2 = g + \tau e \quad (57)$$

Observe that the left hand side of (57) is equal to $t_1 + t_2$. Therefore, equation (57), along with (55), can be used to rewrite the first-order condition (51) as

$$\begin{aligned} \exp[-kc_1] = \\ E\{\exp[-k(y_1 + Y_2 - g + (1-\tau)e - c_1)]\} \end{aligned} \quad (58)$$

To calculate the expectation on the right hand side of (58), the distributions of the random variables Y_2 and e must be specified. It has already been assumed that e has a mean equal to zero and that Y_2 and e are uncorrelated. In addition, assume that Y_2 and e are each normally distributed. Under this assumption, the expectation on the right hand side of (58) can be calculated. Simplifying this expression yields

$$\begin{aligned} c_1 = (1/2)[y_1 + E(Y_2) - g] \\ - (1/4)k[\text{Var}(Y_2) + \text{Var}((1-\tau)e)] \end{aligned} \quad (59)$$

The consumption function in (59) embodies both the optimization of the individual consumer as well as the government's budget constraint. It can be used to examine the effects of various fiscal policies. Note

that the government's budget constraint involves four policy variables: the first period tax t_1 , the second-period head tax t^* , the second-period income tax rate τ , and the total value of government expenditure, g . However, only two of these four variables, namely g and τ , enter the consumption function in (59). Thus, consumption in the first-period does not directly depend on the first-period tax, t_1 , nor on the second-period head tax t^* . This observation immediately suggests a policy change for which the Ricardian Equivalence Theorem applies. Consider a one dollar increase in the first-period tax, t_1 , accompanied by a decrease in the second-period head tax t^* . This change satisfies the government budget constraint. It is clear from (59), that since neither of these tax parameters enters the consumption function, this temporary tax increase has no effect on consumption.

Next, consider a tax change for which the Ricardian Equivalence Theorem does not apply. In particular, consider an increase in the first period tax t_1 that is accompanied by an appropriate decrease in the second-period income tax rate τ , as determined by the government budget constraint. In examining the effects of this tax policy, it is useful to focus on two special cases of the random processes for income. First, consider the case in which there is no uncertainty about future aggregate income Y_2 . In this case, which corresponds to the case considered by Barsky, Mankiw and Zeldes [5], the consumption function in (59) can be written as

$$c_1 = [y_1 + E(Y_2) - g - (1/2)k(1-\tau)^2 \text{Var}(e)]/2 \quad (60)$$

It follows immediately from (60) that the increase in t_1 and the accompanying decrease in τ will reduce first-period consumption, provided

that $\text{Var}\{e\} > 0$. The reason for this reduction in consumption is that the reduction in the future tax rate τ implies that the government will be sharing a smaller fraction of the idiosyncratic income risk. As a consequence, the consumer will face a greater income risk and thus will increase precautionary saving.

Alternatively, consider the case in which aggregate income Y_2 is uncertain but there is no idiosyncratic risk (i.e., $\text{var}\{e\} = 0$). In this case, the consumption function in (59) can be written as

$$c_1 = [y_1 + E(Y_2) - g - (1/2)k\text{Var}(Y_2)]/2 \quad (61)$$

In the absence of idiosyncratic income risk, none of the three tax parameters enters the consumption function in (61). Therefore, an increase in t_1 accompanied by an appropriate decrease in τ has no effect on consumption. Thus, the Ricardian Equivalence Theorem applies in this case. The reason is that, even though there is uncertainty about future aggregate income and about the tax rate on future income, there is no uncertainty about the future tax liability of any consumer. Each consumer pays an extra dollar in taxes in period 1. Therefore, the aggregate tax revenue in period 2 must be reduced by one dollar per capita. Because the idiosyncratic component of income has been assumed to be identically zero, each consumer knows with certainty that his or her second-period tax bill will be equal to the aggregate per capita tax bill. Since the aggregate per capita tax bill will fall by one dollar, each consumer knows with certainty that his or her future taxes will fall by one dollar, exactly offsetting the one dollar increase in period 1 taxes. Therefore, there is no change in the optimal level of first-period consumption.

Changes in the tax rate τ have dramatically different effects depending on whether the uncertainty associated with second period income is idiosyncratic or is common to all consumers. If the second period income risk is idiosyncratic, then an income tax allows consumers to share risks with each other. Therefore, a reduction in the income tax rate would reduce the extent of insurance and would lead to increased precautionary saving. By contrast, if there is no idiosyncratic component to second period income risk, then individual consumers cannot reduce their risks by sharing with other consumers. In this case, the income tax does not provide any insurance and the Ricardian Equivalence Theorem holds. Although aggregate risks cannot be shared across members of a generation, it is possible that aggregate risks could be shared across generations.

Intergenerational Risk Sharing

Virtually all of the risks discussed in previous sections are within-generational risks in the sense that different members of the same generation obtain different realizations of a random variable. Except for problems of adverse selection and moral hazard, these risks could be potentially shared among members of the same cohort. By contrast, this section will focus on risks that cannot be shared among members of the same cohort because all members of a given cohort face the same risk ex post as well as ex ante. To be more specific, this section will examine income shocks that strike all members of a cohort to exactly the same degree. If there is to be any risk sharing, it must be done by sharing risks among two or more generations.

Intergenerational risk-sharing has been studied by Roger Gordon and Hal Varian [14]; the discussion below draws heavily on their analysis and extends their model to allow for population growth. Consider an economy with overlapping generations of consumers, each of whom lives for two periods. Each generation is G times as large as the generation that preceded it. A consumer who is born in period t receives a perfectly storable deterministic endowment w in period t and receives a random endowment e_{t+1} in period $t+1$. Suppose that e_{t+1} has a mean of zero and is identically and independently distributed across generations. For simplicity, consumption is confined to the second period of life. Let c_{t+1} denote the consumption in period $t+1$ of a consumer born in period t . The realized value of the consumer's utility in period $t+1$ is $u(c_{t+1})$, where the utility function $u(\cdot)$ is strictly increasing and strictly concave.

All members of generation t face the same value of the random variable e_{t+1} . Therefore, there is no scope for within-generation risk-sharing. Also, because adjacent generations are simultaneously alive for only one period, there is no scope for private markets to share risks across members of adjacent generations. Therefore, if there is to be intergenerational risk sharing, then a long-lived institution, such as a government, must be involved.

Consider the following scheme to share risks. Suppose that the government levies a tax of τe_{t+1} (where $0 \leq \tau \leq 1$) on each old consumer in period $t+1$, and uses the proceeds of the tax to give a subsidy of $\tau e_{t+1}/G$ to each young consumer in period $t+1$. Of course, if e_{t+1} is negative, then old consumers receive an unlucky realization of income and

the tax levied on old consumers is negative. Thus, if e_{t+1} is negative, the tax system transfers resources from the young consumers to the old consumers. In the presence of this tax system, the consumption of an old consumer in period $t+1$ is

$$c_{t+1} = w + (\tau/G)e_t + (1-\tau)e_{t+1} \quad (62)$$

Thus, this tax scheme spreads the risk associated with the random endowment across two adjacent generations.

Optimal Risk Sharing

Now consider the value of the tax rate τ that maximizes the ex ante utility of a generation t consumer, $E\{u(c_{t+1})\}$, where the expectation is calculated at the beginning of the consumer's life, prior to observing the realization of the random variable e_t . This is the tax rate that a consumer would choose if he had no knowledge of the particular realizations of random income that would occur during his lifetime. The calculation of the optimal value of τ is simplified by using (62) to observe that the expectation of consumption, $E\{c_{t+1}\}$, is invariant to the tax rate τ (because $E\{c_{t+1}\} = w$). Thus, because the utility function $u(\cdot)$ is concave, the optimal tax rate is the rate that minimizes the unconditional variance of consumption. It follows immediately from (62) that the unconditional variance of consumption is

$$\text{Var}\{c_{t+1}\} = [(\tau/G)^2 + (1-\tau)^2] \text{Var}\{e\} \quad (63)$$

The optimal value of τ can be found by differentiating (63) with respect to τ and setting the derivative equal to zero to obtain²¹

$$\tau = 1/[1 + G^{-2}] \quad (64)$$

²¹ The risk sharing scheme is the optimal scheme within the class of schemes that share the risk associated with e_t across two adjacent generations. Gordon and Varian [14] show that sharing the risk across more generations leads to even higher ex ante utility.

Observe from (64) that in the absence of population growth, i.e., with $G = 1$, the optimal value of τ is equal to $1/2$. In this case, optimal risk spreading across pairs of adjacent generations involves each generation having a 50 percent stake in each of the two drawings of e that take place while that generation is alive. More generally, with $G > 1$, the optimal value of τ is greater than $1/2$. Substituting the optimal value of τ from (64) into the expression for consumption (62) yields

$$c_{t+1} = w + [G/(1+G^2)]e_t + [1/(1+G^2)]e_{t+1} \quad (65)$$

If G is greater than one, then the coefficients on e_t and e_{t+1} are each less than $1/2$. Thus, in the presence of population growth, it is possible for generation t to have less than a 50 percent stake in the risk associated with e_t and e_{t+1} . Each old consumer shifts more than half of the old-age income risk to the younger generation. However, because the younger generation has more people than the older generation, it can absorb this increased risk with an increase in risk per person that is smaller than the reduction in risk per old consumer.

Time Consistency

The optimal intergenerational fiscal insurance system presented above was derived under the assumption that this system will remain in force. However, it may turn out that some cohorts may not want to participate in the fiscal risk-sharing arrangement. When a generation is old and receives a positive value of e_{t+1} , this generation would like to sever its participation. This desire to pull out of a system will not be considered here. If these old consumers had, when they were young, voluntarily decided to participate in the risk-sharing arrangement, then

they will not be allowed to renege on their implied contract just because they received a positive value of e_{t+1} .

The interesting question concerning the viability of the fiscal insurance system is whether young consumers in period t will, after learning the value of e_t , choose to participate in this system. The reason they may choose not to participate is that if e_t is negative, they are required to make a net transfer to old consumers. If the value of the risk reduction provided by the fiscal insurance system is less than the value of the required transfer, then these young consumers will choose not to participate. However, if the value of the risk reduction exceeds the value of the required transfer, then young consumers would choose to participate in the fiscal insurance system.²²

Gordon and Varian [14] argue that young consumers in period t will refuse to participate in the fiscal insurance system whenever $e_t < 0$. Their model assumes a constant population size so, for the moment, suppose that $G = 1$. To keep the argument simple, suppose that e_t is a drawing from a finite set and let $-\underline{e} < 0$ be the minimum possible value of e . First, suppose that $e_t = -\underline{e}$. In this case, each young consumer is required to pay $\underline{e}/2$ in taxes. If in the following period, $e_{t+1} = -\underline{e}$, then each generation t consumer would receive a fiscal subsidy of $\underline{e}/2$ and thus would have consumption equal to $w - \underline{e}$, which is the same that

22 This analysis assumes that if a generation decides not to participate in the fiscal insurance system when it is young, then it is precluded from receiving payments from a fiscal insurance system when it is old. Otherwise, if young consumers believed that the choice of whether or not to participate in the fiscal system when young would have no effect on whether they would receive payments when they were old, then young consumers would never choose to pay the fiscal insurance tax.

consumption would have been without participating in the fiscal system. However, if in the following period, $e_{t+1} > -\underline{e}$, then the consumer will either receive a fiscal subsidy less than $\underline{e}/2$ or will pay a tax. Thus, the consumer will end up having paid more into the system than he or she got out of it. Therefore, the best that the young consumer can hope for is to break even by participating in the system, and, in general, the consumer will be worse off ex post. Clearly, such a consumer will choose not to participate in the system in this case.

The argument above establishes that if $e_t = -\underline{e}$, then the generation t consumers will not participate in the fiscal insurance system. Now suppose that there is some value $e^* \geq -\underline{e}$ such that the generation t consumer will choose to participate in the system if and only if $e > e^*$. It is now straightforward to demonstrate, by contradiction, that e^* cannot be negative. Suppose that $e_t = e^* < 0$. In this situation a young generation t consumer is required to pay $|e^*|/2$. But how much will the consumer receive in the following period? If $e_{t+1} < e^*$, then the generation $t+1$ consumers will not join the system, and hence the generation t consumer will receive nothing from the fiscal insurance system. If $e_{t+1} > e^*$, then the generation t consumer will either receive a subsidy smaller than $|e^*|/2$ or will pay a tax. In either of these situations, the consumer is worse off for having participated in the fiscal insurance system. Only if $e_{t+1} = e^*$ will the generation t consumer end up as well off under the fiscal insurance system as without it. Thus, as above, the consumer cannot possibly be made better off by joining the fiscal insurance system and will, in general, be made worse off. Therefore, he or she will not join. Thus, e^* cannot be negative.

Gordon and Varian discuss mechanisms that would avoid the repeal of the intergenerational fiscal insurance system. For example, if large enough costs are imposed on any young cohort that tries to repeal the fiscal insurance system, then no generation will ever repeal the system, and in equilibrium the costs will not have to be borne. For example, if the abandonment of the fiscal insurance system leads to economic or social upheaval, then the young generation may decide that the costs exceed any pecuniary gains from avoiding participation in the fiscal insurance system.

In addition to the mechanisms discussed by Gordon and Varian for sustaining a fiscal insurance system, there is the possibility that population growth can sustain the system. If all generations have the same number of consumers, then the size of the maximum transfer paid by a young consumer is equal to the size of the maximum transfer that this consumer could possibly receive when old. Therefore, as argued above, when a young consumer is required to make the maximum possible transfer, the consumer cannot possibly expect to benefit from participating in the fiscal insurance system. However, if each generation is G times as large as the preceding generation, then the largest transfer that can be received by an old consumer is G times as large as the largest transfer that a young consumer could be required to make. Thus, even if a young consumer had to pay the largest possible transfer, it is still possible that the consumer could receive an even larger transfer in the following period. Depending on the consumer's attitude toward risk, it may turn out that even faced with the largest required payment when young, the consumer would choose to participate in the fiscal insurance system.

To demonstrate that population growth may be able to sustain voluntary participation in the fiscal insurance system, it may be clearer to use a numerical example than an algebraic proof. Suppose that each generation is twice as large as the generation preceding it, i.e., $G = 2$. In this case, it follows from (64) that an optimal fiscal insurance system will set τ equal to $4/5$. Substituting $G = 2$ and $\tau = 4/5$ into the equation for consumption (62) yields

$$c_{t+1} = w + 0.4 e_t + 0.2 e_{t+1} \quad (66)$$

The value of consumption in (66) is based on the assumption that the fiscal insurance system remains intact. As an example, suppose that $w = 11$ and that there are only three possible values for e : -10 , 0 , and 10 . Assume that $e = -10$ and $e = 10$ are equally probable and let $q < 1/2$ denote this common probability. Therefore, the probability that $e = 0$ is $1-2q$. Thus, since consumption, c_{t+1} , depends on e_t and e_{t+1} , there are nine possible values of c_{t+1} which are displayed in Table 1. Each row corresponds to a value of e_t and each column corresponds to a value of e_{t+1} .

Consider a young consumer in period t and suppose that $e_t = -10$, so that this young consumer is faced with paying the largest possible transfer. To determine whether this consumer will choose to participate in the fiscal system, the values of consumption in the first row of Table 1 must be compared with the values of c_{t+1} if the consumer does not participate in the system. These values are shown in Table 2.

Comparing the values of consumption in the first row of Table 1 with the values of consumption in Table 2 reveals that neither row dominates the other in a stochastic dominance sense. By participating in

the fiscal insurance system, the consumer reduces the variance of consumption from $200q$ to $8q$ at the cost of reducing the expected value of consumption from 11 to 7. Whether a consumer views the reduction in risk as worth the price depends on his or her attitude toward risk and on the value of q . Clearly the more aversion the consumer has to risk, the more attractive is the fiscal insurance system.

Suppose that the utility function is $u(c) = -1000\exp[-kc]$ where k is the coefficient of absolute risk aversion. Assume that $q = 0.25$. If $k = 1$, then the expected utility if the consumer participates in the fiscal insurance system is -2.17 ; the expected utility if the consumer does not participate in the system is -91.97 . Thus, the system will be sustained by voluntary participation in this case.

.c. In the example above, all generations voluntarily choose to participate in the fiscal insurance. Because all generations obtain higher utility with the introduction of the fiscal insurance system, such a system is Pareto-improving. In this particular example, the net rate of return on savings is zero, so that using the notation from section II, $R = 1$. Therefore, in this example, $R < G$ so that Golden Rule considerations indicate that a Pareto-improvement could be achieved by increasing consumption and reducing saving. The determination of the optimal fiscal system, which might include both an element of intergenerational risk sharing and an element of lump-sum intergenerational transfers to stimulate consumption, remains an open question for research.

This paper has analyzed the effects of various fiscal policies in situations in which individual consumers face various sorts of risks. Methodologically, the research presented is neoclassical. Although the models employed are very much in the spirit of those embraced by the so-called new classical school of macroeconomics, the results differ quite dramatically from some of the most well-known new classical results. In particular, the Ricardian Equivalence Theorem, which essentially states that lump-sum tax policies have no effect, is an important result that pervades much of the new classical literature. The results reported often deviate importantly from the Ricardian Equivalence Theorem. These departures, and hence the effects of fiscal policy, depend importantly on the availability and the nature of insurance arrangements to protect individual consumers against various types of risk. It is perhaps ironic that the traditional Keynesian model, which emphasizes the effects of fiscal policy, has no place for insurance arrangements to interact with fiscal policy. It is in the neoclassical framework, which is based on the optimizing behavior of consumers facing risk, that the interaction of insurance and fiscal policy becomes apparent. Further research into the nature and evolution of insurance arrangements will help to extend understanding of the effects of fiscal policy.

Table 1

Values of c_{t+1} Under the Fiscal Insurance System

e_{t+1}	-10	0	10
$e_t \backslash$	<hr/>		
-10	5	7	9
<hr/>			
0	9	11	13
<hr/>			
10	13	15	17
<hr/>			

Table 2

Values of c_{t+1} in Absence of Fiscal Insurance

e_{t+1} :	-10	0	10
<hr/>			
c_{t+1} :	1	11	21

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