# DEVALUATIONS IN AN OVERLAPPING GENERATIONS ECONOMY

by

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## 1. Introduction

Despite acceptance of the flexible exchange rate system by some industrialized countries, a significant part of the world community continues to adhere to some sort of pegged exchange rate system. International Monetary Fund (1983) lists 91 countries which operate under a pegged exchange rate system, 37 of which are pegged to the U.S. dollar, 13 to the French franc, 5 to other currencies, 12 to the SDR, and 24 to "other composites." Conversely, only 8 countries are listed as "independently floating." The remaining countries, including the eight members of the European Monetary System (EMS), operate systems somewhere between pegged and flexible exchange rates, either allowing limited flexibility or adjusting the exchange rate regularly according to a set of indicators.

Deciding to maintain a pegged exchange rate does not, however, eliminate changes in the exchange rate and coincident economic effects. For example, IMF (1983) lists 30 "Changes in Exchange Rates of Currencies Pegged to Other Single Currencies or Currency Composites, 1982." All of these changes were devaluations, with the rate of devaluation ranging from 1% to 78%. According to Cooper (1971), 1982 was not an unusual year in that only 13 countries did not devalue their currencies at least once during the period 1947-70.[1]

With exchange rate realignments so frequent and economically important, it is surprising that so little is known about the realignment decision making process. This paper is an attempt to model that process with the decisions based on rational economic behavior, but with the political system also influencing the outcome.

International economists have long been interested in the impact that devaluations have on an open economy.[2] Originally, emphasis was placed on the elasticities of demand for imports and exports and the conditions on these elasticities which permitted devaluations to have a positive impact on the balance of trade. These analyses, however, depended on an assumption of sticky prices and did not consider the capital account of the balance of payments and the capital flows which might occur on account of a devaluation. Perhaps even more important for this paper, the analyses considered the size of the devaluation to be exogenous.

Following the elasticities approach came the absorption approach to the balance of payments. This approach shifted attention away from specific sectors of the economy, such as the import sector, and focused on the economy overall. Devaluations are seen as one means of eliminating trade deficits, but these must be accompanied with deflationary monetary and fiscal policies which help to reduce the absorption of aggregate output by domestic agents. As in the elasticities approach, this approach also ignores capital flows and considers the devaluation decision to be exogenous.

With the advent of the monetary approach to the balance of payments, capital flows became an important determinant in the theory of exchange rate determination. Dornbusch (1973) analyzed the effect of a devaluation from the monetary point of view, but as in the older elasticities approach continued to accept the amount of the devaluation as exogenously given.

Recent research on devaluations has been more concerned with the timing of devaluations. For example, Flood and Garber (1984) consider the case where agents can predict the collapse of a fixed rate regime and act accordingly. To do this they assume, however, that when the fixed rate regime collapses it reverts to a floating exchange rate, something not frequently observed. Blanco and Garber (1984), using a linear money demand function, consider a model with recurrent devaluations in order to obtain estimates of agent's beliefs about the probability that a devaluation will occur in subsequent periods. However, their estimation technique requires the use of the expected exchange rate after devaluation, which they assume is equal to the shadow floating rate plus some random noise. They make this assumption in order to avoid addressing the issue of how devaluations are chosen optimally.

This paper employs a variation of the overlapping generations economy of Samuelson (1958) to analyze the implications of a fixed exchange rate on an economy, and to investigate the process of choosing exchange rates in an optimal sense when agents behave rationally. It is taken as given that the economy is operating under a fixed exchange rate, ignoring any possibility that a flexible exchange rate might be better from a welfare point of view. Some casual empirical justification for making this assumption was offered in the early part of this section, while a theoretical justification is offered in Glen (1987). The paper also assumes that the authorities choose to resolve the balance of payments crisis through a devaluation rather than through a change in domestic monetary policy, the only other alternative to them. The main result is that with heterogeneous agents, optimal devaluations may exceed the level of devaluation needed to eliminate a reserve crisis.

The outline of the paper is as follows. Section 2 develops the model and investigates the characteristics of stationary equilibria. Section 3 uses the model to assess the probability at a point in time that a devaluation will occur in the following period. In section 4 the government is introduced as a democracy and an investigation is performed on the effect that this has on the choice of exchange rate when preferences are logarithmic. Section 5 extends the argument of section 4 for more general preferences and section 6 concludes.

## 2. The Model

This section develops a model of a small open economy which conducts all of its foreign trade with a single large foreign country. In this world only a single perishable internationally traded good exists. Due to the small and open nature of the economy, and to the assumption that only a single good exists, the law of one price can be invoked which dictates that

$$p_t = e_t p_t^*$$

where $p_t$ is the domestic price level, $p_t^*$ is the foreign price level and $e_t$ is the exchange rate expressed in terms of units of domestic currency per unit of foreign currency. For simplicity in what follows it is initially assumed that the foreign price level is constant and normalized to unity. Consequently, the exchange rate and domestic price level are synonymous and will be used interchangeably.

The domestic economy of this country consists of overlapping generations of agents, where an equal number of young agents are born each period and agents live for two periods. At birth each agent is endowed with an exogenously determined amount, $Y_t$, of the consumption good. As a general rule, the endowment process is assumed to be a first-order Markov process.

The domestic monetary, fiscal and exchange rate policies of the country are unilaterally determined by the country's government. For simplicity, assume that no fiscal policy exists. In addition to its ability to dictate domestic monetary and exchange rate policy, assume that the government is able to prohibit domestic agents from holding foreign currency and to prohibit domestic currency from leaving the country. As a result, all trading between domestic agents is done in domestic currency.[3] Similarly, all trade with the outside world is conducted in foreign currency. In order for any foreign trade to occur it is necessary for the government to have reserves of foreign currency, $R_t$, since domestic agents hold no foreign currency. A full explanation of the role played by foreign trade and how such trade is carried out is presented after the decisions of the domestic agents are described.

The preferences of young agents are assumed to be given by $U(C_t, C_{t+1})$, where $C_t$ is period t consumption and $U(\cdot,\cdot)$ is a strictly concave utility function. Assume that $U(\cdot,\cdot)$ satisfies Inada conditions so that consumption is positive in both periods. Given the endowment, $Y_t$, the young agent must decide how much to consume in period t, and how much to trade with old agents for domestic currency, which is the only means of saving for consumption in period t + 1. The young agent's decision is obtained by solving

(1) $$\max_{C_t} E_t U(C_t, C_{t+1})$$

$$\text{s.t.} \quad 0 \le C_t \le Y_t$$

where $E_t$ is the expectation operator conditional on time t information. Assume that the marginal utility of old age consumption is positive so that all savings are consumed during old age. Therefore,

(2) $$C_{t+1} = (Y_t - C_t)w_{t+1}e_t/e_{t+1} = M_t w_{t+1}/e_{t+1}$$

where $w_{t+1}$ represents the gross rate of government transfer payments in period t+1 and $M_t$ is the per capita domestic money supply. During their old age agents consume the real value of their savings which represent the total domestic money supply in the previous period, plus any government transfer payments received at the beginning of the current period.

The preferences of old agents are assumed to be given by $U(C_t^0)$, where $U_c(\cdot) > 0$ and $C_t^0$ is given by the appropriate version of equation (2).

Now that the agents' consumption decisions have been described, consider issues related to the timing of domestic and foreign trade. First, the endowment shock is realized and young agents make their consumption decision as described by (1). The consumption decision of old agents is given by the appropriate version of (2). Added together, $C_t + C_t^0$ represents the aggregate demand for the consumption good. The domestic supply of the good is given by $Y_t$. Any difference between supply and demand must be made up through either price changes, which affect demand, or imports/exports, which affect available supply. In this model it is the government's decision as to which response is appropriate.

In aggregate, the country faces the budget constraint

$$(3) \qquad C_t + C_t^0 \leq Y_t + I_t - X_t$$

where $I_t$ and $X_t$ represent imports and exports respectively. If excess supply (demand) exists, the government can lower (raise) the price level/exchange rate, which from (2) increases (decreases) $C_t^0$ eliminating the surplus. Alternatively, the government purchases the excess supply by printing domestic currency. This is then exported in exchange for foreign currency which is then held in reserve for future use. In the case of excess demand, the government uses its reserves of foreign currency to import goods that are exchanged with agents for domestic currency. In effect, the government is acting as the sole possessor of a storage technology that it makes available to its citizens through the issue of domestic currency.

The role of the government in the economy under investigation is to establish the exchange rate, domestic money supply and the level of transfer payments. The goal of this section is to determine if there exist rules for the government that are time invariant, depend only on the current state of the economy, and bring the economy into equilibrium given that agents form their expectations rationally. Formally, the question is whether there exist rules

(4) $$e_t = e(S_t)$$

(5) $$M_t = M(S_t)$$

(6) $$W_t = W(S_t)$$

where $S_t$ is the state of the economy and where $e(\cdot)$, $M(\cdot)$ and $(\cdot)$ are time invariant[4] such that

(7) $$C_t = \underset{z}{\operatorname{argmax}} E_t\left[U(z, (Y_t - z))w(S_{t+1})e(S_t)/e(S_{t+1})\right]$$

(8) $$C_t^0 = w(S_t)M(S_{t-1})/e(S_t)$$

(3) $$C_t + C_t^0 \leq Y_t + I_t - X_t$$

and

$$R_t \geq \underline{R}, \quad I_t \geq 0, \quad X_t \geq 0, \quad C_t \geq 0, \quad C_t^0 \geq 0 \text{ ,}$$

where $R_t$ is the level of foreign currency reserves held by the authorities and $\underline{R}$ is described below.

The state of the economy, $S_t$, in this case consists of the random variable $Y_t$ and the macrovariables determined in the preceding period, $M_{t-1}$, $e_{t-1}$, $w_{t-1}$ and $R_{t-1}$. A more general specification with stochastic population growth would add the number of young and old agents to the state. Further enrichment of the model to include government consumption (fiscal policy) could also add to the number of state variables in the system.

Given equations (4)-(6), equation (7) states that young agents choose their consumption optimally. Equation (8) provides a similar condition for the old agents. With aggregate consumption defined by the sum of (7) and (8), it must be that the country's budget constraint, (3), is satisfied. The nonnegativity restrictions are straightforward, with the exception of $R_t > \underline{R}$. At any point in time the government holds a finite amount of foreign currency, with the implication that it can only finance a finite amount of foreign trade. Due to considerations not investigated here, it may be unwilling to use all of its reserves to defend the exchange rate, in which case $\underline{R}>0$. Alternatively, the government may have access to foreign currency loans that it is willing to use in defending the exchange rate so that $\underline{R}<0$. In either case, it cannot use more reserves than it has available, this lower bound being represented by $\underline{R}$. The equilibrium given by equations (3) - (8) is assumed to be a rational expectations equilibrium in that the distribution of the state variables used in forming the expectations in (7), as well as the rules (4) - (6) are correctly known by agents.

Note above that, due to the general specification of the young agent's problem, the effect of changes in today's exchange rate on $C_t$ is indeterminant. The reason for this indeterminacy is that a change in today's rate may alter the ratio between today's and tomorrow's rate which, through both income and substitution effects, could alter the optimal consumption plan. In order to avoid this indeterminacy, at this point a specific form for agents' preferences is assumed and it is shown that there exist rules that satisfy equations (3)-(8) and the nonnegativity constraints. Specifically, assume that

$$U(C_t, C_{t+1}) = \log C_t + \beta E_t \log C_{t+1} \qquad 0 < \beta < 1$$

$$U(C_t^0) = \log C_t^0$$

and $\underline{R} = 0$. Rewriting the young agent's problem gives

(9) $$\max_{C_t} \log C_t + \beta E_t \log_{t+1}(Y_t - C_t)\frac{e_t}{e_{t+1}}$$

where equation (2) is used to replace Ct+1. The first order condition for (9) is

(10) $$C_t = \frac{Y_t}{1 + \beta} .$$

Consumption by the old is therefore

(11) $$C_t^0 = \gamma Y_{t-1} w_t e_{t-1}/e_t = M_{t-1} w_t/e_t$$

where $\gamma = \beta/(1+\beta)$. In this model where all imports/exports are financed by foreign reserves it must be that

(12) $$I_t - X_t = R_{t-1} - R_t ,$$

that is, net imports equal the change in foreign reserves. Using (10), (11) and (12) in (3) gives

(3') $$R_t = R_{t-1} + \gamma(Y_t - w_t Y_{t-1} e_{t-1}/e_t) .$$

From (3') it is clear that any change in the exchange rate will be directly reflected in the level of foreign reserves when transfer payments are not adjusted to offset this effect.

Similarly, any change in the level of foreign reserves must result in a corresponding change in the domestic money supply in this model when no transfer payment adjustments are made. To see this, consider an excess demand for the good at the established price level. Assuming the government has adequate reserves, it buys foreign goods with its reserves and sells them to the agents for domestic currency until the excess demand is satisfied. In the process of buying goods it has reduced its stockpile of foreign reserves. At the same time, it has reduced the amount of domestic currency held by domestic agents by the same amount in real terms. Excess supply would be handled in the reverse manner. Algebraically, this can be obtained from the budget constraint by substituting the agents' optimal consumption levels and rearranging to get

(13) $$M_t = w_t M_{t-1} + e_t(R_t - R_{t-1}) \ .$$

From equation (13) it is seen that the government's choices of exchange rate, domestic money supply and transfer payment are not independent. Given two of the three, the model imposes restrictions on what value the third takes when restrictions on reserves are binding. Bearing this in mind, the next step is to verify the existence of stationary equilibria by constructing an example of decision rules for $e_t$, $M_t$ and $w_t$ that satisfy the conditions of (3) - (8), as well as the nonnegativity constraints.

Constructing a stationary equilibrium for this economy where the exchange rate floats is easily done. The main concern of this paper, however, is with devaluations. Construction of a stationary equilibrium in which periodic devaluations are required is therefore our next task. Consider the following rules.

(14) $$e_t = e_{t-1}$$

(15) $$M_t = \gamma Y_t e_t$$

(16) $$w_t \sim G(\cdot)$$

where $w_t$ is assumed to be an exogenous random variable contained in $S_t$ with distribution function $G(\cdot)$. This represents the case in which the government attempts to maintain a fixed exchange rate by varying the level of domestic credit and by making transfers of domestic currency to the current generation of old agents, these transfers given by $w_t M_{t-1}$. When $\underline{R} = 0$, the budget and nonnegativity constraints are satisfied whenever

(17)
$$R_{t-1} \geq \gamma(w_t Y_{t-1} - Y_t) .$$

Violations of (17) imply an infeasible policy and require at least one of the policy functions to be altered.

When (17) does not hold, the government has inadequate reserves to support the current exchange rate. In this paper the authorities are assumed to devalue the currency in order to bring about an equilibrium. This ignores the possibility of altering the transfer payment, however, this may not be unrealistic if the government has committed to a transfer payment program before either the cost of the program or the level of the current endowment is known. A logical question, of course, is how much it should devalue. Ideally, the new exchange rate should be chosen optimally, an issue which is discussed in a later section. For now, assume that the government devalues by the least amount possible. The minimal devaluation will set $R_t = \underline{R} = 0$. The rule that obtains in this case is

(14')
$$e_t = \begin{cases} e_{t-1} & \text{if } R_{t-1} < \gamma(w_t Y_{t-1} - Y_t) \\ \dfrac{w_t M_{t-1}}{\gamma Y_t + R_{t-1}} & \text{otherwise.} \end{cases}$$

This rule has the feature that devaluations are larger for smaller endowment shocks, lower reserve holdings, and larger transfer payments. When consumption by all agents is chosen optimally, it is straightforward to show that this rule satisfies the conditions for an equilibrium.

In the sections that follow it will be assumed that the economy under consideration is adhering to a pegged exchange rate system similar to the one described by (14'). While this is a somewhat arbitrary rule, Glen (1987) shows that, for the case where agents have logarithmic preferences, there is no welfare lost by going from an economy with no domestic currency, i.e. uses only foreign currency, to an economy operating under either a fixed or flexible exchange rate. In fact, welfare of a representative young agent is equivalent under the fixed and flexible exchange rate systems and, in both cases, there is a one-time gain for the economy that occurs due to the introduction of domestic currency. In essence, this gain is seignorage, which the authorities can redistribute to agents if they desire.

This section developed a model of a small country operating under a fixed exchange rate with currency controls. One implication of the model is that, under the rules (14') - (16), it allows assessment of the probability at a point in time that a devaluation will be required in the next period. It is this matter which is investigated in the next section.

## 3. The Probability of a Devaluation Under An Adjustable Peg

One implication of the adjustable peg policy given by equations (14')-(16) is that there always exists the possibility that a devaluation will occur next period.[5] The probability of a devaluation next period is determined by the current state and the distributions that $Y_{t+1}$ and $w_{t+1}$ follow. Specifically, a devaluation will occur only if

$$C_{t+1} + C^{0}_{t+1} > Y_{t+1} + R_t$$

where consumption is determined using $e_t$, and $R_{t+1} = \underline{R} = 0$. Substituting for $C_{t+1}$ and $C^{0}_{t+1}$ demonstrates that the probability of a devaluation at time t+1 given time t information is

$$\Pr\left[\gamma Y_{t+1} - \frac{w_{t+1} M_t}{e_t} < -R_t\right] .$$

The cumulative distribution function of the random variable under consideration is evaluated at minus the level of foreign reserves.

This section concludes by examining the effect of changes in $Y_t$ and $e_t$ on $R_t$ and, therefore, on the probability of a devaluation. From the budget constraint, (3'), the change in reserves due to a change in endowment is given by

$$\frac{\partial R_t}{\partial Y_t} = \gamma\left[1 + w_t Y_{t-1} \frac{e_{t-1}}{e_t^2} \frac{\partial e_t}{\partial Y_t}\right] .$$

There are two cases that need to be considered, each corresponding to one of the two cases given in (14'). If reserves are adequate to maintain the exchange rate then $\partial e_t/\partial Y_t = 0$ and $\partial R_t/\partial Y_t > 0$. The intuition here is that when no change in the exchange rate occurs today, increases in the endowment are translated directly into increases in reserves. The result is that there is a decrease in the probability of a devaluation.

In the case of a devaluation occurring today, (14') implies that

$$\frac{\partial e_t}{\partial Y_t} = \frac{-\gamma w_t M_{t-1}}{(\gamma Y_t + R_{t-1})^2} < 0 .$$

Using this in the expression and rearranging gives $\partial R_t/\partial Y_t = 0$, as it should since, by (14'), devaluations are always minimal and set $R_t = \underline{R}$. Thus, if the government is following (14'), a larger endowment shock results in a lower probability of devaluation tomorrow only when no devaluation occurs today.

Evaluating the effect of changes in $e_t$ on $R_t$ is somewhat easier and will prove useful in the next section. Using the budget constraint, this is

$$\frac{\partial R_t}{\partial e_t} = -\frac{\partial C_t^0}{\partial e_t} = \gamma w_t Y_{t-1} \frac{e_{t-1}}{e_t^2} > 0 .$$

Therefore, increases in the exchange rate reduce the probability of a devaluation next period by increasing the reserve level.

This section has shown that the model produces a reasonable theoretical value for the probability of a devaluation next period. This is used in the next section which addresses the question of how the government chooses the value at which to peg the exchange rate when a devaluation is called for.

## 4. Optimal Exchange Rate Fixing

This section will use the small country model developed above to address the question of how a government chooses the exchange rate under a fixed exchange rate regime. As noted previously, it is assumed that the economy operates under a fixed exchange rate until forces dictate that the currency be devalued. In making this assumption, the question of why countries opt to follow a fixed exchange rate system is ignored, although as discussed above no welfare is lost in doing so. This assumption also eliminates any possibility that the currency will ever be revalued, an event which has certainly occurred for some countries. Including revaluations in the model requires that the nonnegativity constraint on reserves be replaced by the constraint that $\underline{R} < R_t < \overline{R}$, where $\overline{R}$ represents the upper bound on reserves that the authorities are willing to tolerate before the currency is revalued. How $\overline{R}$, as well as $\underline{R}$, is chosen is a candidate for further research. In this model it is assumed that $\overline{R} = \infty$.

The approach taken in this section is to allow the government to choose the exchange rate that solves

$$(18) \qquad \max_{e_t} \; \alpha_t U(C_t, C_{t+1}) + (1 - \alpha_t) U(C_t^0)$$

$$\text{s.t.} \quad 0 \le \alpha_t \le 1 \quad \text{and} \quad R_t \ge \underline{R}$$

Exactly how $\alpha_t$ is determined will depend on the nature of the government. In a democracy with majority rule it might be that $\alpha_t \in \{0, 1\}$. Alternatively, proportional rule might set $\alpha_t$ equal to the proportion of young agents in the economy each period. These represent only two plausible choices for the weighting scheme that the government uses.

By choosing the exchange rate to solve (18), the authority is acting without regard to the effect of the rate of devaluation on future generations. Since agents have no bequest motive in this model, this approach seems reasonable and represents a positive theory of exchange rate devaluation. A more normative and traditional approach might require a social planner to maximize a weighted sum of the utility of all future generations. It is not clear that the result of such a maximization would differ qualitatively from the result described below.

When agents have logarithmic preferences, the solution to (18) is given by the first-order condition

$$E_t\left[\frac{e_t}{e_{t+1}} \frac{\partial e_{t+1}}{\partial e_t}\right] = \frac{\alpha - 1}{\alpha} .$$

While this condition may not at first appear enlightening, a look at the optimization problem when $\alpha = 0$ and $\alpha = 1$ puts things into perspective.

Consider first the problem from the perspective of the old agent. With $C_t^0$ given by (11) and $U_c(\cdot) > 0$, the old agent's utility is decreasing in $e_t$. Consequently, for them the optimal choice of $e_t$ is to select it so that the constraint on reserves is binding. From the budget constraint and the discussion in section two, this is achieved when

$$e_t = \frac{w_t M_{t-1}}{\gamma Y_t + R_{t-1} - \underline{R}} . \tag{19}$$

Intuitively, the reasoning behind the old agent's decision is obvious. Any increase in the exchange rate is directly translated through the law of one price into domestic inflation, which inflates away the value of their savings. In order to maintain as much purchasing power as possible, old agents choose the minimal devaluation possible, which sets reserves at their lower bound.

The young agent's problem is not so immediately obvious. Holding only their endowment of real goods, the devaluation has no impact on current consumption or the current value of savings carried into old age. By affecting the end-of-period reserve holdings, however, changes in the exchange rate do affect the probability of a devaluation next period, which would erode the purchasing power of the young's savings. The young wish to eliminate this source of uncertainty and, therefore, choose the devaluation which best does this. More specifically, the authority's view of the young agent's problem is given by

$$(20) \qquad \max_{e_t} \log C_t + \beta \log(Y_t - C_t) + \beta(\log e_t - E_t \log e_{t+1} + \log w_{t+1})$$

$$\text{s.t.} \quad R_t \geq \underline{R} \; .$$

In the next section it will be shown that in the economy under consideration it is true that

$$\frac{\partial E_t[U(C_t, C_{t+1})]}{\partial e_t} \geq 0 \; .$$

With logarithmic preferences, the young agent's utility is nondecreasing in the current exchange rate. The intuitive reasoning here is that as the exchange rate is increased the probability of a devaluation is diminished (as is the expected value of tomorrow's exchange rate given that a devaluation occurs), thereby reducing the uncertainty regarding the purchasing power of any savings and increasing utility. Exactly how large a devaluation the young desire is not indicated, but the following argument clarifies this issue.

From (20), when the transfer policy is independent of the exchange rate policy, the only terms affected by changes in today's exchange rate are

$$\log e_t - E_t \log e_{t+1} .$$

In section three it was learned that the probability of a devaluation, $x_t$, is given by $x_t = F(-R_t)$, where $F(\cdot)$ is the cumulative distribution function for the random variables in the model. If it is assumed that these exogenous random variables are collectively distributed over some finite interval $(a_t, b_t)$, then $x_t$ is zero whenever

(21) $$-R_t \leq a_t .$$

Using the definition of $E_t \log e_{t+1}$, when $x_t$ is zero the value function in equation (20) becomes

$$\log C_t + \beta E_t \left[ \log(Y_t - C_t) w_{t+1} \right]$$

which is unaffected by changes in et. The optimal exchange rate for young agents is thus to devalue until the probability of a devaluation next period is zero. To find the minimal exchange rate that accomplishes this it is seen from (21) that it must satisfy $a_t = -R_t$.

From the budget constraint

$$e_t = \frac{w_t M_{t-1}}{\gamma Y_t + R_{t-1} + a_t}$$

will set the reserve level such that the probability of a devaluation next period is zero. It is assumed that $-a_t > \underline{R}$, so that this exchange rate exceeds the exchange rate given by equation (19).

Taking into consideration the weight assigned to each of the generations, the government chooses the devalued exchange rate to be

$$\xi_t \frac{w_t M_{t-1}}{\gamma Y_t + R_{t-1} + a_t} + (1 - \xi_t) \frac{w_t M_{t-1}}{\gamma Y_t + R_{t-1} - \underline{R}} \tag{23}$$

where $$\xi_t = \xi\big(\alpha_t, U(\cdot,\cdot)\big) \qquad 0 \leq \xi_t \leq 1 .$$

For strictly positive values of $\xi_t$, the exchange rate given by (23) exceeds the exchange rate given by equation (19) and minimal devaluations are not chosen.

The switch from $\alpha_t$ to $\xi_t$ requires some explanation. The solution to (18) is derived from the first order necessary condition

$$\alpha_t \frac{\partial E_t\big[U(\cdot,\cdot)\big]}{\partial e_t} + (1 - \alpha_t) \frac{\partial U(C_t^0)}{\partial e_t} = 0 .$$

For $\alpha_t$ equal zero the solution is given by (19). When $\alpha_t$ is one the solution is given by (22). Because $U(\cdot)$ is concave and changes in $e_t$ affect both $E_t C_{t+1}$ and $C_t^0$, the optimal choice for $e_t$ may not be the convex combination of (20) and (22) where the coefficients used to combine the two rates are $\alpha_t$ and $1 - \alpha_t$. Instead, when $U(\cdot)$ and $U(\cdot,\cdot)$ are well behaved there will be some well-behaved function, $\xi_t$, which maps $\alpha_t$ and agents' preferences into the unit interval.

This section used the model developed earlier to investigate the problem of how a government chooses a rate of devaluation optimally. The problem was addressed by noting the different effects a devaluation has on the two living generations, allowing each generation to choose its optimal exchange rate, and then having the government choose some weighted average of the two. The next section generalizes this argument to other preferences.

## 5. Extension of Optimal Devaluations to General Preferences

In this section the authorities are assumed to act in the same manner as before, but the assumption of logarithmic utility is relaxed. With this more general setting it is not necessarily true that the authorities will choose a rate of devaluation in excess of the minimal rate described above. The goal of this section then is to determine the conditions on the preferences of agents that result in the optimal rate of devaluation exceeding the minimal rate of devaluation.

Assume that the authority chooses the size of the devaluation to solve

$$\Gamma(\cdot) = \max_{e_t} \alpha_t g(e_t) + (1 - \alpha_t) f(e_t) \tag{24}$$

$$\text{s.t.} \quad \underline{e} \le e_t$$

where $f(e_t) = U^0(C_t^0) = U^0[(Y_{t-1} - C_{t-1}) w_t (e_{t-1}/e_t)]$, $f'(e_t) < 0$

$$g(e_t) = E_t[U(C_t, (Y_t - C_t) w_{t+1}(e_t/e_{t+1}))]$$

and $\underline{e}$ sets $R_t = \underline{R}$. Notice that by stating the problem in this manner it is implicitly assumed that a devaluation of the currency will in fact alleviate any reserve shortage. Mathematically, the assumption is that $\partial R_t/\partial e_t > 0$, which is an assumption that is maintained throughout this section. It will be shown below that this is a sufficient condition to guarantee that if devaluations are chosen optimally as defined by (24), then the resulting choice will exceed the minimal rate of devaluation $\underline{e}$.

The necessary condition for a solution to (24) is that

$$\alpha_t g'(e_t) + (1 - \alpha_t)f'(e_t) + \lambda = 0$$

where $\lambda \geq 0$ is the multiplier associated with the single constraint. It is easy to see that for any $e_t$ that satisfies the constraint with strict inequality, the necessary condition is satisfied whenever

(25) $$\alpha_t = -f'(e_t)/\left[g'(e_t) - f'(e_t)\right] .$$

Thus, any value of the exchange rate that satisfies the constraint can possibly solve (24) if the weight assigned to the old agents satisfies (25). Of course, $\alpha_t$ must also satisfy the constraint that $\alpha_t \in [0, 1]$, so that without more information on the sign of $g'(e_t)$, solutions to (24) may be corner solutions and minimal rates of devaluation are always chosen by the authority. By inspection, however, it is easy to see that since $f'(e_t) < 0$, $\alpha_t \in [0, 1]$ whenever $g'(e_t) \geq 0$. Therefore, solutions to (24) will exceed $\underline{e}$ whenever $\alpha_t \in [0, 1]$ and $g'(e_t) \geq 0$. The task in this section is therefore to determine sufficient conditions to guarantee that $g'(e_t) \geq 0$.

Before proceeding to a discussion of $g'(e_t)$, define the authority's exchange rate policy to be given by the function

$$e_{t+1} = \begin{cases} e_t & \text{if } R(e_t) \geq \underline{R} \\ \hat{e} & \text{otherwise} \end{cases}$$

where $$\hat{e} = \text{argmax}\{\alpha E_t[U(C_{t+1}, C_{t+2})] + (1 - \alpha)U(C^0_{t+1})\} ,$$

where $e_t$ enters the argument in both terms on the right hand side by the definitions of C and $C^0$ given in section 2, $R(e_t)$ is the reserve level resulting from exchange rate et and the time subscript on $\alpha$ has been dropped for simplicity. Under this policy, the authorities maintain a fixed exchange rate as long as reserves allow and devaluations are chosen optimally in the sense that they maximize the utility of the current living generations, where the weight $\alpha$ is given exogenously.

Now define

$$\bar{e} = \text{argmax}\{E_{t+1}[U(C_{t+1}, C^{0}_{t+2})]\} ,$$

where the maximizer is selected subject to the constraint that $e_t \geq \underline{e}$. Under the assumption that $\underline{e}$ sets $R_t = \underline{R}$, $\underline{e}$ is the maximizer for the old agent's utility because it represents the minimal rate of devaluation. Thus, when $\alpha$ is zero $\hat{e}$ will equal $\underline{e}$, and when $\alpha$ is one $\hat{e}$ will equal $\bar{e}$. Assuming for now that $\bar{e} > \underline{e}$, then it must be that

$$\hat{e} = \xi\underline{e} + (1 - \xi)\bar{e}$$

for some $\xi \in [0, 1]$, where the value of $\xi$ depends on the value of $\alpha$.

Under the assumption that $\partial R_t/\partial e_t > 0$, it is also the case that $\partial\underline{e}/\partial R_{t-1} < 0$ since a lower rate of devaluation is needed to set $R_t = \underline{R}$.

It now remains to show that $\bar{e} > \underline{e}$, i.e. $R(\bar{e}) = \bar{R} > \underline{R}$, so that $\hat{e} > \underline{e}$. To do this it is necessary to show that the young agent's expected utility is increasing in the rate of devaluation or, in the terminology of the earlier part of this section, that $g'(e_t) > 0$. An intuitive argument for this goes as follows. Suppose the authority chooses its exchange rate policy this period so that the current rate of devaluation sets $e_t = \hat{e}_t$ with resulting reserve level $R_t = R(\hat{e}_t) > \underline{R}$. Is the young agent better off with this rate of devaluation than if $e_t = \underline{e}$ and $R_t = \underline{R}$? The answer is yes for two reasons. First, with a higher level of reserves, the probability of a devaluation next period is reduced and savings are more secure. Second, for any exchange rate policy pursued next period under which the currency is devalued so that $e_{t+1} = \hat{e}_{t+1}$ and $R_{t+1} = \hat{R}_{t+1}$, the rate of devaluation necessary to achieve this level of reserves, $\hat{R}_{t+1}$, is lower for $R_t = \hat{R}_t$ then when reserves are at their lower bound $\underline{R}$. This is so because by assumption $\partial R_t/\partial e_t > 0$.

Increasing the current value of the exchange rate will impact a young agent's utility only if it effects the distribution of $e_t/e_{t+1}$. Increasing the exchange rate will effect the distribution of this ratio in two ways. First, since by assumption $\partial R_t/\partial e_t > 0$, increasing the exchange rate decreases the probability of a devaluation next period, which increases the value of savings. The second effect is on the value of $\hat{e}$ next period. The next task is to identify the impact of this second effect.

Assume the authority solves (24) next period given some level of reserves $R_1$, with the resulting exchange rate $e_1$. Now increase the reserves available to the authority to some level $R_2 > R_1$. It is easy to show that the authority can do at least as well in this situation as before, so that choosing $\hat{e} = e_1$ and disposing of the extra reserves is one possibility. But these extra reserves can be used to increase consumption by the old and decrease the uncertainty facing the young. This is done by decreasing $\hat{e}$ to $e_2 < e_1$, such that the resulting reserve level $R(e_2) > R(e_1)$. Both agents are better off. Exactly how much $\hat{e}$ is reduced will depend on the weights assigned to the different generations and the distribution of the exogenous variables.

Notice that increasing $\hat{e}$ when more reserves become available does not increase the value of (24), since the optimal choice of $\hat{e}$ before reserves were increased equated the marginal utility of the two agents. Decreasing uncertainty by increasing $\hat{e}$ increases utility for the young agent, but this effect is more than offset by the decrease in consumption and its effect on the utility of the old agent. With more reserves available, however, decreasing the value of $\hat{e}$ increases the utility of both agents through a redistribution of these extra reserves across the generations.

To find the young agent's optimal rate of devaluation, $\bar{e}$, it is easy to see that when the probability of a devaluation next period is zero, $e_{t+1} = e_t$ and the young face no uncertainty. Therefore, the optimal rate of devaluation for the young, $\bar{e}$, sets $R_t$ such that the probability of a devaluation next period is zero. From the economy-wide budget constraint, the probability of a devaluation next period is

$$\Pr[Y_{t+1} - C_{t+1} < Y_t - C_t - R_t + \underline{R}] .$$

When $Y_{t+1} - C_{t+1}$ is distributed over some finite interval [a,b], then this probability is zero whenever

$$R_t \geq Y_t - C_t + \underline{R} - a .$$

Because young agents are indifferent to values of $R_t$ that make the inequality strict, choose

$$\bar{R} = Y_t - C_t + \underline{R} - a .$$

Assuming that $Y_t - C_t - a > \underline{R}$, then $\bar{R} > \underline{R}$, with the result that $\hat{e} > \underline{e}$.

The above arguments have shown that $\partial R_t/e_t > 0$ is a sufficient condition for optimal devaluations to exceed minimal devaluations. The next task is to determine conditions under which this holds.

From the analysis in section 2 it is easily shown that

$$\text{(26)} \qquad \frac{\partial R_t}{\partial e_t} = - \frac{\partial C_t^0}{\partial e_t} - \frac{\partial C_t}{\partial e_t} .$$

The old agent's problem is defined so that

$$\frac{\partial C_t^0}{\partial e_t} = -(Y_{t-1} - C_{t-1})\left[\frac{e_{t-1}}{e_t^2}\right] < 0$$

and the first term on the right hand side of (26) is positive. The change in reserves with respect to a change in the exchange rate is positive if

$$- \frac{\partial C_t^0}{\partial e_t} > \frac{\partial C_t}{\partial e_t}$$

Clearly, this depends on the preferences of the two generations. If young agents do not increase their consumption due to devaluations, as in the case of log utility, then the condition is satisfied. Even some increase in consumption by the young generation can occur, as long as the aggregate level of consumption is decreased by the devaluation, so that the net change in reserves is positive. In general, when agents are risk averse, young agents should decrease their consumption during their youth when reserves increase because of the reduction in uncertainty regarding the purchasing power of money carried into old age, and the desire to optimally smooth consumption across time. In these cases, optimal devaluations will exceed the minimal rate of devaluation needed to set reserves at their lower bound.

## 6. Conclusion

Previous work on the theory of devaluations concentrated on either the effects or the timing of a devaluation, taking the size of the devaluation to be an exogenous variable. This paper has concentrated on the impact that a devaluation has on the different types of agents in a heterogeneous population, and how the influence these groups have may affect the size of a devaluation. when the government is choosing the devaluation optimally. It is found that devaluations may likely exceed the amount needed to avert a reserve crisis, something which is difficult to show in a model where agents are homogeneous.

The heterogeneity introduced by means of the overlapping generations framework may not represent the main elements of an economy affected by a devaluation. Intuitively, however, one can imagine that similar results might obtain if the groups of agents in the economy included importers and exporters, or borrowers and lenders.

## Endnotes

[1]World Currency Yearbook (1984) claims that ". . . more than 2500 full or partial devaluations . . . have been decreed in the post World War 2 era ...."

[2]Cooper (1971) provides a brief summary of the three approaches economists have taken to analyze the effect of devaluations. Unfortunately, he offers no references on the original sources of the theories. Dornbusch (1973) provides references for the monetary approach to devaluations. For references on the elasticities and absorption approaches, see any standard international finance text, such as Rivera-Batiz (1985).

[3]In this model, as in Krugman (1979), if currency substitution is an option for domestic agents, then domestic currency will be held only if the probability of a devaluation is zero, or if revaluations are also probable. If the probability of a devaluation is positive, then, because domestic and foreign currencies are perfect substitutes, no domestic currency will be held and the regime will collapse, unless reserves are adequate. In any event the economy will be demonetized in the sense that, as perfect substitutes, with any positive probability of a devaluation foreign money dominates domestic money as an asset. In the "real world," different currencies are clearly not perfect substitutes. It is to eliminate this that we introduce the "friction" of currency controls. This is of course another extreme; however, for many countries it is a realistic assumption, see IMF (1983).

[4]In assuming that (23) - (25) are time invariant, the analysis is confined to equilibria where price bubbles are nonexistent. This is so because, with no time variation in (23) - (25), a unique price level is determined for every state of the economy, which, by definition, precludes the

existence of price bubbles. For empirical tests of the existence of price bubbles in the foreign exchange market see Meese (1986).

[5]Of course, under appropriate circumstances there would also be a probability of revaluations. In what follows it is assumed that there is never a positive probability of both a revaluation and a devaluation next period. The analysis for the case of revaluations would be symmetric to the case for devaluations and will not be considered mainly because of the prevalence of devaluations throughout history.

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