BLACK-MARKETS FOR FOREIGN CURRENCY

by

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*I wish to thank my committee members, Robert Hodrick, Robert Flood, Deborah Lucas and Nancy Stockey for much patience and many suggestions during the development of this work. As usual, any remaining errors are my own responsibility.

26.1.2

1. Introduction

Governments often impose restrictions on the convertibility of their currencies. As evidence of this, Macedo (1982) reports that only 50 of 140 International Monetary Fund (IMF) member nations have free currency conversion for current account transactions, while only 33 member countries have no restrictions for capital account transactions. When these restrictions are binding, alternative markets develop where, for a premium, agents can obtain the foreign currency denied them through legal means.

This paper investigates these black-markets for foreign currency under the simplifying assumption that their existence is due entirely to the conversion restrictions dictated by the authorities. This specification limits the ability of the paper to address all of the questions related to black-markets for foreign currency, but it does permit a theoretical investigation of some of the issues which arise when black-markets occur.

Among these issues is the question of whether black-market exchange rates satisfy uncovered interest rate parity. Testing this relationship requires equating the expected marginal utility of investments in the two currencies. At the same time, if there is a demand for black-market currency in order to consume foreign currency priced goods, the black-market exchange rate should also equate the expected marginal utility of consumption for the foreign and domestic currency priced consumption goods. This paper develops two models of the black-market exchange rate, one based on investor behavior and the other on consumer behavior. If arbitrage opportunities are to be excluded in equilibrium, neither model should be rejected empirically.

Another issue raised by the existence of black-markets is the relationship between the black-market exchange rate and the equilibrium exchange rate that would arise if the currency restrictions were eliminated.

In this paper the black-market exchange rate has the same functional form as the unrestricted exchange rate, but it will generally not equal the unrestricted rate due to wealth lost through the act of evading currency conversion restrictions.

From a policy perspective, authorities might argue that easing conversion restrictions would cause increased demand for foreign currency and subsequent depreciation of the domestic currency. While this might be true if no black-market exists, with the presence of a black-market most of this excess demand may already be satisfied and easing restrictions might not have the expected effect. In fact, in the model developed in this paper both the official exchange rate and the black-market premium decrease (i.e. the currency appreciates) when the authorities ease conversion restrictions, if the demand for the foreign good is relatively inelastic.

The existence of black-markets for foreign currency, particularly for U.S. dollars, is well documented, with World Currency Yearbook (1984) reporting month-end black-market exchange rates for 115 countries. A review of this source reveals two interesting features. First, black-markets are widespread, existing in a broad range of economies from members of the European Economic Community to the most centralized of economies. Somewhat more interesting, however, is the broad range of black-market premiums, from 1% to an astounding 2,304%.

In addition to considerable cross-sectional variation in the black-market premium, examination of the experience of any one country shows that there also exists significant time variation in the premium. While such time series behavior may be difficult to explain in terms of changing restrictions on currency conversion, it need not be inconsistent with a model in which the existence of the black-market is due to conversion restrictions.

Early work on black-markets was restricted mainly to the study of goods markets and the effects of black-markets on equilibrium prices and quantities. See Michaely (1954) for an early analysis and references. In an international setting, black-markets became associated with the concept of smuggling due to the analysis conducted by Bhagwati and Hansen (1973). Their analysis, however, was still concerned with goods and it was Sheikh (1976) who extended the analysis to black-markets for currency. Various approaches to the concept of black-markets for foreign currency followed, including Macedo (1982), Dornbusch et al. (1983) and Nowak (1984); however, in each case the analysis is restricted to ad hoc supply and demand equations with no explicit modelling of the individual agent's decision problem and the resulting behavior.

The remainder of the paper is organized as follows. In section 2 a relationship is developed between black-market exchange rates and interest rate differentials based on the ideas of efficient markets and the lack of unexploited profit opportunities. This provides an empirically testable hypothesis which is confronted with the data for selected countries. However, this model offers no theoretical explanation for movements in the black-market premium. Therefore, section 3 develops a general equilibrium representative agent model where currency convertibility restrictions force agents to resort to smuggling in order to obtain the foreign currency needed to consume foreign goods. Section 4 analyzes the equilibrium reaction of the representative agent model to exogenous shocks. An empirical test of the model is developed and implemented in section 5. Finally, section 6 contains concluding comments.

2. Black-Market Exchange Rates and Uncovered Interest Rate Parity

The theory of forward exchange rates has been built largely on the assumption that a sufficiently large number of arbitragers exists so that the market will be characterized by interest rate parity (IRP). Empirical evidence seems to support the IRP relationship, especially when transaction costs are included in the calculations, as in Frenkel and Levich (1977) and McCormick (1979).

A similar approach can be taken with respect to the black-market exchange rate when investors are faced with currency convertibility restrictions. This section considers a small economy with a flexible official exchange rate and where the rates of return on both foreign and domestic assets are taken as given by agents operating in the black-market for foreign currency. Assuming agents invest today in order to maximize their wealth available for consumption next period, an efficient black-market would equate the expected marginal utility of investing in either domestic or foreign assets. A black-market exchange rate which did not equate the expected marginal utility of investments would provide expected profits, inducing additional investment in one direction and driving the black-market rate down (up) to offset the demand. Mathematically, this can be written as

$$(1) \qquad E_t\left[U'(C_{t+1})\frac{1 + i_t}{p_{t+1}}\right] = E_t\left[U'(C_{t+1})\frac{e^b_{t+1}}{e^b_t}\,\frac{(1 + i^*_t)k_{t+1}}{p_{t+1}}\right]$$

where $U'(C_{t+1})$ is the marginal utility of consumption at time t+1, p_{t+1} is the domestic price of the consumption good, $i_t(i^*_t)$ is the domestic (foreign) rate of return, e^b_t is the black-market exchange rate expressed as units of domestic currency per unit of foreign currency, and k_{t+1} represents the possible loss in return due to fines, bribes etc. that arise due to transacting in the black-market. Assume that $k_{t+1} \in [0,1]$ and is unknown at the time the black-market currency is purchased. The left-hand side of (1) describes the expected marginal utility of investing in a domestic asset, whereas the right-hand side describes the expected marginal utility of purchasing foreign currency on the black-market, investing in a foreign asset and then converting the foreign currency proceeds back into domestic currency in the black-market. Notice that equation (1) deals with riskless assets whose returns, $i_t(i_t^*)$, are known at time t. Equation (1) should also hold for risky assets, but tests of the appropriateness of (1) will be conducted using riskless assets.

Assuming that all terms are lognormally distributed,[1] appealing to rational expectations and defining ε_{t+1} to be the innovation in the black-market exchange rate uncorrelated with time t information, the expression can be simplified after taking logarithms to

$$\ln(e^b_{t+1}/e^b_t) = \alpha + \beta \ln(1 + i_t)/(1 + i^*_t) + \varepsilon_{t+1} \quad (2)$$

where, due to the unobservable nature of k_{t+1}, it is assumed constant over time and is included in the intercept. Also included in the intercept are variance and covariance terms arising from the assumption of lognormality, which will be constant if the distributions of all random variables are stationary over the period of estimation. The appropriateness of these assumptions will be determined in the tests of misspecification. Here the null hypothesis is that $\beta=1$, and the residuals are both serially uncorrelated and homoscedastic.

Results of tests of equation (2) for five countries are given in table 1. Black-market exchange rates are taken from World Currency Yearbook

(1984). Interest rates are from the International Financial Statistics (IFS) data base. The tests were limited to five countries due to the limited availability of interest rate data, and in an attempt to conform as closely as possible to the assumptions of the model. Regressions were run using 120 monthly observations from the period January 1974 through December 1983.

The regression results provide little evidence in favor of the null hypothesis. The model is rejected in the case of Brazil due to a slope coefficient significantly greater than one. For Ireland the model is rejected due to the presence of heteroscedasticity in the residuals, as determined by a Glesjer test. While the model can not be rejected for the remaining three countries, neither are their coefficients estimated with sufficient precision to be able to reject the hypothesis that they are equal to zero. One possible reason for the weakness of the tests is that the assumption of stationary distributions over the sample period is violated. To test for this, the sample period was divided into two subperiods and the regressions were rerun. Only in the case of Brazil were the estimated coefficients significantly different for the two subperiods. In general, the estimates were such that it was again impossible to reject that the slope coefficients were equal to either zero or one.

Referring back to the casual empirical evidence cited in the introduction, one major shortcoming of this model of the black-market exchange rate is that it provides no explanation for either time or cross-sectional variation in the premium separating the official and black-market exchange rates. Neither does the model provide any explanation for the relationship between the black-market exchange rate and the value that the official exchange rate would take if there were no conversion restrictions. In the next section an alternative general equilibrium model of the black-market is developed in an attempt to more fully describe the black-market premium and its variation over time.

3. A General Equilibrium Model

In the previous section, activity in the black-market was dominated by investors interested only in return differentials between assets denominated in different currencies. One alternative is to consider a black-market used only by consumers so that it is the demand for foreign goods that determines the demand for foreign currency in the black-market. This section develops a model with this feature and uses it to determine the black-market exchange rate. Both the black-market and official rates are flexible; however, due to conversion restrictions, the official market alone does not clear the market for foreign currency.

Consider a small open economy consisting of a large number of identical agents whose preferences are defined over the consumption of a domestic good and a foreign good. Agents enter each period with holdings of domestic and foreign currency carried over from the previous period. At the start of each period agents receive an endowment, Y_t, of the domestic good, where the process generating the endowment is assumed to be exogenous and random. In order to motivate the existence of money in the model, agents are assumed to be unable to consume their endowment. Instead, they must trade their endowment for either domestic or foreign currency, with these proceeds then carried over to the next period where they are used to purchase consumption goods. Consumption this period must be financed by beginning-of-period cash balances, with domestic goods purchased with domestic currency and foreign goods purchased with foreign currency. This is a version of the cash-in-advance constraint used so often in monetary models.

The foreign economy in this model is considered to be large in comparison to the domestic economy and to have supplies of both the foreign and domestic goods which sell at exogenously given terms of trade. Assume that the government has instituted a quota on the amount of foreign currency available. This quota restricts each agent to a supply of legally obtained foreign currency equal to N_t, where the value of the restriction is (possibly) time varying and determined by exogenous factors. Under the assumption that foreign goods must be purchased with foreign currency, this restricts the legally available supply of the foreign good. It is the presence of this quota and the costs of circumventing it which results in the existence of a separate black-market for foreign currency where agents are willing to pay a premium in excess of the official rate.

In choosing their consumption stream, agents are assumed to have preferences given by

$$E_t\left[\sum_{\tau=t}^{\infty} \beta^{\tau-t} U(h_\tau, f_\tau)\right] \tag{3}$$

where

h_τ = consumption of the domestic good at time τ

f_τ = consumption of the foreign good at time τ

$\beta \in (0,1)$

E_t = expectation operator conditional on time t information

and U(.,.) is a utility function with the usual mathematical properties of concavity, continuity and differentiability.

In order to satisfy their demand for the foreign good not met through the legal market, agents are assumed to be able to smuggle domestic goods to the foreign market where they are sold for foreign currency. To prevent all trade from flowing through this black-market, however, the government is assumed to have an enforcement mechanism which induces a cost, s_t, per unit of domestic good smuggled. This cost is taken to be a reduction in the amount of the domestic good that arrives in the foreign market. As such, it is given by the percent lost due to either transportation factors, fines or bribes paid to corrupt officials. It is assumed that $s_t \in [0,1)$ and that $s_t = s(\bar{z}_t)$, where $\bar{z}_t$ is the per capita level of smuggling and $s'(\cdot) > 0$. The time subscript on the cost function implies that the function can vary over time due to exogenous factors that are not modelled here. Regardless of the variability in s_t, it is assumed that the current value of the cost is known with certainty by agents at the time their choice decisions are made.[2]

The total supply of domestic currency is fixed during each period, but it evolves over time according to the rule $\underline{M}_t = \omega_t \underline{M}_{t-1}$, where ω_t is the (gross) rate of increase in the money supply. Assuming that foreign agents hold no domestic currency, the entire supply of domestic currency must be held by the domestic agents. Injections of currency into the economy are assumed to be timed so that agents receive them at the end of the period. Thus, they are unavailable for use in the current period's goods market. The realization of ω_t, however, is revealed at the same time as the realization of Y_t so that agents are aware of the amount of currency they will be receiving.

The only sources of uncertainty in the model are the endowment, Y_t, the foreign currency quota, $\underline{N}_t$, the domestic currency supply, $\underline{M}_t$, and the cost of smuggling function, s_t. To ease notation, these variables will henceforth be referred to as $\Theta_t = \{Y_t, \underline{N}_t, M_t, s_t\}$. Assume that Θ_t has the transition function $G(\Theta_t | \Theta_{t-1})$, which is a Markov process known by agents.

Before proceeding further with the model the following notation is introduced:

p_t = domestic currency per domestic good

p^*_t = foreign currency per domestic good (exogenous)

q_t^* = foreign currency per foreign good (exogenous)

e_t = official exchange rate = domestic currency/foreign currency

e_t^b = black-market exchange rate.

Foreign currency prices, p_t^* and q_t^*, are exogenous under the small country assumption.

Agents participate in three markets in this model, and it is worthwhile to be specific on the timing of those markets and how trade occurs. This is displayed graphically in figure 2, with some further discussion of the markets below.

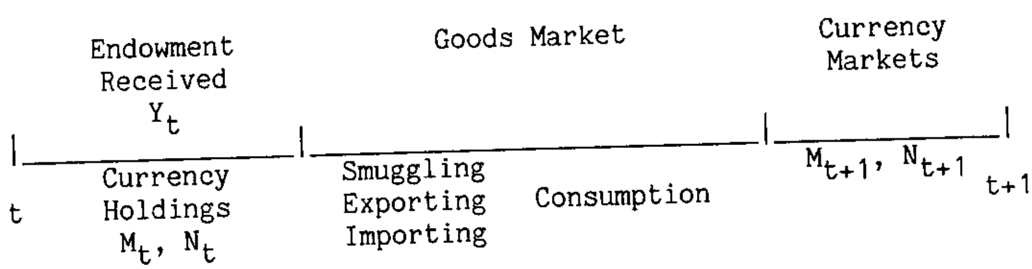

Figure 2

At the start of the period agents receive their endowment and are holding balances of domestic and foreign currencies, M_t and N_t respectively. As discussed above, at that point all uncertainty has been eliminated and the values of the equilibrium prices are known. Agents then go to the goods market where they sell part of their endowment for domestic currency, smuggle part for foreign currency, export part legally, purchase domestic goods for consumption (h_t) and import foreign goods for consumption (f_t). The consumption goods are paid for with their beginning-of-period cash balances. Finally, the currency markets open and agents trade domestic currency obtained in the goods market for foreign currency at the official rate, subject to the quota on foreign currency transactions. The black-market for foreign currency transactions is also open at this time and agents also receive their endowments of domestic currency during this market.

Some discussion on the matter of exports and the country's balance of payments is in order here. By assumption, exports do not incur the loss, s_t, that smuggled goods do. Unlike smuggled goods, however, agents do not receive foreign currency for their exports. Instead, foreign currency proceeds from the sale of exports are required to be exchanged at the central bank for domestic currency at the official rate. While this procedure is an assumption of the model, it is not unrealistic in that a large part of the world community of nations does impose restrictions that require agents to surrender export proceeds.

In the problem described by (3), all relevant information for individual decision making can be characterized by the set $\Phi_t = \{\theta_t, M_t, N_t, \bar{N}_t\}$, where $\bar{N}_t$ is the economy per capita holdings of foreign currency. As all individuals are identical, in equilibrium it will be true that $\underline{M}_t = M_t$ and $\bar{N}_t = N_t$. The state of the economy, therefore, can be characterized by $\Omega_t = \{\theta_t, N_t\}$.

For ease of notation, the purchasing power of domestic currency $\pi_M = 1/p_t$, the official price of foreign currency in terms of domestic goods $\pi_N = e_t/p_t = 1/p_t^*$, and the relative price of the foreign good in the official market $\rho = e_t q_t^*/p_t$ are introduced.

Under the assumptions, (3) defines a value function and the agent's problem can be rewritten as[3]

(4) $$v(\Phi) = \underset{h,f,M',N',\eta',z}{\text{Max}} \left\{U(h, f) + \beta\int v(\Phi')dF(\Omega'|\Omega)\right\}$$

(4a) $$\text{s.t.} \qquad h \le \pi_M M \qquad (\mu_1)$$

(4b) $$\rho f \le \pi_N N \qquad (\mu_2)$$

(4c) $$\pi_M M' + h + z + \pi_N \eta' \leq Y + \pi_M M + \pi_M(\omega' - 1)\underline{M} \quad (\lambda_1)$$

(4d) $$\gamma\pi_N N' + \gamma\pi_N q^* f \leq z + \gamma\pi_N \eta' + \gamma\pi_N N \quad (\lambda_2)$$

(4e) $$\eta' \leq \underline{N} \quad (\nu)$$

where z_t represents the amount of domestic goods smuggled by the individual, η_t represents the legal demand for foreign currency and γ is defined below as $1/(1 - s_t)$ and the letters in parentheses represent the corresponding Lagrange multipliers. In order to simplify notation, unprimed variables are period t variables and primed variables are period t+1 variables.

Equations (4a) and (4b) represent the cash-in-advance constraints for domestic and foreign goods respectively. Equation (4c) is a budget constraint in terms of domestic currency and goods, whereas (4d) presents a similar concept, but for foreign currency and goods. The terms on the left-hand side of (4c) represent the uses of domestic currency based wealth during the period, while the right-hand side presents the sources of that wealth. In this case, wealth can be used to obtain domestic currency (M), for consumption of the domestic good (h), to smuggle for foreign currency (z), or to export for foreign currency (η). In (4d), the uses of foreign currency based wealth include the acquisition of foreign currency (N) and consumption of the foreign good (f). The sources of this wealth include smuggled goods (z), acquisitions in the official market (η) and beginning of period balances. Equation (4e) represents the quota on foreign currency acquisitions.

The official exchange rate, e_t, is taken to be a floating rate in the sense that its level is determined by market forces alone, with no intervention by the government. With a flexible exchange rate, the rate will adjust so that the overall balance of payments is zero, even if the trade account is not balanced. Ultimately, the exchange rate will adjust to the point where agents export enough goods to supply their legal demands of foreign currency. Because imports are not restricted in this model, imports may exceed exports due to the foreign currency obtained by smuggling (or to the drawing down of foreign currency balances held by agents). The offsetting entry in the balance of payments would presumably fall under the category of "statistical discrepancy," since neither the smuggled goods nor the returning foreign currency were recorded. If all movements of goods and money were properly recorded, the entries in the trade account would also include smuggled goods. In this case, the trade account would typically show a surplus with the difference between imports and exports (including smuggled goods) equal to the costs of smuggling. Notice that for simplicity this assumes that no net saving or dissaving of foreign currency occurs. Allowing for net savings of foreign currency implies that the trade account balance will exactly offset the capital account balance plus any statistical discrepancy.

Foreigners have no need to hold domestic currency and will not do so. With no change in official reserve levels, this implies that the value of imported goods is equal to the flow of foreign currency out of the country. Consequently, in order for the balance of payments to be zero, it must be the case that the value of exported goods is equal to the inflow of foreign currency into the country. Therefore, with domestic agents demanding η_{t+1} in legal foreign currency, the amount of domestic goods legally exported to balance the current account is $\eta_{t+1}e_t/p_t$. The value of these exports in foreign currency terms is $\eta_{t+1}e_t p_t^*/p_t$, which must equal the foreign currency demands of domestic agents. Equating the two gives

(5) $$e_t = p_t/p_t^* .$$

Any other official rate would require official reserve flows, a complication that is ignored here. Deviations from (5) would also invite arbitrage activities by domestic and foreign agents and therefore could not be an equilibrium value for the exchange rate.

With the assumption that there is a large number of agents able to participate in smuggling, it should be that the black-market is characterized by zero profits. Intuitively, agents should be indifferent between exchanging currency in the black-market or smuggling a part of their endowment in return for foreign currency. At the margin, the amount of domestic currency obtainable by selling a unit of their endowment in the domestic market is p_t. The foreign currency obtained by smuggling this same quantity of goods is given by $p_t^*(1 - s_t)$. In equilibrium, the black-market exchange rate must equate the two. Mathematically, this is given by

$$e_t^b p_t^*(1 - s_t) = p_t \ .$$

This condition implies that

(6) $$e_t^b = (p_t/p_t^*)[1/(1 - s_t)] = e_t\gamma_t \geq e_t$$

where

$$\gamma_t = \gamma(s_t) \equiv 1/(1 - s_t) \ .$$

In this model, the black-market exchange rate is determined by both exogenous factors, $s_t(\cdot)$, as well as the endogenous factors, e_t and $\bar{z}_t$.

The main obstacle to black-market arbitrage by foreign agents is the timing of markets. That is, with black-market currency trading occurring after the goods markets are closed, no riskless arbitrage can occur. Intertemporal trading for gains in expected utility is a possibility, but it will not threaten the black-market premium if there are sufficiently high costs to foreign agents operating in the black-market. Possible costs include the cost of finding the black-market, as well as possible penalties imposed by domestic authorities on foreign agents caught operating in the black-market. In what follows it is assumed that these costs are at least as large for foreign agents as they are for domestic agents so that the black-market premium is given by equation (5).

It should be made clear that the specification of the black-market exchange rate, (5), depends critically on the assumed timing of the goods and money markets. This timing of markets, originally introduced in Svensson [16], dictates an equivalence between smuggling and black-market currency transactions. This is so because the proceeds from either can be used only in the subsequent period. By altering the timing of the markets so that agents trade first in the currency markets and then in the goods markets, as in Lucas [6], the equivalence between smuggling and black-market transactions is broken since proceeds from smuggling could be used only in the subsequent period, whereas black-market currency would be available for use immediately.

The first-order conditions for the agent's problem are:

(7a) $$U_h(h, f) = \mu_1 + \lambda_1$$

(7b) $$U_f(h, f) = \rho(\mu_2 + \gamma\lambda_2)$$

(7c) $$\beta\int v_M(\Phi')dF(\Omega'|\Omega) = \pi_M\lambda_1$$

(7d) $$\beta\int v_N(\Phi')dF(\Omega'|\Omega) = \gamma\pi_N\lambda_2$$

(7e) $$\gamma\pi_N\lambda_2 = \pi_N\lambda_1 + \nu$$

(7f) $$\lambda_2 = \lambda_1$$

plus the appropriate conditions for each of the five constraints. In addition, there are two envelope conditions which will prove useful,

(7g) $$v_M(\Phi) = \pi_M(\mu_1 + \lambda_1)$$

(7h) $$v_N(\Phi) = \pi_N(\mu_2 + \gamma\lambda_2).$$

Using (7a), (7b), (7g), (7h) and the definitions of ρ and π_N,

$$\pi_M U_h(h, f) = v_M(\Phi)$$

and

$$(1/q^*)U_f(h, f) = v_N(\Phi) .$$

Substituting these expressions into (7c) and (7d) respectively gives

$$\beta\int \pi_{M'} U_h(h', f')dF(\Omega'|\Omega) = \pi_M\lambda_1$$

and

$$\beta\int(1/q^{*\prime})U_f(h', f')dF(\Omega'|\Omega) = \gamma\pi_N\lambda_2 .$$

By virtue of (5) and (7f), this gives

(8) $$e^b = \gamma e = \frac{\gamma\pi_N}{\pi_M} = \frac{\int(1/q^{*\prime})U_f(h', f')dF(\Omega'|\Omega)}{\int\pi_{M'}U_h(h', f')dF(\Omega'|\Omega)} .$$

As with other asset pricing equations, the black-market exchange rate equates the expected value of the marginal utility of black market currency to the expected marginal utility of the alternative investment, domestic currency. Because both currencies are valued only for what they can purchase, the appropriate prices are included in order to convert the currency into consumption good.

Despite the differences in the model, the result, (8), corresponds closely with the value of the floating official exchange rate in Svensson

(1985). This is not really surprising if one looks closely at the agent's problem in the two models. In the present model, the difference between the representative agent's problem, (8), and that of Svensson's (1985), is the existence of the terms involving η. When the quota is binding the marginal activity of agents is in the black-market, so that it is the black-market exchange rate that agents equate to their expectations of next period's marginal utilities.

Notice, however, that the black-market exchange rate in this model is not generally equal to the free-market rate in Svensson's (1985) model because of the wealth effects induced by the cost of smuggling. That is, the arguments of the marginal utility functions and the purchasing power of domestic currency will be different from their free-market values because, due to the cost of smuggling, agents have less wealth than they do in the free-market economy with the same endowment stream. From the agent's necessary conditions (7e) and (7f),

$$\gamma = 1 + \frac{\nu}{\lambda} p^* .$$

Thus, the black-market premium is an increasing function of the conversion constraint multiplier and the foreign currency price of the domestic good. Moreover, the premium exceeds one only when the currency conversion constraint is binding so that $\nu > 0$ and smuggling is taking place. Intuitively, as the conversion constraint becomes more binding, agent's increase the amount of goods smuggled thereby increasing the cost of smuggling and raising the black-market premium. On the other hand, as the foreign currency price of the domestic good (p^*) increases, the resulting improvement in the terms of trade for domestic agents induces additional demand for the foreign good, requiring an increase in the level of smuggling and increasing the black-market premium. The premium is decreasing in the marginal utility of wealth, λ, since an increase in the marginal utility of wealth implies a reduction in the level of wealth and a corresponding decrease in the level of smuggling.

A stationary equilibrium in the model will be characterized by a set of time independent functions for the agent's choice variables and the equilibrium prices, where the arguments of those functions consist only of the state variables in the model, Ω_t. Formally, these functions are

(9a) $$(h_t, f_t, z_t, M_t, N_t, \eta_t) = \Gamma_1(\Omega_t)$$

and

(9b) $$(p_t, e_t, e_t^b) = \Gamma_2(\Omega_t)$$

where Γ_1 and Γ_2 are vector valued functions. In addition to the choice and price functions defined in (9), an equilibrium is also characterized by a law of motion for the stock of foreign and domestic currency held by agents. For the economy overall, this function is given by $\{\underline{M}', \bar{N}'\} = g(\Omega)$.

In order for these functions to define an equilibrium for the economy, they must satisfy the agent's problem (4), given the value function $v(\Phi)$. In addition, they must satisfy the conditions given by equations (5), (6), as well as the cost of smuggling condition

(10) $$s_t = s_t(\bar{z}_t) , \quad s(0) = 0 .$$

Finally, in equilibrium, the domestic currency demands of agents must equal the available supply,

(11) $$M_t = \underline{M}_t .$$

Combining all of the above provides the following

Definition: A stationary equilibrium for the economy consists of:

(i) A value function v(), (ii) price functions (9b), (iii) choice functions (9a), (iv) market conditions (5), (6), (10), and (11), and (v) a function $g(\Omega)$ such that $v(\Phi)$ satisfies (4) when prices and demands are given by (9), (9a) are the choices of agents given the prices (9b), conditions (iv) are satisfied, and the law of motion of the representative agent's currency holdings is consistent with the individual agent's maximizing behavior, i.e. $\{M', N'\} = g(\Omega)$.

Having defined a stationary equilibrium for the economy, it is customary to next prove the existence of such an equilibrium, often through the use of a fixed point theorem. This has not been accomplished due to the nature of the model. In particular, because one of the state variables, the holdings of foreign currency, is also a choice variable, the Schauder fixed point theorem is required which demands that the agent's holdings of foreign currency be given by a sequence of functions forming an equicontinuous family of functions. The problem of proving equicontinuity of the demand functions for foreign currency has prevented the construction of a proof of the existence of an equilibrium in this model.[4] An appendix available from the author presents an analysis of a deterministic version of the model and provides an example of the steady state equilibrium. The next section assumes that an equilibrium exists and then characterizes the reaction of the price level and exchange rates to exogenous shocks.

4. Equilibrium Behavior

This section investigates the behavior of the economy when it is subjected to shocks to the exogenous variables in the model. In order to facilitate the analysis, the investigation is conducted under the assumption that the cash-in-advance constraints are binding. This assumption will hold in this economy if the agent's beliefs about the monetary policy of the government place sufficiently high probability on inflationary states so that the expected return on holding currency is negative, and the variability of the endowment process is low so that smoothing consumption across time is not critical.

Substituting the value of domestic and foreign currency holdings into the utility function, writing the budget constraints as equalities, solving the domestic currency budget constraint for z_t and substituting this into the foreign currency budget constraint, the agent's problem is now

$$\text{(12)} \qquad \underset{\{M_{t+1}, N_{t+1}\}_\tau}{\text{Max}} \; E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} U\left[\frac{M_\tau}{p_\tau}, \frac{N_\tau}{q^*_\tau}\right]$$

$$\text{(12a)} \qquad \text{s.t}\left\{N_{\tau+1} = \left[Y_\tau + \frac{(\omega_{\tau+1} - 1)M_\tau - M_{\tau+1}}{p_\tau}\right] p^*_\tau(1 - s_\tau) + s_\tau N_\tau\right\}_\tau ,$$

where the cost of smuggling function, s_τ, is evaluated at the value of z_t obtained from the agent's domestic currency budget constraint. The first order conditions to this problem can be reduced to

$$\text{(14)} \qquad E_t\left\{U_h\left[\frac{\omega_{t+1}M_t}{p_{t+1}}, H_t\right] \frac{p_t}{p_{t+1}}\right\} =$$

$$E\left\{U_f\left[\frac{\omega_{t+1} M_t}{p_{t+1}}, H_t\right] \frac{1}{q^*_{t+1}} \frac{p^*_t\left(1 - s\left(Y_t - \frac{M_t}{p_t} - \frac{N}{p^*_t}\right)\right)}{q^*_{t+1}}\right\} ,$$

where $H_t = \left(Y_t - \frac{M_t}{p_t}\right)p^*_t\left(1 - s\left(Y_t - \frac{M_t}{p_t} - \frac{N}{p^*_t}\right)\right) + N_t s\left(Y_t - \frac{M_t}{p_t} - \frac{N}{p^*_t}\right) \frac{1}{q^*_{t+1}}$.

Equation (14) is a nonlinear first-order difference equation in the endogenous prices p_t and p_{t+1}. With the exchange rates in the model given by equations

(5) and (6), once the price process has been identified, the dynamics of both the official and black-market exchange rates will follow.

In order to eliminate the problems inherent in nonlinear difference equations, linearize (14) around the deterministic steady-state values of both the endogenous and exogenous variables. After simplifying, the linearized equation is

$$(15) \quad p_t - p = E_t \sum_{i=0}^{\infty} \ell_2^i[\ell_1(\underline{M}_{t+i} - \underline{M}) + \ell_3(p^*_{t+i} - p^*) + \ell_4(Y_{t+i} - Y) +$$

$$\ell_5(\underline{N}_{t+i} - \underline{N}) + \ell_6(\omega_{t+1+i} - \omega) + \ell_7(q^*_{t+1+i} - q^*)] \; ,$$

where the details on the linearization and the values of the coefficients ℓ_i are described in the appendix, and all variables without subscripts are steady-state values of the variables.

For ease of notation in the analysis that follows, the rather cumbersome coefficients on the ratios of second and first derivatives will be replaced with the constants c_i, which are defined in the appendix. In general these coefficients are positive, however, in some cases their sign is indeterminate. In such cases the analysis is conducted as if they were positive, with sufficient conditions to guarantee that they are positive given in the appendix.

Convergence of p_t-p requires that

$$|\ell_2| = \left| \frac{1 - c_1(U_{fh}/U_h) + c_2(U_{hh}/U_h)}{1 + c_3(U_f/U_h) + c_4(U_{hf}/U_h) - c_5(U_{ff}/U_h)} \right| < 1$$

where the values of the coefficients are given in the appendix. Sufficient conditions for this to hold are that $U_{fh} = 0$ and $|(U_{hh}/U_h)c_2| < 1$, in which case $\ell_2 \in (0,1)$.

Under the assumptions that $|\ell_2| < 1$, the model produces intuitive comparative statics for the change in the domestic price level, and therefore the official exchange rate, when changes to the exogenous variables occur. Changes with respect to the money supply process ($\underline{M}_t$ and ω_{t+1}) and the endowment process have the expected signs (positive and negative respectively) and will not be discussed in further detail. More interesting is the price response to changes in the currency conversion restriction, $\underline{N}_t$. Ignoring the effect of ℓ_2^i and assuming $U_{hf} = 0$, the change is given by

$$\partial p/\partial \underline{N} = \ell_5 = \left[1 + c_6(U_{ff}/U_f)\right]/c_7$$

where c_6 and c_7 are described in the appendix. The sign of this can be either positive or negative depending on the concavity of the utility function with respect to consumption of the foreign good, which intuitively is reflected in the elasticity of demand for the foreign good. In effect, relaxing the conversion constraint reduces the relative cost of the foreign good, thereby inducing agents to increase their consumption of it. With more foreign good purchased through the official market, however, the total level of exports needed to finance the new level of consumption may be higher or lower, depending on the elasticity of demand for the foreign good. With low elasticity of demand, total exports will fall resulting in an increase in the level of goods available for domestic consumption and, with a constant money supply, decreasing the domestic price and consequently the official exchange rate. In this case, with decreased smuggling and an increasing cost of smuggling, the black-market premium should also be reduced. With higher elasticity, the official exchange rate increases due to a net increase in the level of exports as agents are induced to consume more foreign goods.

Smuggling could either increase or decrease resulting in an ambiguous effect on the black-market premium in this case.

Again ignoring the effect of ℓ_2^i, changes in the domestic price level due to changes in the foreign prices are given by

$$\partial p/\partial p^* = \ell_3 = [1 + c_8(U_{ff}/U_f)]/c_9$$

$$\partial p/\partial q^* = \ell_7 = [1 + c_{10}(U_{ff}/U_f)]/(-c_{11}) \ .$$

where the values of the constants c_i are defined in the appendix. Again, in both cases the sign of the change depends on the concavity of the utility function with respect to the foreign good and the resulting elasticity of demand for the foreign good. With a high elasticity of demand, an increase in the foreign currency price of the domestic good (p*) induces additional smuggling and reduces the supply of goods in the domestic market, thus depreciating the domestic currency. In the process, with increasing costs of smuggling the black-market premium will also increase, as will the black-market exchange rate. Conversely, for changes in the expected foreign currency price of the foreign good (q*), agents will substitute away from the foreign good, which implies a lower level of smuggling, an appreciated domestic currency and a lower black-market premium.

When the elasticity of demand for the foreign good is sufficiently low, both derivatives change sign. In this case the increase in the foreign currency price of the domestic good (p*) induces additional foreign good consumption, but total exports actually decrease due to the additional foreign currency received for each unit exported. As a result, there are more goods in the domestic market, the currency appreciates and the black-market premium also falls. For changes in the foreign currency price of the foreign good (q*) the opposite occurs. That is, the increase in the expected price results in a movement away from the foreign good toward the domestic good, but the net effect is to increase the level of exports needed to support the new level of consumption. The result is a depreciation of the domestic currency and an increase in the black-market premium.

Having now described the economy from a theoretical perspective, the next task is to confront the model with data to see if it is able to explain time variation in the black-market premium. This is done in the next section.

5. Empirical Formulation of the Model

This section investigates the empirical implications of equation (8). Dividing equation (8) by the official exchange rate and using (5), the value of the black market premium is given by

$$\gamma_t = \frac{e_t^b}{e_t} = \frac{\int P^* \; Q^* \; U_f(\cdot,\cdot)dF(\cdot)}{\int P \; U_h(\cdot,\cdot)dF(\cdot)} , \tag{16}$$

where $P^* = p_t^*/p_{t+1}^*$, $Q^* = p_{t+1}^*/q_{t+1}^*$, and $P = p_t/p_{t+1}$.

Let $\psi_t = (p_t,\, p_t^*,\, q_t^*,\, U_f(\cdot,\cdot),\, U_h(\cdot,\cdot))'$. Due to the multiplicative nature of (16), it is assumed that the vector stochastic process ψ_t has a multivariate lognormal distribution. Using the properties of this distribution, after taking logarithms (16) can be rewritten as

$$\ln\gamma_t = K + E(\ln P^*) + E(\ln Q^*) - E(\ln P) + E(\ln U_f/U_h) \tag{17}$$

where K contains variance and covariance terms and will be constant if the distribution of ψ_t is stationary.

Intuitively, the representation of γ given in (17) is appealing. The first term, $E(\ln P^*)$, represents the expected rate of change in the purchasing power of the foreign currency, which is inversely related to inflation. Thus, as expectations of foreign inflation increase, $E(\ln P^*)$ decreases.

Consequently, agents, expecting lower purchasing power for foreign currency, demand less foreign currency and γ decreases. Similarly, the third term, E(lnP), corresponds to domestic inflation. In this case, however, as expected domestic inflation increases, agents substitute out of domestic currency and into foreign currency, thus causing the premium to increase. The second term, E(lnQ*), accounts for agent's expectations about changes in the worldwide terms of trade. If agents expect an improvement in the terms of trade, i.e. higher E(lnQ*), they will demand more foreign currency today and the premium will increase. The intuition here is that increases in the terms of trade imply a higher relative foreign currency cost of the domestic good which, through (5), will also translate into a higher domestic price for the domestic good. Finally, the fourth term involves the expected marginal rate of substitution. In some sense, this term can be considered to be the time varying risk premium in the equation since it is only when agents are risk averse that it is not constant across time.

Testing the model requires an assumption on the way in which the expectations of agents are formulated since all of the relevant terms in (17) involve expectations of future variables. Assume agents are rational so that

$$E_t(\zeta_{t+1}) = \zeta_{t+1} - \psi_{t+1}$$

where

$$\zeta_{t+1} = \ln\psi_{t+1},$$

$$\psi_{t+1} = (\psi_{p,t+1}, \ldots, \psi_{U,t+1})'$$

and $\psi_{i,t+1}$ is a mean zero normally distributed random error, uncorrelated with time t information.

The specification (17) includes the marginal rate of substitution between domestic and foreign goods. Fortunately, the model provides additional information on this which is useful. From the first-order conditions, (7a), (7b) and (7f),

$$\frac{U_f(t)}{U_h(t)} = \rho_t \gamma_t \left[1 + \frac{\mu_2}{\gamma_t U_h(t)} - \frac{\mu_1}{U_h(t)}\right] .$$

Let

$$\delta_t = \left[1 + \frac{\mu_2}{\gamma_t U_h(t)} - \frac{\mu_1}{U_h(t)}\right] .$$

Replacing the marginal rate of substitution in the regression equation with the above representation, using the definitions of P*, Q*, P, ρ and γ and applying the assumption of rational expectations gives

$$(18) \qquad \ln \frac{e^b_{t+1}}{e_t} = -K + E_t(\ln\delta_{t+1}) + \beta \ln\gamma_t - \varepsilon_{t+1} ,$$

where $\varepsilon_{t+1} = \underline{1}'\psi_{t+1} = \Sigma\psi_{i,t+1}$.

Because $E_t(\ln\delta_{t+1})$ is unobservable, the regression will be run under the assumption that it is a constant. If the assumption is inappropriate, then the regression is misspecified and this should be evident from the results of the tests of misspecification.

Month-end values of the black market exchange rate, e^b, are published for most countries in World Currency Yearbook (1984). Dividing these by the month-end exchange rate in the IFS data base produces the required time series for the premium, γ. Theoretically, this is observed without error, so that an errors-in-variables bias should not be a problem. A review of the premiums produced with these two time series, however, revealed negative (logarithms of the) premiums on occasion for some countries not involved in the reported results. While a negative black-market premium is not possible in this model, such a result would obtain under some circumstances.[5] Perhaps equally likely in this case, however, is the possibility of either measurement error or differences in the timing of measurement of the two exchange rates. Given the high volatility of exchange rates, even measurements on the same day, but at different times during the day, could result in a negative premium. If it is the case that the black-market exchange rates are not measured at the same time as the official rate, the error introduced into the independent variable may cause a bias in the coefficient estimates.

In table 2 equation (18) is estimated for 10 countries, where the null hypothesis is that $\beta = 1$ and the residuals are both serially uncorrelated and homoscedastic. In each case the period of estimation is from 1/74 to 12/83 giving 120 monthly observations.

The model is rejected due to a value of β less than one for Greece, India, Italy, Norway and Portugal. In addition, tests of heteroscedasticity reject the null hypothesis for Brazil, Greece, India, the Philippines and Portugal. The model is rejected for the cases of India, Italy and the Philippines due to the presence of serial correlation in the residuals. Ultimately, only France, Iceland and Ireland conform to the model and the additional assumptions. Note, however, that the slope coefficients for Brazil and the Philippines are not statistically different from one, and it is possible that the heteroscedasticity and serial correlation observed for these countries is a result of the assumption that $E_t(\ln\delta_{t+1})$ is constant when in fact it may represent a time varying risk premium. Given the unobservable nature of δ_t, however, incorporation of a time varying risk premium into the regression is left for future research.

One possible reason for the rejections is due to the assumed stationarity of the distributions of both coefficients over the ten year period under investigation. One test of this assumption is to divide the sample into two subperiods, reestimate the equations and then test to see if the coefficients are equal. The results of this estimation are reported in table 3, where the two subperiods represent 1974-1978 and 1979-1983. Equality of the coefficients is rejected for 4 out of 10 countries at the 5% level, with one additional rejection at the 10% level. Therefore, stationarity could be one reason for rejection. However, closer examination of the coefficients and tests of heteroscedasticity and serial correlation shows that allowing for different coefficients over these two periods does not generally improve the performance of the model.

The model is no longer rejected for the first subperiod in the case of Brazil. This probably coincides with the fact that in 1979 the cruzeiro underwent a "maxidevaluation" and in the early 80's the government started an inflationary policy which induced capital flight from the economy. The model is unable to capture the effect of these and other developments.

Conversely, the model is now rejected for the first subperiod in the cases of France, Iceland and Ireland. One possible reason in the case of France is that it was during the second subperiod when the European Monetary System was initiated and when the socialist government of Mitterand instituted strict currency controls. Prior to this period there may have been insufficient activity in the black market to induce the efficiency assumed in the model.

For the case of Iceland, not only is the model rejected for the first subperiod, but there is also a significant drop in the ability of the model to explain variation as measured by R^2. Prior to 1979 the Icelandic government changed the value of its currency often and by small amounts. Starting in 1981, however, a new policy was initiated with the currency tied to a trade-weighted basket of goods and the value of the currency changed at infrequent intervals and by large amounts. Unfortunately, since these changes seem to represent a move away from the assumptions of the model, it should be more likely to reject the model during the second subperiod then during the first.

Rejection of the model also occurs in the first subperiod in the case of Ireland. Like France, Ireland also joined the EMS in 1979. In addition, Ireland also broke its link to the British pound at about the same time. It is not clear, however, why these actions may have changed the properties of the black market as they seem to have done.

6. Conclusion

This paper has analyzed black-market exchange rates from two theoretical perspectives. In the first, black-market currency was viewed as an asset which allowed investors to invest in foreign currency based assets. Accordingly, its value was determined by the interest rate differential between countries and the expected future black-market exchange rate. While this approach has a certain appeal from an efficient markets perspective, it fails in empirical testing to provide a convincing explanation of the movement in the black-market exchange rate over time, and does not provide an explanation for the black-market premium.

In the consumption-based model of section three, black-market currency was introduced in order to allow agents to circumvent convertibility restrictions. While this approach proved to be empirically successfully for some countries, in other cases there was evidence of misspecification, possibly due to the existence of a time varying risk premium which is unobservable in the model.

In reality, the truth probably lies somewhere in between the two models. That is, the black-market is used for both consumption and investment purposes, with agent's demands for black-market foreign currency driven by both (expected) return differentials and consumption decisions. Of course, this is exactly the problem faced by models of the official exchange rate.

Endnotes

[1]An excellent reference on the logormal distribution is Atchinson and Brown (1969). The assumption that marginal utility and returns are lognormally distributed has been used previously in consumption-based asset pricing models, for example Hansen and Singleton (1983).

[2]See Martin and Penagariya (1984) for an analysis of smuggling when costs are uncertain. Uncertainty could be incorporated into the present model. Its main impact would be on equation (6) below, which would have to be written in expected value terms and include marginal utilities of consumption under the two alternatives. In this case, (6) would become

$$e_t^b = e_t[E_t(1/(1 - s_t))E_t(U_h(\cdot,\cdot)) + \mathrm{cov}(1/(1 - s_t),\ U_h(\cdot,\cdot))]/E_t(U_h(\cdot,\cdot)) \ .$$

[3]Boundness of the return function, $U(\cdot,\cdot)$, is sufficient to guarantee the existence of a value function in this case, although it is not necessary as the case of logarithmic utility proves in the standard growth model.

[4]For an exposition of the Schauder fixed point theorem see Stokey, Lucas and Prescott (1982). For an example of an economy involving endogenous state variables where equilibrium was proven in an alternative fashion see Lucas and Stokey (1985). While Stockman (1980) admitted to having a similar problem and did not prove the existence of an equilibrium, other open economy models of this type, e.g. Lucas (1982) and Svensson (1985), have avoided the existence problem by assuming that agents in both countries are symmetric so that complete risk sharing occurs and the equilibrium holdings of each agent are always half of the total available supply. In effect, these models prove existence by construction, which will not work in this model due to the asymmetric nature of the two economies.

[5]Dornbusch et al. (1983) cite a negative premium in the Brazilian black market as arising from a tight conversion policy where the banks would not purchase unlimited amounts of dollars from agents who were not supposed to be in possession of dollars. In this situation, the black market turns into a vehicle for "laundering" illegal dollars. Given conversion restrictions, a negative premium could also arise in a fixed exchange rate economy where agents were expected the authorities to revalue the currency.

References

Atchinson, J. and J. A. C. Brown: "The Lognormal Distribution," Cambridge University Press, 1969.

Bhagwati, Jagdish and Bent Hansen: "A Theoretical Analysis of Smuggling," The Quarterly Journal of Economics, May 1973, 172-187.

Dornbusch, Rudiger, Daniel V. Dantas, Clarice Pechman, Robert De Rezende Rocha, and Demetrio Simoes: "The Black Market for Dollars in Brazil," The Quarterly Journal of Economics, February 1983, 25-40.

Frenkel, J. A. and R. M. Levich: "Transaction Costs and Interest Arbitrage: Tranquil versus Turbulent Periods," Journal of Political Economy, November 1977.

Hansen, Lars P. and K. J. Singleton, "Stochastic Consumption, Risk Aversion, and the Temporal Behavior of Asset Returns," Journal of Political Economy, April 1983, pp. 249-265.

Lucas, Robert E.: "Interest Rates and Currency Prices in a Two-Country World," Journal of Monetary Economics 10, 1982, 335-359.

Lucas, R. E. and N. L. Stokey: "Money and Interest in a Cash-in-Advance Economy," manuscript, Northwestern University, 1985.

Macedo, Jorge Braga de: "Exchange Rate Behavior with Currency Inconvertibility," Journal of International Economics 12, 1982, 65-81.

Martin, L. and A. Pangariya: "Smuggling, Trade and Price Disparity: A Crime Theoretic Approach," Journal of International Economics, November 1984, pp. 201-217.

McCormick, Frank: "Covered Interest Arbitrage: Unexploited Profits? Comment," Journal of Political Economy, 1979, 411-417.

Michaely, Michael: "A Geometrical Analysis of Black-Market Behavior," American Economic Review, September 1954, 627-637.

Nowak, Michael: "Quantitative Controls and Unofficial Markets in Foreign Exchange," IMF Staff Papers, June 1984, 404-431.

Sheikh, Munir A.: "Black Market for Foreign Exchange, Capital Flows and Smuggling," Journal of Development Economics 3, 1976, 9-26.

Stockman, Alan: "A Theory of Exchange Rate Determination," Journal of Political Economy, 1980, p. 673.

Stokey, N. L., Lucas, R. E. and E. C. Prescott: "Recursive Methods in Economic Dynamics," manuscript, Northwestern University, 1982.

Svensson, Lars E.: "Money and Asset Prices in a Cash-in-Advance Economy," Journal of Political Economy, 1985, 919-944.

Svensson, Lars E.: "Currency Prices, Terms of Trade, and Interest Rates," Journal of International Economics 18, 1985, 17-41.

World Currency Yearbook, International Currency Analysis Inc., 1984.

Appendix

The coefficients in the text, c_i, are given by

$$c_1 = \omega Mp^*(1 - s)/pq^*$$

$$c_2 = \omega M/p$$

$$c_3 = p^*Ms'/pq^*$$

$$c_4 = Mk/p$$

$$c_5 = Mp^*(1 - s)k/pq^*$$

$$c_6 = p^*(1 - s)[s + s'(Y - (M/p)) - (N/p^*)]/q^*s'$$

$$c_7 = (L_2 - R_2)q^*/U_f s'$$

$$c_8 = p^*(1 - s)[(Y - (M/P)) + s'(N/p^*)^2/(1 - s - s'(N/p^*))]/q^*$$

$$c_9 = (L_2 - R_2)q^*/U_f(1 - s - s'(N/p^*))$$

$$c_{10} = [(Y - (M/P))p^*(1 - s) + Ns]/q^*$$

$$c_{11} = (L_2 - R_2)q^{*2}/U_f p^*(1 - s) ,$$

where $L_2 - R_2 = U_h/p + U_{hf}Mk/p^2 + U_f p^*Ms'/q^*p^2 - U_{ff}Mp^*(1 - s)k/q^*p^2$. This will be positive provided $U_{hf} \geq 0$. In what follows, this condition is assumed to hold. The constant K is defined in an appendix describing the linearization of (14) which is available from the author.

The coefficients c_1, c_2 and c_{10} are positive. The coefficients c_3 and c_6 are positive if $s' > 0$. Coefficients c_4, c_5, c_7 and c_{11} are positive if $k > 0$, which is true if $s + s'(Y - (M/P) - (N/p^*)) < 1$. Coefficients c_8 and c_9 are positive if $s + s'(N/p^*) < 1$.

Appendix A

In the model the agent solves

$$v(M, N) = \max_{h,f,M',N',\eta',z} \{U(h, f) + \beta v(M', N')\}$$

s.t.

$$ph \le M \quad (\mu_1)$$

$$q^*f \le N \quad (\mu_2)$$

$$M' + ph + pz + \eta'e \le pY + M \quad (\lambda_1)$$

$$N' + q^*f \le zp^*(1 - s) + \eta' + N \quad (\lambda_2)$$

$$\eta' \le \underline{N} \quad (\nu)$$

where the Greek letters in parentheses represent the corresponding Lagrange multipliers.

In what follows assume that $s(\bar{z}) = s$ (a constant) and that $\eta' = \underline{N}$ $(\nu > 0)$ so that the black market exists. This will be true if $\underline{N}$ is small compared to Y, given s.

Before stating the first-order conditions, notice that

(a) $M = M' = \underline{M}$ (money market equilibrium)

(b) $e^b = e(1/(1 - s))$ (black market equilibrium)

(c) $e = p/p^*$

and (d) $Y = h + z + \eta'/p^*$ (goods market equilibrium).

The first-order conditions are

(1) $U_h(\cdot,\cdot) = p(\mu_1 + \lambda_1)$

(2) $U_f(\cdot,\cdot) = q^*(\mu_2 + \lambda_2)$

(3) $\beta v_M(M', N') = \lambda_1$

(4) $\beta v_N(M', N') = \lambda_2$

(5) $\lambda_2 = e\lambda_1 + \nu$

(6) $p^*(1 - s)\lambda_2 = p\lambda_1$

(7) $v_M(M, N) = \mu_1 + \lambda_1$

(8) $v_N(M, N) = \mu_2 + \lambda_2$

(9) $M' + ph + pz + \eta' e = pY + M$

(10) $N' + q^*f = zp^*(1 - s) + \eta' + N$

(11) $ph \leq \underline{M}$ with equality if $\mu_1 > 0$

(12) $q^*f \leq N$ with equality if $\mu_2 > 0$

where (7) and (8) represent the envelope conditions and equality holds in (9) and (10) since, from (3) and (4), λ_1 and λ_2 are greater than zero.

The task in this section is to show that an equilibrium exists in which the choice variables and prices are stationary functions of the state variables

$$\Omega = \{Y, p^*, q^*, \underline{N}, N, \underline{M}, s\} .$$

Using the first-order conditions, (1) and (7) imply

$$U_h(\cdot,\cdot) = pv_M(\cdot,\cdot)$$

(2) and (8) imply

$$U_f(\cdot,\cdot) = q^*v_N(\cdot,\cdot)$$

and (3) and (4) imply

$$v_M(\cdot',\cdot') = \lambda_1/\beta, \quad v_N(\cdot',\cdot') = \lambda_2/\beta.$$

In the steady state, $M = M'$ and $N = N'$, so that the preceding equations imply

$$\frac{U_h(h,\ f)}{U_f(h,\ f)} = \frac{p\lambda_1}{q^*\lambda_2} .$$

From equation (6) we find that

$$\lambda_1/\lambda_2 = p^*/p(1 - s)$$

so that the marginal rate of substitution can be expressed as

(13) $$\frac{U_h(h, f)}{U_f(h, f)} = \frac{p^*}{q^*}(1 - s) .$$

Given Ω and $U(\cdot,\cdot)$, (13) defines a relationship between h and f. Write this as

(14) $$f = f(\Omega, h) .$$

Solving (d) now for z gives

(15) $$z = z(\Omega, h)$$

and substituting this into (9) shows that

$$M' = M = \underline{M}$$

so that the money market condition is satisfied. Substituting (15) for z in (10) gives

$$N' + q^*f = p^*(1 - s)Y - p^*h(1 - s) - \eta's + N .$$

Using (14) and the preceding equation allows us to write

(16) $$N' = N'(\Omega, h) .$$

By assumption we have

(17) $$\eta' = \underline{N} = \eta'(\Omega) .$$

It remains then to show that h, p, e, and e^b are functions of Ω. In the steady state in a static model we know that (11) and (12) can be replaced by

(11') $$ph = \underline{M}$$

(12') $$q^*f = N .$$

Using (14), (12') implies that

$$q^*f(\Omega, h) = N$$

defines h as a function of Ω

(18) $$h = h(\Omega) .$$

From (11') then we have

(19') $$p = \underline{M}/h = p(\Omega)$$

which implies that

(20) $$e = p/p^* = e(\Omega)$$

and

(21) $$e^b = e[1/(1 - s)] = e^b(\Omega) .$$

We have shown that in the static case there exists an equilibrium steady state consisting of time invariant functions of the state variables. We now present an example of a steady state when preferences are logarithmic.

Example: Logarithmic Preferences

Assume that the preferences of agents are given by

$$U(h, f) = \log h + \log f .$$

The marginal utility functions are thus

$$U_h(\cdot,\cdot) = 1/h , \quad U_f(\cdot,\cdot) = 1/f .$$

Therefore, the relationship between domestic and foreign consumption, (14), is given by

$$f = (p^*/q^*)(1 - s)h .$$

Equation (15) is obtained from (d) and is given by

$$z = Y - h - \eta'/p^* .$$

In a similar fashion, (16) defines N' as

$$N' = Yp^*(1 - s)d + \eta's + N - 2hp^*(1 - s)$$

where $\eta' = \underline{N}$ by assumption.

From (12') and (14) we learn that

$$h = (N/p^*)\left(1/(1 - s)\right)$$

so that

$$f = (p^*/q^*)(1 - s)(N/p^*)\left(1/(1 - s)\right) = N/q^* .$$

This, together with (11') implies that

$$p = \underline{M}/h = (\underline{M}/N)p^*(1 - s)$$

so that

$$e = p/p^* = (\underline{M}/N)(1 - s)$$

and

$$e^b = e\left(1/(1 - s)\right) = \underline{M}/N .$$

Returning to the formula for N' we find that

$$N' = Yp^*(1 - s) + \underline{N}s - N .$$

In the steady state, N = N' implies

$$N = N' = (\tfrac{1}{2})\left[Yp^*(1 - s) + \underline{N}s\right] .$$

Solving now for z we learn that

$$z = (\tfrac{1}{2})\left[Y - (\underline{N}/p^*)\left(1/(1 - s)\right)\right] .$$

Our assumption that $\eta' = \underline{N}$ will be true whenever $z > 0$, which is true in the steady state provided

$$Y - (\underline{N}/p^*)\left[1/(1 - s)\right] > 0$$

or

$$\underline{N} < Yp^*(1 - s).$$

Appendix B

Linearization of the left-hand side of (14) produces

$$U_h + L_1(\underline{M}_t - \underline{M}) + L_2(p_t - p) + L_3E_t(p_{t+1} - p) + L_4(p_t^* - p^*) + L_5(Y_t - Y)$$

$$L_6(\underline{N}_t - \underline{N}) + L_7(\omega_{t+1} - \omega) + L_8E_t(q_{t+1}^* - q^*)$$

where

$$L_1 = U_{hh}\frac{\omega}{p} + U_{hf}\frac{1}{q^*}\left[(Y - \frac{M}{p})\frac{p^*}{p}s' - \frac{p}{p^*}(1 - s) - \frac{Ns'}{p}\right]$$

$$L_2 = U_h\frac{1}{p} + U_{hf}\frac{M}{p^2}k$$

$$L_3 = -U_h\frac{1}{p} - U_{hh}\frac{\omega M}{p^2}$$

$$L_4 = U_{hf}\frac{1}{q^*}\left[(Y - \frac{M}{p})[1 - s - s'\frac{N}{p^*}] + s'\frac{N^2}{p^{*2}}\right]$$

$$L_5 = U_{hf}k$$

$$L_6 = U_{hf}\frac{1}{q^*}\left[s + s'(Y - \frac{M}{p} - \frac{N}{p^*})\right]$$

$$L_7 = U_{hh}\frac{M}{p}$$

$$L_8 = -U_{hf}\frac{1}{q^{*2}}\left[(Y - \frac{M}{p})p^*(1 - s) + \underline{N}s\right] .$$

where

$$k = \frac{p^*}{q^*}\left[1 - s - s'[Y - \frac{M}{p} - \frac{N}{p^*}]\right]$$

$$s = s(\cdot) \text{ and } s' = s'(\cdot)$$

and all functions are evaluated at the steady state values.

Linearization of the right-hand side of (14) produces

$$U_f \frac{p^*(1 - s)}{q^*} + R_1(\underline{M}_t - \underline{M}) + R_2(p_t - p) + R_3E_t(p_{t+1} - p) + R_4(p^*_t - p^*) +$$

$$R_5(Y_t - Y) + R_6(\underline{N}_t - \underline{N}) + R_7(\omega_{t+1} - \omega) + R_8E_t(q^*_{t+1} - q^*)$$

where

$$R_1 = U_f \frac{p^*s'}{q^*p} + U_{fh} \frac{p^*(1 - s)\omega}{q^*p} - U_{ff} \frac{p^*(1 - s)}{q^*p} k$$

$$R_2 = -U_f \frac{p^*M}{q^*p^2} s' + U_{ff} \frac{M}{p^2} \frac{p^*(1 - s)}{q^*} k$$

$$R_3 = -U_{fh} \frac{\omega M}{p^2} \frac{p^*(1 - s)}{q^*}$$

$$R_4 = U_f \frac{1}{q^*} (1-s- \frac{s'N}{p^*}) + U_{ff}[\frac{p^*(1 - s)}{q^{*2}}][(Y - \frac{M}{p})(1-s) - \frac{s'N}{p^*} [Y - \frac{M}{p} - \frac{N}{p^*}]]$$

$$R_5 = -U_f \frac{p^*s'}{q^*} + U_{ff} \frac{p^*}{q^*} (1 - s)k$$

$$R_6 = U_f \frac{s'}{q^*} + U_{ff} \frac{p^*(1 - s)}{q^{*2}} [s + s'(Y - \frac{M}{p} - \frac{N}{p^*})]$$

$$R_7 = U_{fh} \frac{Mp^*(1 - s)}{pq^*}$$

$$R_8 = -U_f \frac{p^*(1 - s)}{q^{*2}} - U_{ff} \frac{p^*(1 - s)}{q^{*3}}[(Y - \frac{M}{p})p^*(1 - s)\underline{N}s] .$$

Equating the two sides and noting that the first terms on each side are equal due to the agent's first-order conditions gives

$$p_t-p = \frac{1}{L_2 - R_2}\{(R_1- L_1)(\underline{M}_t- \underline{M}) + (R_3- L_3)E_t(p_{t+1}- p) + (R_4-L_4)(p^*_t - p^*)$$

$$+ (R_5-L_5)(Y_t-Y) + (R_6-L_6)(\underline{N}_t-\underline{N}) + (R_7-L_7)(\omega_{t+1}-\omega) + (R_8-L_8)E_t(q^*_{t-1}-q^*)\}$$

$$\equiv \ell_1(\underline{M}_t - \underline{M}) + \ell_2 E_t(p_{t+1} - p) + \ell_3(p_t^* - p^*) + \ell_4(Y_t - Y) + \ell_5(\underline{N}_t - \underline{N})$$

$$+ \ell_6(\omega_{t+1} - \omega) + \ell_7 E_t(q_{t+1}^* - q^*) \ .$$

Substituting forward for p_{t+1}, p_{t+2}, etc., produces (15).

Table 1

Black-Market Premiums for the U.S. Dollar - December 1983

Malaysia	1%	Belize	43%
Italy	2%	Philippines	50%
Norway	2%	Egypt	66%
Sweden	2%	Ethiopia	79%
Israel	3%	Paraguay	178%
Peru	3%	Sao Tome	189%
Ireland	4%	Zimbabwe	192%
Spain	4%	Iraq	235%
Bermuda	5%	Bulgaria	289%
Portugal	7%	Bolivia	300%
Barbados	8%	Iran	320%
S. Korea	8%	E. Germany	411%
Yem A.R	8%	Nigeria	457%
Botswana	10%	USSR	480%
Argentina	11%	Albania	809%
Turkey	11%	Poland	815%
France	13%	Nicaragua	845%
Greece	17%	Romania	1255%
Yugoslavia	23%	Laos	1360%
India	28%	Vie Nam	1606%
Iceland	29%	Cuba	2038%
Brazil	37%	N. Korea	2304%

Source: World currency Yearbook (1984)

Table 2

$$\log(e^b_{t+1}/e^b_t) = \alpha + \beta \log[(1 + i_t)/(1 + i^*_t)] + \varepsilon_{t+1}$$

Country	α	β	R^2	G	Ser(4)
Brazil	0.054 (0.006)	3.983 (1.420)	0.06	1.054 (1.531)	0.922 (0.453)
France	0.009 (0.005)	-2.039 (1.603)	0.01	-2.112 (1.703)	0.548 (0.700)
Ireland	0.006 (0.005)	-0.093 (1.080)	0.00	-2.174* (0.738)	0.162 (0.956)
Italy	0.008 (0.007)	-0.444 (1.094)	0.00	-1.393 (0.722)	0.688 (0.601)
Norway	0.001 (0.002)	0.627 (0.955)	0.00	0.985 (1.164)	0.182 (0.947)

Standard errors in parentheses, except for Ser(4) where the number in parentheses is the level of significance.

* - significant at the 5% level

G is a Glesjer test for heteroscedasticity

Ser(4) ~ $F(4,109)$, Ser(4) tests the hypothesis that the first 4 autocorrelation coefficients are zero.

Table 3

$$\ln(e^b_{t+1}/e_t) = \alpha + \beta \ln(e^b_t/e_t) + \varepsilon_{t+1}$$

Country	α	β	G	Ser(4)	R^2
Brazil	0.032 (0.011)	1.055 (0.046)	0.099** (0.027)	1.033 (0.393)	0.81
France	0.004 (0.004)	1.027 (0.090)	0.028 (0.064)	0.454 (0.769)	0.52
Greece	0.033 (0.006)	0.599 (0.101)	0.332** (0.076)	1.126 (0.348)	0.22
Iceland	-0.025 (0.043)	0.984 (0.017)	-0.011 (0.019)	0.081 (0.987)	0.96
India	0.032 (0.010)	0.796 (0.063)	0.108** (0.040)	2.479* (0.048)	0.57
Ireland	0.001 (0.008)	1.004 (0.007)	-0.003 (0.004)	0.149 (0.963)	0.99
Italy	1.195 (0.320)	0.743 (0.069)	0.073 (0.046)	2.702* (0.034)	0.49
Norway	0.010 (0.002)	0.251 (0.161)	0.086 (0.136)	0.237 (0.916)	0.02
Philippines	0.005 (0.007)	1.046 (0.078)	0.444 (0.054)	2.624 (0.038)	0.60
Portugal	0.036 (0.008)	0.730 (0.069)	0.166** (0.050)	0.516 (0.723)	0.48

Standard errors in parentheses, except for Ser(4) where the number in parentheses is the level of significance.

* - significant at the 5% level

** - significant at the 1% level

Ser(4) ~ χ^2_4

Table 3A

$$\ln(e^b_{t+1}/e_t) = \alpha + \beta \ln(e^b_t/e_t) + \varepsilon_{t+1}$$

Country		α	β	G	Ser(4)	R^2	C
Brazil	I	0.046 (0.014)	0.879 (0.069)	-0.015 (0.042)	0.465 (0.760)	0.73	9.365 (0.000)*
	II	0.052 (0.017)	1.057 (0.060)	0.099 (0.033)*	1.239 (0.306)	0.84	
France	I	0.002 (0.005)	0.464 (0.253)	-0.036 (0.199)	2.465 (0.056)	0.05	4.888 (0.009)*
	II	0.018 (0.009)	0.906 (0.131)	-0.036 (0.089)	0.178 (0.948)	0.45	
Greece	I	0.032 (0.005)	0.335 (0.119)	0.033 (0.079)	0.717 0.583	0.11	7.382 (0.000)*
	II	0.053 (0.012)	0.508 (0.149)	0.317 (0.114)*	0.056 (0.993)	0.16	
Iceland	I	0.110 (0.034)	0.746 (0.086)	0.003 (0.057)	0.935 (0.450)	0.56	2.978 (0.054)
	II	-0.161 (0.116)	0.954 (0.033)	-0.056 (0.032)	0.057 (0.993)	0.93	
India	I	0.037 (0.016)	0.737 (0.104)	0.123 (0.069)	3.342 (0.016)*	0.46	0.861 (0.425)
	II	0.033 (0.013)	0.820 (0.079)	0.089 (0.049)	3.302 (0.017)*	0.64	
Ireland	I	-0.018 (0.014)	0.982 (0.015)	0.005 (0.009)	0.413 (0.798)	0.98	1.524 (0.222)
	II	0.002 (0.014)	1.005 (0.010)	-0.014 (0.006)*	0.161 (0.956)	0.99	
Italy	I	0.937 (0.403)	0.798 (0.086)	0.100 (0.060)	2.168 (0.086)	0.59	2.315 (0.103)
	II	3.919 (1.185)	0.153 (0.256)	-0.284 (0.154)	1.763 (0.151)	0.00	

Table 3A (cont'd.)

Norway	I	0.004 (0.004)	0.257 (0.233)	0.230 (0.210)	0.732 (0.573)	0.02	3.483 (0.033)*
	II	0.018 (0.003)	0.139 (0.211)	-0.097 (0.158)	0.885 (0.479)	0.00	
Philippines	I	0.022 (0.007)	0.670 (0.106)	0.029 (0.075)	1.922 (0.121)	0.40	2.68 (0.131)
	II	0.009 (0.013)	1.080 (0.113)	0.497 (0.074)*	3.135 (0.022)*	0.61	
Portugal	I	0.043 (0.016)	0.683 (0.100)	0.119 (0.069)	0.271 (0.854)	0.44	0.810 (0.146)
	II	0.022 (0.008)	0.939 (0.109)	0.183 (0.074)*	2.165 (0.086)	0.56	

Standard errors in parentheses, except for Ser(4) and C where the number in parentheses is the level of significance.

* - significant at the 5% level

Ser(4) ~ χ^2_4

G is a Glesjer test for heteroscedasticity

C is a Chow test for equality of coefficients across time periods and is distributed F(2,115)

The two periods of estimation are 1974-1978 and 1979-1983.