PROGRAM TRADING AND THE BEHAVIOR OF STOCK INDEX FUTURES PRICES

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ABSTRACT

This study examines intraday transaction data for S&P 500 stock index futures prices and the intraday quotes for the underlying index. The data indicate that the futures price changes are uncorrelated, and that the variability of these price changes exceeds the variability of price changes in the S&P 500 index. This excess variability of the futures over the index remains even after controlling for the nonsynchronous prices in the index quotes, which induces autocorrelation in the index changes. We advance and examine empirically two hypotheses regarding the difference between the futures price and its theoretical value: that this "mispricing" increases on average with maturity, and that it is path dependent. Evidence supporting these hypotheses is presented.

1. Introduction

The spectacular growth in the volume of trading in Stock Index Futures contracts reveals the interest in these instruments that is shared by a broad cross-section of market participants. It is generally agreed that the linkage in prices between the underlying basket of stocks and the futures is maintained by arbitrageurs. If this link is maintained effectively, then investors who are committed to trade will recognize these markets as perfect substitutes, and their choice between these markets will be dictated by convenience and their transaction costs. However, researchers have reported substantial and sustained deviation in futures prices from their theoretical values: indeed, Rubinstein (1987, p. 84) concludes that "[t]he growth in index futures trading continues to outstrip the amounts of capital that are available for arbitrage."

Considerable attention has been focused on arbitrage strategies involving stock index futures, and on their effects on markets; especially on the expiration dates of these contracts. By contrast, there is little work on the stochastic behavior of the deviation of futures prices from fair values. In this paper, we study transaction data on Standard and Poor's 500 futures contracts in conjunction with minute by minute quotes of the S&P 500 Index. Our goal is to examine the behavior of these prices in light of the conventional arbitrageur's strategies.

It should be emphasized at the outset that it is extremely difficult to specify a model for the deviations of futures prices from "fair values" -- these deviations are, presumably, affected by the flow of orders as well as by the difference of opinion among participants regarding parameters of the valuation model that provides "fair values." The purpose of this paper is to examine the empirical behavior of these deviations: in doing so we examine the validity of certain proposed hypotheses regarding the stochastic behavior of these deviations, given that market participants will attempt to exploit these as profit opportunities.

In Section 2, we discuss some considerations of the behavior of futures and index prices after describing the well known and commonly used pricing model. Section 3 provides the empirical results, and we conclude in Section 4.

2. Arbitrage Strategies and the Behavior of Stock Index Futures Prices

The arguments underlying the valuation of derivative assets exploit the availability of a replicating portfolio of existing assets whose value coincides with the price of the derivative security at its expiration date. In frictionless markets the availability of a perfect substitute for the derivative asset guarantees that a profit opportunity, if one surfaced, would attract "arbitrageurs" who would quickly close the gap between the price of the asset and of its substitute. The presence of transaction costs implies that the price of the derivative asset could fluctuate within a band around its theoretical value without representing a potential profit opportunity. The width of this band would be dictated by the transaction costs of the most favorably situated arbitrageurs. In the context of the daily settlement prices of stock index futures contracts, this has been examined by Modest and Sundaresan (1983). However, the band could also be affected by the fact that the replicating portfolio of existing assets serves only as a close substitute, and that the temporal behavior of the spread between the market price and a model value is further influenced by alternative trading strategies that will be employed by arbitrageurs. We examine these issues in this section.

It is well known, from the work of Black and Scholes (1973), that the replicating portfolio for an option involves a dynamic, self-financing trading rule which depends on the unobservable volatility parameters for the stochastic process of the underlying asset's price. However, for a forward contract, the replicating portfolio involves a buy-and-hold strategy which, in the absence of random payouts from the underlying asset, depends only on observable quantities. The differences between forward prices and futures prices have been studied extensively (see, for example, Black (1976), Cox, Ingersoll and Ross (1981), Richard and Sundaresan (1981), Jarrow and Oldfield (1981) and French (1984)). With nonstochastic interest rates, forward and futures prices will be equal; however, the replicating portfolio for futures contracts will involve a dynamic trading rule even in this case.[1] In practice, it is generally argued that differences in forward and futures prices are small enough to be safely ignored; indeed, many programs that seek to arbitrage the price differences by trading in stock index futures and in the basket of stocks representing the index employ the forward pricing model adjusted for transaction costs. We begin by examining, briefly, this model for forward prices on stock index portfolios (with and without transaction costs) and we draw implications for the behavior of futures prices over a contract's life.

Forward Contracts on Stock Indexes (No transaction costs)

Consider a forward contract on an index of stocks, where the index represents a capitalization weighted basket of stocks and is a feasible buy and hold portfolio. Assume that markets are perfect and frictionless; that any performance bonds necessary to take a position in the forward market can

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[1] See Cox, Ingersoll & Ross (1981), p. 340.

be posted in interest-bearing assets; that borrowing and lending takes place at the (constant) continuously compounded rate r; and that the basket of stocks representing the index pays dividends continuously at the rate d. Consider the following portfolio, constructed at date t and held until the forward contract expires, at date T:

(A) Buy the basket of stocks at the price S_t (the index price at date t) and continuously reinvest the dividends received until date T;

(B) Borrow $\$S_t$ at t to finance the acquisition in (1);

(C) Sell a forward contract at the currently quoted forward price $G_{t,T}$.

This portfolio is costless at t; and to avoid certain losses or gains at T, it can be shown that

$$G_{t,T} = S_t e^{(r-d)(T-t)} \quad . \tag{1}$$

If the forward price at market $G^m_{t,T}$ is greater than $S_t e^{(r-d)(T-t)}$, then a strategy that buys the index and sells forward contracts will earn riskless profits in excess of the risk-free rate r. If $G^m_{t,T}$ is less, then a strategy that sells the index and buys futures contracts will achieve a financing rate below the risk-free rate.[2]

The Impact of Transaction Costs

Stoll and Whaley (1986) discuss the impact of transaction costs on the index-futures arbitrage strategy, starting with the forward contract pricing relation shown above. The impact of transactions costs is to permit the

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[2]In this case, investors who already own the basket of stocks represented in the index portfolio are in the best position to undertake the arbitrage; investors who are not already in possession of the index basket would be forced to sell stocks short and would be subject to the "up-tick" rule for short sales. For a more complete description of these strategies, see Gould (1987); and for analysis of the hedging costs and effectiveness see Merrick (1987).

futures price to fluctuate within a band around the formula value in relation (1). The width of the band derives from round trip commissions in the stock and futures markets, and the market impact costs of putting on the trade initially. The market impact costs of closing the stock position can be avoided by holding the position until expiration of the futures contract and employing market-on-close orders.[3]

We consider two issues related to this view. First, this line of argument says that the mispricing around the formula value (the band) should not exceed a value (dictated by transaction costs) which is constant over the life of the contract. That is, if the transaction costs are independent of the remaining maturity for the contract, then the width of the band should not vary over time.[4] Second, this argument provides no role for the arbitrageurs when futures prices lie within the band; there is no influence on the trajectory of the futures price as long as it does not stray from its transaction cost based limits. Consider the commonly defined "mispricing,"[5]

$$x_{t,T} = [F_{t,T} - S_t e^{(r-d)(T-t)}]/S_t \quad (2)$$

which is the difference between the market futures price of the stock index

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[3]Beginning with the June 1987 contract, the expiration has shifted to using the opening index price as the cash settlement price for the futures. Therefore, reversal of stock positions would employ market-on-open orders, and these orders also avoid market impact costs.

[4]Modest and Sundaresan (1983) argue that if arbitrageurs lose the interest earnings on a fraction of the proceeds of the short-sale of stocks when their strategy calls for shorting stock, then the band would be asymmetric around the "fair" price and would be wider with more time remaining. However, arbitrageurs rarely use short positions in program driven strategies, because of the up-tick rule; they generally employ the pools of stock they own or control if the futures are underpriced.

[5]We use this term only because we lack a less clumsy alternative--we do not mean to imply that every non-zero level of the "mispricing" is evidence of market inefficiency.

futures contract ($F_{t,T}$) and its theoretical price assuming it is a forward contract, normalized by the index value.[6] The transaction cost limits for $x_{t,T}$ would be given by the sum of the commission costs in stock and futures markets, plus the market impact cost of trading initially in the stocks and in the futures. Sustained deviations of $x_{t,T}$ outside these limits would be evidence of the lack of arbitrage capital. This view implies that $x_{t,T}$ should be clipped above and below at these limits but provides no guidance with respect to its behavior within the boundaries.

We argue that larger deviations in $x_{t,T}$ can persist outside these transaction cost limits for longer times until expiration (T - t). This may occur for several reasons. First, with longer times until expiration, there is increased risk of unanticipated increases or decreases in dividends. These will erode the anticipated profits from an attempt to arbitrage $x_{t,T}$ when it violates these limits. Put another way, programs that seek riskless profits should account for worst case dividend policies. Second, the difference between futures and forward prices, which is embedded in the definition in (2) reflects the unanticipated interest earnings or costs from financing the marking-to-market flows from the futures position. An attempt to replicate the futures contract payoff will require trading in the stocks; and both these will contribute to a wider limit for $x_{t,T}$ with greater times to expiration. Finally, attempts at arbitrage-motivated trading which employ less than the full basket of stocks in the index must allow for a greater margin of error with longer times to expiration. This would arise not only because of the possibility that the value of chosen basket might not track the index

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[6] We work with the mispricing in relative terms because the major components of the determinant of the bounds should be proportional to the level of the index.

accurately, but also because costly adjustments would be necessary prior to expiration. Consequently, wider deviations in $x_{t,T}$ will be required at longer times to maturity to induce arbitrageurs to take a position in these markets. These considerations point up the fact that the "arbitrage" strategies are not risk-free.

There are countervailing forces that serve to provide a narrower trading band, and they stem from the fact that arbitrageurs have the option to either reverse their positions prior to the expiration date, or to roll forward their futures position into the next available maturity.[7] To see this, suppose an arbitrageur views the random mispricing $x_{t,T}$ as an arbitrageable sequence whose current level is observable. She knows that the mispricing disappears at date T so that $x_{T,T} = 0$. The arbitrage strategy conventionally considered is to sell $x_{t,T}$ if it is positive at date t and reverse the position at T, or to buy it if $x_{t,T}$ is negative and to reverse that position at date T, as long as

$$|x_{t,T}| > TC_1 \tag{3}$$

where

TC_1 = Roundtrip Stock Commission + Roundtrip Futures Commission + Market Impact in Futures + Market Impact in Stocks .[8]

However, the arbitrageur knows that it is possible for her, at a future date $s < T$ to (a) reverse her position by paying a market impact cost in both stock and futures markets, or (b) to roll her futures position into the next

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[7] Brennan and Schwartz (1986), (1987) make the argument, as we do here, that the arbitrageurs have the option to close out a position prematurely.

[8] This assumes that the transaction costs are the same for long and short positions in futures, and for purchases and sales in stocks. It is not crucial to the analysis.

maturity and incur commissions and market impact costs only in the futures market. Therefore, the optimal band at which to undertake an opening arbitrage position would be narrower than the optimal band in the absence of strategies (a) and (b). This is because at the current date t, with an arbitrage program trade already in place, the arbitrageur benefits from the option value of closing her position prematurely at perhaps a greater profit than indicated by the current level $x_{t,T}$. Given these two arguments, it is important to examine empirically the behavior of the deviations as a function of time to maturity. We consider this in the next section.

However, this option argument has a further implication. Once an arbitrage trade has been put on, it will be optimal to close that position prior to putting on a new arbitrage program in the reverse direction. Suppose that we had put on an arbitrage at date t (in the past) when $x_{t,T} > 0$, by buying the index basket and selling futures short. Then, to initiate an offsetting trade at the current date, we would incur additional costs TC_2 which is simply the sum of market impact costs in stock and futures markets. If we were to undertake this as a net new position we would need to cover higher costs TC_1. The implication is the stochastic behavior of the mispricing will display properties, over the next interval, that depend on the history of the mispricing until that point. Suppose the historical trajectory of the mispricing $x_{t,T}$ has been positive and large. Then arbitrageurs who had undertaken positions long in index stocks and short futures will undo them when at date $s > T$, the mispricing $x_{s,T}$ had fallen to some negative value. The magnitude of this value will depend upon, among other things, the additional transaction costs from closing the position prematurely (TC_2). If the mispricing never "corrects" itself over the life of the contract, then the burden of the reversing trades will fall at the close of the expiration date. In fact, the direction of market-on-close (or, since the June 1987 expiration, market-on-open) orders on expiration days may be predictable only if the history of the mispricing indicates that the arbitrageurs took positions that were all on one side of the market and did not have the opportunity to reverse or roll forward these positions profitably. In the empirical section, we consider the hypothesis that the behavior of $x_{t,T}$ over time displays non Markov properties: its distribution over the future is dependent on its path in the past.

3. Empirical Evidence

In this section, we present evidence using the intraday prices for the Standard and Poor's 500 futures contract and for the underlying index. This evidence deals with the behavior of the futures and index prices and with the hypotheses regarding the behavior of the mispricing series $x_{t,T}$. We begin first by describing the data employed.

3.1 Data

The futures price data base, obtained from the Chicago Mercantile Exchange, consists of time stamped transaction data for transactions in the S&P 500 futures contracts from April 1982 (the inception of trading) to June 1987. The contracts traded follow the March, June, September, December cycle -- although the nearest contract is typically the most heavily traded. Each transaction record contains, in addition to contract identification, time stamp and price, information that tags that transaction as a sale, a bid, or an offer, and whether it was cancelled, corrected, or a designated open or a close. The size (number of contracts) of the transaction is not available.

These transaction data record only transactions with price changes. Because the trading occurs by open outcry in a continuous market, the time stamping of a consummated transaction will lag by a few seconds, and perhaps by more in periods of heavy trading. In those periods particularly, it is possible to have the records stamped out of sequence. We observed several transactions subsequent to 9:00 a.m. (subsequent to 8:30 a.m. after October 1, 1985) occurring before the transaction that was labelled as the open -- this signified the end of the opening "round" of transactions. Likewise, we observed transactions after the first designated closing "round" transactions, occurring after 3:14 p.m. Almost always, these open and close designated records are also marked as representing a sale (as opposed to a bid or offer).

The data base of S&P 500 Stock Index quotes, time-stamped approximately one minute apart, was also provided to us by the Chicago Mercantile Exchange. This index is updated continuously using transaction prices (the most recent prices as reported) of the component 500 stocks. This data base captures these quotations approximately 60 seconds apart. While traders on the floor of the CME have access to continuously updated series, the index series available contains stale prices, especially for the thinly traded stocks; and the quoted index fails to use the bid or offer side of the market, so that the price at which one can buy or sell the index basket might be higher or lower than the quoted value. These facts must be kept in mind when working with the mispricing series.

In computing the mispricing series, we use quotes that are approximately 15 minutes apart, and we employ the nearest quotes available <u>after</u> the quarter-hour mark. Each contract is followed from the expiration date of the previous contract until its expiration. Because the near futures are heavily traded, and our stock index quotes are clocked one minute apart, this means that our futures price will be stamped almost immediately following the quarter hour mark, while the index quote will be, on average, 30 seconds after the quarter hour mark. The mispricing computed from these quotes will be biased, perhaps slightly, in favor of signalling potential profit opportunities. Conversations with market makers suggest that the time taken to put on a simultaneous program in stock and futures market depends, among other things, on the size of the trade, the composition of the stock basket, and on the depth of the market -- the estimates range from 60 seconds to several minutes. Usually, the futures leg can be executed very quickly. This means that, because we do not compute a separate series that would represent executable profits after recognizing a profit opportunity, the constructed series may be inappropriate to judge the actual profitability of program trades.

In order to construct the mispricing series $x_{t,T}$, we require dividend forecasts for the 500 stocks in the index, and a measure of the interest rate for loans maturing at the expiration date of the futures. We use the realized daily dividend yield of the value weighted index of all NYSE stocks supplied by the Center for Research in Security Prices at the University of Chicago as a proxy for the yield of the S&P 500.[9] Given that the S&P 500 is also value weighted, the CRSP value weighted dividend yield should be a reasonable proxy. Furthermore, given that the average maturity of our futures contracts is 1½ months, the error in employing this series is likely to be quite small. The daily interest data for Treasury Bills and for Certificates of Deposits expiring around the expiration date were kindly supplied by Kidder, Peabody. Throughout the paper we report results using the rates for

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[9]For the March and June 1987 contracts, the CRSP data are not available. We used the daily forecasts of dividend yield (remaining until expiration) on the S&P 500 provided to us by Kidder, Peabody.

Certificates of Deposits: results using the Treasury bill interest rate were also calculated and similar to those with the CD's.

The mispricing series so constructed is available for every quarter hour mark until 4:00 p.m. EST, although the S&P 500 futures contracts continue to trade until 4:15 p.m. This series is constructed starting with the September 1983 contract: we avoided using earlier contracts because prior studies had reported unusual behavior for these.[10] In Section 3.2, we report on the behavior of the futures and index series used to construct the mispricing. Section 3.3 considers the behavior of the actual mispricing series.

3.2 Behavior of futures and index series

In this section we examine the behavior of futures prices and index prices for each of the 16 contracts from September 1983 through June 1987. We present evidence on the autocorrelations and on the variability of futures prices and index prices: our focus is (1) on the extent to which nonsynchronous (or stale) prices are a problem in available index values, and (2) on the relative variability of the prices in two markets.

The results reported in this section employ first differences in the logarithm of the futures price and in the logarithm of the index value over the appropriate interval. By varying the interval length (we use 15, 30, 60, 120 minutes and one trading day) we can assess the importance of stale prices in the index quotes.

Table 1 reports the autocorrelation estimates at eight lags for the price changes for both the futures and the spot series using the 15 minute interval. They are computed from intraday intervals only: overnight and weekend intervals are discarded. The futures series is uncorrelated at all 8

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[10] See for example Figlewski (1984).

lags. By contrast, the index series is positively autocorrelated at the first lag, with first order autocorrelations ranging from 0.038 to 0.41 across the 16 contracts. At lags beyond the first, the index series exhibits autocorrelations close to 0. These results confirm that stale prices are present in the available index quotes. It is noteworthy that the problem of stale prices has diminished over time: the first order autocorrelations for the index series are noticeably smaller in the recent past. This finding may be attributed to increased stock market trading volume in recent years.

The autocorrelations for longer differencing intervals (except for one trading day) are based on estimates which exploit the overlapping nature of the data: these estimators are formed as a function of the estimators for 15 minute intervals. For example, the first order autocorrelation for the 60 minute differencing interval, $\rho_{60}(1)$, is given by

$$\rho_{60}(1) = \frac{\rho(1) + 2\rho(2) + 3\rho(3) + 4\rho(4) + 3\rho(5) + 2\rho(6) + \rho(7)}{4 + 6\rho(1) + 4\rho(2) + 2\rho(3)}$$

where $\rho(j)$ = jth order correlation for 15 minute intervals. Given that our basic series uses 15 minute intervals, this estimator is efficient in exploiting the degree of overlap. We do not report higher order autocorrelations for the longer differencing intervals.

Panels B, C, D, and E of Table 2 present the first order autocorrelations for the longer differencing intervals of 30, 60, 120 minutes and 1 trading day, respectively. Two results emerge from these panels. First, the problem of nonsynchronous data in the index series is mitigated by employing longer differencing intervals: at 30 minutes, the first order autocorrelation in the index series (Panel B) is much smaller than the first order autocorrelation at 15 minutes (Panel A), and the corresponding value at 60 minutes (Panel C) is close to 0. Second, whenever the first order autocorrelation for the index series is high (for any contract, over the longer differencing intervals), the autocorrelation for the futures series also tends to be high. For example, the 120 minute interval data in Panel C provides evidence that, for the September 84 contract, the index series' autocorrelation was 0.19, but the futures autocorrelation was also high at 0.17. This indicates that nonsynchronous data are not the sole source of autocorrelation.

We turn now to the variability of the two data series. If arbitrageurs maintained the link between these markets, then the variability of the two series should be equal: this is in keeping with the "redundant security" view, and it is consistent with the implication from a forward pricing model, as long as interest rates and dividends were nonstochastic. Furthermore, if differences in transaction costs are large between these two markets, it is possible that new information is incorporated with greater speed in one market relative to the other. Therefore, these differences would exhibit themselves when we examine the variability of the two series, especially for the smaller differencing intervals.

Panels A through E of Table 2 report the standard deviations for the two series for the five observation intervals. Panel A reports the standard deviation for the basic 15 minute differencing interval, as well as the variance ratio for the futures and index series. The standard deviations of the futures series are all higher than those for the index, but this might be solely due to nonsynchronous prices rather than a structural feature of the data. The results for longer intervals, in panels B through E, serve to resolve this issue.[11]

If the variability of the two markets is equal, then as the observation interval is lengthened, the variance ratio of the futures series to the index series should approach 1. However, that is not the case. The ratio in most cases is above 1 for all intervals. Table 3 reports some aggregated evidence of the higher variability of the futures market. The average variance ratio for the sixteen contracts is presented for each interval length. The average drops considerably from the 15 minute interval average of 1.56 to the 60 minute interval average of 1.16 but remains flat from the 60 minute interval to the one trading day interval. We can test the hypothesis that the ratio equals 1 by treating the ratios as being independent across contracts. The z-statistics for this test are reported in Table 3. The smallest z-statistic is 4.40 for the 120 minute interval which supports the hypothesis that the futures market is more variable than the spot market.

3.3 The behavior of the mispricing series

We now examine the behavior of the mispricing series. Data for two contracts, December 1984 and March 1987 (employing data 15 minutes apart) are plotted in Figure 1. The graphs display some sharp reversals in the mispricing levels, but the tendency is for the series to stay above or below 0 for substantial lengths of time. A single sharp spike that penetrates a transactions bound (placed, say, at ±0.6%) is more likely to be symptomatic of a lagging and smoothed index than an arbitrage opportunity. The 100 point

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[11]The standard deviations for longer intervals are efficiently computed using information from the 15 minute interval. For example, the 60 minute interval standard deviation, σ_{60}, is computed from

$$\sigma_{60} = \sigma_{15}[4 + 6\rho(1) + 4\rho(2) + 2\rho(3)]^{\frac{1}{2}} .$$

scale interval on the X-axis corresponds to 1500 minutes, or approximately 4 trading days. These graphs provide a visual description of the typical behavior of the mispricing series.

Table 4 reports the means, standard deviations and autocorrelations at 8 lags for the levels and the first differences in the constructed mispricing series. We report the results for the overall time period, June 1983 through June 1987, as well as for the sixteen separate contracts. All statistics are computed using the quarter-hour intervals: overnight and weekend intervals are treated as missing observations and are not included in the computations.

The results for the mispricing levels are in Panel A of Table 4. Over the sixteen contracts, the average mispricing is 0.12%. For the December 1986 contract the average mispricing is the lowest, with a mean of -0.20%; it is the highest for the December 1984 contract (0.78%). These results are consistent with the hypothesis that the forward pricing model gives a downward biased estimate for the futures price, but the short time period and the small number of contracts considered prohibits one from drawing strong conclusions. The overall standard deviation of the mispricing levels is 0.44%. Panel B of Table 4 reports the corresponding results for the first differences in the mispricing series (the "changes"). The mean of these changes is 0 for all the contracts, as one might expect given that the level of the mispricing is constrained by arbitrageurs. The standard deviations are fairly stable across all contracts, that for the overall period being 0.14%.

The series of the mispricing levels is highly autocorrelated (see Panel A, Table 4). For the individual contracts the first-order autocorrelation coefficient ranges from 0.74 to 0.94. For six of the ten contracts the autocorrelation is quite high for all 8 lags reported (a 2 hour time interval) and this indicates a persistence in the computed series. The series does not, as one might have conjectured, fluctuate randomly about 0. The autocorrelation behavior of the first differences in the mispricing are close to 0, except for the first two lags for which they are all negative (Panel B of Table 4). The first-order autocorrelation ranges from -0.34 to -0.08; the second-order autocorrelations are smaller in magnitude and range from -0.14 to -0.01. The negative autocorrelations at low lags are consistent with the implication that when the mispricing deviates from zero it is elastically pulled toward zero by the action of those traders who perceive that transacting in one market is cheaper.

<u>**Mispricing and time-to-expiration**</u>

We now examine the relation between the magnitude of the mispricing and the contract maturity. Lacking a precise functional form, we seek to test the proposition that the magnitude of the mispricing is positively related to time-to-maturity. Define

$x_{t,T}(j)$ = the mispricing at the jth quarter hour mark during day t, for the futures contract maturing at T;

N_t = number of observations in day t;

and

$$Z(t, T) = \text{ABS}\left[\sum_{j=1}^{N_t} x_{t,T}(j)/N_t\right] .$$

Consider the regression model

$$Z(t, T) = \beta_0 + \beta_1(T-t) + \varepsilon(t,T)$$

where $\varepsilon(t,T)$ is a mean zero disturbance. We seek to test the null hypothesis $\beta_1 = 0$ against the alternative $\beta_1 > 0$. For this model, the standard assumption that the error term is i.i.d. is questionable. To avoid reliance on this assumption, we present z-statistics based on the procedure outlined in

Newey and West (1985) in addition to the usual OLS t-statistics. The estimator of β_1 is the same as the OLS estimator but its standard error is corrected for heteroscedasticity and autocorrelation in the error term.[12,13]

The results are presented in Table 5, for each contract individually as well as for all contracts stacked together. For 13 of 16 contracts the estimates of β_1 are positive, with 11 of them having associated z-statistics greater than 2.0. (All 13 contracts have OLS t-statistics greater than 2.0.) In remaining three cases the estimates are negative but not statistically different from 0. Except in a few cases the error autocorrelations are closer to zero. These results imply the magnitude of the mispricing increases with time to maturity. Figure 2 illustrates the cone shaped bounds one obtains for the mean mispricing as a function of time-to-maturity: it is drawn using the estimates from the overall regression. Thus the arbitrage boundaries should depend not only on transaction costs, but also on other factors which are affected by time until expiration, such as the unanticipated costs of marking-to-market flows, dividend uncertainty and risk in tracking the stock index with a partial basket of stocks.

<u>Path Dependence of Mispricing</u>

We now investigate the path dependence of the mispricing series. One implication of this hypothesis is that, conditional on the mispricing having crossed one arbitrage bound, it is less likely to cross the opposite bound.

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[12]These corrections are asymptotic in nature. In order for the asymptotics to be valid, it is necessary to let the number of contracts grow, rather than increase the time to maturity.

[13]We also conducted the regression analysis using the daily mean squared deviation for Z(t, T). The inferences are not sensitive to this change in definition of Z(t, T).

This phenomenon is a result of the fact that arbitrageurs will unwind positions established when the mispricing was outside one bound before it reaches the other bound.[14] To investigate this issue, for each of the ten contracts, we document the number of upper bound and lower bound mispricing violations which occur.[15] A tendency for a given contract to have mostly upper bound violations or mostly lower bound violations (but not both) can be interpreted as evidence consistent with the mispricing being path dependent. Indeed, this is the case. Table 6 documents the number of upper bound and lower bound violations for each contract. For this table bounds of ± 0.6% are selected.[16] With the exception of the September 1983 contract, each contract is dominated by either upper bound violations or lower bound violations. For example, the March 1985 contract violated the upper bound 625 times (using 15 minute observations) and did not violate the lower bound for any of the observations. In contrast, the December 1986 contract violated the -0.6% mispricing bound 233 times and violated the +0.6% bound only 2 times. Table 6 also reports the number of times a given bound was crossed for each contract and the average time (in trading minutes) the mispricing remained outside the bounds. These results indicate that the mispricing remains outside the bounds for a considerable length of time and rules out the possibility that stale prices in the index are a major cause of the observed violations.

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[14]We ignore the possibility that arbitrageurs might roll forward into the next futures contract.

[15]Stoll and Whaley (1986) also document violations of these bounds. They use hourly observations and assume a constant dividend yield.

[16]The results are not overly sensitive to the bound selected. We based the selection of 0.6% on roundtrip stock commission of 0.70, roundtrip futures commission of 0.08, market impact cost in futures of 0.05, market impact in stocks of 0.35, and an index level of 200.

We can develop further evidence of the path dependence of the series by examining conditional probabilities. Consider two possible arguments:

1. The mispricing is path independent, following some stochastic process which is pinned to zero at T; and

2. This mispricing is path dependent; conditional on having crossed an upper (lower) bound, the probability of its hitting the lower (upper) bound is smaller.

An implication of argument 1 is that if the mispricing is currently 0, then it is equally likely to hit an upper or a lower bound independent of the past. Argument 2 has a different implication if the mispricing is 0. It implies that if the mispricing has crossed the upper bound in the past, it is more likely to continue to deviate above 0 and more likely in the future to hit the upper bound than the lower bound. We address this question empirically by identifying all cases where the mispricing crossed the upper (or lower) bound, returned to 0 and then again crossed the upper (or lower) bound. For the sixteen contracts there are 142 such cases. Table 7 reports the number of cases which fell into each of the four categories: upper-upper, upper-lower, lower-upper, and lower-lower. The path independence argument implies the probability of hitting the upper bound given the mispricing is at 0 is the same whether it previously hit the upper or lower bound. We can estimate these conditional probabilities from the results in Table 7. The estimates are:

$$p(x \text{ hitting upper bound} \mid x \text{ has hit lower and has crossed } 0) = 0.36$$

$$p(x \text{ hitting upper bound} \mid x \text{ has hit upper and has crossed } 0) = 0.73$$

This argument should also hold for valuations of the lower bound. For the lower bound the conditional probability estimates are:

$$p(x \text{ hitting lower bound} \mid x \text{ has hit lower and has crossed } 0) = 0.64$$

$$p(x \text{ hitting lower bound} \mid x \text{ has hit upper and has crossed } 0) = 0.27$$

These conditional probabilities differ substantially for the two cases.[17] And the evidence is consistent with the notion that arbitrageurs' option to unwind prematurely introduces path dependence into the mispricing series.

The option to unwind prematurely and the path dependence can also be related to the ability to predict expiration day movements. Even if during the life of the contract there has been substantial positive (negative) mispricing, often there is also some time prior to expiration where the mispricing is negative (positive). (See the September 1984 contract in Figure 1 for an example.) Hence, arbitrageurs will often have had the opportunity to unwind at a profit prior to expiration day, making expiration day predictions based on the identification of mispricing outside the arbitrage bound difficult.

4. Conclusion

We consider the intraday behavior of the S&P 500 futures and index quotes. Comparisons of the autocorrelations of the changes of the log price of these two series indicate that with a 15 minute observation interval, nonsynchronous trading in the stocks in the index poses a problem. As the interval length is increased the autocorrelation disappears, with no evidence of the problem with 60 minute observation intervals. We also examined the relative variability of the futures and spot market. The results indicate that the futures market is more variable than the spot market even after controlling for problems caused by nonsynchronous prices in the observed index.

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[17]The p-value of a chi-square test of the equality of the conditional probabilities is less than 0.001.

Much of the paper focuses on the behavior of the mispricing series -- the difference between the actual futures price and its theoretical value. We advance and examine empirically two hypotheses: the average magnitude of the mispricing increases with time to maturity, and that the mispricing series is path dependent. Evidence supporting these hypotheses is provided. These results have implications for the width of the arbitrage bounds, the selection of a stochastic process to "model" mispricing, the valuation of options related to the mispricing series, and the prediction of expiration day movements of the S&P 500 index.

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FIGURE 1a

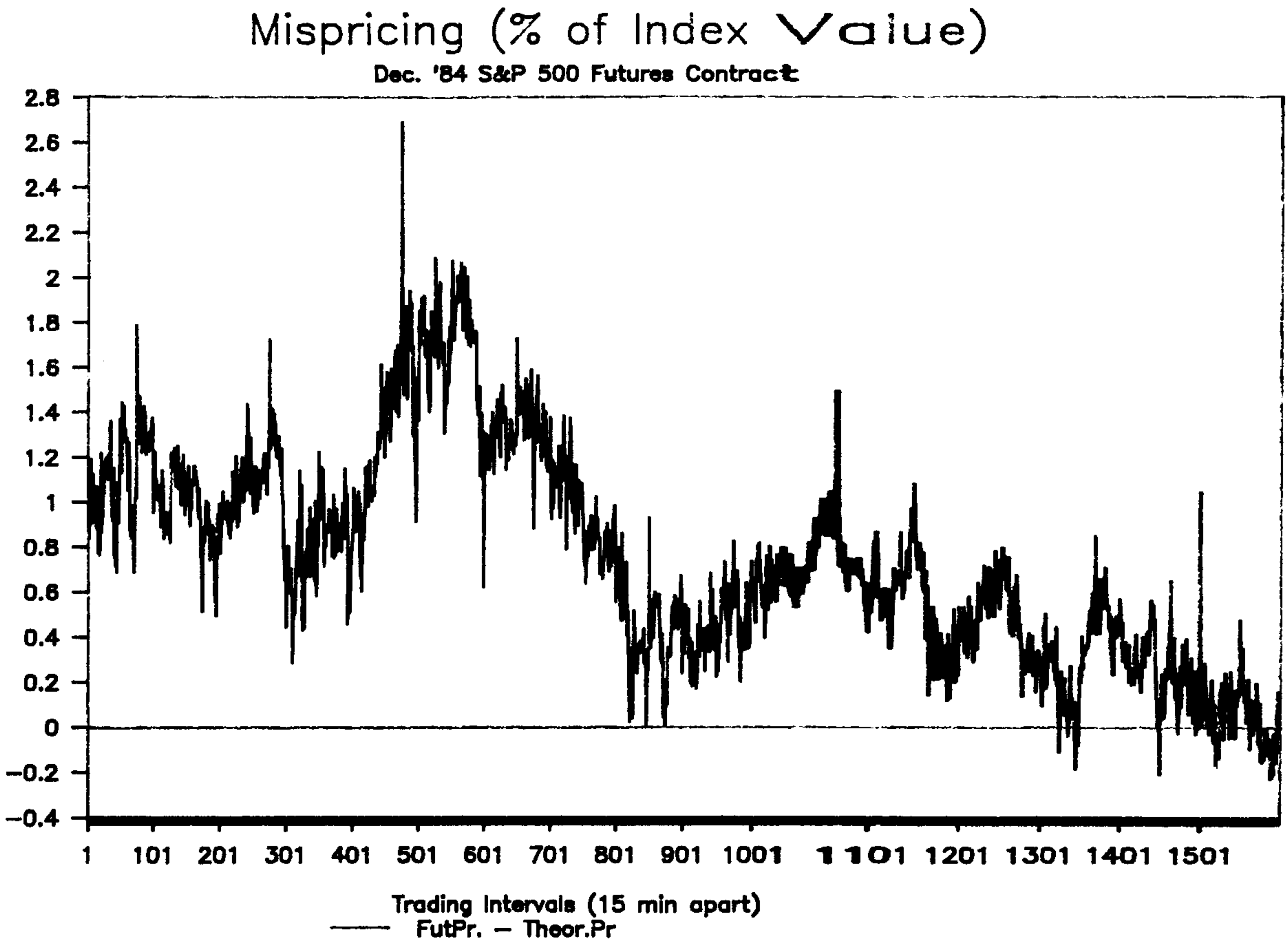
Mispricing (% of Index Value)
Dec. '84 S&P 500 Futures Contract
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
1
101
201
301
401
501
601
701
801
901
1001
1101
1201
1301
1401
1501
Trading Intervals (15 min apart)
FutPr. − Theor.Pr

FIGURE 1b

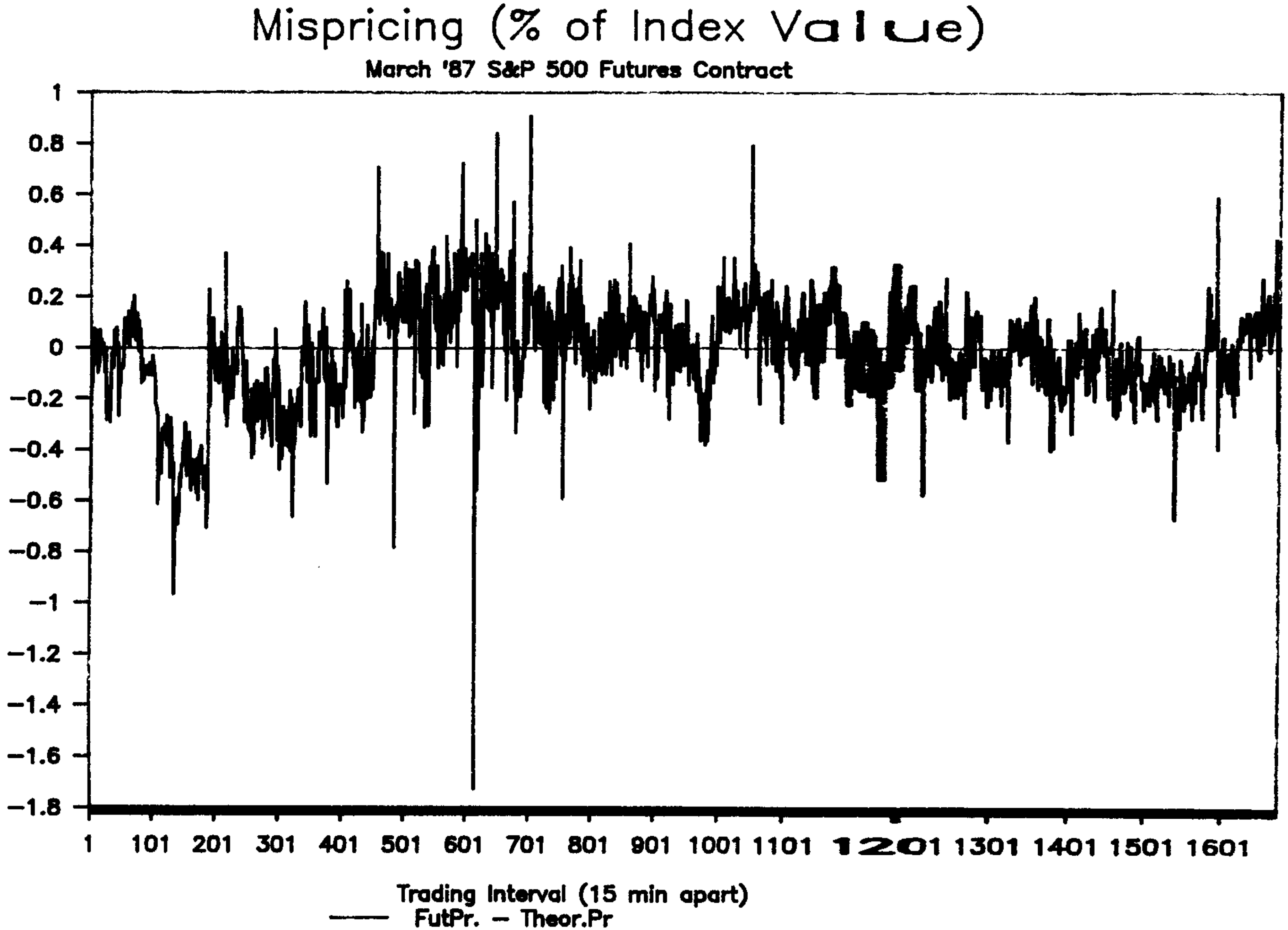
Mispricing (% of Index Value)
March '87 S&P 500 Futures Contract
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
−1.2
−1.4
−1.6
−1.8
1
101
201
301
401
501
601
701
801
901
1001
1101
1201
1301
1401
1501
1601
Trading Interval (15 min apart)
FutPr. − Theor.Pr

FIGURE 2

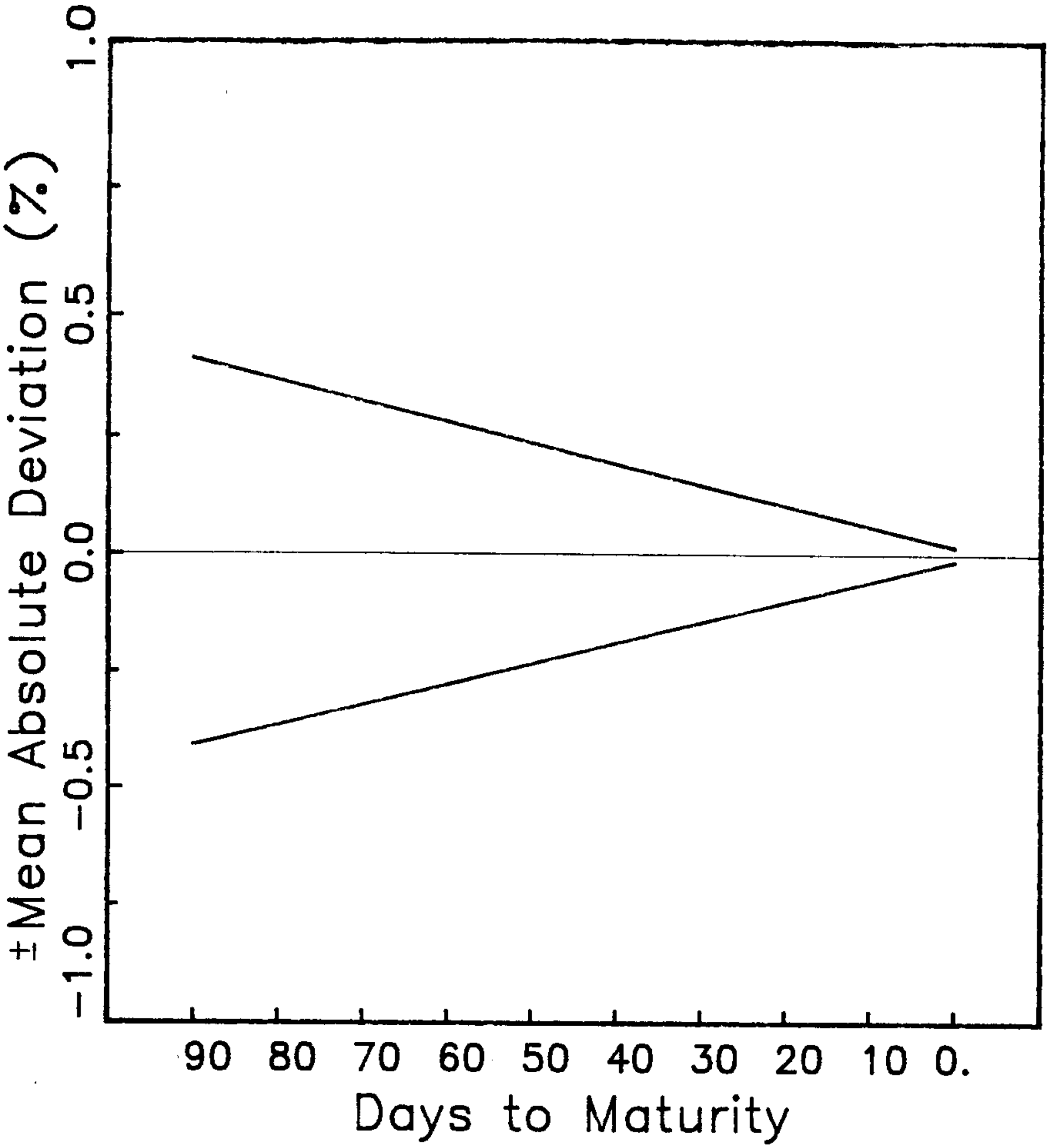
±Mean Absolute Deviation (%)
1.0
0.5
0.0
−0.5
−1.0
90 80 70 60 50 40 30 20 10 0.
Days to Maturity

TABLE 1

Autocorrelations for Changes of the Logarithm of Price in the S&P 500 Futures and Index by Contract, September 1983 to June 1987

Autocorrelations are based on 15 minute observation intervals

Contract	Lag 1	2	3	4	5	6	7	8	No. of Observations
Panel A: S&P 500 futures									
SEP 83	0.02	-0.07	-0.05	-0.01	0.03	-0.01	0.06	0.03	1512
DEC 83	0.00	0.02	0.04	0.00	0.02	0.00	0.02	0.03	1512
MAR 84	0.00	-0.01	-0.04	0.05	0.01	0.04	-0.02	0.03	1488
JUN 84	-0.01	0.03	-0.04	0.00	-0.03	-0.03	0.02	0.01	1440
SEP 84	-0.01	-0.01	0.04	0.00	0.00	-0.04	0.08	-0.04	1632
DEC 84	-0.05	-0.02	0.05	0.04	0.01	0.03	0.00	-0.05	1536
MAR 85	-0.08	0.02	0.08	0.00	0.02	-0.01	0.01	0.01	1368
JUN 85	-0.08	-0.01	0.07	0.04	0.05	0.00	0.02	-0.04	1632
SEP 85	-0.02	0.05	0.03	0.01	0.05	0.01	0.00	-0.03	1512
DEC 85	-0.07	0.02	0.06	0.05	-0.02	0.02	-0.02	0.01	1654
MAR 86	-0.03	0.03	0.01	-0.01	0.01	0.00	0.01	0.00	1612
JUN 86	0.00	0.00	0.01	0.01	0.03	0.04	0.04	-0.04	1638
SEP 86	-0.02	0.03	-0.02	-0.01	0.00	0.04	0.01	0.01	1638
DEC 86	-0.02	-0.01	0.03	-0.02	-0.02	-0.04	0.02	0.01	1664
MAR 87	-0.14	-0.16	0.22	-0.04	-0.04	0.06	0.01	-0.01	1612
JUN 87	0.05	0.00	0.02	0.00	0.03	-0.02	0.03	-0.02	1612
Panel B: S&P 500 index									
SEP 83	0.41	-0.03	-0.09	-0.04	0.02	0.05	0.07	0.06	1512
DEC 83	0.41	0.03	0.01	0.00	0.00	0.00	0.03	0.03	1512
MAR 84	0.31	0.03	-0.05	-0.02	-0.01	0.01	0.05	0.03	1488
JUN 84	0.37	0.04	-0.03	-0.02	0.01	0.03	0.02	0.01	1440
SEP 84	0.29	-0.02	0.00	0.01	-0.05	-0.03	0.03	0.04	1632
DEC 84	0.21	-0.03	0.00	0.04	0.04	0.02	0.03	0.02	1536
MAR 85	0.16	-0.03	0.06	0.03	-0.01	0.00	0.01	0.01	1368
JUN 85	0.18	-0.03	0.02	0.01	0.02	0.02	0.03	0.04	1632
SEP 85	0.25	0.03	0.04	0.05	0.04	0.06	0.02	0.01	1512
DEC 85	0.18	0.02	0.01	-0.03	0.00	0.02	0.01	0.04	1654
MAR 86	0.14	0.00	0.02	0.02	-0.02	0.03	0.00	0.01	1612
JUN 86	0.10	-0.02	-0.01	0.06	0.03	0.07	0.03	0.01	1638
SEP 86	0.04	0.04	0.03	-0.01	-0.02	0.02	0.09	0.02	1638
DEC 86	0.08	0.00	0.01	0.00	-0.01	-0.02	0.02	0.00	1664
MAR 87	0.04	-0.07	0.13	0.01	-0.02	-0.02	0.02	0.01	1612
JUN 87	0.09	0.00	0.00	0.02	0.01	-0.02	-0.04	0.00	1612

TABLE 2

Summary Statistics for the Changes of the Logarithm of Price in the S&P 500 Futures and Index by Contract, September 1983 to June 1987.

Contract	Futures Standard Deviation[a]	Futures First Order Autocorrelation	Index Standard Deviation[a]	Index First Order Autocorrelation	Variance Ratio[b]
Panel A: Data at 15 minute intervals					
SEP 83	0.163	0.022	0.128	0.408	1.634
DEC 83	0.125	-0.001	0.095	0.409	1.739
MAR 84	0.147	-0.005	0.119	0.313	1.536
JUN 84	0.150	-0.010	0.114	0.369	1.715
SEP 84	0.176	-0.011	0.149	0.289	1.410
DEC 84	0.155	-0.055	0.114	0.213	1.858
MAR 85	0.145	-0.078	0.112	0.156	1.664
JUN 85	0.115	-0.079	0.093	0.178	1.506
SEP 85	0.108	-0.020	0.083	0.249	1.683
DEC 85	0.128	-0.065	0.102	0.184	1.575
MAR 86	0.172	-0.029	0.137	0.140	1.576
JUN 86	0.171	-0.005	0.142	0.103	1.460
SEP 86	0.202	-0.018	0.173	0.045	1.362
DEC 86	0.182	-0.021	0.147	0.079	1.520
MAR 87	0.219	-0.136	0.168	0.038	1.703
JUN 87	0.219	0.048	0.214	0.086	1.049
Panel B: Data at 30 minute intervals					
SEP 83	0.233	-0.085	0.214	0.093	1.186
DEC 83	0.177	0.036	0.159	0.170	1.233
MAR 84	0.208	-0.035	0.193	0.121	1.164
JUN 84	0.210	0.010	0.189	0.152	1.240
SEP 84	0.248	0.007	0.239	0.093	1.081
DEC 84	0.213	-0.027	0.177	0.062	1.448
MAR 85	0.197	0.021	0.171	0.063	1.327
JUN 85	0.156	-0.018	0.143	0.055	1.178
SEP 85	0.151	0.052	0.131	0.137	1.321
DEC 85	0.175	0.013	0.157	0.096	1.244
MAR 86	0.239	0.019	0.207	0.067	1.342
JUN 86	0.242	0.002	0.211	0.020	1.318
SEP 86	0.283	0.014	0.251	0.074	1.280
DEC 86	0.254	-0.004	0.216	0.038	1.380
MAR 87	0.287	-0.143	0.241	0.012	1.417
JUN 87	0.318	0.028	0.316	0.039	1.013
Panel C: Data at 60 minute intervals					
SEP 83	0.315	-0.047	0.316	0.029	0.992
DEC 83	0.255	0.051	0.244	0.077	1.092
MAR 84	0.289	0.032	0.289	0.031	1.002
JUN 84	0.299	-0.042	0.287	0.058	1.087
SEP 84	0.352	0.025	0.353	0.019	0.996
DEC 84	0.297	0.083	0.258	0.096	1.326
MAR 85	0.281	0.060	0.249	0.068	1.275
JUN 85	0.218	0.111	0.208	0.075	1.097
SEP 85	0.219	0.089	0.198	0.153	1.223
DEC 85	0.249	0.078	0.232	0.036	1.149
MAR 86	0.342	0.016	0.302	0.054	1.282
JUN 86	0.342	0.064	0.301	0.113	1.294
SEP 86	0.404	0.004	0.367	0.060	1.209
DEC 86	0.359	-0.032	0.312	0.007	1.324
MAR 87	0.376	0.010	0.343	0.060	1.201
JUN 87	0.456	0.041	0.455	0.017	1.001

TABLE 2 (continued)

Contract	Futures Standard Deviation[a]	Futures First Order Autocorrelation	Index Standard Deviation[a]	Index First Order Autocorrelation	Variance Ratio[b]
Panel D: Data at 120 minute intervals					
SEP 83	0.435	0.069	0.454	0.174	0.919
DEC 83	0.369	0.026	0.358	0.084	1.066
MAR 84	0.415	0.044	0.414	0.107	1.003
JUN 84	0.414	-0.004	0.417	0.082	0.984
SEP 84	0.504	0.174	0.503	0.186	1.002
DEC 84	0.437	0.050	0.382	0.129	1.311
MAR 85	0.410	0.091	0.364	0.133	1.265
JUN 85	0.325	0.021	0.305	0.118	1.134
SEP 85	0.322	0.022	0.300	0.087	1.155
DEC 85	0.365	0.022	0.334	0.076	1.195
MAR 86	0.487	0.038	0.438	0.095	1.236
JUN 86	0.499	0.067	0.449	0.115	1.237
SEP 86	0.572	0.124	0.534	0.183	1.146
DEC 86	0.499	0.031	0.443	0.068	1.273
MAR 87	0.535	0.003	0.500	0.003	1.145
JUN 87	0.658	0.009	0.650	-0.040	1.025
Panel E: Data at 1 trading day intervals					
SEP 83	0.887	-0.062	0.874	0.038	1.032
DEC 83	0.703	0.063	0.709	0.081	0.983
MAR 84	0.845	-0.181	0.839	-0.148	1.013
JUN 84	0.790	-0.074	0.799	-0.015	0.978
SEP 84	0.871	0.060	0.871	0.108	1.001
DEC 84	0.760	-0.133	0.677	-0.003	1.259
MAR 85	0.811	-0.144	0.686	-0.030	1.399
JUN 85	0.595	0.122	0.551	0.194	1.164
SEP 85	0.586	0.027	0.495	0.181	1.400
DEC 85	0.730	0.013	0.667	0.137	1.199
MAR 86	0.840	0.109	0.789	0.059	1.134
JUN 86	1.033	0.156	0.909	0.141	1.292
SEP 86	1.231	0.113	1.120	0.121	1.208
DEC 86	1.017	-0.169	0.884	-0.006	1.324
MAR 87	0.873	0.027	0.781	0.077	1.248
JUN 87	1.278	0.034	1.211	0.031	1.114

[a] The standard deviation is reported in percent.

[b] The variance ratio is the variance of the change of log futures price divided by the variance of the change of log index price.

TABLE 3

Aggregate Variance Ratios Results
(Based on 16 contracts, September 1983 to June 1987)

Observation Time Interval	Average Variance Ratio	Cross-Sectional Standard Deviation	z-statistic[a]
15 minutes	1.56	0.19	11.9
30 minutes	1.26	0.12	8.81
60 minutes	1.16	0.12	5.14
120 minutes	1.13	0.12	4.40
1 trading day[b]	1.17	0.14	4.79

[a]The null hypothesis is that the average variance ratio equals 1.

[b]Data uses prices at 3 p.m. Eastern time.

TABLE 4

Summary Statistics on the Levels and First Differences in Mispricing in the S&P 500 Futures Contracts, By Expiration (15 minute Interval Transaction Data, Mispricing in % of Index Value)

Mispricing = Futures Price - Theoretical Forward Price

			Autocorrelations								
	Mean	S.D.	Lag								
Contract	(%)	(%)	1	2	3	4	5	6	7	8	Obs.
Panel A: Statistics on the Levels											
SEP 83	0.01	0.29	0.83	0.73	0.70	0.70	0.70	0.71	0.69	0.67	1575
DEC 83	0.37	0.29	0.86	0.75	0.65	0.56	0.45	0.33	0.21	0.07	1575
MAR 84	0.50	0.36	0.85	0.73	0.62	0.52	0.41	0.28	0.12	-0.04	1550
JUN 84	0.06	0.23	0.81	0.71	0.67	0.66	0.65	0.64	0.63	0.62	1500
SEP 84	0.11	0.32	0.84	0.79	0.76	0.74	0.73	0.71	0.71	0.68	1700
DEC 84	0.78	0.48	0.84	0.71	0.58	0.44	0.27	0.10	-0.11	-0.33	1600
MAR 85	0.64	0.60	0.93	0.87	0.82	0.75	0.69	0.61	0.53	0.44	1425
JUN 85	0.28	0.34	0.91	0.87	0.83	0.79	0.74	0.69	0.64	0.58	1700
SEP 85	0.04	0.28	0.94	0.92	0.90	0.89	0.87	0.85	0.84	0.83	1575
DEC 85	-0.17	0.30	0.91	0.87	0.85	0.83	0.80	0.78	0.76	0.73	1718
MAR 86	0.01	0.30	0.86	0.82	0.81	0.79	0.78	0.78	0.77	0.77	1674
JUN 86	-0.03	0.29	0.85	0.81	0.81	0.80	0.78	0.77	0.77	0.76	1701
SEP 86	-0.16	0.27	0.74	0.66	0.63	0.59	0.56	0.54	0.52	0.48	1701
DEC 86	-0.20	0.34	0.85	0.82	0.80	0.78	0.76	0.74	0.71	0.67	1728
MAR 87	-0.02	0.21	0.65	0.58	0.60	0.59	0.56	0.53	0.52	0.51	1674
JUN 87	-0.11	0.22	0.46	0.34	0.31	0.27	0.25	0.23	0.20	0.17	1674
OVERALL	0.12	0.44	0.93	0.91	0.90	0.89	0.88	0.87	0.86	0.85	26070
Panel B: Statistics on the First Differences											
SEP 83	0.00	0.15	-0.08	-0.14	-0.12	-0.05	-0.02	0.01	0.05	-0.01	1512
DEC 83	0.00	0.11	-0.16	-0.14	-0.06	0.00	-0.01	-0.02	0.03	0.01	1512
MAR 84	0.00	0.14	-0.15	-0.10	-0.11	-0.03	0.00	0.03	-0.06	0.04	1488
JUN 84	0.00	0.13	-0.17	-0.08	-0.10	-0.02	-0.03	-0.01	0.00	-0.03	1440
SEP 84	0.00	0.16	-0.19	-0.06	-0.06	-0.05	0.03	-0.04	0.05	-0.06	1632
DEC 84	0.00	0.13	-0.27	-0.08	0.01	0.04	-0.05	0.04	0.00	-0.06	1536
MAR 85	0.00	0.12	-0.27	-0.08	0.04	-0.01	-0.04	-0.02	0.00	0.03	1368
JUN 85	0.00	0.11	-0.25	-0.06	0.00	0.00	0.00	-0.03	0.02	-0.03	1632
SEP 85	0.00	0.09	-0.26	-0.03	0.00	-0.03	0.02	-0.02	-0.04	0.02	1512
DEC 85	0.00	0.11	-0.26	-0.09	0.01	0.00	-0.01	0.01	-0.02	0.01	1654
MAR 86	0.00	0.14	-0.30	-0.01	0.00	-0.02	-0.03	0.03	-0.03	0.02	1612
JUN 86	0.00	0.14	-0.30	-0.08	0.02	0.01	-0.04	0.01	0.02	-0.06	1638
SEP 86	0.00	0.16	-0.26	-0.05	-0.01	-0.03	0.01	0.04	-0.01	-0.01	1638
DEC 86	0.00	0.15	-0.24	-0.06	-0.03	0.01	-0.01	0.02	0.00	0.00	1664
MAR 87	0.00	0.16	-0.34	-0.10	0.04	0.03	-0.02	0.01	-0.02	0.00	1612
JUN 87	0.00	0.18	-0.20	-0.05	0.04	-0.03	0.02	-0.01	0.03	-0.04	1612
OVERALL	0.00	0.14	-0.23	-0.07	-0.02	-0.01	-0.01	0.01	0.00	-0.02	25062

TABLE 5

Regression Results for the Test of Relation between the Magnitude of the Mispricing and time to expiration S&P 500 Futures Contracts, Daily Data[a]

Contract	β_1[b]	t-statistic[c]	z-statistic[d]	$\rho(1)$	$\rho(6)$	No. of Observations
SEP 83	0.94	2.44	4.80	0.01	0.05	63
DEC 83	-0.22	-0.19	-0.12	0.06	0.36	63
MAR 84	5.20	3.09	2.18	0.12	0.27	62
JUN 84	0.79	3.42	3.03	0.00	-0.08	60
SEP 84	-0.58	-1.31	-1.05	0.01	0.03	68
DEC 84	19.1	5.91	4.00	0.44	0.25	64
MAR 85	35.9	11.13	5.03	0.32	0.24	57
JUN 85	7.85	7.38	2.87	0.06	0.42	68
SEP 85	1.72	4.09	2.54	0.01	0.06	63
DEC 85	1.31	2.26	1.62	0.01	0.09	64
MAR 86	1.53	4.87	2.91	0.00	0.10	62
JUN 86	1.16	2.84	1.55	0.01	0.05	63
SEP 86	-0.11	-0.18	-0.10	0.01	0.01	63
DEC 86	5.13	7.78	4.65	0.02	0.31	64
MAR 87	0.88	3.54	3.17	0.00	-0.06	62
JUN 87	0.85	2.62	2.73	0.00	-0.24	62
OVERALL[e]	4.41	8.88	3.83	0.17	0.57	1008

Notes: [a]The regression model is $Z(t, T) = \beta_0 + \beta_1(T-t) + \varepsilon(t,T)$, where

$$Z(t, T) = ABS\left[\sum_{j=1}^{N_t} x_{t,T}(j)/N_t\right];$$

$x_{t,T}(j)$ = the mispricing at the jth quarter hour mark during day t, for the futures contract maturing at T;

N_t = number of observations in day t.

[b]Scaled by 10^3.

[c]This statistic is the Ordinary Least Squares t-statistic.

[d]This statistic is corrected for heteroscedasticity and autocorrelation in the residuals.

[e] β_0 for the overall regression is 0.014%.

TABLE 6

Mispricing Violations for S&P 500 Index Futures

Number of Upper Bound Violations[a]	Number of Upper Bound Crossings	Average Time Above Upper Bound[b]	Number of Lower Bound Violations[a]	Number of Lower Bound Crossings	Average Time Below Lower Bound[b]	Number of Observations[c]
30	20	23	29	14	31	1575
371	66	84	0	0	NA	1575
631	64	148	0	0	NA	1550
17	9	28	4	4	15	1500
92	44	31	21	15	21	1700
974	61	240	0	0	NA	1600
625	28	335	0	0	NA	1425
271	29	140	0	0	NA	1700
64	24	40	0	0	NA	1575
4	4	15	143	41	52	1718
7	4	26	36	31	17	1674
46	23	30	19	17	17	1701
1	1	15	83	44	28	1701
2	2	15	233	62	56	1728
5	5	15	16	9	27	1674
9	9	15	18	12	23	1674
3149	393	120	602	249	36	26070

upper bound is set at +0.6% and the lower bound at -0.6%.

average time outside the bounds is in trading minutes.

observations are recorded at 15 minute intervals.

TABLE 7

Documentation of Cases where Mispricing crosses ±0.6% boundary, crosses 0.0 and then crosses ±0.6% boundary

Second boundary crossed	Initial boundary crossed: -0.6%	Initial boundary crossed: +0.6%
-0.6%	38	22
+0.6%	22	60

Total Number of Cases: 142.