TESTS OF ASSET PRICING MODELS WITH CHANGING EXPECTATIONS

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1. Introduction

Variables like firm size and price/earnings (P/E) ratios which have been shown to describe cross-sectional return differences may proxy for (as yet) unspecified economic risk factors. An important class of financial valuation models do not fully specify the underlying risk factors, yet still impose structure on the relation of expected returns across assets. If multiple risk-factors do explain expected returns their identification is unique only to within a nonsingular linear transformation in the usual formulations of these models. Thus, it seems natural to ask if the behavior of expected returns is consistent with a general formulation, even if the exact nature of the risk factors is unknown.

Many studies find that expected returns on common stocks change over time and are correlated with predetermined variables such as the day of the week (French (1980) and Gibbons and Hess (1981)), the month of the year (Rozeff and Kinney (1976)), asset price levels (Keim and Stambaugh (1986)), lagged returns (Fama (1965) and Fama and French (1987)), and others. Further, the magnitude of the cross-sectional "anomalies" relative to the Capital Asset Pricing Model (e.g., the size effect) seems to change predictably through time with some of these predetermined variables (e.g., January (Keim (1983)), day-of-the-week (Keim and Stambaugh (1984)), and asset price levels (Keim and Stambaugh (1986))).

Examination of the consistency with financial valuation models of such rate of return "anomalies" requires an approach which recognizes that expected returns change over time. The "standard" approaches to testing asset pricing models in finance (e.g., Fama and MacBeth (1973), Gibbons (1982) or Chan, Chen and Hsieh (1986)) examine unconditional, or long-run average, expected returns. Such methodologies do not focus on the variation over time in conditional expected returns that is allowed by equilibrium models and strongly suggested by the data.

This study presents tests, using the approach of Gibbons and Ferson (1985), which focus on the variation in expected returns over time.[1] This approach examines a multivariate regression model of expected returns, conditional on a set of predetermined information variables, and conducts tests of restrictions on the coefficients that are implied by alternative asset pricing models. This provides a consistent framework for investigating if the weekend, January and size effects imply a rejection of a given model.

The present application of this methodology differs from the Gibbons and Ferson (GF) example in several interesting ways. GF use the Dow Jones 30 common stocks as the test assets and they condition expectations on a limited information set. Our samples of stock portfolios based on firm size and on industry allow us to study expected return behavior in these commonly-used experimental designs. These samples may display a greater cross-sectional dispersion of risk exposures than do the Dow Jones 30. We also examine a wider selection of predetermined information variables than do GF. Finally, we extend the GF tests in several ways. One extension allows for the possibility that the underlying sensitivities of the assets to risk (i.e., the "beta" coefficients) display a January seasonal. Such an hypothesis is examined by a simple modification of the GF cross-equation restriction.

A second extension of the GF approach is to conduct tests against specific alternative hypotheses. Such tests may have more power than the GF tests, which are tests against an unspecified alternative. We propose to test a multiple-factor model of expected returns against the alternative that expected returns are related to a measured attribute of the test portfolios that should not be "priced" according to the model. Such a test can be viewed as an extension of studies which examine the incremental cross-sectional explanatory power of "size," "residual risk," the square of beta, etc., for average returns.

In Section 2 we outline the test methodology. Section 3 briefly illustrates the approach by examining the sensitivity of inferences, in the context of the GF empirical example, to the choice of instrumental variables and sample period. GF do not reject a "one-factor" model of expected returns for the Dow Jones 30. When expected returns are conditioned on more information variables than GF use (or, in a subperiod they did not examine), we find that a two-factor model is required to "explain" these expected returns. Section 4 presents an examination of the expected common stock returns for both size-based and industry-based portfolios. Section 5 presents (preliminary) results illustrating our extensions of the GF tests. Section 6 summarizes and concludes.

2. Methodology

Returns are assumed to obey the regression model

$$\tilde{R}_{it} = \alpha_i + \delta_i' Z_{t-1} + \tilde{u}_{it} \; ; \qquad E(\tilde{u}_{it} | Z_{t-1}) = 0 \quad \forall \; i = 0, 1, \ldots, N \; ; \tag{1}$$

where Z_{t-1} is an L-vector of time-varying, instrumental variables contained in the market's information set at time t - 1.[2] The essence of the GF methodology is to test restrictions on the parameters of the regression system (1) that are imposed by an asset pricing model with the familiar form[3]

$$E(\tilde{R}_{it} | Z_{t-1}) = E(\tilde{R}_{0t} | Z_{t-1}) + \sum_{h=1}^{k} \beta_{ih} E(\tilde{\lambda}_{ht} | Z_{t-1}) \; ; \tag{2}$$

where $\tilde{\lambda}_{ht} \equiv$ one of k (possibly unobservable) risk premiums,

$\beta_{ih} \equiv$ risk measure ("beta") of security i relative to risk factor h, and

$\tilde{R}_{0t} \equiv$ return on a "zero-beta" security.

Following much of the empirical work on the CAPM and APT, the approach assumes the β_{ih} are constant to examine an asset pricing hypothesis of a given dimension, k.[4]

The GF approach derives its power from the assumption that expected returns are changing over time and from the ability to identify instruments Z_{t-1} that are correlated with these expectations. Given the statistical specification (1), then

$$E(\tilde{R}_{it}|Z_{t-1}) = \alpha_i + \delta_i' Z_{t-1}$$

may be substituted into Equation (2). The following parameter restrictions on system (1) are implied (see the appendix for a proof):

$$\delta_i = \sum_{j=0}^{k} c_{ij}\delta_j ,$$

$$\alpha_i = \sum_{j=0}^{k} c_{ij}\alpha_j \qquad (3)$$

and

$$\sum_{j=0}^{k} c_{ij} = 1 , \quad \forall\ i = k + 1, \ldots, N ;$$

where the c_{ij} may be interpreted as ratios of the betas for assets i and j in coefficients of all N + 1 assets may be replicated from only k + 1 assets[6] if the "k-factor" model (2) characterizes expected returns.

The GF approach is well suited to our purpose for several reasons. First, because the approach allows expected returns to change over time it can accommodate seasonality in mean returns (e.g., the January effect) and a nonconstant relation between expected returns and size (as suggested, for example, by Brown, Kleidon and Marsh (1983)). Second, the model may be tested without specifying a market index or the risk factors; only the dimension of the asset pricing hypothesis k must be specified. As a result, the approach avoids some specification problems. For example, security risks may be ranked according to the Capital Asset Pricing Model (CAPM) without the ambiguity, discussed by Roll (1977), that arises from error in measuring the "true" market portfolio. Our inferences about the minimum number of factors required to explain a sample of expected returns should apply to any study that attempts to measure constant betas directly using a specified proxy.

The test methodology can avoid biases caused by some types of measurement errors; e.g., independent measurement errors in the test asset returns, measurement errors in the instruments (provided the measured values are known by the market), missing information variables, and missing assets. The main constraint on the selection of instrumental variables to model conditional expected returns is that the regression system (1) is well specified.

2.1 Choice of Information Variables

We require that the information variables, Z_{t-1}, used in the tests satisfy certain criteria. First, the variables should contain information that is correlated with changes in investor expectations. The econometric tests also assume that Z_{t-1} is known when the market sets prices at t - 1.

variables, with constant coefficients. Finally, the number of information variables L must be at least the number of "factors" k in the equilibrium model.

The approach we follow is to identify, a priori, a set of variables that previous research suggests should be useful predictors and then to conduct specification tests on system (1) to determine an appropriate specification before testing the restrictions. Previous research suggests potential instruments by documenting reliable predictive relations between stock returns and predetermined variables. For example, French (1980) and Gibbons and Hess (1981) find evidence of predictable daily patterns in stock returns; Keim and Stambaugh (1984) conclude these patterns are not explained as measurement errors. Rozeff and Kinney (1976) document a January seasonal in stock returns and Keim (1983) finds that the January seasonal is much more pronounced for small firms. Fama (1965) finds significant autocorrelation in stock returns. Autocorrelations in the returns of portfolios representing large and small firms are studied by Keim and Stambaugh (1986), Fama and French (1987) and others.

In their original empirical example, GF used the Dow Jones 30 common stocks as test assets, and a lagged stock index return and a dummy indicator for Mondays as instruments. We extend their investigation to include portfolios formed by size and industry groups, and to examine instrumental variables that capture the day-of-the-week and turn-of-the-year effects, differences in these effects related to firm size, and autocorrelation in returns. The specific variables are discussed in the following sections.

3. The Gibbons-Ferson Empirical Application

This section illustrates the approach by examining one- and two-factor returns of the thirty securities on the Dow Jones Industrials' List during 1963 to 1985.[7]

Data are obtained from the Center for Research in Security Prices (CRSP) daily files for the period January 2, 1963 to December 31, 1985. Table 1 presents the mean daily rates of return and standard deviation (for each of four subperiods) for an equal-weighted portfolio of the Dow Jones securities, including averages over all months, January only, non-January, turn-of-the-year,[8] Mondays, and Fridays.

The table illustrates day-of-the-week patterns in returns, including the "Monday effect" exploited by GF in their tests. In each subperiod, average Monday returns are negative. Also, Friday returns are greater than average daily returns. Mean daily returns are larger in January versus non-January months in all subperiods except the second, which exhibits a negative average return in January. Mean daily returns are also larger at the turn-of-the-year than at other times, and the magnitudes are quite large in each subperiod. The smallest average return over the six trading days at the turn of the year is in subperiod two--48% on an annualized basis.

GF specialize equation (1) as

$$\tilde{R}_{it} = \alpha_i + \delta_{i1} D_t^m + \delta_{i2} RVM_{t-1} + \tilde{u}_{it}\ , \quad i = 1, \ldots, 30, \quad t = 1, \ldots, T \qquad (4)$$

where $D_t^m = 1$ if day t is Monday, zero otherwise, and RVM_{t-1} is the lagged return on the CRSP value-weighted index. A multivariate regression system is formed by combining the n equations for the test assets. The restrictions implied by a k-factor asset pricing model are given by equation (3). GF use a standard likelihood ratio (LR) statistic to examine the restrictions, applied to the system of equations. The number of restrictions is (n - k - 1)(L - k) where n is the number of assets (= 30), k is the number of factors, (k = 1 or k = 2) and L is the number of predetermined variables (L = 3).

The LR statistic is equal to the number of time series observations multiplied by the natural logarithm of the ratio of the determinants of the restricted and unrestricted multivariate regression systems. LR has an asymptotic chi-square distribution with degrees of freedom equal to the number of restrictions. We also compute the Lagrange multiplier (LM) statistic for each of the models we examine. In the present context, the two produce virtually identical inferences, and we report the LR in our tables.[9]

An examination of the unrestricted regressions (not reported)[10] confirms GF's observation that equation (4) appears to be a reasonably well-specified model for the Dow Jones 30. Like GF, we find that the regressions can detect reliable variation in expected returns in the first three subperiods, covering the interval January 1963 to December 1979. (This corresponds roughly to the overall period of GF: August 1962 to December 1980.) However, the R-square is always less than .07, and in the fourth subperiod, the standard F-test for the regression produces tight-tail p-values smaller than .05 in only twelve out of thirty cases. For every regression in each subperiod, the absolute value of the first-order autocorrelation of the residuals is less than .12, although 22% of these were greater than two standard errors from zero. Chow tests indicate that the hypothesis of constant regression parameters within a subperiod is rejected at the .01 level (.05) in only five (fifteen) of the 120 regressions and these rejections occur uniformly in all subperiods. Finally, White's (1980) test for heteroskedasticity in the residuals of (4) turns up little evidence of heteroskedasticity.

The middle column of Table 2 reports the test statistics for the cross-is not rejected. The likelihood ratio test p-values range from .12 to .65. Aggregating the statistics over 1963 to 1979 by summing across the first three subperiods yields a p-value of .211. These are very similar to the results obtained by GF. However, very different results are observed in the fourth subperiod (1980 to 1985). In this subperiod the chi-square statistic is 82.3 (p-value = .0127). As a result of this large statistic in the fourth subperiod the aggregate (1963 to 1985) chi-square indicates rejection of the one-factor model.

The results of tests of a two-factor model conform very closely to the results of GF. The p-value of the likelihood ratio statistic is in excess of .62 in every subperiod as well as in the combined samples.

Observations at the turn of the year do not seem to be particularly influential for the rejection indicated in the fourth subperiod. Omitting these observations from the tests, we obtain a p-value for the one-factor model of .001 in the fourth subperiod (p-values in excess of .185 are observed in the other three subperiods).

We extend the GF example to study other patterns in expected returns suggested by the data in table 1. Including in equation (4) a dummy indicator for the turn-of-year, the unrestricted regressions (not reported) show a reliable increase in the adjusted R-square relative to equation (4) in each subperiod except the second. White's specification test and other diagnostics suggest a well-specified regression model. The right-hand columns of table 2 report the results of testing the one- and two-factor asset pricing hypotheses, using this expanded set of instruments (labeled (4') in the

In summary, our results indicate that the inability of GF to reject a one-factor model of the daily expected returns of the Dow Jones 30 common stocks is a sample-specific result. A one-factor model can be rejected using the same experimental design in the 1980-1985 subperiod. Using more information to model conditional expected returns, a one-factor model can be rejected even in earlier subperiods. (See Foerster (1987) for further analysis of the expected returns of the Dow Jones 30 stocks.)

4. Tests with Size and Industry Portfolios

One might question the generality of tests of asset pricing models using 30 "blue chip" securities. In order to generalize inferences about the dimension of the expected return model, the test assets' risk sensitivities must span the relevant risks in the economy. GF (p. 231) suggest that "expected returns on the Dow Jones 30 stocks may be better explained by a single-factor model than would the returns on a broader sample of assets." In this section we conduct tests using daily return data and common stock portfolios formed on the basis of firm size and by industry.[11]

4.1 Size-Based Portfolio Results

Data are obtained from the CRSP daily files for the period January 2, 1963 to December 31, 1985. The sample consists of all NYSE and AMEX firms that had returns on the CRSP files during the entire calendar year under consideration. The number of sample firms ranges from a low of 1425 in 1963 to a high of 2516 in 1976. The following procedure is used to create the size-based portfolios. At each year-end (beginning with December 31, 1962), sample firms are ranked from smallest to largest, based on the market value of their common stock. Market value is calculated as the number of shares of portfolio one containing the smallest firms and portfolio ten containing the largest firms. Each portfolio contains approximately the same number of sample firms. For example, in 1976, portfolios one through six contain 252 firms and portfolios seven through ten contain 251 firms, for a total of 2516 in that year. The ranking and portfolio formation is then repeated for each of the twenty-three years.

Daily portfolio returns are computed for day t as

$$R_{p,t} = [\sum_{i=1}^{n}(1 + R_{i,t-1})(1 + R_{i,t}) / \sum_{i=1}^{n}(1 + R_{i,t-1})] - 1 , \qquad (5)$$

where $R_{p,t}$ is the return on portfolio p for day t and $R_{i,t}$ is the return on security i for day t for the n securities contained in portfolio p. That is, the current day's security return is weighted by the previous day's relative gross return. We use equation (5) because Blume and Stambaugh (1983) and Roll (1983a) show that equal-weighted portfolio returns (used in many previous studies) are subject to a statistical bias related to bid-ask spreads. The bias in average returns is negatively related to size of the firm. Blume and Stambaugh and Roll demonstrate that use of a buy-and-hold portfolio strategy reduces the magnitude of the bias. Calculating returns as in equation (5) takes us a step toward a buy and hold return.

Panel A of table 3 presents summary statistics for the overall 1963 to 1985 period for each of the ten size-based portfolios. Average market values (not shown) range from $5 million for portfolio 1 to $1.4 billion for portfolio 10. Panel A illustrates that the familiar size effect (column 2), day-of-the-week effect (columns 3 and 4), January size effect (column 5), and turn-of-the-year effect (column 6) are observed even after attempting to

Our tests are conducted using the same subperiods as in Section 3. With daily data, each subperiod provides a large number of observations, so the test statistics should be close to their asymptotic distributions.[12]

Recent literature, as well as the data in tables 1 and 3 above, suggest that expected returns vary with day-of-the-week and turn-of-the-year effects as well as with lagged values of returns. We examine regressions which use dummy variables to account for these seasonal patterns and also use lagged returns as instruments. Lagged returns of equal-weighted portfolios of (1) all stocks on the NYSE and AMEX and (2) the Dow Jones 30 are included. The equally-weighted NYSE-AMEX portfolio captures the serial correlation of small firms (see, e.g., Fama and French (1987)). The Dow Jones portfolio reflects large firms. We study the following regression model:

$$\tilde{R}_{pt} = \alpha_p + \delta_{p1} D_t^m + \delta_{p2} D_t^F + \delta_{p3} D_t^J + \delta_{p4} REM_{t-1} + \delta_{p5} RDJ_{t-2} \qquad (6)$$

$$+ \delta_{p6} REM_{t-3} + \delta_{p7} (REM_{t-1} {*} D_t^J) + \tilde{u}_{pt}, \; p = 1, \ldots, 10, \; t = 1, \ldots, T ,$$

where REM_{t-j} is the return on the CRSP Equal-Weighted Index and RDJ_{t-j} is the return on an equal-weighted index of the 30 common stocks of the Dow Jones Industrial Index for day t-j. DF_t and DJ_t are dummy indicators for Fridays and Januarys.

Ferson and Keim (1984) employed equation (6) to capture the expected returns of size-based portfolios, after examining several unrestricted regression specifications using data for the 1963 to 1979 period. Consequently, subperiod four (1980 to 1985) in this study serves as a "holdout period" to check their inferences.

Panel A of table 4 reports estimates for regression (6) for each size-based portfolio over the full 1963-1985 period. Consistent with results reported by Keim and Stambaugh (1984), the tables indicate that the "Friday effect" on returns is related to firm size. The estimated coefficients on the Friday dummy in Table 4 decline monotonically in size. The coefficients on the Monday dummy are negative for each portfolio, consistent with the evidence in French (1980) and Gibbons and Hess (1981). These coefficients display no obvious pattern related to size. Since the Friday coefficients are related to size, this evidence is consistent with the evidence in Keim and Stambaugh (1984) and Smirlock and Starks (1986) that the Friday and Monday effects on returns are not explained by offsetting measurement errors. Note that the intercepts in Table 4 show no obvious pattern related to size. This suggests that our information variables capture the cross-sectional differences in average returns of the different size portfolios.[13]

Recall that the GF restrictions imply that the regression coefficients for all of the test assets must be linear combinations of the coefficient vectors of the reference assets. The different cross-sectional patterns in the coefficients for different instruments in table 4 suggest that the sample design should allow tests with some power.

Table 5 reports some cross-equation tests of linear hypotheses on the regression coefficients in table 4 for the size-based portfolios. These tests can reject, for most of the instruments and subperiods, the hypotheses that the coefficients equal zero or are equal across portfolios. These tests provide further information about the behavior of expected returns. Tinic and West (1984), following earlier observations of Rozeff and Kinney (1976), suggest that the expected market risk premium is not different from zero in months other than January. Their tests are based on sample average returns and a specific market portfolio proxy. If conditional expected returns, given market information, equal a "zero-beta" rate in months other than January, then the coefficients for each portfolio in table 4 should be equal given that $D_t^J = 0$. Such a test is not dependent on a specified market proxy and should be more powerful than the tests conducted by Tinic and West.

The hypothesis of zero expected risk premiums outside of January can be rejected in each of the subperiods, based on the tests of equality across portfolios, of the non-January coefficients. Thus, table 5 indicates that the sample of size portfolios displays significant cross-sectional dispersion in conditional expected returns, both in January and in non-January months.

Table 6 contains some additional diagnostics for regression (6) in the subperiods. The test methodology assumes that the predetermined variables are correlated with changes in expected returns. Table 6 indicates that the regressions have reliable explanatory power for the returns of each size portfolio. In some cases as much as 37% of the small firms' return variation is associated with the predetermined variables; for the large firm portfolio the R-squares are never more than 15%. Still these are higher than the R-squares reported for the Dow Jones 30, suggesting that more powerful tests may be possible using the size-based portfolios.

An important maintained assumption of the GF tests is that the regression model coefficients are constant over the test period. If this assumption fails, then the restrictions are no longer implied by the asset pricing hypothesis. The third panel in table 6 reports p-values of the Chow test for constant parameters within a subperiod. The few rejections of the constant parameter hypothesis all occur in subperiods one and three. The tests reject parameter stability only for the smaller firm portfolios in these cases.

The residuals of the regression models do not appear to exhibit much autocorrelation. The highest absolute value of the forty residual autocorrelations reported in table 6

Given the specification of (6), the restrictions implied by the equilibrium model (2) are examined in panel (a) of table 7 for the case of k = 1, 2, and 3. The single-factor asset pricing hypothesis is rejected with p-values less than .0001 in each of the four subperiods. The evidence suggests that the behavior of the conditional expected returns of the ten size portfolios cannot be explained by a constant-beta model with a single factor (plus a zero-beta factor). These results are consistent with the evidence of Ferson and Keim (1984), who reject a single-factor model using equation (6) and size-based portfolios. They compute portfolio returns without the "buy and hold" adjustment (equation (5)), and use daily data for 1963-1979 only.

Tests of the two-factor model in table 7 also indicate a rejection (at the .025 level) in subperiods one, two, and four. In subperiod three the p-value is .098. For the overall 1963 to 1985 period, the aggregate test statistic produces a p-value less than .0001.

A three-factor model of expected returns is not rejected (at the .05 level) in any subperiod. The p-value for the overall 1963 to 1985 period is .164.

4.2 Industry Portfolio Results

In this section, we repeat the tests using daily returns of portfolios formed by industry group. The same securities are used as in forming the size-based portfolios. We group these securities into twelve industry portfolios, using the same combinations of 2-digit SIC codes as in Breeden, Gibbons and Litzenberger (1986). Unlike the size portfolios, the number of firms in each industry portfolio is not approximately the same. For example, in December of 1985 the number ranges from 59 to 251, across the industries.

Panel B of table 3 presents summary statistics for each of the twelve substantially larger than other industries (see Foerster (1987)), exhibits the lowest Monday and highest Friday mean returns. The large Friday returns are in direct contrast to the results in the upper panel, which suggest Friday returns are negatively related to firm size. January and turn-of-the-year returns are highest for the consumer durable portfolio and lowest for utilities.

Panel B of table 4 contains regression estimates based on equation (6) for each of the industry portfolios. Consistent with the size-based results in the upper panel, the coefficients on the Monday dummy are negative and on the Friday dummy are positive. The adjusted R-squares range from .07 (petroleum) to .24 (textiles/trade). Additional diagnostics, similar to those reported in tables 5 and 6 for the size-based portfolios, are calculated but not reported. Cross-sectional F-tests can reject, for most of the instruments and subperiods, the hypotheses that the coefficients equal zero or are equal across industry portfolios. The residuals of the regression models do not appear to exhibit much autocorrelation. Chow tests indicate few rejections of the constant parameter hypothesis, except in the third subperiod (as is the case with the size-based results).

Panel (b) of table 7 presents the results of testing a two- and a three-factor asset pricing model using these portfolios and the same instruments to model expected returns as in the preceding section. The tests can reject both the two-factor model (p-values less than .0001 in each subperiod) and the three-factor model (p-values less than .021 in each subperiod).

5. Extensions of the Tests

This section examines several extensions of the GF restriction, in an attempt to learn more about what is driving the rejections of the asset restrictions (3) on the regression coefficients to allow the intercepts to be different and unrestricted in each equation. If an asset pricing model can be rejected with unrestricted intercepts, we conclude that differences across assets in the variation through time of expected returns around their unconditional means is richer than can be captured by the asset pricing hypothesis. If a rejection is not observed when the intercepts are unrestricted, it suggests that differences in unconditional mean returns are important in the rejections.

Table 8 reports the results of tests using both the size and industry portfolios with unrestricted intercepts. Inferences from the likelihood ratio test statistics are basically the same as those drawn from the test statistics in table 7. Differences in unconditional mean returns do not seem to be important in the rejections in section 4.[15]

A second extension of the tests of expected return models allows for the possibility that the assets' sensitivities to risk shift over time. Rogalski and Tinic (1984) and Keim and Stambaugh (1986), for example, suggest that betas may shift in January.[16] Such an hypothesis can be examined by a simple modification of the GF cross-equation restriction. Assume that the "true" beta coefficients in equation (2) follow a simple switching model. In January the betas are β^J_{ij}, possibly different from β_{ij}, the values during the rest of the year. We modify the unrestricted regression (1) using a dummy indicator for January D^J_t, as follows:

$$R_i = \alpha_i + \alpha^J_i D^J_t + \boldsymbol{\delta}_i' \mathbf{Z} + \boldsymbol{\delta}^{J\prime}_i (D^J_t \mathbf{Z}) + \varepsilon_i, \qquad (7)$$

where the instruments **Z** are the Monday dummy, Friday dummy and lagged market variables in equation (6).

The restrictions on the coefficients of (7) implied by the k-factor asset pricing hypothesis, allowing for shifts in the risk measures at the turn of the year, are:

$$\delta_i = \sum_{j=0}^{k} c_{ij}\delta_j , \quad \delta_i^J = \sum_{j=0}^{k} c_{ij}^J \delta_j^J ,$$

$$\alpha_i = \sum_{j=0}^{k} c_{ij}\alpha_j , \quad \alpha_i^J = \sum_{j=0}^{k} c_{ij}^J \alpha_j^J , \qquad (8)$$

$$\sum_{j=0}^{k} c_{ij} = 1 , \quad \sum_{j=0}^{k} c_{ij}^J = 1 .$$

The results are reported in table 9 for restrictions on both the size and industry portfolio regressions implied by k = 1, 2 and 3. As in table 7, the LR tests reject the one and two factor model for the size portfolios and the two and three factor model for the industry portfolios, suggesting "risk shifts" in January are not driving the results.

A third extension of the GF tests is to examine specific alternative hypotheses. Such tests may have more power than the GF tests, which are tests against an unspecified alternative. We test a model against the hypothesis that unconditional expected returns are related across assets to measured attributes $\mathbf{x}_i$ of the test asset i, given the "loadings" c_{ij} that capture the variation in expected returns over time. Such a test can be viewed as an extension of studies which examine the incremental cross-sectional explanatory power of size, "residual risk," the square of beta, etc., for average returns. Those tests measure the beta of the asset pricing model by the sample (unconditional) covariation of returns with a specified "market portfolio" proxy or "factors." The tests we propose measure the loadings indirectly, through the variation of conditional expected returns over time.

We intend to examine (at least) two alternative attributes. The first is the size-ranking of a size-based test portfolio. In this case x_i is simply a different integer for each asset i. The second attribute is the square of the c_{ij} coefficients, in which case $\mathbf{x}_i$ is a vector of length k ($j = 1, \ldots, k$), containing the squared c_{ij}'s. This is the alternative hypothesis of a nonlinear relation between the indirect risk measures and unconditional mean returns across assets, analogous to the CAPM tests of Fama and MacBeth (1973), who include the square of beta in cross-asset regressions.

6. Conclusions

We examine the behavior of expected returns over time on various common stock portfolios, using the test methodology of Gibbons and Ferson (1985) (GF). GF study expected returns of the Dow Jones 30 common stocks. We extend their investigation to include portfolios formed by size and by industry groups, and we examine instrumental variables that capture the day-of-the-week and turn-of-the-year effects, differences in these effects related to firm size, and autocorrelation in returns.

When expected returns are conditioned on more information than GF used (or, in a subperiod they did not examine), we find that a two-factor, constant beta model is required to "explain" the expected returns of the Dow Jones 30 common stocks. At least two or three factors seem to be required to explain the variation of expected returns on portfolios formed by firm size or by industry. These tests also provide evidence that expected risk premiums are nonzero and differ reliably across size-based portfolios, even in months other than January, in contrast to evidence of Tinic and West (1984). We also sensitivities to risk (i.e., the "beta" coefficients) exhibit a January seasonal.

Further work will extend the methodology to allow tests against specific alternative hypotheses. We intend to investigate a multiple-factor model of expected returns against the alternative that "average" (i.e., unconditional) expected returns are related to size or to the squared "loadings" that capture the variation in expected returns over time. Such a test can be viewed as an extension of studies which examine the incremental cross-sectional explanatory power of "size," "residual risk," the square of beta, etc., for average returns.

Appendix

In this appendix we provide a proof of the restrictions (3). This proof illustrates the general approach we use to derive the other restrictions in the text.

Consider the regression model (1) stacked up across assets, which implies:

$$E(R_1|Z) = \alpha_1 + \delta_1'Z$$

$$E(R_2|Z) = \alpha_2 + \delta_2'Z \qquad \text{(a1)}$$

where R_1 is the K-vector (where K = k + 1) of reference asset returns at time t+1, R_2 are (n - K) test assets, Z is the L-vector of information at time t, and the time subscripts are suppressed to conserve notation. The asset pricing hypothesis (2) implies:

$$E(R_1|Z) = (1,\ \beta_1')\lambda^*(Z)\ ,$$

$$E(R_2|Z) = (1,\ \beta_2')\lambda^*(Z)\ , \qquad \text{(a2)}$$

where

$$\lambda^*(Z) = \left[E(R_0|Z)',\ \lambda(Z)'\right]'\ .$$

Equating the right-hand sides of equations (a1) and (a2) for the reference assets, and assuming that the K × K matrix $(1,\ \beta_1)$ is invertible, we solve for the unobserved expected risk premium:

$$\lambda^*(Z) = (1,\ \beta_1')^{-1}(\alpha_1 + \delta_1'Z)\ . \qquad \text{(a3)}$$

Substituting back into the second line of (a2) from (a3) implies:

$$(\alpha_2 + \delta_2'Z) = (1,\ \beta_2')(1,\ \beta_1')^{-1}(\alpha_1 + \delta_1'Z) \qquad \text{(a4)}$$

Define $\mathbf{C} = (1, \beta_2')(1, \beta_1')^{-1}$. Since (a4) must hold for all values of $\mathbf{Z}$, the coefficients of the test assets are restricted as follows:

$$\alpha_2 = C\alpha_1 \ ; \quad \delta_2' = C\delta_1' \ ; \quad C1 = 1 \ , \tag{a5}$$

where the last restriction that the c_{ij} sum to unity across the factors j for each asset i, follows from the definition of $\mathbf{C}$: $C(1, \beta_1') = (1, \beta_2')$. Equation (a5) is, of course, the same as equation (3) in the text.

Notice that in regression model (1) if $\mathbf{Z}$ is mean-centered, then the slopes are unaffected, but the intercepts α_i may be interpreted as long-run (or unconditional) mean returns; thus the intercept restriction in (3) and (a5) is like the Gibbons (1982) restriction on the unconditional moments of returns.

Footnotes

[1]Several recent studies have examined asset pricing models with a focus on time-varying expected returns. Studies that test linear models of expected return variation include: Hansen and Hodrick (1983) (the forward foreign exchange market); Hansen and Singleton (1983) (a consumption-based model); Stambaugh (1986) (the term structure); Ferson, Kandel, and Stambaugh (1987), Campbell (1987), Chan (1986), and Foerster (1987) (stock market returns). Singleton (1987) reviews studies that adopt nonlinear formulations of asset pricing models which allow time-variation of expected returns.

[2]This is, of course, an assumption about market efficiency. We assume rational (i.e., mathematical) conditional expectations given the "market's" information.

[3]Examples of asset pricing models like equation (2) include those of Sharpe (1964), Black (1972), Merton (1973), Long (1974), Ross (1976) and Breeden (1979), among others.

[4]An alternative interpretation is to view the joint hypothesis as a no-arbitrage assumption and a specification of the behavior of conditional covariances of return with an aggregate marginal utility function. See Gibbons and Ferson (1985) and Ferson (1987) for discussions of this interpretation.

[5]This is strictly true if the unobserved risk factors are chosen to be mutually uncorrelated and the reference assets are mutually uncorrelated; otherwise, the Cij are related to the assets' betas by a linear transformation. See the appendix.

[6]The reference assets must be chosen so that the matrix of their betas and a unit vector is nonsingular; that is, they must span all the relevant

[7]Securities are included if they are on the Dow Jones List at the beginning of the subperiod under analysis. We were able to follow this design with one exception, in subperiod four. Since General Foods was acquired by Phillip Morris on November 2, 1985, complete data for General Foods is not available during this subperiod. Phillip Morris is included in its place.

[8]Turn-of-the-year is defined to include the last trading day and the first five trading days of a year. See Roll (1983b) and Keim (1983).

[9]Asymptotically the LM and LR statistics are equivalent, but the LM statistic produces a smaller value in any finite sample and has been shown to have better small sample properties in some asset pricing contexts (e.g., MacKinlay (1987) and Amsler and Schmidt (1985)). Foerster's (1987) Monte Carlo experiments indicate some tendency for both the likelihood ratio and Lagrange multiplier test statistics to reject too often (to understate p-values), using 480 monthly observations in tests similar to these. The distributions of the test statistics should be better-approximated by the asymptotic distribution in our samples, given the large number of time series observations (1250 or more).

[10]Foerster (1987) presents diagnostics for regression models of the Dow Jones 30 daily expected returns.

[11]Campbell (1987), Chan (1986), Ferson (1987), and Foerster (1987) examine broader portfolios of assets and reject single-factor, constant beta models using the GF methodology and monthly data. Campbell and Ferson both use samples of stock and bond portfolios, Chan uses common stock portfolios formed by firm size and Foerster studies industry-related portfolios. Stambaugh (1987) also rejects a single-factor model for the expected returns of Treasury bills using monthly data.

[12]For example, given ten assets, seven predetermined variables (discussed below) plus an intercept and a k = 5 factor asset pricing hypothesis there are 155 parameters to be estimated. (The matrix of regression coefficients has 8 x 10 = 80 parameters, plus 20 Cijs, plus 55 parameters in the covariance matrix for a total of 155 parameters.) The minimum number of return observations in a subperiod is 1258 x 10 = 12,580; 81 observations of a dependent variable per parameter. Including the independent variables, the number of data points per parameter exceeds 137. This compares quite favorably with the studies of Gibbons (1982) and Stambaugh (1982) who employ monthly data, and indicate some evidence of small sample problems using the LR statistic. A similar accounting reveals fewer than 6 observations per parameter in those studies.

[13]The significant coefficients on the lagged market returns, especially for smaller stocks, might be an indication of infrequent trading which is a problem for small firms in particular. However, Lo and MacKinlay (1987) demonstrate that positive serial correlation in stock index returns is unlikely to be caused by infrequent trading. Further, we reproduced the results in Tables 4 to 7 with NYSE firms only--for which infrequent trading is much less severe. The results are qualitatively and quantitatively the same.

[14]Huizinga and Mishkin (1986) observe that residual autocorrelation in a regression model of expected returns indicates that information, used by the market to form expectations, is omitted from the empirical model. There is no strong evidence from the subperiods that regression (6) omits such information.

[15]Campbell (1987) reaches similar conclusions in his examination of five stock and bond portfolios.

[16]Rogalski and Tinic (1984) examine unconditional betas while Keim and Stambaugh (1986) examine conditional betas.

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Table 1

Summary statistics for daily rates of return for the sample of the Dow Jones 30 including overall, January, February to December, Turn-of-Year, Monday, and Friday averages for each of the four subperiods.[a]

	Mean Daily Return × 10^4 (Standard Deviation × 10^4)							
Time Period	1963 to 1968		1969 to 1973		1974 to 1979		1980 to 1985	
All Months	4.75	(57.56)	0.99	(87.31)	3.48	(96.21)	6.58	(94.27)
January	13.46	(51.12)	-3.16	(66.23)	22.47	(108.77)	13.54	(106.93)
Feb-Dec	3.93	(58.08)	1.37	(88.99)	1.71	(94.80)	5.95	(93.04)
T-o-Y[b]	45.38	(59.08)	19.19	(77.10)	35.68	(119.28)	30.43	(125.35)
Monday	-10.02	(60.75)	-23.09	(94.16)	-0.56	(102.85)	-2.70	(106.95)
Friday	9.35	(48.03)	13.10	(82.26)	9.23	(91.47)	11.27	(83.65)

[a]Subperiods one through four have 1484, 1258, 1514 and 1504 observations respectively.

[b]Turn-of-Year includes the last trading day and the first five trading days of each year.

Table 2

Likelihood ratio tests (p-values) of the restrictions on the coefficients in equations (4) and (4') implied by a k-factor asset pricing model (plus a zero-beta portfolio) for k=1,2. The data consists of daily returns for the 30 Dow Jones Industrials as of the beginning of each subperiod.

$$\tilde{R}_{it} = \alpha_i + \delta_{i1}D_t^m + \delta_{i2}RVM_{t-1} + \tilde{u}_{it} \tag{4}$$

$$\tilde{R}_{it} = \alpha_i + \delta_{i1}D_t^m + \delta_{i2}RVM_{t-1} + \delta_{i3}D_t^T + \tilde{u}_{it} \tag{4'}$$

A. H_0: One-factor model[a]

Subperiod	Equation (4) χ^2_{56} [b]		Equation (4') χ^2_{84}	
1963-1968	68.347	(.1246)	135.919	(.0003)
1969-1973	51.530	(.6446)	87.649	(.3711)
1974-1979	62.594	(.2538)	110.080	(.0297)
1980-1985	82.282	(.0127)	129.927	(.0010)
Overall[d]	264.753	(*)[c]	462.111	(*)

B. H_0: Two-factor model[a]

Subperiod	Equation (4) χ^2_{27}		Equation (4') χ^2_{54}	
1963-1968	14.420	(.9768)	68.254	(.0918)
1969-1973	19.181	(.8633)	44.119	(.8291)
1974-1979	24.177	(.6205)	60.078	(.2651)
1980-1985	18.012	(.9032)	55.801	(.4069)
Overall[d]	75.790	(.9920)	228.252	(.2707)

[a]The k-factor restrictions are

$$\boldsymbol{\delta}_i = \sum_{j=0}^{k} c_{ij}\boldsymbol{\delta}_j$$

$$\alpha_i = \sum_{j=0}^{k} c_{ij}\alpha_j, \qquad i = k+1, \ldots, N ,$$

where $\sum_{j=0}^{k} c_{ij} = 1$

and $\boldsymbol{\delta}_i$ is the vector of regression coefficients for firm i.

[b]The chi-square degrees of freedom represents the number of restrictions.

[c] * indicates p-value < .0001.

[d]The overall chi-square statistic represents the sum of independent chi-square statistics across subperiods, with degrees of freedom equal to the sum of the degrees of freedom in each subperiod.

Table 3

Summary statistics for daily rates of return of ten size-based and twelve industry portfolios of NYSE and AMEX common stocks including overall, January, Turn-of-Year, Monday, and Friday averages from 1963 to 1985.

	Mean Daily Return × 10^4 (Standard Deviation × 10^4)				
	Overall	Monday	Friday	January	T-o-Y.
A. Size Portfolios					
Smallest	10.90 (85.21)	-6.41 (95.47)	31.69 (79.90)	68.94 (115.03)	149.73 (163.20)
2	8.37 (82.91)	-10.21 (95.47)	27.63 (75.26)	49.38 (100.75)	112.82 (140.17)
3	7.78 (82.18)	-12.61 (93.99)	25.82 (74.46)	40.19 (95.82)	90.93 (131.62)
4	6.87 (82.94)	-13.55 (93.08)	23.61 (75.57)	34.30 (94.28)	77.66 (127.30)
5	6.45 (82.51)	-13.50 (94.44)	22.26 (75.54)	29.22 (92.11)	63.59 (118.26)
6	5.98 (80.81)	-13.10 (92.44)	19.45 (73.37)	24.71 (86.93)	53.85 (111.78)
7	75.63 (76.60)	-12.05 (87.64)	17.73 (69.33)	20.66 (80.11)	45.18 (101.40)
8	5.31 (77.06)	-13.28 (86.83)	16.08 (70.28)	15.14 (79.58)	35.10 (99.79)
9	4.83 (75.85)	-11.44 (84.60)	13.93 (69.82)	12.31 (77.08)	29.84 (99.28)
Largest	4.15 (77.15)	-9.76 (85.30)	10.35 (70.74)	6.76 (75.72)	16.79 (87.59)
B. Industry Portfolios					
Petroleum[a] (13,29)	6.71 (107.19)	-25.35 (111.81)	27.90 (96.46)	21.94 (114.83)	49.99 (141.69)
Finance/Real Estate (60-69)	6.14 (74.42)	-11.22 (82.89)	19.83 (67.78)	29.02 (83.60)	75.98 (130.27)
Consumer Durables (25,30,36,37,50,55,57)	7.37 (91.46)	-11.88 (104.58)	23.41 (84.36)	36.47 (101.57)	78.00 (131.70)
Basic Industries (10,12,14,24,26,28,33)	6.45 (77.46)	-11.76 (89.03)	19.55 (70.78)	30.58 (84.81)	68.15 (103.26)
Food/Tobacco (1,20,21,54)	6.56 (62.23)	-7.73 (71.57)	18.56 (56.57)	26.74 ([illegible].61)	60.51 (93.77)
Construction (15-17,32,52)	6.39 (81.14)	-11.13 (91.59)	17.98 (74.33)	32.92 (96.79)	71.12 (126.20)
Capital Goods (34,35,38)	6.82 (88.01)	-13.02 (100.92)	21.49 (79.43)	31.60 (96.14)	66.12 (123.76)
Transportation (40-42,44,45,47)	6.68 (100.32)	-18.56 (106.66)	23.84 (94.61)	30.75 (104.18)	63.62 (124.61)
Utilities (46,48,49)	4.72 (48.93)	-3.11 (54.39)	11.81 (44.54)	15.82 (59.46)	44.39 (83.23)
Textiles/Trade (22,23,31,51,53,56,59)	6.66 (74.95)	-3.18 (87.50)	21.58 (69.31)	32.29 (85.62)	70.53 (112.86)

Table 4

Parameter estimates (t-statistics) for unrestricted OLS regression equation (6) of daily stock returns for the ten size-based and twelve industry portfolios of NYSE and AMEX stocks for 1963 to 1985. Number of observations is 5777.

$$\tilde{R}_{pt} = \alpha_p + \delta_{p1}D_t^m + \delta_{p2}D_t^F + \delta_{p3}D_t^J + \delta_{p4}REM_{t-1} + \delta_{p5}RDJ_{t-2} + \delta_{p6}REM_{t-3} + \delta_{p7}(REM_{t-1} \cdot D_t^J) + \tilde{u}_{pt} \quad (6)$$

	α_p^a	δ_{p1}^a	δ_{p2}^a	δ_{p3}^a	δ_{p4}	δ_{p5}	δ_{p6}	δ_{p7}	Adj R^2
A. Size Portfolios									
Smallest	0.42	-2.67	1.87	3.20	0.50	-0.08	0.14	0.30	.30
	(3.4)	(-10.9)	(7.7)	(8.8)	(35.9)	(-7.2)	(11.4)	(7.7)	
2	0.41	-2.82	1.65	1.79	0.47	-0.08	0.13	0.22	.26
	(3.3)	(-11.4)	(6.8)	(4.9)	(34.0)	(-7.1)	(10.3)	(5.5)	
3	0.54	-3.04	1.44	1.14	0.46	-0.09	0.11	0.17	.23
	(4.3)	(-12.2)	(5.8)	(3.1)	(32.3)	(-7.4)	(8.6)	(4.3)	
4	0.54	-3.03	1.28	0.87	0.43	-0.08	0.09	0.13	.19
	(4.2)	(-11.8)	(5.0)	(2.3)	(29.7)	(-6.8)	(7.2)	(3.1)	
5	0.56	-2.97	1.18	0.62	0.42	-0.08	0.09	0.07	.18
	(4.3)	(-11.5)	(4.6)	(1.6)	(28.9)	(-6.4)	(6.5)	(1.7)	
6	0.60	-2.88	0.91	0.35	0.40	-0.[illegible]8	0.08	0.05	.16
	(4.7)	(-11.2)	(3.6)	(0.9)	(27.6)	(-6.[illegible])	(5.9)	(1.3)	
7	0.60	-2.66	0.80	0.14	0.36	-0.07	0.07	0.04	.14
	(4.8)	(-10.9)	(3.3)	(0.4)	(26.2)	(-6.2)	(5.5)	(1.1)	
8	0.70	-2.78	0.60	-0.29	0.34	-0.07	0.05	0.04	.12
	(5.5)	(-11.2)	(2.4)	(-0.8)	(24.3)	(-6.2)	(4.1)	(0.9)	
9	0.66	-2.44	0.47	-0.35	0.3[illegible]	-0.07	0.03	0.04	.10
	(5.3)	(-9.8)	(1.9)	(-1.0)	(21.4	(-6.0)	(2.6)	(1.0)	
Largest	0.66	-2.05	0.20	-0.42	0.22	-0.05	0.00	0.01	.05
	(5.0)	(-7.9)	(0.8)	(-1.1)	(14.7)	(-4.4)	(0.4)	(0.1)	
B. Industry Portfolios									
Petroleum	0.86	-4.15	1.61	0.36	0.31	-0.09	0.06	0.05	.07
	(4.8)	(-11.7)	(4.6)	(0.7)	(15.4)	(-5.[illegible])	(3.3)	(0.9)	
Finance/Real Estate	0.52	-2.63	0.99	0.38	0.37	-0.08	0.08	0.19	.19
	(4.5)	(-11.4)	(4.3)	(1.1)	(28.8)	(-7.0)	(6.8)	(5.2)	
Consumer Industries	0.56	-2.93	1.22	1.06	0.45	-0.10	0.10	0.11	.17
	(3.9)	(-10.2)	(4.3)	(2.5)	(27.7)	(-7.0)	(6.7)	(2.4)	
Basic Industries	0.58	-2.71	0.91	1.00	0.36	-0.07	0.07	0.06	.15
	(4.7)	(-11.0)	(3.7)	(2.7)	(26.2)	(-5.8)	(6.0)	(1.6)	
Food/Tobacco	0.54	-2.15	0.91	0.69	0.32	-0.07	0.06	0.09	.18
	(5.6)	(-11.1)	(4.8)	(2.4)	(29.5)	(-7.5)	(6.2)	(2.8)	
Construction	0.58	-2.82	0.66	0.72	0.43	-0.06	0.08	0.14	.19
	(4.6)	(-11.2)	(2.7)	(1.9)	(30.3)	(-5.3)	(6.6)	(3.6)	
Capital Goods	0.61	-3.00	1.01	0.83	0.43	-0.[illegible]	0.08	0.08	.16
	(4.4)	(-10.8)	(3.7)	(2.0)	(27.2)	(-6.2)	(5.7)	(1.7)	
Transportation	0.71	-3.52	1.22	1.07	0.38	-0.10	0.06	0.05	.10
	(4.3)	(-10.7)	(3.8)	(2.2)	(20.7)	(-6.2)	(3.4)	(1.0)	
Utilities	0.42	-1.26	0.48	-0.03	0.22	-0.03	0.02	0.13	.14
	(5.3)	(-8.1)	(3.1)	(-0.1)	(24.9)	(-4.4)	(2.0)	(5.3)	
Textiles/Trade	0.41	-2.39	1.20	0.62	0.45	-0.07	0.10	0.12	.24
	(3.7)	(-10.6)	(5.4)	(1.8)	(35.1)	(-7.0)	(8.9)	(3.4)	

Table 5

F-Statistic p-values for the significance of the pre-determined variables in forecasting the returns of the ten size-based portfolios for four subperiods and the overall 1963 to 1985 period.

$$\tilde{R}_{pt} = \alpha_p + \delta_{p,1}D_t^m + \delta_{p,2}D_t^F + \delta_{p,3}D_t^J + \delta_{p,4}REM_{t-1} + \delta_{p,5}RDJ_{t-2} + \delta_{p,6}REM_{t-3} + \delta_{p,7}(REM_{t-1} \times D_t^J) + \tilde{u}_{pt}$$

Null Hypothesis	1963-1968	1969-1973	1974-1979	1980-1985	1963-1985
$\delta_{1,1} = \delta_{2,1} = \ldots = \delta_{10,1} = 0$	*[a]	*	*	*	*
$\delta_{1,2} = \delta_{2,2} = \ldots = \delta_{10,2} = 0$	*	*	.0086	*	*
$\delta_{1,3} = \delta_{2,3} = \ldots = \delta_{10,3} = 0$	.0024	*	*	*	*
$\delta_{1,4} = \delta_{2,4} = \ldots = \delta_{10,4} = 0$	*	*	*	*	*
$\delta_{1,5} = \delta_{2,5} = \ldots = \delta_{10,5} = 0$	.0051	*	*	.0058	*
$\delta_{1,6} = \delta_{2,6} = \ldots = \delta_{10,6} = 0$	*	*	*	*	*
$\delta_{1,7} = \delta_{2,7} = \ldots = \delta_{10,7} = 0$	.1158	*	*	.3246	*
$\delta_{pi} = 0 \quad \forall p = 1, \ldots, 10; \ \forall i = 1, \ldots, 7$	*	*	*	*	*
$\delta_{1,1} = \delta_{2,1} = \ldots = \delta_{10,1}$	*	.0310	*	*	*
$\delta_{1,2} = \delta_{2,2} = \ldots = \delta_{10,2}$	*	*	.0215	*	*
$\delta_{1,3} = \delta_{2,3} = \ldots = \delta_{10,3}$	.0013	*	.0002	*	*
$\delta_{1,4} = \delta_{2,4} = \ldots = \delta_{10,4}$	*	*	*	*	*
$\delta_{1,5} = \delta_{2,5} = \ldots = \delta_{10,5}$	.0028	.0009	.0002	.0037	.0002
$\delta_{1,6} = \delta_{2,6} = \ldots = \delta_{10,6}$	*	*	*	*	*
$\delta_{1,7} = \delta_{2,7} = \ldots = \delta_{10,7}$	.3064	*	*	.2672	*
$\delta_{1,i} = \delta_{2,i} = \ldots = \delta_{10,i} \quad \forall i = 1, \ldots, 7$	*	*	*	*	*
Number of Observations	1484	1260	1516	1517	

[a] * indicates a p-value less than .0001.

Table 6

Selected specification tests of a model of conditional expected returns (Equation (8)) on size-based portfolios constructed from firms on the NYSE and AMEX for four subperiods spanning 1963 to 1985[a]

$$\tilde{R}_{pt} = \alpha_p + \delta_{p1}D_t^m + \delta_{p2}D_t^F + \delta_{p3}D_t^J + \delta_{p4}REM_{t-1} + \delta_{p5}RDJ_{t-2} + \delta_{p6}REM_{t-3} + \delta_{p7}(REM_{t-1} \cdot D_t^J) + \tilde{u}_{pt} \quad (6)$$

	Dependent Variable: Size Decile Portfolio R_{pt}									
	Smallest	2	3	4	5	6	7	8	9	Largest
Adjusted R^2 [b]										
1963-68	.20	.16	.13	.10	.10	.10	.10	.08	.06	.04
1969-73	.37	.32	.28	.25	.24	.23	.21	.20	.17	.14
1974-79	.37	.35	.32	.25	.23	.19	.16	.13	.11	.05
1980-85	.20	.16	.14	.13	.11	.11	.09	.08	.05	.01
1st-Order Auto.[c]										
1963-68	-.018	-.024	-.028	-.008	.009	.023	.040	.051	.076	.098
1969-73	-.030	-.022	-.002	.004	.021	.033	.061	.055	.086	.082
1974-79	.078	-.005	-.021	-.014	.004	.022	.022	.056	.080	.098
1980-85	-.042	-.055	-.046	-.030	-.034	.001	.013	.041	.055	.063
Chow Test[d]										
1963-68	.0001**	.1807	.5551	.8230	.8518	.6598	.8765	.6177	.7471	.5991
1969-73	.0582	.0577	.6437	.5004	.8353	.8444	.8992	.8488	.8436	.7718
1974-79	.0048**	.0011**	.0125*	.0039**	.0119*	.0127*	.0514	.0759	.1126	.1965
1980-85	.6294	.6428	.5734	.6892	.4473	.4920	.5814	.8080	.6437	.5814

[a]Number of observations in each of the four subperiods is 1484, 1260, 1516, and 1517, respectively.

[b]The F-test for the significance of the regression produces a p-value less than .0001 for each gression.

[c]1st-order autocorrelation of residuals. Standard error ranges from approximately .026 (1516 servations) to .028 (1260 observations) if the true autocorrelation is equal to zero.

[d]p-value of the F-test of the hypothesis that a particular regression has constant parameters in the rst and second half of a subperiod. * indicates p-value < .05; ** indicates p-value < .01.

.26.8

Table 7

Likelihood ratio tests of the restrictions[a] on the coefficients in equation (6) implied by a k-factor asset pricing model (plus a zero-beta portfolio) for k=1,2,3. The data consists of daily returns of (a) ten size-based portfolios and (b) twelve industry-based portfolios of NYSE and AMEX common stocks.

$$\tilde{R}_{pt} = \alpha_p + \delta_{p1}D_t^M + \delta_{p2}D_t^F + \delta_{p3}D_t^J + \delta_{p4}REM_{t-1} + \delta_{p5}RDJ_{t-2} + \delta_{p6}REM_{t-3} + \delta_{p7}(REM_{t-1} \cdot D_t^J) + \tilde{u}_{pt} \qquad (6)$$

(a) Size-Based Portfolios

Subperiod	H_0: k=1; χ^2_{56} [b]		H_0: k=2; χ^2_{42}		H_0: k=3; χ^2_{30}	
1963-1968	114.699	(*)[c]	62.668	(.0209)	34.384	(.2657)
1969-1973	209.218	(*)	62.333	(.0224)	41.663	(.0765)
1974-1979	115.549	(*)	54.225	(.0978)	37.448	(.1644)
1980-1985	230.275	(*)	66.364	(.0097)	21.591	(.8685)
Overall[d]	669.741	(*)	245.590	(*)	135.086	(.1639)

(b) Industry-Based Portfolios

Subperiod	H_0: k=2; χ^2_{54}		H_0: k=3; χ^2_{40}	
1963-1968	116.242	(*)	72.810	(.0012)
1969-1973	121.093	(*)	65.196	(.0072)
1974-1979	113.828	(*)	60.213	(.0209)
1980-1985	158.787	(*)	82.087	(*)
Overall[d]	509.950	(*	280.306	(*)

[a]The k-factor restrictions are

$$\boldsymbol{\delta}_i = \sum_{j=0}^{k} c_{ij}\boldsymbol{\delta}_j$$

$$\alpha_i = \sum_{j=0}^{k} c_{ij}\alpha_j \qquad i = k+1, \ldots, N$$

where

$$\sum_{j=0}^{k} c_{ij} = 1$$

and $\boldsymbol{\delta}_i$ is the vector of regression coefficients for firm i.

[b]The chi-square degrees of freedom represent the number of restrictions.

[c] * indicates p-value < .0001.

[d]The overall chi-square statistic represents the sum of independent chi-square statistics across subperiods, with degrees of freedom equal to the sum of the degrees of freedom in each subperiod.

Table 8

Likelihood ratio tests of the restrictions[a] on the coefficients in equation (6) implied by a k-factor asset pricing model (plus a zero-beta portfolio) for k=1,2. The data consist of daily returns of ten size-based portfolios of NYSE and AMEX stocks.

$$\tilde{R}_{pt} = \alpha_p + \delta_{p1}D_t^M + \delta_{p2}D_t^F + \delta_{p3}D_t^J + \delta_{p4}REM_{t-1} + \delta_{p5}RDJ_{t-2} + \delta_{p6}REM_{t-3} + \delta_{p7}(REM_{t-1} \cdot D_t^J) + \tilde{u}_{pt} \qquad (6)$$

(a) Size-Based Portfolios

Subperiod	H_0: k=1; $\chi^{2\,b}_{48}$	H_0: k=2; χ^2_{35}	H_0: k=3; χ^2_{24}
1963-1968	101.792 (*)[c]	56.898 (.0111)	29.307 (.0698)
1969-1973	195.986 (*)	52.843 (.0270)	34.669 (.0735)
1974-1979	110.237 (*)	49.706 (.0509)	34.458 (.0768)
1980-1985	222.607 (*)	62.817 (.0027)	18.767 (.7643)
Overall[d]	630.622 (*)	222.264 (*)	117.202 (.0698)

(b) Industry-Based Portfolios

Subperiod	H_0: k=2; χ^2_{45}	H_0: k=3; χ^2_{32}
1963-1968	99.847 (*)	65.011 (.0005)
1969-1973	86.986 (.0002)	37.231 (.2408)
1974-1979	91.590 (*)	43.947 (.0845)
1980-1985	114.413 (*)	60.323 (.0018)
Overall[d]	392.832 (*	206.062 (*)

[a]The restrictions (3) are modified to allow the intercept to be different and unrestricted in each equation.

[b]The chi-square degrees of freedom represent the number of restrictions.

[c] * indicates p-value less than .0001.

[d]The overall chi-square statistic represents the sum of independent chi-square statistics across subperiods, with degrees of freedom equal to the sum of the degrees of freedom in each subperiod.

Table 9

Likelihood ratio tests of the restrictions in equation (8) on the coefficients in equation (7) implied by a k-factor asset pricing model (plus a zero-beta portfolio) for k=1,2. The data consist of daily returns for ten size portfolios of NYSE and AMEX stocks.

$$R_{pt} = \alpha_p + \alpha_p^J D_t^J + \delta_p' Z_{t-1} + \delta_p^{J\prime}(D_t^J \cdot Z_{t-1}) + \varepsilon_{pt} \qquad (7)$$

(a) Size-Based Portfolios

Subperiod	H_0: k=1; χ^2_{80} [a]	H_0: k=2; χ^2_{56}	H_0: k=3; χ^2_{36}
1963-1968	218.859 (*)[b]	69.737 (.1026)	34.570 (.5366)
1969-1973	238.973 (*)	68.594 (.1204)	42.944 (.1981)
1974-1979	330.708 (*)	70.449 (.0926)	28.560 (.8067)
1980-1985	304.622 (*)	108.400 (*)	—[d]
Overall[c]	1093.162 (*)	317.180 (*)	106.074 (.5344)

(b) Industry-Based Portfolios

Subperiod	H_0: k=2; χ^2_{72}	H_0: k=3; χ^2_{48}
1963-1968	110.829 (.0022)	51.760 (.3293)
1969-1973	118.200 (.0005)	55.378 (.2163)
1974-1979	115.691 (.0008)	52.617 (.3000)
1980-1985	173.170 (*)	86.286 (.0006)
Overall[c]	517.889 (*)	246.041 (*)

[a]The chi-square degrees of freedom represent the number of restrictions.

[b] * indicates p-value less than .0001.

[c]The overall chi-square statistic represents the sum of independent chi-square statistics across subperiods, with degrees of freedom equal to the sum of the degrees of freedom in each subperiod.

[d]Program did not converge. Overall result is based on aggregation over first three subperiods only.