MANAGERIAL INCENTIVES AND CAPITAL STRUCTURE: A GEOMETRIC NOTE

by

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I. INTRODUCTION

The role of debt in resolving the potential conflicts between corporate principals (shareholders) and agents (managers) has been raised by Grossman and Hart (1982) (hereafter G-H) and Jensen (1986). They asserted that if management's stake depends on the continuous operation of a firm, a higher debt which results in a higher bankruptcy possibility may resolve the conflicts.[1]

Interestingly, the above assertion gains support from two recent empirical works, Friend and Lang (1987) and Gonedes, Lang and Chikaonda (1987). They have found empirical evidence for the proposition that, to reduce bankruptcy risks implicit in a higher debt level, management may try to use an amount of debt which is less than the firm's value maximization (hereafter optimum debt) would dictate. In contrast, a higher debt is demanded by shareholders to possibly reduce potential conflicts between them and management.[2] The empirical findings motivate the writing of this paper in the framework of G-H: it is hypothesized that management intends to use a less than optimum amount of debt to reduce bankruptcy risk, but it has to use more debt than it desires to resolve conflicts.

The G-H model is carried out in a world of no taxes and no bankruptcy costs, in which no optimum debt exists in the sense that the optimum debt balances the interest tax shields against costs of bankruptcy until the firm's value is maximized. To analyze the previous hypothesis in a world with the optimum debt, corporate taxes and bankruptcy costs are incorporated into the G-H model. In this extended model, management is assumed to collect funds from investors by issuing debt and equity to implement the predetermined investment projects. Management may intend to increase its perquisites by investing less if the investment level is unobservable to the market. Under the circumstance of the potential conflict, no funds could be collected from investors unless the investors could make inferences about the investment level from the other observable actions taken by management.

Debt creation enables management to effectively bond its promise to pay out future cash flows. If the manager commits the company to a certain amount of debt but can not repay interest and principal, he also loses his perquisites at bankruptcy. To ensure enough cash flow to pay back the company's debt obligation, a certain amount of investment has to be carried out. The market may thus observe the debt level to infer the investment level: thus, the potential conflict can be resolved by the use of debt.

In equilibrium, the same as in the G-H model, a less than optimum amount of investment is carried out, and a corresponding debt which resolves the potential conflict is also derived. Our new theoretical results suggest that management would plan to use an amount of debt which is less than optimum to reduce the bankruptcy risk implicit in a higher debt level. Management has to use more debt than it desires to resolve conflicts: this equilibrium or realized debt may or may not exceed the optimum. To explain our result intuitively, a simple figure is presented. The mathematical proofs are listed in Appendices and footnotes as concisely as possible.

II. THE BASIC MODEL

Consider a corporation in which the manager has discovered a new investment opportunity. The manager has to collect funds H by issuing stocks and bonds to implement the investment projects at date 0. To simplify the model, it is assumed that this firm will be dissolved after date 1.

The firm's pre-tax cash flow X is normally distributed with mean g and variance σ^2.[3] The mean g is assumed to be a concave function on investment I with the properties $g'>0$, $g''<0$, $g'(0)=\infty$. To simplify the model, the investors are assumed to be risk neutral, and the interest rate is assumed to be zero in the subsequent analysis.

The manager invests I out of H and the difference H-I can be treated as the perquisite under the management's discretion. However, when the manager invests I dollars, the equilibrium condition requires that the funds H the manager intends to collect must be equal to the firm's value V(I,B). The firm's value V(I,B) in the sense of Kim (1978) is introduced as follows:

$$(1) \qquad V(I, B) = \int_{-\infty}^{B} (X - K)f(X)dX + \int_{B}^{\infty} \left[B + (1 - t)(X - B)\right]f(X)dX$$

$$= \int_{-\infty}^{S} \left[\sigma z + g(I) - K\right]f(z)dz + \int_{S}^{\infty} \left[B + (1 - t)(\sigma z + g(I) - B)\right]f(z)dz$$

where $S = (B-g)/\sigma$, B is the principal of debt plus interest payments, K is the lump-sum bankruptcy cost, t is the income tax rate, and z is the standardized cash flow. If in a world of no taxes and no bankruptcy costs, t and K are equal to 0 in (1), the firm's value becomes $V(I)=g(I)$: the model is reduced to the original Grossman and Hart one.

(1) states that the operating cash flow X after deducting B will be subject to an income-tax-rate t. If $X > B$, then at date 1, the bondholders will get back B, and the stockholders will get back $(1-t)(X-B)$. If $X < B$, then this firm goes bankrupt with a lump-sum bankruptcy cost K. The stockholders can just exercise their limited liabilities, and the ownership of the bankrupt firm will be transferred to the bondholders. The bondholders have to pay the bankruptcy cost K out of X.

Let the manager have a concave von Neumann-Morgenstern utility function U in which $U'>0$, $U''<0$, $U'(0)=\infty$. It is assumed that this utility is only realized if the firm does not go bankrupt. If the firm goes bankrupt, we suppose the manager also loses his perquisite and so the manager's utility is $U(0)=0$.

The manager maximizes his expected utility function defined as follows before X is known.

$$\text{(2)} \qquad \text{Max } U = E\left[U(I, B)\right] = U(H - I)\left(1 - F(S)\right) \text{ s.t. } 0 \leq I \leq H$$

where F(S) is the cumulative distribution function, which denotes the bankruptcy possibility.

In this model, we assume the investment level is unobservable to the market, thus investors have to make inferences about the investment from the other observable actions taken by the manager, in particular his choice of a debt level.

If the market is rational in the sense that the investors in the market know the utility function $U(H-I)(1-F)$ of the manager, then they can also perceive that, given H and B, the manager will choose an optimal I to maximize his expected utility function (2) as follows:

$$\text{(3)} \qquad U_I = -U'(1 - F) + Ufg'/\sigma = 0$$

U_I denotes the partial derivative of U with respect to I. The assumptions $g'(0)= \infty$ and $U'(0)= \infty$ guarantee the interior solution of I, i.e. $0<I<H$. Rearranging (3) yields:

$$\text{(4)} \qquad U'\sigma/(Ug') = r(S)$$

where $r(S)=f(S)/(1-F(S))$ is the hazard rate, $r'(S)$ is greater than 0 for normal distribution (see Barlow and Proschan (1975), p79).

THEOREM 1: (4) is a necessary and sufficient condition for optimality.

(Proof): See Appendix A.

(4) has a unique solution and there is a unique optimal choice of I for the manager. We write the optimal I as I(H,B). It is easy to show that $I_H>0$, $I_B>0$ (i.e. I(H,B) is a continuous and differentiable function of (H,B)). Given B, the equilibrium requires that the funds H the manager intends to collect must be equal to the firm's value V(I(H,B),B). Replacing H by V yields V(I(V,B),B) = V(B) (Notice, V(B) may not be defined as we will discuss later). Substituting V(B) into I(H,B) yields I(V(B),B)=I(B). Taking V(B) and I(B) back to (2) yields the following:

$$U = U\left[V(I(B),\ B) - I(B)\right]\left(1 - F(S^*)\right) \tag{5}$$

where $S^*=[B-g(I(B))]/\sigma$. An analysis of maximization of (5) is somewhat complicate. V(B) may not be uniquely defined for some B, it also may not be continuous. As suggested by Grossman and Hart, that things become easier if we regard I rather than B as the manager's choice variable. For each $0<I<V=H$, consider the following equation:

$$U'\left(V(I,\ B) - I\right)\sigma/\left(U\left[V(I,\ B) - I\right]g'\right) = r(S) \tag{6}$$

(6) is simply the first order condition for the manager's maximization problem, equation (4), with H set equal to V. Right hand side is the hazard rate which is strictly decreasing in I, but we do not know the properties of left hand side, there may be no solution or more than one solutions. If there is a solution to (6), denoted as B(I), the interpretation of B(I) suggests

that it is the level of debt which sustains the equilibrium (V,I), i.e. $I[V(B(I),I),B(I)]=I$. The reason is that if we set V=H, we see from (6) that (4) is satisfied when B=B(I). It also means that I=I(V,B), i.e. I is an optimum choice for the manager given V and B.

Suppose the solution B(I) is well-defined at $I^0>0$, i.e. (6) can be solved at $I = I^0$. Since r(S) is strictly increasing, (6) can be solved in a neighborhood of I^0. Furthermore, since the left hand side and right hand side of (6) are continuous function, B(I) must be continuous at I^0. Differentiating (6) with respect to I, it is easy to show that B'(I) is finite, therefore, B(I) is also differentiable at I^0.

Having analyzed the relationship between B and I, let us return to the manager's choice of optimal debt level B. Regarding I as the independent variable, we may write the manager's problem as follows:

$$\text{(7)} \qquad \text{Max } U\big[V\big(I,\ B(I)\big) - I\big]\big(1 - F(S^{**})\big)$$

where $S^{**}=[B(I)-g(I)]/\sigma$, and I is restricted to those values for which B(I) is defined. A similar proof as the Lemma 5 in the Grossman and Hart can be carried out to show that there exists at least one solution to (7). Let I be a solution to the manager's overall maximization problem, and we know that B(I) is defined at I, differentiating (7) to get the following necessary condition for optimality:

$$\text{(8)} \qquad dU/dI = -U'(1 - F) + U'(1 - F)(dV/dI) - Uf(B' - g')/\sigma = 0$$

where $dV/dI=V_I+V_BB'$. V_I and V_B are as follows:

$$\text{(9)} \qquad V_I = g'(1 - V_B)$$

$$\text{(10)} \qquad V_B = t(1 - F) - Kf/\sigma$$

where $t(1-F)$ can be explained as the expected tax shield benefits and Kf/σ can be explained as the expected bankruptcy costs.

THEOREM 2: The equilibrium conditions suggest:

$$g' > 1 \tag{11}$$

$$V_B = t(1 - F) - Kf/\sigma = T + G \gtrless 0 \tag{12}$$

where $T=r(S)U/\sigma U' > 0$, $G=(1-g')/(B'-g')<0$.

(Proof): See Appendix B.

In equilibrium, $g'>1$ suggests an underinvestment level I_0 relative to that determined by $g'=1$. As shown in Lemma 1 of Appendix B, $B'(I)>0$, thus a unique debt level B_2 determined by $B(I_0)$ is derived. To gain an economic interpretation of B_2, $V_B=t(1-F)-Kf/\sigma=T+G$ in (12) is decomposed as follows:

$$V_B = t(1 - F) - Kf/\sigma = T > 0 \tag{13}$$

$$V_B = t(1 - F) - Kf/\sigma = T + G \gtrless 0 \tag{14}$$

$T > 0$ can be interpreted as a positive premium (reflected by the positive difference of expected tax shield benefits minus the expected bankruptcy costs) required by management to compensate its bankruptcy risk implicit in a higher debt level. $G<0$ offsets the positive premium T: it suggests that a G amount of premium is demanded by the investors to compensate their informational asymmetry risks.[4]

INSERT FIGURE 1

Equations (13) and (14) can be interpreted intuitively through the aid of Figure 1. In Figure 1, the vertical axis denotes firm's value V, the horizontal axis is debt level B. $V(I_0, B)$ is a concave function of B given the underinvestment level I_0 determined by $g'>1$.[5] V_B is the slope of $V(I_0, B)$, $V_B = 0$ suggests an optimum B^* which maximizes the firm's value under given I_0 (i.e. at B^*, the expected tax shield benefits is equal to the expected bankruptcy costs).

Management intends to demand a positive premium T by using an amount of debt B_1 (determined by (13)) which is less than optimum B^*. In contrast, it has to give up a G amount of premium to meet market requirements by using more debt B_2 (determined by (14)) to resolve conflicts. The realized or equilibrium debt B_2 is greater than B_1 (the one management desires) and less than B_3 (determined by V_B=G), but it may or may not exceed the optimum B^*.

APPENDIX A

The left hand side of (4) is strictly increasing in I, while r(S) in right hand side is strictly decreasing in I. Therefore (4) can hold for at most one value of I. Since (4) is a necessary condition for optimality and the second order derivative U_{II} is less than 0 after we plug in the first order condition (and the first order derivative of hazard rate r(S)). Therefore an optimum certainly exists by Weierstrauss' theorem, this proves that (4) holds if and only if we are at the optimum.

APPENDIX B

Lemma 1: $B' - g' > 0$ ($B' > g' > 0$).

(Proof): Rearranging (8) yields:

(B.1) $$V_B = T + (1 - g')/(B' - g') = T + G$$

where $T=r(S)U/\sigma U'>0$, and $G=(1-g')/(B'-g')$, but we do not know the sign of G. Differentiating (3), substituting $(dV/dI)-1=T(B'-g')$ from (B-1) and going through some arrangements yields:

(B.2.) $$B' - g' = X/Y > 0$$

where

$$X = \sigma^2 U'Ug'' > 0$$

$$Y = \left[\sigma U^2 g'U''r(S)/U' - \sigma U'g'Ur(S) - r'(S)(Ug')^2\right] > 0 .$$ Q.E.D.

Lemma 2: $g' > 1$.

(Proof): Rearranging (8) yields:

(B.3) $$U'\sigma/Ug' = r(S)(B' - g')/\left[dV/dI - 1)g'\right]$$

(6) is rewritten as:

$$U'\sigma/Ug' = r(S) \tag{B.4}$$

(B.4.) must hold at B=B(I), therefore putting (B.3) and (B.4) together, we get

$$(B' - g')/\left[(dV/dI - 1)g'\right] = 1 \tag{B.5}$$

Since $dV/dI = V_I + V_B B'$, substituting dV/dI into (B.5) yields:

$$B' = g'^2(1 - V_B)/(1 - V_B g') \tag{B.6}$$

A necessary condition for I to be an optimal level of investment for the manager is (6) holds. If I is greater than or equal to the optimal I* determined by $g'=1$, then $g'\leq 1$. B' in (B.6) is thus less than or equal to g' if $0<V_B<1$, it clearly contradicts (B.2). Therefore, g' must be greater than 1: (11) is obtained. Q.E.D.

Combining (B.1) and (10) yields (12) in which $T = r(S)U/\sigma U' > 0$ and $G = (1 - g')/(B' - g') < 0$ (since $B' - g' > 0$ by Lemma 1 and $g' > 1$ by Lemma 2): (12) is obtained.

FOOTNOTES

[1]Without actually modelling the idea rigorously, Brander and Lewis (1986, p969) also raised the similar view about the role of a higher debt in motivating management's quality.

[2]Friend and Lang asserted that the negative impact of insider holdings on debt ratio implies the conservative attitude of management toward the use of debt. However, the existence of outside large shareholders demands a higher debt to possibly reduce the potential conflicts between management and shareholders.

Gonedes, Lang and Chikaonda argued that the less dispersion of shareholders, the stronger the shareholders' ability to demand a higher debt to reduce the potential conflicts between the principals and agents.

[3]To simplify the analysis, the cash flow is assumed to be normally distributed. In fact, the same analysis can be carried out for other distributions, such as the Exponential, the Gamma and Weibull with degree of freedom parameter larger than 1, the LaPlace, and the Uniform.

[4]The reason why T is interpreted as the premium to compensate the bankruptcy risk, and G is interpreted as the negative premium to compensate informational asymmetry risk is very simple. Consider the case of no informational asymmetry but with bankruptcy possibility: V_B is equal to T which can be solved by maximizing U(V-I)(1-F) with respect to I and B simultaneously, or simply set G in (12) equal to 0. In the case of no informational asymmetry and no bankruptcy possibility, V_B is equal to 0, which can be solved by maximizing U(V-I) with respect to I and B simultaneously, or simply set T equal to 0.

[5]$V_B=t(1-F)-Kf/\sigma=0$ is the first order condition, rearranging it yields:

$$t\sigma/K=r(S) \tag{5.1}$$

Since the left hand side of (5.1) is a constant, and the right hand side is increasing in B. Therefore, (5.1) can hold for at most one value of B, denoted as B*. The second order condition V_{BB} is negative after we plug in the first order condition (5.1) (and the first order derivative of the hazard rate r(S)): a global maximum is obtained at B* by Weierstrauss theorem.

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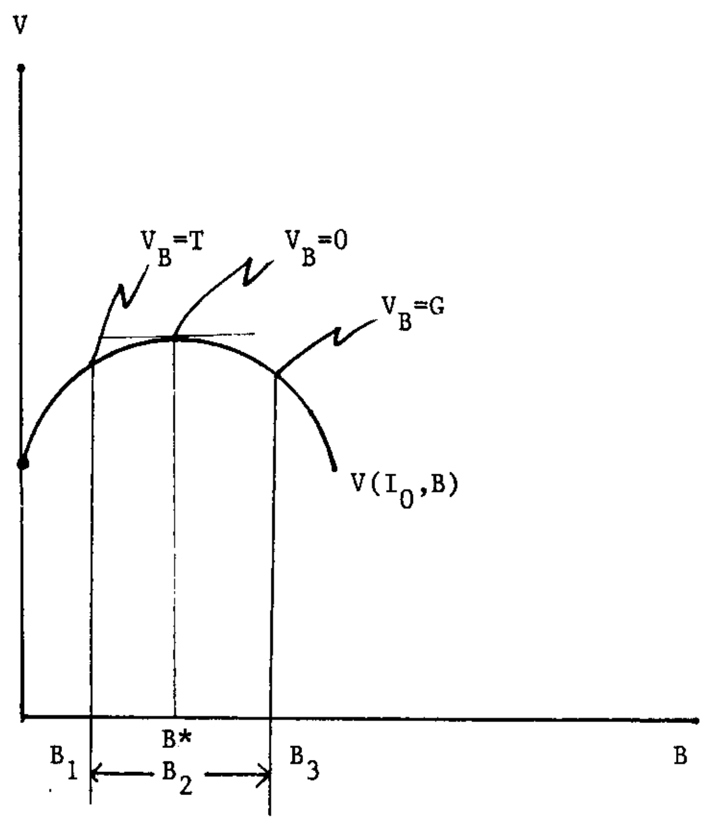

FIGURE 1