PERFORMANCE OF CURRENCY PORTFOLIOS
CHOSEN BY A BAYESIAN TECHNIQUE: 1967-1985

by

Bernard DUMAS and Betrand JACQUILLAT

#18-87

RODNEY L. WHITE CENTER FOR FINANCIAL RESEARCH
The Wharton School
University of Pennsylvania
Philadelphia, PA 19104-6367

April, 1987

PERFORMANCE OF CURRENCY PORTFOLIOS CHOSEN BY A BAYESIAN TECHNIQUE: 1967-1985

Bernard DUMAS
Bertrand JACQUILLAT

Bernard Dumas is professor of Finance at the Wharton School of the University of Pennsylvania and at the Centre HEC-ISA. Bertrand Jacquillat is professor of Finance at Université Paris-Dauphine and at the Centre HEC-ISA. They thank Richard Levich and the Harris Bank for providing them with carefully recorded data on exchange rates and interest rates. They also thank Bruno Solnik for his comments.

Abstract

The hypothesis being tested in this article is that participants in the foreign exchange market are improperly diversified across currencies. If this type of inefficiency were to be verified, it could constitute an explanation of the large volatility of exchange rates: traders who do not fully exploit the potential for diversification unnecessarily restrict the sizes of the positions they do take in the individual currencies, generate thereby a shortage of speculative interest, and, as a consequence, stabilize exchange rates less than they otherwise would. In order to test the hypothesis, a number of implementable portfolio diversification policies are tried out on a large body of data covering nine major currencies and eighteen years of weekly observations. While some policies do produce abnormal returns (over and beyond proper reward for risk), none does so in a statistically significant way. This means that the evidence does not allow one to conclude that market participants are improperly diversified. As a byproduct of this investigation, techniques are found which would allow portfolio managers to earn a proper reward for risk by following a purely mechanical procedure; such techniques may be valuable in a multi-country world where the aggregate portfolio of currencies and securities is unknown and is not supposed to be efficient.

I. INTRODUCTION

It is conceivable that the high volatility of exchange rates, documented by Mussa (1979) and Huang (1981, 1984) among others, could be in part the result of the peculiar (and apparently suboptimal) behavior of the participants in the foreign exchange market. That behavior seems to be at great variance with one of the essential messages of portfolio theory: diversification for risk reduction.

Indeed, casual observation of foreign exchange market practices seems to indicate that industrial and trade customers hardly ever take large[1] open positions in foreign exchange. As for bank traders, they do not seem to pursue any form of concerted diversification policy: trading rooms are decentralized and home and foreign deposits are managed, on a currency-by-currency basis; forecasts of the evolution of individual currencies play a dominant role in management decisions; positions are taken in one currency irrespective of those taken in another, and little account is taken of cross-hedging possibilities.

This situation, if confirmed by the evidence,[2] could explain that there may be a shortage of speculative interest (cf. McKinnon (1979)) in the foreign exchange market, and, as a consequence, could account for some of the apparent excess volatility of spot exchange rates. Furthermore, if the hypothesis were to be verified, a trader could conceivably beat the market, provided he diversifies optimally when his colleagues do not.

---

[1]Large in comparison to their overall involvement with foreign currencies.

[2]Earlier experiments by Bilson (1981) and Hodrick and Srivastava (1984)

In this paper, we explore this possibility by simulating a large family of known mechanical diversification procedures, on an extensive data set covering nine major currencies and eighteen years of weekly data. This endeavor raises several difficulties. The first one pertains to the design of the experiment; in general the choice of a properly diversified portfolio depends on the investor's preferences: risk aversion and consumption tastes, and on the way he will consume the income generated by the portfolio; this last is especially relevant in a world where deviations from Purchasing Power Parity are the rule rather than the exception. In order to solve (or avoid) this difficulty, consideration will be given exclusively to investment policies which would be optimal in the eyes of a logarithmic investor.[3]

The second difficulty arises from the fact that our data set, like all data set, is the result of discrete sampling (data are collected every Friday afternoon), and that the investment vehicles to be considered cover a finite investment period (one month). The choice of optimal portfolios in discrete time cannot, in principle, be handled by means of standard mean-variance theory:[4] we shall therefore have to work out some approximations.

Thirdly, the implementation of an optimal diversification policy requires estimates of the parameters of the probability distribution of rates of return. These parameters being unknown, a mechanical procedure can only use estimates based on past observations. In practice, these estimates are very

---

[3]Bilson (1981) and Hodrick and Srivastava (1984) assumed that the investor targets a given expected return. As Hodrick and Srivastava point out, such a strategy would not be followed by any investor with a given degree of risk aversion.

imprecise: returns being random, the estimated values of expected returns diverge randomly from their true values. These statistical problems have far reaching and disastrous implications for the composition of optimal portfolios: portfolios computed on the basis of such estimates show very large positive or negative positions that no prudent portfolio manager would accept to take. This fact can be easily explained: the optimization program selects massively the assets which have experienced exceptionally high returns in the past, even if these are not statistically significant. In order to overcome this selection inefficiency, we follow Jorion (1985) and use a Bayesian approach; i.e., starting from a prior hypothesis regarding the probability distribution of returns, observations are used to gradually update the statistics.[5] An essential parameter in this updating procedure is the relative confidence placed on the prior hypothesis and on the actual observations. Our experiment will deal with the estimation of expected returns only; we shall find that the prior hypothesis must retain almost 100% weight in this estimation, as otherwise, when past returns are extrapolated to any degree, the ex post results are disastrous.

---

[5]The literature concerning this problem, known as the problem of "estimation risk," includes: Alexander and Resnick (1985), Frost and Savarino (1986), Jobson and Korkie (1980), Klein and Bawa (1976), Son-Nan Chen and Brown (1983). Most of these contributions, in contrast to Jorion (1985) postulate a non-informative prior, so that the decisions made are hardly influenced by the prior. In the international context, to our knowledge, existing contributions include only Jorion (1975) and Eun and Resnick (1987). Eun and Resnick compute and compare the performance of tangent portfolios held by people of different countries. This is not, in our view, an acceptable procedure because these portfolios would be chosen by investors who would differ in risk aversion, if they differed in country of residence.

The fourth and final difficulty must be overcome if we are to demonstrate superior performance: it stems from the absence of an internationally valid performance benchmark. In the realm of currencies, the concept of a world market portfolio awaits a precise definition; furthermore, in a world where countries are separated by Purchasing Power Parity deviations, there is no presumption that the world market portfolio should be an efficient one in the eyes of any investor (cf. Adler and Dumas (1983)). Using this portfolio as a performance benchmark is not justified, contrary to what would happen in the domestic context. Our only choice in this case is to use the one performance measurement technique which does not rely on the knowledge of the market portfolio: that of Cornell (1979), which assumes that past returns are a correct indication of required returns.

The paper is organized as follows: Section 2 presents the portfolio problem and the various approximate solutions. Section 3 introduces the Bayesian approach. Section 4 is devoted to performance and the empirical findings, while Section 5 offers some concluding remarks.

II. OPTIMAL PORTFOLIOS AND APPROXIMATIONS THEREOF

In the international context it is crucial to measure portfolio returns in terms of their purchasing power. Because the same unit of money has a different purchasing power when spent in different countries (deviations from Purchasing Power Parity), investors of different nationalities measure differently real returns from the same security. As a result, they will optimally hold different portfolios even if their investment opportunities are identical, simply because their future consumption opportunities differ. There is no universally efficient set of portfolio policies; it all depends on which national vantage point one adopts. Comparisons between national category of investors: the logarithmic investors (see Sercu (1980) and Adler-Dumas (1983)). When an investor has a logarithmic (Bernoulli) utility function, his current investment decisions are independent of his future decisions (myopic property), including his future consumption decisions. Consequently logarithmic investors are nationless, as it were, and their portfolio policy is optimal irrespective of their country of residence. Furthermore this policy can be optimized based on nominal, rather than real, returns expressed in any currency whatsoever; the result is invariant to the choice of the currency of measurement.[6] This is true equally in continuous or in discrete time.

As a further consequence of the myopic property, logarithmic policies computed in a multiperiod context are identical to those computed in a single-period setting. Hence one can obtain them by means of a (relatively simple) static optimization rather than having to resort to dynamic programming. Because of these practical features and because of the universal value of logarithmic policies, we focus our attention on them exclusively.

In addition to a choice of utility function, we need an assumption regarding the probability distribution of rates of returns.[7] It is out of the question to assume that exchange rate variations are normally distributed; if the $/FF exchange rate were normally distributed, such would not be the case for the FF/$ exchange rate. At any rate, exchange rates cannot become

---

[6]Even if investors have an attitude towards risk which differs from that of the logarithmic person, their optimal portfolio, if it can be revised continuously, is a combination of the logarithmic portfolio and a home currency riskless deposit (see Adler and Dumas (1983)).

[7]An alternative would be to use the empirical frequency distribution observed in the past (cf. Hakansson (1986)). But costly numerical integration

negative. It is much more sensible therefore to assume that exchange rates are lognormally distributed, or more generally that their distribution belongs to the log-stable family. Within that family, the lognormal distribution is the most practical one, and it is postulated in what follows.[8]

2.1. The Simple Case of Two Currencies

Under the twin assumptions of logarithmic utility and lognormal returns, the problem statement for the simple case of two currencies is as follows:

$$\text{(1)} \qquad \underset{w}{\text{Max}} \int_{0}^{+\infty} \log\left[(1 - w)(1 + r) + w \frac{S}{S_0} (1 + r^*)\right] g(S)\, dS \; .$$

In this equation returns are measured in units of one of the two currencies: the measurement-currency. The choice of measurement-currency is arbitrary and immaterial. Symbols have the following meanings:

w = weight devoted to non-measurement-currency deposits in the portfolio;

$1 - w$ = weight devoted to measurement-currency deposits;

r, r^* = interest rates on measurement-currency and non-measurement-currency deposits;

$g(\,)$ = the lognormal density.

---

[8]The lognormal distribution is a highly questionable assumption. Other members of the log-stable family can produce large infrequent jumps of prices, which seems to correspond to observed behavior. Such distributions should therefore induce a more cautious investment policy, a desirable goal in view of the erratic behavior often exhibited by optimal programs (see below). Further, sample estimates of means and variances are unreliable and erratic if in fact the distribution is not lognormal but is another member of the

A change of variable inside the integral (1) will prove convenient.

Let:

$$u = \log\left[\frac{S}{S_0}\,\frac{1 + r^*}{1 + r}\right] ;$$

then equation (1) can be rewritten:

$$\text{(2)} \qquad \underset{w}{\text{Max}} \int_{-\infty}^{+\infty} \log\left[1 - w + w\exp(u)\right] f(u)du \ ,$$

where $f(u)$ is the normal density. Let us also define the expected value and standard deviation of this normal density:

$$\mu = E(u) \qquad \sigma = \text{st. dev. } (u) \ .$$

The reader should note that the variable u, in this Section 2.1., is the log of the gross return ratio between the non-measurement- and the measurement-currencies. (This was done in order to tighten the notation through the forthcoming proofs.) Although μ is akin to the expected excess return of the non-measurement currency deposit, it is not equal to it: it differs from it by $\frac{1}{2}\sigma^2$.

Problem statement (2) is simple enough but there is no known method, other than costly numerical integration and optimization, to compute the solution w. We therefore focus on two approximations to problem (2) and investigate their properties.

The first one will be called the "continuous-time" approximation. Letting μ and σ become infinitely small according to the rules of Itô's calculus (μ/σ^2 remains finite) and taking a Taylor approximation leads to:

$$\text{(3)} \qquad \underset{w}{\text{Max}}\ w\left(\mu + \frac{1}{2}\sigma^2\right) - \frac{1}{2}w^2\sigma^2$$

(4) $$w = \frac{1}{2} + \frac{\mu}{\sigma^2}$$

The second approximation is due to Ohlson (1972), and is described in Ziemba and Vickson (1975, page 356). It has been applied by Ohlson and Ziemba (1976) to the case of power utility functions. It amounts to assuming that $1 - w + w \exp(u)$ is lognormally distributed. This is distinctly incorrect: since $\exp(u)$ is lognormal, so is $w \exp(u)$ but the addition of the constant $1 - w$ suffices to destroy lognormality. The approximation will be close for: $0 \ll w \ll 1$. For $w < 0$ or $w > 1$, however, there will be a marked difference between the true and the approximate distributions: while the true one can produce negative values of $1 - w + w \exp(u)$, the approximate one cannot (see Figure 1).

Now, the objective function (2) is equivalent to:

(5) $$\max_{w} \int_{-\infty}^{+\infty} \log(x(u)) f(u) du$$

where $x = \exp(\alpha u)\left[(1 - w) + w \exp(u)\right]$. And this is true for any number α.

Following Ohlson, x is approximately lognormal and one can compute explicitly the value of this new objective function:

$$E(\log x) = 2 \log E(x) - \frac{1}{2} \log E(x^2)$$

Picking: $$\alpha = -\left(\frac{1}{2} + \frac{\mu}{\sigma^2}\right)$$

one gets (up to constant) a one-term expression:

$$E(\log x) = -\frac{1}{2} \log\left[(1 - w)^2 + 2w(1 - w)\exp(-\mu - \frac{1}{2}\sigma^2) + w^2 \exp(-2\mu)\right] .$$

(6) $$\min_{w} \; (1 - w)^2 + 2w(1 - w)\exp\left(- \mu - \frac{1}{2}\,\sigma^2\right) + w^2 \exp(-2\mu)$$

with the solution:

(7) $$w = \frac{-1 + \exp(-\mu - \sigma^2/2)}{-1 + 2\exp(- \mu - \sigma^2/2) - \exp(-2\mu)} \, .$$

Many elements of a comparison between the exact solution to (2) and the two approximate solutions (4) and (7) are displayed in Figure 2 which graphs the variations of the optimal weight as a function of μ. The following features are apparent:

a. All three solutions give $w = \frac{1}{2}$ for $\mu = 0$.

b. In a neighborhood of $\mu = 0$, the Ohlson solution is tangent and therefore close to the exact solution. Such is not the case for the continuous-time approximation unless $\sigma^2 \to 0$.

c. The exact solution has the property that: $0 < w < 1$. This is because $1 - w + w \exp(u)$ may otherwise be negative with the result that the exact objective function (2) would be undefined.

d. This property is not satisfied by the two approximate solutions. Indeed the continuous-time approximation is linear in μ and is therefore unbounded. The Ohlson approximate solution may also fall outside the (0, 1) range but is at least bounded. It is not, however, monotonically increasing with respect to μ, which is undesirable. There may be some advantage therefore to imposing the constraints $0 \leq w \leq 1$ on the two surrogate optimization problems (3) and (6), as had been suggested by Ohlson.

e. In addition, as one varies the degree of uncertainty of future returns and lets $\sigma^2 \to \infty$ to represent "knowing little about the future," the exact and the continuous-time solutions converge to the "neutral" or "non-committal" policy: $w = \frac{1}{2}$. But the Ohlson solution does not have this property unless μ is close to 0. Indeed the limit of the Ohlson solution is:

$$w = \frac{-1}{-1 - \exp(-2\mu)}$$

2.2. The General Case of n + 1 Currencies

When there are n + 1 currencies rather than just two, the n + 1st being the measurement currency, the two approximate formulations are given by[9]:

a. Continuous-time

Problem Statement:

(3') $$\underset{\{w_i\}}{\text{Max}} \sum_{i=1}^{n+1} w_i\left(\mu_i + \frac{1}{2}\sigma_{ii}\right) - \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij} \; ; \; \sum_{i=1}^{n+1} w_i = 1$$

Solution:

(4') $$w_i = \sum_{j=1}^{n} s_{ij}\left(\mu_i + \frac{1}{2}\sigma_{ii} - \mu_{n+1}\right) \qquad i = 1, \ldots, n$$

(5') $$w_{n+1} = 1 - \sum_{j=1}^{n} w_j$$

(8) where $u_i = \log\left(\frac{S_i}{S_{0i}}(1 + r_i)\right)$; $\quad i = 1, \ldots, n$

$\mu_i = E(u_i)$; $\quad i = 1, \ldots, n$

$\sigma_{ij} = \text{cov}(u_i, u_j)$; $\quad i, j = 1, \ldots, n$

$\mu_{n+1} = \log(1 + r_{n+1})$;

s_{ij} = the elements of the matrix inverse of $[\sigma_{ij}]$

b. Ohlson

Problem Statement:

(6') $$\underset{\{w_i\}}{\text{Min}} \sum_{i=1}^{n+1}\sum_{j=1}^{n+1} w_i w_j \beta_{ij} \; ; \; \sum_{i=1}^{n+1} w_i = 1$$

---

[9]The reader should beware that the following formulae incorporate a slight change of notation: whereas, in Section 2.1., u was the log of the gross return ratio between nonmeasurement-currency deposits over the

Solution:

$$(7') \qquad w_i = \sum_{j=1}^{n+1} b_{ij} \Big/ \sum_{j=1}^{n+1} \sum_{j=1}^{n+1} b_{ij} \; ; \quad i = 1, \ldots, n+1$$

where: $\beta_{ij} = \exp\left(-\mu_i - \mu_j - \frac{1}{2}(\sigma_{ii} + \sigma_{jj} - 2\sigma_{ij})\right) \; ; \quad i, j = 1, \ldots, n+1$

b_{ij} = the elements of the matrix inverse of $[\beta_{ij}]$

$\sigma_{n+1,i} = 0.$

Both methods, as well as the exact one, produce the "neutral" result: $w_i = 1/(n+1)$ if:

$$(9) \qquad \mu_i = \mu_{n+i}$$

and

$$(10) \qquad \sigma_{ii} = \sigma_{jj} = 2\sigma_{ij} \quad \text{for all } i \text{ and } j .$$

Condition (10) implies that all currencies have the same individual risk and that all currency pairs have a correlation of .5. It is invariant under a change of measurement-currency: if all currencies have equal risk and correlations of .5 when measured against the dollar, the same will be true when measured against the franc. Condition (9) plus (10) taken jointly is also invariant.

III. INPUTS TO THE PORTFOLIO-CHOICE PROGRAM: ALTERNATIVE STATISTICAL TECHNIQUES

Two solution methods have been outlined which lead to two investment strategies: the continuous-time and the Ohlson strategies. These may be applied without the constraint $w_i \geq 0$ as in (4') and (7'), or with this constraint, using an appropriate algorithm.[10] So, we have already four possible strategies. But one essential problem remains: the estimation of the parameters μ_i and σ_{ij}. Additional strategy choices will be produced by different estimation methods.

Estimation risk (over and above pure investment risk) arises when the parameters of the probability distributions of returns are not known exactly. The most satisfactory way to handle this problem is to adopt a Bayesian viewpoint. One may start from a non-informative prior (where the parameters are assumed uniformly distributed to represent "knowing little"), record observations and obtain posterior distributions. Predictive distribution for future returns may then be derived based on past returns.[11] For sufficiently large samples, the estimates of the parameters are very close to classical sample estimates of means, variances and covariances. Hence Bayesian approaches predicated on a non-informative prior make almost no difference to the end result (cf. Klein and Bawa (1976)).

Another approach to the estimation of a vector of independent expected returns was proposed by James and Stein (1971) with the objective of minimizing the sum of the squared errors taken over all the elements of the vector. This procedure can be interpreted as a Bayesian one where, instead of positing a non-informative prior, one assumes an informative one, conveying the prior hypothesis that all expected returns are zero. Jorion (1985) recently generalized this technique to allow interdependent returns and the

---

[10]We use below the Frank-Wolfe (1956) algorithm, with the equal-weights policy as initial solution.

[11]The notion that a predictive distribution of future returns may be based on past sample returns is not inconsistent, from a statistical point of

prior hypothesis that all expected returns are equal but unknown.[12] The posterior estimates of expected returns on individual investments were found to be a weighted average of the own sample means and of the average return on the minimum-variance portfolio. The effect of this estimation on portfolio policy is to shrink the efficient set towards the minimum-variance portfolio. Jorion found that this policy was distinctly more successful _ex post_ than the classical ones which use simply the sample averages. He concluded that "the merit of diversification rests in lowering risk, rather than pursuing higher returns." Further, this reduction in _ex post_ risk was achieved despite the fact that no account was taken of the estimation risk linked to risk parameters (i.e. straight sample variances and covariances were used).

Although we intend to retain Jorion's approach, we must object to his choice of prior information which is not advisable in the international context. The "shrinkage" towards the minimum-variance portfolio introduces a behavior which is country-specific,[13] so that it cannot be appropriate for a universal investor such as the logarithmic one considered here. Further, investors of different countries cannot simultaneously be postulating that expected _real_ returns on all securities are equal, as their respective assumptions would be incompatible when there are random deviations from Purchasing Power Parity.

---

[12]When all expected returns are assumed equal, the investor holds the minimum-variance portfolio, no matter what the value of this common expected return may be. As far as the portfolio decision is concerned, it matters little whether the common value is known or unknown.

[13]The minimum-variance portfolio of a U.S. investor is very different from the minimum-variance portfolio of a French investor (see Adler and Dumas

A more "neutral" prior should produce a shrinkage towards the portfolio which assigns equal weights to all currencies. Hence, considering what was said, the prior information could be chosen to be:

(9) $$\mu_i = \mu_{n+1} \qquad \text{for all } i \text{ ;}$$

(1) $$\sigma_{ii} = \sigma_{jj} = 2\sigma_{ij} \qquad \text{for all } i, j \text{ .}$$

The first part of this assertion says that the expected value of the <u>logarithm</u> of gross returns (see the definitions (8)) from all currency deposits should be equal, and <u>not</u> that all expected rates of returns are equal. The prior hypothesis so formulated presents the major advantage that the same prior[14] may be used irrespective of the unit in which gross returns are measured, and therefore irrespective of the country of residence of the investor.

More specifically, following Ando and Kaufman (1965), we assume that the prior distribution of the parameters μ_i, and σ_{ij} is Normal-Wishart. This category of distributions has four parameters which are:

- a vector interpreted as the prior expected value of the vector of μ_i's; in accordance with (9), we choose a vector of equal numbers (where the common value denoted m' will be immaterial),
- a number reflecting the confidence placed on this prior expected value; we choose a number which is large next to the number of observations used to compute portfolios. In what follows, λ will denote the confidence placed on the observations relative to that placed on the prior; λ will therefore assume small values between 0 and 0.05 (0.1 at the most);
- a matrix interpreted as the prior expected value of the inverse of the variance-covariance matrix of returns; this matrix can be chosen in accordance with (10);
- a number reflecting the confidence placed in this prior hypothetical matrix; we choose a number which is negligible next to the number of

---

[14]We noted above already that, if (9) is true when gross returns are

observations to be used in computing portfolios. This means that we believe that observations give a reliable estimate of the variance-covariance matrix, almost without regard for the prior[15]; as a consequence, (10) will play no role in our calculations.

The portfolio simulations to be performed below will consider various possible values for the number of observations used in portfolio choice, and various possible values of λ, the relative confidence placed on past observations. Call m' the common prior expected value of rates of returns and m_i the sample mean return on investment i; the posterior expected value on investment i can then be shown to be (cf. Ando and Kaufman (1965) or any textbook on Bayesian statistics)[16]:

$$m''_i = \lambda m_i + (1 - \lambda')m' \quad . \tag{11}$$

This posterior estimate is used in place of the parameter μ_i, in the formulae of Section 2.2 governing portfolio choice, while the sample estimates of variances and covariances are used in place of σ_{ij}.

4. PERFORMANCE

The several strategies outlined in the previous section have been simulated on a data set containing nine currencies: the United States and Canadian dollars, the British Pound, the Belgian and French francs, the German Mark, the Italian Lira, the Dutch guilder and the Swiss franc. These data were taken from the Harris Bank Weekly Newsletter. There are 955 weekly records from January 6, 1967 to June 28, 1985. The exchange rates are spot

---

[15]There is no logical inconsistency in using a large number for the prior confidence in the expected values while using a small number for the prior

closing end-of-week bid rates. The interest rates are one-month eurocurrency rates quoted each week. One month, in the eurocurrency markets is 30 or 31 days, which is not exactly four weeks. This slight discrepancy between the frequency of observations on exchange rates and the period of investment in deposits was ignored.[17] The weekly data on exchange rates were used to estimate the parameters of the exchange rate processes, in order to use as large a sample of nonoverlapping data as possible. Since the period of investment is one month rather than a week, the parameters of the exchange rate processes were multiplied by four on the assumption that exchange rates follow approximately a geometric Brownian motion. This last assumption affects only the calculation of optimal portfolios, not the measurement of the *ex post* performance. Of course, at each point in time, only past data were used in the calculation of the investment policy to be implemented during the following month. The performance of each policy was evaluated on a succession of nonoverlapping monthly periods so that we obtain each time four different estimates of performance, corresponding to four investment cycles.

A first look at the data and at the investment terrain during the period under investigation is afforded by Table 1 which shows the performance of individual currency investments, in terms of the average excess dollar return (over a pure dollar investment) and of the standard deviation of the excess dollar return. The best investment performance was provided by the Belgian franc with an average excess return of 0.154% per month for a standard

---

[17]Considering the high uncertainty which is characteristic of exchange

deviation of 2.57% per month.[18] The worst investment performance over the period was that of the Canadian dollar with a negative average excess return equal to -0.053% per month and a standard deviation of 0.95% per month. But, of course, buying and holding individual currencies cannot serve as a proper benchmark with which to compare various investment strategies, since an investor would have had no a priori knowledge allowing him or her to choose a currency to buy and hold.

A more acceptable or neutral benchmark is provided by a policy of investing in equal amounts of the various currencies.[19] Its performance is displayed as the last row of Table 1: the average excess return is 0.043% per month and the standard deviation is 1.68% per month. The Belgian franc, the Swiss franc and the German mark are the only individual currencies with an average excess return larger than that of the equally weighted portfolio; but, because of diversification, the equally weighted portfolio has a lower standard deviation.[20]

In principle, performance must be measured relative to a portfolio which is presumed to be efficient. In domestic capital market equilibrium, the market portfolio, if and when it can be identified, provides such a benchmark. But, in international capital market equilibrium, the world market portfolio, in the presence of deviations from Purchasing Power Parity, is not

---

[18] In annualized terms, this is an average excess return of 1.85% p.a. and a standard deviation of 8.9% p.a. We chose generally to present the results of monthly investments on a per month basis, rather than a per annum basis, because annualization involves multiplying the average return by 12 and the standard deviation by the square root of 12; the comparison between annualized risk and return may then be misleading.

[19] Recall that this is the policy which, as a Bayesian, our investor would

supposed to be efficient (cf. Adler and Dumas (1983)). At any rate, identifying it would be even more problematic than in the domestic context. There exists one technique, originally suggested by Cornell (1979), which does not rely on any particular Capital Asset Pricing Model, or any measurement of the market portfolio, and yet allows appraisal of performance. It requires, however, the knowledge of the managed fund's portfolio weights at all times--which, in our case, is not a problem--and the assumption that at any point in time the average of recent rates of return on each security provides a reliable indication of the required return on that security, given its risk. The way the market evaluates the said risk is a moot issue, because reliance on the past average allows one to bypass the CAPM.

The Cornell measure of performance, which we shall call the "abnormal rate of return," represents the difference, over any investment period, between the actual rate of return on the managed portfolio and the rate of return expected by the market on this portfolio, the latter being estimated using the sample average of each security. More specifically, let w_{it} be the portfolio weights chosen at time t and let R_{it} be the <u>ex post</u> rate of return on investment i over the period [t, t + 1]. Also, define $\bar{R}_{it}$ as the average of rate of return observations of investment i prior to time t. The abnormal rate of return during investment period [t, t + 1] is defined as:

$$\Sigma_i w_{it}[R_{it} - \bar{R}_{it}] \; . \tag{12}$$

Once each period's abnormal return is known, one is at liberty to summarize the investment record by means of various statistics (mean etc.).[21]

It is debatable whether the calculation of $\bar{R}_{it}$ in the Cornell measure should incorporate some Bayesian prior. It is likely that a Bayesian technique, similar to the one which has been advocated for the computation of portfolios, would improve the statistical efficiency of the estimation of expected returns. But the need to apply such a technique in the present context is less pressing. When the purpose of expected return estimation is portfolio choice, a small degree of inefficiency in the estimation in expected return estimation translates into a sizeable inefficiency in the estimation of optimal portfolio weights, because optimization emphasizes past occurrences of extraordinary returns. But when the purpose of expected rate of return estimation is performance evaluation, an error in the estimate of expected return translates into an equal error in the measurement of performance. While experiments with Bayesian Cornell measures may be advisable, they will not be conducted here: in what follows, we incorporate classical sample averages into equation (12).[22]

We are now ready to examine the performance of Bayesian investment policies computed on the basis of past statistics, as outlined in the previous section. The choice is between the two approximations of the logarithmic problem (continuous-time vs. Ohlson approximation) with or without nonnegativity weight constraints on the portfolio composition. We also vary the weight (lambda) placed on past observations of returns as against the prior hypothesis (9), as well as the number of observations (N weeks) used at each point in time to compute the statistics on which portfolio choice is based. A complete account of these experiments is shown in Table 2; it leads to the following conclusions.

---

[22]These sample average values of $\bar{R}_{it}$ will be based on the same number of weekly observations which serves to compute portfolio composition.

Consider first the investment policies which place a weight equal to zero on past observations (lambda = 0). These policies do not place any bet on the future expected evolution of exchange rates as they rely entirely on the neutral prior hypothesis (9), as far as the estimation of expected returns is concerned. They rely on past observations for the estimation of variances and covariances only, so that past information is used mostly to reduce risk exposure. These prudent and nonspeculative policies produce a very satisfactory performance which is displayed in panel A of Table 2. Their standard deviations are about equal to that of the equally weighted portfolio (see Table 1), but their average excess returns are about twice as large and equal to 0.1% per month (1.2% per year). These policies, which require no foresight, produce an average excess return almost equal to that achievable by a person who would have had the foresight of holding the Swiss franc throughout (see Table 1); but the risk involved is markedly less (approximately 1.7% per month, rather than 2.53% per month). They are also the only ones which produce positive average abnormal returns, although a standard t test would indicate that they are not significantly positive.

Panel B of Table 2 gives the results of policies which place some weight on past returns in the estimation of expected returns, and impose no restriction on portfolio composition. They generally lead to disastrous results, with negative average excess returns[23,24] and worse negative average

---

[23]This means of course that it would have been preferable to stay with the dollar throughout, rather than implement such policies. Although holding the dollar may seem like a natural fallback policy to an American investor, we should consider that, for someone with no particular home currency preference, buying and holding the dollar is really a policy which requires foresight, while the policies examined in Table 2 require none whatsoever.

[24]One must be careful not to pay undue attention to the sign of the average excess return. This sign changes depending on whether one measures

abnormal returns. Such policies extrapolate the past evolution of exchange rates; they are overly sensitive to past signals and produce highly volatile portfolio compositions which are themselves responsible for high ex post variances.[25] If one is to place any weight on past observations in the estimation of expected returns, it is essential to impose some restriction on portfolio compositions as a protection against these excessive reactions.[26]

When this is done, by means of non negativity constraints, performance is again excellent (cf. panel C of Table 2).[27] This is particularly true as far as excess returns are concerned: these policies can produce average excess return two to four times larger than the policy of holding the Belgian franc, for instance,[28] with a similar standard deviation of excess return. Abnormal returns, however, are always negative indicating no ability to "beat the

---

return. ½ variance would produce a ranking of policies which is invariant to the choice of the measurement currency. In practice, the ½ variance hardly ever affects the ranking. Hence average excess return can be used to determine approximately how much better one policy is compared to another. We are grateful to B. Solnik for warning us against "overinterpretation" of the sign of the excess return.

[25]There is a legitimate question as to whether the ex post unconditional variance is an appropriate measure of risk when portfolio composition varies over time, or whether one should look at each period's portfolio variance conditional on the portfolio weights being chosen, and then perhaps take an average of these.

[26]It is because we anticipated this result that panel B contains experiments based on the Ohlson technique only: recall from Section 2 and Figure 2 that the Ohlson portfolio weights, in contrast to those obtained by the continuous-time approximation, are at least bounded. This precaution proved illusory.

[27]Recall from Section 2 and Figure 2 that the exact logarithmic portfolio, under the assumption of logarithmically distributed rates of returns, never contains any security held negatively; this justifies imposing nonnegativity constraints as an add on to the approximate techniques used in the experiments of panel C.

market." As for the approximation technique used, it seems to affect performance negligibly.[29]

5. CONCLUSION

Even though we have discovered some automatic portfolio policies, which require no foresight, which are readily implementable by portfolio managers, and which seem to produce satisfactory risk-return results, we have failed to prove that participants in the foreign exchange market are insufficiently diversified. In order to prove this, one would have needed to find statistically significant and positive abnormal returns for at least some portfolio diversification strategies. This never happened.

It can be argued, however, that we, perhaps, did not look in the right place, and that we should not have expected to find such a result considering the restricted data set which has been used. Even though we have experimented with nine major currencies, these were the U.S. dollar, the Canadian dollar and seven European currencies. Within the group of the first two currencies and the group of the last seven, correlations are strong, so that the scope for diversification may be limited. A wider data set including the yen,[30] other currencies, stocks, etc., ... may have produced different results. The search can be continued. This paper has served to illustrate how several major problems of portfolio choice can be solved, when the time comes to implement expected utility-maximization theory.

---

[29]The almost indistinguishable results of using the continuous-time and the Ohlson approximations are presented as one, even when both were tried out.

REFERENCES

M. Adler and B. Dumas, "International Portfolio Choice and Corporation Finance: A Synthesis," Journal of Finance 38 (June 1983).

G. S. Alexander and B. Resnick, "More on Estimation Risk and Simple Rules for Optimal Portfolio Selection," Journal of Finance, March 1985, 125-133.

A. Ando and G. M. Kaufman, "Bayesian Analysis of the Independent Multi-Normal Process: Neither Mean nor Precision Known," American Statistical Association Journal (March 1967), 347-358.

J. F. D. Bilson, "The Speculative Efficiency Hypothesis," Journal of Business 54 (July 1981), 435-452.

B. Cornell, "Asymmetric Information and Portfolio Performance Measurement," Journal of Financial Economics 7 (December 1979), 381-390.

C. S. Eun and B. G. Resnick, "International Diversification under Estimation Risk: Actual versus Potential Gains," in S. J. Khoury and A. Ghosh, Eds., Recent Developments in International Banking and Finance, Vol. 1, D. C. Heath and Company, 1987, 135-148.

M. Frank and P. Wolfe, "An Algorithm for Quadratic Programming," Naval Research Logistics Quarterly 3 (1956), 95-110.

P. A. Frost and J. E. Savarino, "Portfolio Size and Estimation Risk," Journal of Portfolio Management 12 (Summer 1986), 60-64.

N. H. Hakansson, "On Optimal Myopic Portfolio Policies, With and Without Serial Correlation of Yields," The Journal of Business 44 (1971), 324-334.

N. H. Hakansson and R. Grauer, "1968-1984 Returns on Internationally Diversified Portfolios of Stock and Bonds, With and Without Leverage," working paper University of California, Berkeley, 1986.

R. J. Hodrick and S. Srivastava, "An Investigation of Risk and Return in Forward Foreign Exchange," Journal of International Money and Finance 3 (1984), 5-29.

R. D. Huang, "The Monetary Approach to Exchange Rate in an Efficient Foreign Exchange Market: Tests Based on Volatility," The Journal of Finance 36 (March 1981), 31-42.

__________, "Some Alternative Tests of Forward Exchange Rates as Predictors of Future Spot Rates," Journal of International Money and Finance 3 (1984), 153-167.

W. James and C. M. Stein, "Estimation with Quadratic Loss," in J. Neyman (ed.)

J. D. Jobson and B. Korkie, "Estimation for Markowitz Efficient Portfolios," Journal of the American Statistical Association 75 (September 1980), 544-554.

P. Jorion, "International Portfolio Diversification with Estimation Rise," The Journal of Business 58, 3 (July 1985), 259-278.

R. W. Klein and V. S. Bawa, "The Effect of Estimation Risk on Optimal Portfolio Choice," Journal of Financial Economics 3 (June 1976), 215-232.

R. McKinnon, R., Money in International Exchange: The Convertible Currency System, New York: Oxford University Press, 1979.

M. Mussa "Empirical Regularities in the Behavior of Exchange Rates and Theories of the Foreign Market," in Policies for Employment, Prices, and Exchange Rates, K. Brunner and A. H. Meltzer, eds., Carnegie-Rochester Series on Public Policy, Vol. 11.

J. A. Ohlson, "Optimal Portfolio Selection in a Log-Normal Market when the Investor's Utility Function is Logarithmic," Research Paper No. 117, Graduate School of Business, Stanford University, 1972.

J. A. Ohlson and W. T. Ziemba, "Portfolio Selection in a Lognormal Market when the Investor has a Power Utility Function," Journal of Financial and Quantitative Analysis 11 (March 1976), 57-72.

P. Sercu, "A Generalization of the International Asset Pricing Model," Revue de l'Association Francaise de Finance 1 (June 1980), 91-135.

Son-Nan Chen and S. J. Brown, "Estimation Risk and Simple Rules for Optimal Portfolio Selection," Journal of Finance (September 1983), 1087-1093.

B. W. Stuck, "Explicit Solutions to Some Single-Period Investment Problems for Risky Log-Stable Stocks," Journal of Financial Economics 3 (June 1976), 277-295.

W. T. Ziemba and R. G. Vickson, Stochastic Optimization Models in Finance, Academic Press, 1975.

TABLE 1

BENCHMARK

(Worst observed performance out of the four monthly cycles, each covering approximately 160 months of investment.)

Policy	AVERAGE EXCESS RETURN % per month	STANDARD DEVIATION % per month
Canada	-0.053	0.95
England	0.002	2.41
Belgium	0.154	2.57
France	-0.005	2.54
Germany	0.078	2.69
Italy	0.016	2.23
Netherlands	0.043	2.53
Switzerland	0.118	2.86
United States	0	0
Equal Weights	0.043	1.68

TABLE 2

PERFORMANCE OF COMPUTED POLICIES*

(Worst observed performance out of the four monthly cycles.)

	PARAMETERS**		EXCESS RETURN			ABNORMAL RETURN		
TION	Lambda	N	Average % per month	Standard Deviation % per month	CYCLE	Average % per month	Standard Deviation % per month	CYCLE
	0	40	0.107	1.8	3	0.089	1.86	3
	0	60	0.097	1.71	2	0.077	1.89	3
	0	80	0.096	1.73	2	0.058	1.87	3
ICTED WEIGHTS								
	0.01	60	-0.046	4.64	1	-0.428	4.7	1
	0.02	60	-0.278	9.96	1	-1.059	10.04	1
	0.03	60	-0.604	16.09	1	-1.734	16.22	1
	0.04	60	-0.874	21.63	1	-2.425	21.79	1
	0.05	60	-1.11	27.18	1	-3.085	27.37	1
	0.1	60	-2.363	54.5	1	-6.447	54.84	1
E WEIGHTS								
JOUS	0.01	40	0.277	2.01	3	-0.148	2.1	3
	0.02	40	0.399	2.31	3	-0.232	2.4	3
	0.03	40	0.498	2.53	3	-0.229	2.46	4
	0.04	40	0.536	2.49	2	-0.306	2.59	2
	0.05	40	0.561	2.57	2	-0.312	2.67	2
	0.1	40	0.598	2.67	2	-0.316	2.84	3
JOUS	0.01	60	0.178	2.1	1	-0.115	2.13	1
	0.02	60	0.246	2.24	1	-0.239	2.27	1
	0.03	60	0.287	2.41	1	-0.31	2.44	1
	0.04	60	0.319	2.53	1	-0.359	2.58	1
	0.05	60	0.342	2.59	1	-0.369	2.63	1
	0.1	60	0.377	2.75	1	-0.42	2.79	1

*Over approximately 150 months of investment.

*Lamda = weight based on past observations, as against the prior hypothesis, when computing lios. N = number of weekly observations used in calculating portfolios, and also in estimating ed return in the Cornell measure of abnormal performance.

FIGURE 1

Correct and approximate distributions of 1-w+w exp (u) for w>1

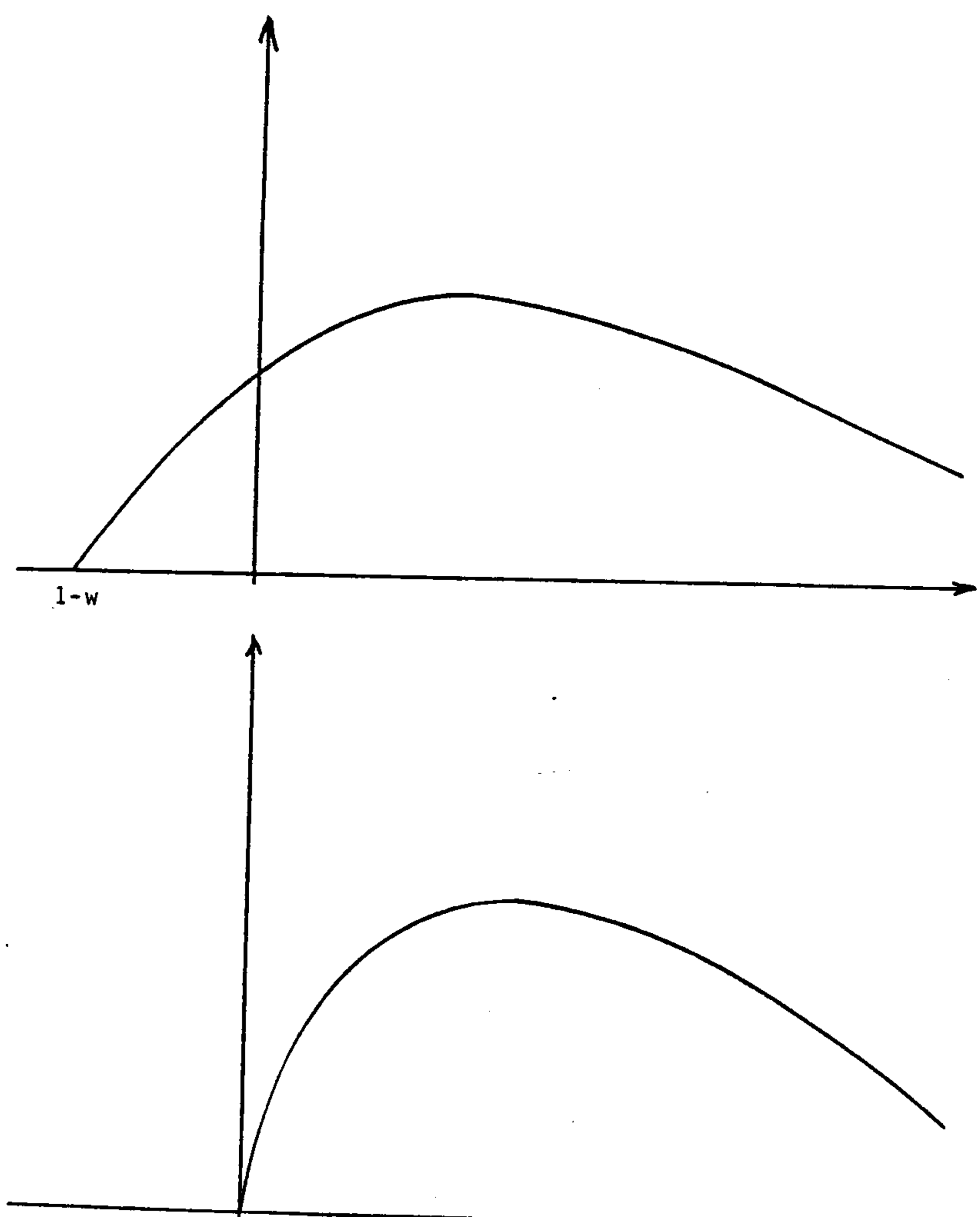

FIGURE 2 - Comparison of exactly and approximately optimal portfolio weights