A SIMPLE SPECIFICATION TEST OF THE RANDOM WALK HYPOTHESIS

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(13-87)

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October 1986

Latest Revision: April 1987

We propose a simple test for the random walk hypothesis using variance estimators derived from data sampled at different frequencies. This Hausman-type specification test exploits the linearity of the variance of random walk increments in the observation interval by comparing the (per unit time) variance estimates obtained from distinct sampling intervals. Test statistics are derived for both the i.i.d. Gaussian random walk and the more general uncorrelated but possibly heteroscedastic random walk. Monte Carlo experiments indicate that although the finite-sample behavior of our specification test is comparable to that of the Dickey-Fuller t-test and the Box-Pierce Q-statistic under the i.i.d. null, our test is more reliable than either of these tests under a heteroscedastic null. We also perform simulation experiments to compare the power of all three tests against two interesting alternative hypotheses: a stationary mean-reverting Markov process which has been interpreted as a 'fads' model of asset prices, and an explosive non-Markovian process which exhibits essentially the opposite time series properties. By choosing the sampling frequencies appropriately, the variance ratio test is shown to be as powerful as the Dickey-Fuller and Box-Pierce tests against both alternatives. As an empirical illustration, we perform our test on weekly stock market data from 1962 to 1985 and strongly reject the random walk hypothesis for several stock indexes.

*Department of Finance, Wharton School, University of Pennsylvania, Philadelphia, PA, 19104-6367. We thank Chris Cavanagh, Whitney K. Newey, Ken Singleton, Mark Watson and seminar participants at M.I.T., Northwestern University, Princeton University, Stanford University, UCLA, University of Chicago, University of Michigan, and the University of Pennsylvania for helpful comments. We are grateful to Stephanie Hogue, Elizabeth Schmidt, and Madhavi Vinjamuri for preparing the manuscript. Research support from the National Science Foundation (Grant No. SES-8520054) and the University of Pennsylvania Research Fund is gratefully acknowledged. Any errors are of course our own.

1. INTRODUCTION.

Whether or not an economic time series follows a random walk has long been a question of great interest to economists. Although its origins lie in the modelling of games of chance, the random walk hypothesis is also an implication of many diverse models of rational economic behavior.[1] In this paper, we propose an alternative test of the random walk hypothesis which is particularly simple to implement. The test is based upon a comparison of variance estimators obtained from data sampled at different frequencies, and exploits the fact that the variance of random walk increments is linear in the sampling interval. Therefore the variance of, for example, quarterly differences must be three times as large as the variance of monthly differences. Comparing the (per unit time) variance estimates from quarterly-to monthly data within a given time span will then yield an indication of the plausibility of the random walk specification. Such a comparison is formed quantitatively along the lines of the Hausman [1978] specification test and is quite easy to perform.

Since the random walk model is a special case of what has come to be known as the unit root hypothesis, a few comments concerning the relation of our variance-based tests to those in the unit root literature may be appropriate. Due to the nonstationarity of time series with unit roots, it is now well known that the standard regression test statistics do not possess the usual limiting distributions. Nevertheless such limiting distributions have been tabulated numerically by Dickey and Fuller [1979, 1981] so that unit root tests may still be performed using their significance points.[2] However, because these tabulated distributions depend critically upon nuisance parameters (e.g. the drift of the random walk), there has been some statistic is the invariance of its limiting distribution to the value of any nuisance parameters. In particular, the asymptotic distributions of the proposed statistics are all Gaussian and do not depend upon whether the drift is zero or not. This is in sharp contrast to the usual regression t-statistic, whose limiting distribution depends discontinuously upon the presence or absence of a non-zero drift.[3] Of course, it must be emphasized that our variance-based statistic tests only the random walk special case of the unit root hypothesis, whereas Phillips [1985, 1986], Phillips and Perron [1986], and Perron [1986] show that the Dickey-Fuller distributions are asymptotically appropriate for a more general class of processes.[4]

In Section 2, we develop our specification test statistics and derive their limiting distributions under two distinct forms of the random walk null hypothesis: a random walk with independent and identically distributed increments, and with uncorrelated but weakly dependent heterogeneously distributed heteroscedastic increments. In order to obtain consistent estimates of our test statistics' asymptotic variance in the latter case, we employ the results of White [1980] and White and Domowitz [1984]. Since our inferences are based entirely upon asymptotic approximations, we perform Monte Carlo experiments to deduce the quality of those approximations in finite samples and report those results in Section 3. The simulation experiments are conducted under two null hypotheses: the i.i.d. Gaussian random walk and a random walk with uncorrelated but heteroscedastic increments. For purposes of comparison, we also report corresponding results for the Box-Pierce Q-statistic and the Dickey-Fuller t-statistic. The results indicate that, although the three test statistics are comparable under the i.i.d. null, the variance ratio test is generally more reliable than the other two in the three tests under two specific alternative hypotheses which have particular economic significance: a mean-reverting Markov process which has been interpreted as a 'fads' model of asset prices, and an explosive non-Markovian process which is an empirically more relevant alternative for stock returns. We show that, although the Dickey-Fuller test is more powerful than the Box-Pierce against the first alternative and vice-versa against the second, the variance ratio test is as powerful as both tests against both alternatives for appropriately chosen sampling intervals. As an illustrative empirical example, we perform our test on weekly stock market data from 1962 to 1985 and report those results in Section 5. For several stock indexes the random walk hypothesis is strongly rejected. We conclude in Section 6.

2. THE SPECIFICATION TEST.

We begin by defining our null hypothesis explicitly; let X_t denote a stochastic process which satisfies the following recursive relation:

$$X_t = \mu + X_{t-1} + \epsilon_t \qquad E[\epsilon_t] = 0 \quad \text{for all } t . \tag{1a}$$

or

$$\Delta X_t = \mu + \epsilon_t \qquad \Delta X_t \equiv X_t - X_{t-1} \tag{1b}$$

where the drift μ is an arbitrary parameter. The notion of a random walk considered in this paper essentially reduces to the restriction that the disturbances ϵ_t are serially uncorrelated, or that innovations are unforecastable.[5] In Section 2.1, we develop our test under the restrictive null hypothesis that the ϵ_t's are independently and identically distributed normal random variates. However, since the unforecastability of increments is in fact the hypothesis of interest, we construct a heteroscedasticity-robust test statistic in Section 2.2 for the weaker null hypothesis that the disturbances are serially uncorrelated.

2.1 I.I.D. GAUSSIAN DISTURBANCES.

Let the null hypothesis H_1 denote the case where the ϵ_t's are i.i.d. normal random variables with variance σ_0^2 hence:

$$H_1 : \epsilon_t \quad \text{i.i.d.} \quad N(0, \sigma_0^2) \ . \tag{2}$$

Note that in addition to homoscedasticity, we have made the assumption of independent Gaussian increments as in Dickey and Fuller [1979, 1981], Evans and Savin [1981, 1984], and Bhargava [1986]. An example of such a specification is the exact discrete-time process X_t obtained by sampling the following well-known continuous-time process at equally spaced intervals:

$$dX(t) = \mu dt + \sigma_0 dW(t) \ . \tag{3}$$

This process is usually referred to as arithmetic Brownian motion with drift and corresponds to a lognormal diffusion price process often used in continuous-time models of financial asset prices.

Suppose we obtain nq + 1 observations $X_0, X_1, \ldots, X_{nq}$ of X_t where both n and q are arbitrary integers greater than one. Considering the following estimators for the unknown parameters μ and σ_0^2:

$$\hat{\mu} \equiv \frac{1}{nq} \sum_{k=1}^{nq} [X_k - X_{k-1}] \equiv \frac{1}{nq} [X_{nq} - X_0] \ . \tag{4}$$

$$\hat{\sigma}_a^2 \equiv \frac{1}{nq} \sum_{k=1}^{nq} [X_k - X_{k-1} - \hat{\mu}]^2 \tag{5}$$

The estimator $\hat{\sigma}_a^2$ is simply the sample variance of the first-difference of X_t; it corresponds to the maximum likelihood estimator of the parameter σ_0^2 and therefore enjoys the usual consistency, asymptotic normality and efficiency properties. Now consider the following alternative estimator of σ_0^2. Suppose we consider the variance of q-th differences of X_t which, under H_1, is q times the variance of first-differences. By dividing by q, we obtain the estimator $\hat{\sigma}_b^2(q)$ which also converges to σ_0^2 under H_1, where:

$$\hat{\sigma}_b^2(q) \equiv \frac{1}{nq} \sum_{k=1}^{n} [X_{qk} - X_{qk-q} - q\hat{\mu}]^2 . \tag{6}$$

We have written $\hat{\sigma}_b^2(q)$ as a function of q (which we term the aggregation value) to emphasize the fact that a distinct alternative estimator of σ_0^2 may be formed for <u>each</u> q. Under the null hypothesis of a Gaussian random walk, the two estimators $\hat{\sigma}_a^2$ and $\hat{\sigma}_b^2(q)$ should be "close", therefore a test of the random walk may be constructed by computing the difference $J_d(q) = \hat{\sigma}_b^2(q) - \hat{\sigma}_a^2$ and checking its proximity to zero. Alternatively, a test may also be based upon the dimensionless centered variance ratio $J_r(q) \equiv \frac{\hat{\sigma}_b^2(q)}{\hat{\sigma}_a^2} - 1$, which converges in probability to zero as well.[6]

In order to formally base a test upon $J_d(q)$, the sampling distribution of the difference is obviously required. Since $\hat{\sigma}_a^2$ is asymptotically efficient under H_1, we may use Hausman's [1978] insight that the asymptotic variance of $J_d(q)$ is simply the difference of the asymptotic variances of $\hat{\sigma}_b^2(q)$ and $\hat{\sigma}_a^2$. Moreover, a straightforward application of the delta-method yields the corresponding asymptotic distribution of the centered variance ratio $J_r(q)$ hence we have (proofs of all the theorems are given in the Appendix):

<u>Theorem 1</u>: Under the null hypothesis H_1, the statistics $J_d(q)$ and $J_r(q)$ have the following asymptotic distributions:

$$\sqrt{nq}\ J_d(q) \overset{a}{\sim} N(0,\ 2(q - 1)\sigma_0^4) \tag{7a}$$

$$\sqrt{nq}\ J_r(q) \overset{a}{\sim} N(0,\ 2(q - 1))\ . \tag{7b}$$

For each q, the statistics $J_d(q)$ and $J_r(q)$ provide a comparison of the variance of increments over the sampling interval h to the variance of increments over qh. Under the null hypothesis H, the latter must be q times the former quantity.

Since $J_d(q)$ and $J_r(q)$ are based upon variances computed from non-overlapping q-th differences of X(t), a natural extension is to consider corresponding test statistics based upon overlapping differences. In this way the variance estimator of q-th differences is based upon nq-q+1 observations instead of only n observations in the non-overlapping case, possibly yielding a more efficient estimator. Therefore, we define the following estimator $\hat{\sigma}_c^2(q)$ of σ_0^2:

$$\hat{\sigma}_c^2(q) = \frac{1}{nq^2} \sum_{k=q}^{nq} [X_k - X_{k-q} - q\hat{\mu}]^2\ . \tag{8}$$

Using $\hat{\sigma}_c^2(q)$, define the corresponding difference and ratio test statistics as:

$$M_d(q) = \hat{\sigma}_c^2(q) - \hat{\sigma}_a^2 \qquad M_r(q) = \frac{\hat{\sigma}_c^2(q)}{\hat{\sigma}_a^2} - 1\ . \tag{9}$$

We then have the following result:

Theorem 2: Under the null hypothesis H_1, the statistics (9) have the following asymptotic distributions:

As expected, the asymptotic variances of M_d and M_r are less than their non-overlapping counterparts. In particular, the asymptotic relative efficiency (ARE) of M to J is simply:[7]

$$\text{ARE} = \frac{3q}{2q-1} \,. \tag{11}$$

For q = 2, the M statistics are twice as efficient as the J's. Alternatively, when q = 2 twice as many observations are required with J as with M in order to obtain the same limit distribution.[8] However, the ARE decreases monotonically with q and is bounded below by 1.5. This suggests that M may be the preferred test statistic.

In order to develop some intuition for these variance ratios, observe that for an aggregation value q of 2, the $M_r(q)$ statistic may be re-expressed as:

$$M_r(2) = \hat{\rho}(1) - \frac{1}{4n\sigma_a^2}\left[(X_1 - X_0 - \hat{\mu})^2 + (X_{2n} - X_{2n-1} - \hat{\mu})^2\right] \tag{12}$$

hence for q = 2 the $M_r(q)$ statistic is approximately the first-order autocorrelation coefficient estimator $\hat{\rho}(1)$ of the differences of X. More generally, we have the following Corollary to Theorem 2:

Corollary 2.1:

$$M_r(q) \doteq \frac{2(q-1)}{q}\hat{\rho}(1) + \frac{2(q-2)}{q}\hat{\rho}(2) + \ldots + \frac{2}{q}\hat{\rho}(q-1) + o_p(n^{-\frac{1}{2}}) \tag{13}$$

where $o_p(n^{-\frac{1}{2}})$ denotes terms which are of order smaller than $n^{-\frac{1}{2}}$ in probability.

Specifically, variance ratios computed with an aggregation value q are (approximately) linear combinations of the first q - 1 autocorrelation coefficient estimators of the first differences with arithmetically declining weights.[9]

One further adjustment which may improve the finite-sample behavior of the test statistics is to use unbiased estimators $\bar{\sigma}_a^2$, $\bar{\sigma}_b^2$, and $\bar{\sigma}_c^2$ in computing $J_i(q)$ and $M_i(q)$, $i = d, r$. In particular, we have:

Proposition: The following are unbiased estimators for σ_0^2:

$$\bar{\sigma}_a^2 \equiv \frac{1}{(nq-1)} \sum_{k=1}^{nq} (X_k - X_{k-1} - \hat{\mu})^2 \tag{14a}$$

$$\bar{\sigma}_b^2 \equiv \frac{1}{(nq-q)} \sum_{k=1}^{n} (X_{qk} - X_{qk-q} - q\hat{\mu})^2 \tag{14b}$$

$$\bar{\sigma}_c^2 \equiv \frac{1}{m} \sum_{k=q}^{nq} (X_k - X_{k-q} - q\hat{\mu})^2 , \qquad m \equiv q(nq - q + 1)(1 - \frac{q}{nq}) . \tag{14c}$$

We denote the resulting adjusted specification test statistics $\bar{J}_i(q)$ and $\bar{M}_i(q)$, respectively, where $i = d, r$. Of course, although the variance estimators $\bar{\sigma}_a^2$, $\bar{\sigma}_b^2$, and $\bar{\sigma}_c^2$ are unbiased, only the adjusted variance difference is unbiased; the variance ratio is not.

2.2 WEAKLY DEPENDENT HETEROGENEOUS DISTURBANCES.

Since there is already a growing concensus that many economic time series possess time-varying volatilities, a rejection of the random walk hypothesis due to the presence of heteroscedasticity would not be of much interest. We therefore wish to derive a version of our specification test of the random converge to one in probability even with heteroscedastic disturbances. Heuristically, this is simply due to the fact that the variance of the sum of uncorrelated increments must still equal the sum of the variances despite heteroscedasticity. Of course, the asymptotic variance of the variance ratios will clearly depend upon the type and degree of heteroscedasticity present. However, by controlling the degree of heterogeneity and dependence of the process, it is possible to obtain consistent estimators of the asymptotic variances. Specifically, in order to relax the i.i.d. Gaussian restriction of the ϵ_t's, we follow White's [1980] and White and Domowitz's [1984] use of mixing and moment conditions to derive heteroscedasticity-consistent estimators of our variance ratio's asymptotic variance. More formally, we require the following assumptions on $\{\epsilon_t\}$:

(A1) For all t, $E[\epsilon_t] = 0$, $E[\epsilon_t \epsilon_{t-\tau}] = 0$ for any $\tau \neq 0$.

(A2) $\{\epsilon_t\}$ is ψ-mixing with coefficients $\psi(m)$ of size $r/(2r-1)$ or is α-mixing with coefficients $\alpha(m)$ of size $r/(r-1)$, $r > 1$ such that for all t and for any $\tau \geq 0$, there exists some δ for which:

$$E|\epsilon_t \epsilon_{t-\tau}|^{2(r+\delta)} < \Delta < \infty . \quad (2)$$

(A3) $\lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} E[\epsilon_t^2] = \sigma_0^2 < \infty$.

(A4) For all t, $E[\epsilon_t \epsilon_{t-j} \epsilon_t \epsilon_{t-k}] = 0$ for any non-zero j, k where $j \neq k$.

These assumptions then form our second null hypothesis H_2. Assumption (A1) is the essential property of the random walk which we wish to test. Assumptions (A2) and (A3) are restrictions on the degree of dependence and heterogeneity of heteroscedasticity including deterministic changes in the variance (due, for example, to seasonal components) as well as Engle's [1982] ARCH processes (in which the conditional variance depends upon past information). Assumption (A4) implies that the sample autocorrelations of ε_t are asymptotically uncorrelated. Although this assumption may be weakened considerably, it would be at the expense of computational simplicity since in that case the asymptotic covariances of the autocorrelations must be estimated and taken into account in Corollary 3.1 below.[10]

Under the conditions (A1)-(A4), we may obtain heteroscedasticity-consistent estimators of the asymptotic variance of the first-difference autocorrelations of X_t. Applying Corollary 2.1 above then yields a heteroscedasticity-consistent estimator of the asymptotic variance of our variance ratios M and $\bar{M}$. In particular, we have:

<u>Theorem 3</u>: Denote by $\delta(j)$ the asymptotic variance of the j-th order autocorrelation coefficient $\hat{\rho}(j)$ of ΔX_t. Let ε_t satisfy Assumptions (A1)-(A4) above. Then

(a) The statistics $J_d(q)$, $J_r(q)$, $M_d(q)$, $M_r(q)$, $\bar{M}_d(q)$, $\bar{M}_r(q)$ all converge almost surely to 0 for all q as T increases without bound.

(b) The following is a heteroscedasticity-consistent estimator of $\delta(j)$:

$$\hat{\delta}(j) = \frac{\sum_{k=j+1}^{nq} (X_k - X_{k-1} - \hat{\mu})^2 \cdot (X_{k-j} - X_{k-j-1} - \hat{\mu})^2}{\left[\sum_{k=1}^{nq} (X_k - X_{k-1} - \hat{\mu})^2\right]^2} . \qquad (15)$$

Corollary 3.1: Let $\theta(q)$ denote the asymptotic variance of the variance ratios $\sqrt{nq}\, M_r(q)$ and $\sqrt{nq}\, \bar{M}_r(q)$ under H_2. Then the following is a consistent estimator of $\theta(q)$:

$$\hat{\theta}(q) \equiv \sum_{j=1}^{q-1} \left[\frac{2(q-j)}{q}\right]^2 \cdot \hat{\delta}(j) . \tag{16}$$

Tests of H_1 and H_2 may then be based upon the normalized variance ratios z_1 and z_2 respectively where:

$$z_1 \equiv \sqrt{nq}\, \bar{M}_r(q) \cdot \left(\frac{2(2q-1)(q-1)}{3q}\right)^{-\frac{1}{2}} \stackrel{a}{\sim} N(0,\ 1) . \tag{17a}$$

$$z_2 \equiv \sqrt{nq}\, \bar{M}_r(q) \cdot \hat{\theta}^{-\frac{1}{2}}(q) \stackrel{a}{\sim} N(0,\ 1) . \tag{17b}$$

An equivalent and somewhat more intuitive method of arriving at formula (15) is to consider the regression of the increments ΔX_t on a constant and the j-th lagged increment ΔX_{t-j}. The estimated slope coefficient is then simply the j-th autocorrelation coefficient and the estimator $\hat{\delta}(j)$ of its variance is numerically identical to White's [1980] heteroscedasticity-consistent covariance matrix estimator. Note that White [1980] requires independent disturbances whereas White and Domowitz [1984] allow for weak dependence (of which uncorrelated errors is, under suitable regularity conditions, a special case).[11]

3. FINITE-SAMPLE PROPERTIES OF THE TEST STATISTICS UNDER H_1 AND H_2.

Since the inferences proposed above are based upon asymptotic arguments, simulation experiments are required in order to determine the reliability of indicate that tests based upon the other statistics proposed in Section 2 generally yield less reliable inferences hence, in the interest of brevity, we only report the results for $\bar{M}_r(q)$.[12] For purposes of comparison, we also report the results of Monte Carlo experiments performed for the Box-Pierce Q-statistic (BP-Q) and the Dickey-Fuller t-statistic (DF).

Before turning to the simulations, we develop the following inequalities for the variance of $\bar{M}_r(q)$ which provide considerable intuition for the Monte Carlo results. Recall that $\sqrt{nq}\,\bar{M}_r(q) \overset{a}{\sim} N\left(0, \frac{2(2q-1)(q-1)}{3q}\right)$, hence the variance of the test statistic $\bar{M}_r(q)$ is given by V, where:

$$V \equiv \frac{2(2q-1)(q-1)}{3nq^2} = \frac{4}{3n} \cdot \left[\frac{q^2 - \frac{3}{2}q + 1}{q^2}\right] . \tag{18}$$

Note that the rational function of (18) enclosed in brackets is, for all natural numbers q, bounded between ½ and 1 and monotonically increasing in q. Therefore for fixed n, this implies upper and lower bounds V_U and V_L for the variance V. Table 1 displays such upper and lower bounds for several values of n. A bound may also be derived for the statistic $\bar{M}_r(q)$ itself; since variances are nonnegative, it is clear that $1 + \bar{M}_r(q) \geq 0$, hence a lower bound for $\bar{M}_r(q)$ is -1. This yields the following lower bound on the asymptotically standard normal test statistic z_1:

$$\text{Inf}[z_1] = \frac{\text{Inf}[\bar{M}_r(q)]}{\text{Inf}[\sqrt{V}]} = -\frac{1}{\sqrt{V_L}} \tag{19}$$

Table 1 also displays this lower bound for several values of n. Note that n does not represent the sample size (which is given by nq), but is the number in computing the variance ratio statistic $\bar{M}_r(q)$. We shall return to these lower bounds later.

Tables 2-8 summarize the results of the Monte Carlo experiments. Tables 2a,b report the finite-sample behavior of the z_1 and z_2 statistics respectively under the homoscedastic i.i.d. null hypothesis H_1. Tables 3 and 4 report corresponding results for z_1 and z_2 under two specific heteroscedastic null hypotheses in which the variance of the increments follow a stationary AR(1) process. Tables 5a,b present finite-sample properties of the Box-Pierce Q-statistic with and without heteroscedasticity corrections respectively under the homoscedastic null. Table 6 reports the empirical quantiles of the Dickey-Fuller t-statistic under the homoscedastic null. Tables 7a,b compare the empirical sizes of the variance ratio, Box-Pierce, and Dickey-Fuller tests with and without heteroscedasticity corrections respectively, under one heteroscedastic alternative. Tables 8a,b perform the same comparison but for a more extreme form of heteroscedasticity. All simulations are based upon 20,000 replications.[13]

3.1 FINITE SAMPLE PROPERTIES OF z_1 AND z_2 UNDER H_1 AND H_2.

When the true data-generating process (DGP) is an i.i.d. random walk, the results in Tables 2a,b show that the empirical size of 5 percent tests based upon either the z_1 or z_2 statistics are close to their nominal value for aggregation values less than one-eighth of the sample size.[14] However, Tables 3a,b indicate that when the true DGP is heteroscedastic, the z_1-statistic rejects far too often. Specifically, in Table 3a the variance of the random walk disturbances is assumed to follow a stationary AR(1) process of the form:

$$\ln \sigma_t^2 = \psi \ln \sigma_{t-1}^2 + \zeta_t \qquad \zeta_t \text{ i.i.d. } N(0,1) \tag{20a}$$

$$\ln \sigma_0^2 \sim N\left(0, \frac{1}{1-\psi^2}\right) \tag{20b}$$

where the parameter ψ is set to 0.50.[15] Observe that for sample sizes of 512, the empirical size of a 5 percent test with $q = 2$ is 14.7 percent. However, as the results in Table 3b indicate, the empirical size of a test based upon the heteroscedasticity-robust statistic z_2 is much closer to the nominal value for aggregation values less than one-fourth the sample size. For example, the 5 percent z_2 test with $q = 2$ and 512 observations has an empirical size of 4.9 percent. Tables 4a,b provide the same comparisons when the autoregressive coefficient ψ is 0.80 (which increases the unconditional volatility of the variance). Under this more severe form of heteroscedasticity, z_1 induces even more rejections. For example, with a sample size of 512 and $q = 2$, the empirical size of a 5 percent test is now 34.7 percent. Nevertheless using the z_2 statistic to perform the same test yields a rejection rate of 4.2 percent, considerably closer to its nominal 5 percent level.

3.2 COMPARISON WITH THE BOX-PIERCE Q-STATISTIC.

Since the variance ratio $\bar{M}_r(q)$ is asymptotically equivalent to a specific linear combination of the first q-1 autocorrelations of the increments ΔX_t, it is natural to compare the finite sample behavior of $\bar{M}_r(q)$ with the corresponding Box-Pierce Q-statistic (BP-Q). This is done in Tables 5a,b. Table 5a reports the empirical size of tests based on z_1 and the Q-statistics at the 1, 5, and 10 percent levels under a homoscedastic DGP.[16] Not surprisingly, for $q = 2$ their sizes are comparable since the $\bar{M}_r(q)$ statistic is approximately equal to the first-order serial correlati statistics differ. In particular, although the Q-statistic does not reject often enough at the 5 percent level as q approaches one-eighth the sample size, the size of the z_1 test is relatively closer to its nominal value. For example, in sample sizes of 1024, the empirical size of the Q-statistic test is 2.8 percent using 127 autocorrelations whereas the z_1 test has a size of 5.2 percent with q = 128. Nevertheless, note that the Q-statistic seems to perform somewhat better than z_1 at the 1 percent level, and both are comparable at the 10 percent level. Table 5b reports similar size comparisons of the heteroscedasticity-robust statistics z_2 and BP-Q* under the same homoscedastic data-generating process, with similar results.

3.3 COMPARISON WITH THE DICKEY-FULLER t-TEST.

Since the sampling theory for both the Q and z statistics are asymptotic in nature, the actual size of any test based upon these statistics will of course differ from their nominal values in finite samples. Although the results in Tables 2-5 indicate that such differences may not be large for reasonable aggregation values, it may nevertheless seem more desirable to base tests upon the regression t-statistic for which Fuller [1976], Dickey and Fuller [1979, 1981], and Nankervis and Savin [1985] have tabulated the exact finite sample distrbution. However, due to the dependence of t-statistic's distribution upon the drift μ, a time trend must be included in the regression of X_t on X_{t-1} in addition to a constant term in order to yield a sampling theory for the t-statistic which is independent of μ. Although it has been demonstrated that the t-statistic from such a regression converges in distribution to the Dickey-Fuller distribution, there may be some discrepancies in finite samples. Table 6 presents the empirical quantiles of

$$X_t = \mu + \omega t + \beta X_{t-1} + \nu_t \ . \tag{21}$$

A comparison of these quantiles with those given in Fuller [1976, Table 8.5.2] suggest that there may be some significant differences for small samples, but for sample sizes of 500 or greater the quantiles in Table 6 are almost identical to those of Dickey and Fuller.

An additional comparison which may be performed across the variance ratio, Box-Pierce, and Dickey-Fuller statistics is their respective finite sample properties under heteroscedasticity. Table 7a reports the empirical sizes of the z_1, BP-Q, and DF statistics under the DGP (20) for $\psi \equiv 0.50$. Not surprisingly, DF is much better behaved than z_1 and BP-Q statistics since the DF statistic has been shown by Phillips [1986] and Phillips and Perron [1986] to be robust to heteroscedasticity whereas the other two statistics are not. However, once we use z_2 and BP-Q* to perform our tests, they both compare favorably to DF in Table 7. Moreover, for the more severe case of heteroscedasticity associated with $\psi \equiv 0.80$, both the z_2 and BP-Q* tests have empirical sizes closer to the nominal size than the DF test. For example, Table 8b reports empirical sizes of 5.2, 4.7, and 7.9 percent for the z_2, BP-Q*, and DF tests respectively.

Several conclusions may be drawn from these simulation experiments. First, although the finite sample properties of z_1 do not differ considerably from their asymptotic counterparts when the true data generating process is an i.i.d. random walk, significant discrepancies may arise in the presence of heteroscedasticity. This, however, may be corrected by using the statistic z_2

comparable for 10 percent tests and smaller for the Q statistic at the 1 percent level. Finally, the discrepancies between empirical and nominal sizes for tests based upon z_2, BP-Q*, and the Dickey-Fuller t-statistic are similar under mild heteroscedasticity, but for a more volatile variance process the DF test may yield less reliable inferences than either the variance ratio or Box-Pierce tests.

4. POWER.

In this section, we explore the power of our variance ratio test against two specific alternatives. For computational convenience, both these alternatives are formulated in continuous time although their discrete-time analogues will become apparent. The first is the well-known mean-reverting Ornstein-Uhlenbeck process which has often been used as a model of interest rates.[17] For contrast, we also consider a second alternative hypothesis which is nonstationary and exhibits essentially the opposite time series properties. For reasons which will become evident in the exposition below, we call the first alternative a 'price fads' model and the second a 'returns fads' model.

4.1 POWER AGAINST A MEAN-REVERTING ALTERNATIVE.

As an alternative to the random walk model for asset prices, several recent studies have examined what Shiller [1981] describes as a 'fads' model: market prices fluctuate according to investors' fads which have exponentially decaying influence. In discrete time, this hypothesis has been implemented by supposing that deviations from the rational expectation of the continuous time, one representation of the fads model is given by the Ornstein-Uhlenbeck (O.U.) process:

$$K_1: \qquad dX(t) = -\gamma_p\left[X(t) - \alpha_p\right]dt + \sigma_p dW(t) \qquad \gamma_p > 0 \tag{22}$$

where $X(t)$ denotes the log-price process $\ln P(t)$, as in Shiller and Perron [1985]. In order to develop some intuition for the empirical implications of this alternative, we report some of its population moments (all conditional upon $X(0) = X_0$):

$$E_0\left[X(t)\right] = \alpha_p + (X_0 - \alpha_p)e^{-\gamma_p t} \tag{23a}$$

$$\mathrm{Var}_0\left[X(t)\right] = \frac{\sigma_p^2}{2\gamma_p}\left(1 - e^{-2\gamma_p t}\right) \tag{23b}$$

$$\mathrm{Cov}_0\left[X(t_1),X(t_2)\right] = \frac{\sigma_p^2}{2\gamma_p}\left(1 - e^{-2\gamma_p t_1}\right)e^{-\gamma_p(t_2-t_1)} \qquad t_1 \le t_2 \tag{23c}$$

$$\mathrm{Corr}_0\left[X(t_1),X(t_2)\right] = \left|\frac{1 - e^{-2\gamma_p t_1}}{1 - e^{-2\gamma_p t_2}}\right|^{\frac{1}{2}} e^{-\gamma_p(t_2-t_1)} \qquad t_1 \le t_2 \,. \tag{23d}$$

From (23a), we see that for large t the log-price $X(t)$ tends to its steady-state value of α_p. Note that $X(t)$ is a stationary process if $E[X_0] = \alpha_p$ and $\mathrm{Var}[X_0] = \sigma_p^2/2\alpha_p$, and is a Gaussian process assuming X_0 is Gaussian. Since the process is Gaussian and Markov, these moments completely characterize its finite-dimensional distributions which, in turn, are given by products of its conditional distributions:

Since (22) possesses the following solution on $[0,T]$:[18]

$$X(t) = X(0)e^{-\gamma_p t} + \alpha_p(1 - e^{-\gamma_p t}) + \sigma_p \int_0^t e^{-\gamma_p(t-s)} dW(s) \tag{25}$$

its discrete-time representation is given by the recursive relation:

$$X_k = \alpha_p + \psi(h)[X_{k-1} - \alpha_p] + \zeta_k \ . \tag{26}$$

where $X_k \equiv X(kh)$, $\psi(h) \equiv e^{-\gamma_p h}$, $\zeta_k \equiv \int_{kh-h}^{kh} e^{-\gamma_p(kh-s)} dW(s)$, $E[\zeta_k] = 0$, $E[\zeta_k^2] = \frac{\sigma_p^2}{2\gamma_p}(1 - e^{-2\gamma_p h})$ and ζ_k is Gaussian for all k. Observe that for more finely sampled data the autoregressive coefficient $\psi(h)$ is closer to unity and is the continuous-time analogue of Phillips's [1986] discrete-time 'near-integrated' time series.

Further intuition for the empirical properties of the 'price fads' model (22) may be obtained by examining the steady-state first-order autocorrelation coefficient $\rho(1)$ of continuously compounded returns over arbitrary holding periods τ. More formally, let $R(t,t+\tau) \equiv X(t+\tau) - X(t)$. Then we have:

$$\rho(1) \equiv \lim_{t\to\infty} \frac{COV\left[R(t,t+\tau),\ R(t+\tau,t+2\tau)\right]}{VAR\left[R(t,t+\tau)\right]} = -\frac{1 - e^{-\gamma_p \tau}}{2} \ . \tag{27}$$

Observe that the first-order autocorrelation of returns is always negative under the price fads alternative, approaches -0.50 for longer holding periods τ, and approaches 0.00 for shorter holding periods. Table 9 presents values of $\rho(1)$ for 1 to 12 holding-period returns under three distinct parameter values γ_p corresponding to autoregressive coefficients $\psi(h)$ of 0.95, 0.96, and 0.99 (per period, i.e., for h = 1) respectively in equation (26). Entries in short holding-period returns, the price fads alternative may exhibit significant (negative) autocorrelation for longer holding periods.

In order to determine the power of our test against K_1 via simulation experiments, we must choose values for the parameters (α_p, σ_p, γ_p). Since the power results are obviously sensitive to our selection, we choose parameter values which correspond roughly to reasonable empirical values of weekly stock returns data and which yield an interesting range of power across sample sizes and aggregation values. Therefore, we set the unconditional variance of weekly returns to 0.0004 and assume that the weekly first-order autocorrelation coefficient of log-prices is 0.96 (implying a weekly steady-state first-order autocorrelation of -0.020 for returns). These two assumptions imply σ_p = 0.0202 and γ_p = 0.041 at h = 1 week.[19] Since the value of α_p does not affect our test statistics, we set it to zero without loss of generality. Using these values for (α_p, σ_p, γ_p), we simulate 20,000 realizations of the price series for a variety of sample sizes, compute the z-statistics as well as the Box-Pierce and Dickey-Fuller statistics, and summarize the empirical power of those tests in Tables 10-11.

Tables 10a,b report the power of 10, 5, and 1 percent tests against K_1 based upon the z_1, Dickey-Fuller, and Box-Pierce statistics for different sample-size/aggregation-value combinations. Table 10a uses a base observation period of h = 1 (week), whereas Table 10b uses a base observation period of h = 4 (weeks). The critical values of all three test statistics were empirically determined by simulation under the homoscedastic null.

An interesting pattern emerges from Table 10a. For a given number of observations, the power of the test increases with the aggregation value q. The explanation for this pattern lies in the behavior of serial correlations resembles a random walk as the observation interval h decreases. This is confirmed by the first-order serial correlations which grow farther away from 0 as the observation interval increases. Therefore, under this alternative it becomes easiest to detect departures from the random walk by comparing the most coarsely-sampled data to the finest. This corresponds exactly to using larger aggregation values q. A further demonstration of this property is given in Table 10b which reports the power of the variance ratio test using a base observation period of 4 weeks ($h = 4$). Note that within a <u>fixed</u> calendar time span of 512 weeks, the power of a 5 percent test based on 512 weekly observations using $q = 2$ is 6.6 percent, whereas the same test based on 128 monthly observations using $q = 32$ is 46.5 percent!

Although for larger aggregation values the power of the variance ratio test exceeds that of the Dickey-Fuller t-test, the difference is generally not significant. However, the variance-based test clearly dominates the Box-Pierce Q-test. For example, with a sample of 512 observations the power of a 5 percent variance ratio test is 46.5 percent ($q = 64$) whereas the power of the Box-Pierce Q-statistic (using 63 autocorrelations) is 8.4 percent! Against the alternative K_1, we conclude that the variance ratio test (with appropriate q) is roughly comparable to the Dickey-Fuller t-test, but is considerably more powerful than the Box-Pierce test.

Although the critical values of the test statistics in Tables 10a,b were determined via simulation under the i.i.d. null hypothesis, it may also be of interest to compare power when asymptotic critical values are used. In to determine rates of rejection. From the entries in Table 11a, it is evident that the asymptotic critical values yield less powerful tests; a 5 percent test with a sample size of 512 and $q = 32$ has 34.4 percent power using critical values of the empirical distribution whereas the asymptotic critical values yield 14.7 percent power. Table 11b reports similar results for the heteroscedasticity-robust statistic z_2. Also, note that the power does not increase with q monotonically but declines after some point. This is not surprising since we have shown in Section 3 and Table 1 that the lower bound of z_1 increases as q becomes large relative to the number of observations nq. More specifically, observe that under K_1 the increments ΔX_t display <u>negative</u> serial correlation hence the variance ratio of coarser to finer sampled data converges in probability to a value less than unity. This implies that rejections of the null hypothesis ought generally to be due to extreme negative realizations of z_1. However, if q is equal to half the sample size ($n = 2$), Table 1 shows that the lower bound of z_1 is -1.72 hence the test will never reject the null hypothesis! In view of this danger, we restrict q to be no more than one-eighth the sample size in all our power simulations.

4.2 POWER AGAINST A NONSTATIONARY ALTERNATIVE.

One of the implications of K_1 is that log-prices are positively autocorrelated at all lags (see (23d)). Letting $R(t_1, t_2) \equiv X(t_2) - X(t_1)$ denote the (continuously compounded) total return for the holding period $[t_1, t_2]$, it may readily be shown that returns over any two non-overlapping finite holding-periods are always <u>negatively</u> autocorrelated. Specifically, the autocovariance function for returns is given by:

$$\frac{\sigma_p^2}{2\gamma_p} e^{-\gamma_p(t_1+t_2+t_3+t_4)}[e^{\gamma_p(t_1+t_2)} + 1][e^{\gamma_p t_2} - e^{\gamma_p t_1}][e^{\gamma_p t_3} - e^{\gamma_p t_4}] \le 0 \quad (28)$$

where $0 \le t_1 \le t_2 \le t_3 \le t_4$. This result is not surprising since, loosely speaking, if the process is to be mean-reverting then large changes from the steady-state mean α_p ought to be followed by smaller ones. It does, however, seem to be inconsistent with the empirically observed positive serial correlation in weekly stock returns.[20] We therefore consider another closely related alternative hypothesis under which returns are positively autocorrelated. Heuristically, this consists of modelling instantaneous returns as an O.U. process and deriving the log-price process by integration. More formally, let R(t) denote the instantaneous return of a security at time t with price P(t). Then we have:

$$R(t) = \frac{\dot{P}(t)}{P(t)} + \frac{D(t)}{P(t)} = \dot{X}(t) + \frac{D(t)}{P(t)} \quad (29)$$

where D(t) is the dividend flow of the security at time t. For simplicity, we assume that $D(t) \equiv 0$ for all t so that the return consists solely of capital appreciation.[21] Observe that if the log-price process X(t) were any type of diffusion, the instantaneous return R(t) is no longer well-defined since the sample paths of P(t) are nowhere differentiable. However, if we begin by first specifying the dynamics of R(t), then equation (29) may be used to <u>define</u> the log-price process X(t). Specifically, we have:

$$dR(t) = -\gamma_r(R(t) - \alpha_r)dt + \sigma_r dW \qquad \gamma_r > 0 \quad (30a)$$

$$X(t) \equiv X_0 + \int_0^t R(s)ds . \quad (30b)$$

this alternative instantaneous returns are mean-reverting whereas log-prices are explosive. The moments of the log-price process under the returns fads model K_2 are given by:

$$E[X(t)] = X_0 + \alpha_r t + \frac{1}{\gamma_r}[R(0) - \alpha_r][1 - e^{-\gamma_r t}] \tag{31a}$$

$$Var[X(t)] = \frac{\sigma_r^2}{\gamma_r^2} t + \frac{\sigma_r^2}{2\gamma_r^3}[1 - e^{-\gamma_r t}][e^{-\gamma_r t} - 3] \tag{31b}$$

$$Cov[X(t_1),X(t_2)] = \frac{\sigma_r^2}{\gamma_r^2} t_1 + \frac{\sigma_r^2}{2\gamma_r^3}[2e^{-\gamma_r t_1} + 2e^{-\gamma_r t_2} - 2 - e^{-\gamma_r (t_2 - t_1)} - e^{-\gamma_r (t_1 + t_2)}], \qquad t_1 \le t_2 . \tag{31c}$$

Moreover, it may readily be shown that under this alternative, non-overlapping finite-holding period returns $R(t_i, t_j)$ are positively autocorrelated at all lags since for $0 \le t_1 \le t_2 \le t_3 \le t_4$ we have:

$$Cov[R(t_1,t_2),R(t_3,t_4)] = \frac{\sigma_r^2}{2\gamma_r^3} e^{-\gamma_r(t_1+t_2+t_3+t_4)}[e^{\gamma_r(t_1+t_2)} - 1][e^{\gamma_r t_2} - e^{\gamma_r t_1}][e^{\gamma_r t_4} - e^{\gamma_r t_3}] \ge 0 . \tag{32}$$

Note also that, unlike the price fads model, the returns fads model generates sample paths of $X(t)$ which possess (mean-square) first derivatives.[22] Moreover, the first-order autocorrelation pattern of continuously compounded returns across holding periods differs considerably between K_1 and K_2. To see this, we calculate the steady-state first-order autocorrelation of returns under K_2 to be:

$$\rho(1) \equiv \lim \frac{COV[R(t,t+\tau),\ R(t+\tau,t+2\tau)]}{\quad} \qquad [1 - e^{-\gamma_r \tau}]^2$$

As the holding period τ increases $\rho(1)$ approaches zero, and as the holding period becomes smaller $\rho(1)$ approaches unity. The last three columns of Table 9 display values of $\rho(1)$ for 1 to 12 holding-period returns for $\gamma_r = 2$, 6, and 11. In contrast to the autocorrelations of the price fads model, under a returns fad the first-order autocorrelation is positive for all holding periods. Moreover, as the holding period increases from 1 to 12 periods the autocorrelation (for $\gamma_r = 2$) declines from 32.9 percent to 2.2 percent. This implies that for a process with $\gamma_r = 2$ when h = 1 week, the autocorrelation of weekly returns is quite large whereas the corresponding monthly or quarterly returns exhibit considerably smaller autocorrelation. This observation will play an important role in explaining the empirical results of Section 5.

Although $X(t)$ is no longer a Markov process under K_2, the Markov property may be restored by considering the return and log-price processes jointly, i.e.,

$$dZ(t) \equiv d\begin{bmatrix} R(t) \\ X(t) \end{bmatrix} = \left\{ \begin{bmatrix} -\gamma_r & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} R(t) \\ X(t) \end{bmatrix} + \begin{bmatrix} \gamma_r \alpha_r \\ 0 \end{bmatrix} \right\} dt + \begin{bmatrix} \sigma_r \\ 0 \end{bmatrix} dW \, . \quad (34)$$

Since the vector process $Z(t)$ is Markov and Gaussian, its finite-dimensional distributions are completely characterized by its first two moments and, for purposes of simulating discrete observations of Z, we calculate its conditional distribution to be:

$$Z(t_2) | Z(t_1) \sim MVN(\mu_Z, \Sigma_Z) \quad (35a)$$

where

$$\mu_Z = \begin{bmatrix} \alpha_r + [R(t_1) - \alpha_r] e^{-\gamma_r \tau} \\ [illegible] \end{bmatrix} , \quad (35b)$$

$$\Sigma_Z = \begin{bmatrix} \frac{\sigma_r^2}{2\gamma_r}\left(1 - e^{-2\gamma_r \tau}\right) & \frac{\sigma_r^2}{2\gamma_r^2}\left[1 - 2e^{-\gamma_r \tau} + e^{-2\gamma_r \tau}\right] \\ \bullet & \frac{\sigma_r^2}{\gamma_r^2}(t_2 - t_1) + \frac{\sigma_r^2}{2\gamma_r^3}\left[1 - e^{-\gamma_r \tau}\right]\left[e^{-\gamma_r \tau} - 3\right] \end{bmatrix} \tag{35c}$$

where $\tau \equiv t_2 - t_1$. The power of our specification test against alternative K_2 may then be determined numerically by simulation using relation (35).

As in the price fads simulation, we select parameter values for $(\alpha_r, \sigma_r, \gamma_r)$ which correspond roughly to reasonable empirical quantities and which yield an interesting range of power. The entries in Table 9 suggest that $\gamma_r = 6$ might be a plausible alternative to the random walk. Using the same value of the weekly returns variance of 0.0004 as in the price fads simulation, this implies a value of 0.0693 for σ_r when h = 1 period. Finally, since α_r is the steady-state mean of returns, we set it to 0.004 when h = 1 week, corresponding approximately to the empirical average weekly return on stocks.

Tables 12-13 report the empirical power of our test against the K_2 alternative. The organization of Tables 12-13 corresponds exactly to that of Tables 10-11. Tables 12a,b report power results against alternative K_2 based upon the z_1, Dickey-Fuller, and Box-Pierce statistics; Table 12a uses a base observation period of h = 1, whereas Table 12b sets h = 4. Empirical critical values are used in Tables 12a,b and asymptotic critical values are used in Tables 13a,b.

It is apparent f

has comparable power to the variance ratio test, and both strongly dominate the Dickey-Fuller t-test. For example, with a sample size of 512 a 5 percent variance ratio test has 60.8 percent power ($q = 2$), Box-Pierce has 62.1 percent power (essentially the first-order autocorrelation coefficient), but Dickey-Fuller has only 5.8 percent power!

The pattern of decreasing power in q is understandable in light of the autocorrelation patterns of Table 9. Unlike the price fads alternative, the returns fads alternative behaves more like a random walk with coarser sampling. Therefore, a comparison of variance estimators based on coarser to finer data is less able to reveal returns fads. Table 12b reinforces this point; since the base observation period is $h = 4$, the process is closer to a random walk than the observations used in Table 12a, implying less powerful tests. Indeed, the results of Tables 12a,b imply that the simple first-order serial correlation coefficient of the increments (using the most finely sampled data) would yield a more powerful test than variance ratios with higher aggregation values.

Tables 13a,b report the power of tests based upon z_1 and z_2 respectively using asymptotic critical values. In contrast to the price fads case, against the returns fads there is no loss of power in using asymptotic critical values. Whereas the difference in power arising from using empirical versus asymptotic critical values may be as large as 41 percentage points for the 5 percent test against the price fads alternative, the largest difference between 5 percent tests of Tables 12a and 13a is about 6 percentage points.

5. AN EMPIRICAL EXAMPLE USING WEEKLY STOCK RETURNS.

obtained from the Center for Research in Security Prices (CRSP) daily returns file over the 1216-week period from September 6, 1962 to December 26, 1985. A more complete empirical investigation is conducted in Lo and MacKinlay [1987]. Tables 14a,b report results for the entire sample period as well as various sub-periods, aggregation values ranging from 2 to 64, and for base observation intervals of 1 and 4 weeks.

Table 14a presents the results for a one-week base observation period. That is, the aggregation value q is measured in one-week units hence the denominator of $\bar{M}_r(q)$ is always the variance estimator of weekly returns in Table 14a. The entries reported in the main rows are the actual estimated variance ratios (i.e., $\bar{M}_r(q) + 1$). The values enclosed in parentheses immediately below the main rows are the corresponding z_1-statistics and the second set of parenthetical entries are the z_2-statistics which are robust to heteroscedasticity. Panel A presents results for the CRSP equal-weighted index, and Panel B reports similar results for the CRSP value-weighted index. Within each panel, the first row reports results for the entire 1216-week sample period, the next two give the results for the two 608-week sub-periods, and the last four rows contain results for the four 304-week sub-periods.

It is clear from Panel A that the random walk null hypothesis may be rejected at all the usual significance levels for the entire sample period and all sub-periods. Moreover, the rejections are not due to heteroscedasticity since the z_2-statistics also reject the heteroscedastic random walk. Also, note that the estimates of the variance ratio are larger than 1.0 for all cases. Specifically, consider the entries in the first column of Table 14a's consistent estimate of the first-order serial correlation coefficient of weekly returns. The entry in the first row, 1.30, implies that the first-order autocorrelation for weekly returns is approximately 30 percent. Since the 1.30 ratio is based upon 1216 observations, the standard test of the first-order autocorrelation coefficient (based upon the standard error $1/\sqrt{1216} = 0.03$) easily rejects the random walk hypothesis at any significance level. In addition, note that the variance ratios increase with q. This implies that there is positive first-order autocorrelation at lower frequencies as well. For example, the variance ratio for the entire sample with q = 4 is 1.64. Since 1.64 exceeds 1.30, this implies that one-fourth of the estimated variance of four-week returns exceeds one-half the estimated variance of two-week returns, or that two-week returns are also positively correlated.

Although the variance ratios increase with q, note that the magnitude of the z_1 and z_2 statistics do not. Indeed, the test statistics seem to decline with q hence the significance of the rejections becomes weaker as coarser-sample variances are compared to weekly variances. This pattern is inconsistent with the price fads alternative K_1 in which the power is an increasing function of q. If price fads were indeed present in the data, we should observe more significant rejections for larger q. Moreover, since price fads imply negative serial correlation of returns, we should also observe variance ratios less than 1.0. However, the results of Table 14a are clearly inconsistent with these implications and support those of the returns fads alternative instead: positive serial correlation which declines for longer holding-periods, implying variance ratios greater than 1.0 and weaker

Although the rejection of the random walk hypothesis is much weaker for the value-weighted index as Panel B of Table 14a indicates, nevertheless the general patterns persist. The variance ratios also exceed 1.0 and the z_1 and z_2 statistics decline as q increases. Note that the rejections for the value-weighted index are primarily due to the first 304 weeks of the sample period.

Table 14b presents the variance ratios using a base observation period of 4 weeks hence the first entry of the first row, 1.15, is the variance ratio of the eight-week returns to four-week returns, etc. Note that with a base interval of a month, we generally do not reject the random walk model even for the equal-weighted index. This result lends further support to the returns fads alternative K_2 since, as Table 9 shows, the weekly-sampled process can deviate considerably from a random walk whereas increments of the monthly-sampled process may be very close to white noise.

These empirical results indicate that weekly stock returns simply do not conform to a random walk process. Moreover, the pattern of rejections suggests that the mean-reverting price fads alternative is not a plausible model for deviations from the null hypothesis, whereas the returns fads model seems to be more consistent with the data. Although neither alternative hypothesis is grounded in any formal model of economic behavior, the empirical results would suggest that mean-reverting models of prices may be a less fruitful line of investigation.

Since the rejections are stronger for the equal-weighted index, this suggests that the smaller capitalization stocks are driving much of the inferences. Because the market for these "small" stocks is generally thinner than for larger capitalization issues, a natural objection to our empirical results is whether or not the rejections are due merely to mismeasurement of market's micro-structure. Although these issues are beyond the scope of our empirical example, they are examined in more detail in Lo and MacKinlay [1987]. The results in that investigation indicate that our rejections of the random walk cannot be explained by the usual infrequent trading arguments, nor are they due to the use of nominal instead of real or excess returns. Given the extraordinary difference in volatility of nominal returns relative to inflation and T-bill rates, it should be obvious that the use of nominal, real, or excess returns in volatility-based tests will yield practically identical results.

6. CONCLUSION.

In this paper, we have proposed a simple variance-based specification test of the random walk hypothesis. Although the finite-sample properties of this test are comparable to those of the Dickey-Fuller and Box-Pierce tests under the homoscedastic random walk null hypothesis, our test statistic yields more reliable inferences than the other tests under a heteroscedastic random walk. Further Monte Carlo experiments were performed in order to deduce the power of our test against two interesting alternative hypotheses. Against the price fads alternative, the variance ratio test (with larger aggregation values q) and Dickey-Fuller t-test are of comparable power; both dominate the Box-Pierce Q-test. Against the returns fads alternative, the variance ratio test (with smaller q) and the Box-Pierce test are of comparable power; both dominate the Dickey-Fuller t-test. The simplicity and versatility of the variance ratio test suggests that it may be of more practical use than the other two. Using the heteroscedasticity-robust variance ratio test ...

Type II errors and must be determined on a case-by-case basis. Moreover, our results emphasize the truism that the reliability of inferences depend intimately upon the alternative hypothesis of interest. As the empirical evidence demonstrates, although we reject the random walk hypothesis, it is not rejected in the direction of a mean-reverting alternative as in Fama and French [1986] but is more consistent with our second non-stationary alternative. Other applications of our specification test must obviously re-examine the power issue on a case-by-case basis. Although we have shown that our simple test has advantages over other more standard tests of unit roots under specific null and alternatives, there are of course many other situations in which those tests may possess more desirable properties. Consequently, in order to select a sensible testing strategy, one must consider not only the null hypothesis but also the most relevant alternative hypothesis.

FOOTNOTES

[1]See, for example, Gould and Nelson [1974], Hall [1978], Lucas [1978], Shiller [1981], Kleidon [1986], Marsh and Merton [1986], etc.

[2]Other important contributions in this substantial and still growing literature are Dickey [1976], Fuller [1976], Evans and Savin [1981, 1984], Sargan and Bhargava [1983], Nankervis and Savin [1985], Schwert [1985], Bhargava [1986], Cavanagh [1986], Stock and Watson [1986a, 1986b].

[3]See, for example, Nankervis and Savin [1985] and Perron [1986]. Of course, this dependence upon the drift may be eliminated by the inclusion of a time trend in the regression. However, this requires the estimation of an additional parameter and may affect the power of the resulting test. We perform power comparisons explicitly in Section 4.

[4]In particular, they show that the Dickey-Fuller significance points are asymptotically correct even in the presence of weakly dependent heterogeneously distributed disturbances which satisfy certain mixing and moment conditions.

[5]Note that the hypothesis of interest here is the uncorrelatedness of the innovations, whereas the general hypothesis of a unit root focuses on the nonstationarity of the process induced by the unit slope coefficient. In particular, the seminal papers by Phillips [1985], Phillips and Perron [1986], and Perron [1986] have extended the applicability of unit root tests to cover the case of disturbances which are weakly dependent.

[6]The use of variance ratios is, of course, not new. Most recently, Campbell and Mankiw [1987], Cochrane [1986], Fama and French [1986], French and Roll [1986], and Huizinga [1986] have all computed variance ratios in a variety of contexts. However, those studies do not provide any formal sampling theory for our statistics. Specifically, Cochrane [1986], Fama and French [1986], and French and Roll [1986] all rely upon Monte Carlo simulations to obtain standard errors for their variance ratios under the null. Campbell and Mankiw do derive the asymptotic variance of the variance ratio but only under the assumption that the variance parameter is known. Specifically, they use Priestley's [1981, p. 463] expression for the asymptotic variance of the estimator of the spectral density of ΔX_t at frequency zero with a Bartlett window as the appropriate asymptotic variance of the variance ratio. But Priestley's result is for the <u>non-normalized</u> spectral density estimator, i.e., it gives the asymptotic variance of only the numerator of the variance ratio. If the population variance parameter were known, then Campbell and Mankiw's [1987] expression would be appropriate. However, under the more common assumption that the variance is unknown, the asymptotic correlation between the numerator and the denominator of the variance ratio must be accounted for in calculating the ratio's limiting distribution (e.g. by the delta-method). In this paper, we develop the formal sampling theory of the variance ratio statistics for this more

$$\pi f(0) = \gamma(0) + 2\cdot \sum_{k=1}^{\infty} \gamma(k)$$

where $\gamma(k)$ is the autocovariance function. Dividing both sides by the variance $\gamma(0)$ then yields:

$$\pi f^*(0) = 1 + 2\cdot \sum_{k=1}^{\infty} \rho(k)$$

where f^* is the normalized spectral density and $\rho(k)$ is the autocorrelation function. Now in order to estimate the quantity $\pi f^*(0)$, the infinite sum on the right-hand side of the preceding equation must obviously truncated. If, in addition to truncation, the autocorrelations are weighted using Newey and West's [1987] procedure, then the resulting estimator is formally equivalent to our $M_r(q)$ statistic. Although he does not explicitly use this variance ratio, Huizinga [1986] does employ the Newey and West [1987] estimator of the normalized spectral density.

[7]See, for example, Lehmann [1983, p. 345, Theorem 2.1].

[8]See Lehmann [1983, Chapter 5.2].

[9]Note the similarity between the variance ratio and the Box-Pierce Q-statistic which is a linear combination of the squared autocorrelations with all the weights set identically equal to unity. It is this similarity which motivates our comparison of the variance ratio to the Q-statistic in Sections 3 and 4.

[10]Specifically, since it is shown in Corollary 2.1 that the variance ratio statistic is asymptotically equivalent to a linear combination of autocorrelations, its asymptotic variance is simply the asymptotic variance of the linear combination of autocorrelations. If (A4) obtains, this variance is equal to the weighted sum of the individual autocorrelation variances. If (A4) is violated, then the autocovariances of the autocorrelations must also be estimated. This is readily accomplished using, for example, the approach in Newey and West [1987]. Note that an even more general (and possibly more exact) sampling theory for the variance ratios may be obtained using the results of Dufour and Roy [1985]. Again, this would sacrifice much of the simplicity of our asymptotic results.

[11]Taylor [1984] also obtains this result under the assumption that the multivariate distribution of the sequence of disturbances is symmetric.

[12]The complete set of results are available from the authors upon request.

[13]Null simulations were performed in single-precision FORTRAN on a DEC VAX 8600 using the random number generator GGNML of the IMSL subroutine library. Power simulations were performed on an IBM 3081 also in single-precision FORTRAN using GGNML.

unreliable inferences in even the largest samples, we do not report results for larger values of q. The complete set of simulation results are available from the authors upon request.

[15] More formally, let the random walk disturbance ϵ_t be given by the relation $\epsilon_t \equiv \sigma_t \lambda_t$ where λ_t is i.i.d. N(0, 1) and σ_t satisfies relation (20) (it is assumed that λ_t and ζ_t are independent). The empirical studies of French, Schwert, and Stambaugh [1985] and Poterba and Summers [1986] posit such a process for the variance. Note that σ_t^2 cannot be interpreted as the unconditional variance of the random walk disturbance ϵ_t since σ_t^2 is itself stochastic and does not correspond to the unconditional expectation of any random variable. Rather, conditional upon σ_t^2, ϵ_t is normally distributed with expectation 0 and variance σ_t^2. If, in place of (20), the variance σ_t^2 were re-parameterized to depend only upon exogenous variables in the time t - 1 information set, this would correspond exactly to Engle's [1982] ARCH process.

The unconditional moments of ϵ_t may be readily deduced by expressing the process explicitly as a function of all the disturbances:

$$\epsilon_t = \lambda_t \sigma_0^{\psi^t} \cdot \prod_{k=1}^{t} \exp[\tfrac{1}{2} \psi^{t-k} \zeta_k]$$

Since σ_0, λ_t, and ζ_k are assumed to be mutually independent, it is apparent that ϵ_t is serially uncorrelated at all leads and lags (hence Assumption (A1) is satisfied) but is non-stationary and temporally dependent. Moreover, it is evident that $E[\epsilon_t^2 \epsilon_{t-j} \epsilon_{t-k}] = 0$ for all t and for $j \neq k$ hence Assumption (A4) is also satisfied. A straightforward calculation yields the moments of ϵ_t:

$$E[\epsilon_t^{2p}] = E[\sigma_0^{2p\psi^t}] \cdot \frac{(2p)!}{p!2^p} \exp[\frac{p}{2} \frac{1 - \psi^{2t}}{1 - \psi^2}]$$

$$E[\epsilon_t^{2p+1}] = 0 , \qquad p = 0, 1, 2, \ldots$$

From these expressions it is apparent that, for $\psi \in (0, 1)$, ϵ_t possesses bounded moments of any order and is unconditionally heteroscedastic; similar calculations for the cross-moments yield Assumption (A2). Finally, the following inequality is easily deduced:

$$\frac{1}{n} \sum_{k=1}^{n} E[\epsilon_t^2] < \exp[\frac{5}{2(1 - \psi^2)}] < \infty$$

thus Assumption (A3) is verified. Note that the kurtosis of ϵ_t is given by:

$$\frac{E[\epsilon_t^4]}{(E[\epsilon_t^2])^2} = 3 \cdot \frac{E[\sigma_0^{4\psi^t}]}{(E[\sigma_0^{2\psi^t}])^2} \geq 3$$

by Jensen's inequality. This implies that, as for Engle's [1982] stationary ARCH process, the distribution of ϵ_t is more peaked and possesses fatter tails than that of a normal random variate. However, when $\psi = 0$ or as t increases without bound, the kurtosis of ϵ_t is equal to that of a Gaussian process.

[16]Since the Box-Pierce statistic is the sum of squares, tests based on the Q-statistic are one-sided, whereas the z-statistic tests are two-sided.

[17]See, for example, Vasicek [1977] and Cox, Ingersoll, and Ross [1985a, 1985b].

[18]The "solution" is in the sense of Itô [1951]; that is, the stochastic integral in (25) is an Ito integral.

[19]Of course, the value of h depends upon the time units used to measure data hence our results may be interpreted more generally as applicable to the case where h = 1 period, where the actual length of the period is arbitrary. However, since our empirical investigations employ weekly data, we choose parameter values with the implicit understanding that h = 1 means one week. Therefore, although our results also apply to cases where h = 1 corresponds to one year, the particular parameter values we have chosen would be quite implausible for such a sampling interval.

[20]See Section 5 and Lo and MacKinlay [1987].

[21]This, of course, entails no loss of generality if all dividends are reinvested in the security or if the dividend-price ratio is a nonstochastic function of time.

[22]Note that the mean-square differentiability of X(t) may not be consistent with any continuous-time equilibrium model of asset prices. Specifically, Harrison, Pitbladdo, and Schaefer [1984] show that continuous-time price processes in frictionless markets with continuous sample paths must be of unbounded variation to rule out arbitrage. We therefore do not advocate the returns fads process as an economically reasonable alternative to the lognormal diffusion; its use is merely to illustrate the power of our test against an alternative in which returns are positively autocorrelated. Note, however, that the returns fad model may be an appropriate model for aggregate wealth (e.g., in a single-good representative agent model).

where

$$V_b = \mathrm{Var}\left[\frac{1}{h\sqrt{nq}} \sum_{k=1}^{n} (\varepsilon_k^2 - q\sigma_0^2 h)\right] = \frac{1}{h^2 nq} \sum_{k=1}^{n} \mathrm{Var}\left[\varepsilon_k^2 - q\sigma_0^2 h\right] \tag{A1-4a}$$

$$= \frac{1}{h^2 nq} \cdot nq^2 \sigma_0^4 h^2 = 2q\sigma_0^4 \ . \tag{A1-4b}$$

Since $\hat{\sigma}_a^2$ is the maximum-likelihood estimator of σ_0^2 under the null hypothesis (1), it is asymptotically efficient. Therefore, following Hausman's (1978) approach, we conclude that the asymptotic variance of $\sqrt{nq}\,(\hat{\sigma}_b^2 - \hat{\sigma}_a^2)$ is simply the difference $V_b - V_a$ of the asymptotic variances of $\sqrt{nq}\,(\hat{\sigma}_b^2 - \sigma_0^2)$ and $\sqrt{nq}\,(\hat{\sigma}_a^2 - \sigma_0^2)$ respectively. Thus we have

$$\sqrt{nq}\ J_d(r) \equiv \sqrt{nq}\ (\hat{\sigma}_b^2 - \hat{\sigma}_a^2) \overset{a}{\sim} N(0,\ 2(q-1)\sigma_0^4) \ . \tag{A1-5}$$

The asymptotic distribution of the ratio then follows by applying the "delta-method" to the quantity $\sqrt{nq}\ \left(g(\hat{\sigma}_a^2, \hat{\sigma}_b^2) - g(\sigma_0^2, \sigma_0^2)\right)$ where $g(u,v) \equiv \frac{v}{u}$, hence:

$$\sqrt{nq}\ J_r(q) = \sqrt{nq}\ \left(\frac{\hat{\sigma}_b^2}{\hat{\sigma}_a^2} - 1\right) \overset{a}{\sim} N(0,\ 2(q-1)) \ . \tag{A1-6}$$

Proof of Theorem 2:

In order to derive the limiting distributions of $\sqrt{nq}\ M_d$ and $\sqrt{nq}\ M_r$, we require the asymptotic distribution of $\sqrt{nq}\ (\hat{\sigma}_c^2 - \sigma_0^2)$. Because $\hat{\sigma}_c^2$ is computed using overlapping observations, its asymptotic behavior is somewhat more complicated to deduce. Our approach is to re-express this variance estimator as a function of the autocovariances of the $(X_k - X_{k-q})$ terms and then employ well-known limit theorems for autocovariances. Consider then the quantity:

$$\hat{\sigma}_c^2 = \frac{1}{nq^2h} \sum_{k=q}^{nq} [X_k - X_{k-q} - q\hat{\mu}h]^2 = \frac{1}{nq^2h} \sum_{k=q}^{nq} [\sum_{j=1}^{q} (X_{k-j+1} - X_{k-j} - \hat{\mu}h)]^2 \quad \text{(A2-1a)}$$

$$= \frac{1}{nq^2h} \sum_{k=q}^{nq} [\sum_{j=1}^{q} \hat{\eta}_{k-j+1}]^2 \quad \text{(A2-1b)}$$

where $\hat{\eta}_{k-j+1} \equiv X_{k-j+1} - X_{k-j} - \hat{\mu}h$. But then we have:

$$\hat{\sigma}_c^2 = \frac{1}{nq^2h} \sum_{k=q}^{nq} [\sum_{j=1}^{q} \hat{\eta}^2_{k-j+1} + 2 \sum_{j=1}^{q-1} \hat{\eta}_{k-j+1}\hat{\eta}_{k-j} + 2 \sum_{j=1}^{q-2} \hat{\eta}_{k-j+1}\hat{\eta}_{k-j-1} + \ldots + 2\hat{\eta}_k\hat{\eta}_{k-q+1}] \quad \text{(A2-2a)}$$

$$= \frac{1}{nq^2h} \Big[q \sum_{k=1}^{nq} \hat{\eta}_k^2 - \sum_{k=1}^{q-1} \Big([q - k]\hat{\eta}_k^2 + k\hat{\eta}^2_{nq-q+k+1}\Big)$$

$$+ 2(q - 1) \sum_{k=2}^{nq} \hat{\eta}_k\hat{\eta}_{k-1} - 2 \sum_{k=2}^{q-1}\Big([q - k]\hat{\eta}_k\hat{\eta}_{k-1} + [k - 1]\hat{\eta}_{nq-q+k+1}\hat{\eta}_{nq-q+k}\Big)$$

$$+ 2(q - 2) \sum_{k=3}^{nq} \hat{\eta}_k\hat{\eta}_{k-2} - 2 \sum_{k=3}^{q-1}\Big([q - k]\hat{\eta}_k\hat{\eta}_{k-2} + [k - 2]\hat{\eta}_{nq-q+k+1}\hat{\eta}_{nq-q+k-1}\Big)$$

$$+ \ldots$$

$$+ 2 \sum_{k=q}^{nq} \hat{\eta}_k\hat{\eta}_{k-q+1}\Big] \quad \text{(A2-2b)}$$

$$= \hat{\gamma}(0) - o_p(n^{-\frac{1}{2}}) + \frac{2(q - 1)}{q} \hat{\gamma}(1) - o_p(n^{-\frac{1}{2}}) + \frac{2(q - 2)}{q} \hat{\gamma}(2) - o_p(n^{-\frac{1}{2}})$$

$$+ \ldots + \frac{2}{q} \hat{\gamma}(q - 1) \quad \text{(A2-2c)}$$

where $\hat{\gamma}(j) \equiv \frac{1}{nqh} \sum_{k=j+1}^{nq} \hat{\eta}_k\hat{\eta}_{k-j}$ and $o_p(n^{-\frac{1}{2}})$ denotes a quantity which is of order smaller than $n^{-\frac{1}{2}}$ in probability. Now define the $q \times 1$ vector $\hat{\gamma} \equiv [\hat{\gamma}(0)\ \hat{\gamma}(1)\ \ldots\ \hat{\gamma}(q - 1)]'$. A standard limit theorem for sample

$$\sqrt{nq}\ (\hat{\gamma} - \sigma_0^2 e_1) \overset{a}{\sim} N(0,\ \sigma_0^4[I_q + e_1e_1']) \tag{A2-3}$$

where e_1 is the $q\times1$ vector $[1\ 0\ \ldots\ 0]'$ and I_q is the identity matrix of order q. Returning to the quantity $\sqrt{nq}\ (\hat{\sigma}_c^2 - \sigma_0^2)$, we have:

$$\sqrt{nq}\ (\hat{\sigma}_c^2 - \sigma_0^2) = \sqrt{nq}\ \Big[\big(\hat{\gamma}(0) - \sigma_0^2\big) + \frac{2(q-1)}{q}\ \hat{\gamma}(1) + \ldots + \frac{2}{q}\ \hat{\gamma}(q-1)\Big] - \sqrt{nq}\ o_p(n^{-\frac{1}{2}})\ . \tag{A2-4}$$

Applying the 'delta-method' to (A2-4) in view of equation (A2-3) then yields the following result:

$$\sqrt{nq}\ (\hat{\sigma}_c^2 - \sigma_0^2) \overset{a}{\sim} N(0,\ V_c) \tag{A2-5a}$$

where

$$V_c \equiv 2\sigma_0^4 + \Big[\frac{2(q-1)}{q}\Big]^2\sigma_0^4 + \ldots + \Big(\frac{2}{q}\Big)^2\sigma_0^4 \tag{A2-5b}$$

$$= 2\sigma_0^4\Big[1 + \frac{2}{q^2}\Big(\sum_{j=1}^{q-1} j^2\Big)\Big] \tag{A2-5c}$$

$$V_c = 2\sigma_0^4\Big[\frac{2q}{3} + \frac{1}{3q}\Big]\ . \tag{A2-5d}$$

Given the asymptotic distributions (A1-1) and (A2-5), Hausman's (1978) method may be applied in precisely the same manner as in Theorem 1 to yield the desired result:

$$\sqrt{nq}\ M_d(q) \overset{a}{\sim} N\Big(0,\ \frac{2(2q-1)(q-1)}{3q}\ \sigma_0^4\Big)$$

$$\sqrt{nq}\ M_r(q) \overset{a}{\sim} N\Big(0,\ \frac{2(2q-1)(q-1)}{3q}\Big)\ .$$

Proof of Theorem 3:

(a) We prove the result for $\bar{M}_r(q)$; the proofs for the other statistics follow almost immediately from this case. Define the increment process $Y_t \equiv X_t - X_{t-1}$ and define $\hat{\rho}(\tau)$ as:

$$\hat{\rho}(\tau) \equiv \frac{\frac{1}{T}\sum_{t=\tau}^{T}(Y_t - \hat{\mu})\cdot(Y_{t-\tau} - \hat{\mu})}{\frac{1}{T}\sum_{t=1}^{T}(Y_t - \hat{\mu})^2} \equiv \frac{A(\tau)}{B(\tau)} . \tag{A3-1}$$

Consider first the numerator $A(\tau)$ of $\hat{\rho}(\tau)$:

$$A(\tau) \equiv \frac{1}{T}\sum_{t=\tau}^{T}(Y_t - \hat{\mu})\cdot(Y_{t-\tau} - \hat{\mu}) = \frac{1}{T}\sum_{t=\tau}^{T}(\mu - \hat{\mu} + \varepsilon_t)\cdot(\mu - \hat{\mu} + \varepsilon_{t-\tau}) \tag{A3-2a}$$

$$= \frac{T-\tau+1}{T}(\mu-\hat{\mu})^2 + (\mu-\hat{\mu})\cdot\frac{1}{T}\sum_{t=\tau}^{T}\varepsilon_t + (\mu-\hat{\mu})\cdot\frac{1}{T}\sum_{t=\tau}^{T}\varepsilon_{t-\tau} + \frac{1}{T}\sum_{t=\tau}^{T}\varepsilon_t\varepsilon_{t-\tau} . \tag{A3-2b}$$

Since $\hat{\mu} \xrightarrow{a.s.} \mu$, the first term of (A3-2b) converges a.s. to zero as $T \to \infty$. Moreover, under Assumption (A2) it is apparent that $\{\varepsilon_t\}$ satisfies the conditions of White's [1984] Corollary 3.48, hence Assumption (A1) implies that the second and third terms of (A3-2b) also vanish a.s. Finally, because $\varepsilon_t\varepsilon_{t-\tau}$ is clearly a measurable function of the ε_t's, $\{\varepsilon_t\varepsilon_{t-\tau}\}$ is also mixing with coefficients of the same size as $\{\varepsilon_t\}$. Therefore, under (A2) Corollary 3.48 of White [1984] may also be applied to $\{\varepsilon_t\varepsilon_{t-\tau}\}$ for which (A1) implies that the fourth term of (A3-2b) converges a.s. to zero as well. By similar arguments, it may also be shown that

$$B(\tau) \equiv \frac{1}{T}\sum_{t=1}^{T}(Y_t - \hat{\mu})^2 \xrightarrow{a.s.} \sigma_0^2 . \tag{A3-3}$$

Therefore, we have $\hat{\rho}(\tau) \overset{a.s.}{\to} 0$ for all $\tau \neq 0$, hence we conclude:

$$\bar{M}_r(q) \overset{a.s.}{\to} 0 \text{ as } T \to \infty .$$

(b) By considering the regression of increments ΔX_t on a constant and lagged increments ΔX_{t-j}, this follows directly from White and Domowitz [1984].

Proof of Corollary 3.1:

This result follows trivially from Corollary 2.1 and Assumption (A4).

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Table 1

Upper and lower bounds on the standard deviation of z_1 and lower bound on z_1 itself, for several values of n.

n	V_L	V_U	$\mathrm{Inf}[z_1]$
2	0.333	0.667	-1.732
4	0.167	0.333	-2.449
8	0.083	0.167	-3.464
16	0.042	0.083	-4.899
32	0.021	0.042	-6.928
64	0.010	0.021	-9.798

Table 2a

f Monte Carlo experiments for the z_1-statistic associated with $\bar{M}_r(q)$ under the null hypothesis of a random walk with
stic disturbances. Each set of rows corresponding to a given sample size forms a separate and independent simulation
t based upon 20,000 replications.

gation ue q	Mean[1]	Standard[2] Deviation	Skewness[3]	Excess Kurtosis[4]	Studentized Range	Maximum Observation	Minimum Observation	Size[5] of 1% Test	Size[5] of 5% Test	Size[5] of 10% Test
2	0.007	1.030	0.058	-0.180	7.241	3.942	-3.518	0.010	0.056	0.110
4	0.005	1.061	0.689	0.461	7.671	5.600	-2.536	0.019	0.056	0.107
2	0.015	1.008	0.028	-0.060	8.247	4.113	-4.200	0.010	0.052	0.103
4	0.010	1.036	0.482	0.230	8.489	5.888	-2.908	0.014	0.055	0.106
8	0.012	1.074	0.935	1.139	8.134	6.544	-2.193	0.026	0.057	0.100
2	0.009	0.999	0.039	-0.078	7.809	3.662	-4.143	0.009	0.050	0.100
4	0.003	1.015	0.344	0.075	7.440	4.568	-2.983	0.012	0.051	0.100
8	-0.003	1.026	0.622	0.508	7.579	5.021	-2.760	0.016	0.050	0.098
16	-0.007	1.057	0.985	1.449	8.270	6.675	-2.065	0.022	0.053	0.090
2	0.015	1.007	0.068	-0.036	7.236	3.521	-3.763	0.010	0.052	0.103
4	0.006	1.008	0.213	0.056	8.493	4.958	-3.607	0.011	0.052	0.102
8	0.006	1.024	0.420	0.213	8.118	5.366	-2.950	0.013	0.051	0.102
16	0.005	1.043	0.688	0.652	8.082	5.948	-2.482	0.019	0.053	0.101
32	0.005	1.077	1.079	1.711	9.671	8.250	-2.170	0.026	0.056	0.096
2	0.009	1.006	0.013	-0.045	7.376	3.677	-3.742	0.010	0.051	0.102
4	0.013	1.002	0.175	0.004	7.488	3.949	-3.552	0.010	0.051	0.099
8	0.011	1.014	0.338	0.188	7.605	4.464	-3.247	0.012	0.053	0.100
16	0.006	1.025	0.498	0.342	7.359	4.749	-2.792	0.014	0.052	0.101
32	0.007	1.044	0.746	0.758	7.762	5.515	-2.588	0.019	0.055	0.099
64	0.007	1.083	1.162	2.046	8.745	7.412	-2.059	0.027	0.057	0.093
2	0.002	1.004	0.010	-0.028	8.395	4.059	-4.367	0.010	0.049	0.102
4	-0.003	1.002	0.099	-0.020	7.691	4.257	-3.452	0.010	0.050	0.100
8	-0.005	0.999	0.193	0.012	7.775	4.274	-3.495	0.010	0.048	0.099
16	-0.008	1.001	0.321	0.205	7.989	4.797	-3.203	0.011	0.050	0.098
32	-0.005	1.013	0.494	0.431	8.295	5.457	-2.949	0.014	0.050	0.095
64	0.001	1.036	0.711	0.746	7.720	5.403	-2.593	0.018	0.051	0.097
28	0.001	1.075	1.121	2.115	8.702	7.335	-2.021	0.024	0.052	0.090

dard error for the estimate of the mean is $1/\sqrt{20{,}000} = 0.0071$.

dard error for the estimate of the standard deviation is (based on the normal approximation) $1/\sqrt{2(20{,}000)} = 0.0050$.

dard error for the skewness is (based on the normal approximation) $\sqrt{6/20{,}000} = 0.0173$.

dard error for the excess kurtosis is (based on the normal approximation) $\sqrt{96/20{,}000} = 0.0693$.

e standard binomial approximation, the standard error SE(α) for the estimate of the size-α test is given by $\sqrt{\alpha(1-\alpha)/20{,}000}$ hence
007, SE(0.05) = 0.0015, and SE(0.10) = 0.0021.

Table 2b

ts of Monte Carlo experiments for the z_2-statistic associated with $\bar{M}_r(q)$ under the null hypothesis of a random walk with
cedastic disturbances. Each set of rows corresponding to a given sample size forms a separate and independent simulation
iment based upon 20,000 replications.

Aggregation Value q	Mean[1]	Standard[2] Deviation	Skewness[3]	Excess Kurtosis[4]	Studentized Range	Maximum Observation	Minimum Observation	Size[5] of 1% Test	Size[5] of 5% Test	Size[5] of 10% Test
2	0.015	1.081	0.093	-0.361	7.147	3.814	-3.914	0.011	0.067	0.131
4	0.008	1.122	0.661	0.296	7.342	5.509	-2.728	0.024	0.071	0.131
2	0.018	1.033	0.051	-0.189	7.879	3.559	-4.581	0.010	0.056	0.112
4	0.011	1.066	0.470	0.140	7.906	5.538	-2.886	0.016	0.061	0.118
8	0.012	1.113	0.917	1.047	8.421	6.745	-2.624	0.028	0.063	0.112
2	0.009	1.011	0.038	-0.156	7.470	3.893	-3.656	0.008	0.051	0.105
4	0.004	1.029	0.337	0.033	7.335	4.652	-2.893	0.013	0.053	0.105
8	-0.003	1.045	0.618	0.495	7.648	5.138	-2.855	0.017	0.054	0.105
16	-0.008	1.085	0.984	1.460	8.376	6.950	-2.136	0.025	0.058	0.098
2	0.016	1.012	0.069	-0.073	7.627	3.652	-4.068	0.011	0.053	0.105
4	0.005	1.015	0.208	0.033	8.330	4.660	-3.794	0.011	0.053	0.105
8	0.005	1.033	0.413	0.192	7.924	5.310	-2.877	0.013	0.054	0.107
16	0.005	1.056	0.683	0.631	7.979	5.921	-2.508	0.020	0.056	0.105
32	0.005	1.100	1.074	1.693	9.651	8.438	-2.173	0.027	0.060	0.102
2	0.009	1.009	0.010	-0.064	7.431	3.634	-3.862	0.010	0.051	0.102
4	0.013	1.005	0.172	-0.006	7.511	3.975	-3.573	0.010	0.051	0.099
8	0.011	1.018	0.336	0.181	7.444	4.408	-3.171	0.012	0.054	0.101
16	0.006	1.031	0.497	0.341	7.402	4.832	-2.801	0.015	0.054	0.103
32	0.007	1.055	0.746	0.764	7.795	5.621	-2.599	0.020	0.057	0.103
64	0.007	1.103	1.161	2.051	8.823	7.625	-2.106	0.030	0.060	0.098
2	0.002	1.005	0.010	-0.030	8.467	3.939	-4.570	0.010	0.049	0.102
4	-0.003	1.004	0.100	-0.027	7.703	4.191	-3.540	0.010	0.051	0.100
8	-0.005	1.001	0.193	0.009	7.802	4.331	-3.481	0.010	0.048	0.100
16	-0.008	1.004	0.321	0.204	8.017	4.860	-3.192	0.011	0.050	0.099
32	-0.005	1.018	0.493	0.428	8.236	5.424	-2.963	0.014	0.051	0.097
64	0.001	1.045	0.710	0.739	7.701	5.422	-2.625	0.018	0.053	0.100
128	0.001	1.093	1.119	2.108	8.716	7.472	-2.058	0.025	0.054	0.095

standard error for the estimate of the mean is $1/\sqrt{20{,}000} = 0.0071$.

standard error for the estimate of the standard deviation is (based on the normal approximation) $1/\sqrt{2(20{,}000)} = 0.0050$.

standard error for the skewness is (based on the normal approximation) $\sqrt{6/20{,}000} = 0.0173$.

standard error for the excess kurtosis is (based on the normal approximation) $\sqrt{96/20{,}000} = 0.0693$.

r the standard binomial approximation, the standard error $SE(\alpha)$ for the estimate of the size-α test is given by $\sqrt{\alpha(1-\alpha)/20{,}000}$ hence
0.0007, $SE(0.05) = 0.0015$, and $SE(0.10) = 0.0021$.

Table 3a

Monte Carlo experiments for the z_1-statistic associated with $\bar{M}_r(q)$ under the null hypothesis of a random walk with
astic disturbances, where the heteroscedasticity is of the form $\ln \sigma_t^2 = \psi \ln \sigma_{t-1}^2 + \zeta_t$, ζ_t i.i.d. N(0,1), ψ = 0.50. Each
s corresponding to a given sample size forms a separate and independent simulation experiment based upon 20,000
ns.

ation e q	Mean[1]	Standard[2] Deviation	Skewness[3]	Excess Kurtosis[4]	Studentized Range	Maximum Observation	Minimum Observation	Size[5] of 1% Test	Size[5] of 5% Test	Size[5] of 10% Test
2	-0.009	1.164	0.013	-0.219	7.401	3.942	-4.671	0.024	0.093	0.161
4	-0.008	1.145	0.697	0.423	6.950	5.338	-2.622	0.028	0.071	0.132
2	0.009	1.214	0.022	-0.005	7.458	4.510	-4.546	0.037	0.107	0.175
4	0.001	1.184	0.546	0.346	7.421	5.699	-3.084	0.029	0.088	0.155
8	-0.002	1.152	0.979	1.344	8.742	7.836	-2.230	0.030	0.066	0.119
2	-0.006	1.274	-0.018	0.069	8.581	5.540	-5.388	0.043	0.123	0.195
	-0.012	1.220	0.368	0.222	9.184	7.325	-3.885	0.033	0.104	0.174
	-0.011	1.156	0.691	0.764	8.814	7.375	-2.811	0.028	0.077	0.138
	-0.008	1.126	1.085	1.816	8.404	7.357	-2.108	0.030	0.063	0.106
2	0.011	1.320	0.021	0.237	9.679	6.047	-6.727	0.053	0.134	0.207
	0.002	1.257	0.341	0.297	8.318	6.051	-4.404	0.041	0.112	0.183
	-0.004	1.179	0.562	0.646	8.748	7.050	-3.259	0.029	0.087	0.152
	-0.003	1.129	0.817	1.234	8.710	7.103	-2.734	0.025	0.067	0.122
	-0.002	1.117	1.207	2.640	10.333	9.420	-2.123	0.029	0.057	0.096
2	0.011	1.360	0.031	0.201	9.112	6.336	-6.061	0.058	0.147	0.223
	0.011	1.295	0.310	0.284	8.271	6.233	-4.477	0.046	0.125	0.201
	0.011	1.208	0.464	0.385	8.574	6.586	-3.769	0.033	0.101	0.169
	0.011	1.136	0.619	0.610	8.321	6.373	-3.079	0.026	0.076	0.136
	0.007	1.096	0.793	0.890	7.884	6.107	-2.531	0.024	0.064	0.115
	0.003	1.096	1.105	1.746	8.870	7.576	-2.145	0.027	0.058	0.097
	0.009	1.366	0.005	0.261	10.047	6.680	-7.041	0.059	0.148	0.222
	0.003	1.290	0.195	0.188	8.326	5.971	-4.767	0.047	0.128	0.197
	-0.002	1.202	0.292	0.248	9.162	6.796	-4.219	0.031	0.101	0.167
	0.001	1.123	0.393	0.293	8.738	6.342	-3.471	0.021	0.079	0.139
	0.008	1.083	0.567	0.503	8.093	5.861	-2.901	0.020	0.063	0.119
	0.009	1.075	0.803	1.017	9.148	7.181	-2.651	0.023	0.057	0.106
	0.012	1.103	1.190	2.077	8.682	7.513	-2.060	0.030	0.061	0.096

rd error for the estimate of the mean is $1/\sqrt{20,000}$ = 0.0071.

rd error for the estimate of the standard deviation is (based on the normal approximation) $1/\sqrt{2(20,000)}$ = 0.0050.

rd error for the skewness is (based on the normal approximation) $\sqrt{6/20,000}$ = 0.0173.

rd error for the excess kurtosis is (based on the normal approximation) $\sqrt{96/20,000}$ = 0.0693.

standard binomial approximation, the standard error SE(α) for the estimate of the size-α test is given by $\sqrt{\alpha(1-\alpha)/20,000}$ hence
7, SE(0.05) = 0.0015, and SE(0.10) = 0.0021.

Table 3b

... experiments for the z_2-statistic associated with $\bar{M}_r(q)$ under the null hypothesis of a random walk with ...bances, where the heteroscedasticity is of the form $\ln \sigma_t^2 = \psi \ln \sigma_{t-1}^2 + \zeta_t$, ζ_t i.i.d. $N(0,1)$, $\psi = 0.50$. Each ...ding to a given sample size forms a separate and independent simulation experiment based upon 20,000

...an[1]	Standard[2] Deviation	Skewness[3]	Excess Kurtosis[4]	Studentized Range	Maximum Observation	Minimum Observation	Size[5] of 1% Test	Size[5] of 5% Test	Size[5] of 10% Test
...14	1.133	-0.019	0.262	12.299	4.679	-9.255	0.015	0.071	0.141
...05	1.154	0.645	0.416	10.490	5.243	-6.862	0.030	0.076	0.133
...14	1.055	0.004	-0.268	9.297	3.429	-6.382	0.008	0.055	0.118
...06	1.073	0.462	0.001	9.427	4.689	-5.422	0.016	0.061	0.116
...01	1.109	0.897	0.939	9.565	6.891	-3.719	0.028	0.063	0.108
...01	1.035	-0.066	0.363	14.288	3.586	-11.199	0.007	0.051	0.109
...06	1.038	0.300	0.050	11.417	3.998	-7.855	0.013	0.053	0.106
...06	1.047	0.631	0.423	9.275	4.751	-4.965	0.018	0.053	0.102
...07	1.089	1.039	1.549	8.938	6.547	-3.183	0.027	0.059	0.097
...12	1.011	0.045	-0.259	7.525	3.682	-3.923	0.008	0.047	0.102
...04	1.016	0.312	-0.055	7.364	4.424	-3.057	0.010	0.049	0.101
...02	1.024	0.513	0.336	7.895	5.037	-3.045	0.014	0.050	0.099
...02	1.051	0.771	0.953	8.124	6.059	-2.478	0.020	0.052	0.098
...02	1.094	1.174	2.440	10.521	8.967	-2.548	0.027	0.055	0.092
...08	1.014	0.038	-0.239	7.352	3.798	-3.658	0.008	0.049	0.105
...08	1.018	0.264	-0.072	7.965	4.365	-3.747	0.011	0.053	0.104
...09	1.024	0.418	0.148	7.567	4.557	-3.193	0.013	0.052	0.105
...09	1.033	0.580	0.438	8.482	5.672	-3.090	0.016	0.054	0.103
...05	1.049	0.757	0.734	7.823	5.695	-2.513	0.020	0.056	0.101
...01	1.089	1.084	1.670	9.063	7.652	-2.218	0.026	0.057	0.096
...07	0.998	0.007	-0.126	7.753	4.003	-3.738	0.008	0.046	0.097
...03	0.999	0.165	-0.040	7.500	4.086	-3.406	0.009	0.050	0.100
...01	1.006	0.264	0.073	7.931	4.479	-3.500	0.010	0.050	0.100
...01	1.009	0.371	0.188	8.370	5.230	-3.216	0.011	0.050	0.101
...07	1.025	0.555	0.454	8.243	5.458	-2.993	0.015	0.051	0.100
...08	1.053	0.797	1.005	9.551	7.397	-2.657	0.021	0.054	0.099
...2	1.107	1.183	2.053	8.833	7.703	-2.072	0.030	0.061	0.097

... the estimate of the mean is $1/\sqrt{20{,}000} = 0.0071$.

... the estimate of the standard deviation is (based on the normal approximation) $1/\sqrt{2(20{,}000)} = 0.0050$.

... the skewness is (based on the normal approximation) $\sqrt{6/20{,}000} = 0.0173$.

... the excess kurtosis is (based on the normal approximation) $\sqrt{96/20{,}000} = 0.0693$.

...omial approximation, the standard error SE(α) for the estimate of the size-α test is given by $\sqrt{\alpha(1-\alpha)/20{,}000}$ hence ... = 0.0015, and SE(0.10) = 0.0021.

Table 4b

onte Carlo experiments for the z_2-statistic associated with $\bar{M}_r(q)$ under the null hypothesis of a random walk with tic disturbances, where the heteroscedasticity is of the form $\ln \sigma_t^2 = \psi \ln \sigma_{t-1}^2 + \zeta_t$, ζ_t i.i.d. $N(0,1)$, $\psi = 0.80$. Each corresponding to a given sample size forms a separate and independent simulation experiment based upon 20,000 .

ion q	Mean[1]	Standard Deviation[2]	Skewness[3]	Excess Kurtosis[4]	Studentized Range	Maximum Observation	Minimum Observation	Size[5] of 1% Test	Size[5] of 5% Test	Size[5] of 10% Test
	-0.012	1.215	-1.625	30.470	29.894	4.319	-32.004	0.019	0.074	0.148
	-0.014	1.156	0.558	0.819	12.990	5.094	-9.924	0.032	0.076	0.128
	0.003	1.072	-0.109	0.163	10.578	3.421	-7.916	0.009	0.051	0.113
	-0.001	1.077	0.541	0.234	10.233	4.797	-6.227	0.018	0.060	0.110
	-0.004	1.107	1.103	1.539	9.212	6.962	-3.233	0.031	0.066	0.103
	0.016	1.036	-0.027	0.056	13.963	4.001	-10.465	0.006	0.046	0.107
	0.016	1.042	0.488	0.058	12.073	5.116	-7.464	0.013	0.053	0.105
	0.019	1.061	0.927	0.972	10.360	7.374	-3.621	0.023	0.059	0.098
	0.017	1.096	1.315	2.384	10.620	8.905	-2.738	0.031	0.062	0.094
	0.004	1.019	-0.009	-0.224	11.634	3.966	-7.889	0.006	0.045	0.100
	-0.002	1.019	0.394	-0.150	6.936	4.048	-3.021	0.011	0.048	0.101
	0.002	1.028	0.717	0.466	7.615	5.036	-2.793	0.017	0.051	0.095
	-0.003	1.041	0.983	1.294	8.527	5.915	-2.960	0.023	0.053	0.087
	-0.009	1.079	1.303	2.529	8.913	6.950	-2.668	0.029	0.057	0.086
	0.003	1.010	-0.005	-0.393	8.180	3.559	-4.700	0.006	0.042	0.098
	0.004	1.012	0.372	-0.140	8.378	4.484	-3.992	0.009	0.047	0.099
	0.007	1.016	0.644	0.389	8.888	6.105	-2.928	0.016	0.051	0.093
	0.008	1.031	0.860	0.950	9.203	6.853	-2.635	0.020	0.052	0.092
	0.004	1.053	1.066	1.662	9.470	7.533	-2.442	0.024	0.055	0.090
	0.007	1.097	1.353	2.643	8.858	7.512	-2.207	0.031	0.061	0.090
	0.015	1.009	0.046	-0.351	7.170	3.775	-3.460	0.008	0.044	0.098
	0.008	1.010	0.346	-0.101	7.741	4.394	-3.428	0.009	0.049	0.098
	0.004	1.017	0.540	0.262	8.643	5.671	-3.119	0.014	0.050	0.100
	0.004	1.023	0.687	0.609	8.560	6.034	-2.726	0.016	0.051	0.097
	0.004	1.034	0.822	0.947	8.613	6.235	-2.669	0.020	0.053	0.094
	-0.002	1.047	0.980	1.523	9.468	7.489	-2.429	0.023	0.053	0.091
	-0.006	1.080	1.286	2.896	10.408	9.167	-2.077	0.028	0.056	0.084

error for the estimate of the mean is $1/\sqrt{20{,}000} = 0.0071$.

error for the estimate of the standard deviation is (based on the normal approximation) $1/\sqrt{2(20{,}000)} = 0.0050$.

error for the skewness is (based on the normal approximation) $\sqrt{6/20{,}000} = 0.0173$.

error for the excess kurtosis is (based on the normal approximation) $\sqrt{96/20{,}000} = 0.0693$.

ndard binomial approximation, the standard error $SE(\alpha)$ for the estimate of the size-α test is given by $\sqrt{\alpha(1-\alpha)/20{,}000}$ 0007, $SE(0.05) = 0.0015$, and $SE(0.10) = 0.0021$.

Table 5a

Empirical size of nominal 1, 5, and 10 percent tests of the random walk null hypothesis with homoscedastic disturbances using the z_1-statistic corresponding to $\bar{M}_r(q)$ vs. the Box-Pierce Q-statistic (BP-Q) with q-1 autocorrelations. Each set of rows corresponding to a given sample size forms a separate and independent simulation experiment based upon 20,000 replications.

Sample Size	q	1 Percent Test		5 Percent Test		10 Percent Test	
		z_1	BP-Q	z_1	BP-Q	z_1	BP-Q
32	2	0.010	0.006	0.056	0.044	0.110	0.093
32	4	0.019	0.007	0.056	0.035	0.107	0.073
64	2	0.010	0.008	0.052	0.047	0.103	0.094
64	4	0.014	0.008	0.055	0.040	0.106	0.084
64	8	0.026	0.010	0.057	0.039	0.100	0.073
128	2	0.009	0.010	0.050	0.051	0.100	0.099
128	4	0.012	0.009	0.051	0.046	0.100	0.092
128	8	0.016	0.011	0.050	0.044	0.098	0.087
128	16	0.022	0.011	0.053	0.041	0.090	0.076
256	2	0.010	0.009	0.052	0.049	0.103	0.099
256	4	0.011	0.010	0.052	0.049	0.102	0.095
256	8	0.013	0.011	0.051	0.047	0.102	0.092
256	16	0.019	0.012	0.053	0.048	0.101	0.092
256	32	0.026	0.011	0.056	0.042	0.096	0.077
512	2	0.010	0.010	0.051	0.050	0.102	0.100
512	4	0.010	0.009	0.051	0.046	0.099	0.093
512	8	0.012	0.009	0.053	0.046	0.100	0.092
512	16	0.014	0.010	0.052	0.045	0.101	0.090
512	32	0.019	0.010	0.055	0.043	0.099	0.083
512	64	0.027	0.008	0.057	0.032	0.093	0.062
1024	2	0.010	0.010	0.049	0.051	0.102	0.099
1024	4	0.010	0.010	0.050	0.050	0.100	0.096
1024	8	0.010	0.010	0.048	0.048	0.099	0.096
1024	16	0.011	0.010	0.050	0.046	0.098	0.092
1024	32	0.014	0.010	0.050	0.045	0.095	0.089
1024	64	0.018	0.010	0.051	0.043	0.097	0.081
1024	128	0.024	0.006	0.052	0.028	0.090	0.053

Table 5b

Empirical size of nominal 1, 5, and 10 percent tests of the random walk null hypothesis with homoscedastic disturbances using the z_2-statistic corresponding to $\bar{M}_r(q)$ vs. the heteroscedasticity-consistent Box-Pierce Q-statistic (BP-Q*) with q-1 autocorrelations. Each set of rows corresponding to a given sample size forms a separate and independent simulation experiment based upon 20,000 replications.

Sample Size	q	1 Percent Test		5 Percent Test		10 Percent Test	
		z_2	BP-Q*	z_2	BP-Q*	z_2	BP-Q*
32	2	0.011	0.006	0.067	0.049	0.131	0.107
32	4	0.024	0.009	0.071	0.044	0.131	0.097
64	2	0.010	0.007	0.056	0.048	0.112	0.102
64	4	0.016	0.009	0.061	0.046	0.118	0.096
64	8	0.028	0.013	0.063	0.053	0.112	0.100
128	2	0.008	0.009	0.051	0.052	0.105	0.103
128	4	0.013	0.010	0.053	0.049	0.105	0.101
128	8	0.017	0.012	0.054	0.053	0.105	0.103
128	16	0.025	0.017	0.058	0.060	0.098	0.110
256	2	0.011	0.009	0.053	0.050	0.105	0.100
256	4	0.011	0.011	0.053	0.050	0.105	0.098
256	8	0.013	0.012	0.054	0.050	0.107	0.101
256	16	0.020	0.015	0.056	0.059	0.105	0.110
256	32	0.027	0.019	0.060	0.068	0.102	0.119
512	2	0.010	0.010	0.051	0.050	0.102	0.100
512	4	0.010	0.009	0.051	0.047	0.099	0.095
512	8	0.012	0.011	0.054	0.047	0.101	0.095
512	16	0.015	0.011	0.054	0.050	0.103	0.100
512	32	0.020	0.014	0.057	0.056	0.103	0.107
512	64	0.030	0.018	0.060	0.066	0.098	0.118
1024	2	0.010	0.010	0.049	0.051	0.102	0.100
1024	4	0.010	0.010	0.051	0.050	0.100	0.097
1024	8	0.010	0.010	0.048	0.049	0.100	0.098
1024	16	0.011	0.010	0.050	0.050	0.099	0.098
1024	32	0.014	0.012	0.051	0.052	0.097	0.103
1024	64	0.018	0.016	0.053	0.062	0.100	0.114
1024	128	0.025	0.021	0.054	0.071	0.095	0.123

Table 6

uantiles of the (Dickey-Fuller) t-statistic associated with the hypothesis $\beta = 1$ in ion $X_t = \mu + \omega t + \beta X_{t-1} + \varepsilon_t$. Each row corresponds to a separate and independent experiment based upon 20,000 replications.

5	0.010	0.025	0.050	0.100	0.900	0.950	0.975	0.990	0.995
7	-4.456	-4.043	-3.731	-3.361	-1.222	-0.887	-0.598	-0.246	-0.013
9	-4.188	-3.860	-3.570	-3.243	-1.230	-0.906	-0.620	-0.279	-0.019
4	-4.087	-3.777	-3.492	-3.186	-1.241	-0.918	-0.635	-0.273	-0.040
5	-3.990	-3.684	-3.424	-3.135	-1.241	-0.910	-0.649	-0.276	-0.049
3	-3.973	-3.676	-3.424	-3.131	-1.233	-0.903	-0.611	-0.299	-0.032
0	-3.959	-3.663	-3.425	-3.130	-1.252	-0.951	-0.673	-0.319	-0.054

Table 7a

Empirical size of nominal 1, 5, and 10 percent tests of the random walk null hypothesis under heteroscedastic disturbances using the z_1-statistic corresponding to $\bar{M}_r(q)$, the Box-Pierce Q-statistic (BP-Q) with q-1 autocorrelations, and the Dickey-Fuller t-statistic (DF). The specific form of heteroscedasticity is given by $\ln \sigma_t^2 = \psi \ln \sigma_{t-1}^2 + \zeta_t$, ζ_t i.i.d. N(0,1), $\psi \equiv 0.50$. Each set of rows corresponding to a given sample size forms a separate and independent simulation experiment based upon 20,000 replications.

Sample Size	q	1 Percent Test			5 Percent Test			10 Percent Test		
		z_1	BP-Q	DF	z_1	BP-Q	DF	z_1	BP-Q	DF
32	2	0.024	0.014	0.023	0.093	0.069	0.073	0.161	0.133	0.124
32	4	0.028	0.010	0.023	0.071	0.047	0.073	0.132	0.094	0.124
64	2	0.037	0.029	0.019	0.107	0.094	0.066	0.175	0.158	0.116
64	4	0.029	0.023	0.019	0.088	0.078	0.066	0.155	0.134	0.116
64	8	0.030	0.016	0.019	0.066	0.053	0.066	0.119	0.099	0.116
128	2	0.043	0.039	0.016	0.123	0.115	0.061	0.195	0.184	0.111
128	4	0.033	0.032	0.016	0.104	0.103	0.061	0.174	0.169	0.111
128	8	0.028	0.025	0.016	0.077	0.080	0.061	0.138	0.134	0.111
128	16	0.030	0.020	0.016	0.063	0.055	0.061	0.106	0.095	0.111
256	2	0.053	0.050	0.012	0.134	0.129	0.058	0.207	0.200	0.111
256	4	0.041	0.043	0.012	0.112	0.122	0.058	0.183	0.192	0.111
256	8	0.029	0.033	0.012	0.087	0.096	0.058	0.152	0.161	0.111
256	16	0.025	0.023	0.012	0.067	0.073	0.058	0.122	0.127	0.111
256	32	0.029	0.018	0.012	0.057	0.053	0.058	0.096	0.091	0.111
512	2	0.058	0.056	0.010	0.147	0.146	0.049	0.223	0.220	0.099
512	4	0.046	0.051	0.010	0.125	0.138	0.049	0.201	0.218	0.099
512	8	0.033	0.038	0.010	0.101	0.113	0.049	0.169	0.183	0.099
512	16	0.026	0.029	0.010	0.076	0.086	0.049	0.136	0.146	0.099
512	32	0.024	0.020	0.010	0.064	0.065	0.049	0.115	0.116	0.099
512	64	0.027	0.013	0.010	0.058	0.044	0.049	0.097	0.077	0.099
1024	2	0.059	0.058	0.012	0.148	0.148	0.054	0.222	0.222	0.105
1024	4	0.047	0.057	0.012	0.128	0.148	0.054	0.197	0.226	0.105
1024	8	0.031	0.039	0.012	0.101	0.116	0.054	0.167	0.193	0.105
1024	16	0.021	0.029	0.012	0.079	0.095	0.054	0.139	0.160	0.105
1024	32	0.020	0.022	0.012	0.063	0.073	0.054	0.119	0.130	0.105
1024	64	0.023	0.016	0.012	0.057	0.058	0.054	0.106	0.100	0.105
1024	128	0.030	0.009	0.012	0.061	0.033	0.054	0.096	0.061	0.105

Table 7b

Empirical size of nominal 1, 5, and 10 percent tests of the random walk null hypothesis under heteroscedastic disturbances using the z_2-statistic corresponding to $\bar{M}_r(q)$, the heteroscedasticity-consistent Box-Pierce Q-statistic (BP-Q*) with q-1 autocorrelations, and the Dickey-Fuller t-statistic (DF). The specific form of heteroscedasticity is given by $\ln \sigma_t^2 = \psi \ln \sigma_{t-1}^2 + \zeta_t$, ζ_t i.i.d. $N(0,1)$, $\psi \equiv 0.50$. Each set of rows corresponding to a given sample size forms a separate and independent simulation experiment based upon 20,000 replications.

Sample Size	q	1 Percent Test			5 Percent Test			10 Percent Test		
		z_2	BP-Q*	DF	z_2	BP-Q*	DF	z_2	BP-Q*	DF
32	2	0.015	0.003	0.023	0.071	0.036	0.073	0.141	0.098	0.124
32	4	0.030	0.005	0.023	0.076	0.035	0.073	0.133	0.080	0.124
64	2	0.008	0.004	0.019	0.055	0.039	0.066	0.118	0.098	0.116
64	4	0.016	0.006	0.019	0.061	0.037	0.066	0.116	0.087	0.116
64	8	0.028	0.009	0.019	0.063	0.045	0.066	0.108	0.092	0.116
128	2	0.007	0.005	0.016	0.051	0.043	0.061	0.109	0.098	0.111
128	4	0.013	0.007	0.016	0.053	0.039	0.061	0.106	0.087	0.111
128	8	0.018	0.008	0.016	0.053	0.045	0.061	0.102	0.090	0.111
128	16	0.027	0.013	0.016	0.059	0.052	0.061	0.097	0.096	0.111
256	2	0.008	0.007	0.012	0.047	0.045	0.058	0.102	0.096	0.111
256	4	0.010	0.007	0.012	0.049	0.042	0.058	0.101	0.089	0.111
256	8	0.014	0.009	0.012	0.050	0.044	0.058	0.099	0.093	0.111
256	16	0.020	0.010	0.012	0.052	0.050	0.058	0.098	0.097	0.111
256	32	0.027	0.016	0.012	0.055	0.057	0.058	0.092	0.106	0.111
512	2	0.008	0.007	0.010	0.049	0.047	0.049	0.105	0.102	0.099
512	4	0.011	0.008	0.010	0.053	0.045	0.049	0.104	0.094	0.099
512	8	0.013	0.009	0.010	0.052	0.047	0.049	0.105	0.097	0.099
512	16	0.016	0.010	0.010	0.054	0.050	0.049	0.103	0.098	0.099
512	32	0.020	0.012	0.010	0.056	0.054	0.049	0.101	0.104	0.099
512	64	0.026	0.016	0.010	0.057	0.060	0.049	0.096	0.111	0.099
1024	2	0.008	0.008	0.012	0.046	0.046	0.054	0.097	0.096	0.105
1024	4	0.009	0.009	0.012	0.050	0.049	0.054	0.100	0.100	0.105
1024	8	0.010	0.008	0.012	0.050	0.046	0.054	0.100	0.095	0.105
1024	16	0.011	0.010	0.012	0.050	0.051	0.054	0.101	0.100	0.105
1024	32	0.015	0.012	0.012	0.051	0.053	0.054	0.100	0.102	0.105
1024	64	0.021	0.014	0.012	0.054	0.057	0.054	0.099	0.107	0.105
1024	128	0.030	0.019	0.012	0.061	0.068	0.054	0.097	0.119	0.105

Table 8a

Empirical size of nominal 1, 5, and 10 percent tests of the random walk null hypothesis under heteroscedastic disturbances using the z_1-statistic corresponding to $\bar{M}_r(q)$, the Box-Pierce Q-statistic (BP-Q) with q-1 autocorrelations, and the Dickey-Fuller t-statistic (DF). The specific form of heteroscedasticity is given by $\ln \sigma_t^2 = \psi \ln \sigma_{t-1}^2 + \zeta_t$, ζ_t i.i.d. N(0,1), $\psi \equiv 0.80$. Each set of rows corresponding to a given sample size forms a separate and independent simulation experiment based upon 20,000 replications.

Sample Size	q	1 Percent Test			5 Percent Test			10 Percent Test		
		z_1	BP-Q	DF	z_1	BP-Q	DF	z_1	BP-Q	DF
32	2	0.056	0.039	0.043	0.152	0.117	0.106	0.230	0.190	0.162
32	4	0.042	0.031	0.043	0.102	0.099	0.106	0.181	0.165	0.162
64	2	0.091	0.078	0.042	0.195	0.176	0.101	0.275	0.254	0.161
64	4	0.069	0.089	0.042	0.162	0.194	0.101	0.246	0.278	0.161
64	8	0.054	0.068	0.042	0.104	0.149	0.101	0.178	0.216	0.161
128	2	0.136	0.127	0.041	0.249	0.237	0.100	0.332	0.320	0.163
128	4	0.111	0.164	0.041	0.223	0.301	0.100	0.306	0.397	0.163
128	8	0.072	0.148	0.041	0.166	0.271	0.100	0.248	0.357	0.163
128	16	0.058	0.095	0.041	0.103	0.181	0.100	0.173	0.251	0.163
256	2	0.182	0.176	0.032	0.302	0.293	0.093	0.383	0.376	0.153
256	4	0.154	0.253	0.032	0.273	0.399	0.093	0.355	0.491	0.153
256	8	0.106	0.246	0.032	0.219	0.391	0.093	0.307	0.482	0.153
256	16	0.064	0.180	0.032	0.147	0.301	0.093	0.230	0.381	0.153
256	32	0.050	0.101	0.032	0.092	0.184	0.093	0.154	0.247	0.153
512	2	0.223	0.220	0.026	0.347	0.343	0.079	0.429	0.425	0.135
512	4	0.198	0.322	0.026	0.322	0.473	0.079	0.405	0.565	0.135
512	8	0.146	0.331	0.026	0.266	0.486	0.079	0.347	0.578	0.135
512	16	0.092	0.263	0.026	0.196	0.408	0.079	0.279	0.497	0.135
512	32	0.058	0.176	0.026	0.127	0.288	0.079	0.204	0.366	0.135
512	64	0.047	0.098	0.026	0.085	0.172	0.079	0.135	0.224	0.135
1024	2	0.264	0.261	0.020	0.391	0.390	0.073	0.468	0.467	0.126
1024	4	0.231	0.389	0.020	0.360	0.540	0.073	0.442	0.627	0.126
1024	8	0.181	0.414	0.020	0.307	0.575	0.073	0.391	0.661	0.126
1024	16	0.121	0.345	0.020	0.237	0.504	0.073	0.322	0.590	0.126
1024	32	0.072	0.241	0.020	0.164	0.379	0.073	0.246	0.469	0.126
1024	64	0.044	0.150	0.020	0.104	0.254	0.073	0.175	0.328	0.126
1024	128	0.037	0.076	0.020	0.071	0.139	0.073	0.118	0.188	0.126

Table 8b

Empirical size of nominal 1, 5, and 10 percent tests of the random walk null hypothesis under heteroscedastic disturbances using the z_2-statistic corresponding to $\bar{M}_r(q)$, the heteroscedasticity-consistent Box-Pierce Q-statistic (BP-Q*) with q-1 autocorrelations, and the Dickey-Fuller t-statistic (DF). The specific form of heteroscedasticity is given by $\ln \sigma_t^2 = \psi \ln \sigma_{t-1}^2 + \zeta_t$, ζ_t i.i.d. N(0,1), $\psi \equiv 0.80$. Each set of rows corresponding to a given sample size forms a separate and independent simulation experiment based upon 20,000 replications.

Sample Size	q	1 Percent Test			5 Percent Test			10 Percent Test		
		z_2	BP-Q*	DF	z_2	BP-Q*	DF	z_2	BP-Q*	DF
32	2	0.019	0.002	0.043	0.074	0.034	0.106	0.148	0.091	0.162
32	4	0.032	0.005	0.043	0.076	0.033	0.106	0.128	0.078	0.162
64	2	0.009	0.002	0.042	0.051	0.033	0.101	0.113	0.089	0.161
64	4	0.018	0.005	0.042	0.060	0.035	0.101	0.110	0.080	0.161
64	8	0.031	0.009	0.042	0.066	0.038	0.101	0.103	0.083	0.161
128	2	0.006	0.004	0.041	0.046	0.037	0.100	0.107	0.094	0.163
128	4	0.013	0.005	0.041	0.053	0.034	0.100	0.105	0.080	0.163
128	8	0.023	0.008	0.041	0.059	0.039	0.100	0.098	0.081	0.163
128	16	0.031	0.011	0.041	0.062	0.047	0.100	0.094	0.091	0.163
256	2	0.006	0.004	0.032	0.045	0.040	0.093	0.100	0.094	0.153
256	4	0.011	0.006	0.032	0.048	0.039	0.093	0.101	0.086	0.153
256	8	0.017	0.009	0.032	0.051	0.041	0.093	0.095	0.086	0.153
256	16	0.023	0.010	0.032	0.053	0.046	0.093	0.087	0.088	0.153
256	32	0.029	0.013	0.032	0.057	0.052	0.093	0.086	0.099	0.153
512	2	0.006	0.005	0.026	0.042	0.040	0.079	0.098	0.096	0.135
512	4	0.009	0.007	0.026	0.047	0.040	0.079	0.099	0.088	0.135
512	8	0.016	0.009	0.026	0.051	0.043	0.079	0.093	0.088	0.135
512	16	0.020	0.010	0.026	0.052	0.047	0.079	0.092	0.094	0.135
512	32	0.024	0.010	0.026	0.055	0.052	0.079	0.090	0.101	0.135
512	64	0.031	0.014	0.026	0.061	0.059	0.079	0.090	0.111	0.135
1024	2	0.008	0.007	0.020	0.044	0.042	0.073	0.098	0.097	0.126
1024	4	0.009	0.007	0.020	0.049	0.041	0.073	0.098	0.091	0.126
1024	8	0.014	0.008	0.020	0.050	0.041	0.073	0.100	0.089	0.126
1024	16	0.016	0.009	0.020	0.051	0.044	0.073	0.097	0.095	0.126
1024	32	0.020	0.009	0.020	0.053	0.049	0.073	0.094	0.097	0.126
1024	64	0.023	0.011	0.020	0.053	0.051	0.073	0.091	0.098	0.126
1024	128	0.028	0.017	0.020	0.056	0.063	0.073	0.084	0.111	0.126

Table 9

steady state values of first-order autocorrelations of returns generated by price and
s models respectively, under several alternative parameter values and holding periods.

PRICE FADS STEADY STATE $\rho(1)$			RETURNS FADS STEADY STATE $\rho(1)$		
$\gamma_p = 0.051$	$\gamma_p = 0.041$	$\gamma_p = 0.010$	$\gamma_r = 2.000$	$\gamma_r = 6.000$	$\gamma_r = 11.000$
-0.025	-0.020	-0.005	0.329	0.099	0.050
-0.049	-0.039	-0.010	0.160	0.045	0.024
-0.071	-0.058	-0.015	0.099	0.029	0.016
-0.093	-0.075	-0.020	0.071	0.022	0.012
-0.113	-0.092	-0.025	0.056	0.017	0.009
-0.132	-0.109	-0.029	0.045	0.014	0.008
-0.151	-0.124	-0.034	0.038	0.012	0.007
-0.168	-0.139	-0.039	0.033	0.011	0.006
-0.185	-0.154	-0.043	0.029	0.009	0.005
-0.201	-0.168	-0.048	0.026	0.008	0.005
-0.216	-0.181	-0.052	0.024	0.008	0.004
-0.230	-0.194	-0.057	0.022	0.007	0.004

Table 10a

Power of the specification test $\bar{M}_r(q)$ (using statistic z_1), the Dickey-Fuller t-test (D-F), and the Box-Pierce Q-test (BP-Q) against the price fads alternative with parameters $(\alpha_p, \gamma_p, \sigma_p)$ = (0.000, 0.041, 0.020) with h = 1 for various sample sizes and aggregation values. All simulations are based upon 20,000 replications using empirical critical values obtained from simulations under the homoscedastic null hypothesis.

Sample Size	q	Power - 1% Test		Power - 5% Test		Power - 10% Test	
		z_1	BP-Q	z_1	BP-Q	z_1	BP-Q
32	2	0.009	0.009	0.044	0.046	0.092	0.098
32	4	0.010	0.009	0.045	0.046	0.090	0.097
32	D-F	0.011		0.050		0.094	
64	2	0.010	0.010	0.049	0.050	0.100	0.102
64	4	0.010	0.009	0.049	0.051	0.097	0.100
64	8	0.008	0.008	0.046	0.050	0.097	0.105
64	D-F	0.008		0.041		0.086	
128	2	0.010	0.010	0.049	0.052	0.097	0.102
128	4	0.010	0.011	0.054	0.054	0.108	0.108
128	8	0.011	0.012	0.052	0.058	0.106	0.110
128	16	0.011	0.011	0.054	0.059	0.101	0.113
128	D-F	0.009		0.042		0.090	
256	2	0.012	0.013	0.056	0.060	0.108	0.114
256	4	0.018	0.012	0.062	0.061	0.119	0.121
256	8	0.020	0.014	0.083	0.067	0.151	0.124
256	16	0.029	0.013	0.105	0.062	0.187	0.123
256	32	0.032	0.013	0.128	0.061	0.222	0.121
256	D-F	0.030		0.123		0.215	
512	2	0.015	0.017	0.066	0.068	0.123	0.127
512	4	0.024	0.019	0.089	0.084	0.156	0.152
512	8	0.040	0.021	0.141	0.086	0.227	0.163
512	16	0.080	0.022	0.227	0.093	0.343	0.167
512	32	0.142	0.020	0.344	0.087	0.490	0.162
512	64	0.199	0.018	0.465	0.084	0.636	0.151
512	D-F	0.186		0.477		0.650	
1024	2	0.026	0.024	0.093	0.093	0.161	0.165
1024	4	0.056	0.033	0.166	0.118	0.258	0.207
1024	8	0.122	0.035	0.298	0.137	0.407	0.236
1024	16	0.269	0.035	0.489	0.148	0.627	0.252
1024	32	0.510	0.034	0.751	0.134	0.847	0.232
1024	64	0.764	0.026	0.925	0.107	0.968	0.196
1024	128	0.854	0.022	0.979	0.092	0.994	0.170
1024	D-F	0.913		0.993		0.998	

Table 10b

Power of the specification test $\tilde{M}_r(q)$ (using statistic z_1) and the Dickey-Fuller t-test (D-F) against the price fads alternative of Table 10a with h = 4 for various sample sizes and aggregation values. All simulations are based upon 20,000 replications using empirical critical values obtained from simulations under the homoscedastic null hypothesis.

Sample Size	q	Power (1%)	Power (5%)	Power (10%)
32	2	0.010	0.051	10.102
32	4	0.011	0.053	0.103
32	D-F	0.010	0.047	0.092
64	2	0.017	0.072	0.134
64	4	0.026	0.094	0.175
64	8	0.030	0.120	0.213
64	D-F	0.028	0.109	0.193
128	2	0.027	0.104	0.180
128	4	0.066	0.204	0.307
128	8	0.120	0.313	0.452
128	16	0.182	0.446	0.600
128	D-F	0.153	0.427	0.616
256	2	0.067	0.198	0.306
256	4	0.210	0.399	0.542
256	8	0.424	0.694	0.807
256	16	0.698	0.900	0.958
256	32	0.823	0.973	0.992
256	DF	0.877	0.988	0.998

Table 11a

Power of the specification test $\bar{M}_r(q)$ (using z_2) against the price fads alternative with parameters $(\alpha_p, \gamma_p, \sigma_p) = (0.000, 0.041, 0.020)$ with h = 1 for various sample sizes and aggregation values using asymptotic critical values. All simulations are based upon 20,000 replications.

Sample Size	q	Power (1%)	Power (5%)	Power (10%)
32	2	0.009	0.050	0.107
32	4	0.014	0.045	0.092
64	2	0.010	0.051	0.104
64	4	0.010	0.046	0.095
64	8	0.014	0.035	0.075
128	2	0.011	0.053	0.102
128	4	0.007	0.044	0.103
128	8	0.005	0.032	0.086
128	16	0.005	0.016	0.054
256	2	0.012	0.057	0.110
256	4	0.010	0.056	0.113
256	8	0.005	0.052	0.124
256	16	0.001	0.038	0.119
256	32	0.000	0.004	0.077
512	2	0.015	0.066	0.123
512	4	0.019	0.081	0.151
512	8	0.019	0.109	0.204
512	16	0.015	0.143	0.280
512	32	0.002	0.147	0.358
512	64	0.000	0.029	0.318
1024	2	0.023	0.090	0.158
1024	4	0.042	0.145	0.243
1024	8	0.074	0.243	0.368
1024	16	0.123	0.401	0.572
1024	32	0.175	0.611	0.791
1024	64	0.074	0.750	0.928
1024	128	0.000	0.428	0.943

Table 11b

Power of the specification test $\bar{M}_r(q)$ (using z_2) against the price fads alternative of Table 11a for various sample sizes and aggregation values using asymptotic critical values. All simulations are based upon 20,000 replications.

Sample Size	q	Power (1%)	Power (5%)	Power (10%)
32	2	0.010	0.063	0.126
32	4	0.018	0.058	0.115
64	2	0.010	0.056	0.112
64	4	0.012	0.051	0.105
64	8	0.015	0.041	0.087
128	2	0.010	0.055	0.109
128	4	0.007	0.047	0.107
128	8	0.006	0.036	0.092
128	16	0.005	0.018	0.064
256	2	0.012	0.057	0.112
256	4	0.010	0.058	0.117
256	8	0.005	0.056	0.127
256	16	0.001	0.044	0.128
256	32	0.000	0.008	0.092
512	2	0.014	0.067	0.124
512	4	0.019	0.082	0.150
512	8	0.020	0.111	0.206
512	16	0.016	0.148	0.286
512	32	0.003	0.159	0.372
512	64	0.000	0.045	0.353
1024	2	0.023	0.090	0.158
1024	4	0.041	0.145	0.243
1024	8	0.075	0.246	0.370
1024	16	0.125	0.404	0.575
1024	32	0.183	0.616	0.794
1024	64	0.089	0.764	0.932
1024	128	0.000	0.519	0.954

Table 12a

Power of the specification test $\overline{M}_r(q)$ (using statistic z_1), the Dickey-Fuller test (D-F), and the Box-Pierce Q-test (BP-Q) against the returns fads alternative with parameters $(\alpha_r, \gamma_r, \sigma_r)$ = (0.004, 6.0, 0.069) with h = 1 for various sample sizes and aggregation values. All simulations are based upon 20,000 replications using empirical critical values obtained from simulations under the homoscedastic null hypothesis.

Sample Size	q	Power - 1% Test		Power - 5% Test		Power - 10% Test	
		z_1	BP-Q	z_1	BP-Q	z_1	BP-Q
32	2	0.017	0.024	0.075	0.094	0.136	0.165
32	4	0.016	0.017	0.065	0.071	0.120	0.132
32	D-F	0.005		0.031		0.070	
64	2	0.033	0.040	0.113	0.133	0.194	0.226
64	4	0.023	0.025	0.083	0.097	0.157	0.171
64	8	0.016	0.020	0.068	0.080	0.127	0.149
64	D-F	0.009		0.045		0.093	
128	2	0.061	0.075	0.182	0.210	0.282	0.316
128	4	0.042	0.043	0.145	0.144	0.226	0.233
128	8	0.027	0.030	0.094	0.110	0.167	0.186
128	16	0.019	0.023	0.075	0.091	0.130	0.162
128	D-F	0.010		0.054		0.113	
256	2	0.155	0.174	0.346	0.374	0.475	0.497
256	4	0.112	0.095	0.246	0.244	0.368	0.362
256	8	0.052	0.056	0.159	0.180	0.254	0.281
256	16	0.029	0.038	0.101	0.129	0.175	0.218
256	32	0.016	0.031	0.073	0.107	0.132	0.192
256	D-F	0.016		0.070		0.127	
512	2	0.371	0.381	0.608	0.621	0.723	0.736
512	4	0.225	0.233	0.458	0.467	0.589	0.591
512	8	0.108	0.143	0.281	0.334	0.398	0.465
512	16	0.055	0.091	0.167	0.241	0.262	0.358
512	32	0.033	0.066	0.105	0.179	0.179	0.284
512	64	0.022	0.043	0.076	0.147	0.137	0.235
512	D-F	0.014		0.058		0.113	
1024	2	0.741	0.735	0.888	0.888	0.935	0.938
1024	4	0.546	0.559	0.776	0.774	0.853	0.859
1024	8	0.299	0.375	0.542	0.632	0.656	0.745
1024	16	0.143	0.243	0.311	0.482	0.433	0.612
1024	32	0.066	0.148	0.178	0.344	0.272	0.478
1024	64	0.037	0.085	0.105	0.238	0.177	0.361
1024	128	0.019	0.057	0.075	0.176	0.133	0.286
1024	D-F	0.015		0.067		0.122	

Table 12b

Power of the specification test $\bar{M}_r(q)$ (using statistic z_2) and the Dickey-Fuller t-test (D-F) against the returns fads alternative of Table 12a with h = 4 for various sample sizes and aggregation values. All simulations are based upon 20,000 replications using empirical critical values obtained from simulations under the homoscedastic null hypothesis.

Sample Size	q	Power (1%)	Power (5%)	Power (10%)
32	2	0.010	0.052	0.102
32	4	0.010	0.051	0.102
32	D-F	0.010	0.051	0.099
64	2	0.012	0.054	0.107
64	4	0.010	0.050	0.100
64	8	0.009	0.049	0.101
64	D-F	0.009	0.050	0.098
128	2	0.012	0.057	0.115
128	4	0.012	0.056	0.108
128	8	0.011	0.052	0.107
128	16	0.011	0.054	0.101
128	D-F	0.009	0.047	0.101
256	2	0.019	0.078	0.139
256	4	0.019	0.063	0.123
256	8	0.013	0.058	0.114
256	16	0.012	0.054	0.105
256	32	0.011	0.050	0.102
256	D-F	0.011	0.055	0.108

Table 13a

Power of the specification test $\bar{M}_r(q)$ (using z_1) against the returns fads alternative with parameters $(\alpha_r, \gamma_r, \sigma_r) = (0.004, 6.0, 0.069)$ with $h = 1$ for various sample sizes and aggregation values using asymptotic critical values. All simulations are based upon 20,000 replications.

Sample Size	q	Power (1%)	Power (5%)	Power (10%)
32	2	0.017	0.081	0.150
32	4	0.006	0.032	0.093
64	2	0.031	0.116	0.198
64	4	0.005	0.056	0.128
64	8	0.006	0.023	0.073
128	2	0.068	0.196	0.293
128	4	0.017	0.107	0.203
128	8	0.002	0.046	0.123
128	16	0.006	0.020	0.067
256	2	0.158	0.350	0.475
256	4	0.063	0.224	0.347
256	8	0.012	0.104	0.211
256	16	0.002	0.040	0.119
256	32	0.006	0.019	0.064
512	2	0.367	0.606	0.724
512	4	0.187	0.434	0.575
512	8	0.058	0.231	0.366
512	16	0.010	0.103	0.211
512	32	0.002	0.040	0.120
512	64	0.006	0.016	0.061
1024	2	0.725	0.885	0.933
1024	4	0.492	0.749	0.842
1024	8	0.207	0.475	0.620
1024	16	0.058	0.241	0.383
1024.	32	0.011	0.104	0.215
1024	64	0.002	0.040	0.116
1024	128	0.005	0.015	0.060

Table 13b

Power of the specification test $\bar{M}_r(q)$ (using z_2) against the returns fads alternative of Table 13a for various sample sizes and aggregation values using asymptotic critical values. All simulations are based upon 20,000 replications.

Sample Size	q	Power (1%)	Power (5%)	Power (10%)
32	2	0.017	0.090	0.169
32	4	0.009	0.045	0.114
64	2	0.030	0.118	0.205
64	4	0.006	0.060	0.139
64	8	0.008	0.029	0.088
128	2	0.065	0.196	0.297
128	4	0.017	0.108	0.206
128	8	0.003	0.050	0.130
128	16	0.007	0.023	0.078
256	2	0.151	0.349	0.474
256	4	0.060	0.222	0.348
256	8	0.013	0.106	0.214
256	16	0.002	0.044	0.125
256	32	0.006	0.021	0.073
512	2	0.357	0.606	0.722
512	4	0.182	0.429	0.573
512	8	0.057	0.232	0.367
512	16	0.011	0.106	0.213
512	32	0.002	0.043	0.125
512	64	0.006	0.018	0.069
1024	2	0.721	0.884	0.933
1024	4	0.488	0.748	0.841
1024	8	0.205	0.475	0.618
1024	16	0.059	0.242	0.384
1024	32	0.012	0.105	0.218
1024	64	0.002	0.042	0.121
1024	128	0.005	0.017	0.068

TABLE 14a

Variance ratio test $\bar{M}_r(q)$ of the random walk hypothesis for CRSP equal and value weighted indexes using a one-week base observation interval (h = 1 week) for the sample period September 6, 1962 to December 26, 1985 and sub-periods. The actual variance ratios are reported in the main rows, with the z and z* statistics given in parentheses in rows immediately below each main row.

*Indicates significance at the 5 percent level

Time period	Number nq of base observations	Number q of base observations aggregated to form variance ratio: 2	4	8	16	32	64
A. Equal-Weighted CRSP NYSE-AMEX Index							
620906-851226	1216	1.30 (10.29)* (7.51)*	1.64 (11.94)* (8.87)*	1.94 (11.08)* (8.48)*	2.05 (8.30)* (6.59)*	2.22 (6.66)* (5.52)*	2.23 (4.71)* (4.05)*
620906-740501	608	1.31 (7.53)* (5.38)*	1.62 (8.23)* (6.03)*	1.92 (7.64)* (5.76)*	2.09 (6.10)* (4.77)*	2.37 (5.28)* (4.32)*	
740502-851226	608	1.28 (7.02)* (5.32)*	1.65 (8.51)* (6.52)*	1.93 (7.75)* (6.13)*	1.91 (5.07)* (4.17)*	1.74 (2.84)* (2.45)*	
620906-680703	304	1.32 (5.66)* (4.12)*	1.68 (6.29)* (4.77)*	1.92 (5.44)* (4.23)*	2.07 (4.26)* (3.45)*		
680704-740501	304	1.29 (4.99)* (4.03)*	1.58 (5.36)* (4.44)*	1.83 (4.90)* (4.18)*	1.87 (3.46)* (3.04)*		
740502-791219	304	1.29 (5.12)* (3.80)*	1.71 (6.58)* (5.02)*	2.01 (5.93)* (4.66)*	1.91 (3.60)* (2.93)*		
791220-851226	304	1.26 (4.61)* (3.99)*	1.49 (4.55)* (3.83)*	1.66 (3.91)* (3.46)*	2.00 (3.94)* (3.63)*		
B. Value-Weighted CRSP NYSE-AMEX Index							
620906-851226	1216	1.08 (2.96)* (2.33)*	1.16 (2.94)* (2.31)*	1.22 (2.59)* (2.07)*	1.22 (1.71) (1.38)	1.35 (1.94) (1.60)	1.31 (1.17) (1.00)
620906-740501	608	1.15 (3.66)* (2.89)*	1.22 (2.87)* (2.28)*	1.27 (2.22)* (1.79)	1.32 (1.78) (1.46)	1.42 (1.61) (1.37)	
740502-851226	608	1.05 (1.13) (0.92)	1.12 (1.57) (1.28)	1.18 (1.50) (1.24)	1.10 (0.56) (0.46)	1.01 (0.06) (0.05)	
620906-680703	304	1.20 (3.55)* (2.87)*	1.29 (2.71)* (2.19)*	1.32 (1.90) (1.55)	1.29 (1.15) (0.96)		
680704-740501	304	1.12 (2.12)* (1.86)	1.18 (1.69) (1.49)	1.22 (1.32) (1.18)	1.30 (1.18) (1.08)		
740502-791219	304	1.00 (-0.01) (-0.01)	1.11 (1.07) (0.87)	1.21 (1.21) (0.99)	1.14 (0.57) (0.47)		
791220-851226	304	[illegible]	[illegible]	[illegible]			

TABLE 14b

Variance ratio test $\overline{M}_r(q)$ of the random walk hypothesis for CRSP equal and value weighted indexes using a four-week base observation interval (h = 4 weeks) for the sample period September 6, 1962 to December 26, 1985 and sub-periods. The actual variance ratios are reported in the main rows, with the z and z* statistics given in parentheses in rows immediately below each main row.

*Indicates significance at the 5 percent level.

Time period	Number nq of base observations	Number q of base observations aggregated to form variance ratio			
		2	4	8	16
A. Equal-Weighted CRSP NYSE-AMEX Index					
620906-851226	304	1.15	1.19	1.30	1.30
		(2.63)*	(1.80)	(1.74)	(1.20)
		(2.26)*	(1.54)	(1.52)	(1.07)
620906-740501	152	1.13	1.23	1.40	
		(1.64)	(1.54)	(1.67)	
		(1.39)	(1.32)	(1.46)	
740502-851226	152	1.15	1.11	1.02	
		(1.86)	(0.73)	(0.10)	
		(1.68)	(0.64)	(0.09)	
620906-680703	76	1.11	1.20		
		(0.92)	(0.95)		
		(0.80)	(0.87)		
680704-740501	76	1.12	1.15		
		(1.01)	(0.71)		
		(0.90)	(0.64)		
740502-791219	76	1.16	1.07		
		(1.43)	(0.30)		
		(1.23)	(0.27)		
791220-851226	76	1.02	1.21		
		(0.21)	(1.00)		
		(0.29)	(1.10)		
B. Value-Weighted CRSP NYSE-AMEX Index					
620906-851226	304	1.05	1.00	1.11	1.07
		(0.79)	(0.00)	(0.64)	(0.28)
		(0.75)	(0.00)	(0.57)	(0.26)
620906-740501	152	1.02	1.04	1.12	
		(0.27)	(0.29)	(0.50)	
		(0.26)	(0.26)	(0.46)	
740502-851226	152	1.05	0.95	0.89	
		(0.64)	(-0.34)	(-0.46)	
		(0.63)	(-0.31)	(-0.42)	
620906-680703	76	1.00	1.02		
		(0.02)	(0.08)		
		(0.02)	(0.08)		
680704-740501	76	1.02	1.05		
		(0.18)	(0.22)		
		(0.18)	(0.21)		
740502-791219	76	1.12	0.98		
		(1.07)	(-0.11)		
		(1.01)	(-0.10)		
791220-851226	76	0.90	0.95		
		(-0.89)	(-0.24)		
		(-0.95)	(-0.23)		