PONZI GAMES AND RICARDIAN EQUIVALENCE

by

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Abstract

Under certain conditions, a government can run a "rational Ponzi game," i.e., issue debt and never repay any interest or principal. Does Ricardian equivalence hold with respect to a tax cut that is financed by such a scheme? We study this question using overlapping generations models in which generations are linked by gifts or bequests. We find that although there are multiple equilibria, Ricardian equivalence can hold, and often seems to be the most natural outcome.

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1. Introduction

The literature on Ricardian Equivalence (REQ) tells us that under certain assumptions, debt-financed decreases in taxes will have no effect on the real economy. The argument relies on an intertemporal government budget constraint stating that decreases in current taxes (holding spending constant) must be matched by future tax increases of equal present value. If the private sector internalizes the future tax liabilities implied by current debt, a tax cut today will not increase the wealth of those receiving the tax cut, and there will be no effect on aggregate demand.

In a recent paper (O'Connell and Zeldes (1986)), we used the term "rational Ponzi game" to describe a situation in which an agent (e.g., a government) borrows a dollar and finances all payments of interest and principal by issuing new debt. We showed that arrangements of this sort require an infinity of agents, and that when they are feasible they play a role very similar to that of fiat money or asset price bubbles. In the present paper, we ask the following question: does the choice between using borrowing or taxes to finance government expenditures have real effects if the debt constitutes a rational Ponzi game? In other words, does Ricardian Equivalence fail if the government gives a tax cut today and never increases future taxes?

The intuitive answer to this question would seem to be yes, on two grounds. First, in the most familiar economies in which rational Ponzi games are feasible (e.g., versions of Samuelson's (1958) overlapping generations model without altruism), REQ fails even with respect to debt that is repaid. To pursue the question of whether the repayment of debt matters for REQ, we therefore must find an economy in which (1) rational Ponzi games are feasible, and yet (2) REQ cannot be ruled out with respect to debt that is repaid.

Since the overlapping generations model with altruism (first studied by Barro (1974)) is the only example we know of, we use it for the analysis.

The second, perhaps more fundamental reason for supposing that a tax cut financed by a rational Ponzi game should affect aggregate demand, is that such a policy represents an increase in net wealth for the private sector as a whole. Assuming the private sector has a positive marginal propensity to consume out of net wealth, this increase in wealth should raise aggregate demand.

The notion that REQ should fail because the increased bonds are an increase in net wealth has been disputed by Carmichael (1982) in an overlapping generations model with altruism running from children to parents (the gift economy). Carmichael's argument is that net wealth does not matter--REQ should hold as long as the government policy does not change the opportunity set of the current generation. We find that this is correct in the limited sense that REQ may hold. The possibility of REQ is a case in point of the general result of Bernheim and Bagwell (1985) stating that when market participants (current and future) are linked by active resource transfers, the set of equilibria is not affected by government tax or price policy.[1] There are generally multiple equilibria, however, and there typically exist sets of equilibria across which tax cuts financed by rational Ponzi games increase current consumption.

In the bequest economy (altruism running from parents to children), Douglas Gale (1983) has pointed out that although current generations take the welfare of all future generations into account, so that the infinite sequence

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[1]More precisely, they show that the set of equilibria is unaffected by small policy changes if the "corner constraints" on transfers (e.g., the constraint that bequests be nonnegative) are not binding in the initial equilibrium.

of generations ends up behaving like a single infinitely lived dynasty, there is nothing to prevent the dynasty from being on the lending side of a rational Ponzi game. Individual saving decisions can respond not only to the desire for current and future consumption by the dynasty, but also to a desired limiting value of the dynasty's discounted bequest. In this case, we show that REQ may hold or fail depending on the form of the dynasty's preferences.

The paper is organized as follows. In section 2, we introduce a simple Nash equilibrium concept for the gift and bequest economies and characterize the set of equilibria with stationary consumption patterns. Many of these equilibria, as we show in section 3, are consumption inefficient, implying that the government can extract resources via a rational Ponzi game by pushing out inefficient storage. We also discuss the interpretation of the bequest equilibria as solutions to an infinite horizon programming problem. In section 4, we raise the issue of REQ. This involves examining the possibilities when the government transfers the resources extracted by a rational Ponzi game to individuals currently alive. We prove two propositions that characterize the conditions for REQ in the gift and bequest economies. When REQ holds, it does so because the debt accompanying the tax cut pushes out gifts or adds to bequests. In both cases, this raises the interesting possibility that Ponzi games that leave interest rates unchanged may be feasible even in consumption efficient equilibria. In these equilibria, while the government is unable to extract resources from the economy via a rational Ponzi game, it can nonetheless give a tax cut to the current generation and never raise taxes in the future. We end section 4 with a discussion of the problem of selecting among equilibria in the gift economy. Section 5 concludes the paper.

2. The OG model with altruism

Individuals live for two periods. The population grows at rate n, with subscript "t" denoting the generation born in period t. We normalize the size of generation 0 to equal 1. In period 1, there are two generations alive: the initial old (generation 0) and the initial young (generation 1). Interest is accumulated on end of period assets (s_t) between periods, with r_{t+1} being the rate between t and t+1.

Gifts are given by the current young and received in the same period by the current old; bequests are left by parents in the second period of life and received by their children in the same period.[2] Throughout the paper we consider two separate economies, one in which there is a gift motive and one in which there is a bequest motive. We do not consider economies in which gift and bequest motives exist simultaneously. We denote the per-capita gift or bequest received by generation t by g_{t+1} and b_{t-1}, respectively; the time subscript refers to the generation giving the gift or bequest. Figure 1 shows the timing of these intergenerational transfers.

Our basic model is a perfect foresight, constant returns to scale storage economy in which individuals are born with no assets but receive certain endowments (e_1, e_2) in the two periods of life. Many of the results have direct analogs in the Diamond (1965) neoclassical production economy. We will restrict the analysis to the set of equilibria in which consumption patterns of successive generations are identical. This simplifies matters considerably without sacrificing any of the points we wish to make. We call such

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[2]The timing of bequest income differs from Barro (1974), where bequests (plus interest) are received in the second period of life. Having agents receive bequests in the first period is formally identical (under perfect foresight) to having them receive bequests in the second period but allowing them to borrow against anticipated bequests. The timing may matter if this

equilibria "quasi-steady states" because although the interest rate and consumption patterns are stationary, the gift or bequest per capita may be changing.

2.1 Equilibrium

In the gift economy, the generation born at t splits its endowment when young among consumption, saving (= storage), and a per-capita gift of g_t to parents.[3] Consumption when old is the sum of endowment, assets (storage plus interest) and the gift received from children. In the simplest version of the gifts economy, the maximized utility of the generation born at t (V_t) depends on its own consumption and the discounted maximized utility of its parents in the following way:[4]

$$(1) \qquad V_t(g_{t+1}) = \max_{\{c_{1t}, g_t\}} u(c_{1t}, c_{2t}) + \theta V_{t-1}(g_t)$$

$$\text{s.t.} \quad \text{(i)} \quad c_{1t} + s_t \le e_1 - (g_t/(1+n))$$

$$\text{(ii)} \quad c_{2t} \le e_2 + s_t(1+r_{t+1}) + g_{t+1}$$

$$\text{(iii)} \quad s_t \ge 0, \; g_t \ge 0 \; .$$

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[3]We examine the behavior of the generation as a whole, and thereby do not deal with the strategic issues that arise when 1+n children are separately giving gifts to the same parent. For a further discussion of this, see Abel (1985).

[4]We omit government taxes and transfers from the budget constraints for simplicity. The budget constraints with taxes and transfers included appear in the Appendix.

The per-capita utility of parents is discounted by a factor $\theta \equiv 1/(1+\delta)$; we follow Carmichael (1982) and others in assuming $\delta > 0$.[5] The time preference of the household as regards its own consumption is subsumed in the utility function $u(\cdot,\cdot)$.

In the bequests case, it is the utility of children rather than parents that is discounted by the current generation:

$$(2) \qquad V_t(b_{t-1}) = \max_{\{c_{1t}, b_t\}} u(c_{1t}, c_{2t}) + \theta V_{t+1}(b_t)$$

s.t. (i) $c_{1t} + s_t \leq e_1 + b_{t-1}$

(ii) $c_{2t} \leq e_2 + s_t(1+r_{t+1}) - b_t(1+n)$

(iii) $s_t \geq 0,\ b_t \geq 0.$

Unless the economy has a finite terminal date, optimal behavior of the current generation is not uniquely determined by either (1) or (2). The reason is simple: in order to determine its own behavior, each generation must make a conjecture about the behavior of the next generation. In the gift case, the young must know what gift g_{t+1} to expect from their offspring; in the bequest case, the old must know what value $V_{t+1}(b_t)$ their offspring attach to a bequest of size b_t. If the economy is open-ended, there is no terminal condition to tie down the sequence of conjectures, and the behavior of the current generation is indeterminate (see Gale (1983)). This is true even though the recursions (1) and (2) imply a sequence of first-order conditions, so that one can derive a set of necessary conditions for a quasi-steady

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[5]There has been some controversy over the appropriate specification of the discount factor. See Carmichael (1982), Burbridge (1983,1984), Buiter and

state. Without a terminal condition, these conditions do not suffice to tie down the solution. Some restriction must therefore be imposed in order to give each generation a well-posed optimization problem.

In order to explore the range of equilibria consistent with the basic recursions (1) and (2), we will use an equilibrium concept that yields the same sequence of first-order conditions as (1) and (2) but in which the optimal behavior of each successive generation is well defined. We define an equilibrium as a Nash equilibrium in which each generation optimizes taking the transfer of its parents and children as given, and in equilibrium all conjectured gifts or bequests are confirmed by experience.[6] There will generally be many such equilibria. More formally:

Definition 1: A **gift equilibrium** is a sequence $\{(r_2,g_1,c_{2,0},c_{1,1}), (r_3,g_2,c_{2,1},c_{1,2}), \ldots, (r_{t+1},g_t,c_{2,t-1},c_{1t}), \ldots\}$ of interest rates, gifts, and consumption levels such that

(a) $g_t > 0$ for all $t \geq 1$, i.e., all gifts are strictly positive,

(b) the gift and consumption pattern of generation t, given (i) the interest rate sequence $\{r_t\}$, (ii) the gift given by and first period consumption of parents (g_{t-1} and $c_{1,t-1}$), and (iii) the gift to be received from children g_{t+1}, solves the following problem for t=2,3,...:

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[6]Carmichael (1982) refers to a "modified Nash equilibrium" concept developed in his earlier unpublished work. He does not, however, discuss the multiple equilibrium issues raised in this section. This omission is important, since (as we show in this section) the Ricardian equivalence proposition in his 1982 article need not hold under our equilibrium concept

$$(3) \qquad \max_{\{c_{1t}, g_t\}} [u(c_{1t}, c_{2t}) + \theta u(c_{1,t-1}, c_{2,t-1})]$$

s.t.

(i) $c_{1t} + s_t \le e_1 - (g_t/(1+n))$

(ii) $c_{2t} \le e_2 + s_t(1+r_{t+1}) + g_{t+1}$

(iii) $s_t \ge 0, \quad g_t \ge 0$

(iv) $c_{1,t-1} + s_{t-1} \le e_1 - (g_{t-1}/(1+n))$

(v) $c_{2,t-1} \le e_2 + s_{t-1}(1+r_t) + g_t$

(c) given g_2, the gift g_1 given by the initial young solves (3) with s_0 given.[7]

Definition 2: A **bequest equilibrium** is a sequence $\{(r_2, b_0, c_{2,0}, c_{1,1}), (r_3, b_1, c_{2,1}, c_{1,2}), \ldots, (r_{t+1}, b_{t-1}, c_{2,t-1}, c_{1t}), \ldots\}$ of interest rates, bequests, and consumption levels such that

(a) $b_t > 0$ for all $t \ge 0$, i.e., all bequests are strictly positive,

(b) the bequest and consumption pattern of generation t, given (i) the interest rate sequence $\{r_t\}$, (ii) the bequest received from parents, b_{t-1}, and (iii) the bequest to be left by children, b_{t+1}, and their second period consumption $c_{2,t+1}$, solves the following problem for t=1,2,...:

[7] If the utility function $u(\cdot,\cdot)$ is not additively separable, then the first period consumption of the initial old, $c_{1,0}$, is an additional initial condition for a gift or bequest equilibrium.

$$(4) \qquad \max_{\{c_{1t}, b_t\}} [u(c_{1t}, c_{2t}) + \theta u(c_{1,t+1}, c_{2,t+1})]$$

$$\text{s.t.} \quad \text{(i)} \quad c_{1t} + s_t \leq e_1 + b_{t-1}$$

$$\text{(ii)} \quad c_{2t} \leq e_2 + s_t(1+r_{t+1}) - b_t(1+n)$$

$$\text{(iii)} \quad s_t \geq 0, \quad b_t \geq 0$$

$$\text{(iv)} \quad c_{1,t+1} + s_{t+1} \leq e_1 + b_t$$

$$\text{(v)} \quad c_{2,t+1} \leq e_2 + s_{t+1}(1+r_{t+2}) - b_{t+1}(1+n)$$

(c) given b_1, the bequest b_0 given by the initial old solves (4) with s_0 given.

Gift and bequest equilibria are defined by considering the behavior of any generation t. The household takes as predetermined the gift or bequest of adjacent generations and the saving of the generation that will receive its transfer. This truncates the optimization problem facing each generation after a single period: in the bequest case, the household effectively views the utility of its grandchildren as independent of its own action, and in the gift case, the household views the gift to be given by its grandchildren as independent of its own action. Each generation's behavior is therefore well defined, and an equilibrium holds when all conjectures are consistent with one another and with the initial condition s_0. It is easy to verify that the first-order conditions characterizing a gift or bequest equilibrium are identical to those characterizing the recursions (1) and (2).

2.2 Quasi-steady states

(5) $$u_{1,t} - (1+r_{t+1})u_{2,t} \geq 0, \text{ with equality if } s_t > 0,$$

(6) $$u_{1,t} - (1+n)\theta u_{2,t-1} \geq 0, \text{ with equality if } g_t > 0,$$

where r_{t+1} is the return on storage and $u_{i,t}$ is the marginal utility of period i (i = 1,2) consumption for generation t. These conditions have familiar interpretations. Equation (5) characterizes the consumption-saving margin. Equation (6) characterizes the margin between saving and increasing the gift: the individual will equalize the marginal return to saving and gift giving unless this would require a negative gift. If a negative gift is implied, no gift is given and the inequality is strict; otherwise (6) holds with equality.

In the bequest case, the first-order conditions (with similar interpretations) take the form

(7) $$u_{1,t} - (1+r_{t+1})u_{2,t} \geq 0, \text{ with equality if } s_t > 0,$$

(8) $$u_{2,t} - [\theta/(1+n)]u_{1,t+1} \geq 0, \text{ with equality if } b_t > 0.$$

Since we are focusing on equilibria with strictly positive gifts or bequests, conditions (6) and (8) must hold with equality.

A stationary consumption pattern requires that the marginal utilities of first and second period consumption be equalized across successive generations. If the condition that storage be nonnegative is not binding, so that (5) and (7) also hold with equality, a quasi-steady state requires that the return on storage satisfy

(9) $$(1 + r^G) = (1 + n)/(1 + \delta)$$

in the gift economy and

(10) $$(1 + r^B) = (1 + n)\ (1 + \delta)$$

in the bequest economy, as pointed out by Carmichael (1982). We impose these restrictions for the remainder of the paper.

Note that when the "nonnegative storage" condition is not binding, each generation faces a consolidated budget constraint of the form

$$c_{1t} + c_{2t}/(1+r^G) = e_1 + e_2/(1+r^G) + g_{t+1}/(1+r^G) - g_t/(1+n)$$

(11) $\qquad t = 1,2,\ldots$

$$c_{1t} + c_{2t}/(1+r^B) = e_1 + e_2/(1+r^B) + b_{t-1} - b_t[(1+n)/(1+r^B)] \ .$$

It will be useful to define e^i and c_t^i as the present value of lifetime endowment and consumption in the gift (i = G) or bequest (i = B) economy, at the appropriate rate of return on storage:

(12) $$c_t^i \equiv c_{1t} + c_{2t}/(1+r^i), \ e^i \equiv e_1 + e_2/(1+r^i) \qquad i = G,B.$$

We assume that the utility functions are additively separable. This implies that the first period consumption of the initial old is irrelevant; the only predetermined variable that enters the equations is s_0, the saving of the initial old. This simplifies the analysis without changing anything essential.

Characterizing the set of quasi-steady state equilibria is slightly more straightforward in the gift case, so we start there. Consider the behavior of the first k decision making generations $(1,\ldots,k)$, for an arbitrary k. What restrictions does our definition of equilibrium impose on the set of consumption levels $\{c_{2,0}, (c_{1,1},c_{2,1}), \ldots, (c_{1,k},c_{2,k})\}$, savings decisions $\{s_1,\ldots,s_k\}$, and transfers $\{g_1,\ldots,g_{k+1}\}$, relevant to these generations? First, the process of consolidating the budget constraints to arrive at (11) eliminates the saving levels s_t for generations one through k, and leaves us with k equations in the remaining 3k+2 unknowns. The first order conditions (5) and (6) for each generation then provide an additional 2k equations (assuming that the nonnegativity conditions on saving and gifts are not binding). The budget constraint of the initial old supplies a final equation in $c_{2,0}$, the initial transfer g_1, and the exogenous initial value for s_0.

For any $k \geq 1$, then, and given the initial condition s_0, our definition of equilibrium gives us 3k+1 equations in 3k+2 unknowns. With the additional restriction that $g_t \leq e_1$ (since the young have no initial assets), any sequence of non-negative gifts satisfying these restrictions for all $k > 0$ is a quasi-steady state gift equilibrium.

To analyze the bequest case, again consider the behavior of the first k decision making generations (0,...,k-1). What restrictions does equilibrium imply on the 3k+2 unknowns $\{c_{2,0}, (c_{1,1},c_{2,1}),\ldots, (c_{1,k},c_{2,k})\}, \{b_0,\ldots,b_k\}$? As in the gift case, we have the consolidated budget constraints (t=1,...,k) and the budget constraint of the initial old. We also have the intergenerational first-order condition (8) for generations 0,...,k-1, and the intertemporal first-order condition (7) for generations 1,...,k. These 3k+1 equations again exhaust the restrictions imposed by equilibrium on the 3k+2 unknowns. Any sequence of non-negative bequests satisfying these restrictions for all $k > 0$ is a quasi-steady state bequest equilibrium.

Our equation counting exercise indicates that equilibrium is underdetermined in both the gift and bequest economies: there are multiple candidates for quasi-steady state gift and bequest equilibria given the initial condition s_0. We show in the next section that in each case a continuum of these candidates satisfy the requirement of nonnegative transfers. This multiplicity of equilibria corresponds to the indeterminacy of optimal behavior of the initial generation in the basic recursions (1) and (2). We also show that "most" of the gift and bequest equilibria are dynamically inefficient, and that rational Ponzi games are feasible in these equilibria (see also O'Connell and Zeldes (1986)).

3. Rational Ponzi games with gifts and bequests

In this section, we characterize the sets of quasi-steady state gift and bequest equilibria. This leads directly to an analysis of the feasibility of rational Ponzi games that extract resources from either economy. This analysis is useful for two reasons. First, it applies the general principles developed in O'Connell and Zeldes (1986) to altruistic economies. Second, it puts us in a position to answer (in section 4) the REQ question: what happens if these resources are transferred to individuals currently alive in the form of a tax cut? We begin by looking at the asymptotic behavior of gifts and bequests.

Combining (11) and (12) and using (9) or (10), we have the following recursions relating successive equilibrium anticipated transfers and the discounted value of the stationary consumption pattern (c_G or c_B):

$$(13) \qquad \begin{aligned} g_{t+1} &= \theta g_t + \theta(1+n)(c^G - e^G) \\ b_t &= \theta^{-1} b_{t-1} - \theta^{-1}(c^B - e^B) \end{aligned} \qquad t = 1,2,3,\ldots$$

Using this to solve for the time paths of gifts and bequests in a quasi-steady state, we get

$$(14) \qquad g_t = \theta^{t-1} g_1 + (1 + r^G)(c^G - e^G)\,[(1 - \theta^{t-1})/(1 - \theta)],$$

and

$$(15) \qquad b_t = \theta^{-t}[b_0 + (e^B - c^B)(1 - \theta^t)/(1 - \theta)],$$

Recall that $\theta \equiv (1+\delta)^{-1} < 1$. The limiting behavior of gifts and bequests is therefore given by

(16) $$\lim_{t\to\infty} g_t = (1+r_G)(c^G - e^G)/(1-\theta) ,$$

(17) $$\lim_{t\to\infty} [(1+n)/(1+r^B)]^{t+1} b_t = \theta[b_0 - (c^B-e^B)/(1-\theta)] .$$

Per-capita gifts, in other words, converge (monotonically) to a finite limit over time. Since the interest rate is smaller than the population growth rate, however, the aggregate gift, $G_t = (1+n)^{t-1} g_t$, grows faster than the interest rate asymptotically. Per-capita bequests, on the other hand, <u>diverge</u> over time as long as $b_0 > (c^B-e^B)/(1-\theta)$; but the discounted value of the aggregate bequest converges to a finite limit. The aggregate bequest $B_t = (1+n)^{t+1} b_t$ therefore grows asymptotically at the rate of interest. In the case where $b_0 = (c^B-e^B)/(1-\theta)$, the per capita bequest is constant and the discounted aggregate bequest goes to zero. In both the gift and bequest case, given the saving s_0 of the initial old, equilibria can be indexed either by the discounted value of stationary consumption (c^G or c^B) or by the initial gift or bequest (g_1 or b_0).

Figures 2 and 3 illustrate quasi-steady state equilibria in the gift and bequest economies. For convenience, we assume that the utility function $u(\cdot,\cdot)$ is homothetic, so that the expansion path associated with the return on storage is a ray from the origin. Point A gives the steady state equilibrium in the economy without intergenerational altruism, asset bubbles or rational Ponzi games. We call this economy the "benchmark" economy. We have restricted preferences and endowments to be such that storage is positive at both r^G and r^B in the benchmark economy.

3.1. Gifts and Ponzi Games

If the return on storage is $r^G < n$, the benchmark (non-altruistic) equilibrium (point A in Figure 2) is dynamically inefficient. This equilibrium is Pareto dominated, for example, by the golden rule (point GR), which can be reached starting at A in a variety of ways: (i) by the introduction of a fixed amount of fiat money (Samuelson (1958)), (ii) by a variety of rational Ponzi games (O'Connell and Zeldes (1986)) or (iii) by a perfectly anticipated tax policy (e.g., unfunded social security) in which the government forces each young person to transfer an amount τ_{GR} to its parents. Each of these mechanisms generates a Pareto improvement by offering each generation the most favorable intertemporal terms of trade the economy can provide, i.e., (1+n).

A subset of the set of dynamically inefficient allocations in a dynamic economy is the set of consumption inefficient allocations. These are allocations from which Pareto improvements are possible that leave the consumption of all generations in all periods at least as high as in the initial equilibrium.[8] Pareto improvements of this type rely on the existence of inefficient physical storage, i.e., storage that grows asymptotically at least as fast as the interest rate. In these equilibria, the government can at some date extract real resources via a rational Ponzi game without altering either consumption levels or the subsequent return on saving in the economy. The government simply offers successive generations an alternative (paper)

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[8]In the absence of a storage technology, the benchmark equilibrium would be at the endowment point. This is an example of an equilibrium that is consumption efficient but (given our assumption that storage would be positive at r^G) dynamically inefficient (Samuelson (1958)). Pareto improvements are possible, but they involve a reduction in first period consumption for all generations.

vehicle for saving, pushing out some portion of inefficient storage in each period.

Now consider the case in which a gift motive is present. Beginning in a quasi-steady state gift equilibrium, can the government extract resources from the economy by running a rational Ponzi game? Gifts are essentially the third mechanism above, with the exception that (1) they are done voluntarily, and (2) in a quasi-steady state, the marginal return on saving must remain at r^G. The latter requirement means that government debt, if introduced, must pay return r^G.

Figure 2 shows that storage per capita is positive asymptotically in most of the gift equilibria. Quasi-steady state gift equilibria that can be reached starting at A lie on the segment AE. The benchmark equilibrium at A can be thought of as a borderline gift equilibrium with zero gifts throughout and per capita storage constant at s_A (although gifts are not strictly positive, the constraints $g_t \geq 0$ are not binding). To reach an intermediate point like D, the initial young begin with a gift of $g_1 > 0$, which is followed by a sequence of gifts rising asymptotically to g_D. Storage falls to s_D asymptotically; since s_D is strictly positive, the government can run a rational Ponzi game that pushes out storage and leaves everything else unchanged.

The equilibrium at E is the only consumption efficient quasi-steady state gift equilibrium that can be reached starting at A. At E, storage per capita is driven to zero asymptotically, so there is no room for a rational Ponzi game that extracts resources at interest rate r^G and leaves consumption unchanged. The equilibrium at E is still dynamically inefficient, however; a variety of mechanisms (including Ponzi games paying rate n) exist that could push the economy to the golden rule without lowering the value of $u(\cdot,\cdot)$ for any transitional generation (note that the golden rule is not a gift equilibrium, however, since with $u_{1,t}/u_{2,t} = (1+n)$, the constraint that gifts be nonnegative is binding (cf. equation (6)), and gifts are zero).

By transferring resources from the young to the old in an economy that would be consumption inefficient in the absence of a gift motive (or a bubble -- like fiat money -- to drive out the inefficient storage and possibly raise the real interest rate), gifts partially alleviate the consumption inefficiency of the original economy. In effect, although individuals continue to view the return on saving as r^G, each generation (asymptotically) can be thought of as holding two assets: (i) claims of s_D (on storage) bearing return r^G, and (ii) claims of $g_D/(1+n)$ (on the future gift received) bearing rate of return n. As indicated in equation (16), holdings of the latter asset make possible an increase of lifetime consumption above the benchmark value e^G.

3.2 Bequests and Ponzi games

Consider now the bequest economy. Since $r^B > n$, the benchmark (non-altruistic) equilibrium (point A in Figure 3) is dynamically efficient. Rational Ponzi games and asset bubbles are therefore ruled out in the absence of a bequest motive (O'Connell and Zeldes (1986), Tirole (1985)). What happens when the bequest motive is added?

Starting at the benchmark equilibrium (where $s_0 = s_A$), a central planner choosing bequests in order to attain the most favorable stationary consumption pattern would clearly choose $b_t = 0$ for all t. In fact, this borderline bequest equilibrium (which leaves the economy at A) is the stationary solution to the bequests recursion (15) for $s_0 = s_A$ and the only solution starting at A that is consumption efficient. Given the saving of the initial old, however, there exists a continuum of other and $s_A(1+r^B) + e_2$ (the available resources of the initial old). These equilibria imply stationary consumption patterns on the line segment OA and are therefore consumption inefficient.

Any positive initial bequest means a reduction of second period consumption relative to the benchmark case for the initial old. Since we are looking at quasi-steady states, this implies lower consumption in both periods for all subsequent generations. At an equilibrium like D, for example, the initial bequest of $b_0 > 0$ is followed by a sequence of growing bequests whose net effect is to lower the value of lifetime consumption to $c^B(D) < e$. In direct contrast to the gift case, a higher initial bequest implies lower quasi-steady state consumption. By transferring resources from the old to the young, bequests are serving the opposite role from an asset price bubble or rational Ponzi game.

It is instructive to look briefly at the case where the initial old have assets exceeding the level in the benchmark equilibrium (i.e., $s_0 > s_A$). In this case, the unique stationary solution to (15) has a strictly positive bequest and is again consumption efficient, implying a stationary consumption pattern at a point like F in Figure 3. Since the per capita bequest is constant, the limit in equation (17) is zero. Solving for c^B, we get

$$c^B = e^B + (r-n)\left[b_0/(1+r)\right] . \tag{18}$$

In the stationary bequest equilibrium, in other words, each generation consumes its endowment plus that portion of the interest on the initial bequest that is left over after accounting for population growth.

As in the case where $s_0 = s_A$, however, there are also consumption inefficient equilibria with diverging per capita bequests. If s_0 is such that the consumption efficient equilibrium is at F, then any equilibrium between O and F is inefficient and can be reached with a suitable initial bequest. In these equilibria, the limiting term in (17) is positive: both the initial bequest and some portion of feasible stationary lifetime consumption in each period is passed on in the form of an ever-increasing per capita bequest. By the budget constraint of the young, an aggregate bequest growing at the rate of interest means aggregate storage growing at the rate of interest (recall that first period consumption per capita is constant).

The consumption inefficiency of the equilibria with positive initial bequests is dramatic. Each generation voluntarily reduces lifetime consumption in order to leave a larger per capita bequest than it received from its parents--but these bequests are never converted into consumption! The initial old, in other words, do not foresee an increase in the consumption of future generations above the benchmark level. Rather, they leave a positive bequest ($b_0 > 0$) because they anticipate that their children will leave a larger positive bequest ($b_1 > b_0$). The behavior of the initial young (and all subsequent generations) is in turn rationalized by the same argument.

Rational Ponzi games are clearly feasible in the consumption inefficient bequest equilibria. Government debt that is perpetually rolled over can simply push out private storage and form part of the terminal bequest.

3.3 Bequests and infinite horizon behavior

Rational Ponzi games can exist in the bequest economy not by providing an alternative to outright intergenerational transfers, but by providing an alternative vehicle for saving in an economy that is accumulating a positive discounted aggregate bequest. This possibility is puzzling for two reasons. First, Barro has observed that when bequests are positive, the current generation can be viewed as maximizing "dynasty" utility over an infinite

(19)
$$\max_{\{b_0;\ (c_{1t},b_t):t=1,2,\ldots\}} \sum_{t=0}^{\infty} \theta^t u(c_{1t},c_{2t})$$

s.t.

$$s_0 = (c_{2,0} - e_2)/(1+r^B) + [(1+n)/(1+r^B)]b_0$$

$$\left(\frac{1+n}{1+r^B}\right)b_0 = \sum_{t=1}^{\infty} [(1+n)/(1+r^B)]^t(c_t - e) + \lim_{t\to\infty} [(1+n)/(1+r^B)]^{t+1}b_t$$

It is easy to verify that the behavior of the dynasty in (19) satisfies the bequest economy recursion (2). In effect, the behavior of an infinity of agents has been "consolidated" into that of a single infinitely lived agent. As argued more fully in O'Connell and Zeldes (1986), an agent with preferences given in (19) will necessarily choose a consumption efficient sequence $\{c_t\}_{t=0}^{\infty}$. Such a sequence will satisfy the transversality condition

(20)
$$\lim_{t\to\infty}[(1 + n)/(1 + r^B)]^{t+1}b_t \le 0 .$$

The dynasty will ensure, in other words, that the limiting term in the budget constraint--which is the discounted aggregate bequest--is nonpositive. This will rule out being on the lending side of a rational Ponzi game. Since the economy is composed of only a finite number of dynasties, each satisfying (20), it follows that the government cannot run a rational Ponzi game.

Second, although Weil (1987) examines a neoclassical production economy (cf. Diamond (1965)) rather than the storage economy examined here, our claim that rational Ponzi games can exist in a bequest equilibrium appears to violate the spirit of his result that bubbles and bequests cannot simultaneously exist. Weil argues that bequests can be positive only if the equilibrium in the corresponding non-altruistic economy is dynamically efficient. Since the latter condition rules out asset bubbles (Tirole

(1985)) -- and, via our earlier arguments, rational Ponzi games -- one gets the conclusion that bubbles and bequests can not simultaneously exist.

How, then, can we get bequest equilibria that are consumption inefficient and in which the dynasty will participate in a rational Ponzi game? The answer is that the preferences in (19) are not the only infinite horizon preferences consistent with the period-by-period altruism in (2) (Gale (1983)). In writing (19), Barro implicitly imposed the terminal condition

$$\lim_{t\to\infty} \theta^t V_t(b_{t-1}) = 0. \tag{21}$$

This condition selects the dynamically efficient central planner's equilibrium from among the set of possible equilibria. But consider the following example due to Gale (1983): for some value of $\mu \geq 0$, let the current generation (generation 0) solve the problem

$$\max_{\{b_0;(c_{1t},b_t):t=2,\ldots\}} \sum_{t=0}^{\infty} \theta^t u(c_{1t},c_{2t}) + \mu \lim_{t\to\infty} \theta^{t+1} b_t, \tag{22}$$

subject to the dynasty budget constraint in (19). To solve this by dynamic programming, we define V_t as the value of the program (22) starting at time t. Bellman's Principle of Optimality then states that V_t satisfies the recursion (2). In other words, solutions to (22) are valid bequest equilibria.

With the interest rate on storage given by the modified golden rule (10), it is easy to show that solutions to (22) imply stationary consumption, and that for μ sufficiently high, the dynasty will choose a consumption inefficient quasi-steady state equilibrium. The reason is simple: with $\mu > 0$ and $\theta = (1+n)/(1+r^B)$, the objective function in (22) places a positive value on the discounted value of the "terminal" aggregate bequest. With preferences of this form, the transversality condition

$$\lim_{t\to\infty} \theta^{t+1} b_t \leq 0 \tag{23}$$

is no longer a necessary condition for an optimal consumption plan. The dynasty's taste for the terminal bequest leads it to reduce consumption in all periods in order to accumulate assets to be passed on from generation to generation.

The failure to impose (23) also explains the possible coexistence of rational Ponzi games--or asset bubbles like valued fiat money--and bequests. These assets, which grow at the rate of interest, can simply form part of the terminal bequest. A key assumption in Weil's analysis is therefore that the dynasty chooses $\mu = 0$, leading it to impose the transversality condition (23).

In Gale's example, the value function is linear in the discounted terminal bequest. We generalize this to the case where the discounted terminal bequest enters via any increasing concave function w. The problem then becomes:

$$\max_{\{b_0;(c_{1t},b_t):t=1,2,\ldots\}} \sum_{t=0}^{\infty} \theta^t u(c_{1t},c_{2t}) + w(\lim_{t\to\infty} \theta^{t+1} b_t) , \tag{24}$$

and the analysis proceeds as before.

4. Ponzi Games and Ricardian Equivalence

We are now in a position to assess the case for REQ when tax cuts are financed by rational Ponzi games. What happens, in other words, when the government runs a rational Ponzi game and returns the extracted resources to the initial old in the form of a tax cut?[9] The following propositions treat the gift and bequest case in turn.

Proposition 1: In the storage economy with a gift motive, start in a quasi-steady state equilibrium with positive gifts (the "initial equilibrium"), and give a lump sum tax cut to the current old. Let the tax cut be financed by a rational Ponzi game, starting with borrowing from the current young. Then

(i) there exists a quasi-steady state gift equilibrium in which the higher level of bonds held by the young is exactly offset by lower levels of gifts, so that interest rates and the consumption of all generations (including the current old) are the same as in the initial equilibrium.

(ii) if the initial equilibrium has positive asymptotic storage per capita, there exists a continuum of other gift equilibria with the higher level of bonds that consumption dominate the initial equilibrium.

Proposition 2: In the storage economy with a bequest motive, start in a quasi-steady state in which households are choosing the sequence of value functions implied by (24) subject to the budget constraints in (19). Let the government give the current old a tax cut and finance the tax cut by borrowing from the current young.

(i) (Gale (1983)) If the sequence of value functions is affected by government financial policy, Ricardian equivalence (REQ) will in general fail. This is true whether the initial borrowing is to be repaid by future tax increases or rolled over forever.

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[9]We point out at the end of this section that there are circumstances under which such an experiment is feasible even though it would not be possible for the government to run a rational Ponzi game and simply keep the

(ii) Suppose that the value functions are <u>not</u> influenced by government financial policy. Then

(a) if the debt is to be repaid by future tax increases, REQ holds.

(b) if the debt is to be rolled over forever, and the value functions are linear in the discounted terminal bequest then REQ holds.

(c) if the debt is to be rolled over forever, the value functions are increasing and strictly concave in the discounted terminal bequest, and the initial equilibrium is consumption inefficient, then REQ fails.

Proofs: Appendix 1.

Consider Proposition 1. In the first set of equilibria, demand for government debt varies exactly with supply at given interest rates. This part of the proposition supports Carmichael (1982). Ponzi games work just like gifts; they serve to transfer resources from young to old. If the government gives a tax cut to the current old (by, for example, dropping bonds on them) and never raises future taxes, one possible resulting equilibrium is for the young to buy the bonds from the old and cut their gift by the exact same amount. The total transfer of funds from young to old is the same as before, but the purchase of bonds is higher and the gift is lower. In this case, Ponzi game crowds out an equal amount of gifts, and all consumptions remain unchanged. In other words, Ricardian equivalence holds with respect to a tax cut with no future increase in taxes.

Part (i) of the Proposition implies that any sequence of consumption patterns and interest rates that was an equilibrium before the tax cut and Ponzi game is an equilibrium afterwards. One can readily show the converse: any quasi-steady state equilibrium with the tax cut and Ponzi game can also be reached in the absence of both. In other words, the set of equilibria is unchanged by the government policy. An observation of this sort led Carmichael (1982) and Bernheim and Bagwell (1986) to a presumption in favor of REQ.

Given the multiplicity of equilibria, however, any presumption about the effects of government financial policy must rely on some implicit or explicit way of choosing among the alternative Nash equilibria. By construction, all of these equilibria are valid solutions to the gift recursion (1). Part (ii) of the Proposition simply points out that the REQ experiment can in principle lead to an increase in consumption for all generations as long as the original equilibrium was consumption inefficient. As we discuss further in Section 4.1, a mechanism that chooses among equilibria based on consumption levels or net real transfers will preserve REQ; the mechanism of Part (ii) of the proposition is implicitly one that focuses (to some degree) on the level of the outright gift, rather than the net real transfer from children to parents.

Next consider Proposition 2, which describes the bequest case. If there is no way of tying down the equilibrium sequence of value functions, then REQ (or any other proposition about comparative statics) need not hold, although again the set of quasi-steady state Nash equilibria will be unaffected by the running of a rational Ponzi game. However, the value function interpretation of the problem may prove useful in choosing among equilibria. Each value function corresponds to a different way of selecting among feasible equilibria. Part (ii) of the Proposition investigates the case in which the choice of value function is unaffected by the introduction of the rational Ponzi game. If the value function is linear in the terminal bequest, REQ holds independently of whether the debt is repaid or rolled over forever. In this case, it is optimal for the dynasty to pass the entire increased debt along as a higher bequest. Real variables are unchanged, but the utility of each generation rises as a result of the increased terminal bequest.

If the value function is concave in the terminal bequest, however, it matters for REQ whether or not the debt is a Ponzi game. REQ will hold for debt that is repaid, but not for debt that is rolled over forever. A tax cut accompanied by the initiation of a rational Ponzi game will lead to an increase in both the constant per capita level of consumption (recall that we began in a consumption inefficient quasi-steady state) and the terminal bequest. The increase in consumption will come from a fall in storage, so that the terminal bequest will rise by less than the increase in debt. REQ fails in this bequest case even though the set of Nash equilibria is unaffected by the government policy. This failure of the REQ presumption brings out the importance of Bernheim and Bagwell's assumption that individuals ultimately draw utility only from the consumption of other individuals. In our setup, individuals may also care about the level of the discounted terminal bequest (although we continue to assume that individuals do not care, per se, about the level of transfers from themselves to their children).

Before proceeding, it is worth pointing out that Propositions 1 and 2 lead to an interesting modification of our intuition regarding rational Ponzi games and related phenomena like asset bubbles or fiat money. They show that there are cases in which it is impossible to run a rational Ponzi game and extract resources from the economy (without changing the interest rate and/or consumption levels) but in which it is perfectly possible to run a rational Ponzi game if the extracted resources are returned in the form of a tax cut on the initial old. These cases are the consumption efficient quasi-steady state equilibria in the gift and bequest economies. In the gift case, bonds replace gifts one-for-one, leaving the consumption efficient consumption path unchanged; in the bequest case, the bonds are passed from generation to generation, adding one-for-one to the growing discounted aggregate bequest.[10]

4.1 Selecting among equilibria in the gift economy

In the bequest economy, the infinite horizon value functions provide a useful way of thinking about the REQ issue. To the extent that the central planner's problem is the "natural" choice for the value function of current and subsequent generations, rational Ponzi games are ruled out and REQ holds. In the gift economy, one can think of the indeterminacy in the problem as residing in the discounted terminal gift rather than the discounted terminal value function. Is there a counterpart in the gift economy to the restriction in the bequest economy that $\theta^t V_t(b_{t-1})$ go to zero as t goes to infinity? If so, does REQ hold if we impose this restriction?

The obvious candidate among possible quasi-steady state gift equilibria is the equilibrium with maximal quasi-steady state consumption. This equilibrium (which is the only consumption efficient quasi-steady state) is supported by the maximum initial gift, followed by a sequence of gifts that rises to the maximum steady state gift. Starting in this equilibrium, Ricardian equivalence holds with respect to a current tax cut only if the criterion used to select among the resulting equilibria is the level of consumption, rather than the asymptotic gift per capita. Equivalently, successive generations must focus on the equilibrium with the maximal limiting

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[10]The multiplicity of equilibria in these economies also raises the possibility that the introduction of a rational Ponzi game might shift the economy from one equilibrium to another. This means that rational Ponzi games that extract resources from an economy cannot be ruled out even starting in the consumption efficient equilibrium. Such a scheme would be feasible if the Ponzi game induced a shift from the consumption efficient equilibrium to one

net real transfer of resources per capita, where the net transfer is the sum of outright gifts and purchases of bonds from parents. Under this assumption, the increased bonds will simply crowd out outright gifts one for one, leaving consumption of all generations unchanged. This is true whether the rational Ponzi game is a one-shot deal, in which case even the rolled over debt disappears per capita asymptotically since $r < n$, or if we contemplate an increase in per capita debt (allowing the government to increase its steady state non-interest deficit), financed by a new rational Ponzi game each period. In the latter case, the government debt does not disappear (per capita) asymptotically and therefore crowds out a portion of the terminal per capita gift.

5. Conclusions

Our goal in this paper has been to examine the case for Ricardian Equivalence when tax cuts are financed by borrowing that is never repaid. In cases where REQ fails with respect to debt that is repaid, there will of course also be an effect on aggregate demand if the debt is floated with no implied increase in future taxes. But in the OG model with altruism, where (under certain conditions) REQ holds with respect to debt that is repaid, we find that REQ need not continue to hold if the debt is perpetually rolled over. There exist pairs of equilibria across which the government financing decision has real effects. REQ can hold, in the sense that there exist equilibria with different levels of taxes and identical values of consumption and interest rates; in fact the set of quasi-steady state Nash equilibria is unchanged by the introduction of a Ponzi game.

In both the gift and bequest economy, multiplicity of equilibria makes it difficult to say anything definitive about a comparative statics exercise like the substitution of debt for tax finance. In the gift case, we can make two observations. First, while there is no obvious infinite horizon analog to the period-by-period optimization done by households, any selection criterion that depends on net real transfers between generations (consumption efficiency is an example of such a criterion) will preserve REQ. Second, and more fundamentally, the REQ question itself does not depend (in the gift economy) on whether or not debt is perpetually rolled over. To the extent that REQ holds with respect to debt that is repaid, it also holds with respect to debt that constitutes a rational Ponzi game.

In the bequest case, while the set of Nash equilibria is also unchanged by a REQ exercise, the value function formulation provides a set of alternative ways of selecting among these equilibria. One of these is the central planner's objective function, which places a zero value on the terminal bequest and rules out rational Ponzi games. There are other alternatives, however, in which Ponzi games are feasible and in which REQ may or may not hold depending on the exact form of the preferences.

Figure 1

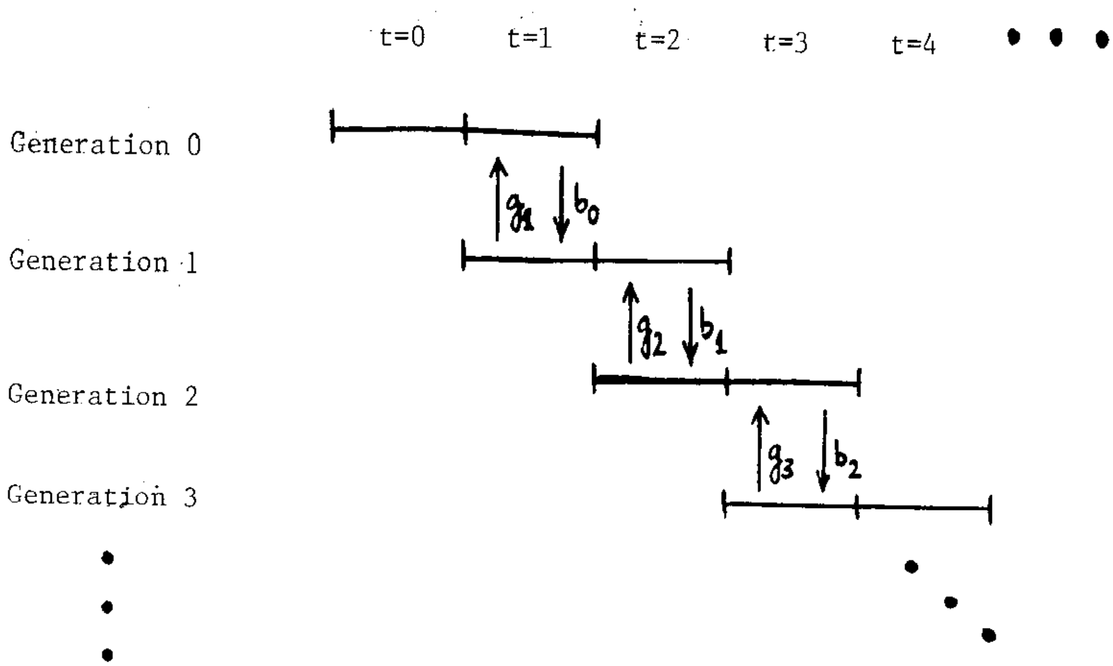


Figure 2

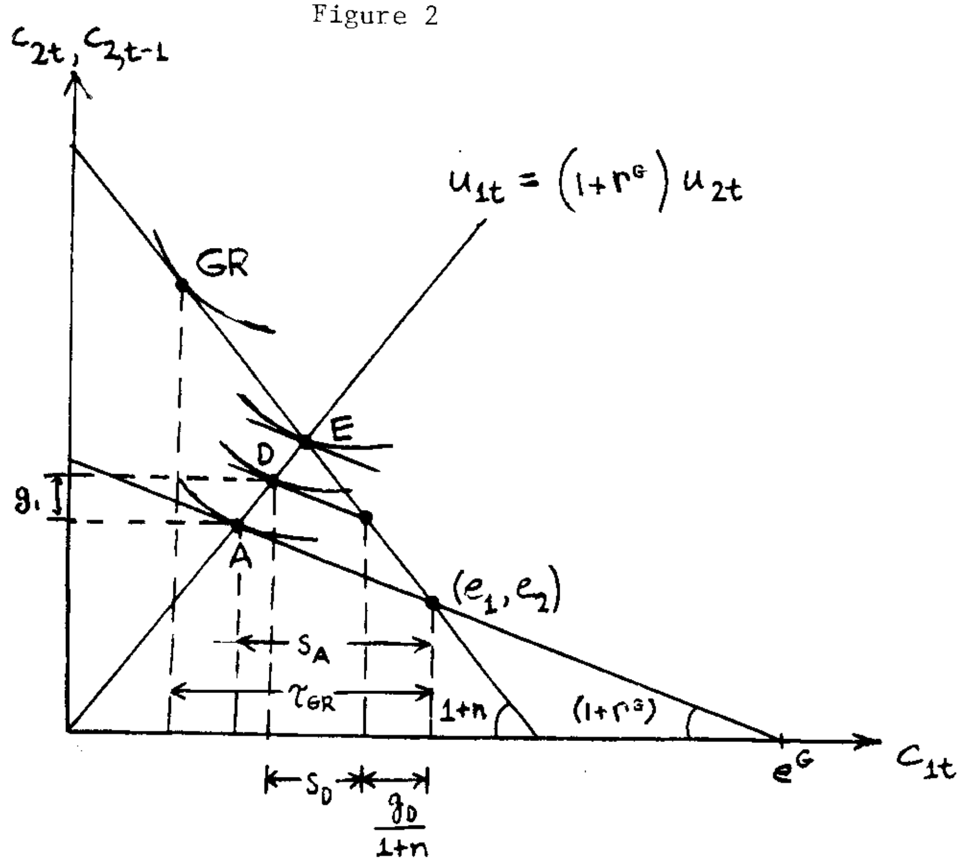


Figure 3

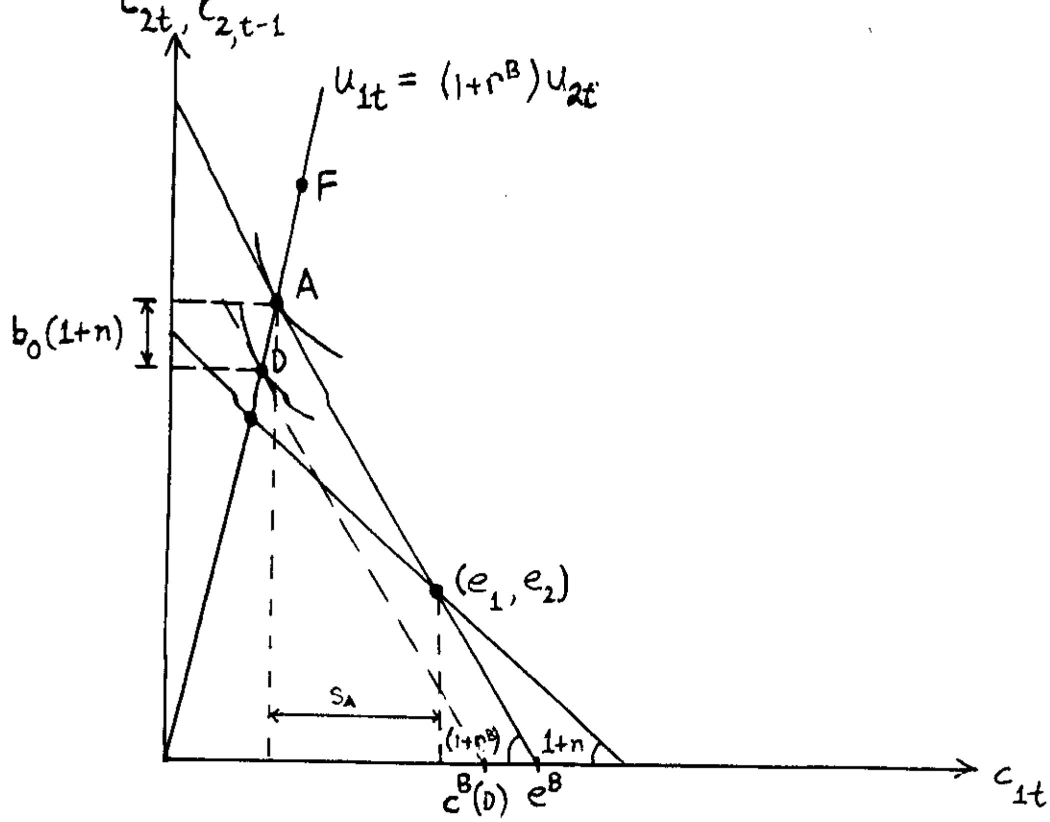
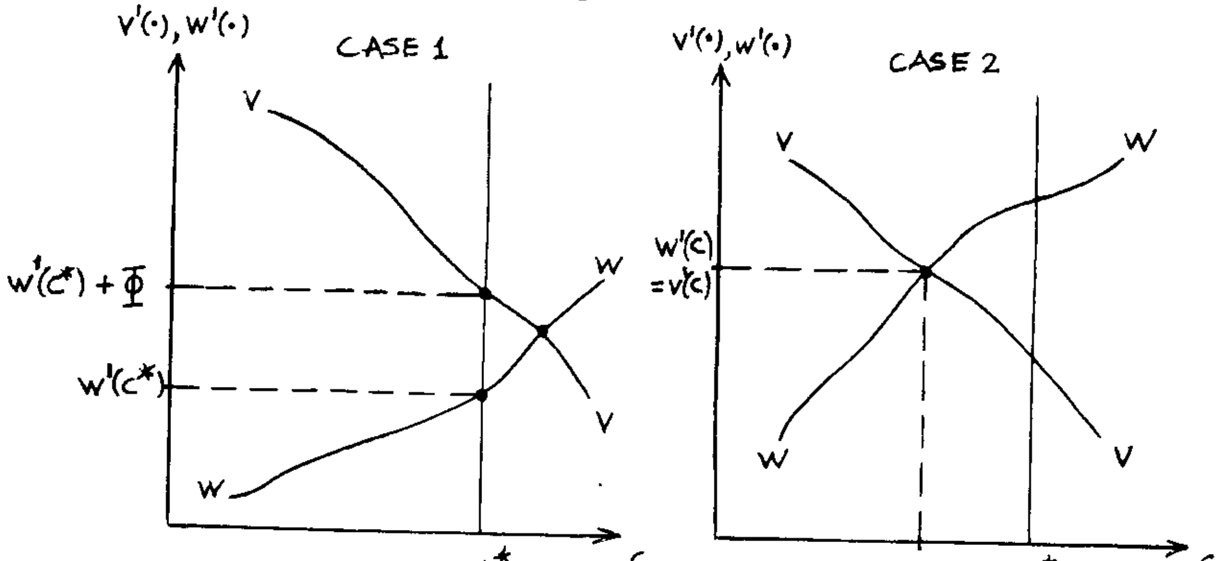


Figure 4



Appendix

Proofs of Ricardian Equivalence Propositions

1. Proposition 1

Let τ_t be the net lump-sum tax levied on generation t when old (the tax occurs in period t+1). For simplicity, we assume that the government does not tax the young. Let d_t be the holdings of real government bonds (debt) acquired by generation t when young (s_t will denote storage, as before). All variables are per-capita. When debt and taxes are included, the budget constraints relevant to generation t in a gift equilibrium (cf. equation (3)) become:

$$\text{(A1)}\quad \begin{aligned} &\text{(i)} && c_{1t} + s_t + d_t \le e_1 - (g_t/(1+n)) \\ &\text{(ii)} && c_{2t} \le e_2 - \tau_t + (s_t + d_t)(1+r^G) + g_{t+1} \\ &\text{(iii)} && s_t \ge 0, \quad g_t \ge 0 \\ &\text{(iv)} && c_{2,t-1} \le e_2 - \tau_{t-1} + (s_{t-1} + d_{t-1})(1+r^G) + g_t. \end{aligned}$$

We assume for simplicity that government spending is zero. The government budget constraint in period t is therefore

$$\text{(A2)}\qquad d_t = (1+r_t)d_{t-1} - \tau_{t-1}.$$

An equilibrium is a sequence of gifts $\{g_1, g_2, \ldots\}$ and consumption pairs $\{c_{2,0}, (c_{1,1}, c_{2,1}), \ldots, (c_{1t}, c_{2t}), \ldots\}$ satisfying the first-order conditions (5) and (6) for all generations. We denote these sequences in the initial equilibrium by $\{g\}^0$ and $\{c\}^0$, respectively, and the corresponding sequences of storage and bonds by $\{s\}^0$ and $\{d\}^0$. The initial equilibrium is a function of the sequence $\{\tau\}^0$ of taxes.

Part (i): To establish this part of Proposition 1, we need to construct an equilibrium in which all government debt is willingly held, the interest rate is equal to the rate on storage, and consumption patterns are the same as in the original equilibrium. The first-order conditions then hold in each period, and we have an equilibrium.

Suppose then that the government cuts taxes on the initial old by an amount $\alpha < g_1$, and leaves all future taxes unchanged. We denote this tax sequence $\{\tau\}'$. We claim that the gift sequence

(A3) $\{g\}^1 \equiv \{g_1^0 - \alpha,\ g_2^0 - [(1+r^G)/(1+n)]\alpha,\ g_3^0 - [(1+r^G)/(1+n)]^2\alpha,\ \ldots\}$

is an equilibrium, with debt holdings $\{d\}^1$ given by

(A4) $\{d_1^0 + [1/(1+n)]\alpha,\ d_2^0 + [(1+r^G)/(1+n)^2]\alpha,\ d_3^0 + [(1+r^G)^2/(1+n)^3]\alpha,\ \ldots\}$

and consumption sequence $\{c\}^0$.

To see this, first note that with $\alpha < g_1$ and $r^G < n$, the sequence $\{g\}^1$ of gifts is strictly positive. Second, note that with the tax policy given, the sequences $\{g\}^1$, $\{\tau\}^1$, $\{d\}^1$, $\{c\}^0$, and $\{s\}^0$ jointly satisfy the set of budget constraints (A1)(i)-(iii). Note that the total transfer of resources from children to parents (gift plus purchase of bonds) is identical in the two equilibria. The sequence of bonds increases by the amounts that gifts decrease; in other words, the current young and all future young cut their gift by exactly the amount needed to acquire the bonds associated with the rational Ponzi game. ||

Part (ii) of the Proposition follows from the following argument. With positive asymptotic storage in the initial equilibrium, there exists a continuum of equilibria with lower initial storage and higher initial gift and asymptotic gift. All of these equilibria have higher ...

consumption. By the argument in Part (i) above, these are also equilibria of the economy with the tax cut given to the initial old. To reach these equilibria requires that the initial young finance the purchase of the Ponzi game bonds only partially by a decrease in their gift; part of the financing comes out of a reduction in storage. Since the initial gift is cut by less than the tax cut, the consumption of the initial old rises, as does the consumption of all generations. The "total" gift (gift plus purchase of Ponzi game bonds) rises in each period from the baseline. Some storage is therefore crowded out asymptotically relative to the baseline; this is what makes the rise in stationary consumption possible. ||

2. Proposition 2

In consolidated form (cf. equation (19)), budget constraints for successive generations in the bequest economy, with debt and taxes included, imply:

$$\text{(A5)} \quad d_0 + s_0 = \frac{(c_{2,0} - e_2)}{1 + r} + \sum_{t=1}^{\infty} \theta^t (c_t - e^B) - \sigma(\{\tau_t\}) + \lim_{t\to\infty} \theta^{t+1} b_t ,$$

where $\sigma(\{\tau_t\})$ denotes the present value as of time 0 of aggregate taxes implied by the sequence $\{\tau_t\}$:

$$\text{(A6)} \quad \sigma(\{\tau_t\}) = \left(\frac{1}{1+r}\right) \sum_{t=0}^{\infty} \theta^t \tau_t .$$

Part (i): See Gale (1983).

Part (ii): (a) Debt that is repaid does not affect the present value of taxes and therefore does not affect the budget set of the dynasty, i.e., the set of consumption sequences and terminal bequests satisfying (A2). It therefore cannot affect the optimal consumption path. ||

(b) To establish parts (b) and (c), we use the following useful observation due to Gale (1983): at a fixed interest rate, the maximum of $u(c_{1t}, c_{2t})$ is an increasing, concave function of the discounted value of lifetime expenditure on consumption. We denote this function by $v(\cdot)$: $v(c_t) \equiv \max_{\{c_{1t}, c_{2t}\}} u(c_{1t}, c_{2t})$, subject to $c_t = c_{1t} + c_{2t}/(1+r^B)$. The dynasty's problem can then be written as follows (noting that $(1+n)/(1+r^B) = \theta$):

$$\text{(A7)} \quad \underset{\{c_{2,0};c_t:t=1,2,\ldots\}}{\text{Max}} \quad u(c_{1,0}, c_{2,0}) + \sum_{t=1}^{\infty} \theta^t v(c_t) + w(\lim_{t\to\infty} \theta^{t+1} b_t)$$

s.t. the consolidated budget constraint (A5)

The standard terminal constraint ruling out the dynasty running a Ponzi game is that the discounted aggregate bequest (defined as z) be nonnegative:

$$\text{(A8)} \quad z \equiv \lim_{t\to\infty} \left(\frac{1+n}{1+r}\right)^{t+1} b_t = (d_0+s_0) - (c_{2,0}-e_2)/(1+r) - \sum_{t=1}^{\infty} (c_t-e^B)\theta^t - \sigma(\{\tau_t\}) \geq 0.$$

We impose constraint (A8) in order to give the dynasty a well-posed problem. As discussed in O'Connell and Zeldes (1986), a dynasty with $w(\cdot) \equiv 0$ will impose the transversality condition $z \leq 0$, in which case z would equal 0.

When we add constraint (A8), the dynasty chooses the sequence $\{c_t\}$ to maximize the Lagrangian:

$$\text{(A9)} \quad L = u(c_{1,0}, c_{2,0}) + \sum_{t=1}^{\infty} \theta^t v(c_t)$$

$$+ w\Big((d_0-s_0) - (c_{2,0}-e_2)/(1+r) - \sum_{t=1}^{\infty} (c_t-e^B)\theta^t - \sigma\Big)$$

$$+ \Phi \cdot \Big((d_0-s_0) - (c_{2,0}-e_2)/(1+r) - \sum_{t=1}^{\infty} (c_t-e^B)\theta^t - \sigma\Big)$$

where $\Phi \geq 0$ is the shadow price on (A8). The first-order conditions are

(A10) $$u_{2,0} \cdot (1+r^B) = w'(z) + \Phi$$

$$v'(c_t) = w'(z) + \Phi \qquad t=1,2, \ldots$$

Since z does not depend on t, solutions to the dynasty's problem imply stationary consumption; they are the quasi-steady states discussed in the text. The condition for an optimal consumption plan is therefore

(A11) $$v'(c) = w'(z(c,\sigma,d_0+s_0)) + \Phi.$$

Figure 4 shows the solution for consumption. By concavity of $v(\cdot)$, the marginal utility of c is a decreasing function of c (the vv schedule). Since z is linearly decreasing in c, concavity of w implies that w' is nondecreasing in c (the ww schedule). The vertical line at c* denotes the highest possible stationary consumption plan that does not violate the terminal condition (A8); this is the consumption efficient steady state. Feasible consumption patterns lie on or to the left of the vertical line.

There are two cases to consider. If vv and ww intersect to the right of c* (Case 1), the shadow price Φ on the terminal condition must be positive; the taste for the terminal bequest is not strong enough to induce the dynasty to choose a consumption inefficient solution. The solution in this case (or when the intersection is exactly at c*) is c*, with a discounted terminal bequest of zero. In Case 2, vv and ww intersect to the left of c*, and the equilibrium is consumption inefficient. In this case (A8) is not binding, so θ is zero, and the dynasty equalizes the marginal utility of lifetime consumption with the marginal cost of additional consumption in terms of the lower terminal bequest. Proposition 2 restricts attention to Case 2.

Note that a tax cut to the young financed by a rational Ponzi game implies a fall in σ, the present value of taxes owed by the dynasty. The budget constraint (A2) states that this can be split between a rise in consumption and a rise in the discounted per-capita terminal bequest. We can now prove parts (b) and (c).

(b) if $w(\cdot)$ is linear, then w' is independent of c, so the solution for c in (A11) is independent of the present value of taxes. The tax cut will simply be used to purchase the corresponding bonds, which will then be rolled over perpetually and increase the terminal bequest. The "utility" of the dynasty rises, but there are no effects on the consumption path. In Figure 4, the ww schedule is a horizontal line that does not shift with the fall in σ. ||

(c) if $w(\cdot)$ is strictly concave, then optimal consumption in (A11) is a decreasing function of σ. In Figure 4, the ww schedule shifts to the right with a fall in σ. Both optimal consumption and the optimal terminal bequest rise; the tax cut financed by a rational Ponzi game thus has real effects. ||

References

Abel, Andrew B. (1985), "Gifts and Bequests: Comment and Extension," mimeo, The Wharton School, University of Pennsylvania, October.

Barro, Robert (1974), "Are Government Bonds Net Wealth?" Journal of Political Economy November/December: 1095-1118.

Bernheim, B. Douglas, and Kyle Bagwell (1985), "Is Everything Neutral?" mimeo, Stanford University, revised May.

Buiter, Willem H., and Jeffrey Carmichael (1984), "Government Debt: Comment," American Economic Review 74, No. 4, September: 762-765.

Burbridge, John B. (1984), "Government Debt: Reply," American Economic Review 74, No. 4, September: 766-767.

__________ (1983), "Government Debt in an Overlapping Generations Model with Bequests and Gifts," American Economic Review 73, No. 1, March: 222-227.

Carmichael, Jeffrey (1982), "On Barro's Theorem of Debt Neutrality: The Irrelevance of Net Wealth," American Economic Review 72, No. 1, March: 202-213.

Diamond, Peter (1965), "National Debt in a Neoclassical Growth Model," American Economic Review 55: 1126-1150.

Gale, Douglas (1983), Money: in Disequilibrium (Cambridge, Cambridge University Press).

Hamilton, James, and Ronald W. Michener (1986), "The Tax Obligations Associated with Government Deficits in Overlapping Generations Models," mimeo, University of Virginia, March.

O'Connell, Stephen A., and Stephen P. Zeldes (1986), "Rational Ponzi Games," Rodney L. White Center Working Paper #18-86, University of Pennsylvania, August.

Samuelson, Paul A. (1958), "An Exact Consumption Loan Model With and Without the Social Contrivance of Money," Journal of Political Economy 66: 467-482.

Tirole, Jean (1985), "Asset Bubbles and Overlapping Generations: A Synthesis," Econometrica 53, No. 5, September: 1071-1100.

Weil, Philippe (1987), "Love Thy Children: Reflections on the Barro Debt Neutrality Theorem," forthcoming, Journal of Monetary Economics.