CAPITAL STRUCTURE AND IMPERFECT COMPETITION IN PRODUCT MARKETS

by

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Abstract

A linear duopoly model is used to consider investment and financing decisions. Bankruptcy is assumed to cause a delay in investment which is not costly in itself. However, the imperfect competition in the product market means this delay puts the bankrupt firm at a strategic disadvantage which forces it to either partially or completely liquidate. Since this is costly, firms use only a limited amount of debt despite the corporate tax advantage it enjoys. Equilibrium can be symmetric or asymmetric. In the latter case similar firms have different capital structures.

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1. Introduction

It is often acknowledged that the question of how firms choose their capital structure has not been satisfactorily answered yet: current theories seem to be unable to explain the financing decisions firms actually make.

The debate has mainly centered around two apparent empirical regularities. Firstly, many firms paying corporate taxes use only a small amount of debt: debt ratios are typically of the order of 20 to 30 percent (Taggart (1985)). Secondly, similar firms in the same industry often have significantly different capital structures (Remmers et al. (1974)).

The most widely taught theory of capital structure is still perhaps Modigliani and Miller (1958) extended to include corporate taxes and bankruptcy costs. This argues that firms trade off the corporate tax advantage of debt arising from interest being tax deductible, against the costs of bankruptcy. Given the first regularity, these costs must be large for the theory to be plausible. In many early papers liquidation costs, which are potentially large, are the main costs the authors seem to have in mind. A possible reason is that bankruptcy is often followed by liquidation: Stanley and Girth (1971) found only 12½ percent of corporate bankrupts reorganized and of these 60 percent were ultimately unsuccessful. However, as Haugen and Senbet (1978) stress liquidation is a capital budgeting decision whereas bankruptcy is a transfer of assets: standard models do not explain why bankruptcy should cause liquidation. Given this, the trade-off theory is difficult to reconcile with the first observation since the other costs appear to be small. As Myers (1984) argues, it also seems inconsistent with the second since it suggests similar firms should have similar capital structures.

These and other difficulties have led to alternative theories being another is to invoke asymmetric information. These are again difficult to reconcile with the empirical evidence. (For a survey see Myers (1984).)

A theory of capital structure is presented below where bankruptcy causes partial or complete liquidation. It is based on imperfections in firms' product markets: most corporations, particularly large ones, operate in product markets with only a few firms. For simplicity, a linear duopoly model with a Cournot market structure is analyzed. In addition, it is assumed that because of the way the bankruptcy laws operate, which is described below, the effect of bankruptcy is to delay investment decisions. This delay is not costly in itself. However, given the imperfectly competitive product market, it means the bankrupt firm is at a strategic disadvantage and is forced to contract if fixed costs are small or liquidate if they are large. Thus when choosing the amount of debt to use, firms trade-off the corporate tax advantage against these liquidation costs. The nature of equilibrium also depends on the fixed costs of capacity. If these are small, the equilibrium is symmetric so both firms adopt the same strategy. If they are large the equilibrium is asymmetric: one firm goes for the tax advantage of debt and the other goes for the strategic advantage of equity. Thus, even though the firms are similar, they can have different capital structures.

In a recent paper, Brander and Lewis (1985) also consider investment and financing decisions in a duopoly. In their model, even though there is no tax advantage to debt, equilibrium may involve positive debt levels because bankruptcy costs effectively allow firms to precommit to a particular output. Their results provide an interesting contrast to those obtained here.

The paper proceeds as follows. Section 2 outlines the model. In Section 3 equilibrium is analyzed. Section 4 discusses the consistency of the model

2. The Model

There are two identical firms denoted j and k. The market structure is Cournot: each period the firms produce quantities q_j and q_k, place them on the market and together these determine the market clearing price p.

Demand for the industry is stochastic. The demand curve has the form

$$p = \alpha - \beta(q_j + q_k) \tag{1}$$

where the intercept α is a random variable but the slope β is a constant. The density function of α is linear:

$$f(\alpha) = \alpha - \alpha_\ell \qquad \text{for } \alpha_\ell \le \alpha \le \bar{\alpha} \tag{2a}$$

$$= \alpha_\ell + 2 - \alpha \qquad \text{for } \bar{\alpha} \le \alpha \le \alpha_\ell + 2 \tag{2b}$$

where $\bar{\alpha} = \alpha_\ell + 1$ (see Figure 1). It is assumed that α_ℓ is sufficiently large that in equilibrium

$$p > 0 \tag{3}$$

for all possible realizations of α. The α's in each period are independent.

A firm's output is equal to its capacity which is determined by its initial investment I_j. The cost of capacity is a linear function:

$$\begin{aligned} I_j &= \phi + \mu q_j \qquad &\text{for } q_j > 0 \\ &= 0 \qquad &\text{for } q_j = 0 \end{aligned} \tag{4}$$

where ϕ is the fixed cost and μ ($< \bar{\alpha}$) is the constant marginal cost.

There are two periods t = 0, 1, 2. The sequence of events in each period is the following.

(i) Initially firms make their investment decisions which determine their capacities. At the same time they make their financing decisions. The decisions and use the proceeds to pay the costs of their investments. All this is done before α is realized.

(ii) Output is produced; α is realized.

(iii) The two firms' quantities, together with α, determine the price. Revenue pq is received. This is used to pay the corporate income tax and make payments on the securities issued at stage (i).

Capital markets are perfectly competitive and frictionless. Investors are risk neutral and securities sell for their expected value.

Firms have two financing instruments: debt and equity. Debt is a promise to repay D at the end of the period or the firm goes bankrupt. Equity receives the profits (if any) remaining at the end of the period after taxes and payments to debtholders.

At the start of each period, before firms make their investment and financing decisions, they are entirely owned by equityholders. When firms finance their investment they issue debt with par value D. If the debt is risky, its market value D^M will be less than D. The remainder required to finance the investment, $I - D^M$, is raised by issuing equity. Since the capital markets are perfect, the original shareholders total expected return is the expected value of the firm. Hence their objective when they make the investment and financing decisions is to maximize this.

The corporate tax rate is τ $(0 < \tau \leq 1)$. Investment costs are tax deductible. So is interest. For ease of notation, it is assumed the (after-tax) real interest rate is zero and all the variables are in real terms. This greatly simplifies the expressions by allowing discounting terms to be omitted, without altering the results significantly. There is a positive nominal rate of interest so that for a firm using debt D, δD $(0 < \delta \leq 1)$ is tax deductible. This means there is a tax bias in favor of debt. The taxes paid by a firm, T, are:

$$T = \tau\left[pq - (\phi + \mu q) - \delta D\right] . \qquad (5)$$

Taxes are paid when the tax base is positive and subsidies are received when it is negative. To prevent firms from issuing an infinite amount of debt D, and claiming the infinite subsidy this would give rise to, there is an upper bound D_u on the amount of debt that is tax deductible:

$$D \le D_u . \qquad (6)$$

This corresponds to the limited loss carryover provisions in the actual code.

For simplicity there are no personal taxes on either debt or equity income or equivalently the marginal personal rate on equity income is the same as the marginal personal rate on debt income.

Bankruptcy occurs when the total receipts at the end of the period from selling the goods less any tax payments (or plus any subsidies) are insufficient to repay the debt:

$$pq - T \le D . \qquad (7)$$

This condition assumes that equityholders do not have the opportunity to raise funds against future profits and use them to repay the debt and possibly prevent bankruptcy. This is the simplest case to consider: if future profits were included in the bankruptcy condition, there would still be a determinate bankruptcy point and a similar analysis to that below would be possible.

The condition also assumes that the equityholders cannot repurchase the debt at its current market value just before bankruptcy as suggested by Haugen and Senbet (1978). The justification for this is that it will be difficult there is a free-rider problem. Given some people accept the repurchase offer from the firm, anybody who holds out and refuses the offer can expect the value of their securities to increase. This is because if the firm were then to go bankrupt its assets would be divided between fewer bonds and so the payoff on each would be increased. Roe (1987) terms this the "buoying up effect." The end result of it is that standard debt cannot be repurchased.

It is not possible to use forms of debt contract which allow conversion to equity just before bankruptcy or other forms of debt contract which permit bankruptcy to be avoided. The reason is the restrictions placed on bond contracts by the Trust Indenture Act of 1939. Roe (1987) points out the purpose of these restrictions is to precipitate bankruptcy. The proponents of the law suggested that to prevent minority bondholders being cheated, it is necessary to ensure that any change in a bond's terms occur in bankruptcy under the supervision of the court rather than before bankruptcy. Without such protection it was argued that equityholders would obtain a majority of a firm's bonds and vote to expropriate the minority bondholders. In fact for these reasons it was standard practice prior to 1939 to issue bonds which did not allow bankruptcy to be avoided. Thus not only is debt of this type not observed currently, it was rare historically.

The effect of bankruptcy in the model is to cause a delay in a bankrupt firm's investment decision. The basis for this is the operation of the bankruptcy laws. With the current Bankruptcy Reform Act of 1978 reorganizations occur under Chapter 11 of the code. This replaced Chapter X of the previous code, which was intended primarily for large corporations, and Chapter XI, which was for smaller businesses.

After bankruptcy occurs a plan of reorganization must be agreed upon.

investments or sales of assets. The plan must be approved by two-thirds of each class of claimant. If it is rejected, an amended plan is produced and the process repeated until one is approved. Once the plan is agreed upon the judge must also give his consent to it. The firm can then emerge from bankruptcy and continue operations normally. Although it is possible for those in control of the firm to sell assets or raise money outside of the plan, the approval of the judge must be obtained and this is only given in special cases. Moreover, even if this is obtained it can be challenged by the securityholders of the firm. If any claimants' bargaining power is reduced by the change they will have an incentive to do this. (For a description and examples of the operation of Chapter 11 see Baird and Jackson (1985).)

The bankruptcy process is usually protracted. Roe (1987) gives a figure of 3 years for a typical reorganization of a large corporation. Altman (1971) found for a sample of 90 firms that the average time under the old Chapter X was 2¼ years. Stanley and Girth (1971) give a figure of 1½ years under the old Chapter X1. It is this lengthy process and the restrictions placed on the firm while it is going on which is the basis for the assumption that the investment decisions of the bankrupt and solvent firm are not simultaneous.

Thus in the model, the effect of bankruptcy at the end of a period is to delay the investment decision at the start of the next period. If both firms are in the same position at the end of period 1 so they are either both solvent or both bankrupt, they make their decisions simultaneously at the start of period 2 and so play a Nash game. However, if one firm is solvent and the other is bankrupt so its investment decision is delayed, they play a Stackelberg game with the solvent firm as leader and the bankrupt one as

3. Equilibrium

A rational expectations perfect Nash equilibrium concept is used. The focus of the analysis is on pure-strategy equilibria, which always exist in this version of the model. Other possible types of equilibria are discussed in Section 5.

The solution procedure involves solving backwards by first looking at the possibilities in period 2 and then analyzing the decisions for period 1 given that the firms play equilibrium strategies in the period 2 subgames. The equilibrium of the model depends crucially on the level of the fixed cost of capacity ϕ. These determine the outcome of the Stackelberg subgame the firms play in period 2 if one of them goes bankrupt at the end of period 1. If there are no fixed costs, it is always profitable for the follower to remain in business at a reduced size. However, if these are sufficiently large, it is optimal for the leader to expand and make the profits of the follower fall below the level of the fixed costs. This causes the follower to liquidate and the leader becomes a monopolist. The magnitude of fixed costs therefore determines the marginal cost of bankruptcy and hence the form of equilibrium.

In Subsection A the case where $\phi = 0$ is analyzed; the opposite extreme where the fixed costs are large is looked at in B, and C deals with the intermediate case. Finally D contains a brief summary.

A. No fixed costs of capacity

In this subsection it is assumed there is only a marginal cost of capacity so

$$\phi = 0 . \tag{8}$$

First, consider decisions for period 2, which are made at t = 1. Since that firms will always use as much debt as possible. To save having to repeatedly write a lump sum subsidy component, τD_u, in profit expressions, it is simplest to assume that in period 2 $D_j = D_u = 0$. At $t = 1$ the only decision firms are then concerned with is the level of investment I or equivalently their period 2 capacity q_2.

There are two possibilities at $t = 1$. Either both firms are solvent or both are bankrupt in which case they play a Nash game. Alternatively, one firm is solvent and the other is bankrupt in which case they play a Stackelberg game. Consider the Nash game first. Each firm chooses its second period capacity to maximize expected profits given the other firm's capacity:

$$\max_{q_{2j}} \; (1 - \tau)[\bar{\alpha} - \beta(q_{2j} + q_{2k}) - \mu]q_{2j} \tag{9}$$

taking q_{2k} as given.

The first order condition implies

$$q_{2j} = \frac{\bar{\alpha} - \mu}{2\beta} - \frac{q_{2k}}{2} . \tag{10}$$

Similarly for k. Solving these simultaneously gives the standard results

$$q_{2j} = \frac{\bar{\alpha} - \mu}{3\beta} \; ; \quad E\pi_{2j} = (1 - \tau)\,\frac{Z}{9} \tag{11}$$

where $E\pi_{2j}$ is the expected profit of j in period 2 and

$$Z = \frac{(\bar{\alpha} - \mu)^2}{\beta} . \tag{12}$$

The other possibility is that one firm, say firm k, is solvent and the other, j, is bankrupt. Firm k acts as leader and makes its investment decision while j is tied up in bankruptcy court. When j finally makes its investment decision it takes k's as given. Firm k takes this into account decision as follower is

$$\underset{q_{2j}}{\text{Max}} \quad (1 - \tau)\left[\bar{\alpha} - \beta(q_{2j} + q_{2k}) - \mu\right]q_{2j} \tag{13}$$

taking q_{2k} as given. Hence

$$q_{2j} = \frac{\bar{\alpha} - \mu}{2\beta} - \frac{q_{2k}}{2} . \tag{14}$$

Then k's decision as leader is

$$\underset{q_{2k}}{\text{Max}} \quad (1 - \tau)\left\{\bar{\alpha} - \beta\left[q_{2k} + \left(\frac{\bar{\alpha} - \mu}{2\beta} - \frac{q_{2k}}{2}\right)\right] - \mu\right\}q_{2k} . \tag{15}$$

It follows

$$q_{2k} = \frac{\bar{\alpha} - \mu}{2\beta} ; \quad E\pi_{2k} = (1 - \tau)\ \frac{Z}{8} \tag{16}$$

$$q_{2j} = \frac{\bar{\alpha} - \mu}{4\beta} ; \quad E\pi_{2j} = (1 - \tau)\frac{Z}{16} . \tag{17}$$

The effect of a single firm going bankrupt can be seen by comparing (11) with (16) and (17). The leader expands its capacity above that in the Nash case. The follower is at a strategic disadvantage, and as a result has a lower capacity than before. The bankrupt firm is worse off since its profits are reduced from $(1 - \tau)Z/9$ to $(1 - \tau)Z/16$ but the solvent firm is made better off: its profits are increased from $(1 - \tau)Z/9$ to $(1 - \tau)Z/8$. Hence the delay in a firm's investment decision, although not costly in itself, means bankruptcy is undesirable.

Next consider the decisions of the firms at $t = 0$. In period 1 it is assumed that $D_u > 0$. The analysis will be concerned with interior solutions such that (6) does not bind.

In period 1 firm j's expected profits are where q_j and q_k are the firms' period 1 capacities and D_j its period 1 debt.

The realization of the demand parameter α at the end of period 1 determines whether or not the firms go bankrupt and hence the type of game played and the expected profits in period 2. In order to derive an expression for j's expected profits in period 2 evaluated at $t = 0$, it is necessary to define the level of demand, α_j^*, such that firm j goes bankrupt at the end of period 1. Using (1), (5), (7) with an equality, and (8),

$$\alpha_j^* = \left(\frac{1 - \tau\delta}{1 - \tau}\right) \frac{D_j}{q_j} + \beta(q_j + q_k) - \frac{\tau}{1 - \tau}\mu . \tag{19}$$

For values of α above α_j^*, firm j is solvent; for values below it is bankrupt. Similarly for k. If $\alpha_j^* \geq \alpha_k^*$ then

$$E\pi_{2j} = (1 - \tau)\left[\int_{\alpha_\ell}^{\alpha_k^*} \frac{Z}{9} f(\alpha)d\alpha + \int_{\alpha_k^*}^{\alpha_j^*} \frac{Z}{16} f(\alpha)d\alpha + \int_{\alpha_j^*}^{\alpha_\ell + 2} \frac{Z}{9} f(\alpha)d\alpha\right] . \tag{20}$$

As illustrated in Figure 2, for α between α_ℓ and α_k^* both firms go bankrupt and in period 2 they play a Nash game so j's expected profits are $(1 - \tau)Z/9$. From α_k^* to α_j^* firm j is bankrupt but k is not: they play a Stackelberg game with j as a follower so its expected profits are $(1 - \tau)Z/16$. For α above α_j^* they again play a Nash game.

For $\alpha_j^* \leq \alpha_k^*$,

$$E\pi_{2j} = (1 - \tau)\left[\int_{\alpha_\ell}^{\alpha_j^*} \frac{Z}{9} f(\alpha)d\alpha + \int_{\alpha_j^*}^{\alpha_k^*} \frac{Z}{8} f(\alpha)d\alpha + \int_{\alpha_k^*}^{\alpha_\ell + 2} \frac{Z}{9} f(\alpha)d\alpha\right] . \tag{21}$$

This is the same as when $\alpha^* > \alpha^*$ except that between α^* and α^*, j is the

Since the objective of the initial shareholders is to maximize the total value of the firm as explained in Section 2, firm j's decision problem at t = 0 for choosing its period 1 capacity and debt is

$$\max_{q_j, D_j} E\pi_j = E\pi_{1j} + E\pi_{2j} \tag{22}$$

taking q_k and D_k as given.

The equilibria of interest are those where firms are at interior optima and so attention is restricted to these. Two types are possible. The first is symmetric with $\alpha_j^* = \alpha_k^*$ so both have the same capacity and debt. In the other $\alpha_j^* > \alpha_k^*$ (say) so that it is asymmetric: one goes for the tax advantage of debt and the other goes for the equity advantage of being leader if demand is low. These are analyzed in turn.

In a symmetric equilibrium, each firm has the same level of debt and capacity. For an equilibrium to exist each firm must perceive that if it increases or reduces its debt or capacity, taking those of the other firm as given, then its profits will fall.

First consider firm j's debt decision. Differentiating (19) gives:

$$\frac{\partial \alpha_j^*}{\partial D_j} = \left(\frac{1 - \tau\delta}{1 - \tau}\right) \frac{1}{q_j} \tag{23}$$

$$\frac{\partial \alpha_k^*}{\partial D_j} = 0 \ . \tag{24}$$

Hence if j increases its debt a small amount then $\alpha_j^* > \alpha_k^*$ and so (20) is the relevant expression for period 2's profits. Thus where

$$m_F = (1 - \tau\delta)\frac{7}{144}Z . \qquad (26)$$

The first term, $\tau\delta$, in (25) is the tax benefit in period 1 from increasing debt. The second term is the loss in expected profits in period 2 arising from the fact that j will go bankrupt in more states. In these states, instead of having expected profits of $(1 - \tau)Z/9$ when playing a Nash game, as Stackelberg follower j only has $(1 - \tau)Z/16$ so that the difference between them is $(1 - \tau)(7/144)Z$. The reason $f(\alpha)$ is given by (2a) is that this is necessary for there to be an interior solution; otherwise $E\pi_j$ would be convex in D_j.

In contrast if j reduces its debt then $\alpha_j^* < \alpha_k^*$ and (21) is the relevant expression for period 2 profits. Thus

$$\frac{\partial E\pi_j^-}{\partial D_j} = \tau\delta - m_L \frac{(\alpha_j^* - \alpha_\ell)}{q_j} \qquad (27)$$

where

$$m_L = (1 - \tau\delta)\frac{2}{144}Z . \qquad (28)$$

It can be seen

$$m_F > m_L \qquad (29)$$

and

$$\frac{\partial E\pi_j^+}{\partial D_j} - \frac{\partial E\pi_j^-}{\partial D_j} = (m_L - m_F)\frac{(\alpha_j^* - \alpha_\ell)}{q_j} < 0 \qquad (30)$$

so that $E\pi_j$ is kinked at $\alpha_j^* = \alpha_k^*$. Moreover, the kink is concave so that it can correspond to a maximum.

Changes in capacity can be similarly analyzed. Since these affect the levels of demand the two firms go bankrupt at, there is again a kink: an

This means the kink is again concave so that it can correspond to a maximum. These changes of capacity and the other features of the symmetric equilibria of the model are considered in detail in Section I of the Appendix.

The second type of equilibrium that could exist is asymmetric with $\alpha_j^* \neq \alpha_k^*$. This possibility is considered in Section II of the Appendix. It is shown formally there that an asymmetric equilibrium cannot exist when $m_F > m_L$. To see why this is, suppose for the sake of discussion

$$\alpha_j^* > \alpha_k^* , \tag{31}$$

In such an equilibrium firm j goes bankrupt in states that k doesn't. Consider firm j's debt decision. If the firm increases its debt, α_j^* rises and its payoff in the marginal states switches from the Nash level of $(1 - \tau)Z/9$ to the Stackelberg follower level of $(1 - \tau)Z/16$. Given (31), a small reduction in debt leads to the opposite, namely a switch from the Nash level to the Stackelberg follower level. Thus the left- and right-hand derivatives are equal.

$$\frac{\partial E\pi_j^+}{\partial D_j} = \frac{\partial E\pi_j^-}{\partial D_j} = \frac{\partial E\pi_j}{\partial D_j} = \tau\delta - m_F \frac{(\alpha_j^* - \alpha_\ell)}{q_j} = 0 . \tag{32}$$

In contrast to the symmetric case, the right and left-hand derivatives are equal and there is no kink. Similarly for k. If it increases its debt level, its payoff in the marginal states switches from the Nash level of $(1 - \tau)Z/9$ to the Stackelberg leader level of $(1 - \tau)Z/8$. If it reduces its debt level the change is reversed.

$$\frac{\partial E\pi_k^+}{\partial D_k} = \frac{\partial E\pi_k^-}{\partial D_k} = \frac{\partial E\pi_k}{\partial D_k} = \tau\delta - m_L \frac{(\alpha_k^* - \alpha_\ell)}{q_k} = 0 . \tag{33}$$

for the two firms must be equal:

$$m_F \frac{(\alpha_j^* - \alpha_\ell)}{q_j} = m_L \frac{(\alpha_k^* - \alpha_\ell)}{q_k} . \qquad (34)$$

Now $m_F > m_L$ and $\alpha_j^* > \alpha_k^*$. The only possibility if there is to be an equilibrium is that q_j is sufficiently larger than q_k. It is shown in Section II of the Appendix that this is not possible.

The basic problem is that firm j not only has a high cost of bankruptcy m_F, it also has a high marginal probability of going bankrupt, $(\alpha_j^* - \alpha_\ell)$. In contrast, k has both a low cost of bankruptcy, m_L, and a low marginal probability of bankruptcy, $(\alpha_k^* - \alpha_\ell)$. Firm j's marginal bankruptcy cost of debt is therefore higher than k's. However, in equilibrium they must be equal since both firms receive the same marginal benefit from the tax shield provided by debt. Thus no asymmetric equilibrium can exist.

The results of this section are summarized by the following proposition.

Proposition 1

When $\phi = 0$ the only type of equilibrium that exists is symmetric with $\alpha_j^* = \alpha_k^*$. At each firm's optimum in such equilibria, profits are a nondifferentiable function of capacity and debt.

B. High fixed costs of capacity

In this subsection the fixed cost of capacity is such that

$$\frac{Z}{16} \leq \phi \leq \frac{Z}{9} . \qquad (35)$$

As before, in order to find the equilibrium it is necessary to first consider investment decisions at t = 1. If both are solvent or both are bankrupt the firms play a Nash game similarly to (9) except with the inclusion of ϕ. Each firm's optimal capacity is again as in (12) and their expected profits are

$$E\pi_{2j} = (1 - \tau)\left[\frac{Z}{9} - \phi\right] . \qquad (36)$$

When one firm is solvent (say k) and the other is bankrupt (say j) they play a Stackelberg game with k as leader and j as follower. In this case if $q_{2k} = (\bar{\alpha} - \mu)/2\beta$ as in (16) then it follows that the highest profits j can obtain with a positive output are

$$E\pi_{2j} = (1 - \tau)\left[\frac{Z}{16} - \phi\right] . \qquad (37)$$

The left-hand inequality of (35) implies $E\pi_{2j} \leq 0$ and j's optimal strategy in this case is to liquidate so that

$$q_{2j} \ ; \quad E\pi_{2j} = 0 . \qquad (38)$$

The q_{2k} in (16) was derived on the assumption that $q_{2j} = (\bar{\alpha} - \mu)/4\beta$. However, it turns out even if $q_{2j} = 0$ that k can do no better than $q_{2k} = (\bar{\alpha} - \mu)/2\beta$. This can easily be seen from the first order condition for k's problem as a monopolist. Thus k's profits when it is the Stackelberg leader are

$$E\pi_{2k} = (1 - \tau)\left[\frac{Z}{4} - \phi\right] . \qquad (39)$$

The analysis of decisions at t = 0 is then similar to that in A. Expected profits in the first period $E\pi_{1j}$ are as in (18) except for the

Profits are of this form since between α_j^* and α_k^*, j is forced to liquidate and receives nothing in the second period. If $\alpha_j^* \le \alpha_k^*$, j is the leader between α_j^* and α_k^* and receives $(1 - \tau)(Z/4 - \phi)$:

$$E\pi_{2j} = (1-\tau)\Big[\int_{\alpha_\ell}^{\alpha_j^*}\Big(\frac{Z}{9} - \phi\Big)f(\alpha)d\alpha + \int_{\alpha_j^*}^{\alpha_k^*}\Big(\frac{Z}{4} - \phi\Big)f(\alpha)d\alpha + \int_{\alpha_k^*}^{\alpha_j^*}\Big(\frac{Z}{9} - \phi\Big)f(\alpha)d\alpha\Big] \,. \tag{41}$$

For symmetric equilibria the partial derivatives of $E\pi_j$ with respect to D_j are the same as in (25) and (27) except now

$$m_F = (1 - \tau\delta)\Big[\frac{4}{36}Z - \phi\Big] \tag{42}$$

$$m_L = (1 - \tau\delta)\,\frac{5}{36}Z \,. \tag{43}$$

In contrast to (29),

$$m_F < m_L \,. \tag{44}$$

Hence there cannot be any symmetric equilibria since from (30) the kinks at firms' optima are now convex and correspond to minima rather than maxima.

Next consider the possibility of an asymmetric equilibrium. The analysis is similar to that in A except now m_F and m_L are given by (42) and (43) respectively. Since $m_F < m_L$ it is possible for (34) to be satisfied so an asymmetric equilibrium can exist. This is shown formally in Section II of the Appendix. The basic reason is that firm j which has a high marginal probability of bankruptcy now has a low marginal cost of bankruptcy; firm k, which has a low marginal probability, has a high cost. It is therefore possible for both firms to have a marginal bankruptcy cost of debt equal to its marginal tax benefit.

The results of this subsection are summarized by the following

Proposition 2

For $Z/16 \leq \phi \leq Z/9$, the equilibria are asymmetric. If $\alpha_j^* > \alpha_k^*$ (say), then for realizations of α in period 1 such that $\alpha_k^* < \alpha < \alpha_j^*$, the bankrupt firm j liquidates at t = 1 and k becomes a monopolist in period 2.

C. Intermediate fixed costs of capacity

This subsection considers the case where

$$0 < \phi < \frac{Z}{16}. \tag{45}$$

With no fixed costs of capacity, a bankrupt firm acting as follower chooses a smaller capacity in the equilibrium at t = 1 than it would if it were not bankrupt. With a high fixed cost, a bankrupt firm acting as follower liquidates. With intermediate fixed costs, it is shown both of these are possible: below a certain level ϕ_1^* the follower stays in the market; above it, the firm leaves. Similarly, the equilibrium is symmetric for ϕ below a certain level $\phi_2^* (> \phi_1^*)$ and asymmetric above.

Consider the decision of a bankrupt firm, say j, at t = 1 given that the solvent firm k has already chosen capacity q_{2k}. It can either set $q_{2j} = 0$, or it can set it so that (14) is satisfied and profits are maximized given a positive capacity. Its profits in the latter case are

$$E\pi_{2j} = (1 - \tau)\left[\beta\left(\frac{\bar{\alpha} - \mu}{2\beta} - \frac{q_{2k}}{2}\right)^2 - \phi\right]. \tag{46}$$

Hence if

$$\left(\frac{\bar{\alpha} - \mu}{\beta} - q_{2k}\right)^2 \leq \frac{4\phi}{\beta} \tag{47}$$

the firm liquidates; otherwise it stays in the market.

In Subsection B it was shown that if $\phi > Z/16$ and k chooses the monopoly strategy is also to choose a capacity of $(\bar{\alpha} - \mu)/2\beta$ but now j remains in the market. For intermediate ϕ, k has two possible courses of action. Firstly, it could set q_k so that (47) is satisfied: this pushes j out of the market and k becomes a monopolist. Alternatively, it could allow j to remain in the market. In order to determine which of these it should do, it is necessary to find the profitability of each.

If k is a monopolist its profits are

$$E\pi_{2k} = (1 - \tau)\left[(\bar{\alpha} - \mu - \beta q_{2k})q_{2k} - \phi\right] . \tag{48}$$

It follows that for k to be able to make a positive profit $q_{2k} < (\bar{\alpha} - \mu)/\beta$. Given this, (47) is satisfied if

$$q_{2k} \geq \frac{\bar{\alpha} - \mu}{\beta} - 2\left(\frac{\phi}{\beta}\right)^{\frac{1}{2}} . \tag{49}$$

When $\phi < Z/16$ the right hand side of this inequality is greater than $(\bar{\alpha} - \mu)/2\beta$. For such q_{2k}, k's profits as a monopolist are a decreasing function of capacity. Hence if k wants to force j to liquidate it should choose q_k so that (49) is satisfied with an equality. Its expected profits are then

$$E\pi^0_{2k} = (1 - \tau)\left[2\left(\frac{\phi}{\beta}\right)^{\frac{1}{2}}(\bar{\alpha} - \mu) - 5\phi\right] . \tag{50}$$

If k allows j to remain in the market it can be shown similarly to the case where $\phi = 0$ that its optimal capacity is again $(\bar{\alpha} - \mu)/2\beta$ and its expected profits are

$$E\pi^+_{2j} = (1 - \tau)\left(\frac{Z}{8} - \phi\right) . \tag{51}$$

It follows from these that for ϕ satisfying (45)

$$\phi_1^* = 0.0054\ Z\ . \tag{53}$$

All this implies the following proposition.

Proposition 3

In the Stackelberg equilibrium at $t = 1$, if $0 \leq \phi \leq \phi_1^*$ the solvent firm chooses a capacity of $(\bar{\alpha} - \mu)/2\beta$ and the bankrupt firm stays in the market at a reduced size. For $\phi_1^* < \phi < Z/16$ it is optimal for the solvent firm to expand capacity above $(\bar{\alpha} - \mu)/2\beta$ and force the bankrupt firm to liquidate.

The other question of interest concerns whether the equilibria at $t = 0$ in this intermediate case are symmetric or asymmetric. It can be seen from the analyses of the previous subsections that this is determined by whether m_F is above or below m_L. If $m_F > m_L$ the equilibrium is symmetric; if $m_F < m_L$ it is asymmetric.

For $\phi \leq \phi_1^*$, m_F and m_L are as in (26) and (28) and the equilibria are symmetric. For $\phi > \phi_1^*$

$$m_F = (1 - \tau\delta)\left(\frac{Z}{9} - \phi\right) \tag{54}$$

$$m_L = (1 - \tau\delta)\left[2\left(\frac{\phi}{\beta}\right)^{\frac{1}{2}}(\bar{\alpha} - \mu) - \frac{Z}{9} - 4\phi\right]\ . \tag{55}$$

Hence

$$m_F \gtreqless m_L \qquad \text{as} \qquad \phi \lesseqgtr \phi_2^* \tag{56}$$

where

$$\phi_2^* = 0.0198\ Z\ . \tag{57}$$

This gives the following.

Proposition 4

D. Summary

The results of this section are summarized diagrammatically in Figure 3. The crucial determinant of the form of equilibrium is the fixed cost of capacity. For $0 \le \phi \le \phi_1^*$ a solvent firm acting as a Stackelberg leader at $t = 1$ chooses a capacity of $(\bar{\alpha} - \mu)/2\beta$. The bankrupt firm acting as follower is forced to reduce its size relative to the equilibrium if it were not bankrupt, but still remains in the market. For $\phi_1^* < \phi < Z/16$ the solvent firm expands its capacity above $(\bar{\alpha} - \mu)/2\beta$ in order to force the bankrupt firm to liquidate. For $Z/16 \le \phi \le Z/9$ the solvent firm produces the monopoly output which is again $(\bar{\alpha} - \mu)/2\beta$ and the bankrupt firm's best response is to liquidate.

As far as the equilibrium at $t = 0$ is concerned, it is symmetric so that firms have the same capacity, use the same amount of debt, and so on if $0 \le \phi \le \phi_2^*$. It is asymmetric if $\phi_2^* < \phi \le Z/9$: one firm goes for the tax advantage of debt and the other goes for the equity advantage of being able to force the debt firm out of the market if demand turns out to be low.

4. Relationship to Empirical Evidence

The model developed in this paper involves a number of simplifying assumptions such as there being two periods, linear demand and density functions and so on. These are designed to allow theoretical results to be derived. Their disadvantage is that, as it stands, the model is an unsuitable basis for an empirical investigation. However, it is possible to demonstrate that the theory is broadly consistent with the two regularities mentioned in the introduction as well as with a number of other observations.

The first was that most taxpaying firms use predominantly equity finance:

Table 1

Debt Ratios in Various Regulated and Unregulated Industries in 1962.

Industry	Debt as % of Total Financing
Railroads	49.0
Electric and Gas Utilities	54.9
Mining	24.6
Industrials	32.5

(From Table 2 of Schwartz and Aronson (1967))

and as a result they will be forced to either partially or completely liquidate. Since liquidation costs can be significant, this implies the marginal bankruptcy costs of debt can be large. Even at low debt ratios the model therefore provides a rationale for why marginal bankruptcy costs might be large enough to offset the tax advantage of debt.

The second observation was that similar firms in the same industry often have different capital structures. In asymmetric equilibria of the model the two firms can have capital structures that differ significantly, which is consistent with this.

One of the implications of the model is that other things equal, debt ratios will be lower in industries where strategic considerations are important than in industries where they are not. Much of the empirical literature on capital structure undertaken in the late 1960's and early 1970's focussed on whether the use of debt varied across industries. Schwartz and Aronson (1967) found that in the industries they considered which are regulated, namely railroads and utilities, the debt ratios are much higher than in the unregulated industries, mining and industrials (see Table 1).This strategic interaction between firms within regulated industries, whereas in unregulated industries there typically is.

There is some evidence that the difference in capital structures between regulated and unregulated industries was even more pronounced before the introduction of the corporate income tax in 1909. Dewing (1914) writes (p.6):

> The reason why upwards of 40% of the railway mileage of the country passed into receivers' hands during the depression following the panic of 1893 was that a large part of the capitalization of these railroads was represented by bonds carrying fixed charges enforceable at law. The reason why so few of the industrial consolidations failed, in the legal sense of the word, during the depressions of 1903 and 1907 was that practically all their entire capitalization was in the form of stocks with no legally enforceable claims to interest and profits.

In addition to the differences in capital structure, this quotation also indicates the incidence of bankruptcy was much higher among railroads than industrial firms. Despite this, liquidations of railroads were rare whereas liquidations of industrial firms were not.

A recent illustration of the problems and delays involved during bankruptcy is provided by Braniff Airways. During the late 1970's and early 1980's Braniff was a major carrier operating over 100 aircraft and employing around 9,500 people. In May 1982 it filed for bankruptcy under Chapter 11. A plan of reorganization was eventually approved in December 1983. During this period its major competitors at its Dallas-Fort Worth base, American and Delta Airlines, expanded their operations there. When it finally emerged from bankruptcy Braniff was much smaller in size, operating only around 20 aircraft and employing less than 2,000 people. Even at this reduced size it is not clear the company is viable (Wall Street Journal (1984)).

5. Concluding Remarks

The model can be extended in a number of ways. It is assumed above that the firms' assets depreciate completely within one period. This implies that the only costs of liquidating are the profits the firm would have obtained if it had remained solvent. However, the analysis can also be applied to the case where liquidation involves selling the assets of the firm and this is costly. If in the second period a firm can sell its assets at some constant marginal cost, and there is a fixed cost of maintenance, a similar theory to that above holds. The industry considered has only two firms, but the analysis can be applied to the case where there are more than two. The main difference would be that in the Stackelberg equilibrium, there would be multiple leaders and followers. In general, the more firms there are, the less the strategic interaction between them and the less the disadvantage of going bankrupt becomes. Another assumption of the basic case is that the only debt a firm can issue is short term, lasting for one period. Provided that there is a positive nominal interest rate so that a possibility of bankruptcy at the end of the first period exists, it is clear that the results obtained above still hold even if long term debt can be used. (For a more detailed description of these extensions see Allen (1986).)

The analysis above is concerned with pure-strategy equilibria which always exist in this linear version of the model. In general, mixed-strategy equilibria may exist and these might have higher ex ante profits for the firms. Also, equilibria with coordinated strategies in which one or other firm randomly dominates the industry may exist which might have yet higher ex ante profits. The standard way in which these types of equilibria are made economically meaningful is the use of a "purification" argument (see, e.g., associating the random choice over the possible strategies with private information of the agents. The original equilibrium with stochastic behavior then corresponds to a pure-strategy equilibrium of the new game. Finding extensions of the model above with interesting specifications of private information which might implement such mixed-strategy and correlated equilibria is a topic for future research.

In conclusion, this paper models bankruptcy as causing a delay in investment which in itself is costless. If the product market is imperfectly competitive, this delay puts a bankrupt firm at a strategic disadvantage. In particular, the bankrupt firm is either partially or completely pushed out of the market and forced to liquidate by the solvent one. In such cases the total costs of bankruptcy include these costs of liquidation. In equilibrium firms determine their optimal capital structure by weighing the tax advantages of debt against its bankruptcy and liquidation costs.

For many years the most widely taught theories of capital structure have involved a trade-off of one sort or another. In essence the model presented above is also a trade-off theory. It is more complex than traditional theories in that it suggests firms take account of their strategic position within an industry. The effect of this is that bankruptcy leads to liquidation. As a result the theory provides a rationale for why liquidation costs should be included in bankruptcy costs and hence why these may be large. It is also consistent with the fact that similar firms in the same industry often have such different capital structures.

Appendix

I. Symmetric Equilibria

In this section symmetric equilibria of the model where firms are at interior optima are analyzed. It is shown that such equilibria can only exist when $m_F > m_L$.

The effect of changes in debt on profits are given in (25) and (27).

Next consider changes in capacity. Now,

$$\frac{\partial \alpha_j^*}{\partial q_j} = - \left(\frac{1 - \tau\delta}{1 - \tau}\right) \frac{D_j}{q_j^2} + \beta < \frac{\partial \alpha_k^*}{\partial q_j} = \beta \ . \tag{A1}$$

For increases in capacity (21) is therefore the relevant expression for $E\pi_{2j}$. Using the fact that $\alpha_j^* = \alpha_k^*$ it can be shown:

$$\frac{\partial E\pi_j^+}{\partial q_j} = (1 - \tau)\left[\bar{\alpha} - \mu - \beta(2q_j + q_k)\right] + m_L(\alpha_j^* - \alpha_\ell) \frac{D_j}{q_j^2} \ . \tag{A2}$$

For reductions in capacity (20) is relevant, so

$$\frac{\partial E\pi_j^-}{\partial q_j} = (1 - \tau)\left[\bar{\alpha} - \mu - \beta(2q_j + q_k)\right] + m_F(\alpha_j^* - \alpha_\ell) \frac{D_j}{q_j^2} \ . \tag{A3}$$

Hence

$$\frac{\partial E\pi_j^+}{\partial q_j} - \frac{\partial E\pi_j^-}{\partial q_j} = (m_L - m_F)(\alpha_j^* - \alpha_\ell) \frac{D_j}{q_j^2} < 0 \ . \tag{A4}$$

There is again a kink which is concave so that it can correspond to a maximum.

Similarly for k.

the point could not be a maximum.

When firm j increases its debt it goes bankrupt in more states than k and becomes the follower in these. When it decreases its capacity firm j also goes bankrupt in more states than k. Hence one extreme equilibrium is where $\partial E\pi_j^+/\partial D_j$ and $\partial E\pi_j^-/\partial q_j$ are both zero. This involves bankruptcy in the fewest possible states so that $\alpha_j^* = \alpha_k^*$ is at a minimum. Solving (25) and (A3) simultaneously and using (19) it can be shown:

$$\alpha_j^* - \alpha_\ell = \frac{\tau\delta\left[(1 - \tau\delta)(\bar{\alpha} - \mu) + \tau\delta\alpha_\ell + \tau^2\delta\mu/(1 - \tau)\right]}{(3 - \tau\delta)\beta m_F - (\tau\delta)^2} \tag{A5}$$

$$q_j = \frac{m_F(\alpha_j^* - \alpha_\ell)}{\tau\delta} \tag{A6}$$

$$D_j = \left(\frac{1 - \tau}{1 - \tau\delta}\right)\left[\alpha_j^* - 2\beta q_j + \tau\mu/(1 - \tau)\right]q_j \ . \tag{A7}$$

The other extreme involves $\partial E\pi_j^-/\partial D_j$ and $\partial E\pi_j^+/\partial q_j$ both equal to zero. The equilibrium is as in (A5)-(A7) except m_F is replaced by m_L. It involves bankruptcy in the most states so that $\alpha_j^* = \alpha_k^*$ is at a maximum.

It can be seen that there always exist solutions to (A5)-(A7). However, these only correspond to an interior equilibrium if

$$0 \leq \alpha_j^* - \alpha_\ell \leq 1 \ . \tag{A8}$$

Otherwise equilibrium involves corner solutions to the firms' debt or capacity choice problems.

II. Asymmetric Equilibria

In this section it is shown that asymmetric equilibria where firms are at interior optima can only exist when $m_F < m_L$. As in the text $\alpha_j^* > \alpha_k^*$.

conditions corresponding to debt choices are given by (32) and (33). For capacity choices they are:

$$\frac{\partial E\pi_j}{\partial q_j} = (1-\tau)\left[\bar{\alpha}-\mu-\beta(2q_j+q_k)\right] - \left(\frac{1-\tau}{1-\tau\delta}\right)\beta m_F(\alpha_j^*-\alpha_k^*) + m_F(\alpha_j^*-\alpha_\ell)\frac{D_j}{q_j^2} = 0 \qquad (A9)$$

$$\frac{\partial E\pi_k}{\partial q_k} = (1-\tau)\left[\bar{\alpha}-\mu-\beta(q_j+2q_k)\right] + \left(\frac{1-\tau}{1-\tau\delta}\right)\beta m_L(\alpha_j^*-\alpha_k^*) + m_L(\alpha_k^*-\alpha_\ell)\frac{D_k}{q_k^2} = 0\,. \qquad (A10)$$

Solving these simultaneously it can be shown

$$\alpha_j^* - \alpha_\ell = C\left(\beta m_F + \frac{\beta m_L}{\tau\delta} - \tau\delta\right) \qquad (A11)$$

$$\alpha_k^* - \alpha_\ell = C\left(\beta m_L + \frac{\beta m_F}{\tau\delta} - \tau\delta\right) \qquad (A12)$$

$$q_j = m_F\,\frac{(\alpha_j^* - \alpha_\ell)}{\tau\delta} \qquad (A13)$$

$$q_k = m_L\,\frac{(\alpha_k^* - \alpha_\ell)}{\tau\delta} \qquad (A14)$$

$$D_j = \left(\frac{1-\tau}{1-\tau\delta}\right)\left[\alpha_j^* - \beta(q_j + q_k) + \tau\mu/(1-\tau)\right]q_j \qquad (A15)$$

$$D_k = \left(\frac{1-\tau}{1-\tau\delta}\right)\left[\alpha_k^* - \beta(q_j + q_k) + \tau\mu/(1-\tau)\right]q_k \qquad (A16)$$

where

$$C = \{(1-\tau\delta)\bar{\alpha} - \left[1 - \tau\delta - \tau^2\delta/(1-\tau)\right]\mu + \tau\delta\alpha_\ell\}/ \qquad (A17)$$

$$\left[(\tau\delta - 2\beta m_F/\tau\delta)(\tau\delta - 2\beta m_L/\tau\delta) - \beta^2(m_F - m_L/\tau\delta)(m_L - m_F/\tau\delta)\right]\,.$$

If $C < 0$ then only corner solutions are possible. This follows from the fact that given (2) and (31) it is necessary for an interior solution that

This along with (A11) and (A12) implies that if $C < 0$ then

$$\tau\delta > (\beta m_L)^{\frac{1}{2}} \quad ; \quad \tau\delta > (\beta m_F)^{\frac{1}{2}} \ . \tag{A19}$$

Also (32), (33), and (A18) imply

$$q_j \leq \frac{m_F}{\tau\delta} \quad ; \quad q_k \leq \frac{m_L}{\tau\delta} \ . \tag{A20}$$

Using (A19) and (A20) together with the definitions of m_F and m_L, it can be shown that for all ϕ such that $0 < \phi < 2/9$,

$$\bar{\alpha} - \mu - \beta(q_j + 2q_k) > 0 \ . \tag{A21}$$

Taking this along with (3) and (31), it follows that (A10) cannot be satisfied since all the terms on the left-hand side are positive. Hence for interior asymmetric solutions it is necessary that

$$C > 0 \ . \tag{A22}$$

From (A11) and (A12)

$$\alpha_j^* - \alpha_k^* = - C(\frac{1}{\tau\delta} - 1)(m_F - m_L) \ . \tag{A23}$$

Using (29) and (A22), then since $0 < \tau\delta \leq 1$ it follows that if $m_F > m_L$, (A25) cannot be satisfied and no asymmetric equilibrium can exist.

Next consider the case in Section 3B where $m_F < m_L$. Here (A23) means that an interior asymmetric equilibrium can exist provided $\tau\delta$ is sufficiently small so that $C > 0$ and $\tau\delta \leq \beta(m_L + m_F/\tau\delta)$. For large $\tau\delta$ either one or both firms set $D_j = D_u$ in equilibrium and are at a corner solution to their optimization problems.

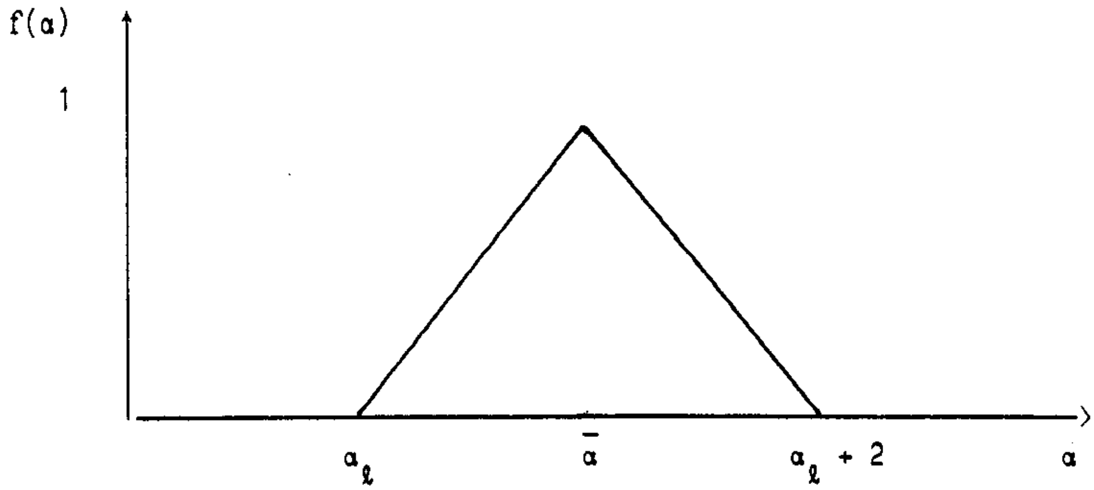

Figure 1

Density function for α.

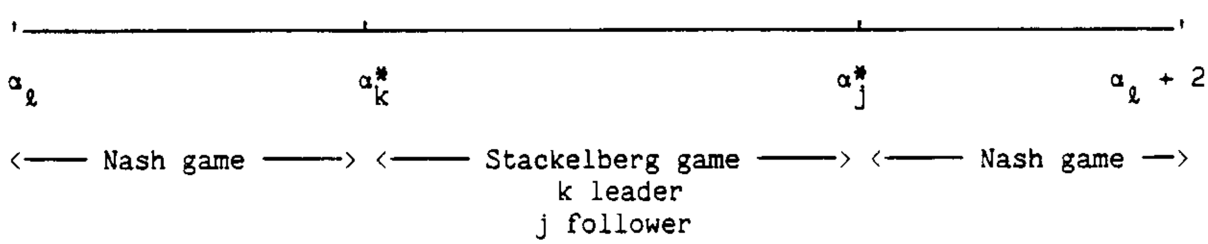

Figure 2

Relationship between first period α
and the second period game.

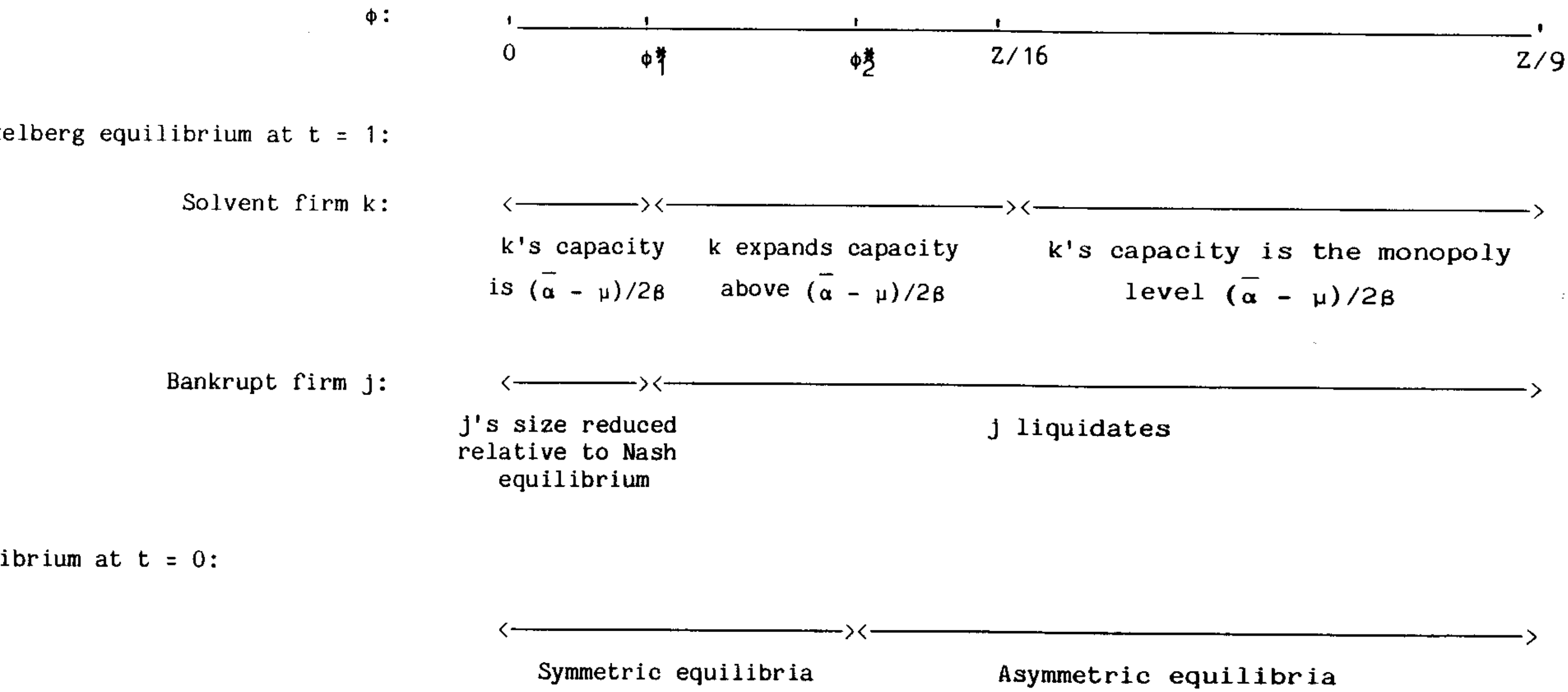

Figure 3

The relationship between fixed costs and equilibrium.

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