AN ANALYSIS OF FISCAL POLICY UNDER OPERATIVE AND INOPERATIVE BEQUEST MOTIVES

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September 1986

revised, March 1987

This paper was prepared for the Pinhas Sapir Conference on the Economic Effects of the Government's Budget, Tel Aviv University, December 22-24, 1986. Research support from the National Science Foundation and the Sloan Foundation is gratefully acknowledged. I thank Allan Drazen and Philippe Weil for helpful comments.

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Abstract

This paper presents a general equilibrium model with logarithmic preferences and technology. If the non-negativity constraint on bequests is strictly binding, then the bequest motive is characterized as inoperative. After determining the conditions for operative and inoperative bequest motives, the paper examines the effect of pay-as-you-go social security on the stochastic evolution of the capital stock. If the non-negativity constraint on bequests is strictly binding, then an increase in social security reduces the unconditional long-run expected capital stock. If the social security taxes and benefits are large enough, then the non-negativity constraint ceases to bind, and further increases in social security have no effect. This paper extends previous analyses by examining bequest behavior outside of the steady state and by allowing a non-degenerate cross-sectional distribution in the holding of capital.

It is well-known that if a consumer has an infinite horizon, then his consumption and saving behavior is invariant to changes in the timing of lump-sum taxes that leave the present value of his taxes unchanged. Barro (1974) argued that if consumers have operative altruistic bequest motives, then they behave as if they have infinite horizons. This important insight implies that the Ricardian Equivalence Theorem, which is the proposition that changes in the timing of lump-sum taxes have no effect, can hold even in an economy in which consumers have finite lifetimes. Since the appearance of Barro's seminal paper, there have been several challenges to the Ricardian Equivalence Theorem that have shown that even with an operative altruistic bequest motive, lump-sum tax changes can have an effect. Such effects arise if there are not complete insurance markets for stochastic fluctuations in labor income (Barsky, Mankiw and Zeldes(1986)), if there are pre-existing nonlinear taxes on wealth or property income (Abel (1986)), or if new consumers who do not receive bequests from current domestic consumers enter the economy (Weil(1986)).

This paper also analyzes a reason for departure from Ricardian Equivalence but focusses on a different channel than the research cited above. The assumption that the altruistic bequest motive is operative, which is a maintained hypothesis in the work mentioned above, will be critically examined in this paper. Specifically, I will assume that individual consumers are indeed altruistic with respect to their heirs and I will then determine, in a specific model, how strong the bequest motive must be in order to be operative. I will then show that lump-sum fiscal policy affects whether the bequest motive is operative and I will analyze the effects of fiscal policy when the bequest motive is not operative.

The question of whether the bequest motive is operative has received some attention in the literature. Drazen (1978) presented conditions on equilibrium marginal rates of substitution which must hold for the bequest motive to be operative, but in his general non-separable formulation of altruism, there is no single parameter, or set of paramters, which measures the strength of the bequest motive. In an elegantly simple analysis, Weil (1984) derived a lower bound for the strength of the bequest motive in order for there to be positive bequests in the steady state. This lower bound is not stated directly in terms of preferences and technology, but rather is expressed in terms of the steady state marginal product of capital in an economy without bequests. Weil's analysis is extended in Abel (1987) to determine conditions under which the gift motive (from child to parent) or the bequest motive (from parent to child) or neither will be operative. However, Weil's analysis and the extension are both confined largely to steady states. Weil did explore bequest behavior outside of the steady state but did not find a clean set of conditions which guarantee that the bequest motive would be operative for every generation, even for specific examples of preferences and technology. It must be emphasized, as noted by Weil, that the Ricardian Equivalence Theorem in general requires that the bequest motive be operative for every generation. Therefore, the determination of conditions under which the Ricardian Equivalence Theorem holds along the transition path remains an important open question.

In this paper I derive conditions for operative bequests everywhere along the transition path for a specific structure of preferences and technology. In particular, I restrict attention to a logarithmic utility function and a Cobb-Douglas production function and present conditions under which the bequest motive will always be operative, regardless of the initial level of capital intensity. It should be recalled that Weil also considered the case with a logarithmic utility function and a Cobb-Douglas production function but was able to show that the bequest motive is always operative only under the assumption that the initial capital stock was above a certain critical level. The difference between Weil's example and the model in this paper is that, unlike Weil's specification, my specification of the Cobb-Douglas production function essentially assumes complete depreciation of capital in one period. The importance of this assumption is simply technical: it implies that in equilibrium the logarithm of the capital stock follows a linear (stochastic) difference equation whereas in Weil's specification the evolution of the capital is not log-linear. This log-linearity permits the derivation of simple conditions for operative bequests.

Recently, Cukierman (1986) and Feldstein (1986) have each analyzed binding non-negativity constraints on bequests when individual consumers face uncertainty. In Cukierman (1986), young consumers are uncertain about their state health in old age (modeled formally as uncertainty about tastes) and thus are uncertain about whether they will ultimately want to leave a positive bequest. In Feldstein (1986) young consumers are also uncertain about whether they will ultimately want to leave a positive bequest but this uncertainty arises from uncertainty about income in old age. The prospect that each young consumer may ultimately face a strictly binding constraint on bequests leads to a violation of the Ricardian Equivalence Thereom because consumers who are constrained would prefer shifting taxes onto future generations.

In this paper I introduce uncertainty by making the production function subject to random shocks. It turns out that with the specfication of preferences and technology in this paper, each young consumer knows whether his bequest motive will be operative in old age. Although this formulation ignores an interesting aspect of the individual decision problem that is emphasized in the partial equilibrium analyses of Cukierman and Feldstein, it permits aggregation across consumers with heterogeneous wealth. I exploit the easy aggregation in the model to analyze the general equilbirium effects of policy on endogenous factor prices as well as on the individual's decision problem.

In addition to introducing uncertainty, this analysis departs from the now-standard representative consumer framework by allowing for a non-degenerate cross-sectional distribution of capital holdings. In this simple model, I can study the evolution of the distribution of wealth. In addition, I can determine how much variation in wealth is compatible with the requirement that all consumers have operative bequest motives.

In section I I present a model with a stochastic production function and with altruistic consumers. I then derive the optimal saving and consumption rules for an artificial decision problem in which the old consumer is given control over the wage income of this children. This artificial decision problem ignores the non-negativity constraint on bequests. In section II I analyze conditions under which bequests will in fact be non-negative and present restrictions which are sufficient to guarantee that the bequest motive will be operative in every period. In section III I analyze individual and aggregate behavior when the bequest motive is not operative and in section IV I present restrictions which guarantee that the bequest motive will never be operative. The relation between dynamic efficiency and the possibility of operative bequests is analyzed in section V. Section VI examines a laissez faire economy without taxes or transfers and analyzes bequest behavior in the presence of cross-sectional variation in wealth. In section VII, I confine attention to a representative consumer economy and examine the effects of changes in the tax and transfer system on the stochastic evolution of the capital stock. Section VIII concludes the paper.

I. Consumption and Capital Accumulation in the Absence of Non-negativity Constraints

Consider an overlapping generations economy in which each consumer lives for two periods and gives birth to n heirs at the beginning of the second period. Each consumer inelastically supplies one unit of labor when young and does not work when old. A consumer born at the beginning of period t receives a competitive wage W_t and consumes an amount c_t^y in period t when he is young and consumes c_{t+1}^o in period t+1 when he is old. Let k_t be the stock of capital (per worker) held by an individual family at the beginning of period t. This capital is actually owned by the old consumers in the family and represents their saving from the previous period. Let R_t be the gross rate of return on capital held from period t-1 to period t. Therefore the capital stock held by the family evolves according to

$$nk_{t+1} = R_t k_t + W_t - c_t^y - c_t^o/n \tag{1}$$

Equation (1) states that the amount of capital carried into period t+1 is equal to the family's total income in period t, $R_t k_t + W_t$, minus the consumption of the young and old consumers. Let K_t, C_t^y, and C_t^o, denote the economy-wide average values of k_t, c_t^y and c_t^o, respectively. With competitive factor markets, all families face the same wage rate, W_t, and the same return to capital R_t. Therefore, since (1) is linear in k_t, c_t^y and c_t^o,

$$nK_{t+1} = R_t K_t + W_t - C_t^y - C_t^o/n \tag{2}$$

Suppose that consumers have altruistic bequest motives. Letting U_t be the utility of a consumer born at the beginning of period t, we specify the utility function to be

$$U_t = E_t\{ u(c_t^y, c_{t+1}^o) + \delta U_{t+1}\} \qquad 0 \leq \delta < 1 \tag{3}$$

where $E_t\{\ \}$ denotes the expectation conditional on information available at the beginning of period t. The parameter δ, which is assumed to be nonnegative and less than one, measures the strength of the bequest motive. One goal of this analysis is to determine how large δ must be in order for the bequest motive to be operative.

Now suppose that the utility function is logarithmic

$$u(c_t^y, c_{t+1}^o) = \beta \ln c_t^y + (1-\beta) \ln c_{t+1}^o \qquad 0 < \beta < 1 \tag{4}$$

and that the production function is Cobb-Douglas

$$Y_t = \psi_t K_t^\alpha \tag{5}$$

where Y_t is output per worker and ψ_t is a positive random variable. With the Cobb-Douglas production function in (5) the competitive wage rate is

$$W_t = (1-\alpha)\psi_t K_t^\alpha \tag{6}$$

and the competitive rate of return on capital is

$$R_t = \alpha \psi_t K_t^{\alpha-1} \tag{7}$$

It is useful first to consider the artifical decision problem in which the old consumer maximizes his altruistic utility function (3) subject to the family's budget constraint in (1). This problem is artificial in that the old consumer is allowed to consume some or all of the wage income of his children. This decision problem can be solved using the value function

$$V(k_t, \psi_t, K_t) = \max[(1-\beta)\ln c_t^o + \delta\beta \ln c_t^y + \delta E_t\{V(k_{t+1}, \psi_{t+1}, K_{t+1})\}] \tag{8}$$

where the maximization in (8) is with respect to c_t^o, c_t^y and k_{t+1} and is subject to the family and aggregate capital accumulation constraints in (1) and (2).

The value function is the expected present value of utility from the old consumer's own consumption when old plus the utility the old consumer obtains from his heirs' utility. The value function is a solution to the functional equation in (8). I have used the method of undetermined coefficients to solve (8). Because the solution procedure is neither novel nor instructive, and because the solution is easily verified, I will simply present a solution to the functional equation

$$V(k_t, \psi_t, K_t) = [(1-\beta+\delta\beta)/(1-\delta)] \ln[(1-\alpha)K_t + (1-\delta)\alpha k_t] + \phi \ln K_t + J(\psi_t) + D \quad (9)$$

where $\phi = -(1-\beta+\delta\beta)(1-\alpha)/[(1-\delta)(1-\delta\alpha)]$

and $J(\psi_t) = [(1-\beta+\delta\beta)/(1-\delta\alpha)] \ln \psi_t + \delta E_t\{J(\psi_{t+1})\}$

and D is an unimportant constant.

Using equation (9) it is straightforward but tedious to derive the optimal consumption and capital accumulation for the artificial decision problem and to derive the behavior of aggregate consumption and capital accumulation. The behavior of an individual family is given by

$$c_t^y = [\delta\beta/(1-\beta+\delta\beta)] \psi_t K_t^{\alpha-1}\{(1-\alpha)K_t + (1-\delta)\alpha k_t]\} \quad (10)$$

$$c_t^o/n = [(1-\beta)/(1-\beta+\delta\beta)] \psi_t K_t^{\alpha-1}\{(1-\alpha)K_t + (1-\delta)\alpha k_t]\} \quad (11)$$

$$nk_{t+1} = \alpha\delta\psi_t K_t^{\alpha-1} k_t \quad (12)$$

To illustrate the "artificial" nature of this problem, observe that if $\delta = 0$, then (10)-(12) imply that $c_t^y = 0$, $c_t^o = n(W_t + R_t k_t)$ and $k_{t+1} = 0$. That is, if the consumer does not care about the utility of his children, then he will consume the family's entire income, including his children's wages. He would neither save nor allocate any current consumption to his children. Clearly, this allocation would imply a negative bequest. As shown in section II, the consumer will make a positive bequest if δ is sufficiently large.

Because the optimal consumption and capital accumulation decision rules are linear in k_t, it is easy to aggregate these rules to obtain

$$C_t^y = [\delta\beta(1-\delta\alpha)/(1-\beta+\delta\beta)] \psi_t K_t^{\alpha} \quad (13)$$

$$C_t^o/n = [(1-\beta)(1-\delta\alpha)/(1-\beta+\delta\beta)] \psi_t K_t^{\alpha} \quad (14)$$

$$nK_{t+1} = \alpha\delta\psi_t K_t^{\alpha} \quad (15)$$

By distinguishing an individual family's holding of capital, k_t, from average capital per worker, K_t, I have allowed for cross-sectional variation in k_t. Observe that with the assumed specification of preferences and technology, the cross-sectional distribution of capital remains fixed over time. More precisely, dividing (12) by (15) yields $k_{t+1}/K_{t+1} = k_t/K_t$ so that any initial inequality in the distribution of capital is preserved forever.

II. The Nonnegativity Constraint on Bequests

The formal analysis to this point has ignored any nonnegativity constraint on bequests. In this section we determine how strong the bequest motive must be in order to be operative, i.e., for the non-negativity constraint to be non-binding. Let y_t^0 be the disposable resources available to an old consumer in period t and let b_t be the bequest left by this consumer so that

$$b_t = y_t^0 - c_t^0 \tag{16}$$

Suppose that there is a permanent tax and transfer system which taxes wage income at rate τ and uses the tax revenues τW_t to finance a lump-sum transfer of $n\tau W_t$ to each old consumer in period t. If τ is positive, then the tax and transfer scheme is a pay-as-you-go social security system. Because labor supply is inelastic, the tax is non-distortionary. Furthermore, since the taxes paid by the young consumers in each family are equal to the transfers received by the old consumers in that family, this scheme has no effect on the present value of taxes paid by any family. Therefore, if the bequest motive is always operative, then the path of consumption and capital accumulation is invariant to τ. The optimal consumption and capital accumulation rules presented for the artificial decision problem is section I continue to hold in the presence of this tax and transfer scheme.

The disposable resources of an old consumer consist of the gross return on his capital as well as the fiscal subsidy he receives so that

$$y_t^0 = n\psi_t K_t^{\alpha-1}\{(1-\alpha)\tau K_t + \alpha k_t\} \tag{17}$$

Substituting (11) and (17) into (16) yields an expression for the bequest that an old consumer in period t would like to leave

$$b_t = (1-\beta+\delta\beta)^{-1} n\psi_t K_t^{\alpha} \{(1-\alpha)[\tau\delta\beta - (1-\beta)(1-\tau)] + \alpha\delta(k_t/K_t)\} \tag{18}$$

The desired bequest will be positive if and only if the term in curly brackets on the right hand side of (18) is positive. Recall that in an equilibrium in which all families have operative bequests, each family's k_t/K_t is constant over time so that the condition that the right hand side of (18) be positive is time-invariant.

It is convenient to express the condition that the bequest motive be operative in terms of how strong the bequest motive must be as measured by δ. Define $\delta^0 = \delta^0(\tau, k_t/K_t)$ where

$$\delta^0(\tau, k_t/K_t) = (1-\alpha)(1-\beta)(1-\tau)/[\alpha k_t/K_t + (1-\alpha)\tau\beta] \qquad (19)$$

Observe that $\delta > \delta^0$ is a necessary and sufficient condition for the right hand side of (18) to be positive. Therefore, if $\delta > \delta^0(\tau, k_t/K_t)$ for all relevant values of k_t/K_t, then the bequest motive is operative for all families.

III. Inoperative Bequests

In this section I analyze the dynamic behavior of an economy in which the bequest motive is not operative. It might appear that to analyze an economy with an inoperative bequest motive, one can simply analyze the behavior of a standard Diamond (1965) model. However, in general, this strategy would not be appropriate because it is possible that if though the bequest motive is currently inoperative, it may become operative at some date in the future. Therefore, to describe the dynamics of an economy with a currently inoperative bequest motive, I must use a procedure that allows for the possibility that the bequest motive will be operative at some future date(s). It turns out that for the particular preferences and technology assumed in this paper, the bequest motive will always be operative or will always be inoperative in a representative consumer economy, but this is a result to be derived from studying the model and should not be assumed at the outset.

The decision problem facing an old consumer in period t can be solved using the value function. Indeed, the functional equation (8) applies to consumers with inoperative bequest motives. However, because the constraint $b_t \geq 0$ is binding, the solution to the functional equation $V(k_t, \psi_t, K_t)$ differs from that in (9). It can be verified that the value function in this case is

$$V(k_t, \psi_t, K_t) = (1-\beta)\ln[(1-\alpha)\tau K_t + \alpha k_t] + d\ln K_t + H(\psi_t) + E \qquad (20)$$

where $\quad d = [\delta\alpha - (1-\alpha)(1-\beta)]/(1-\alpha\delta)$

$$H(\psi_t) = [1 - \beta + \delta + \delta d]\ln\psi_t + \delta E_t\{H(\psi_{t+1})\}$$

and E is an unimportant constant which depends on the parameters of preferences and technology as well as on the tax rate τ.

Using the value function in (20) it is straightforward to derive the optimal consumption and capital accumulation for an individual family with capital per worker k_t

$$c_t^y = \beta(1-\alpha)(1-\tau)[(\alpha+(1-\alpha)\tau)/(\alpha+(1-\alpha)\beta\tau)]\psi_t K_t^\alpha \quad (21)$$

$$c_t^o/n = y_t^o/n = \psi_t K_t^{\alpha-1}[\alpha k_t + (1-\alpha)\tau K_t] \quad (22)$$

$$nk_{t+1} = [\alpha(1-\alpha)(1-\beta)(1-\tau)/(\alpha+(1-\alpha)\beta\tau)]\psi_t K_t^\alpha \quad (23)$$

With a binding constraint on bequests, the consumption of each old consumer is equal to his disposable income y_t^o. If there is cross-sectional variation in y_t^o, and if all consumers face binding nonnegativity constraints on bequests, then there will of course be cross-sectional variation in c_t^o; however, there will be no cross-sectional variation in the consumption of young consumers or in the accumulation of capital for the next period. Any cross-sectional variation in wealth is eliminated in one period. The reason for this strong result is that the only source of cross-sectional variation is the variation in the initial holdings of capital. If all consumers leave zero bequests, then this inequality in the distribution of wealth is not transmitted to subsequent generations. It should be noted that this result contrasts sharply with the result that under operative bequests any inequality in the distribution of wealth is preserved forever.

Aggregate consumption and capital accumulation are easily calculated from (21) - (23) to be

$$C_t^y = \beta(1-\alpha)(1-\tau)[(\alpha+(1-\alpha)\tau)/(\alpha+(1-\alpha)\beta\tau)]\psi_t K_t^\alpha \quad (24)$$

$$C_t^o/n = \psi_t K_t^\alpha[\alpha+(1-\alpha)\tau] \quad (25)$$

$$nK_{t+1} = [\alpha(1-\alpha)(1-\beta)(1-\tau)/(\alpha+(1-\alpha)\beta\tau)]\psi_t K_t^\alpha \quad (26)$$

IV. The Nonnegativity Constraint on Bequests

I now determine under what conditions the nonnegativty constraint on bequests will be binding. Formally, I could derive the optimal consumption and capital accumulation rules by substituting (1) and (2) into (20) and then performing the indicated maximization in the functional equation (8). Letting θ_t be the Lagrange multiplier associated with the constraint $b_t = y_t^0 - c_t^0 \geq 0$, the Kuhn-Tucker conditions are

$$\delta\beta/c_t^y = \delta\alpha(1-\beta)/[(1-\alpha)\tau K_{t+1} + \alpha k_{t+1}]/n \tag{27}$$

$$(1-\beta)/c_t^0 = \delta\alpha(1-\beta)/[(1-\alpha)\tau K_{t+1} + \alpha k_{t+1}]/n^2 + \theta_t \tag{28}$$

$$\theta_t(y_t^0 - c_t^0) = 0 \tag{29}$$

$$\theta_t \geq 0 \tag{30}$$

Conditions (27) and (28) are obtained by differentiating with respect to c_t^y and c_t^0, respectively. Substituting (27) into (28) yields the simpler expression

$$(1-\beta)/c_t^0 = \delta\beta/nc_t^y + \theta_t \tag{31}$$

When the nonnegativity constraint is strictly binding, the Lagrange multiplier θ_t is positive. In this case, (31) indicates that the appropriately-weighted marginal utility of the old consumer's consumption exceeds the appropriately-weighted marginal utility of the young consumer's current consumption. Thus, the appropriately-weighted sum of utilities could be increased if some consumption could be shifted from the young consumer to the old consumer by a negative bequest. However, because the nonnegativity constraint is binding, this reallocation is not possible.

When the non-negativity constraint binds, $c_t^0 = y_t^0$ and $\theta_t > 0$ so that (31) implies that

$$(1-\beta)\, c_t^y > \delta\beta\, y_t^0/n \tag{32}$$

Substituting (17) and (21) into (32) motivates the definition of δ^c, the critical value of the bequest motive parameter, as $\delta^c = \delta^c(\tau,k_t/K_t)$ where

$$\delta^c(\tau,k_t/K_t) \equiv [(1-\beta)(1-\alpha)(1-\tau)/(\alpha+(1-\alpha)\beta\tau)] \times [(\alpha+(1-\alpha)\tau)/(\alpha k_t/K_t + (1-\alpha)\tau)] \tag{33}$$

Observe that $\delta < \delta^c(\tau,k_t/K_t)$ is a necessary and sufficient condition for (32) to hold. This condition is applicable in an equilibrium in which all old consumers face binding non-negativity constraints.

V. Dynamic Efficiency

Abel, Mankiw, Summers and Zeckhauser (1986) have shown that if, in a competitive stochastic economy, the rate of return on some asset is always less than the growth rate of the aggregate capital stock, then the economy is dynamically inefficient in the sense that it suffers from an inefficient overaccumulation of capital. If the rate of return on some asset always exceeds the growth rate of the aggregate capital stock, then the economy is dynamically efficient.

To determine whether the economy is dynamically efficient, observe from (7) and (15) that the growth rate of the aggregate capital stock in the economy with operative bequests, G_t^o, is

$$G_t^o \equiv (nK_{t+1}/K_t)^o = \delta R_t^o \tag{34}$$

where the superscript "o" denotes the equilibrium value of a variable in the economy with operative bequests. Under the assumption that δ is less than 1, it follows immediately from (34) that $R_t^o > G_t^o$ for all t, and hence the economy with operative bequests is dynamically efficient. Of course, the dynamic efficiency of the economy in which the bequest motive is always operative is to be expected because the consumers behave as if they have infinite horizons.

The growth rate of the aggregate capital stock in the economy without bequests, G_t^c, can be calculated from (7), (19) and (26) to be

$$G_t^c \equiv (nK_{t+1}/K_t)^c = \delta^o(\tau,1)\ R_t^c \tag{35}$$

where the superscript "c" denotes the equilibrium value of a variable in the economy with constrained bequests. Equation (35) implies that the ratio G_t^c/R_t^c is constant and equal to $\delta^o(\tau,1)$. If $\delta^o(\tau,1)$ is less than or equal to one, then the economy without operative bequests is efficient; if $\delta^o(\tau,1)$ is greater than one, the economy is inefficient. Because in this model dynamic inefficiency implies $\delta^o(\tau,1) > 1$ and because a positive bequest requires $\delta > \delta^o(\tau,1)$, there is no admissable vlaue of δ under with bequests will be positive if the no-bequest economy is inefficient. This result was originally derived by Weil (1984). Weil's result is more general in that it is not restricted to logarithmic utility and Cobb-Douglas production functions; however, his result is less general in that he did not consider stochastic economies and, more importantly, his result could not be applied everywhere along the transition path.

VI. Bequests in the Absence of Fiscal Transfers

In this section I examine bequest behavior in the competitive economy without fiscal transfers ($\tau = 0$). Observe from (19) and (33) that when $\tau = 0$

$$\delta^{o}(0, k_t/K_t) = \delta^{c}(0, k_t/K_t) = (1-\alpha)(1-\beta)K_t/(\alpha k_t) \tag{36}$$

Although the critical values $\delta^{o}(\tau, k_t/K_t)$ and $\delta^{c}(\tau, k_t/K_t)$ are not, in general, equal to each other, equation (36) states that in the absence of taxes, these critical values are equal for all values of k_t/K_t. The critical value is a declining function of k_t/K_t which illustrates that a stronger bequest motive is required in order to induce a poorer consumer to leave a positive bequest.

The critical values of the bequest parameter were derived under the alternative assumptions that all consumers have operative bequest motives or that all consumers face binding nonnegativity constraints on their bequests. However, if there is a non-degenerate cross-sectional distribution of capital holding, then there is a range of values of δ for which neither of these assumptions is satisfied. This range depends on the range of values of k_t/K_t in the population as illustrated in Figure 1. If δ is greater than $(1-\alpha)(1-\beta)K_t/\alpha k_t^{min}$, where k_t^{min} is the minimum value of k_t, then the bequest is operative for all families and hence the

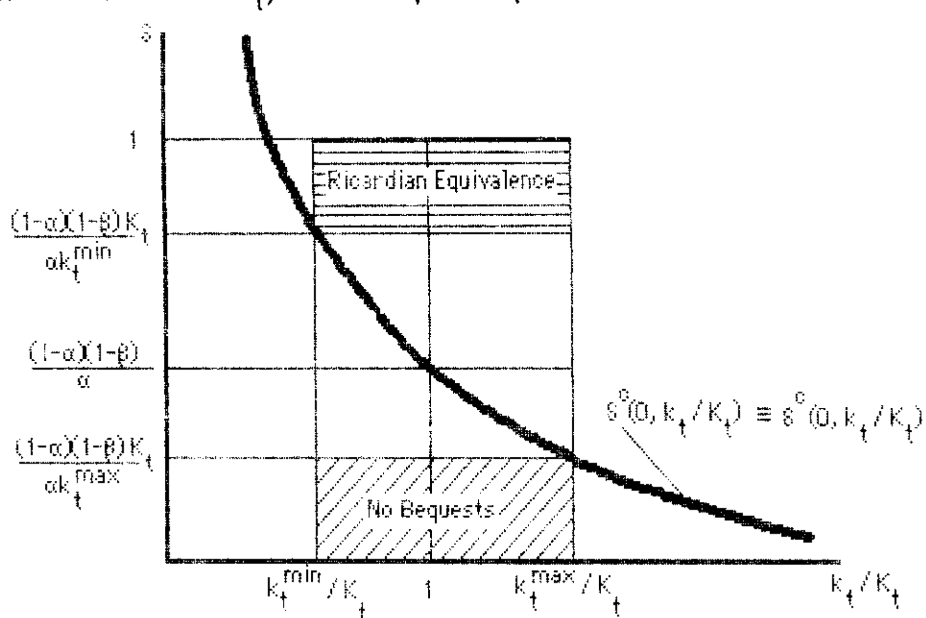

Figure 1

Ricardian Equivalence Theorem holds. This range of values of δ and the corresponding values of k_t/K_t are shown in the shaded region of Figure 1 labelled Ricardian Equivalence. Alternatively, if the bequest motive is sufficiently weak so that δ is less than $(1-\alpha)(1-\beta)K_t/\alpha k_t^{max}$, not even the richest families in the economy will leave positive bequests. In this case, there will be no bequests. As discussed earlier, if there are no positive bequests, then any inequality in the distribution of capital is eradicated completely after one period and k_t/K_t equals one for all families in subsequent periods.

VII. The Effects of Taxes and Transfers in a Representative Consumer Economy

In this section we analyze the effects of permanent changes in τ, the tax rate on labor income which finances pay-as-you-go social security. For simplicity I consider only representative consumer economies, i.e., economies in which the cross-sectional distribution of wealth is degenerate so that $k_t/K_t \equiv 1$ for all families. It follows immediately from (19) and (33) that if $k_t/K_t = 1$, the critical values of the bequest parameter, δ^0 and δ^c, are equal to each other.

To derive the stochastic process governing the evolution of the capital stock in the economy with operative bequests, substitute (7) into (15) and take logarithms to obtain

$$\ln K_{t+1}{}^0 = \ln \delta + \ln(\alpha/n) + \alpha \ln K_t{}^0 + \ln \psi_t \tag{37}$$

The unconditional mean of the aggregate capital stock per worker, $E\{\ln K^0\}$, is equal to $[\ln \delta + \ln(\alpha/n) + E\{\ln \psi\}]/(1-\alpha)$ where $E\{\ln \psi\}$ is the unconditional mean of $\ln \psi$.

If the bequest motive is not operative, then it follows from (7) and (35) that

$$\ln K_{t+1}{}^c = \ln \delta^0(\tau,1) + \ln(\alpha/n) + \alpha \ln K_t{}^c + \ln \psi_t \tag{38}$$

The stochastic process followed by the capital stock in the absence of bequests ,(38), is identical to the stochastic process followed by the capital stock the presence of bequests, (37), except that the unconditional mean, $E\{\ln K^c\}$, is equal to $[\ln \delta^0(\tau,1) + \ln(\alpha/n) + E\{\ln \psi\}]/(1-\alpha)$ rather than to $[\ln \delta + \ln(\alpha/n) + E\{\ln \psi\}]/(1-\alpha)$.

Now consider the effects of permanent changes in τ on the stochastic process for capital. According to (37), if the bequest motive is operative, then the stochastic process for capital is invariant to τ, as predicted by the Ricardian Equivalence Theorem. If the bequest motive is inoperative, then according to (38) all autocovariances of the stochastic process for capital are invariant to τ. However, since $\delta^0(\tau,1)$ is a decreasing function of τ, the unconditional mean of $\ln K_t$ is a decreasing function of τ. Thus, if the bequest motive is inoperative, then a permanent increase in pay-as-you-go social security reduces the long-run expected value of $\ln K_t$. However, if the tax rate τ becomes sufficiently large, then eventually old consumers will have sufficiently large disposable resources that the bequest motive becomes operative. At this point, further increases in τ would have no effect.

The effects of changes in τ are illustrated in Figure 2.

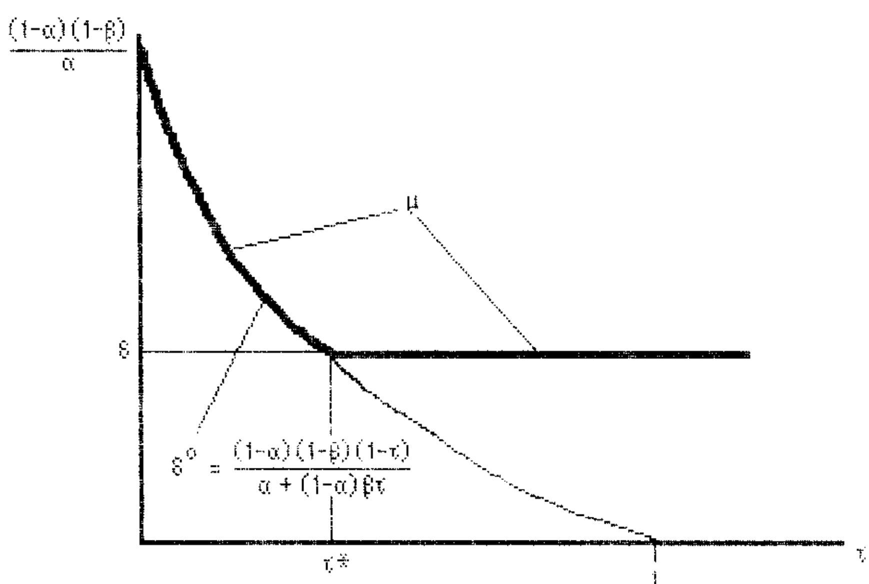

Figure 2

Define μ as

$$\mu \equiv \exp[(1-\alpha)E\{\ln K\} - \ln(\alpha/n) - E\{\ln\psi\}] \tag{39}$$

and observe that μ is an increasing function of the unconditional expectation of $\ln K$. If the bequest motive is operative, then, as discussed above, μ is equal to δ and $\delta > \delta^0(\tau,1)$. Alternatively, if the bequest motive is not operative, then μ is equal to $\delta^0(\tau,1)$ and $\delta < \delta^0(\tau,1)$. Therefore, we have

$$\mu = \max[\delta, \delta^0(\tau,1)] \tag{40}$$

The heavy line in Figure shows the value of μ as a function of the tax rate τ. This figure is drawn under the assumption that in the absence of taxes, the bequest motive is inoperative. This assumption is illustrated in the figure by the fact that $\delta^0(0,1) > \delta$. As the tax rate τ is increased from zero, the value of μ, and hence the expected long-run capital stock, falls monotonically as predicted from previous analyses of the Diamond model. Eventually when τ reaches τ^*, the old consumers are receiving large enough transfers that the bequest motive now becomes operative. Any increases in τ beyond τ^* will have no effect on the stochastic behavior of the capital stock.

VIII. Conclusion

Barro's (1974) formulation of intergenerationally altruistic consumers has become the basis for a widely used framework to study competitive economies with overlapping generations of consumers. Much of the subsequent work in this tradition has been conducted in deterministic models with a representative consumer in each generation, and the bequest motive is often simply assumed to be operative. The model in this paper was developed to relax these three sets of restrictions with the goal of understanding channels by which lump-sum taxes and transfers can affect economic activity. I derived conditions for the bequest motive to be operative and expressed these conditions in two different ways: first, I expressed the conditions in terms of the parameters of preferences and technology; then, as in Weil (1984), I expressed these conditions in terms of the rate of return on capital and the growth rate of the capital stock in an economy without bequests.

After determining conditions under which the altruistic bequest motive will be operative, I then examined the effects of a pay-as-you-go social security system financed by a proportional tax on (exogenous) wage income. If the bequest motive is initially inoperative, then the introduction of social security increases the consumption of old consumers and reduces the unconditional capital stock. Further increases in social security will continue to reduce the unconditional capital stock until eventually old consumers receive a large enough transfer that the bequest motive becomes operative. Once this point is reached, further increases in social security have no effect on consumption or capital accumulation.

An additional feature of the model examined in this paper is that we can examine an economy with cross-sectional variation in the distribution of capital holdings. If the bequest motive is operative for all families, then, in the particular model examined in this paper, the initial inequality in the distribution of capital holdings is preserved forever. By contrast, if the bequest motive is inoperative, then any inequality in capital holdings is eradicated after one generation.

This paper departs from the representative consumer framework and presents conditions which guarantee that the bequest motive will always be operative for all consumers or, alternatively, will always be inoperative for all consumers. An interesting extension of this research would be to analyze the behavior of an economy in which some consumers have operative bequest motives while other consumers face binding constraints on bequests. At this stage, we can say that the Ricardian Equivalence Theorem would not hold in such an economy, but the effects of fiscal policy in such an economy merit further study.

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