# OPERATIVE GIFT AND BEQUEST MOTIVES

by

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Abstract

The Ricardian Equivalence Theorem, which is the proposition that changes in the timing of lump-sum taxes have no effect on consumption or capital accumulation, depends on the existence of operative altruistic motives for intergenerational transfers. These transfers can be bequests from parents to children or gifts from children to parents. In order for the Ricardian Equivalence Theorem to hold, one of these transfer motives must be operative in the sense that the level of the transfer is not determined by a corner solution resulting from a binding non-negativity constraint. This paper derives conditions that determine whether the bequest motive will be operative, the gift motive will be operative, or neither motive will be operative in a model in which consumers are altruistic toward their parents and their children.

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In a pioneering paper, Barro (1974) demonstrated that if consumers have operative altruistic bequest motives, then a reduction in lump-sum taxes accompanied by the issue of an equal amount of government bonds has no effect on the allocation of resources. Barro stressed that this result, which has come to be known as the Ricardian Equivalence Theorem, requires that the bequest motive be operative. In this context, the term "operative" means that equilibrium bequests are determined by tangency conditions rather than by corner solutions such as may arise from binding non-negativity constraints. If the bequest motive is not operative, then the Ricardian equivalence result presented by Barro does not hold and there are important effects associated with the government's choice between debt finance and taxes.

More recently, Buiter (1979) and Carmichael (1982) have analyzed the altruistic gift motive in which consumers obtain utility from the utility of their parents and thus may be motivated to give resources to their parents. Their analyses confirm Barro's claim (p. 1104) that if the gift motive is operative, then the Ricardian Equivalence Theorem holds. If the gift motive is not operative, then the Ricardian Equivalence Theorem fails to hold.

Because the Ricardian Equivalence Theorem depends on an operative motive for private intergenerational transfers, it is important to determine the conditions under which either transfer motive will be operative. Several papers have studied whether the bequest motive is operative in a variety of different models[1] but the literature does not contain an analysis of the conditions which determine whether the gift motive is operative. In this paper, I will study the conditions for an operative gift motive. However, rather than confine the analysis to a model in which consumers have only a gift motive, I will assume that individual consumers have two-sided transfer motives. That is, I will assume that individual consumers have both a gift motive and a bequest motive as in Burbidge (1983), Buiter and Carmichael(1984) and Burbidge (1984). In the steady state equilibrium, the gift motive may be operative, the bequest motive may be operative, or neither motive may be operative. If either of the intergenerational transfer motives is operative, then the Ricardian Equivalence Theorem holds; however, if neither motive is operative, then changes in the timing of lump-sum taxes have important effects on the intertemporal and intergenerational allocation of resources.[3]

The major goal of this paper is to determine conditions under which each of the intergenerational transfer motives is operative if individual consumers have two-sided transfer motives. As a prerequisite to this analysis, I will discuss, in section I, appropriate restrictions on the gift motive and the bequest motive. In section II, I will discuss the restrictions on two-sided transfer motives implied by intergenerational consistency. The specification of the motives for intergenerational transfers has important implications for a wide range of issues extending beyond the effects of fiscal policy, including the intergenerational transmission of inequality[4] and for the behavior of financial markets, especially markets for life insurance and annuities.[5] In section III I discuss the endogenous determination of equilibrium factor prices and then describe the steady state equilibrium. The conditions under which one or the other of the transfer motives is operative are derived in section IV. Concluding remarks are presented in section V.

## I. A Two-Sided Transfer Motive

In this section I present a two-sided transfer motive and discuss appropriate restrictions on the parameters of the transfer motive. Consider a representative consumer economy in which each consumer lives for two periods. A generation t consumer is born at the beginning of period t, consumes $c_{1t}$ in period t at age 1 and consumes $c_{2t+1}$ in period t+1 at age 2. Let $u_t = u(c_{1t},c_{2t+1})$ be the utility that a generation t consumer obtains directly from his own consumption. Defining $u_{1t}$ as $\partial u(c_{1t},c_{2t+1})/\partial c_{1t}$ and $u_{2t+1}$ as $\partial u(c_{1t},c_{2t+1})/\partial c_{2t+1}$, assume that $u_{1t} > 0$, $u_{2t+1} > 0$ and that $u_{1t}(0,\cdot) = \infty = u_{2t+1}(\cdot,0)$. Also, assume that $u(\cdot,\cdot)$ is strictly concave and that $c_{1t}$ and $c_{2t+1}$ are normal goods.

In addition to obtaining utility directly from his own consumption, a generation t consumer obtains utilility from the consumption of his parents and from the consumption of all of his descendents. In particular, I will use the Buiter-Carmichael(1984) generalization of the Burbidge(1983) two-sided utility function

$$v_t = u_t + \alpha u_{t-1} + \sum_{j=1}^{\infty} \beta^j u_{t+j} \qquad (1)$$

The parameter $\beta$ measures the strength of the bequest motive and satisfies the restriction $0 \leq \beta < 1$. The assumption that $\beta$ must be less than one is the standard assumption in the literature[6] and is necessary and sufficient for the transversality condition to hold in the steady state with constant per capita consumption. The nonnegative parameter $\alpha$ measures the strength of the gift motive. There is no compelling reason to restrict $\alpha$ to be less than one.[7,8] I will show in section II that intergenerational consistency (defined below) places an upper bound on the admissable values of $\alpha$, but depending on the value of $\beta$, this upper bound may be greater than, equal to, or less than one.

The two-sided utility function in (1) nests both the one-sided altruistic bequest motive and the one-sided gift motive. The one-sided altruistic bequest motive is often specified recursively as

$$v_t = u_t + \beta v_{t+1} \qquad (2)$$

When $\alpha=0$, the utility function in (1) is consistent with the recursively specified altruistic bequest motive in (2).[9]

The one-sided gift motive is often specified recursively as $v_t = u_t + \alpha v_{t-1}$ which can be rewritten as

$$v_t = u_t + \alpha u_{t-1} + \alpha^2 v_{t-2} \qquad (3)$$

From the point of view of the generation t consumer with the one-sided gift motive in (3), the utility of his grandparent, $v_{t-2}$, is fixed; maximization of the utility function in (3) is equivalent to maximization of the utility function in (1) when $\beta = 0$. Thus, the utility function in (1) nests the one-sided altruistic bequest motive and the one-sided altruistic gift motive.[10]

Before presenting the consumer's budget constraint, it is necessary to describe the demographic composition of dynastic families. Each consumer lives for two periods and has $n \geq 1$ children at the beginning of the second period of his life. This assumption follows the standard convention of ignoring the fact that it takes two people from different families to produce children.[11] In the model, each consumer has n children and has one parent.[12]

Let $g_t$ be the gift given by a generation t consumer to his parent who is a generation t-1 consumer. This gift is made during period t which is the only period during which both generations are alive. Because the generation t consumer has one parent and n children, this consumer gives a gift of $g_t$ in period t and receives gifts totalling $n\, g_{t+1}$ in period t+1.

Let $b_t$ be the bequest given by a generation t consumer to each of his n children (generation t+1 consumers) in period t+1. The generation t consumer receives a bequest $b_{t-1}$ from his parent in period t. In addition to receiving the bequest $b_{t-1}$ in period t, the generation t consumer inelastically supplies one unit of labor in period t and receives the real wage rate $w_t$ in period t. The generation t consumer is retired in period t+1. Letting $R_{t+1}$ be the gross rate of return on saving held from period t to period t+1, the budget constraint of a representative period t consumer is

$$[c_{1t} + g_t]R_{t+1} + c_{2t+1} + n\, b_t = [w_t + b_{t-1}]R_{t+1} + n\, g_{t+1} \qquad (4)$$

The left hand side of (4) contains the generation t consumer's expenditure on his own consumption in the two periods of his life plus the expenditure on bequests to children and a gift to his parent. The right hand side of (4) contains the three sources of the generation t consumer's resources: labor income, bequest received from his parent and the gifts received from his children.

I use the standard Nash assumption that in choosing optimal values of consumption, bequests, and gifts, the consumer takes as given the actions of all other members of his dynastic family. In particular, in choosing $g_t$, the generation t consumer takes as given the gifts given by his siblings to their common parent. The maximization problem of a representative generation t consumer is to maximize (1) subject to (4), the non-negativity constraints[13] $g_t \geq 0$ and $b_t \geq 0$ and subject to the given values of the decisions of all other members of the dynastic family. Recalling that $u_{1t}$ and $u_{2t+1}$ are the derivatives of $u_t(c_{1t}, c_{2t+1})$ with respect to its first and second arguments, respectively, the first-order conditions are

$$u_{1t} = R_{t+1}\, u_{2\,t+1} \tag{5}$$

$$u_{1t} \geq \alpha\, u_{2\,t} \quad \text{(holds with equality if } g_t > 0) \tag{6}$$

$$u_{2\,t+1} \geq (\beta/n)\, u_{1\,t+1} \quad \text{(holds with equality if } b_t > 0) \tag{7}$$

Equation (5) characterizes the optimal intertemporal allocation of the consumer's own consumption over his lifetime. If the consumer reduces $c_{1t}$ by one unit he suffers a utility loss of $u_{1t}$. However, if this unit of the consumption good is saved, then $c_{2\,t+1}$ can be increased by $R_{t+1}$ units which increases utility by $R_{t+1} u_{2\,t+1}$. At the optimum, the utility loss in period t is equated to the utility gain in period t+1, as indicated by (5).

Equation (6) characterizes the optimal gift $g_t$. In period t, the generation t consumer can reduce his own consumption by one unit, suffering a utility loss of $u_{1t}$, and can increase the gift $g_t$ by one unit, increasing his parent's utility by $u_{2t}$. The increase in parent's utility raises the generation t consumer's utility by $\alpha\, u_{2t}$. If the optimal gift is at an interior optimum ($g_t > 0$), then the utility loss ($u_{1t}$) from the reduction in $c_{1t}$ will equal the utility gain ($\alpha\, u_{2t}$) from the increased gift. If, at $g_t = 0$, the utility loss from reduced consumption exceeds the utility gain from an increased gift, then the consumer will not make a positive gift, and the non-negativity constraint on the gift binds strictly. It is worth noting that if, for some unspecified reason, siblings jointly decide on the level of the gift to give to their common parent, or equivalently, if each consumer is assumed to have 1/n parents, then the first-order condition (6) must be amended to

$$u_{1t} \geq \alpha\, n\, u_{2\,t} \quad \text{(holds with equality if } g_t > 0) \tag{6'}$$

Equation (6') corresponds to the first-order condtion derived by Carmichael (1982) and is consistent with the conditions in Buiter and Carmichael (1984).

Equation (7) characterizes the optimal bequest $b_t$. The generation t consumer can reduce $c_{2\,t+1}$ by one unit and increase the bequest to each child by 1/n, which increases the utility of each child by $(1/n)u_{1\,t+1}$. If the bequest motive is operative ($b_t > 0$), then the utility loss from decreased consumption is equal to the utility gain from increasing the bequest. If the non-negativity constraint binds strictly, then the inequality in (7) holds strictly.

## II. Intergenerational Consistency under a Two-Sided Motive

In this section I discuss the conditions under which the decisions of different generations within a family are "intergenerationally consistent." There are two aspects of intergenerational consistency. First, there is the notion of dynamic consistency introduced by Strotz (1956). Strotz showed that for a particular formulation of the intertemporal utility function in which the discount factor between two periods depends only on the length of time between the two periods, and not on calendar time, the consumption plan will be dynamically inconsistent unless the discount factors are geometrically declining. In the context of the utility function in (1), it is important that the weights on $u_{t+j}$ are geometrically declining for $j = 0,1,2,\ldots$. If these weights were not geometrically declining, then the consumption plan would suffer from dynamic inconsistency in Strotz's sense, if the bequest motive were operative.

The second notion of intergenerational consistency is that the first-order conditions of parents and their children should not contradict each other. More precisely, consider the first-order condition characterizing the optimal gift from a child to a parent at time t (equation(6)) and the first-order condition characterizing the optimal bequest from a parent to a child at time t (equation (7) with the time subscript decremented by 1). If both of these first order conditions are to hold, then

$$u_{1t} \geq \alpha u_{2t} \geq (\beta\alpha/n)\, u_{1t} \qquad (8)$$

Because $u_{1t}$ is assumed to be positive, equation (8) implies that

$$\beta\alpha \leq n \qquad (9)$$

Equation (9) along with the restrictions $0 \leq \beta < 1$ and $\alpha \geq 0$ describe the admissable values of the parameters $\alpha$ and $\beta$ under the restriction that the two-sided transfer motive is intergenerationally consistent.

## III. Competitive Factor Prices and Steady State Equilibrium

In the previous sections I analyzed the behavior of an individual dynastic family taking as given the factor prices $w_t$ and $R_t$. These factor prices, which are determined endogenously in competitive factor markets, depend on the productive technology. Let $Y_t$ be gross output in period t. This output is homogenous and can either be consumed or used as capital in the following period. The level of output is determined by a neoclassical linearly homogeneous production function $Y_t = F(K_t, N_t)$ where $K_t$ is the aggregate stock of capital and $N_t$ is the number of young consumers who each supply one unit of labor. The production function $F( , )$ is a gross production function in the sense that the aggregate capital stock, $K_{t+1}$, is equal to output, $Y_t$, minus total consumption, $N_t c_{1t} + N_{t-1} c_{2t}$, in period t. The production function can be written in intensive form as $y = f(k)$ where y is the output-labor ratio, k is the capital-labor ratio, $f' > 0$ and $f'' < 0$.

In competitive factor markets, each factor is paid its marginal product

$$R_t = R(k_t) \equiv f'(k_t) \tag{10}$$

$$w_t = w(k_t) \equiv f(k_t) - k_t f'(k_t) \tag{11}$$

The steady state is characterized by constant values of consumption for both young consumers and old consumers. Therefore, $u_{1t}$ and $u_{2t}$ are each constant in the steady state. Equations (5) - (7) imply that in the steady state the interest rate R must satisfy the following condition

$$\alpha \leq R \leq n/\beta \tag{12}$$

If one of the transfer motives is operative, then the steady state interest rate is at one of the boundaries in (12). In particular,

$$R = n/\beta \qquad \text{if } b > 0 \tag{13a}$$

$$R = \alpha \qquad \text{if } g > 0 \qquad (13b)$$

Since $\beta$ is restricted to be less than one, equation (13a) yields the well-known result that a steady state with operative bequests is undercapitalized relative to the Golden Rule (i.e., $R>n$). However, since $\alpha$ can be less than, greater than, or equal to n, equation (13b) implies that a steady state with an operative gift motive can be either overcapitalized, undercapitalized, or at the Golden Rule. This result is contrary to the result in Carmichael (1982) that a steady state with an operative gift motive is overcapitalized. Carmichael's overcapitalization result follows from his assumption that the gift parameter $\alpha$ must be less than one and from his implicit assumption that siblings jointly determine the gifts to their common parent according to (6'). Under this pair of assumptions $R = n\alpha < n$ in the steady state with operative gifts.

Finally, observe from (13a,b) that if $\alpha\beta < n$, then either bequests or gifts must be equal to zero in the steady state. In the case with $\alpha\beta = n$, which is on the boundary of the admissable region of the parameter space, and which corresponds to Burbidge's specification[14], it is possible for both gifts and bequests to be positive in the steady state. However, as shown below in section IV, the direction of net intergenerational transfers will be determinate in this case. Also note that with $\alpha\beta = n$, the range of possible values for the steady state interest rate in (12) is degenerate: the steady state interest rate is equal to $n/\beta = \alpha$ regardless of the level of government debt which is serviced by lump-sum taxes. Finally, since at least one of the transfer motives is operative, the Ricardian Equivalence Theorem holds in this case, as argued by Burbidge.

## IV. When Are the Transfer Motives Operative?

The neutrality of government debt requires that one of the transfer motives is operative both before and after the change in government debt, and, furthermore, that the same motive is operative after the change as before the change. Since the Ricardian Equivalence theorem rests on the existence of an operative transfer motive, the question of when one of the transfer motives will be operative takes on great importance. In this section, I extend Weil's analysis of the one-sided bequest motive in (2) to the case of the two-sided utility function in (1).

Recall that $K_{t+1}$ is the total stock of capital at the beginning of period t+1. All of this capital is held by generation t consumers and, furthermore, this is the only asset held by these consumers. Therefore, letting $s_t$ denote the saving of a representative generation t consumer, it follows that $K_{t+1} = N_t s_t$ which can be written as

$$nk_t = s_t \tag{14}$$

Rather than determine the saving of a generation t consumer as the solution to an infinite-horizon maximization problem, I will follow Weil's approach and ask the following question: How much would a generation t consumer save if he earns a wage income $w_t$, receives a bequest $b_{t-1}$ from his parent, receives gifts totalling $ng_{t+1}$ from his n children, earns a rate of return $R_{t+1}$, and, in addition, if he is arbitrarily required to leave a bequest of $b_t$ to each of his children and to give a gift of $g_t$ to his parent? Although I cannot answer this question explicitly at this level of generality, the saving function will have the following form

$$s_t = s\,(b_{t-1} - g_t + w_t \;,\; n(g_{t+1} - b_t) \;,\; R_{t+1}) \tag{15}$$

The saving function in (15) depends on first-period income, second-period income, and the rate of return to saving. Under the assumption that $c_{1t}$ and $c_{2t+1}$ are both normal goods, $s(.,.,.)$ is increasing in its first argument and is decreasing in its second argument. Substituting the competitive factor prices (10, 11) into (15), then substituting the resulting expression into (14) and restricting attention to the steady state yields

$$h(k, b-g) \equiv s(b - g + w(k)\,,\, n(g-b)\,,\, R(k)) - nk = 0 \tag{16}$$

We follow Diamond (1965) and confine attention to locally stable steady states (i.e., steady states for which $h_k < 0$). To avoid any complications which may arise from multiple locally stable steady states, I follow Weil (1984) and assume that there is a unique locally stable steady state. Let $k = k^*(z)$ be the steady state capital labor ratio when $b - g = z$.

As a point of reference, consider the steady state of the Diamond (1965) economy in which consumers have neither a bequest motive nor a gift motive. Let $k^D$ denote the steady state capital-labor ratio in the Diamond economy. Because $b = g = 0$ in the Diamond economy, it follows that

$$k^D \;=\; k^*(0) \tag{17}$$

Recall that the saving function s( , , ) is increasing in its first argument and is decreasing in its second argument. Therefore, it follows from the definition of $h(k, z)$ in (16) that $h_z(k, z) > 0$ and hence $k^*(z)$ is an increasing function of z.[15] Because $k^{*\prime}(z) > 0$ and $R'(k) < 0$, equation (17) implies that

$$b-g \gtreqless 0 \quad \text{as} \quad k \gtreqless k^D \quad \text{as} \quad R \lesseqgtr R^D \tag{18}$$

I now present simple conditions which are sufficient for each type of transfer motive to be operative. Essentially, in order for a transfer motive to be operative, it must be sufficiently strong. Proposition 1, which provides a sufficient condition for operative bequests, is due to Weil(1984); Proposition 2, which provides a sufficient condition for operative gifts, is new.

**Proposition 1**. If $\beta > n/R^D$, then $b > 0$.

Proof: If $\beta > n/R^D$, then (12) implies that $R^D > n/\beta \geq R$. Therefore, (18) implies that $b-g > 0$, which along with the non-negativity constraint on g, implies that $b > 0$. q.e.d.

**Proposition 2**. If $\alpha > R^D$, then $g > 0$.

Proof: If $\alpha > R^D$, then (12) implies that $R^D < \alpha \leq R$. Therefore, (18) implies that $b-g < 0$, which along with the non-negativity constraint on b, implies that $g > 0$. q.e.d.

If both transfer motives are sufficiently weak, then there will be no transfers in either direction. Precise conditions are given by

**Proposition 3**. If $\beta \leq n/R^D$, $\alpha \leq R^D$, and $\alpha\beta < n$, then $b = g = 0$.

Proof (by contradiction): Suppose that $b > 0$ so that (13a) implies that $R = n/\beta \geq R^D$. Therefore, (18) implies that $b-g \leq 0$ which implies that $g > 0$. However, if $g > 0$, then (13b) implies that $R = \alpha$ which contradicts the statements above that $R = n/\beta$ and $\alpha\beta < n$. Therefore, $b = 0$. A similar line of argument proves that $g=0$. q.e.d.

Finally, we consider the case in which $\alpha\beta = n$, which corresponds to the case considered by Burbidge.[16] In general, it is possible for there to be both positive gifts and positive bequests in the steady state. Nevertheless, one can determine whether the net flow of intergenerational transfers is from parents to children ($b-g > 0$), from children to parents ($b-g < 0$), or zero.

**Proposition 4**. If $\alpha\beta = n$, then $b-g \gtreqless 0$ as $R^D \gtreqless n/\beta = \alpha$.

Proof: Suppose that $R^D > n/\beta$. It follows from (12) that $R^D > R$ which, according to (18), implies that $b-g > 0$. Similarly, $R^D < \alpha$ implies that $R^D < R$ which, according to (18) implies that $b-g < 0$. Finally, $R^D = n/\beta = \alpha$ implies that $R^D = R$ which implies that $b-g = 0$. q.e.d.

The results concerning when the transfer motives will be operative are summarized in Figures 1 and 2. The distinction between Figures 1 and 2 is that the utility function u(.,.) and the production function f( ) are such that the steady state of the Diamond economy is efficient in Figure 1 but is inefficient in Figure 2. If the Diamond economy is efficient, then Figure 1 indicates that either the gift motive or the bequest motive could be operative; if neither motive is sufficiently strong, then neither motive will be operative. If the Diamond economy is inefficient, then Figure 2 indicates that, for admissable values of $\beta$, the bequest motive cannot be operative, which is consistent with Weil's (1984) results. However, the gift motive can be

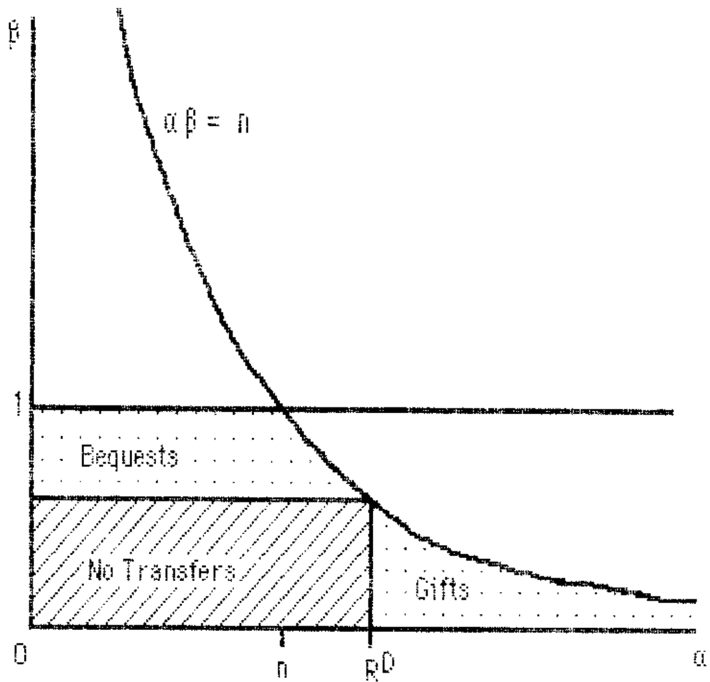

Figure 1 - Diamond steady state is efficient

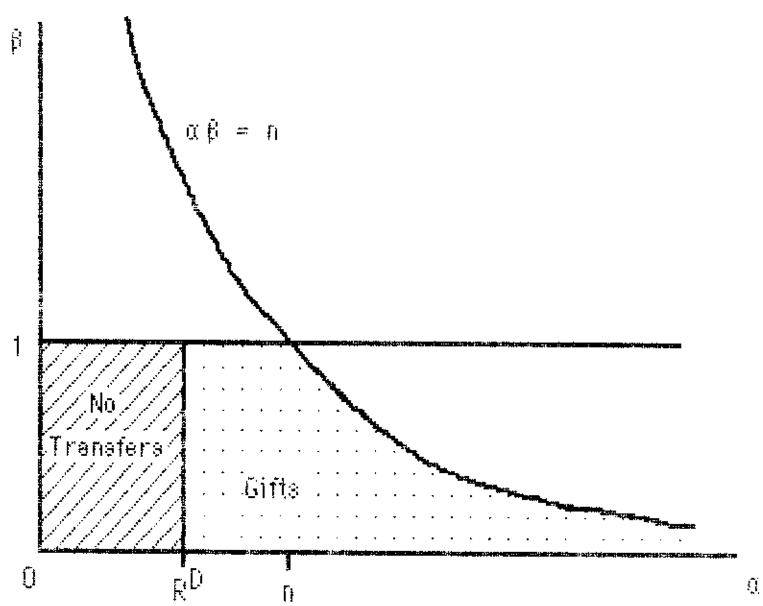

Figure 2 - Diamond steady state is inefficient

operative if it is sufficiently strong. Again, if neither motive is sufficiently strong, then neither will be operative.

The conditions for operative transfer motives are stated in terms of $R^D$, the steady state interest rate in the Diamond model. It was Weil's insight to recognize that the $R^D$ provides a useful summary of the utility function u( , ) and the production function f( ) for determining whether a transfer motive will be operative. Nevertheless, it would be useful to state the conditions for operative bequests in terms of underlying preferences and technology. As a step toward this goal, I will relate $R^D$ to consumer behavior expressed in terms of the average propensity to consume and to the production function expressed in terms of the capital share of income. Then, for a specific example I will express $R^D$ directly in terms of the parameters of preferences and technology.

Let $\sigma_t$ denote $s_t/w_t$, the average propensity to save out of wage income and let $\phi_t$ denote the capital share in income, $R_t k_t/y_t$. Because the production function is assumed to be linearly homogeneous, the labor share in income, $w_t / y_t$, is equal to $1 - \phi_t$ so that

$$w_t = [(1-\phi_t)/\phi_t] R_t k_t \tag{19}$$

It follows from (19) and the definition of the average propensity to save, $\sigma_t$, that

$$s_t = \sigma_t [(1-\phi_t)/\phi_t] R_t k_t \tag{20}$$

Equating the left hand side of (14) to the right hand side of (20) in the steady state of the Diamond economy yields

$$nk^D = \sigma[(1-\phi)/\phi] R^D k^D \tag{21}$$

It follows immediately from (21) that

$$R^D = n\phi/[\sigma(1-\phi)] \tag{22}$$

It follows from (22) that in the Diamond economy, the steady state interest rate tends to be large when either the capital share in income, $\phi$, is large or the average propensity to save, $\sigma$, is small. Of course, the capital share, $\phi$, and the average propensity to save, $\sigma$, are, in general, endogenously determined. However, there is a special case in which both $\phi$ and $\sigma$ are exogenous parameters. If the utility function is logarithmic, $u(c_{1t}, c_{2t+1}) \equiv (1-\sigma)\ln c_{1t} + \sigma \ln c_{2t+1}$, $0 < \sigma < 1$, then the average propensity to save out of wage income is constant and equal to $\sigma$. If the production function is Cobb-Douglas, $f(k) \equiv Ak^{\phi}$, $0 < \phi < 1$ and $A > 0$, then the capital share in income is constant and equal to $\phi$. In this special case, the expression for $R^D$ on the right hand side of (22) is simply a function of the parameters of preferences and technology. Substutiting this expression for $R^D$ in Propositions 1 - 4 delivers, for this example, a complete characterization, in terms of the parameters of preferences and technology, of situations in which the transfer motives will be operative or inoperative.

## V. Concluding Remarks

The effects of changes in the timing of lump-sum taxes depend crucially on whether the motives for intergenerational transfers are operative. In this paper I have derived conditions which determine whether the bequest motive is operative, the gift motive is operative or neither motive is operative. When neither motive is operative, then changes in the timing of lump-sum taxes affect the intertemporal and intergenerational allocation of resources.

The formal results presented in Propositions 1 - 4 and summarized in figures 1 and 2 apply only to the steady state of a representative consumer economy. Future research should be devoted to extending the analysis to the transition path outside the steady state and should analyze economies with interesting heterogeneity. The reason for extending the analysis to the transition path is that the Ricardian Equivalence Theorem requires that all consumers in all generations be linked by operative intergenerational transfer motives. If some generation has no operative intergenerational transfer motive, then at least some changes in the timing of lump-sum taxes will affect the intertemporal allocation of resources. The magnitude of the effect would depend on, among other things, the extent and sort of heterogeneity among consumers. For example, heterogeneity with respect to initial wealth or labor income may lead to a situation in which some consumers have operative bequest motives while other consumers in their cohort face binding constraints. In this situation, the Ricardian Equivalence Theorem would not hold; the extent of the departure from the Ricardian Equivalence Theorem, i.e., the magnitude of the effect of fiscal policy, would depend on the proportion of consumers who face binding constraints. In a subsequent paper, Abel (1986) I have begun to explore some of these issues. However, the model in that paper is restricted to Cobb-Douglas technology, logarithmic utility with a bequest motive but no gift motive, and the heterogeneity is restricted to initial wealth. In addition to analyzing more general utility and production functions, future research should analyze the effects of fiscal policy in the presence of heterogeneous labor productivity, secular productivity growth and two-sided transfer motives.

## Footnotes

1. See Drazen(1978), Weil(1984), Cukierman(1986), Cukierman and Meltzer(1986), Feldstein(1986) and Abel(1986).

2. Recently, Kimball has extended the analysis in this paper to analyze the conditions under which there will be an operative bequest motive under two-sided altruism.

3. As pointed out by Carmichael, in order for the Ricardian Equivalence to hold, the same transfer motive must be operative both before and after the change in fiscal policy.

4. See Abel (1985), Kotlikoff, Shoven and Spivak(1984), and Tomes(1981).

5. See, for example, Fischer(1973), Friedman and Warshawsky(1984).

6. See, for example, Buiter(1979) , Buiter and Carmichael (1984), Carmichael(1982), Burbidge(1983,1984), Weil(1984).

7. Buiter and Carmichael (1984) note that the specification of the gift motive as $v_t = u_t + \alpha v_{t-1}$ implies that $v_t = \Sigma_{j=0}^{\infty} \alpha^j u_{t-j}$. They argue that if $\alpha > 1$, then the utility $v_t$ is unbounded as t approaches infinity. However, even if $\alpha \geq 1$, the maximation of (3) subject to the constraints on the generation t consumer is a well-defined maximization problem.

Alternatively, Buiter and Carmichael point out that if $v_t$ is constant over time, then the "steady state utility function" is $v(c_1,c_2) \equiv u(c_1,c_2)/[1-\alpha]$, where $c_i$ is the steady state consumption of consumers of age i. They observe that if $\alpha > 1$, then "the model has the peculiar characteristic that the steady-state utility function v(·) has the opposite properties to the consumption utility u(·); for example, if u(·) is positive and increasing in $c_1$ and $c_2$, v(·) is negative and *decreasing* in $c_1$ and $c_2$."(p.763) However, the "steady state utility function" v(·) is not a useful construct. Samuelson (1968) showed that the steady state capital stock is lower than the Golden Rule capital stock if consumption is allocated to maximize the weighted sum of utility of all generations, with declining weights on future generations (which is formally identical to the problem faced by consumers with a bequest motive in (2)). Maximization of the "steady state utility function" led Buiter (1979) to conclude erroneously that if either the bequest motive or the gift motive is operative, then a competitive economy would attain the Golden Rule in the steady state and that "lump-sum taxation and debt policy will not affect the *steady state* capital-labor ratio if there are both bequest and gift motives." (p.425)

8. In an interesting analysis of consumption and gift behavior under a specific assumption about expectations of future gifts, Hori and Tsukamoto (1985) analyze the case in which $\alpha > 1$ as well as the case in which $\alpha < 1$.

9. Gale (1983) has pointed out that there is an infinity of infinite-horizon utility functions which are consistent with the recursive formulation in (2). By starting with equation (1) as the specification of preferences, I am explicitly choosing one solution, a practice which is followed, at least implicitly, in an overwhelming majority of the literature.

10. The relation between the utility function in (1) and "two-sided altruism" is discussed in Kimball (1986).

11. Bernheim and Bagwell (1984) have recently provided a stimulating analysis of the implications of marriage and altruism for the efficacy of fiscal policy.

12. This point has not been appreciated in the gift motive literature. In fairness to Carmichael, it must be noted that he seemed to be aware of this point and avoided its implications by treating the "descendents and forebearers as though there were only one of each; the descendent will be n times 'bigger,' and the forebearer n times 'smaller' than the individual." (1979, fn 2). Subsequently, Buiter and Carmichael (1984, p. 763, fn. 2) recognized that each consumer has one, rather than 1/n,(set of) parent(s). They use this observation to make an insightful comment on Burbidge's specification of the utility function, but they ignore this observation in deriving optimal individual behavior under the Nash assumption.

13. The assumption that the marginal utility of consumption at each age becomes infinite as the level of consumption approaches zero implies that any non-negativity constraints on consumption will not be binding.

14. Actually, Burbidge departed from the Nash assumption in determining an individual

consumer's optimal gift and thus arrived at the analogue of (6') rather than (6). Under this assumption, the boundary of the admissable region of parmater values is $\alpha\beta = 1$ rather than $\alpha\beta = n$. Adjusting Burbidge's analysis to incorporate the Nash assumption would amend his assumption to $\alpha\beta = n$.

15. Formally, $h(k^*(z), z) \equiv 0$ which implies that $k^{*\prime}(z) = -h_z/h_k > 0$.

16. See footnote 14.

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