ASSET PRICING IN A MACROECONOMIC CONTEXT

by

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February 1987

Abstract

This paper develops a framework that integrates macroeconomic analysis and consumption-oriented asset valuation. All behavioral relations and valuation formulae are derived as results of optimal decisions of individuals or firms. Using the endogeneity of consumption and returns in the macroeconomic context, it is shown how the valuation premia on stocks and nominal bonds relative to riskfree indexed bonds depend on the fundamental stochastic shocks to technology and preferences.

1. Introduction

The objective of the paper is to develop a macroeconomic model of asset valuation. From a macroeconomic perspective, most determinants of capital asset pricing such as consumption and interest rates are endogenous variables. We derive the valuation of stocks, nominal bonds, and indexed bonds (as examples of interesting assets) in terms of the fundamental shocks to preferences and technology and show how the covariance matrix of the valuation-relevant variables depend on the structure of the macroeconomy.[1]

The valuation of nominal and real bonds has been analyzed extensively by Fischer (1975), Landskroner and Liviatan (1981), Levhari and Liviatan (1976), Liviatan and Levhari (1977), and Svensson (1985). Fischer (1975) also considers the covariances of stock and bond returns in this context, but uses exogenous diffusion processes on prices. Landskroner and Liviatan impose the quantity theory of money as a macroeconomic restriction relating output and inflation, but they do not derive it within the model. Svensson derives a more general monetary model and analyzes asset prices as functions of exogenous processes of output and money supply. Our analysis differs from these papers in that we simultaneously derive asset valuation equations and a macroeconomic model with endogenous output, prices, and interest rates. The use of macroeconomic relations also distinguishes this approach from equilibrium models in financial economics, e.g. Abel (1986), Cox, Ingersoll, Ross (1985), Lucas (1978).

The essay draws heavily upon the literature in both macroeconomics and finance. For asset valuation, we follow the usual CAPM approach, see Sharpe (1964), Lintner (1965), Merton (1973), Rubinstein (1976), Breeden (1979). We derive the macroeconomic model from basic assumptions about preferences and technology, but we write it in two versions which are designed to have features close to the conventional classical and Keynesian system, see e.g. Sargent (1979). In both fields we concentrate on "standard" models with well known properties. The focus is on the combination of these approaches.

The basic model of a monetary economy is derived by making assumptions on the behavior of utility maximizing individuals, who supply labor and demand goods, money, and nonmonetary assets. The transactions demand for money is introduced by assuming that one of the goods can only be purchased at times when credit markets have closed for the period. Under this condition, the only means of payment is money, similar to Lucas (1984).[2] Overlapping generations of individuals live for two generations and therefore have a one-period horizon for investments, which leads to the usual CAPM formulation. The model also contains a competitive firm sector that produces output from labor inputs and a government that issues money and bonds. With competitive spot markets for labor, this generates a model with classical features, e.g. neutrality of money. To model real effects of monetary disturbances, we also derive a "Keynesian" version, which may be motivated by nominal wage contracts. Economic fluctuations occur because of shocks to productivity and to individual demand for goods and money.

The results on asset valuation are developed in three steps. First, we derive a consumption CAPM formula, which expresses valuation premia in terms of the covariance of returns and consumption. Since individuals consume all wealth in their second period, this is close to the Sharpe/Lintner market-CAPM, where the market includes bonds in addition to shocks. Second, we analyze the determinants of consumption in the macroeconomic context and conclude that a major factor is aggregate output. Third, the covariance of output (as a determinant of consumption) with returns depends on the inflation. Returns to stocks vary due to stochastic dividends and stochastic prices of shares, which are a function of output and the level of (indexed) interest rates, respectively. Hence, we have to analyze the covariance matrix of output, inflation, and interest or discount rates. This matrix is generated by the fundamental shocks to technology and preferences.

In the classical version of the model, the main findings are that shocks to productivity induce a negative relation of output and inflation and of output and interest rates. In contrast, real demand shocks induce a positive relation between both pairs of variables. In the Keynesian version, shocks to money demand also induce a positive correlation of output and inflation, but now the relation of output and interest rates is negative. Quite surprisingly, the qualitative implications for real demand and supply shocks are the same in the classical version. As a result, an economy subject to frequent supply shocks should have low values (high interest rates) for nominal bonds relative to indexed bonds when compared to an economy subject mainly to demand shocks of either type. Premia on stocks are presumably negative in either economy due to the pro-cyclical nature of their returns. Their value may be somewhat higher, however, in an economy subject to real demand shocks (due to the negative relation of output and discount rates).

The paper is organized as follows. In Section 2, we develop the basic model of the economy in its classical version. Subsequently, in Section 3, we present the valuation of asset in terms of consumption and in terms of shocks. In Section 4, we extend the model to include real effects of monetary shocks and real balance effects of nominal assets. The conclusions are summarized in Section 5.

2. A Macroeconomic Model

In this section we derive the main model. It consists of three sectors--consumers, firms, and the government. In the consumer sector we have overlapping generations of individuals who live for two periods. They work and consume part of their income when they are "young" and save the rest for consumption for their second period. The assumption of overlapping generations naturally regenerates the macroeconomic setting every period while keeping each individual decision problem simple. Firms produce output, using labor and fixed capital. The government issues bonds and money and levies lump-sum taxes to finance the interest payments on debt.

To generate money demand, we assume that credit markets open only at the beginning of the period. The young generation consumes two goods. One of these goods can only be purchased after markets have closed for the period. Money is the only medium of exchange to buy this good. All other activity takes place when markets are open.

2.1. Consumers

First, we have to introduce some notation. Assume that individuals of generation t live in periods t and t + 1. They consume a good c in real amounts c_t^t and c_{t+1}^t. In period t, they also supply labor l_t at real wage w_t and demand a good c_t^m, which is only available after credit markets have closed.[3] It can be purchased with money at the relative price p_t^m, which implies that consumers demand real money $M_t = p_t^m \cdot c_t^m$. In the terminology of Lucas (1984), c is a "credit-good" and c^m a "money-good."[4] Consumers also receive transfers τ_t, which will be determined later.

Individuals may save by holding three types of securities. A riskfree, one-period discount bond B_t pays one unit of consumption in t + 1 and is sold payoff $1 - \pi_{t+1}$. The rate of inflation π_{t+1} between t and t + 1 is defined as $\pi_{t+1} = \frac{p_{t+1} - p_t}{p_{t+1}}$, where p_t is the price level of c_t in terms of money.[5] We assume $n_t < 1$ so that bonds are preferred to money as a store of value (in order to exclude tedious distinctions of different cases). We only consider one period bonds, i.e. exclude an analysis of the term structure. This is just for expositional convenience; the issue is analyzed in Bohn (1986, chapter 3). Finally, we have shares of firms F_t, which are traded at prices f_t and represent a right to dividends from period t + 1 on. They also define the ownership of the firm sector. Let f_t be the value of a share in t (ex dividend) and denote dividends by d_{t+1}. Then the total payoff of stocks is $f^d_{t+1} = d_{t+1} + f_{t+1}$. For simplicity, we concentrate on these two risky assets, although one could value any kind of asset using the same principles. The price of the nominal bond and the price of a share can also be expressed relative to the price of the indexed bond, $n_t = \phi_{nt} \cdot b_t$, $f_t = \phi_{ft} \cdot b_t$, respectively.

Consumers have preferences over goods and leisure, which are additively separable. To generate interesting macroeconomic fluctuations, we need some noise in aggregate demand. In reality, this noise may be due to factors such as government policy, changes in investment opportunities, or shifts in individual behavior. As a simple device to generate economic fluctuations, we assume that preferences for first period consumption vary for each generation (hence over time) in two dimensions. Formally, let x_t and v_t be i.i.d. random variables with mean zero that affect marginal utility. x_t is a shock that increases demand for c^t_t and v_t is a shock that increases demand for c^m_t but reduces demand for c^t_t. They will be used later to motivate shocks to aggregate goods and money demand. Then individuals of generation t maximize

$$V(c_t^t;\ x_t,\ v_t) + V^m(c_t^m;\ x_t,\ v_t) - W(l_t;\ x_t,\ v_t) + E_tU(c_{t+1}^t)\ , \tag{1}$$

subject to

$$c_{t+1}^t = B_t + (1 - \pi_{t+1})N_t + f_{t+1}^d F_t \tag{2}$$

$$w_t l_t + \tau_t = c_t^t + p_t^m c_t^m + b_t B_t + \phi_{nt} b_t N_t + \phi_{ft} b_t F_t \tag{3}$$

We assume that V, V^m, and U are concave in the respective consumption good and that W is convex in l_t (the disutility of labor increases).

In period $t + 1$, generation t consumes all of its wealth, i.e. the returns from assets.[6] In period t, individuals choose actions $(c_t^t,\ c_t^m,\ l_t,\ B_t,\ N_t,\ F_t)$ as functions of prices, transfers, and shocks $(b_t,\ \phi_{nt},\ \phi_{ft},\ w_t,\ p_t^m,\ \tau_t,\ x_t,\ v_t)$. The optimal choice is characterized by the constraint (3) and first order conditions (subscripts indicate partial derivatives)

$$b_t V_c(c_t^t) = E_t U'(c_{t+1}^t) \tag{4}$$

$$b_t \phi_{nt} V_c(c_t^t) = E_t(1 - \pi_{t+1}) \cdot U'(c_{t+1}^t) \tag{5}$$

$$b_t \phi_{ft} V_c(c_t^t) = E_t f_{t+1}^d \cdot U'(c_{t+1}^t) \tag{6}$$

$$p_t^m V_c(c_t^t) = V_c^m(c_t^m) \tag{7}$$

$$w_t V_c(c_t^t) = W_l(l_t) \tag{8}$$

Equations (7) and (8) implicitly describe the relation of consumption, money demand, and labor supply within a period and imply optimal decisions c_t^t, c_t^m, l_t, B_t, N_t, F_t as functions of wages, interest rates, tax rates, inflation, and the values of the shocks. To obtain definite signs on the derivatives of the individual demand and supply functions, we make 3 sets of assumptions. Our main assumption is that substitution effects caused by changes in relative prices are always larger than income effects.[7] Second, we assume that money goods are only a "small" part of the economy so that direct effects of price changes on c_t, l_t, or B_t dominate possible indirect effects through changes in the demand for money goods c_t^m. Third, we simplify by assuming constant absolute risk aversion, R_a, in second period consumption $U(c_{t+1}^t)$. Then the demand for risky assets, stocks F_t and nominal bonds N_t, depends only on the relative values, ϕ_{ft}, and ϕ_{nt}.[8]

The derivatives of the supply and demand functions then have the following signs: c_b, c_w, c_{pm}, $c_\tau > 0$; c_b^m, c_w^m, $c_\tau^m > 0$; $c_{pm}^m < 0$; $l_w > 0$; and l_b, l_{pm}, $l_\tau < 0$. This is shown in Appendix 1. Essentially, an increase in the discount rate b_t has an intertemporal substitution effect that increases current goods demand and discourages labor supply. An increase in the relative price of money goods shifts consumption away from money goods and also discourages labor supply. Higher wages w_t increase the incentives to work and allow higher consumption of all goods. Finally, transfers have a positive income effect that raises consumption and labor supply. Thus, none of the signs of the derivatives are really surprising. The main point is that the functions can be used to characterize the macroeconomy in the same model that also determines asset pricing. We will return to these behavioral relation in more detail after describing the macroeconomic setting.

2.2. Firms

Firms produce output Y_t using labor L_t and fixed capital K as inputs. We assume that all firms are identical and competitive, so that we can concentrate on one representative, price-taking firm. Suppose further that production is affected by a productivity shocks u_t, i.i.d. with mean zero, and assume that we have a Cobb-Douglas production function

$$Y_t = f(K, L_t, u_t) = K^{1-\alpha} \cdot L_t^{\alpha} \cdot u_t , \tag{9}$$

where $0 < \alpha < 1$. The fixed factor capital here only motivates decreasing returns to labor.

Produced goods can be used for any type of consumption, i.e.

$$Y_t = c_t^t + c_t^m + c_t^{t-1} . \tag{10}$$

Firms pay real wages w_t per unit labor and (by definition) get a real price of one for consumption goods c_t^t and c_t^{t-1}. Recall that good c_t^m is purchased with money <u>after</u> markets close in t. Thus, revenues from sales of c_t^m are only available to pay wages or dividends in periods t + 1. Since nominal claims N_t are valued at n_t in period t, we deduce that competitive firms set a relative price $p_t^m \equiv \frac{1}{n_t}$ to be indifferent between selling "credit" and "money" goods. For simplicity, we assume that firms just issue nominal bonds of face value $N_t^f = \frac{1}{n_t} c_t^m = M_t$, so that all t-revenues are distributed in t, and the bonds can be retired in t + 1 with the money earned later in the period. Thereby we exclude considerations of nominal values from the firm problem.

The current profit of the firm, paid out as dividend to shareholders, is then

$$d_t = c_t^t + c_t^{t-1} + c_t^m - w_t L_t = (1 - \alpha) \cdot Y_t .$$

Ownership of the firm represents a right to the profit-share of each period's output (or equivalently, aggregative income). The value f_t of the firm is the present value of these dividends, where we can normalize the number of shares to $F_t = \bar{F} = 1$. For every combination of wage w_t and shock u_t, the firm maximizes its value, which leads to supply and labor demand functions $Y_t = Y(w_t, u_t)$ and $L_t = L^S(w_t, u_t)$, where derivatives are $Y_w, L_w^S < 0$ and $Y_u, L_u^S > 0$.

2.3. Government

We are mainly interested in the behavior and interaction of private agents in the economy. Therefore, we model the government as relatively "passive" and make simplifying assumptions that are convenient in the rest of the model.

It is important for asset valuation that prices and the rate of inflation can fluctuate in response to shocks. To have the most simple setting, we assume that the real money supply follows the process $M_t = M \cdot (1 - \pi_t)$, where M is constant. This may be motivated by assuming that the stock of nominal money is set one period in advance.[9] The government issues money by executing open market operations and by distributing the seignorage from issuing money to individuals. Specifically, the government purchases an amount of nominal bonds, N_t^g, at a price $n_t = \phi_{nt} \cdot b_t$, that is sufficient to offset its monetary liabilities, $N_t^g = M_t$. The cost of these bonds is $n_t \cdot N_t^g = n_t M_t < M_t$, hence the seignorage $(1 - n_t)M_t = \tau_t$ can be distributed as a lump sum transfer to individuals. Thus, the government has always a balanced budget and zero net liabilities. This specific policy serves as a useful benchmark; modifications will be considered later (see Section 4).

2.4. Equilibrium

The complete model consists of six markets, namely those for goods, money, labor, stocks, indexed bonds, and nominal bonds. To obtain aggregate supply of labor and aggregate demand for goods and money, we can substitute the equations for transfers and the price of money goods into the decision rules for each generation's representative individual. In Appendix 2, we show that these are functions of wages, inflation and discount rates and can be written in the form

$$C_t = c_t^t + c_t^m = C_t(b_t,\ w_t,\ \phi_{nt},\ \phi_{ft},\ x_t)\ ,\quad C_b > 0,\ C_w > 0,\ c_x > 0\ , \tag{11}$$

$$l_t = L^s(b_t,\ w_t,\ \phi_{nt},\ \phi_{ft}),\quad L_b^s < 0,\ L_w^s > 0\ , \tag{12}$$

$$M_t = \frac{1}{n_t} c_t^m = M^d(b_t,\ w_t,\ \phi_{nt},\ \phi_{ft},\ v_t),\quad M_b^d > 0,\ M_w^d > 0,\ M_v^d > 0\ . \tag{13}$$

The signs of these functions are not surprising in general and follow from the individual optimizing decision. We assume that the shocks to preferences x_t and v_t are defined so that x_t just summarizes all variations in preferences that affect real goods demand, and that v_t reflects the changes in preferences that affect money demand (for details, see Appendix 2). We also assume that the different shocks x_t, v_t, and u_t are jointly independent with variances denoted by $(\sigma_x^2,\ \sigma_v^2,\ \sigma_u^2)$.

Consumption of generations t - 1 is given by equation (2). After inserting the equations for profits, the tax rule, and the stocks of bonds and money, we obtain

$$c_t^{t-1} = (1 - \alpha)Y_t + \bar{F} \cdot \phi_{ft} \cdot b_t\ . \tag{14}$$

Thus, second period consumption is equal to the market value of stocks.

Given these assumptions, the macroeconomic equilibrium conditions are

$$Y_t = C_t(b_t,\ w_t,\ \phi_{nt},\ \phi_{ft},\ x_t) + c_t^{t-1}(Y_t,\ b_t,\ \phi_{ft})\ , \tag{15}$$

$$M \cdot (1 - \pi_t) = M_t^d(b_t,\ w_t,\ \phi_{nt},\ \phi_{ft},\ v_t)\ , \tag{16}$$

$$L_t^s(b_t,\ w_t,\ \phi_{nt},\ \phi_{ft}) = L_t^d(w_t,\ u_t)\ , \tag{17}$$

$$Y_t = Y_t(w_t,\ u_t)\ , \tag{18}$$

$$N_t + N_t^g = N_t^f\ , \tag{19}$$

and

By Walras' law, we can omit the market for indexed debt. Equations (15), (16), and (17) are conventional equilibrium conditions for the goods, money, and labor market, respectively. Equation (18) is the supply function, and (19) and (20) are necessary for equilibrium on asset markets. Since all assets are held by consumers, the last two equations are satisfied if and only if the first order conditions (5) and (6) hold.

Substituting equation (4) into (5) and (6), we get the relative values of "risky" nominal bonds and stocks in terms of the price of the riskfree indexed bonds (the discount factor)

$$\phi_{nt} = \frac{n_t}{b_t} = \frac{E_t(1 - \pi_{t+1}) \cdot U'(c^t_{t+1})}{E_t U'(c^t_{t+1})} = E_t(1 - \pi_{t+1}) + \mathrm{cov}\left(1 - \pi, \frac{U'}{E_t U'}\right) , \quad (21)$$

$$\phi_{ft} = \frac{f_t}{b_t} = \frac{E_t f^d_{t+1} \cdot U'(c^t_{t+1})}{E_t U'(c^t_{t+1})} = E_t f^d_{t+1} + \mathrm{cov}\left(f^d, \frac{U'}{E_t U'}\right) . \quad (22)$$

The value of a risky asset depends on its expected value and the covariance of the payoff with marginal utility of future consumption. We have a system of six equations, (15) to (18) and (21) to (22), in six endogenous variables, namely inflation, wages, output, the discount factor, and the relative values of stocks and nominal bonds. To find equilibrium levels of the endogenous variables we will exploit some special properties of the model. Given a fixed shock of capital and zero net liabilities of the government, the state of the economy is completely characterized by the three i.i.d shocks x_t, v_t, u_t. There are no relevant dynamic links between periods. This has two implications: All variables in the model are uncorrelated over time, and the right-hand sides of equations (21) and (22) are constants. Hence, the relative values of nominal bonds, $\phi_{nt} = \phi_n$, and of stocks, $\phi_{ft} = \phi_f$, are other four variables are determined by (15) - (18) as functions of the contemporaneous shocks.[10]

We assume that an equilibrium of this economy exists and is unique. Our main interest is to analyze the properties of the equilibrium mapping of the fundamental shocks (u_t, x_t, v_t) to prices (b_t, w_t, π_t), output Y_t and (via equation (14)) consumption. This analysis allows us to make an endogenous determination of the covariance structure of all these variables directly in terms of the fundamental exogenous shocks. In turn, the covariance structure determines the values of ϕ_n and ϕ_f.

The model works essentially as follows. The market clearing wage is found in the labor market. Through the firms' supply curve, this determines income and the supply of goods. Then, one obtains a real rate of interest (a discount factor b_t) in the goods market, which feeds back to labor supply because of an intertemporal substitution effect. Finally, money market equilibrium gives the rate of inflation.

We can directly verify that the reduced form effects have signs as given in Table 1 (the formulae are given in Appendix 3).

Table 1:

Effect of on	x_t	v_t	u_t
Y_t	+	0	+
π_t	+	-	-
b_t	-	0	+
w_t	-	0	+

Intuitively, if an increase in productivity u_t occurs, firms increase requires intertemporal substitution, which must be induced by a fall in interest rates, i.e. higher s_t. This fall in interest rates also reduces labor supply and partially offsets the raise in income and wages. Finally, higher wages and discount rates increase money demand and therefore reduce the rate of inflation. The direct effect of an increase in real consumer demand x_t is an increase in interest rates, a fall in s_t, which restores the goods market equilibrium. Second, this intertemporal substitution effect increases labor supply, which lowers wages and raises supply. Higher interest rates also decrease the demand for money, which leads to higher inflation. Higher demand for money v_t clearly reduces the rate of inflation. As expected in a classical setting, we have no real effects generated by the money market.

Overall, the model behaves like the textbook model of a classical economy, but with two important modifications. First, we include the valuation of risky assets as an integral part of the model. This makes consumption and returns, the conventional determinants of asset prices, endogenous. This fact will be used to relate the values of assets to macroeconomic variables. Ultimately, all economic fluctuations (including those that affect the covariances of returns with consumption) must depend on the fundamental shocks to preferences and technology. This model allows us to formulate this dependency explicitly (see below). Second, our model is derived from microeconomic optimization. In the context of multiple assets, this shows how different interest rates enter in specific macroeconomic equations. Hence, changes in risk (variances of shocks) may affect macroeconomic performance through changes in relative values. This phenomenon is analyzed in more detail in Bohn (1986, chapter 2).

3. Asset Valuation

We can use the macroeconomic model to interpret the asset valuation equations (21) and (22). Equilibrium consumption is a function of macroeconomic variables. Therefore, we can express the relation between consumption and returns in terms of the variances and covariances of macroeconomic variables, namely output, inflation, and interest rates. These variables are endogenous in the macromodel (15) to (18). Hence risk premia are functions of the variances of the fundamental shocks to the macroeconomy.

3.1. Asset Valuation in Terms of Macroeconomic Variables

To simplify, suppose all three fundamental shocks are normally distributed. Then equations (21) and (22) can be approximately written as[11]

$$\phi_{nt} = E_t(1 - \pi_{t+1}) - R_a \cdot \text{cov}(1 - \pi_{t+1}, c^t_{t+1}) \ , \tag{23}$$

$$\phi_{ft} = E_t f^d_{t+1} - R_a \cdot \text{cov}(f^d_{t+1}, c^t_{t+1}) \ . \tag{24}$$

The premium of nominal bonds or stocks depends on the covariance of returns with consumption.[12] In particular, this covariance is with the consumption of the old generation which holds assets.

One could price any arbitrary asset in this way. If its net supply were zero, it would not affect any of the macroeconomic relations and could be easily included in the model. If its net supply were not zero, it would cause obvious wealth effects. For notational simplicity only, we restrict our model to nominal bonds and stocks, which are economically interesting assets. The results on valuation derived in this essay hold analogously for assets in general. (In Bohn (1986, chapter 3) this analysis is applied to the term structure of interest rates.)

The crucial, next step is to observe that asset values as well as equilibrium consumption are endogenous variables in a macroeconomic context. It would not be correct to postulate exogenous stochastic processes for these variables. Here, the relevant consumption is given by equation (14). By substituting this equation into (23) and (24) we obtain

$$\phi_n = E_t(1 - \pi) + R_a \cdot \left[(1 - \alpha) \cdot \sigma_{Y,\pi} + \phi_f \cdot \sigma_{b,\pi}\right] , \qquad (25)$$

$$\phi_f = E_t f^d + R_a \cdot \left[- (1 - \alpha)^2 \cdot \sigma_Y^2 - 2 \cdot \phi_f (1 + \alpha) \cdot \sigma_{Y,b} - \phi_f^2 \cdot \sigma_b^2\right] \qquad (26)$$

where $\sigma_{a,b}$ denotes the covariance of (a, b).

Equations (25) and (26) use two observations on consumption. First, notice that a major component of stock returns, dividends, are proportional to aggregate output. Hence, consumption is related to output, which is in line with macroeconomic tradition. Second, consumption is equal to the stock market value, i.e. we obtain a CAPM valuation formula. This result is due to the simple assumptions on firm and government behavior and will be generalized in Section 4.2. In contrast, the first observation is very general: Aggregate income or output must be somehow distributed. This is independent of whether or not we have a stock market and dividends, and whether aggregate income is distributed through stock returns or in some other way. Here, the share of the old generation is $(1 - \alpha)$.

In summary, risk premia on assets depend mainly on the covariance of its returns with aggregate output and to some extent on the covariance with interest rates. However, these covariances are endogenous. Ultimately, we want to express them in terms of the variances of the fundamental stochastic shocks affecting production and preferences.

3.2. Asset Valuation in Terms of Fundamental Shocks

To determine the covariances between the endogenous variables, we have to study the covariances between output, inflation and the discount rate, because they influence the valuation of assets. From the macroeconomic model (15) to (18), we see that any covariance between endogenous variables is a weighted sum of the variances of the fundamental shocks. The values of these covariances depend on what types of stochastic fluctuations are dominant in a given economy. For example, for the covariances of output with inflation and with the discount rate we have

$$\sigma_{Y,\pi} = \frac{dY_t}{dx_t} \cdot \frac{d\pi_t}{dx_t} \cdot \sigma_x^2 + \frac{dY_t}{du_t} \cdot \frac{d\pi_t}{du_t} \cdot \sigma_u^2 + \frac{dY_t}{dv_t} \cdot \frac{d\pi_t}{dv_t} \cdot \sigma_v^2 , \qquad (27)$$

where the weights on variances are the reduced form derivatives of the endogenous variables with respect to a shock, which have signs as indicated in Table 1 (for exact formulae, see Appendix 3). We can see that fluctuations in real demand make a positive contribution to $\sigma_{Y,\pi}$, since $\frac{dY_t}{dx_t} > 0$ and $\frac{d\pi_t}{dx_t} > 0$, and that supply shocks tend to generate a negative sign of the covariance, because $\frac{\partial Y_t}{\partial u_t} > 0$ and $\frac{\partial \pi_t}{\partial u_t} < 0$. Monetary shocks have no effect in the "classical setting," because they do not affect output ($\frac{\partial Y}{\partial v_t} = 0$); this will be modified in Section 4.

Using this kind of argument, we now look separately at the covariance patterns induced by each type of shock.

In an economy with relatively stable supply sector, where most economic fluctuations are caused by changes in real demand, the important covariances in (25) and (26) are $\sigma_{Y,\pi} > 0$, $\sigma_{Y,b} < 0$. The variances are clearly positive.

Then the premium on nominal bonds is influenced by two factors. First, nominal bonds are a hedge against fluctuations in output (i.e. personal income). Nominal bonds have this property because of the positive covariance of output and inflation, which implies a negative relation of output and nominal returns. This property increases the value of bonds. Second, the value of bonds and the (ex dividend) value of stocks are positively related, which lowers the value of nominal bonds. Notice that this effect is proportional to the value of stocks held. The effect is small, if the market capitalization is small. In contrast, the first effect, through aggregate output, should be fairly robust with respect to the exact structure of the economy: independent of the type of assets traded or whether or not income is paid via dividends, aggregate output must be distributed to individuals as aggregate income. This effect is likely to dominate the others, and nominal bonds will probably be valued higher than indexed bonds in an economy with real demand shocks.

The valuation of stocks is determined by the variance of its return. The variance of stock returns is determined by the variances of output (via dividends) and discount rates and by their covariance. The risk created by the variance of the output in t + 1 clearly lowers stock values. Intuitively, we know that stock returns vary with the business cycle, and therefore stocks demand higher average rates of returns than riskfree bonds. In addition, we have to consider the variance of the sales price of stock in t + 1, $\phi_f^2 \cdot \sigma_b^2$, as well as the covariance of output and the discount factor, $\sigma_{Y,b}$. A positive variance of the price of stocks further reduces stock values. The covariance $\sigma_{Y,b}$ is negative, which implies that future profits are discounted at a lower interest rate, i.e. f_{t+1} is high, in situations when dividends are low. This correlation stabilizes total stock returns and therefore increases stock values.

A different pattern of covariances is induced in an economy with major supply shocks. There the signs of the covariances are $\sigma_{Y,\pi} < 0$ and $\sigma_{Y,b} > 0$. Valuation has the same determinants as in the previous case. The premium on stocks will again be negative because of the procyclical variation of returns. The premium is reduced even further because of the positive covariance of output and the discount factor. Concerning the valuation of nominal bonds, bonds no longer function as hedges against output variations. If output falls due to a productivity shock, inflation is increased and nominal bonds have a low payoff. Hence, for an individual the holding of bonds increases fluctuations in income. For comparison, when inflation is high and nominal bonds have low payoffs in an economy with demand shocks, output is probably high and individuals have large incomes. Therefore, the value of nominal bonds is lower in an economy with supply shocks than in one with demand shocks.

In summary, depending on the types of shocks occurring in the economy, we get different relative values of nominal bonds, stocks, and indexed bonds. Stocks are almost always traded at a discount relative to indexed bonds because of their procyclical returns. The size of the discount is influenced by several factors that depend on the source of shocks to the economy. The valuation of nominal bonds depends crucially on the relative magnitude of the demand and supply shocks. If supply shocks are more significant, nominal bonds are valued lower than indexed bonds. In an economy with many large demand shocks, however, the value of nominal bonds may even be higher than that of indexed bonds.

4. Extensions

Real effects of the monetary sector are commonly associated with such a modification of the model. We also allow for real balance effects induced by redistributive government policies.

4.1. A Model with Phillips-Curve

Suppose individuals and firms sign wage contracts before the start of period t. The contract specifies a fixed nominal wage and allows firms to choose the amount of labor input later, after observing the period-t shocks. An exact model (similar to Fischer (1977), Hall (1983)) is derived in Appendix 4. Aggregate supply of firms then depends on the innovation in inflation and real and money demand depends on income instead of the wage rate. In equilibrium, we require four markets to clear, namely the markets for goods, money, nominal bonds, and stocks.[13] We can replace the capital market equilibrium conditions by the conditions for relative discount rates, (21) and (22), which are still valid. Equilibrium is characterized by the following equations (in addition to (21) and (22)):

$$Y_t = C(b_t, Y_t, \phi_{nt}, \phi_{ft}, x_t) \; ; \; C_b, C_Y, C_x > 0 \; , \; C_Y < 1 \; , \tag{28}$$

$$M \cdot (1 - \pi_t) = M^d(b_t, Y_t, \phi_{nt}, \phi_{ft}, v_t) \; ; \; M_b^d, M_Y^d, M_v^d > 0 \; , \tag{29}$$

$$Y_t = Y^S(\pi_t - E_{t-1}\pi_t, u_t) \; ; \; Y_\pi^S, Y_u^S > 0 \; . \tag{30}$$

These equations determine the macrovariables (Y_t, b_t, π_t) as functions of the shocks (x_t, v_t, u_t) with derivatives as indicated in Table 2.[14]

Table 2:

Effect of $\downarrow$ on	x_t	v_t	u_t
Y_t	+	-	+
π_t	+	-	-
b_t	-	-	+

Increases in real demand, represented by x_t, again allow a fall in the riskfree discount rate. Now this has no effect on labor supply. Instead, higher demand tends to increase prices, which increases output via the supply curve. As in the version with spot markets, the real wage rate falls, but this is due to inflation at constant nominal wages.

Increases in goods supply induced by higher u_t also have effects in the same direction as before, but now they work through a decrease in the rate of inflation, which increases the real supply of money. This increase reduces the rate of interest, i.e. increases b_t. Thus, although the way in which some of the effects arise is completely different, the classical and the Keynesian version have very similar qualitative behavior in reaction to these stochastic shocks.

The main difference between the classical and the Keynesian version of the model is in the effect of monetary disturbances. Here, a decrease in the demand for money v_t not only increases the rate of inflation, but it also raises real output through the supply curve and lowers interest rates to restore equilibrium in the goods market. This version gives us a model with rational individual behavior that has all the familiar properties of the Keynesian IS-LM system with a Lucas supply curve.

The existence of real effects of monetary shocks complicates asset valuation. While the reactions to real demand and supply shocks are similar in both versions of the model, significant differences in the covariance structure of the key variables Y_t, b_t and π_t arise if monetary shocks are the dominant source of economic fluctuations. Then Table 2 implies that all three covariances $\sigma_{Y,\pi}$, $\sigma_{Y,b}$, and $\sigma_{\pi,b}$ are positive. For the valuation of nominal bonds, it follows that bonds are hedges against cyclical fluctuation, as in the case of variations in goods demand. The premium on stocks is again negative, due to the procyclical nature of stock returns, which is increased by the positive covariance between the output and the discount factor.

In comparison to the results of the classical version, real effects of monetary shocks lead to a higher valuation of nominal bonds. Otherwise valuation is similar, which we can take as an indication of the robustness of our results with respect to specification issues.

A model with real effects of monetary shocks may be empirically important, if changes in the monetary system cause changes in the covariance structure of macrovariables. For example, consider the October 1979 shift in the Federal Reserve operating procedures. Suppose we interpret the change in operating procedures as a shift towards targeting money stocks rather than interest rates. This change modifies the stochastic properties of the macroeconomic process.

We maintain that a money stock oriented policy must be set one period in advance, for example, due to lags in observing and controlling money stocks. This requirement prevents a precise fixing of price level and inflation. Therefore, we consider monetary rules of the form

$$M_t = M \cdot (1 - \pi_t) - m \cdot (b_t - \beta) ,$$

where the constant m indicates the degree of interest rate targeting. The case of m = 0 corresponds to the "monetarist" policy of fixing the supply of nominal money ex ante that we have analyzed so far, whereas $m \to \infty$ is equivalent to pegging the discount factor at β. A positive value of m implies that the money market equilibrium (equation (28)) is more sensitive to changes in b_t than before.[15]

As one should expect, interest rate targeting reduces fluctuations in interest rats, and it automatically offsets disturbances caused by money demand shocks (v_t). With extreme interest rate targeting, i.e. in the limit as $m \to \infty$, only real demand shocks have positive effects on output and inflation. Supply shocks still decrease inflation, but their positive effect on output vanishes. Intuitively, we know that supply shocks affect demand and interest rates through their effect on inflation and on the real money supply. With interest rate targeting however, upward pressure on interest rates automatically increases money supply and therefore dampens the negative effect on output. On the other hand, if we have a contractionary shocks from the goods market that also decreases interest rates, an interest rate-oriented monetary policy responds with a contraction of nominal money supply and thereby increases the effect on output and inflation.

Overall, interest rate targeting seems to magnify effects of real demand shocks but to dampen or offset those from supply or monetary sources. This finding is completely in line with textbook analysis. Here, this result becomes important because of the implications for risk and asset valuation. With interest targeting, the covariance of output and inflation is likely to be positive for almost any combination of shocks because only real demand reduced interest rate targeting likely reduced the value of numerical bonds by more than what one would expect just from the change in money supply growth.

This example demonstrates another important point: Variances and covariances of macroeconomic variables are not only influenced by changes in the variance of shocks, but also by shifts in systematic policy or other shifts in economic structure that change the reduced form of the model.

4.2. Policy Effects

In the basic model of Sections 2 and 3, consumption of the old generation is equal to the stock market value (equation (14)), so that the consumption asset pricing model (equations (23) - (24)) reduces to the market model (equations (25) - (26)). The valuation of risky assets must be modified, if individuals hold net amounts of other risky assets besides stocks. In particular, if the government debt policy differs from the simple benchmark policy described in Section 2.3, individuals hold net amounts of bonds.

Suppose the government has constant (potentially nonzero) real net liabilities B in real bonds and N in nominal bonds. Then its nominal assets are $N^g_t = M_t - N$, real assets are $B^g_t = -B$, and transfers are (seignorage minus interest on the debt)

$$\tau_t = (1 - n_t) \cdot M_t - (1 - b_t) \cdot B - (1 - \pi_t - n_t) \cdot N . \qquad (31)$$

A generalized debt policy with net nominal government debt, N > 0, has two implications for the macroeconomic structure. First, there is a real balance effect. Transfers to the young increase with inflation. The consumption of the old generation is

$$c_t^{t-1} = (1 - \alpha) \cdot Y_t + F \cdot \phi_{ft} \cdot b_t + B + N \cdot (1 - \pi_t) , \qquad (32)$$

and falls with inflation. Since the marginal propensity to consume out of first period income is less than 1, the net effect of inflation on demand is negative.

As a result, monetary shocks have some effect on output even in the classical model. A monetary shock v_t that increases money demand reduces the rate of inflation (equation (16)), increases goods demand through the real balance effect and therefore increases output. Thus, if real balance effects are large, monetary shocks may induce a negative correlation between output and inflation (or reduce the positive correlation induced by a Phillips-curve).

The second effect of net nominal debt concerns asset valuation. Consumption of the old generation depends on the value of nominal bonds, i.e. the relevant portfolio of risky assets includes nominal bonds in addition to stocks. Then the relative values of nominal bonds and stocks are

$$\phi_n = E_t(1 - \pi) + R_A \cdot \left[(1 + \alpha)\sigma_{Y,\pi} + \phi_f\sigma_{b,\pi} - N\sigma_\pi^2\right] \tag{33}$$

$$\phi_f = E_t f^d + R_A \cdot \left[-(1 - \alpha)\sigma_Y^2 - 2\phi_f(1 - \alpha)\sigma_{Y,b} - \phi_f^2\sigma_b^2 + N \cdot \left((1 - \alpha)\sigma_{Y,\pi} + \phi_f \cdot \sigma_{b,\pi}\right)\right] \tag{34}$$

Compared to (25) and (26), both equations contain an additional term (proportional to N) representing the covariance of the respective return with inflation. If nominal bonds are part of the portfolio, the value of nominal bonds is clearly reduced. The effect on stock values depends on the covariance of inflation with output and the discount factor.

In an economy with real demand shocks, output and inflation are positively correlated, i.e. returns of stocks and nominal bonds are negatively correlated. This increases the value of stocks (and, as discussed before, nominal bonds), because the risk of the total portfolio is reduced.

In an economy with supply shocks, the correlation of output and inflation and of stock values and nominal bonds is likely negative. Then total portfolio risk is high. Stocks are not a hedge against inflation and therefore valued lower than in an economy with demand shocks.

Redistributive effects could also be induced by other government policies, e.g. by direct transfers/taxes to the old generation. Then the correlation properties of the transfers/taxes are important for asset valuation. Issuing nominal government debt is an example of such a policy that has potentially powerful effects on the valuation of assets.[16]

5. Summary

The main objective of the essay has been to integrate capital asset pricing and macroeconomic modeling. Analyzing the CAPM in a macroeconomic context allows us to determine stochastic processes of consumption and asset returns endogenously. Ultimate determinants are shocks to preferences and production technology.

The valuation of assets has been derived in three steps. First, we show how individual optimization implies that the value of assets depends negatively on the covariance of their returns with consumption. Second, from the macroeconomic model, we can identify factors that influence consumption. A key component of consumption seems to be aggregate output or income. In addition, there may be effects of intertemporal redistribution which depend on the precise assumptions about the taxes and debt policy. Thus, the covariance of aggregate output with returns should be a main determinants of valuation.

move parallel to output, we get the conventional result that stocks have a higher rate of return than bonds.

Third, variances and covariances of output, inflation, and other returns can be reduced to linear combinations of the variances of fundamental shocks that generate the economic fluctuations. Shocks to production and to individual preferences generate important economic fluctuations that affect the real demand for goods. Negative shocks to production reduce output and real wages, which implies lower demand for goods and money and higher interest rates and inflation. Therefore, an economy influenced by supply shocks has a negative covariance of output and inflation, hence a low value of nominal bonds relative to indexed bonds. In contrast, an increase in goods demand, which raises interest rates, increases both inflation (via reduced demand for money) and output (via intertemporal substitution in labor supply). If economic fluctuations are mainly due to this type of shocks, the covariance of output and inflation is positive, and therefore the relative value of nominal bonds is higher than in a situation with supply shocks.

In a version with Phillips-Curve, shocks to the monetary sector have real effects. Quite surprisingly, the effects of real demand and supply shocks are qualitatively the same as in the classical version. In addition, monetary shocks also induce a positive relation of output and inflation and therefore tend to increase the value of nominal bonds.

We also demonstrated that a monetary policy of interest rate targeting reduces the effect of supply disturbances, makes the correlation of output and inflation positive, and generates a high value of nominal bonds. Thus, a switch to less interest rate targeting, as e.g. in October 1979, may reduce the value of nominal bonds (increase interest rates) even without any change

Finally, asset valuation may be modified, if government policy redistributes between generations. An important case may be the issue of nominal debt that is financed by taxes on the young generation. Then net nominal debt is held by the old generation, the value of nominal bonds lower because of a risk premium and stocks are valued according to whether or not they are hedges against inflation. In particular, if supply shocks are frequent, stocks do not hedge against inflation and are valued relatively low.

Footnotes

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[1]We also show that private demand for goods and money is affected by the interest rates on different assets and hence their relative valuation. This implies that changes in variances of shocks influence the mean level of key variables such as output, inflation, and interest rates. This issue is analyzed in Bohn (1986, chapter 2).

[2]An alternative formulation has been used by Svensson (1985), who adds precautionary demand for money by assuming that goods markets operate before credit markets open in the next period. Both specifications generate an interest-elastic demand for money, but Lucas' assumption that credit markets open first is more convenient for our purposes.

[3]Alternatively, one could assume that market for c^m is spatially separated from the other markets. This arrangement would have the same implications for money demand.

[4]Unlike Lucas we assume that c_t^t is paid in period t, not later. This assumption does not affect any results.

[5]This definition seems most to be convenient here instead of the more conventional $\frac{p_{t+1} - p_t}{p_t}$. However, both definitions are monotonically related and very similar for small rates of inflation.

[6]None of our results depends on the fact that individuals must hold and trade assets to have any nonzero consumption in their second period. If individuals had other income in period t + 1, e.g. inherit the ownership of firms rather than buying shares in t, all important results could be maintained. Also, our setting differs from many other overlapping generation models in that money is held because it has a real role in facilitating real consumption. As a store of value, money is clearly dominated by nominal bonds.

[7]This assumption can always be satisfied by assuming that the relevant second derivatives of the utility functions 4V, V^m, W, U) are sufficiently small.

[8]To be exact, this is true in a neighborhood of the equilibrium price (see appendix 1).

[9]As an example, suppose monetary policy works only with a lag so that the nominal stock of money M_t to be supplied in period t must be fixed in t - 1.

$(1 - E_{t-1}\pi_t)$, hence policy will set nominal supply $\bar{M}_t = \frac{M^* \cdot p_{t-1}}{(1 - E_{t-1}\pi_t)}$. Since all variables are i.i.d. (and there is no reason to introduce autocorrelation into the model by making money supply a function of past variable), expected inflation is constant and therefore nominal supply proportional to prices p_{t-1}. Hence, real supply is $M_t = \frac{\bar{M}_t}{p_{t-1}} \cdot (1 - \pi_t) = \frac{M^*}{(1 - E_{t-1}\pi_t)} \cdot (1 - \pi_t)$, which means that the real stock of money must fluctuate with the rate of inflation between t and t - 1 and that $M = \frac{M^*}{(1 - E_{t-1}\pi_t)}$.

[10]This arrangement simplifies the analysis considerably, although it is probably one of the least desirable properties of the model. As motivation, suppose one period represents a rather long interval, say a decade or the duration of one business cycle. Then, one shock represents the realization of a boom or a recession, which cannot be anticipated years in advance in magnitude or timing.

[11]As a linear approximation, the endogenous variables in the macroeconomy (described by equations (15) to (18)) and the consumption of the old generation (equation (14)) are normal. For the covariance of any return x (where $x = 1 - \pi$, f^d) with marginal utility, we have $\text{cov}[x, U'(c)] = (E_tU'') \cdot \text{cov}(x, c)$, see Rubinstein (1976). In addition, we use that absolute risk aversion is constant, $R_a = \frac{E_t(-U'')}{E_tU'}$. Alternatively, one could take a Taylor series of marginal utility (i.e. assume quadratic utility) to derive the same valuation formula without assuming normality.

[12]Notice that the word "premium" is sometimes used differently in financial economics (CAPM) and in the macroeconomic literature (on the term structure). For our study, as in financial economics, a positive premium on long bonds means a high initial value, i.e. (given the payoff) a low rate of interest. In the context of the term structure this positive premium would correspond to a negative "liquidity" premium on interest rates.

[13]The market for indexed bonds is also in balance by Walras' law.

[14]Table 2 is obtained by taking the total differential of (28) - (30).

[15]Notice that in the classical version interest rate targeting is not a sensible policy, because monetary policy has essentially no influence on real interest rates. Such a policy could only magnify the effect of real fluctuations on inflation (see Friedman (1968)). Hence, we restrict the discussion to the Keynesian case.

[16]Note that the Ricardian neutrality proposition (Barro (1974)) is not applicable in our model because of finite lives and no bequests.

References

ABEL, ANDREW [1986], "Stock Prices under Time-Varying Dividend Risk: An Exact Solution in an Infinite-Horizon General Equilibrium Model," The Wharton School, November.

BREEDEN, DOUGLAS T. [1979], "An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities," Journal of Financial Economics, vol. 7, 265-296.

BOHN, HENNING [1986], "Financial Markets and the Macroeconomy," Doctoral Dissertation, Graduate School of Business, Stanford University.

COX, JOHN C., JONATHAN E. INGERSOLL, AND STEPHEN A. ROSS [1985], "An Intertemporal General Equilibrium Model of Asset Prices," Econometrica, vol. 53, March, 363-384.

FISCHER, STANLEY [1975], "The Demand for Index Bonds," Journal of Political Economy, vol. 88, no. 3, 509-534.

_________ [1977], "Long-Term Contracts, Rational Expectations, and the Optimal Money Supply Rule," Journal of Political Economy, vol. 85, no. 1, 191-206.

FISCHER, STANLEY AND ROBERT MERTON [1984], "Macroeconomics and Finance: The Role of the Stock Market," Carnegie-Rochester Conference Series on Public Policy, vol. 20, 57-108.

FRIEDMAN, MILTON [1968], "The Role of Monetary Policy," AER, vol. 58, March, 1-17.

HALL, ROBERT E. [1983], "The Wage Adjustment Process," unpublished, November.

LANDSKRONER, YORAM AND NISSAN LIVIATAN [1981], "Risk Premia and the Sources of Inflation," Journal of Money, Credit, and Banking, vol. 13, no. 2, May, 205-214.

LEVHARI, DAVID AND NISSAN LIVIATAN [1976], "Government Intermediation in the Indexed Bonds Market," American Economic Review, vol. 66, May, 186-192.

LINTNER, JOHN [1965], "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets," Review of Economics and Statistics, vol. 47, February, 13-37.

LIVIATAN, NISSAN AND DAVID LEVHARI [1977], "Risk and the Theory of Indexed Bonds," American Economic Review, vol. 67, June, 366-375.

LUCAS, ROBERT E. [1972], "Expectations and the Neutrality of Money," Journal of Economic Theory, vol. 4, 103-124.

_________ [1978], "Asset Prices in an Exchange Economy," Econometrica, vol.

________ [1980], "Equilibrium in a pure Exchange Economy," Economic Inquiry, vol. 18, April, 203-220.

________ [1984], "Money in a Theory of Finance," Carnegie-Rochester Conference Series on Public Policy 21, 9-46.

MERTON, ROBERT C. [1973], "An Intertemporal Asset Pricing Model," Econometrica, vol. 41, no. 5, September, 867-886.

POOLE, JAMES [1970], "Optimal Choice of Monetary Policy Instruments in a Simple Stochastic Macro Model," Quarterly Journal of Economics, 197-216.

RUBINSTEIN, MARK [1976], "The Valuation of Uncertain Income Streams and the Pricing of Options," Bell Journal of Economics, 407-425.

SARGENT, THOMAS J. [1979], "Macroeconomic Theory," Academic Press.

SHARPE, WILLIAM F. [1964], "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk," Journal of Finance, vol. 19, September, 425-442.

STULZ, RENE [1986], "Interest Rates and Monetary Policy Uncertainty," Journal of Monetary Economics, vol. 17, 331-348.

SVENSSON, LARS E. O. [1985], "Money and Asset Prices in a Cash-in Advance Economy," Journal of Political Economy, vol. 93, 919-944.

Appendix

A.1. Individual Behavior

Individual behavior is determined by the first order conditions (4) to (8) and the constraints (2) and (3). Substituting (4) into (5) and (6), we get the differential

$$\begin{pmatrix} EU''(1-\pi-\phi_{nt}) & EU''(1-\pi)(1-\pi-\phi_{nt}) & EU''f^d(1-\pi-\phi_{nt}) \\ EU''(f^d-\phi_{ft}) & EU''(1-\pi)(f^d-\phi_{ft}) & EU''f^d(f^d-\phi_{ft}) \end{pmatrix} \cdot \begin{pmatrix} dB_t \\ dN_t \\ dF_t \end{pmatrix}$$

$$= EU' \cdot \begin{pmatrix} d\phi_{nt} \\ d\phi_{ft} \end{pmatrix}$$

From equations (21) and (22) we see that the "dB_t" terms are zero, hence N_t and F_t are determined by ϕ_{nt} and ϕ_{ft}. Define $x_n = EU''(1-\pi) \cdot \frac{dN_t}{d\phi_{nt}} + EU''f^d \cdot \frac{dF_t}{d\phi_{nt}}$ and $x_f = EU''(1-\pi) \cdot \frac{dN_t}{d\phi_{ft}} + EU''f^d \frac{dF_t}{d\phi_{ft}}$ for reference below.

Taking the total differential, we get four equations for c_t^t, c_t^m, l_t, and B_t as implicit functions of $(b_t, p_t^m, \phi_{nt}, \phi_{ft}, w_t, \tau_t, x_t, v_t)$. The differential is

$$\begin{pmatrix} -s_t(-V_{cc}) & 0 & 0 & (-EU'') & [-EU''(1-\pi)] & (-EU''f^d) \\ -p_t^m(-V_{cc}) & (-V_{cc}^m) & 0 & 0 & 0 & 0 \\ -w_t(-V_{cc}) & 0 & -W_{11} & 0 & 0 & 0 \\ 1 & p_t^m & -w_t & b_t & n_t & f_t \end{pmatrix}$$

$$\cdot \left(dc_t^t \;\; dc_t^m \;\; dl_t \;\; dB_t \;\; dN_t \;\; dT_t\right)^T$$

$$
= \begin{matrix}
-V_c & 0 & 0 & x_n & x_f & 0 & -b_tV_{cx} & -b_tV_{cv} \\
0 & -V_c & 0 & 0 & 0 & 0 & p_t^mV_{cx}+V_{cx}^m & p_t^mV_{cv}+V_{cv}^m \\
0 & 0 & -V_c & 0 & 0 & 0 & -w_tV_{cx}+W_{1x} & -w_tV_{cv}+W_{1v} \\
-B_t-\phi_{nt}N_t-\phi_{ft}F_t & -c_t^m & l_t & -b_tN_t & -b_tF_t & 1 & 0 & 0
\end{matrix}
$$

$$
\cdot \left(db_t \;\; dp_t^m \;\; dw_t \;\; d\phi_{nt} \;\; d\phi_{ft} \;\; dx_t \;\; dv_t\right)^T
$$

The determinant of this system is $\frac{1}{\Delta} = b_t(-V_{cc})[(-V_{cc}^m)W_{11}b_t] + p_t^m(-V_{cc})$ $[p_t^mW_{11}(-EU'')] + w_t(-V_{cc})[(-EU'')(-V_{cc}^m)w_t] + (-EU'')(-V_{cc}^m)W_{11} > 0$, and we get the following comparative static effects.

A raise in transfers raises disposable income; hence, it increases consumption and asset demand and increases labor supply:

$$\frac{\partial c_t^t}{\partial \tau_t} = \Delta \cdot (-EU'')W_{11}(-V_{cc}^m) > 0$$

$$\frac{\partial c_t^m}{\partial \tau_t} = \Delta \cdot (-EU'')W_{11}\, p_t^m(-V_{cc}) > 0$$

$$\frac{\partial l_t^t}{\partial \tau_t} = -\Delta \cdot (-EU'')(-V_{cc}^m)w_t(-V_{cc}) < 0$$

$$\frac{\partial B_t^t}{\partial \tau_t} = \Delta \cdot W_{11}b_t(-V_{cc}^m)(-V_{cc}) > 0$$

Tax changes represent a typical income or wealth effect. Wealth effects for changes in any other variable are proportional to this one. Also, note that $\frac{\partial c_t^m}{\partial \tau_t} = c_\tau^m < \frac{1}{p_t^m}$.

An increase in wages w_t increases the incentive to work and raises

$$\frac{\partial c_t^t}{\partial w_t} = \Delta \cdot V_c(-EU'')(-V_{cc}^m)w_t + l_t \cdot \frac{\partial c_t^t}{\partial \tau_t} > 0$$

$$\frac{\partial c_t^m}{\partial w_t} = \Delta \cdot V_c(-EU'')(-V_{cc})\frac{1}{n_t} W_{11} + l_t \cdot \frac{\partial c_t^m}{\partial \tau_t} > 0$$

$$\frac{\partial l_t}{\partial w_t} = \Delta \cdot V_c\left[s_t(-V_{cc})(-V_{cc}^m)s_t + (-EU'')(-V_{cc}^m)\frac{1}{n_t^2} + (-EU'')(-V_{cc}^m)\right] + l_t \frac{\partial l_t}{\partial \tau_t}$$

$$\frac{\partial B_t}{\partial w_t} = \Delta \cdot V_c s_t(-V_{cc})(-V_{cc}^m)w_t + l_t \cdot \frac{\partial B_t}{\partial \tau_t} > 0$$

An increase in the discount rate b_t induces an intertemporal substitution effect that raises current consumption and lowers labor supply and asset demand:

$$\frac{\partial c_t^t}{\partial b_t} = \Delta \cdot V_c(-V_{cc}^m)b_t W_{11} - (B_t + \phi_{nt}N_t + \phi_{ft}F_t) \cdot \frac{\partial c_t^t}{\partial \tau_t} > 0$$

$$\frac{\partial c_t^m}{\partial b_t} = \Delta \cdot V_c b_t W_{11}(-V_{cc}) \cdot (p_t^m)^2 - A \frac{\partial c_t^m}{\partial \tau_t}$$

$$\frac{\partial l_t}{\partial b_t} = \Delta \cdot V_c b_t(-V_{cc})(-V_{cc}^m)w_t - A \frac{\partial l_t}{\partial \tau_t} < 0$$

$$\frac{\partial B_t}{\partial b_t} = \Delta \cdot V_c\left[(-V_{cc}^m)W_{11} + W_{11}(-V_{cc})(p_t^m)^2 + w_t^2(-V_{cc})(-V_{cc}^m) - A \frac{\partial B_t}{\partial \tau_t} < 0\right.$$

An increase in the price of money goods causes substitution towards the money-good c_t^m and reduces the demand for N_t relative to other assets.

$$\frac{\partial c_t^t}{\partial p_t^m} = \Delta \cdot V_c(-EU'')p_t^m - c_t^m \frac{\partial c_t^t}{\partial \tau_t} W_{11} > 0$$

$$\frac{\partial c_t^m}{\ } \quad V_{cc}\ 2 \quad 2 \quad m \frac{\partial c_t^t}{\ }$$

$$\frac{\partial l_t}{\partial p_t^m} = -\Delta \cdot V_c p_t^m(-EU'')(-V_{cc}^m)w_t - c_t^m \cdot \frac{\partial l_t}{\partial \tau_t} > 0$$

$$\frac{\partial B_t}{\partial p_t^m} = \Delta \cdot V_c b_t W_{11}(-V_{cc}) \cdot p_t^m - c_t^m \frac{\partial B_t}{\partial \tau_t}$$

Also,

$$\frac{\partial M_t^d}{\partial p_t^m} = c_t^m + p_t^m \cdot \frac{\partial c_t^m}{\partial p_t^m} < 0.$$

The effects that shocks x_t and v_t have on demand depend on the signs of their effects on marginal utility (as can be seen in the differential in Appendix 1). We want x_t to represent a disturbance in the goods market and v_t to be disturbance term in the money market. Hence, we assume that V_x (hence c_t^t) increases in x_t. In addition, x_t must change V_c^m and W_1 enough so that labor supply and money demand stay constant, i.e. so that the substitution effect towards c_t^t is offset. Similarly, we assume V_c^m increases in v_t and raises c_t^m, and other marginal utility terms are affected in a way that offsets all side effects on labor supply and total goods demand $c_t^m + c_t^t$. To get a zero effect on <u>total</u> demand, this calculation implies that c_t^t must actually decrease somewhat. The effects of ϕ_{nt} and ϕ_{ft} turn out to be irrelevant; therefore they are not computed. Then, individual behavior can be summarized in Table A.1.

Table A.1:

Effect of on	b_t	p_t^m	w_t	τ_t	x_t	v_t
c_t^t	+	+	+	+	+	-

A.2. Aggregate Behavior

First, $p_t^m = \frac{1}{\phi_{nt} \cdot b_t}$ depends on ϕ_{nt} and b_t. The total effects of b_t on aggregate c_t^t, c_t^m, l_t, and B_t are therefore the sums of the direct effects derived in Appendix 1 and $\left(- \frac{1}{\phi_{nt} b_t^2}\right)$ times the effect of p_t^m. Using the results of Appendix 1, these total effects are $\partial c_t^m/\partial b_t > 0$, $\partial M_t^d/\partial b_t > 0$, $\partial B_t/\partial b_t < 0$ (directly from Table A.1) and $\partial c_t^t/\partial b_t = \Delta \cdot V_c \cdot W_{cc} \cdot \frac{1}{b_t} \cdot \left[b_t^2(-V_{cc}^m) - (p_t^m)^2(-EU'')\right]$, $\partial l_t/\partial b_t = -\Delta \cdot V_c \cdot \frac{W_t}{b_t} \cdot (-V_{cc}) \cdot \left[b_t^2(-V_{cc}^m) - (p_t^m)^2(-EU'')\right]$. The sign of the last two derivatives is ambiguous, but if $|V_{cc}^m|$ is sufficiently large (as implied by the assumption of a "small" monetary sector), then $\partial c_t^t/\partial b_t > 0$, $\partial l_t/\partial b_t < 0$, hence $\partial C_t/\partial b_t > 0$.

Second, to get aggregate consumer behavior as a function of prices alone, we include the effects of equilibrium variations in transfers, which are

$$\tau_t = (1 - n_t)M_t = \frac{1 - \phi_{nt} b_t}{\phi_{nt} b_t} \cdot c_t^m(b_t, \phi_{nt}, \phi_{ft}, w_t, \tau_t, x_t, v_t)$$

We get

$$d\tau_t = \frac{c_t^m}{\phi_{nt} b_t^2}\, db_t + \frac{c_t^m}{\phi_{nt}^2 b_t}\, d\phi_{nt} + \frac{1 - n_t}{n_t}\, dc_t^m ,$$

where c_t^m itself depends on τ_t, but (as noted above) with a derivative $c_\tau^m < 1/p_t^m < 1$. By assumption, the parameters of the model are such that c_t^m is small and that indirect effects through c_t^m on c_t^t, l_t, and B_t are smaller than the direct effects. Therefore, the aggregate behavioral functions have the same sign of derivatives as indicated above.

A.3. Reduced Form of the Classical Version

The reduced form of the model of Section 2 gives the value of the four key macroeconomic variables (Y_t, b_t, w_t, π_t) for any realization of shocks (x_t, v_t, u_t). Equation (15) - (18) imply

$$Y_t(w_t, u_t) = C_t(b_t, w_t, x_t, \phi_t, \phi_{nt}, \phi_{ft}) + (1 - \alpha)Y_t + \phi_{ft} \cdot b_t ,$$

$$L_t^s(b_t, w_t, \phi_{nt}, \phi_{ft}) = L_t^d(w_t, u_t) ,$$

or (subscripts indicating derivatives)

$$\begin{pmatrix} \alpha Y_w - C_w & -C_b - \phi_{ft} \\ L_w^s - L_w^d & L_b^s \end{pmatrix} \begin{pmatrix} dw_t \\ db_t \end{pmatrix} = \begin{pmatrix} -\alpha Y_u & C_x \\ L_u^d & 0 \end{pmatrix} \begin{pmatrix} du_t \\ dx_t \end{pmatrix}$$

The determinant of the system (Δ) is positive (using Appendix 2), hence $dw_t/dx_t < 0$, $db_t/dx_t < 0$, which implies (use (21), (22)) $dY_t/dx_t > 0$ and $d\pi_t/dx_t > 0$. Also, $dw_t/du_t = \frac{1}{\Delta}[-\alpha Y_u L_b^s + L_u^d(C_b + \phi_{ft})] > 0$ and

$$\frac{db_t}{du_t} = \frac{1}{\Delta} \cdot [L_u^d(\alpha Y_w - C_w) + Y_u \cdot (L_w^s - L_w^d)$$

$$= \frac{1}{\Delta} \cdot [\alpha Y_u L_w^s - C_w L_u^d + \alpha(Y_w L_u^d \cdot L_w^d Y_u)] .$$

Cobb-Douglas technology implies $Y_u = \frac{w_t}{\alpha} L_u$ and $Y_w = \frac{w_t}{\alpha} L_w + \frac{L_t}{\alpha}$, and consumer optimization implies $w_t L_w^s - C_w = b_t \cdot dB_t/dw_t - l_t > 0$ (assuming the substitution effect dominates the income effect. Therefore, $Y_w L_u^d - L_w^d Y_u = L_u^d \frac{L_t}{\alpha} > 0$ and $\alpha Y_u L_w^s - C_w L_u^d = L_u^d(w_t L_w^s - C_w) > 0$, hence $db_t/du_t > 0$. Substituting into (16) and (17),

$$\frac{dY_t}{du_t} = Y_u + Y_w \frac{dw_t}{du_t} = \frac{1}{\Delta} \cdot \left[Y_u C_w (-L_b^s) + (c_b + \phi_{ft}) Y_u (-L_w^d) + (c_b + \phi_{ft}) \cdot (L_u^d Y_w - L_w^d Y_u) \right] > 0$$

$$\frac{d\pi_t}{du_t} = - \frac{M_b^d}{M} \frac{db_t}{du_t} - \frac{M_w^d}{M} \frac{dw_t}{du_t} < 0 \ .$$

A.4 Wage Contracts

As a motivation for wage contracts, suppose workers need some firm-specific training before they start producing. To be precise, assume that generation t already lives in t - 1 to get this training and is not economically active in any relevant way. Because the training is specific to a firm, we can no longer have spot markets for labor in period t, but workers and firms must contract for period t labor supply in t - 1. If contingent contracts were possible, this requirement would not necessarily change the allocation. However, we assume that contingencies in contracts are extremely costly and therefore concentrate on a type of contract that seems to be widely used (see e.g. Fischer (1977), Hall (1983)): Workers and the firm agree ex ante on a nominal wage rate w^n per unit labor that is paid in all contingencies, and the firm has the right to decide later how much labor is actually supplied.

Notice that the price level p_{t-1} is known when period t nominal wages are negotiated. In our model, inflation and all other variables are i.i.d. and therefore unaffected by past levels of prices. The nominal wage level must be set proportional to p_{t-1} to exclude money illusion. The labor market effectively determines a value for $w^* = E_{t-1} \frac{1}{p_t} w^n \approx \frac{1}{p_{t-1}} \cdot (1+E_{t-1}\pi_t) \cdot w^n$. By the i.i.d. structure, no information about the state of nature in t is known, so that w^* is a constant. For the stochastic properties of the model it is important that w^* is a constant, but we need not know how it is

Given w^*, the real wage in period t is $w_t = \frac{1}{p_t} \cdot w^n \approx w^* \cdot (1 - \pi_t + E_{t-1}\pi_t)$ and it depends on the unexpected change in the rate of inflation (cf. Lucas (1972)). This Phillips-curve relation provides a channel through which monetary events can affect output. Note, however, that policy still cannot permanently affect average output, since such an attempt would affect the rational expectation of inflation.

The behavior of firms is the same as in the classical version with the modification that w_t is now a given function of inflation. Substituting this function into labor demand and goods supply, we get $L_t = L^d(\pi_t - E_{t-1}\pi_t, u_t)$ and $Y_t = Y^*(\pi_t - E_{t-1}\pi_t, u_t)$, where L^d_π, Y^s_π, L^d_u, $Y^s_u > 0$. Since firms have the right to determine labor supply, labor demand determines actual labor input. From the Cobb-Douglas technology, we then get the income of worker as $w_t \cdot l_t = \alpha \cdot Y_t$.

The only change in an individual's constraints comes from on the labor market. Given w^* and the fact that the firm chooses how much labor is required, individuals face a given disposable income $Y^d_t = w_t \cdot l_t + \tau_t = \alpha Y_t + \tau_t$. Since preferences are additively separable and l_t exogenous to individuals, we can omit the $W(l_t; x, v)$ expression in the maximization problem. That is, in period t individuals maximize $V(c^t_t; x, v) + V^m(c^m_t, x, v) + E_t U(c^t_{t+1})$ subject to (2) and

$$Y^d_t = c^t_t + \frac{1}{n_t} c^m_t + b_t B_t + \phi_{nt} b_t N_t + \phi_{ft} b_t F_t \quad (39)$$

The first order conditions are identical to (4) to (7) in the model with spot labor markets; only the condition (8) for optimal labor supply is missing. Constraints and first order conditions now define a mapping from $(b_t, \phi_{nt}, \phi_{ft}, Y^d_t, x_t, v_t)$ to $(c^t_t, c^m_t, B_t, N_t, F_t)$. As in the spot market prices and income, and finally obtain macroeconomic demand functions for goods and money (analogous to appendix 1),

$$C_t = c_t^t + c_t^m + c_t^{t-1} = C(b_t, Y_t, \phi_{nt}, \phi_{ft}, x_t) , \tag{40}$$

$$M_t = \frac{1}{n_t} c_t^m = M^d(b_t, \phi_{nt}, \phi_{ft}, Y_t, \pi_t, v_t) , \tag{41}$$

where all derivatives are positive. Here, the only important difference to the classical model is that output (as a determinant of disposable income) enters into demand functions instead of the wage rate.