CONTRACTS TO SELL INFORMATION

by

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Abstract

There is often a reliability problem when information is sold since anyone can claim to have superior knowledge. Optimal strategies which allow the seller to overcome this problem are considered in the context of a standard one-period two-asset model. It is shown that when the seller's risk aversion is unobservable, an information market exists and both the seller and buyers are better off. However, because of the reliability problem the seller cannot obtain the full value of his information. This provides an incentive for intermediation since an intermediary may be able to capture some of the remaining returns.

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1. Introduction

Hirshleifer (1971) suggested that one way individuals might profit from private information about the future returns to securities would be to sell it. However he did not pursue this possibility further, pointing to the fact that there is a significant reliability problem when information is sold (p. 565): ". . . it may not be possible for an informed individual to authenticate possession of valuable foreknowledge for resale purposes. After all anyone could claim to have such knowledge."

Leland and Pyle (1977) argued that this problem could be overcome if information were sold indirectly. An intermediary could gather information and use it to invest in a portfolio of assets. He would be able to signal that he was informed by putting a large proportion of his wealth in the equity of the fund. No uninformed person would find it worthwhile imitating this because of the risk he would he would have to bear. The problem of resale is also avoided by this arrangement since the intermediary does not need to reveal his portfolio to investors. Leland and Pyle were thus able to provide a theory of intermediation which does not depend on differences in transaction costs.

The purpose of this paper is to consider feasible and optimal strategies for the direct sale of information when buyers cannot observe whether or not the seller is informed. A standard one-period two-asset model is used. The returns to the risky asset are normally distributed and there is a continuum of traders who have heterogeneous exponential utility functions. The other important assumption is that the transmission of information from a seller to a buyer takes a finite time. This restricts the possibilities for resale: initially it is assumed there is only sufficient time for information to be sold once.

There is an informed person who is the only one to observe a particular signal about the return to the risky asset. Initially, before the person observes his information, he makes a proposal to prospective buyers. For each possible signal he gives a corresponding portfolio he will invest in and a total payment for information that he will receive from the buyers. Each portfolio and corresponding payment are chosen in such a way that no uninformed person would want them: they are constrained so that a seller without any information would always be better off just investing in the two assets. This set of portfolios and payments therefore identifies the seller as informed. Next the buyers, who all pay an equal share of the total payment, decide whether or not the information is worth purchasing. Provided there are enough of them, the average cost will be sufficiently small to make it worth buying. Finally, the seller observes his signal and announces it: this determines the portfolio position he must take and the total payment he receives for the information. The optimal portfolios and payments are such that he always correctly announces his observation.

In the reference case where the seller can directly establish he's informed, he can charge each buyer a positive amount. Since there is a continuum of traders the value of the information is unbounded. If the seller can't directly establish he's informed but his risk aversion is observable, he can demonstrate he's informed effectively enough that his utility is again unbounded. If his risk aversion is unobservable, his utility is bounded as he can capture only part of the value of the information. Nevertheless, he is better off selling his information rather than using it to just invest in the two assets. Thus an information market always exists in the model.

The more risk tolerant the seller, the easier it is for him to establish he's informed so he can extract more from buyers. Also the better the information that buyers have about the seller's risk aversion, in particular the greater the lower bound on this, the greater the amount that can be charged. If only a safe and a risky asset are available, then the more precise his information, the more the seller can charge and the better off he is. However, if state-contingent securities exist, then even though greater precision makes the seller better off, the amount charged for it may be reduced.

The model is extended to the case where sufficient time remains before asset markets meet for a buyer to resell his information. Similarly to the original seller, the buyer will also be better off selling his information than just investing. There is thus a motive for intermediation that does not arise from differences in transaction costs: although the original seller is unable to obtain the full value of his information, the existence of resale possibilities allows an intermediary to extract some of the remaining uncaptured returns. When risk aversion can take any positive value this intermediation is profitable just because the risk-bearing capacity of the sellers is increased: it is equivalent to making the original seller less risk averse. However, when there is a positive lower bound on risk aversion, it is also profitable for risk tolerant people because they can demonstrate they're informed more effectively. This theory contrasts with Leland and Pyle (1977) where part of the justification of intermediation is to prevent resale: here intermediation occurs because resale allows more to be extracted from buyers.

A number of other recent papers have focussed on the information reliability problem and intermediation. Bhattacharya and Pfleiderer (1985) have considered the problem that mutual fund and other institutional investors have ensuring their portfolio is managed by people who have the ability to acquire good information about security returns. Ramakrishnan and Thakor (1984) have looked at a model where firms issuing new securities hire information producers, whose effort cannot be perfectly observed, to certify their value. The relationship of these papers to the model here is considered further below.

The information reliability problem is just one of the important differences between markets for information and those for other commodities. In a series of papers, Admati and Pfleiderer (1985a, 1985b, 1986) consider a number of different aspects of the link between the market for information about an asset and the price of that asset. Allen (1986a, 1986b) develops a framework to distinguish between the quantity and quality of information which can be used in general equilibrium models with information markets. These studies all assume it is possible to observe directly whether or not a person is informed. They thus avoid the reliability problem considered in this paper and focus on difficulties that are avoided here.

The paper proceeds as follows. Section 2 describes the model in detail. Section 3 considers the reference situation where it is possible to directly observe whether or not a seller is informed. In Section 4 the case where this is not possible is investigated. In Section 5 the form of the optimal contract when state-contingent securities are available is analyzed. Section 6 extends the model to give a theory of intermediation. Finally, Section 7 contains concluding remarks.

2. The Model

A standard one-period, two-asset model is used. Each trader is assumed to be endowed with (possibly different) stocks of the two types of security: $\bar{M}$ of the riskless asset and $\bar{X}$ of the risky asset. When trade occurs a person buys M and X of the two assets respectively. The price of the safe asset is normalized at unity and the price of the risky asset is P. A trader's budget constraint is therefore

$$W_0 = \bar{M} + P\bar{X} = M + PX \ . \tag{1}$$

The safe asset yields R and the risky asset ρ. At the end of the period, when the asset returns are received, a trader's wealth is

$$W_1 = RM + \rho X = RW_0 + (\rho - RP)X \ . \tag{2}$$

Each person has an exponential utility function which depends on wealth at the end of the period:

$$V(W_1) = -\exp(-aW_1) \ . \tag{3}$$

Everybody is risk averse. Initially, it is assumed the distribution of a has positive density for all $a > 0$ and is unbounded above.

The return on the risky asset ρ has the unconditional distribution $N(E\theta, \sigma_\rho^2)$ with distribution function denoted $F(\rho)$.

There is a person, denoted I for informed, who can observe a signal θ before asset markets meet where

$$\rho = \theta + \varepsilon \ . \tag{4}$$

The variables θ and ε are independent and distributed $N(E\theta, \sigma_\theta^2)$ and $N(0, \sigma_\varepsilon^2)$ respectively so that

$$\sigma_\rho^2 = \sigma_\theta^2 + \sigma_\varepsilon^2 \ . \tag{5}$$

The precision of the informed person's signal is therefore $1/\sigma_\varepsilon^2$. For simplicity, I's cost of observing θ is taken to be zero. (The only difference if there was a positive cost would be that the person would have to decide whether it was worthwhile becoming informed, and choose the optimal value of σ_θ^2 if a range was possible at different costs. Apart from this the analysis below would be similar.) The distribution function of his prior on ρ having observed θ, is $F(\rho|\theta)$. Nobody else observes θ directly or any other variable correlated with θ, except ρ. The assets and trades of the informed person and any payments he receives for his information are observable. They can therefore be contracted upon.

The model assumes rational expectations: the structure of the economy is known to all the participants. In particular, they know ρ is $N(E\theta, \sigma_\rho^2)$, they are aware that everybody has an exponential utility function and they know the distribution of a. They also know there is an informed person.

There is a continuum of traders so that the actions of any single trader or finite group of traders have no effect on prices. This implies all transactors are price-takers and prices convey no information about θ provided only a finite number of traders become informed through the information market. The purpose of this assumption is to isolate the information market from the market for the risky asset. This means the information seller does not have to take into account the effect of his actions on the price of the risky asset and hence on the value of his information. Admati and Pfleiderer (1985b) have looked at the complex issues this link raises in the absence of the reliability problem. The model considered here is a polar opposite with the reliability problem being analyzed in the absence of any interaction between the markets.

Taking this separation together with the fact that everybody is risk averse implies there is a positive risk premium on the risky asset:

$$\zeta = E\theta - RP > 0 . \tag{6}$$

Finally, an important assumption is that there is a finite amount of time taken for the transmission of information between the seller and buyers. It is assumed initially that there is only sufficient time between the informed person's receipt of his information and the meeting of asset markets for information to be sold once. Hence any buyers of information can only use it to speculate with: they cannot resell it.

3. Informed Person Identifiable

For the purposes of comparison it is helpful to start by considering the ex ante value of the information to the seller in the case where he can directly establish that he will become informed and what his signal is. Given there is no reliability problem, the seller can charge each buyer a fixed fee, η, equal to the ex ante value of the information. This is found by comparing the buyer's expected utility when uninformed to that when he is informed.

When the buyer has no information about θ, his expected utility can be found in the usual way using (2), (3) and the moment generating function for the normal distribution:

$$EV_u = -\exp[-a(RW_0 + \zeta X_u - \frac{a\sigma_\rho^2 X_u^2}{2})]. \tag{7}$$

This gives the standard result that his optimal demand for the risky asset is

$$X_u = \frac{\zeta}{a\sigma_\rho^2}. \tag{8}$$

Hence substituting back into (7),

$$EV_u = -\exp[-a(RW_0 + \frac{\zeta^2}{2a\sigma_\rho^2})]. \tag{9}$$

Similarly for an informed investor,

$$X_i = \frac{\theta - RP}{a\sigma_\varepsilon^2} \tag{10}$$

and hence conditional on θ,

$$EV_i(\theta) = -\exp\left(-a\left[RW_0 + \frac{(\theta - RP)^2}{2a\sigma_\varepsilon^2}\right]\right) . \tag{11}$$

Taking expectations over θ,

$$EV_i = -\exp\left[-a\left(RW_0 + \frac{\zeta^2}{2a\sigma_\rho^2} + \eta\right)\right] \tag{12}$$

where

$$\eta = \frac{1}{2a} \log \frac{\sigma_\rho^2}{\sigma_\varepsilon^2} . \tag{13}$$

Thus η is the ex ante value of signal to the buyer and is the most he would be prepared to pay for it. As might be expected, it is higher the less risk averse the investor and the more precise the information.

Since there is a continuum of investors that can each be charged η, the value of the information to the seller is unbounded.

Proposition 1

When there is no reliability problem, the ex ante value of the information to the seller is unbounded.

The purpose of considering the case where it can be directly established whether or not the seller is informed is to provide a reference point. In contrast, it is shown below that when the informed person is not identifiable and his risk aversion is unobservable, the value of the information to the seller is bounded.

4. Informed Person Not Identifiable

In this section it is assumed that person I cannot directly establish he's informed. Given this, it is not possible for him to charge a fixed fee to buyers since anybody, no matter what their information, would find this worthwhile to do. This is the reliability problem discussed in the introduction. The only publicly observable variable that is correlated with the information is the payoff to the risky asset ρ. This means that the person can establish that he's informed by initially choosing a portfolio for each possible θ and restricting the total payment he receives for the information in such a way that any uninformed person would be worse off than if he did not try to sell information.

The sequence of events in the information market is thus the following.

(i) For each possible θ, the seller announces a portfolio with $X(\theta)$ in the risky asset (and $W_0 - PX(\theta)$ in the safe asset) and a corresponding total payment from all buyers of $\Phi(\theta)$.

(ii) There are n buyers who decide whether or not to purchase information given that they each pay $\Phi(\theta)/n$. The contract becomes binding at this point if they do decide to buy.

(iii) The seller observes θ. He then announces $\hat{\theta}$. This determines the portfolio he must buy $\left(X(\hat{\theta}),\ W_0 - PX(\hat{\theta})\right)$ and the corresponding total payments from the buyers $\Phi(\hat{\theta})$.

(iv) Asset markets meet.

(v) The payoff to the risky asset is realized. The seller receives the return from his portfolio $\left(X(\hat{\theta}),\ W_0 - PX(\hat{\theta})\right)$. He consumes this together with the total payment $\Phi(\hat{\theta})$ for the information.

It is assumed at stage (ii) that buyers pay an equal proportionate share, $1/n$, of the total payment to the seller $\Phi(\theta)$. It can be seen below that this is not crucial. All that matters is the level of $\Phi(\theta)$: how it is split between buyers is irrelevant.

For ease of notation it will be assumed that $W_0 = 0$.

The approach taken in analyzing the optimal contract is to first consider the $[X(\theta), \Phi(\theta)]$ which maximize the seller's utility subject only to the constraint that they establish he's informed. It is subsequently shown that when risk aversion is unobservable, these optimal $[X(\theta), \Phi(\theta)]$ also give the correct incentives for the seller to truthfully announce θ at stage (iii) and that he is better off selling information than just trading. It is also demonstrated that n can be chosen so that the buyers will be prepared to purchase the information. This approach is analytically simpler than initially considering the full optimization problem with all the relevant constraints.

(A) Risk aversion observable

Suppose first that a person's degree of risk aversion a is observable. The case where it is unobservable is considered below.

The main problem in selling information involves establishing its reliability. To do this the contract for selling the information must be such that it demonstrates the seller does indeed have superior knowledge: it must establish that he will become informed. If a person were uninformed and just invested in the two assets then his utility would be as in (9). In order to demonstrate that he's informed, person I's investment in the risky asset, $X(\theta)$, and the payment for the information, $\Phi(\theta)$, must be such that if he were uninformed he would be worse off. For a person with risk aversion a it is necessary that for every θ, the pair $[X(\theta), \Phi(\theta)]$ are such that

$$- \int_{-\infty}^{\infty} \exp\left[- a\left(X(\theta)(\rho - RP) + \Phi(\theta)\right)\right] dF(\rho) \leq - \exp\left[- \frac{\zeta^2}{2\sigma_\rho^2}\right] . \tag{14}$$

(It is assumed in the usual way that when indifferent the seller acts in the buyers' interests.) The inequality must be satisfied for every θ. Otherwise an uninformed person would find it worthwhile to announce the set at stage (i) and then announce the value at stage (iii) which would make him better off than (9). Simplifying (14) gives the equivalent form:

$$\Phi(\theta) \leq \frac{\zeta^2}{2a\sigma_\rho^2} - \zeta X(\theta) + \frac{a\sigma_\rho^2 X^2(\theta)}{2} . \tag{15}$$

Clearly this condition on $[X(\theta), \Phi(\theta)]$ is sufficient to identify the seller as informed. Any uninformed person who suggested a set which satisfied it for all θ would be no better off than if he had just invested in the two assets. In addition, (15) is also a necessary condition. Suppose it weren't satisfied. In that case the seller could be either informed or uninformed. Since there is only one informed person but a continuum of uninformed people, a buyer's prior it is the informed person would be zero and his prior it is an uninformed person would be one. Buyers are therefore unwilling to purchase information unless (15) is satisfied. It is both necessary and sufficient for the information reliability problem to be overcome.

The seller's problem in choosing each pair $[X(\theta), \Phi(\theta)]$ at stage (i) to demonstrate he's informed is therefore:

$$\underset{X(\theta),\Phi(\theta)}{\text{Max}} \; EV_I = - \int_{-\infty}^{\infty} \exp\left[- a_I\left(X_I(\theta)(\rho - RP) + \Phi(\theta)\right)\right] dF(\rho|\theta) \tag{16}$$

subject to (15).

Using (15) with $a = a_I$ to substitute for Φ in (16) and evaluating the integral gives

$$EV_I = -\exp\left[-a_I\left(\frac{\zeta^2}{2a_I\sigma_\rho^2} + (\theta - E\theta)X + \frac{a_I\sigma_\theta^2X^2}{2}\right)\right]. \tag{17}$$

It follows from this that EV_I is unbounded since it is convex in X. The reason is that in (15) Φ is a quadratic function of X. Although taking larger positions in the risky asset means the seller bears more risk, this is more than offset by the fact that a greater total charge for the information can be made. This gives:

<u>Proposition 2</u>

If risk aversion is observable, the value of the information to the seller is unbounded, even though it cannot be directly observed whether or not he is informed.

<u>(B) Risk aversion unobservable: $0 < a < \infty$</u>

Suppose next that a seller's degree of risk aversion is unobservable but it is known that it can take any positive value. In this case the constraint (15) must be satisfied for all possible a, not just a_I. It can be seen that for any X the right hand side of (15) is strictly convex in a. Hence, there exists a minimizing value of a which is denoted a*. Intuitively, the reason a minimum exists is the following. For a > a*, the payment Φ can be higher because, for a given X, the more risk averse a person, the worse off he is. For a < a*, Φ can be higher because a less risk averse person could be taking a larger position if he were uninformed and so would be better off.

Provided (15) is satisfied for the minimizing value a* it will be satisfied for all feasible a. Differentiating the right hand side of (15) gives

$$a^{*2} = \frac{\zeta^2}{X^2\sigma_\rho^2} \, . \tag{18}$$

There are then two possibilities depending on whether X is positive or negative. Since $a^* > 0$,

$$a^* = \frac{\zeta}{X\sigma_\rho^2} \qquad \text{for } X > 0 \, . \tag{19}$$

Substituting this in (15) gives

$$\Phi(\theta) = 0 \, . \tag{20}$$

Utility is no longer unbounded and from (16) the optimal X and EV_I are as in (10) and (11). The certainty equivalent value (i.e., the amount with certainty that would give the same level of expected utility) of the position is

$$C^+(\theta) = \frac{(\theta - RP)^2}{2a_I\sigma_\varepsilon^2} \, . \tag{21}$$

Thus information cannot be charged for when $X > 0$ and the person's demand for the risky asset and expected utility are the same as if the person were just trading and not trying to sell information. This can be understood in the following way. An uninformed person has demand $\zeta/a\sigma_\rho^2$. Depending on a, this can vary between 0 and ∞. Hence it is not feasible to have positive X and at the same time charge anything for it. If a charge were made there would always be uninformed people who would find it worthwhile to pretend to be informed: in addition to the speculative position they would have taken anyway they receive a positive payment.

$$a^* = -\frac{\zeta}{X\sigma_\rho^2} \qquad \text{for } X < 0 . \tag{22}$$

In this case,

$$X(\theta) = \frac{(\theta - RP - 2\zeta)}{a_I\sigma_\varepsilon^2} \tag{23}$$

$$\Phi(\theta) = -2\zeta X(\theta) \tag{24}$$

$$C^-(\theta) = \frac{(\theta - RP - 2\zeta)^2}{2a_I\sigma_\varepsilon^2} . \tag{25}$$

Here it is possible to make a positive charge for the information. Given $\zeta > 0$, uninformed people do not want to take a short position no matter what their degree of risk aversion. If they did so they would be strictly worse off than if they just invested in the two assets. This means that the informed person can charge for the information.

It can be seen from comparing (10) and (23) that it is better for the information seller to hold less of the risky asset (i.e., a larger short position) than he would if he were just investing. This is because a reduction in X at $X = (\theta - RP)/a_I\sigma_\varepsilon^2$ does not directly alter his utility very much since, ignoring the effect on Φ, $\partial EV_I/\partial X = 0$. However, it allows an increase in Φ since for an uninformed person $\partial EV_u/\partial X > 0$ at this point. Thus it is optimal for X to be below $(\theta - RP)/a_I\sigma_\varepsilon^2$.

Whether the seller does better having X positive or negative depends on the condition

$$C^+(\theta) \gtrless C^-(\theta) , \tag{26}$$

which is equivalent to

$$\theta \lesseqgtr E\theta . \tag{27}$$

For above average signals the information seller takes a long position in the risky asset and for below average θ they take a short position.

These results give:

Proposition 3

When a person's degree of risk aversion is unobservable and can take any positive value, the seller's utility is bounded: he can capture only a portion of the value of the information.

For $\theta \geq E\theta$, the optimal set of $[X(\theta), \Phi(\theta)]$ that identifies the seller as informed is given by (10) and (20) and the certainty equivalent these provide to the seller is given by (21). Similarly for $\theta < E\theta$, they are given by (23)-(25), respectively.

At stage (iii) the seller observes θ and then announces $\hat{\theta}$. The contract specifies that he receives $[X(\hat{\theta}), \Phi(\hat{\theta})]$. Hence his decision problem at that point is

$$\max_{\hat{\theta}} EV_I = -\int_{-\infty}^{\infty} \exp[-a(X(\hat{\theta})(\rho - RP) + \Phi(\hat{\theta}))]dF(\rho|\theta) . \tag{28}$$

It can straightforwardly be shown that the solution to this problem is

$$\hat{\theta} = \theta . \tag{29}$$

The reason it is optimal for the seller to announce the value of θ he observes is that the constraint (15) is satisfied for every θ. This means it's never worthwhile for the seller to choose $\hat{\theta}$ so that $[X(\hat{\theta}), \Phi(\hat{\theta})] \neq [X(\theta), \Phi(\theta)]$: a better pair cannot exist since otherwise it would have been chosen at stage (i).

Proposition 4

Given the contract in Proposition 3 the seller chooses $\hat{\theta} = \theta$ at stage (iii).

Figure 1 illustrates $\Phi(\theta)$. The greater the deviation of θ below $E\theta$, the lower X and the more costly it would be for an uninformed person to imitate the seller. This means more can be charged for the information. For θ above $E\theta$ nothing can be charged since, as explained above, any positive demand is consistent with being uninformed.

The dotted curve in Figure 2 (representing (21)) illustrates an informed person's certainty equivalent when he just uses his information to trade with. In contrast, the solid curve (representing (21) and (25)) illustrates the certainty equivalent for an information seller. It can be easily shown it is symmetric about $E\theta$ rather than RP. The value of the signal to the seller depends only on its deviation from $E\theta$, not whether it is good or bad news.

When an informed person is just investing in the two assets his demand switches from negative to positive at θ = RP. However, when selling information it switches at $\theta = E\theta$. The reason is that for $RP < \theta < E\theta$ it is better to sell short and charge a positive price for the information than just take a small long position and not charge anything. Thus even though $X(\theta)$ and $\Phi(\theta)$ are discontinuous at $\theta = E\theta$, $C(\theta)$ is not.

An implication of Proposition 3 which is brought out by Figure 2 is that, independent of his information, person I is at least as well off selling information as trading. Moreover, for $\theta < E\theta$ he is strictly better off. Thus, ex ante before θ is known, he always wants to sell information.

Proposition 5

The informed person is always better off selling information rather than just investing in the two assets.

Differentiating (23) gives that for $\theta < E\theta$

$$\frac{a_I}{\Phi}\frac{\partial \Phi}{\partial a_I} = \frac{\sigma_\varepsilon^2}{\Phi}\frac{\partial \Phi}{\partial \sigma_\varepsilon^2} < 0 . \tag{28}$$

The less risk averse the seller and the greater the precision of his information, the more that he can charge in total. This is because as a_I and σ_ε^2 fall the informed person is willing to sell short a greater amount of the risky asset: as a result he can charge more for the information since it would be more costly for an uninformed person to imitate him.

It is immediate from (21) and (25) that

$$\frac{a_I}{C}\frac{\partial C}{\partial a_I} = \frac{\sigma_\varepsilon^2}{C}\frac{\partial C}{\partial \sigma_\varepsilon^2} < 0 . \tag{29}$$

As might perhaps be expected, the less risk averse the seller, the higher Φ and the better off he is. Similarly for changes in the precision of his information.

Proposition 6

The less risk averse the seller and the more precise his information, the more he can charge and the greater the certainty equivalent value of the information.

Next consider the market from the buyers' point of view. In order for them to be able to decide whether or not to purchase the information at stage (ii) they must have some idea what its precision is. If this is directly observable or they have exogenous information then they can make their decision on the basis of this. When such information is not available buyers know from the set announced at stage (i):

$$X(E\theta) = \frac{\zeta}{a_I\sigma_\varepsilon^2} \,. \tag{30}$$

Since ζ is known, it is possible for them to deduce $a_I\sigma_\varepsilon^2$ but they cannot separately identify a_I and σ_ε^2.

Proposition 7

When $0 < a < \infty$ and the seller can only invest in the safe and the risky asset, the precision of his information cannot be deduced from the contract by buyers.

It is shown below that when there is a positive lower bound on risk aversion, it is possible for buyers to deduce an upper bound on σ_ε^2 and this can be used in the decision of whether to purchase or not. When state-contingent securities are traded it is possible for buyers to separately identify a_I and σ_ε^2.

An alternative interpretation of Proposition 7 is possible. If the seller were to announce his precision and his risk aversion at stage (i), he would only have a strict incentive to correctly announce $a_I\sigma_\varepsilon^2$: he would be indifferent about separately identifying a_I and σ_ε^2.

The buyers will be prepared to purchase information if their expected utility at stage (ii) is greater than when they're uninformed. When a buyer purchases information the certainty equivalent of being informed, conditional on θ, is

$$C^B(\theta) = \frac{(\theta - RP)^2}{\ldots} - \frac{\Phi(\theta)}{\ldots} \tag{31}$$

This is illustrated in Figure 3 by the solid curve. It is discontinuous at $\theta = E\theta$. The dotted curve is the certainty equivalent of being informed at no cost: $(\theta - RP)^2/a_B\sigma_\varepsilon^2$. When n is increased the effect is to move the $\theta < E\theta$ portion of $C_i^B(\theta)$ closer to this. In the limit as $n \to \infty$ they coincide.

If the buyer were to remain uninformed his demand would be as in (8). Using this to calculate his expected utility conditional on θ, gives the certainty equivalent of being uninformed as

$$C_u^B(\theta) = \frac{(\theta - RP)\zeta}{a_B\sigma_\rho^2} - \frac{\zeta^2\sigma_\varepsilon^2}{2a_B\sigma_\rho^4} . \tag{32}$$

This is the solid straight line in Figure 3. It is tangent to $(\theta - RP)^2/a_B\sigma_\varepsilon^2$ at θ^0 where

$$RP < \theta^0 = E\theta - \frac{\sigma_\theta^2}{\sigma_\rho^2}\zeta < E\theta . \tag{33}$$

Otherwise it lies everywhere below it. It can be seen that for θ sufficiently near θ^0, the buyer is worse off ex post from purchasing information. However, for large deviations from θ^0 the reverse is true. As n increases the range of θ for which buyers are worse off becomes arbitrarily small. Hence:

<u>Proposition 8</u>

There always exists a finite n such that buyers will be prepared to purchase information at stage (ii).

The assumption of a continuum of traders means that the seller can ignore the effect of his actions on the price of the risky asset. If the model involved a finite number of traders, the price of the risky asset would convey information about θ to the uninformed. In this case, the buyers' appropriate comparison of the expected utility from buying the information would be with the expected utility given the information about θ deduced from the price. Provided there is enough noise in the supply or demand for the risky asset, a similar result to that above will hold. In such cases, the information buyers' influence on the price will be sufficiently small that the seller can still choose n large enough to make it worthwhile for them to buy the information. When the noise is insufficient for this, the constraint in the seller's full decision problem that buyers have to be better off purchasing information will bind. This introduces risk sharing issues into the analysis and prevents closed form solutions from being found. In the extreme case where there is no noise the risky asset's price would perfectly reveal θ and it would not be possible to sell information (see, e.g., Grossman and Stiglitz (1980), Hellwig (1980) and Diamond and Verrecchia (1981)).

Finally taking Proposition 5 together with Proposition 7 gives:

Proposition 9

When the degree of risk aversion is unobservable and can take any positive value an information market always exists.

(C) Risk aversion unobservable: $0 \leq a_L < a < a_U \leq \infty$

Finally, consider the intermediate case where risk aversion is unobservable but instead of being able to take any positive value it is known that there is a positive lower bound, a_L, and a finite upper bound, a_U.

Suppose first that $a_L > 0$ but $a_U = \infty$ as before. Since the right hand side of (15) is convex in a, then whenever $a^* \geq a_L$ it is still sufficient (and necessary) that the constraint (15) be satisfied for a^*. Using (19) and (22), it can be seen $a^* \geq a_L$ for θ such that

$$\theta^- = E\theta - \left(\frac{a_I\sigma_\varepsilon^2}{a_L\sigma_\rho^2} - 1\right)\zeta \le \theta \le \theta^+ = E\theta + \left(\frac{a_I\sigma_\varepsilon^2}{a_L\sigma_\rho^2} - 1\right)\zeta . \qquad (34)$$

For these θ, Proposition 3 is relevant. For θ outside this range $a^* < a_L$. In this case convexity implies that it is sufficient (and necessary) that (15) be satisfied for a_L: if it holds for a_L it will hold for all feasible a. It can then be shown

$$EV_I = -\exp\left[-a_I\left(\frac{\zeta^2}{2a_L\sigma_\rho^2} + (\theta - E\theta)X - \left(a_I\sigma_\varepsilon^2 - a_L\sigma_\rho^2\right)\frac{X^2}{2}\right)\right] . \qquad (35)$$

When $a_I\sigma_\varepsilon^2 \le a_L\sigma_\rho^2$ the situation is similar to that in Subsection A where risk aversion is observable: the coefficient on X^2 is negative so that the expression is convex in X. By taking a large enough position the seller can reach any level of utility. This will tend to happen when the lower bound on risk aversion is high, the seller is not very risk averse or his information is very precise.

However, when

$$a_I\sigma_\varepsilon^2 > a_L\sigma_\rho^2 \qquad (36)$$

the seller's utility is bounded and his optimal demand is

$$X(\theta) = \frac{\theta - E\theta}{a_I\sigma_\varepsilon^2 - a_L\sigma_\rho^2} . \qquad (37)$$

Substituting this in (15) it follows that for $\theta < \theta^-$ and $\theta > \theta^+$,

$$\phi(\theta) = \frac{(\theta - \theta^+)^2 a_L\sigma_\rho^2}{2\left(a_I\sigma_\varepsilon^2 - a_L\sigma_\rho^2\right)^2} . \qquad (38)$$

Figure 4 illustrates Φ. The amount that the seller charges is given by the solid curve. For θ between θ^- and θ^+, Φ is as in Proposition 3 but for θ outside this range it is given by (38).

When $a_L = 0$, it is not possible to charge anything for $\theta > E\theta$ because any positive demand is consistent with being uninformed. If $a_L > 0$, then for demands such that $(\theta - RP)/a_I\sigma_\varepsilon^2 > \zeta/a_L\sigma_\rho^2$, or equivalently $\theta > \theta^+$, the person can charge something: any uninformed person who took such a position would be strictly worse off. As in Subsection B the seller's demand, (37), is greater in absolute value than the demand of a person who is just trading the two assets, (10). Holding Φ constant, $\partial EV_I/\partial X = 0$ for an informed person at the demand given by (10), but is negative for somebody who is informed. Hence it is optimal for the seller to have a higher X since this allows him to charge more. Similarly for $\theta < \theta^-$.

Using (38) in (35), the certainty equivalent value of the information to the seller corresponding to (37) and (38) is

$$C(\theta) = \frac{\zeta^2}{2a_L\sigma_\rho^2} + \frac{(\theta - E\theta)^2}{2(a_I\sigma_\varepsilon^2 - a_L\sigma_\rho^2)} \,. \tag{39}$$

Otherwise the certainty equivalent is as in Proposition 3. It is illustrated in Figure 5. The solid curve again represents the certainty equivalent the seller actually obtains.

What if $a_U < \infty$? It can be shown using (19) and (22) and the appropriate expressions for $X_I(\theta)$ from Proposition 3 that

$$a^* < a_I \leq a_U \,. \tag{40}$$

Thus the upper bound never binds and it has no effect on the analysis.

Intuitively, it might be expected that when the informed person's demand is sufficiently small it would be costly for the uninformed to imitate and so a positive charge could be made. This would happen whenever

$$\frac{\theta - RP}{a_I \sigma_\varepsilon^2} < \frac{\zeta}{a_U \sigma_\rho^2} . \tag{41}$$

However, this argument is incorrect. Inequality (41) is equivalent to

$$\theta < E\theta - \left(1 - \frac{a_I \sigma_\varepsilon^2}{a_U \sigma_\rho^2}\right)\zeta . \tag{42}$$

Since $a_U \sigma_\rho^2 > a_I \sigma_\varepsilon^2$, (42) only holds for $\theta < E\theta$. As explained above in Subsection B, for $RP < \theta < E\theta$ it is better in any case for the seller to take a short position and charge for the information rather than have a small positive demand. This means the upper bound has no effect on the analysis.

The counterpart to Propositions 2 and 3 is

Proposition 10

When risk aversion is unobservable and lies in the range (a_L, a_U) the seller's utility is unbounded if $a_I \sigma_\varepsilon^2 \leq a_L \sigma_\rho^2$. Otherwise the contract for selling information is the same as in Proposition 3 except that for $\theta < \theta^-$ and $\theta > \theta^+$, $X(\theta)$, $\Phi(\theta)$ and $C(\theta)$ are given by (37)-(39), respectively.

It can straightforwardly be shown that the counterparts to Propositions 4 and 5 also hold in this context provided (36) is satisfied. The only additional result is that for $\theta < \theta^-$ and $\theta > \theta^+$,

$$\frac{\partial \Phi}{\partial a_L} ; \frac{\partial C}{\partial a_L} > 0 . \tag{43}$$

As might be expected, the higher a_L, the nearer θ^+ and θ^- are to $E\theta$ and the more effectively the seller can demonstrate he's informed.

Proposition 11

The greater a_L the more that the seller can charge for the information and the better off he is.

From the buyer's point of view, the analysis is again similar. The main difference is that it's now possible to deduce an upper bound on the seller's precision. As before, $\omega = a_I\sigma_\varepsilon^2$ can be found from the contract specified at stage (i). Combining this with the fact that $a_L \leq a_I$ it follows

$$\frac{1}{\sigma_\varepsilon^2} \leq \frac{a_L}{\omega} . \tag{44}$$

If the buyers do not know the seller's actual precision, they can use a_L/ω when deciding whether or not to buy. In this case the seller would need to increase n above the level necessary if his precision were known.

The counterpart to Proposition 7 is

Proposition 12

When $0 < a_L < a < \infty$, buyers can deduce an upper bound on the seller's precision which can be used in their purchase decision.

These are the only differences from the previous subsection, the other results are similar.

D. Summary

The purpose of Subsection A is to demonstrate that by itself the reliability problem is not sufficient to bound the seller's utility. However, if in addition, risk aversion is unobservable and can take any positive value as in Subsection B, the seller's utility is bounded and he can extract only part of the value of the information. Nevertheless, he will always be better off selling information than just investing in the two assets. By making the number of people he sells to sufficiently large, he can also ensure they're willing to buy the information. Thus an information market always exists.

When risk aversion is unobservable but lies in a limited range, as in Subsection C, only the lower bound has any effect on the analysis. If the seller is not very risk averse relative to this bound, or his information is precise enough that $a_I\sigma_\varepsilon^2 \leq a_L\sigma_\rho^2$, then the seller's utility is again unbounded. However, if the inequality is reversed then the results are similar to the case where risk aversion can take any positive value.

5. State-Contingent Securities

In the previous sections it has been assumed that the only securities that can be purchased are the safe and risky assets. This section investigates what happens when, in addition, securities contingent on ρ are available. These securities provide one unit of consumption in state ρ and nothing otherwise. Their price is denoted $p(\rho)$.

It is well known that when agents have exponential utility functions and are symmetrically informed, the allocation of consumption is the same whether there are just the safe and risky assets or whether there are state-contingent securities. Using this result, it can be shown that when traders believe the return to the risky asset has the distribution $N(E\theta,\ \sigma_\rho^2)$ and the price of the risky asset is P, then $p(\rho)$ is given by

$$Rp(\rho) = \frac{1}{(2\pi)^{\frac{1}{2}}\sigma_\rho} \exp\left[-\frac{(\rho - RP)^2}{2\sigma_\rho^2}\right] . \tag{45}$$

As above, when information is sold there are only a finite number of informed people. Since there is a continuum of uninformed people, (45) is the price function faced by traders here.

Similarly to Section 3, an uninformed trader's demand for state-contingent securities is given by

$$\underset{x_u(\rho)}{\mathrm{Max}} - \int \exp[- a x_u(\rho)] dF(\rho) \tag{46}$$

subject to the budget constraint

$$\int x_u(\rho) p(\rho) d\rho = 0 \; . \tag{47}$$

(This assumes that $W_0 = 0$ as before.) The solution is

$$x_u(\rho) = \frac{(E\theta - RP)}{a\sigma_\rho^2} (\rho - RP) \; . \tag{48}$$

An informed trader's problem is similar except $dF(\rho)$ is replaced by $dF(\rho|\theta)$. Here,

$$x_i(\rho, \theta) = \frac{1}{2a} \left[- \frac{\sigma_\theta^2}{\sigma_\rho^2 \sigma_\varepsilon^2} \left(\rho^2 - (RP)^2\right) + 2\left(\frac{\theta}{\sigma_\varepsilon^2} - \frac{RP}{\sigma_\rho^2}\right)(\rho - RP) + \frac{\sigma_\theta^2}{\sigma_\varepsilon^2}\right] \; . \tag{49}$$

In contrast to (48), which is linear, an informed trader's demand is a quadratic function of ρ. He exploits the information by taking a relatively long position in the more likely states where the density function of $N(\theta, \sigma_\varepsilon^2)$ lies above that of $N(E\theta, \sigma_\rho^2)$ and a relatively short position where it lies below.

The certainty equivalent corresponding to (48) is the same as in (9). For (49),

$$c_i(\theta) = \frac{(\theta - RP)^2}{\cdots} + \frac{\sigma_\theta^2}{\cdots} - \frac{1}{\cdots} \log \frac{\sigma_\rho^2}{\cdots} \tag{50}$$

It can be readily shown that the sum of the last two terms is positive. As might be expected, this means the value of the information to the trader is strictly increased by the existence of state-contingent securities.

Next, consider the problem of selling information when the seller's degree of risk aversion is unobservable and can take any positive value as in Section 4B. The other cases dealt with in Sections 4A and 4C can be similarly analyzed.

The seller's problem if he is to identify himself as informed is

$$\max_{x(\rho,\theta),\phi(\theta)} \; - \int \exp[- a_I(x(\rho,\ \theta) + \phi(\theta))]dF(\rho|\theta) \tag{51}$$

subject to

$$- \int \exp[- a(x(\rho,\ \theta) + \phi(\theta))]dF(\rho) \le - \exp[- \frac{\zeta^2}{2\sigma_\rho^2}] \quad \text{for all } a \tag{52}$$

and the budget constraint (47).

Similarly to before, it can be seen that the left-hand side of (52) is a concave function of a. Let the maximizing value be a*, where

$$\int[x(\rho,\ \theta) + \phi(\theta)]\exp[- a^*(x(\rho,\ \theta) + \phi(\theta))]dF(\rho) = 0\ . \tag{53}$$

The constraint (52) can then be represented by (53) together with the inequality in (52) with a = a*: if it is satisfied for the value that maximizes the left-hand side it is satisfied for all a. Even though an explicit expression for a* can no longer be found it is possible to show (see Appendix):

Proposition 13

The information seller's demands for the state-contingent securities are

$$x(\rho,\ \theta) = \frac{1}{2(a_I - a^*)}\left[-\frac{\sigma_\theta^2}{\sigma_\rho^2\sigma_\varepsilon^2}\left(\rho^2 - (RP)^2\right) + 2\left(\frac{\theta}{\sigma_\varepsilon^2} - \frac{E\theta}{\sigma_\rho^2}\right)(\rho - RP) + \frac{\sigma_\theta^2}{\sigma_\varepsilon^2}\right] \quad (54)$$

where a positive value of a^* is given uniquely by

$$\xi_1\ (a^*,\ a_I) = a^*\sigma_\theta^2 + \frac{a^{*2}\sigma_\rho^2(\theta - E\theta)^2}{A} - A\left[\frac{\zeta^2}{\sigma_\rho^2} + \log\left(1 + \frac{a^*\sigma_\theta^2}{A}\right)\right] = 0 \quad (55)$$

and

$$A = a_I\sigma_\varepsilon^2 - a^*\sigma_\rho^2 \geq 0\ . \quad (56)$$

The corresponding payments for information are

$$\phi(\theta) = \frac{1}{2(a_I - a^*)}\left\{\frac{a^*\sigma_\theta^4}{A\sigma_\varepsilon^2} + \frac{(\theta - E\theta)^2}{A}\left[\frac{a^*\sigma_\rho^2(a_I - a^*)}{A} + a_I\right] + \frac{\zeta^2}{\sigma_\rho^2} - \frac{(\theta - RP)^2}{\sigma_\varepsilon^2}\right\} > 0\ . \quad (57)$$

The certainty equivalents are

$$c(\theta) = \frac{1}{2}\left[\frac{(\theta - E\theta)^2}{A} + \frac{\zeta^2}{a^*\sigma_\rho^2} + \frac{1}{a^*}\log\left(1 + \frac{a^*\sigma_\theta^2}{A}\right) - \frac{1}{a_I}\log\left(1 + \frac{a_I\sigma_\theta^2}{A}\right)\right]\ . \quad (58)$$

These results can be compared with those in Proposition 3. In that case, if $\theta > E\theta$ then $X > 0$ and an informed investor cannot charge anything for his information. This is because any positive demand is consistent with being uninformed, so an information seller cannot distinguish himself from the uninformed. If $\theta < E\theta$ then $X < 0$ and it is costly for the uninformed to imitate no matter what their risk aversion is. This means the seller can charge for the information.

In contrast, with state-contingent securities an uninformed person's optimal position is a linear function of ρ, irrespective of their risk aversion, whereas an informed person chooses a quadratic position. An uninformed person would always be worse off taking a quadratic position. Hence an information seller can distinguish himself from the uninformed and charge something for his information whatever the value of θ.

It can be shown using (55)-(57) that $\lim_{\theta\to+\infty} \phi(\theta) = 0$, $\lim_{\theta\to-\infty} \phi(\theta) = +\infty$ and $d\phi/d(\theta-E\theta) < 0$. These imply $\phi(\theta)$ has the form shown in Figure 6. Although a positive amount can be charged for every θ, there is still an asymmetry between those values above $E\theta$ and those below. The reason for this is similar to before. When an informed person observes a high θ he wants to take a large long position on average. However, uninformed traders with low risk aversion also want to take a large long position. For such people the cost of switching from a linear to a quadratic position is small. Hence an information seller cannot charge very much when θ is high. For low θ an informed person takes a short position on average. The cost to an uninformed person of imitating this is large no matter what their risk aversion, so the seller can charge more than for higher θ.

Comparing (49) and (54), it can be seen that there are two differences. Firstly, the term $1/2a$ is replaced by $1/2(a_I - a^*)$. As before, the seller takes a more extreme position than he would if he were just trading. The loss in utility from the greater risk is more than offset by the fact that a higher charge can be made. Secondly, the coefficient of ρ switches from $(\theta/\sigma_\varepsilon^2 - RP/\sigma_\rho^2)$ to $(\theta/\sigma_\varepsilon^2 - E\theta/\sigma_\rho^2)$. More can be charged when the seller takes a short position. For $RP\sigma_\varepsilon^2/\sigma_\rho^2 < \theta < E\theta\sigma_\varepsilon^2/\sigma_\rho^2$, it is better to do this rather than take a small long position and charge less.

As far as the certainty equivalents are concerned, the seller's utility again depends only on the deviation of θ from $E\theta$. This follows from (58) and the fact that (55) implies a^* is symmetric about $E\theta$. The seller is, of course, better off selling information when state-contingent securities are available than when there are just the safe and risky assets. However, it can easily be demonstrated that the total amount charged is not always higher when there are state-contingent securities (e.g. $\sigma_\varepsilon^2 = 0.1$, $\sigma_\theta^2 = 0.9$, $a_I = 1$, $\theta = E\theta = 2$, $RP = 1$: $\Phi(2) > \phi(2)$). In such cases, it is not worth it for the seller to take an extreme position to separate himself from the uninformed and charge a high amount. Instead, it is better to take a less extreme position and charge less.

The other results are similar to Propositions 4-9 in Section 4B with two exceptions. The first concerns the effect of a change in the precision of the seller's information on the amount that he charges. In Proposition 6, the greater the precision, the more that is charged. It is shown in the Appendix that the derivative of (57) with respect to σ_ε^2 can have either sign:

Proposition 14

An increase in the precision of the seller's information can lead to an increase or a decrease in the amount charged.

Even though ϕ can either rise or fall, it is still true that the seller is always better off the greater the precision of his information. The reason that the amount he charges can fall is the following. As explained above, for θ such that $RP\sigma_\varepsilon^2/\sigma_\rho^2 < \theta < E\theta\sigma_\varepsilon^2/\sigma_\rho^2$ it is better for the seller to take a short position and charge a large amount rather than take a long position and charge less. As precision increases (i.e. σ_ε^2 goes down) $E\theta\sigma_\varepsilon^2/\sigma_\rho^2$ falls. This means that for θ in this region it is no longer worth taking a short position and charging more, so ϕ can fall and thus $d\phi/d\sigma_\varepsilon^2 > 0$ is possible.

The other result that differs from Section 4B concerns the buyers' knowledge of the seller's precision. With just a safe and a risky asset it is not possible to separately identify the seller's risk aversion and his precision. However, the fact that with state-contingent securities the seller takes a quadratic rather than a linear position means that they can be distinguished. This is formally proved in the Appendix and gives the following counterpart to Proposition 7.

Proposition 15

When state-contingent securities are available, an information buyer can uniquely deduce the seller's precision from the portfolios and payments announced at stage (i).

An alternative interpretation of the model, which is equivalent to the one presented here, is the following. At stage (i) the seller specifies payment schedules of the form

$$\pi(\rho, \theta) = x(\rho, \theta) + \phi(\theta) . \tag{59}$$

At stage (iii) he announces $\hat{\theta}$ and at stage (v) receives $\pi(\rho, \hat{\theta})$. The buyers can purchase $x(\rho, \theta)$ at zero net cost (from (47)) to offset $\pi(\rho, \theta)$ so that their net payment is $\phi(\theta)$ as before. The arrangement is exactly equivalent to that considered above.

Bhattacharya and Pfleiderer (1985) have analyzed a model which is similar in nature to this payment schedule interpretation. They consider the question of how portfolio owners can ensure that only managers with the ability to produce precise estimates of future returns are hired, by adopting a principal-agent approach to the problem. The portfolio owners (the principal) maximize their surplus subject to a reservation utility constraint on the manager (the agent) which is determined by an exogenously given earnings opportunity. In contrast to here, the assumption in their paper is that there are many potential managers who compete away any surplus. It is therefore not necessary that risk aversion be unobservable. They derive the approximately optimal contract in the case where the portfolio owners are effectively risk neutral and show it also involves a reward schedule which is a quadratic function of the payoff to the risky asset. Their results provide an interesting contrast to those obtained here.

6. Intermediation

It was shown in Sections 4 and 5 that in situations where a person's information cannot be directly verified and their degree of risk aversion is unobservable, information can nevertheless be sold. However, the seller can only charge a finite amount despite the fact that the value of the information is unbounded. So far it has been assumed that there is only sufficient time for information to be sold once. In this section the model is extended to the case where sufficient time remains to allow buyers to resell the information. This means they can become intermediaries and obtain some of the value of the information that the original seller is unable to extract. The model is therefore able to provide a theory of intermediation which does not depend on transaction costs.

If the contract the original seller uses and his announcement $\hat{\theta}$ are observable to second-stage buyers, possibly ex-post, then an intermediary can use this to verify the information he sells. In this case there's no need for the intermediary to have a portfolio which signals he's informed; he can charge a fixed fee as in Section 3 where it's directly observable whether the seller is informed or not.

The more interesting situation is where the first-stage contract and announcement of $\hat{\theta}$ cannot be observed at any point by the second-stage buyers. In this case the intermediaries have to verify their information in the same way that the original seller does. It follows from Proposition 4 that the intermediaries will always be better off selling the information than just trading on their own account.

<u>Proposition 16</u>

If a buyer of information has sufficient time before asset markets meet to act as an intermediary and resell his information, it will always be worthwhile for him to do this.

The possibility of reselling information thus makes intermediaries strictly better off than if they were only able to use the information to speculate. The reason this is possible is that they are able to extract more of the value of the information from the rest of the investors than the original seller can alone. Similarly to Proposition 6, at least part of the reason for this is that increasing the number of sellers increases the amount of risk that can be borne. The question is whether the risk bearing capacity of the sellers is the only determinant of the amount that can be extracted. In other words, how is the level of rents that can be extracted by a group related to the level that can be extracted by a single less risk averse individual with the same risk bearing capacity.

With exponential utility functions and normally distributed random returns, the risk bearing capacity of a single individual with risk aversion a is the same as that of a group of m individuals, each with risk aversion ma, in the following sense. The certainty equivalent amount required to compensate the single individual for bearing the risk associated with a random return r is the same as the total amount required to compensate the group of m, if they each bear the risk associated with r/m.

Consider first the case where risk aversion can take any positive value: $0 < a < \infty$. With just a safe and a risky asset, it follows from (20) and (24) that the amount that can be charged by a single seller with risk aversion a_I is the same as that which can be charged by a homogeneous group of sellers each with risk aversion ma_I. The analysis when state-contingent contracts are available is similar. It follows from (55) that if $\xi_1(a^*, a_I) = 0$ then $\xi_1(ma^*, ma_I) = 0$. Given this, it can be seen from (57) that the amount a group can charge is the same as the amount an individual with the same risk bearing capacity can. Thus intermediation is profitable purely because of the fact it allows risk to be spread further. The people who can extract the most are those with the lowest risk aversion, but this is only because of their risk-bearing capacity.

Next, consider the case analyzed in Section 4C where there is a positive lower bound on the degree of risk aversion: $0 < a_L < a < \infty$. It can be shown using (34) and (38) that the amount that can be charged by a single person with risk aversion a_I is <u>not</u> the same as the total for a homogeneous group of size m, each with risk aversion ma_I. For the individual θ^-, θ^+ and the total amount that can be charged are given by (34) and (38) together with (20) and (23). For the group of m people, the counterpart of (34) is

$$\theta^- = E\theta - \left[\frac{a_I \sigma_\varepsilon^2}{(a_L/m)\sigma_\rho^2} - 1\right]\zeta \le \theta \le \theta^+ = E\theta + \left[\frac{a_I \sigma_\varepsilon^2}{(a_L/m)\sigma_\rho^2} - 1\right]\zeta \qquad (60)$$

and using (38) the total amount they can charge is

$$m\phi(\theta) = \frac{(a_L/m)\sigma_\rho^2}{2[a_I\sigma_\varepsilon^2 - (a_L/m)\sigma_\rho^2]^2}\{\theta - E\theta - [\frac{a_I\sigma_\varepsilon^2}{(a_L/m)\sigma_\rho^2} - 1]\zeta\}^2 . \quad (61)$$

Comparing these with (34) and (38), it can be seen they're the same except a_L is replaced by a_L/m. Using (43), it follows that in this case the individual can charge more than the group even though they have the same risk bearing capacity. This is because the less risk averse individual can make better use of the lower bound on a when demonstrating he's informed than the more risk averse people in the group. As before, the least risk averse people can extract the most as intermediaries. However, this is not only because of increased risk bearing capacity, but also because here less risk averse people have a comparative advantage in establishing they're informed.

Proposition 17

When $0 < a < \infty$ and there is intermediation, the total amount that can be extracted from buyers depends only on the total risk bearing capacity of the sellers. However, when $0 < a_L < a < \infty$, a risk tolerant individual can extract more than a group with the same risk bearing capacity.

In recent years, a number of other theories of intermediation have been suggested which, like Leland and Pyle (1977), are not based on differences in transaction costs (see e.g. Chan (1983), Diamond (1984)). The one most closely related to the model here is Ramakrishnan and Thakor (1984). Their model involves firms which issue new securities, hiring information producers to certify the value of these. In contrast to here, they assume that the utility functions of the information producers are publicly known and that there is a stochastic ex-post indicator of the effort expended in acquiring the information. These assumptions enable information reliability to be ensured, by conditioning the information producer's payment on the signal of effort. The main result of the paper is to show that provided information producers can monitor each other directly, it is better for them to form an intermediary rather than operate individually since this permits diversification of the risk associated with the effort indicator. In contrast, in the model here, there is no direct ex-post indicator of whether the seller is informed, only the indirect indicator provided by the return on the risky asset. Also, it is not necessary that sellers can monitor each other costlessly: they can use the contracts to signal to each other, as well as to the buyers, that they're informed. Nevertheless, one of the reasons intermediation is profitable is similar: it allows the risk bearing capacity of the sellers to be expanded.

7. Concluding Remarks

This paper is concerned with deriving feasible and optimal strategies for selling information. To make the analysis tractable exponential utility functions and normally distributed returns are assumed. These allow restrictions on portfolios and the corresponding payments which are both necessary and sufficient to ensure the seller is informed, to be derived. In more general models, simple sufficient conditions can be fairly easily found. For example, it can be shown that requiring the expected value of the payment schedule in terms of the uninformed distribution, to be zero, is sufficient to identify the seller as informed. However, finding conditions which are also necessary is much more complex in such situations.

An important feature of the model is that the time horizon is only one period. In any multiperiod model, reputation considerations [illegible] play a role in overcoming the reliability problem associated with information sales. This is an important topic for future research.

In conclusion, the main results of the paper are the following. Firstly, the information reliability problem is not sufficient to rule out the existence of monopolistic markets where information is sold directly. In fact, in the model presented an information market always exists. Secondly, the operation of such markets depends critically on the information buyers have about the risk aversion of the seller and the securities that are available. Finally, the view of information markets presented here, leads to a theory of intermediation which is not based on transaction costs. The original seller is able to capture only a portion of his information's value from buyers. Intermediaries make profits because they can capture some of the remaining value.

Appendix

Proof of Proposition 13

The maximization problem (51) can be solved straightforwardly to give (54) and (57). Substituting these in (53) gives (55). However, (55) alone does not necessarily uniquely define a*. For example, in the case where $\sigma_u^2 = 2$, $\sigma_\varepsilon^2 = \sigma_\theta^2 = 1$, $a_I = 1$, $\zeta = 1$ and $\theta = E\theta = 1$, it can readily be seen $a^* = 0.37$ and $a^* = 1.76$ are both solutions to (55). Nevertheless, combining (55) with (56) does give a unique optimal value of a*. This is shown in two steps: first, there does exist a unique value satisfying (55) and (56), and second, this value yields a strictly higher expected utility than any value of a* not satisfying (56).

A value of a* satisfying (55) and (56) exists since ξ_1 is continuous in this range, $\xi_1(0) < 0$ and $\lim_{a^* \to a_I\sigma_\varepsilon^2/\sigma_\rho^2} \xi_1(a^*) > 0$. The value is unique because given (56), it can be shown that whenever $\xi_1(a^*) = 0$, $\xi_1'(a^*) > 0$.

The second result follows from the expression for the expected utility of the information seller. The certainty equivalent of selling information is given by (58). In order for the variance in the integration to be positive and hence for the expected utility integral to exist, either $a^* < a_I\sigma_\varepsilon^2/\sigma_\rho^2 (< a_I)$ or $a^* > a_I (> a_I\sigma_\varepsilon^2/\sigma_\rho^2)$. Let

$$\ell(a) = \frac{1}{a^*}\log(1 + a^*\nu) - \frac{1}{a}\log(1 + a\nu) \quad . \tag{1A}$$

where

$$\nu = \sigma_\theta^2/A \quad . \tag{2A}$$

It can be seen $\ell(a^*) = 0$. For $\nu = 0$, it can be shown $\partial\ell/\partial a = 0$. For $\nu > 0$, $\partial^2\ell/\partial a\partial\nu > 0$ and so for $a^* < a_I\sigma_\varepsilon^2/\sigma_\rho^2$, $\partial\ell/\partial a > 0$ and $\ell(a_I) > 0$. For $\nu < 0$, $\partial^2\ell/\partial a\partial\nu < 0$ and so for $a^* > a_I$, $\partial\ell/\partial a > 0$ but now $\ell(a_I) < 0$. The remaining terms in (58) are clearly greater when $a^* < a_I\sigma_\varepsilon^2/\sigma_\rho^2$ than when $a^* > a_I$. Hence any solution with a^* satisfying (56) is strictly better than any other feasible solution not satisfying (56) and the proposition is demonstrated.

Proof of Proposition 14

Differentiating (57) and rearranging, it can be shown that

$$\frac{d\phi}{d\sigma_\varepsilon^2} = \frac{1}{a_I - a^*}\{\phi\frac{da^*}{d\sigma_\varepsilon^2} + \frac{a^*\sigma_\theta^4}{2A\sigma_\varepsilon^2}[(\frac{1}{a^*} + \frac{\sigma_\rho^2}{A})\frac{da^*}{d\sigma_\varepsilon^2} - \frac{2}{\sigma_\theta^2} - \frac{a_I}{A} - \frac{1}{\sigma_\varepsilon^2}]$$

$$+ \frac{a_I(\theta-E\theta)^2}{A^3}[(a_I-a^*)\sigma_\varepsilon^2\sigma_\rho^2\frac{da^*}{d\sigma_\varepsilon^2} - (a_I-a^*)a^*\sigma_\rho^2 - \frac{a_IA}{2}] + \frac{(\theta-RP)^2}{2\sigma_\varepsilon^4}\} \quad . \qquad (3A)$$

where

$$\frac{\sigma_\varepsilon^2}{a^*}\frac{da^*}{d\sigma_\varepsilon^2} = 1 + \frac{a^*\sigma_\theta^2A^2}{2a_I\sigma_\varepsilon^2a^*\sigma_\rho^2(a_I - a^*)(\theta - E\theta)^2 + a_Ia^*\sigma_\theta^4A} \quad . \qquad (4A)$$

To see that both signs are possible consider the example with $\sigma_\varepsilon^2 = 0.9$, $\sigma_\theta^2 = 0.1$, $a_I = 1$, $E\theta = 2$ and $RP = 1$. For $\theta = 1.5$, $\frac{d\phi}{d\sigma_\varepsilon^2} = 1.48 > 0$ and for $\theta = 2$, $\frac{d\phi}{d\sigma_\varepsilon^2} = -0.71 < 0$ as required.

Proof of Proposition 15

To demonstrate the proposition, it is necessary to show that there is a one-to-one correspondence between σ_θ^2 and a_I and the set of $[x(\rho, \theta), \phi(\theta)]$ announced at stage (i). It follows directly from Proposition 13 that for each σ_θ^2 and a_I there is a unique set. It then remains to show that for each set the values of σ_θ^2 and a_I can be uniquely identified.

To see this it is helpful to rewrite (54) in the form

$$x(\rho,\ \theta) = -\ \alpha\rho^2 + 2\beta\rho + \gamma\ . \tag{5A}$$

Then consider what can be deduced, given a single $x(\rho,\ \theta)$ (i.e., a particular α, β, γ) and $\phi(\theta)$. Using (5A) and (54) to give the definition of α and rearranging,

$$-\ \frac{(a_I - a^*)\sigma_\varepsilon^2}{\sigma_\theta^2} = -\ \frac{1}{2\alpha\sigma_\rho^2} = Z \tag{6A}$$

where Z (< 0 from (56)) is a constant which is observable to the buyer. Using (6A) together with the definition of β allows (55) to be written in the form

$$\xi_2(a^*) = (a^* + Z)\left[\frac{\xi^2}{\sigma_\rho^2} + \log\left(\frac{Z}{a^* + Z}\right)\right] + a^* - a^{*2}\,\frac{(\beta/\alpha - E\theta)^2}{\sigma_\rho^2(a^* + Z)} = 0\ . \tag{7A}$$

Since $\xi_2(0) < 0$, $\lim_{a^*\to -Z} \xi(a^*) > 0$ and $\xi_2'(a^*) > 0$, this equation enables the unique value of a^* satisfying (56) to be found. However, it is not then possible to use the remaining equations to solve for σ_θ^2 and a_I. This is because substituting into the definition of γ and $\phi(\theta)$ using the definitions of α and β and (6A) gives expressions which are independent of σ_θ^2 and a_I.

However, it follows from (55) that $da^*/d(\theta-E\theta)^2 < 0$. Hence, given any two schedules (denoted by the subscripts 1 and 2) from a set, it is possible to use (7A) to find the values of a_1^* and a_2^*. Then using the corresponding coefficients α_1 and α_2 together with the definition of α gives two simultaneous equations which can be solved uniquely to give

$$\sigma_\theta^2 = \frac{2\alpha_1(a_I - a_1^*)\sigma_\rho^4}{1 + 2\alpha_1(a_I - a_1^*)\sigma_\rho^2}\ . \tag{8A}$$

Hence the proposition is proved.

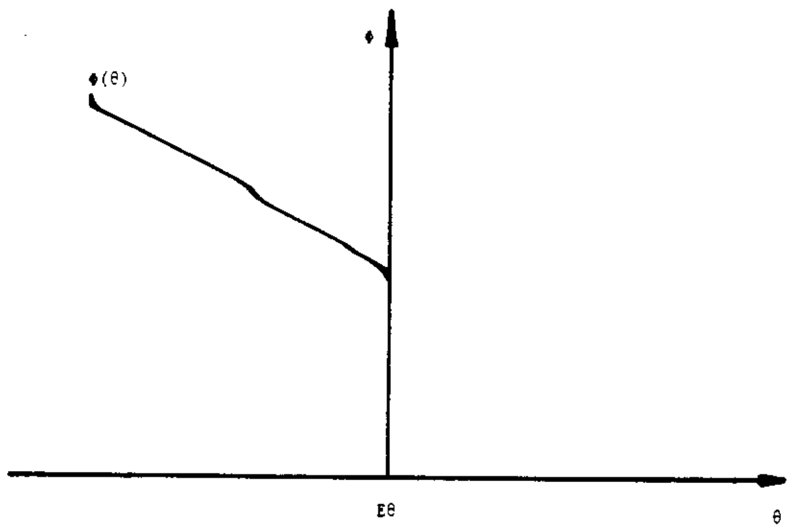

Figure 1

The total charge for information

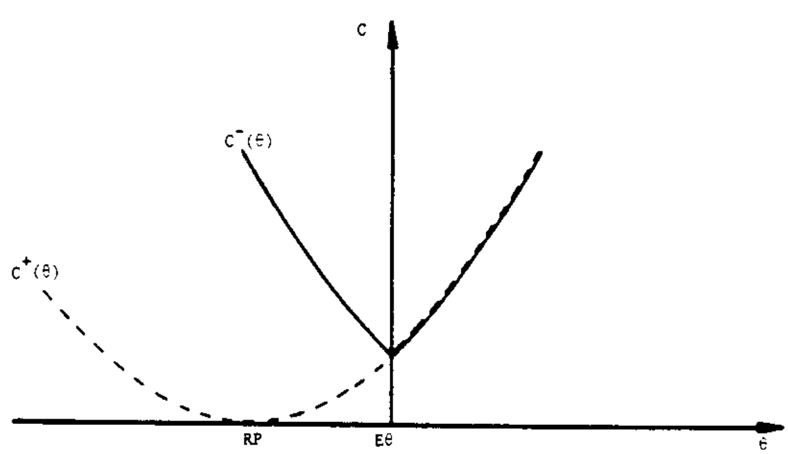

Figure 2

The informed person's certainty equivalents

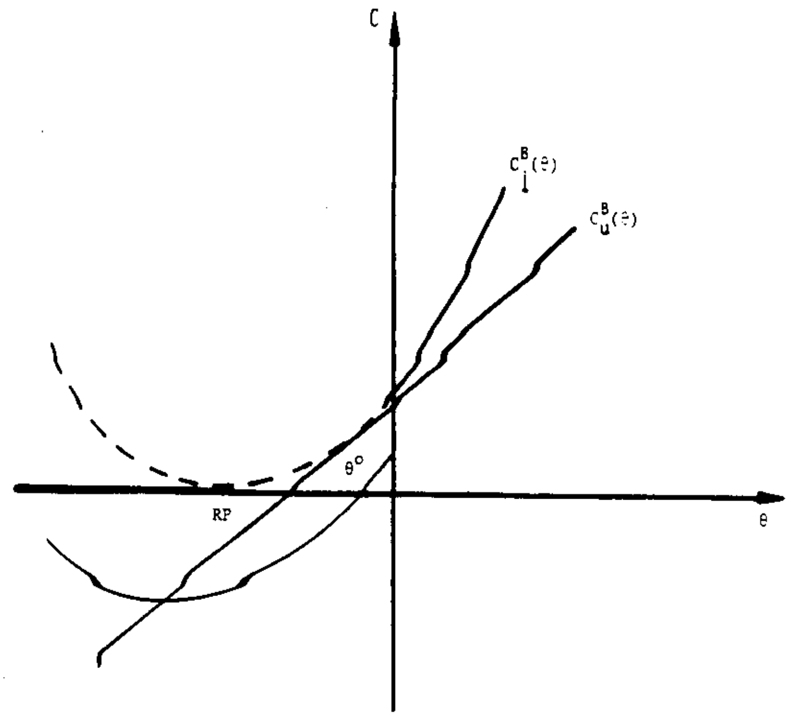

Figure 3

The buyers' decision

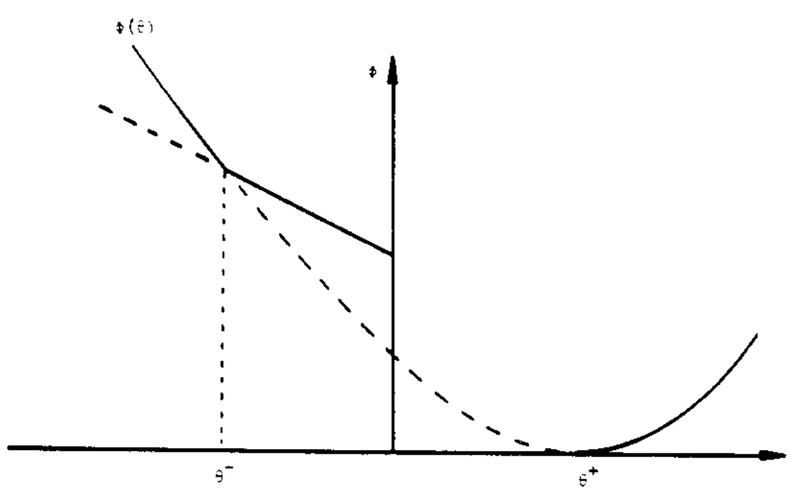

Figure 4

The total charge for information when $a_L > 0$

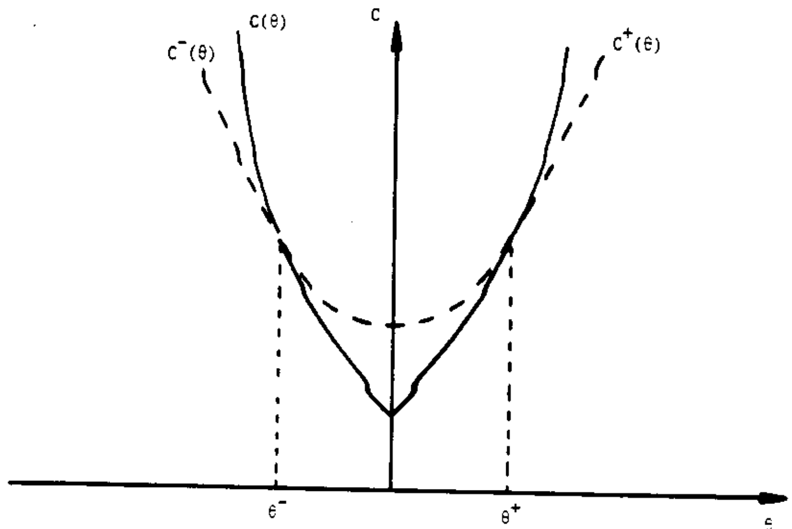

Figure 5

The certainty equivalent of the seller when $a_L > 0$

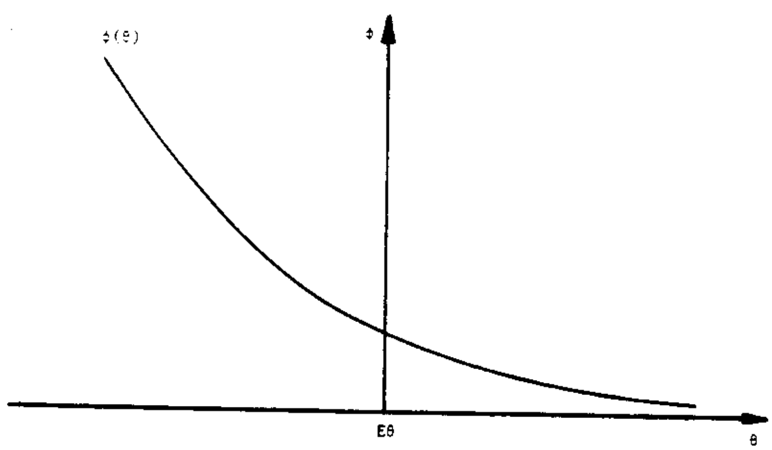

Figure 6

The total charge for securities with state-contingent securities

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