THE INTERACTION OF CORPORATE AND GOVERNMENT FINANCING IN GENERAL EQUILIBRIUM

by

Simon Benninga & Eli Talmor

28-86

RODNEY L. WHITE CENTER FOR FINANCIAL RESEARCH
The Wharton School
University of Pennsylvania
Philadelphia, PA 19104

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by

Simon Benninga
Hebrew University, Jerusalem, Israel

and

Eli Talmor
Tel-Aviv University, Israel
University of Pennsylvania

First draft: October 1986

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*We wish to thank Bernard Dumas, Robert Litzenberger, Roland Portrait and the participants in finance seminars at Wharton and Rutgers for helpful comments.

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ABSTRACT

This paper develops a general equilibrium model for analyzing the interaction of corporate financial and production decisions, consumers' behavior and government financing. We use the model to investigate how changes in income tax rates and government debt policy affect production, interest rates, and consumer welfare. We also show how changes in the different tax rates affect other tax rates in general equilibrium.

The Interaction of Corporate and Government Financing in General Equilibrium

1. Introduction

The purpose of this paper is to relate the taxation of personal and corporate incomes to the financing of the government budget. Two major complications must be accounted for in any discussion of this issue: First, the various kinds of taxation which are levied on different forms of income must be considered; in this paper we consider ordinary income taxation, personal taxes on income from capital, and the corporate income tax. Second, in order to establish the characteristics of a general equilibrium, we must consider how these various types of income taxation affect private and corporate behavior.

The relation between taxation and corporate capital structure has spawned a vast and contentious literature in which only recently some measure agreement has been reached. Modigliani and Miller (1958) showed that in the absence of taxation, debt and equity financing are perfect substitutes. When a corporate tax is introduced into their 1958 model, Modigliani and Miller (1963) argued that since interest is a tax deductible expense for the corporation, issuing debt increases shareholder wealth.

The Modigliani-Miller (1963) model explains the incremental value of debt at the corporate level, but leaves open the question of how equilibrium is determined when corporate interest expenses are tax deductible while outflows to shareholders are not. A solution to this problem is proposed in Miller (1977), which recognizes that the tax asymmetry between debt and equity is a two-edged sword. The advantage of debt over equity financing at the corporate level is reversed at the investor's personal level, at which interest is taxed as ordinary income while equity income is taxed at an effectively much lower rate. Once both corporate and personal taxes are considered in a capital market equilibrium, the pre-tax interest on corporate debt is grossed up, yielding capital structure irrelevancy for the individual firm. Facing differential pre-tax rates of return between debt and equity, investors cluster into two tax clienteles: Investors at a personal tax bracket below the corporate tax rate find it desirable to invest in taxable bonds, while investors at a higher personal tax bracket than the corporate rate buy equity and tax-exempt municipal bonds.

Miller's derivation of a financial market equilibrium does not require the modelling of either a government sector nor a real private sector (production and consumption). Miller's 1977 theorem takes as given real supply and demand, tax rates, and the market risk-free tax-exempt rate of interest. Given these assumptions, Miller proves that there exists an equilibrium in the financial markets. As it stands the model is, however, unable to predict the consequences of changes in government policy (tax rates and borrowing). For example, it is not clear what will happen to interest rates and production if one or more of the tax rates in the Miller equilibrium are changed. It is also not clear how the various tax rates and government debt are related, nor what would be the additional consequences on a particular tax rate of changing other policy variables.[1]

In this paper we address these questions. We employ a two-period model and assume that individuals are taxed differentially on different kinds of income (equity income is taxed at lower rates than other kinds of personal income). When coupled with a tax on corporate income, this gives rise to a Miller (1977) equilibrium in the non-government sector of the economy. In Section 2 of the paper we set out our basic model and prove the characteristics of the equilibrium. In Section 3 we derive the general framework for the comparative statics of the model. We use these results in the two succeeding sections: In Section 4 we examine the effects on the equilibrium of the model of changes in the corporate tax rate. In Section 5 we examine the effects of changing the personal tax rates. In Section 6 we discuss the question of crowding out in the framework of our model. In Section 7 we discuss the applicability of our results in light of the current reform of the United States tax system.

2. The Model

2.1 Basic definitions

We consider a model of the type which incorporates the taxation features of the Miller (1977) model. The model can also accomodate the extensions of DeAngelo and Masulis (1980a, 1980b) and Ross (1985). The model has two types of consumers, two dates (Date 0 and Date 1) and perfect certainty. A single good is used for both consumption and production.

The system of income taxation assumed in the model incorporates, albeit in a highly simplified and stylistic manner, features common to most tax codes: Differential taxation of interest income on corporate bonds and other privately-issued obligations (through an ordinary income tax) and of equity income (through an effectively lower capital gains tax), the existence of riskless bonds whose interest is not taxed (municipal bonds), and the tax deductibility of their interest expenses by corporations.[2]

In the model we shall assume that there is no tax whatever levied on a consumer's income from equity. Interest income from corporate bonds is taxed at the consumer's personal tax rate, but interest on government bonds is not taxed. A flat rate of personal taxation levied on earned income (including interest income from corporate bonds) is t_{p1} for consumer 1 and t_{p2} for consumer 2. Corporations pay income taxes at a rate t_c, and we shall assume that $t_{p1} > t_c > t_{p2}$. When convenient we shall refer to consumer 1 as the high-tax consumer and consumer 2 as the low-tax consumer. Both consumers can buy either corporate debt or government bonds, but neither consumer is allowed to short-sell either kind of debt (i.e. borrow on terms similar to those of the government and the corporation).

Representative individual 1 owns a firm which has a productive technology. The firm's before-tax income at Date 0 is $\bar{f}$ (from inputs purchased previous to the start of the model). At date 0 the firm purchases inputs z which will be used to produce output f(z) at Date 1. The production function f is assumed to be non-convex: $f'(z) \geq 1$ and $f''(z) \leq 0$. At Date 0 the firm issues an amount D of corporate bonds, which bear interest of i at Date 1. The corporate income tax is levied on the firm's income net of purchases of inputs; in addition, interest expenses are recognized for tax purposes at the firm level. The firm's dividend at Date 0 to its shareholders (in this case, individual 1) is thus

$$E_0 = (1 - t_c)[\bar{f} - z] + D .$$

The firm's dividend at Date 1 is

$$E_1 = (1 - t_c)f(z) - D\big(1 + i(1 - t_c)\big) .$$

The firm behaves competitively, taking the market interest rate i as given.

We denote by ω_{kj} consumer k's goods endowment at Date j and shall assume that this endowment is taxable at the consumer's ordinary income tax rate.[3] Let c_{kj} denote the consumption of consumer k at Date j, and let B_k and D_k represent consumer k's purchases at Date 0 of government and corporate debt to be redeemed at Date 1. We denote the interest rate paid on government bonds (tax-exempt) by r and the interest (taxable) on corporate bonds by i. Let

$U(c_{10}, c_{11})$ and $V(c_{20}, c_{21})$ be the concave utility function maximized by individuals 1 and 2, respectively. Then the maximization problem for each type of consumer is:

Individual 1:

$$\max U(c_{10}, c_{11})$$

subject to

$$c_{10} = (1 - t_{p1})\omega_{10} + E_0 - B_1 - D_1$$
$$c_{11} = (1 - t_{p1})\omega_{11} + E_1 + B_1(1 + r) + D_1(1 + i(1 - t_{p1})).$$
$$B_1, D_1 \geq 0.$$

Individual 2:

$$\max V(c_{20}, c_{21})$$

subject to

$$c_{20} = (1 - t_{p2})\omega_{20} - B_2 - D_2$$
$$c_{21} = (1 - t_{p2})\omega_{21} + D_2(1 + i(1 - t_{p2})) + B_2(1 + r)$$
$$B_2, D_2 \geq 0.$$

Note that we make no allowance for the low-tax consumer to buy equity; it follows from Theorem 0 below that in equilibrium he would not want to.

2.2. Equilibrium in the private sector

In this subsection we define basic equilibrium relationships in the model and derive other relations. It is clear that in equilibrium the amount of government and corporate debt issued must equal the amounts purchased by consumers:

$$D_1 + D_2 = D$$
$$B_1 + B_2 = B.$$

Theorem 0 (Miller 1977): Suppose $t_{p1} > t_c > t_{p2}$ and suppose that in equilibrium $f'(z) > 1$. Then in a competitive equilibrium:

1. $i = r/(1 - t_c)$.
2. $D_1 = 0$, $B_1 \geq 0$.
3. $D_2 \geq 0$, $B_2 = 0$.

Proof:

Optimization of the firm investment decision is given by the first-order condition $f'(z) = 1 + r$. If in equilibrium $f'(z) > 1$, then $r > 0$. Since neither equity income nor the interest on government bonds is taxed, it follows by arbitrage that the relevant discount rate for both is r (the interest on government bonds). Since the firm behaves competitively it will act to maximize the shareholders' net present value of debt issuance given the market interest rate i. This net present value is

$$D\{1 - \frac{1 + i(1 - t_c)}{1 + r}\} .$$

As long as $i < r/(1 - t_c)$, all firms will find it profitable to continue to issue more debt. But this debt issuance will in itself push up the market interest rate i. In the resulting equilibrium the relation between the two interest rates is $i = r/(1 - t_c)$.

Next we note that reprsentative consumer 1 (the stockholder) gets an after-tax return of r from his ownership in the firm and from purchases of government bonds. His after-tax return from corporate bonds, however, is

$$i(1 - t_{p1}) = \frac{r(1 - t_{p1})}{(1 - t_c)} ,$$

which is less than r, since $t_{p1} > t_c$. Consumer 1 will thus not find it profitable to purchase corporate bonds. This proves part 2 of the Theorem. The proof of part 3 is similar.

qed

Theorem 0 is a restatement of the Miller (1977) equilibrium. In this equilibrium: 1) Firms do not profit from the issuance of debt, since the net present value of debt issued is zero. 2) The equilibrium total amount of corporate debt is unique. 3) Ownership of corporate equity and corporate debt is segmented between two clienteles: High-personal-tax consumers buy corporate equity and tax-free government bonds and low-personal-tax consumers buy corporate debt. For the limiting case $r = i = 0$ (which will happen if in equilibrium $f'(z) = 1$), consumers in all tax brackets will be indifferent among the various securities. 4) It follows from Theorem 0 that the net bondholder return from corporate debt exceeds the interest on government bonds. This leads to a bondholder surplus of $D[i(1 - t_{p2}) - r]$, or by using the equilibrium relationship between i and r, $Di(t_c - t_{p2})$. As we show shortly, this bondholder surplus is financed by a government wealth transfer.

2.3. Financing the government

The government consumes amount G of the model's single good at Date 0 and uses G to produce a public good.[4] Throughout the paper we shall assume G to be fixed, so that only the financing of G is at issue. The government levies taxes on incomes and issues bonds at Date 0 in order to finance this consumption. The government budget constraint determines the relation between the various taxes and government bond issues B. It follows from Theorem 0 that all of the government's bonds will be purchased at Date 0 by the high-tax consumers. Thus the government budget constraints at Date 0 and Date 1 are given by:

$$G = t_c(\bar{f} - z) + t_{p1}\omega_{10} + t_{p2}\omega_{20} + B \tag{1}$$

$$(1 + r)B = t_c\big(f(z) - iD\big) + t_{p2}(iD + \omega_{21}) + t_{p1}\omega_{11} . \tag{2}$$

Note that at Date 1 the government has to repay not only its debt but also finance the bondholder surplus $iD(t_c - t_{p2})$.

Government expenses are financed by various income taxes and by the sales of government bonds. These means of financing are related through the two government budget constraints. We eliminate B by combining the Date 0 and Date 1 government budget constraints:

$$G = t_c\Big[\bar{f} - z + \frac{\big(f(z) - iD\big)}{1 + r}\Big] + t_{p1}\Big[\omega_{10} + \frac{\omega_{11}}{1 + r}\Big] + t_{p2}\Big[\omega_{20} + \frac{\omega_{21} + iD}{1 + r}\Big] . \tag{3}$$

The economic interpretation of this equation recalls the Ricardian equivalence proposition (see Barro (1974)). Ultimately all government consumption must be financed out of taxes, and the breakdown of government financing into debt and direct taxes does not affect the economy's real equilibrium. Moreover, equation (3) leads to the following additional irrelevance proposition:

Theorem 1: The economy's real equilibrium is uniquely determined by any two of the three income tax rates (t_c, t_{p1}, t_{p2}). The third tax rate and the amount of government debt are determined by the simultaneous solution of the government budget equations (1) and (2).

In the remainder of this paper we wish to consider the relation between the various tax rates of the model and the economy's real equilibrium. Theorem 1 guarantees that it is sufficient to consider only two tax rates and that the third tax rate and the amount of government debt are then endogenous. We choose to eliminate t_{p1} and B from the consumers' budget constraints. Substituting the government budget constraints into the consumers' budget constraints gives:

<u>for consumer 1</u>:

$$c_{10} = \omega_{10} + \bar{f} - z + D - G + t_{p2}\omega_{20} \tag{4}$$

$$c_{11} = \omega_{11} + f(z) + t_{p2}(\omega_{21} + iD) - D(1 + i) \tag{5}$$

<u>for consumer 2</u>:

$$c_{20} = (1 - t_{p2})\omega_{20} - D \tag{6}$$

$$c_{21} = (1 - t_{p2})\omega_{21} + D(1 + i(1 - t_{p2})) \tag{7}$$

The above equations have an interesting economic interpretation: Since individual 1 buys all of the government debt in period 0, he finances all of government consumption G except for that portion directly financed by individual 2's taxes in period 0. In period 1 individual 1 receives back all of his taxes, <u>plus</u> those taxes paid by individual 2.[5]

3. Altering the Government Financing Mix

Almost all debates on public policy which deals with taxation have to do with the effects on the economy of changing the way in which the government finances its consumption. We employ our model to derive the relevant results in a general equilibrium framework.[6]

In this section we give the general derivation of the model's comparative statics. In subsequent sections we shall use these technical results to explore the effects of changing the various tax parameters of the model.

We denote partial derivatives of the utility functions of the consumers with respect to consumption at each date by

$$U_0 \equiv \partial U/\partial c_{10}, \quad U_1 \equiv \partial U/\partial c_{11}, \quad V_0 \equiv \partial V/\partial c_{20}, \quad V_1 \equiv \partial V/\partial c_{21} \ .$$

Maximizing each consumer's utility subject to his budget constraints gives the first-order conditions for optimal consumption and savings:

$$\frac{U_1}{U_0} = \frac{1}{1 + r} \tag{8}$$

$$\frac{V_1}{V_0} \leq \frac{1}{1 + i(1 - t_{p2})} \tag{9}$$

Optimality for firm investment and financial decisions gives:

$$f'(z) = 1 + r \tag{10}$$

$$i = r/(1 - t_c) \ . \tag{11}$$

Since the high-tax consumer is the owner of the firm, (8) and (10) are essentially two aspects of the same maximization problem (i.e. optimal investment). Note that (11) rules out a pure exchange equilibrium. Because it is difficult to imagine an economically meaningful solution to the model's equilibrium without production, we have assumed throughout that (8) and (10) are fulfilled with equality.[7] This, however, does not rule out situations in which the low-tax consumer may decide not to purchase any corporate debt; hence the weak inequality in equation (9).[8]

Using double subscripts on utility functions for the appropriate second derivatives, the second-order conditions for sufficiency are:

For individual 1:

$$\lambda \equiv \frac{U_{00}}{1 + r} - 2U_{01} + (1 + r)U_{11} < 0.$$

For the firm:

$$f''(z) \leq 0.$$

For individual 2:

$$\lambda' \equiv \frac{V_{00}}{1 + i(1 - t_{p2})} - 2V_{01} + [1 + i(1 - t_{p2})]V_{11} < 0.$$

Consider first the high-tax consumer. Totally differentiating (4), (5), (10), and (11) produces

$$dc_{10} = -dz + dD + \omega_{20}dt_{p2} \qquad (12)$$

$$dc_{11} = f'(z)dz + (\omega_{21} + iD)dt_{p2} - (1 - t_{p2})(idD + Ddi) - dD \qquad (13)$$

$$dr = f''(z)dz \qquad (14)$$

$$di = \frac{dr + idt_c}{(1 - t_c)} \qquad (15)$$

Totally differentiating (8) and using (14) yields

$$\lambda_1(1 + r)dc_{10} + \lambda_2 dc_{11} = U_1 f''(z)dz \qquad (16)$$

where

$$\lambda_1 \equiv \frac{U_{00}}{1 + r} - U_{10}$$

$$\lambda_2 \equiv U_{01} - (1 + r)U_{11} .$$

Assuming a non-negative cross derivative of consumption over time, $U_{01} \geq 0$, then $\lambda_1 < 0$ and $\lambda_2 > 0$. Furthermore, it follows that

$$\lambda = \lambda_1 - \lambda_2 < 0 .$$

Next, we substitute (12), (13), (14), (15) and (10) into (16):

$$\left[\lambda(1+r) + f''(z)\left(U_1 + \frac{\lambda_2(1 - t_{p2})}{(1 - t_c)} D\right)\right]dz + \left[\lambda_2\left(1 + i(1-t_{p2})\right) - \lambda_1(1+r)\right]dD \quad (17)$$

$$+ \frac{\lambda_2(1 - t_{p2})iD}{(1 - t_c)} dt_c - \left[\lambda_1(1 + r)\omega_{20} + \lambda_2(\omega_{21} + iD)\right]dt_{p2} = 0 .$$

We write this last equation as:

$$a_1 dz + a_2 dD + a_3 dt_c + a_4 dt_{p2} = 0 . \quad (18)$$

where $a_1 < 0$, $a_2 > 0$, and $a_3 \geq 0$ (with equality holding when $f'(z) = 1$, and consequently $i = 0$). The sign of a_4 is ambiguous.

Repeating the analysis for the second individual, we differentiate (6), (7), and (9),

$$dc_{20} = -\omega_{20} dt_{p2} - dD \quad (19)$$

$$dc_{21} = -(\omega_{21} + iD)dt_{p2} + \left(1 + i(1 - t_{p2})\right)dD + (1 - t_{p2})Ddi \quad (20)$$

$$dV_0 = \left(1 + i(1 - t_{p2})\right)dV_1 + V_1(1 - t_{p2})di - V_1 i dt_{p2} \quad (21)$$

Analogously to (16), we obtain for individual 2

$$\lambda_1'\left(1 + i(1 - t_{p2})\right)dc_{20} + \lambda_{21}' dc_{21} = V_1(1 - t_{p2})di - V_1 i dt_{p2} \quad (22)$$

where

$$\lambda_1' \equiv \frac{V_{00}}{1 + i(1 - t_{p2})} - V_{01} < 0$$

$$\lambda_2' \equiv V_{01} - \big(1 + i(1 - t_{p2})\big)V_{11} > 0 \ ,$$

and also

$$\lambda' = \lambda_1' - \lambda_2' < 0 \ .$$

Substituting (19), (20), (14), and (15) into (22)

$$(V_1 - \lambda_2' D)\frac{(1 - t_{p2})}{(1 - t_c)}\big(f''(z)dz + idt_c\big) + \lambda'\big(1 + i(1 - t_{p2})\big)dD \qquad (23)$$

$$+ \big[\lambda_1'(1 + i(1 - t_{p2}))\omega_{20} + \lambda_2'(\omega_{21} + iD) - V_1 i\big]dt_{p2} = 0 \ .$$

We rewrite this last equation as

$$b_1 dz + b_2 dD + b_3 dt_c + b_4 dt_{p2} = 0 \qquad (24)$$

It is clear that $b_2 < 0$ and that the sign of b_4 is ambiguous. To sign b_1 and b_3 we need the following lemma:

Lemma: $V_1 > \lambda_2' D$.

Proof:

We expand the function $V_1\big(c_{20} - D,\ c_{21} + D\big(1 + i(1 - t_{p2})\big)\big)$ around the point (c_{20}, c_{21}) by the Taylor theorem and ignore terms of second order or higher:

$$V_1\big(c_{20} - D,\ c_{21} + D\big(1+i(1-t_{p2})\big)\big) = V_1(c_{20}, c_{21}) - V_{10}D + V_{11}D\big(1+i(1-t_{p2})\big)$$

$$= V_1 - \lambda_2' D \ .$$

The left-hand side of the above expression is positive, since it is a marginal utility term which is positive throughout. Thus it follows that $V_1 - \lambda_2' D > 0$.

qed

It follows from the Lemma that $b_1 \leq 0$ and $b_3 \leq 0$. Note that $b_1 = 0$ if and only if the production function is linear (i.e., $f''(z) = 0$). Note also that $b_3 = 0$ if and only if $i = 0$; an interesting case for which this occurs is the case of pure storage, $f(z) = z$.

Totally differentiating equation (3) and using (14) and (15) we obtain

$$- \{G + zt_c - (t_{p1}\omega_{10} + t_{p2}\omega_{20}) + \frac{(t_c - t_{p2})}{(1 - t_c)} D\} f''(z)dz \tag{25}$$

$$- (t_c - t_{p2})idD + \left[(\bar{f} - z)(1 + r) + f(z) - \frac{(1 - t_{p2})}{(1 - t_c)} iD\right]dt_c$$

$$+ \left[\omega_{10}(1 + r) + \omega_{11}\right]dt_{p1} + \left[\omega_{20}(1 + r) + \omega_{21} + iD\right]dt_{p2} = 0$$

The coefficient of dz simplifies (using equation (1)) to:

$$-\{B + t_c\bar{f} + \frac{(t_c - t_{p2})}{(1 - t_c)} D\} f''(z),$$

which is non-negative. Equation (25) can thus be written as

$$g_1 dz + g_2 dD + g_3 dt_c + g_4 dt_{p1} + g_5 dt_{p2} = 0 , \tag{26}$$

where $g_1 \geq 0$, $g_2 \leq 0$, g_4 and g_5 are strictly positive, and g_3 is generally ambiguous but can be proved to be positive for any set of realistic parameters.

We can summarize the results of this section as follows:

$$\begin{pmatrix} a_1 & a_2 \\ b_1 & b_2 \end{pmatrix}\begin{pmatrix} dz \\ dD \end{pmatrix} = -\begin{pmatrix} a_3 & a_4 \\ b_3 & b_4 \end{pmatrix}\begin{pmatrix} dt_c \\ dt_{p2} \end{pmatrix} \qquad \begin{matrix} (18) \\ (24) \end{matrix}$$

$$g_1 dz + g_2 dD + g_3 dt_c + g_4 dt_{p1} + g_5 dt_{p2} = 0 \; . \qquad (26)$$

All comparative statics of the equilibrium must satisfy these three equations simultaneously. In the following sections, we examine exogenous changes in t_c and t_{p2}, and solve the first two equations simultaneously to obtain the equilibrium changes in z and D. We may then solve for changes of the other economic variables in the model such as interest rates and the government debt. Finally, the required equilibrium change in t_{p1} is determined for each case by equation (26).

4. The Consequences of Changing the Corporate Income Tax

In this section we explore the impact of changes in the corporate tax rate t_c. Solving (18) and (24) simultaneously, while holding t_{p2} constant and assuming a positive amount of corporate debt, D, we show that increasing the corporate tax rate t_c causes:

--an increase in investment z,

--a decrease in the non-taxable interest rate r,

--an increase in the interest rate i paid on corporate bonds,

--an increase in corporate borrowing D,

--a decrease in the value of the firm,

--a decrease in the value of the firm's equity;

--a decrease in the utility of the high-tax consumer and an increase in the utility of the low-tax consumer

If D = 0 then changing t_c would be inconsequential to the economy.

4.1. The effects on production and interest rates

We now prove the following theorem, which relates changes in the corporate tax rate to changes in production, corporate debt, and interest rates:

Theorem 2: Except for the case where $i = r = 0$ (which will occur if and only if $f'(z) = 1$) an increase in the corporate tax rate t_c will cause an equilibrium increase in corporate investment z and an increase in D. When $f'(z) = 1$, a change in t_c will not affect the real equilibrium of the economy.

Proof:

Substituting $dt_{p2} = 0$ into (18) and (24), it follows that:

$$\frac{dD}{dt_c} = \frac{a_1 b_3 - a_3 b_1}{a_2 b_1 - a_1 b_2} \tag{27}$$

$$\frac{dz}{dt_c} = \frac{a_3 b_2 - a_2 b_3}{a_2 b_1 - a_1 b_2} \tag{28}$$

The denominator of both these expressions is negative. Now consider the numerator of (27):

$$a_1 b_3 - a_3 b_1 = (V_1 - \lambda_2' D)\frac{(1 - t_{p2})}{(1 - t_c)}\, i[\lambda(1 + r) + U_1 f''(z)]$$

This expression is negative for $i > 0$. When $i = 0$, the numerator of (27) is zero. Therefore $dD/dt_c > 0$ unless $f'(x) = 1$, in which case $dD/dt_c = 0$.

Similarly, the numerator of (28) is negative except for the case where $i = 0$; for this case $a_3 = 0$ and $b_3 = 0$.

qed

The economic interpretation of Theorem 2 will be explained subsequently. An immediate corollary of the increase in z is:

Corollary: If the production function f(z) is concave, then an increase in the corporate tax rate will cause a decrease in the real interest rate.

Proof:

The real interest rate is determined by $r = f'(z) - 1$. Since by the previous theorem, an increase in the corporate tax rate causes z to increase, this means that r decreases.

qed

The effects of an increase in t_c on the rate of interest paid on corporate bonds, i, is more difficult to gauge. Rewriting (15),

$$\frac{di}{dt_c} = \frac{dr/dt_c + i}{1 - t_c} \; . \qquad (15')$$

If $i = 0$, then from the previous theorem and (15') it follows that $di/dt_c = 0$. Now consider the case where $i > 0$; in this case there are two counteracting effects on i. A direct effect is due to the increase in t_c, and an indirect one is due to the decrease in r. We wish to compare the magnitude of these two effects. The next theorem shows that the effects on i due to t_c dominates that caused by z:

Theorem 3: If $f'(z) > 1$, then an increase in the corporate tax rate causes an equilibrium increase in the taxable interest rate i. That is, $di/dt_c > 0$.

Proof:

Suppose that $f'(z) > 1$, in which case $i > 0$. We first note that $f''(z)b_3 = ib_1$ and that $b_2 < 0$. Next, it is trivial to verify that $f''(z)a_3 > ia_1$. Thus

$$f''(z)(a_2b_3 - a_3b_2) > (a_2b_1 - a_1b_2)i \ .$$

Since the right-hand side of this last expression is negative, it follows that

$$f''(z)\frac{a_2b_3 - a_3b_2}{a_2b_1 - a_1b_2} < i \ .$$

Substituting in (28), and using (14), we find that $dr/dt_c > -i$. This proves that (15') is positive.

qed

The result of Theorem 3 provides the economics behind Theorem 2. As t_c increases the interest rate on corporate debt, $i = r/(1 - t_c)$, increases. Consequently, there are a price effect and a wealth effect (see below) on the low-tax consumer, both cause him to increase his lending. Thus D goes up, which triggers the firm in equilibrium to increase its investment.

4.2. The effects on firm value of an increase in the corporate income tax

The results of Theorem 2 may be used to gain some insight into the effect of an increase in the corporate income tax on the value of the firm and the value of the firm's equity.

Consider first the value of the firm at Date 0: This is the discounted value of the firm's production:

$$\text{value of the firm} = \frac{(1 - t_c)f(z)}{1 + r} = \frac{(1 - t_c)f(z)}{f'(z)} \ . \tag{29}$$

The value of the firm's equity at Date 0 is the value of the firm less the value of its debt:

$$\text{value of the equity} = \frac{(1 - t_c)f(z)}{f'(z)} - D \tag{30}$$

To determine the effect of a change in the corporate tax rate, we take the derivative of (29) with respect to t_c:

$$\{1 - \frac{f''(z)f(z)}{[f'(z)]^2}\}(1 - t_c)\frac{dz}{dt_c} - \frac{f(z)}{f'(z)} . \qquad (31)$$

We showed in Theorem 2 that in most cases z increases when t_c increases. Thus the sign of expression (31) can be either positive or negative, though it is much more likely that the sign is negative, since the second term in this expression is the value of the firm before corporate taxes. It follows that in general the value of the firm will <u>decline</u> when the corporate tax increases.

To gauge the effect of a change in t_c on the value of the firm's equity, note that for all cases except $i = r = 0$, D increases with t_c. It follows that the value of the firm's equity will usually decrease when the corporate income tax is raised.

4.3. <u>The welfare effects of an increase in the corporate income tax</u>

Next, we examine the effect of increasing t_c on the utility of the two representative consumers. We shall show that an increase in t_c causes the welfare of the producer to decrease and the welfare of the low-tax consumer to increase.

These results are perhaps not surprising, in view of the fact that the high-tax consumer owns the firm whose taxes have been raised. The mechanism by which the utility loss of the producer occurs is surprising, however, since we assuming that t_{p2} does not change. The real reason why a welfare transfer occurs is that although the net present value of corporate debt is zero when viewed from the perspective of the firm, when we "look through" the firm and the government, the net present value of corporate debt from the point of view of the firm's shareholders is actually negative. It is this effect which causes the welfare of the firm's shareholders to fall when the corporate tax rate is raised.

Theorem 4: The effect of an increase in the corporate income tax on the welfare of the firm's shareholder (producer) is negative if and only if $f'(z) > 1$. If $f'(z) = 1$, there is no change in the welfare of the producer.

Proof:

Totally differentiating the utility of individual 1 gives

$$\frac{dU}{dt_c} = U_0 \frac{dc_{10}}{dt_c} + U_1 \frac{dc_{11}}{dt_c}$$

$$= U_0\left(-\frac{dz}{dt_c} + \frac{dD}{dt_c}\right) + U_1\left\{(1 + r)\frac{dz}{dt_c}\right.$$

$$\left. - \left(1 + i(1 - t_{p2})\right)\frac{dD}{dt_c} - \frac{(1 - t_{p2})}{(1 - t_c)} D\left(\frac{dr}{dt_c} + i\right)\right\}$$

This simplifies to become

$$\frac{dU}{dt_c} = -U_1\left\{i(t_c - t_{p2})\frac{dD}{dt_c} + (1 - t_{p2})D \frac{di}{dt_c}\right\} .$$

By Theorems 2 and 3 this is negative if and only if $f'(z) > 1$. If $f'(z) = 1$, this expression is zero.

qed

The intuitive explanation for the negative welfare effects of an increase in t_c on the firm's owners is as follows: Integrating the government budget constraints into consumer 1's budget constraints (as in (4) and (5)), we see that the effective, after tax, net present value of D to the high-tax individual is given by

$$D - \frac{D(1 + i(1 - t_{p2}))}{1 + r} = \frac{Di(t_{p2} - t_c)}{1 + r} .$$

This expression is negative when $r > 0$. Thus, for any level of corporate taxation the rich individual experiences a welfare loss through his borrowing from the poor individual. In Miller (1977) and the literature that follows, there is no consideration of how the bondholder surplus is financed. As was pointed out in our discussion of equation (2), the bondholder surplus must be financed by the government at Date 1. In a general equilibrium this financing involves a wealth transfer from the high-tax to the low-tax consumer. An increase in t_c causes a welfare loss to the rich individual because both D and i go up, and r goes down. The high-tax individual is thus triply punished in this case.

Since the economy we are discussing is a zero-sum game between two types of consumers, and since Theorem 4 shows that an increase in the corporate income tax decreases the welfare of the high-tax consumes, it is not surprising that such a tax change increases the welfare of the low-tax individuals. Formally:

Corollary: If $f'(z) > 1$, the equilibrium effect of an increase in t_c on the welfare of the low-tax consumers is positive.

Proof:

By Theorem 4 increasing t_c results in an increase in i. Even if the second individual did nothing, this would increase his welfare. Since he changes D, this can only increase his welfare more.

qed

Remark: It is clear that for the low-tax individual, $dc_{20}/dt_c < 0$ and $dc_{21}/dt_c > 0$. Note, however, that we cannot sign the changes caused by an increase in t_c in c_{10} and c_{11}.

Finally we consider the case where condition (9) holds with inequality, implying $D = 0$. In such a case there is no bondholder surplus, and as is evident from our discussion above, changing t_c would have no real effects on the economy. It will simply be offset by a commensurable change in t_{p1} so to maintain a balanced government budget. From (26), $dt_{p1}/dt_c = -g_3/g_4 = -[(\bar{f} - z)(1 + r) + f(z)]/[\omega_{10}(1 + r) + \omega_{11}] < 0$.

5. The equilibrium effects of changing the low tax rate, t_{p2}

In this section we discuss the equilibrium consequences of changing the income tax rate of the low-tax consumer, t_{p2}. Raising t_{p2} induces a direct transfer of wealth to the high-tax consumer and tends to induce a reduction in the low-tax consumer's lending to the firm. It is difficult to get precise analytical results for general utility functions for this case. We therefore examine in detail the case of time-separable power utility functions, achieving results of considerable generality.

When both consumers have a time-additive power utility function, they maximize:

$$U(c_0, c_1) = H_1(c_0) + \delta_1 H_1(c_1) ,$$
$$V(c_0, c_1) = H_2(c_0) + \delta_2 H_2(c_1) ,$$

where

$0 \le \delta_k \le 1$ is consumer k's ($k = 1, 2$) pure time preference factor and

$$H_k(c) = \frac{c^{1-\gamma_k}}{1 - \gamma_k}$$

We shall refer to δ and γ as the parameters of the power-utility functions.

5.1. A general derivation for the case of identical utility functions

Theorem 5: If both consumers have time-additive power utility functions with identical parameters, then $dz/dt_{p2} \leq 0$ and consequently $dr/dt_{p2} \geq 0$, with equality if and only if $f'(z) = 1$.

Proof:

We solve (18) and (24) simultaneously, holding t_c constant. This gives the following general expression for dz/dt_{p2}:

$$\frac{dz}{dt_{p2}} = \frac{a_2 b_4 - a_4 b_2}{a_1 b_2 - a_2 b_1} . \tag{32}$$

Note that the denominator of this expression is positive.

We now determine the sign of the numerator. Define $\bar{r} = (1 - t_{p2})i$. Writing out the numerator of (32) using (17), (18), (23), and (24) gives

$$\begin{aligned} &\left[\lambda_2(1 + \bar{r}) - \lambda_1(1 + r)\right]\left[\lambda_1'(1 + \bar{r})\omega_{20} + \lambda_2'(\omega_{21} + iD) - V_1 i\right] \qquad (33)\\ &+ \left[\lambda_1(1 + r)\omega_{20} + \lambda_2(\omega_{21} + iD)\right]\lambda'(1 + \bar{r}) \\ &\equiv A_1 A_2 - A_3 , \end{aligned}$$

where

$$A_1 = \left[\omega_{20}(1 + \bar{r}) + \omega_{21} + iD\right] ,$$

$$A_2 = \left[\lambda_2\lambda_1'(1 + \bar{r}) - \lambda_1\lambda_2'(1 + r)\right] ,$$

$$A_3 = V_1 i\left[\lambda_2(1 + \bar{r}) - \lambda_1(1 + r)\right] .$$

Clearly $A_1 > 0$. When $i > 0$, $A_3 > 0$, and when $i = 0$, $A_3 = 0$.

To prove the theorem, we shall show that for the case of additive power utility functions with identical parameters $A_2 < 0$. To see this note that the optimal solutions for consumption satisfy:

$$c_{11} = c_{10}[\delta(1 + r)]^{1/\gamma} \tag{34}$$

$$c_{21} = c_{20}[\delta(1 + \bar{r})]^{1/\gamma} \tag{35}$$

This gives

$$\lambda_2 = -\theta\lambda_1(1 + r)^{1-1/\gamma}$$

$$\lambda_2' = -\theta\lambda_1'(1 + \bar{r})^{1-1/\gamma}$$

where

$$\theta \equiv \delta^{-1/\gamma} .$$

Thus A_2 becomes in this case

$$A_2 = -\theta(1 + r)^{1-1/\gamma}\lambda_1\lambda_1'(1 + \bar{r}) + \theta(1 + \bar{r})^{1-1/\gamma}\lambda_1\lambda_1'(1 + r)$$

$$= \lambda_1\lambda_1'(1 + r)(1 + \bar{r})\theta[(1 + \bar{r})^{-1/\gamma} - (1 + r)^{-1/\gamma}] .$$

When $r > 0$, this term is negative, since $(1 + \bar{r}) > (1 + r)$. For $r = 0$, A_2 is zero. This proves the Theorem.

qed

5.2. The economics of Theorem 5

The economic logic of the results of Theorem 5 is complex. To clarify the intuition behind the theorem, we analyze two simple examples. In the first example $r = 0$, and we explain why $dz/dt_{p2} = 0$. In the second example $r > 0$, and we show why this causes $dz/dt_{p2} < 0$.

Example 1: As in the theorem, both consumers maximize power utility functions with identical parameters. In this example, we assume that $\delta = 1$

and that $f'(z) = 1$; this second assumption guarantees that $r = i = 0$. Thus both consumers will wish to set consumption equal at both dates.

Under these conditions consumer 2 will choose D so as to equate consumption in both periods, so that

$$D = \begin{cases} \dfrac{(1 - t_{p2})(\omega_{20} - \omega_{21})}{2} & \text{if this is} > 0 \\ 0 & \text{otherwise} \end{cases}$$

All relevant economic insights of this example can be seen by considering the following case: Government spending $G = 0$, so that taxes on consumer 2 constitute a direct transfer to consumer 1; and the corporate income tax is zero. (Since the point of this example is the effect of taxation on production, the simplifying assumptions do not cause any loss in generality. They are made only in order not to complicate the notation.) Furthermore, suppose that $\omega_{10} = \omega_{11}$, so that in the absence of loans from consumer 2, consumer 1 will set $z = 0$. Suppose that $\omega_{20} = 200$ and $\omega_{21} = 100$.

We examine two levels of t_{p2}. First, if $t_{p2} = 0$, consumer 2 lends $D = 50$. To prevent this cash inflow at Date 0 from distorting his optimal consumption pattern, consumer 1 sets his investment $z = 50$. Now suppose we raise t_{p2}, so that $t_{p2} = 0.10$. In this case $D = 45$; however, net transfers from consumer 2 to consumer 1 will be:

at Date 0: $t_{p2}\omega_{20} + D = 65$

at Date 1: $t_{p2}\omega_{21} - D = -35$.

By keeping $z = 50$, consumer 1 maintains his desired balance between consumption at the two dates. Hence $dz/dt_{p2} = 0$, as stated in Theorem 5.

The fundamental economic reason for $dz/dt_{p2} = 0$ in this case is that when $f'(z) = 1$, the low-tax consumer lends at a rate which is the economy's marginal rate of transformation. On the other hand in an economy with differential taxation, the Miller Theorem implies that when $f'(z) > 1$, the low-tax consumer lends at a rate higher than the economy's marginal rate of transformation, and this causes, as the subsequent example will illustrate, $dz/dt_{p2} < 0$.

Example 2: We now consider the case in which $f'(z) > 1$ and $t_c > 0$, so that $i \geq \bar{r} = i(1 - t_{p2}) > r > 0$. As before, both types of consumers maximize power utility functions, with identical parameters. Together consumer 2's optimal consumption ratio (35) and his budget constraints yield an optimal borrowing of:

$$D = \begin{cases} \dfrac{(1 - t_{p2})\{[\delta(1 + \bar{r})]^{1/\gamma}\omega_{20} - \omega_{21}\}}{(1 + \bar{r}) + [\delta(1 + \bar{r})]^{1/\gamma}} & \text{if this is} > 0 \\ 0 & \text{otherwise .} \end{cases} \tag{36}$$

Note from (34) and (35) that consumer 2's desired ratio of consumption at Date 1 to Date 0 is higher than that of consumer 1, since $\bar{r} > r$.

Suppose $\gamma = \delta = 1$ (this is the case of a time-additive logarithmic utility function), the corporate tax rate is 0.50, and the production function is $f(z) = 1.1z$, so that $r = 0.10$ and $i = 0.20$. As in the previous example, we assume that $\omega_{20} = 200$ and $\omega_{21} = 100$, and that government expenditure is zero. Finally, $\omega_{11} = 1.1\omega_{10}$, so that in the absence of transfers from consumer 2, consumer 1 would set $z = 0$.

Given consumer 2's choice of D, consumer 1's budget constraints will be:

$$c_{10} = \omega_{10} + t_{p2}\omega_{20} + D - z$$

$$c_{11} = \omega_{11} + t_{p2}\omega_{21} - D(1 + 0.2(1 - t_{p2})) + 1.1z.$$

Since $\omega_{11} = 1.1\omega_{10}$, this gives an optimal choice of investment

$$z = \frac{t_{p2}[1.1\omega_{20} - \omega_{21}] + [2.1 + \bar{r}]D}{2.2} .$$

Note that there are two effects on z: Each dollar of wealth transferred from consumer 2 to consumer 1 at Date 0 via direct taxation of consumer 2 causes an increase in z of 0.5. On the other hand, each dollar D loaned by consumer 2 to consumer 1 at Date 0 causes an increase in z of $(2.1 + \bar{r})/2.2$. Since $\bar{r} > r = 0.10$, this second effect is never less than unity. Thus an additional dollar of lending to the firm triggers an increase in investment of more than one dollar.

The difference between this example and the previous one is that in the previous example $\bar{r} = r = 0$, so that (ceteris paribus) an additional dollar of lending to the firm causes an increase in investment of exactly one dollar. In the present case the rate of return paid to the low-tax consumer is greater than the marginal rate of production; this bondholder surplus causes that $\partial z/\partial D > 1$.

To see the impact of changes in t_{p2}, suppose first that $t_{p2} = 0$. In this case consumer 2 will set D = 58.33, requiring consumer 1 to repay him 1.2D = 70 at Date 1. This will cause consumer 1 to set z = 60.98. As noted above, because of the bondholder surplus ($\bar{r} > r$), consumer 1 has to invest in production more than D.

If we now make $t_{p2} = 0.1$, consumer 2 will set D = 51.86, requiring consumer 1 to repay him 1.18D = 61.2 at Date 1. Note that the net-of-tax bondholder surplus had decreased. Net transfers from consumer 2 to consumer 1 at Dates 0 and 1 are:

Date 0: taxes--an increase of 20, D--a decrease of 9.12.

Date 1: taxes--an increase of 10, $D(1 + \bar{r})$--a decrease of 10.76.

The increase in taxation causes an increase in z of (22-10)/2.2 = 5.45. The decrease in debt causes a decrease in z of [(2.4)(58.33) - (2.28)(51.86)]/2.2 = 7.24. These net out to result in z = 59.20, which is less than the optimal z = 60.98 for the case of $t_{p2} = 0$.

5.3. Extensions

a) The assumption that both consumers have the same risk and time preference parameters is sufficient for the results of Theorem 5, but it is by no means necessary. For the results of Theorem 5 to hold it is sufficient that the marginal rate of time substitution of consumer 2 be greater than that of consumer 1: $[\delta_1(1 + r)]^{1/\gamma_1} < [\delta_2(1 + \bar{r})]^{1/\gamma_2}$. If the opposite inequality holds, the results may be reversed. Indeed, in the Appendix to the paper we give an example where $dz/dt_{p2} > 0$ when $\gamma_2 > \gamma_1$.

b) The proof of Theorem 5 assumes that the first-order conditions for maximization hold for both consumers, but it can be shown that the results of the Theorem also hold when consumer 2 is at a corner solution, so that D = 0. In this case the ratio of consumer 2's Date 1 endowment to his Date 0 endowment is more than $[\delta(1 + \bar{r})]^{1/\gamma}$. Taxation of consumer 2's endowment results in relative increases in consumer 1's wealth at both Date 0 and Date 1. However, the ratio of the increase at Date 1 to the increase at Date 0 is larger than optimally desired. Consumer 1 will adjust for this imbalance by cutting production. Thus $dz/dt_{p2} < 0$.

c) Another interesting issue is the effect of an increase in t_{p2} on the after-tax interest rate $\bar{r} = i(1 - t_{p2})$. In general it is clear that when the production function is either linear or close to linear, an increase in t_{p2} must result in a decrease in $\bar{r}$. If the economy's production is on a highly concave section of the production function, the decrease in production caused by an increase in t_{p2} can conceivably cause an increase in i large enough to offset the increase in taxation. To examine the likelihood of this case, we ran extensive numerical simulations; invariably, simulations which resulted in "normal" interest rates had $d\bar{r}/dt_{p2} < 0$. However, pathological examples can be found for which $d\bar{r}/dt_{p2} > 0$. An example is given in the Appendix to the paper.

d) Finally, we examine the behavior of D when the tax rate t_{p2} goes up. Differentiating the expression given for D in (36), yields $\partial D/\partial t_{p2} < 0$ when the production function is linear. The economics behind this are clear: When the production function is linear and the corporate tax rate is fixed, i--the rate paid on corporate bonds--is not a function of z. Thus when t_{p2} is raised, both the reduction in wealth and the reduction in the desired ratio of Date 1 to Date 0 consumption of consumer 2 combine to lower D. If the production function is non-linear, it might be possible that when t_{p2} increases, $\bar{r}$ will increase enough so that D also increases. Such a case would seem to be of pathological interest only; in point of fact, although we have found examples where an increase in t_{p2} causes an increase in $\bar{r}$, we have found no examples in which the increase in $\bar{r}$ was large enough to cause an increase in D.

5.4. The effect of firm value of a change in t_{p2}

Recall that the value of the firm and the value of the firm's equity are given by:

$$\text{value of the firm} = \frac{(1 - t_c)f(z)}{1 + r} = \frac{(1 - t_c)f(z)}{f'(z)} \cdot \tag{29}$$

$$\text{value of the equity} = \frac{(1 - t_c)f(z)}{f'(z)} - D \tag{30}$$

We now prove the following theorem:

Theorem 6: If both the high-tax and the low-tax consumers maximize power utility functions with identical parameters, and if $f'(z) > 1$, then the value of the firm will fall when t_{p2} is increased. The change in the value of the firm's equity is ambiguous for this case.

Proof:

Taking the derivative of the value of the firm with respect to t_{p2}, we get:

$$\{1 - \frac{f''(z)f(z)}{[f'(z)]^2}\}(1 - t_c)\frac{dz}{dt_{p2}} . \tag{37}$$

We showed in Theorem 5 that when both types of consumers maximize power utility functions with identical parameters, then $dz/dt_{p2} < 0$. It thus follows from (37) that under these conditions the value of the firm will fall. Since D also falls, the change in the value of the equity is ambiguous.

qed

6. Does Government Borrowing Constitute Crowding Out?

An issue which has been the subject of much discussion in both the academic and popular press is the impact of government borrowing on the ability of corporations to raise debt financing and the cost of this financing. In this section we attempt to shed light on this question using the framework of our model.

In a general equilibrium framework one must connect the volume of government borrowing to the values of other policy parameters in the model, i.e., tax rates. This involves a discussion of the tradeoff between government borrowing and the rates of taxation levied on corporations and consumers. In the context of our model, both government and corporate borrowing are determined simultaneously in the system along with the other tax parameters. We shall use the comparative statics of the model to determine under which conditions increases in government borrowing are accompanied by decreases in corporate borrowing ("crowding out") and vice versa.

To examine this question rigorously, we now derive analytically the relation between the comparative statics of B and D.

From the government budget equation (1) at Date 0 we get,

$$\omega_{10}dt_{p1} = t_c dz - (\bar{f} - z)dt_c - dB - \omega_{20}dt_{p2} \tag{38}$$

The government budget equation at Date 1 (equation 2) yields

$$(1 + r)dB + Bdr = \left[f(z) - iD\right]dt_c - (t_c - t_{p2})(idD + Ddi) + \omega_{11}dt_{p1}$$

$$+ t_c(1 + r)dz + (iD + \omega_{21})dt_{p2}$$

Which, using (14) and (15), becomes

$$(1 + r)dB + \left[Bf''(z) - t_c(1 + r) + \frac{(t_c - t_{p2})}{(1 - t_c)}Df''(z)\right]dz \tag{39}$$

$$= \left[f(z) - \frac{(1 - t_{p2})}{(1 - t_c)} Di\right]dt_c - (t_c - t_{p2})idD) + \omega_{11}dt_{p1}$$

$$+ (\omega_{21} + iD)dt_{p2}$$

Combining (38) and (39),

$$[(1 + r) + \frac{\omega_{11}}{\omega_{10}})(dB - t_c dz) + f''(z)[B + \frac{(t_c - t_{p2})}{(1 - t_c)} D]dz$$

$$+ [iD \frac{(1 - t_{p2})}{(1 - t_c)} - f(z) + (\bar{f} - z)\frac{\omega_{11}}{\omega_{10}}]dt_c + i(t_c - t_{p2})dD$$

$$+ (\frac{\omega_{11}}{\omega_{10}} \omega_{20} - \omega_{21})dt_{p2} = 0$$

which we rewrite as

$$\alpha_1 dz + \alpha_2 dD + \alpha_3 dt_c + \alpha_4 dt_{p2} + \alpha_5 dB = 0 \ , \qquad (40)$$

where $\alpha_1 < 0$, $\alpha_2 \geq 0$, $\alpha_5 > 0$, and the signs of α_3 and α_4 are ambiguous.

Changes in B and D must be accompanied by a change in at least one of the model's tax rates. To analyze "crowding out," we shall therefore examine separately the relation between changes in B and D and a change in each of the model's three income tax rates.

Case 1: The change in B is accompanied by an adjustment of the corporate tax rate t_c. In this case $dt_{p2} = 0$, and we obtain from (40):

$$\frac{dB}{dD} = \frac{dB/dt_c}{dD/dt_c} = - \frac{1}{\alpha_5} [\alpha_2 + \frac{\alpha_3 + \alpha_1 dz/dt_c}{dD/dt_c}] \ .$$

Since $dD/dt_c > 0$, $dz/dt_c > 0$, and $\alpha_3 \gtrless 0$, the sign of dB/dD is ambiguous.

Case 2: The change in B is accompanied by a change in t_{p2}. In this case $dt_c = 0$. In a manner similar to that of Case 1 above, we can show that the sign of dB/dD is ambiguous also in this case.

Case 3: The third potential case is that dB is accompanied only by a change in t_{p1}. However, as we showed in Section 2, any adjustment in both B and t_{p1}

must be accompanied by a change in either t_c and/or t_{p2}. These two cases have already been shown above to be ambiguous.

We can conclude therefore that there is no clear-cut linkage between government and corporate borrowing. Any change in government borrowing must be accompanied by some change in taxation. Irrespective of whether the corporate or the personal tax rates are changed, we find that the sign of dB/dD depends on the specific equilibrium values of the economy, and is thus ambiguous. In the Appendix to this paper we present two non-pathological examples, one of which exhibits "crowding out," and the second of which exhibits "crowding in."

Crowding out is sometimes referred to the relation between the size of government debt and market interest rates. In a similar way it can be shown that ambiguous results are obtained also under such interpretation of crowding out.

There is no support, therefore, for the intuitive notion of crowding out. Government borrowing per se has no effect on the economy's real equilibrium.[9] Therefore the importance of a change in government borrowing is only in its impact on the rates of taxation.

7. Concluding Remarks

We have constructed an integrated framework in which both personal and corporate incomes are taxed, and in which personal tax rates depend on the source of the income in question. Taxes and non-taxable government bonds are used to finance government consumption. Using the model we have examined the changes in equilibrium parameters which stem from changes in the government's financing policies.

The major assumptions of our model relate to the structure of the tax rates on the various types of income. We assume that the highest personal tax rate levied on ordinary personal incomes exceeds the corporate tax rate, and we further assume that the tax rate on equity income is effectively zero. These assumptions roughly correspond to the United States tax code over the years until 1986 and to the tax codes of many other capitalist democracies. The new U.S. Federal tax code appears to violate these assumptions. The tax reform measures eliminate the distinction between capital gains and ordinary income, and lower the maximum Federal income tax rate on personal incomes to below the marginal rate on corporate income (28% versus 34%). Nevertheless, we wish to claim that our model is still a useful tool for analyzing the dynamics of a general equilibrium, even under the overall tax bill. The reasons for this are various:

First, it is suggested in the literature that the equilibrium differential return between tax-exempt and taxable debt will reflect a corporate tax rate less than the statutory rate. Among other reasons for this is partial utilization of tax shelters under uncertainty (DeAngelo and Masulis (1980b); Cordes and Sheffrin (1981, 1983); Barnea, Haugen, and Talmor (1987)). For example, Cooper and Franks (1983) and Warren and Auerbach (1982) provide detailed discussions of factors which limit the ability of firms to market redundant deductions.

Second, under the overall tax bill as income increases, tax deductions are gradually phased out, increasing the effective maximum tax rate on personal income up to 38-40%.

Third, even though the new tax reform legislation does not differentiate between capital gains and ordinary income, an effective differentiation will remain which stems from several factors. Capital gains are taxed only upon realization, and not when they actually occur. By postponing the realization of capital gains investors gain the time value of money on their deferred tax liability. Moreover, by employing a portfolio timing strategy by which capital losses are realized relatively close to the time they occur, investors can reduce further their effective capital gains tax (Constantinides (1983)). Even though realized capital gains will be taxed at the regular tax rate, the fact that the timing of realization is a choice factor of the individual means that he can wait until his marginal tax rate is lower to realize the capital gain. Finally the tax code allows a write-up in the base at the time stocks are inherited. This means that if there is a bequest motive in the individual's utility function, he can lower the tax rate even further.

In other countries the distinction between debt and equity income is very radical, since capital gains are typically not taxed at all, while interest is taxed as an ordinary income. For example, individuals are taxed on interest income at up to 60% in the United Kingdom, up to 75% in Japan and up to 56% in Germany. Both Germany and Japan do not tax individuals on capital gains, while in the United Kingdom capital gains taxes for holding periods over one year are effectively negligible.

APPENDIX: A COMPENDIUM OF EXAMPLES

In this appendix we present five examples which show that some changes in the model's parameters can have ambiguous effects. The examples we consider are:

A.1. $dz/dt_{p2} < 0$. This implies that $dr/dt_{p2} > 0$. In principle when $dz/dt_{p2} < 0$, $i(1 - t_{p2})$ can either increase or decrease. In most examples we find that $d[i(1 - t_{p2})]/dt_{p2} < 0$; this is the case in example A.1.

A.2. $dz/dt_{p2} > 0$. This implies that $dr/dt_{p2} < 0$ and $d[i(1 - t_{p2})]/dt_{p2} < 0$. We use power utility functions for this example, in which we make the marginal rate of substitution of consumer 2 less than that of consumer 1: $[\delta_1(1 + r)]^{1/\gamma_1} > [\delta_2(1 + \bar{r})]^{1/\gamma_2}$.

A.3. In this example we show a case where $d[i(1 - t_{p2})]/dt_{p2} > 0$. The case used is one where equilibrium production z* is on an extremely concave portion of the production function. In this case a small decrease in equilibrium z causes a large increase in i, so that $i(1 - t_{p2})$ increases. The extremeness of the results suggest that this case is in some sense pathological. It is clear from our example, however, that it may occur whenever the production function has the property that $f'(z) \to \infty$ when $z \to 0$.

A.4. and A.5. We present two non-pathological examples in which a change in t_{p2} causes government and corporate borrowing to move either in opposite directions (Example A.4) or in the same direction (Example A.5.). In the literature (popular and professional) these phenomena are often termed "crowding out" and "crowding in" respectively. We show that both phenomena can be considered "normal" in equilibrium.

1. Example 1: $dz/dt_{p2} < 0$

In this example an increase in the tax rate levied on the low-tax consumer causes a decrease in investment. The endowments in the example are:

$$\bar{f} = 3300$$
$$\omega_{10} = 7000$$
$$\omega_{11} = 2300$$
$$\omega_{20} = 4000$$
$$\omega_{21} = 4000$$

The production function employed is $f(z) = z + z^{1/1.4}$.

Both the high-tax and the low tax consumers maximize additive power utility functions with identical parameters $\gamma = 2$ and $\delta = 1$. Government consumption is 8500 at Date 0, and the corporate tax rate is 0.45.

Solutions to the model are (see also Figure I):

t_{p1}	0.7869	0.7452	0.7036	0.6620	0.6205	0.5790	0.5375	0.4961	0.4546
t_{p2}	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4
D	250.26	230.06	210.22	190.81	171.90	153.55	135.85	118.89	102.73
i	0.3360	0.3384	0.3405	0.3422	0.3436	0.3445	0.3451	0.3453	0.3451
r	0.1848	0.1861	0.1873	0.1882	0.1890	0.1895	0.1898	0.1900	0.1898
B	1506.6	1598.3	1689.5	1780.5	1871.2	1961.7	2052.0	2142.2	2232.3
z	113.36	110.58	108.24	106.34	104.87	103.82	103.20	103.01	103.23
f(z)	142.70	139.41	136.63	134.37	132.62	131.38	130.64	130.41	130.67
c_{10}	1936.9	2119.4	2301.9	2484.4	2667.0	2849.7	3032.6	3215.8	3399.5
c_{11}	2108.3	2308.3	2508.3	2708.2	2908.1	3108.1	3308.0	3508.0	3708.2
c_{20}	3749.7	3569.9	3389.7	3209.1	3028.0	2846.4	2664.1	2481.1	2297.2
c_{21}	4334.3	4104.0	3874.6	3646.3	3419.1	3193.2	2968.6	2745.5	2524.0

2. Example 2: $dz/dt_{p2} > 0$

In this example an increase in the tax rate levied on the low-tax consumer causes an increase in investment. The parameters of the example are:

$\bar{f} = 1700$
$\omega_{10} = 6900$
$\omega_{11} = 3200$
$\omega_{20} = 8000$
$\omega_{21} = 1000$

Government consumption is 9000. The corporate tax rate is 0.45. As in the previous example, the production function is $f(z) = z + z^{1/1.4}$. The parameters of the power utility functions of the two consumers are: $\delta_1 = \delta_2 = 1$, and $\gamma_1 = 2$, $\gamma_2 = 8$. This causes consumer 2's marginal rate of substitution over time to be <u>less</u> than that of consumer 1, which, as suggested in Section 5, will cause z to increase with t_{p2}.

Solutions to the model are given below (see also Figure II):

t_{p1}	0.8554	0.8068	0.7584	0.7102	0.6623	0.6146	0.5671	0.5199	0.4729
t_{p2}	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4
D	3286.8	3132.0	2976.2	2819.4	2661.7	2503.1	2343.4	2182.8	2021.2
i	0.1559	0.1559	0.1558	0.1558	0.1557	0.1557	0.1556	0.1556	0.1555
r	0.0858	0.0857	0.0857	0.0857	0.0857	0.0856	0.0856	0.0856	0.0855
$\underline{B}$	3083.3	3019.1	2953.4	2886.3	2817.7	2747.6	2676.0	2602.8	2528.2
r	0.1559	0.1480	0.1402	0.1324	0.1246	0.1167	0.1089	0.1011	0.0933
z	1668.5	1669.4	1670.5	1671.7	1673.2	1674.8	1676.6	1678.5	1680.7
f(z)	1868.8	1869.7	1870.9	1872.3	1873.9	1875.6	1877.5	1879.7	1882.0
c_{10}	1218.3	1462.5	1705.7	1947.7	2188.5	2428.3	2666.8	2904.2	3140.5
c_{11}	1269.5	1523.9	1777.3	2029.4	2280.4	2530.1	2778.7	3026.0	3272.1
c_{20}	4713.1	4467.9	4223.7	3980.5	3738.2	3496.8	3256.5	3017.1	2778.7
c_{21}	4799.2	4545.7	4293.6	4042.8	3793.4	3545.4	3298.8	3053.6	2809.9

3. Example 3: $d[i(1 - t_{p2})]/dt_{p2} > 0$

In "normal" solutions to the model, the low-tax investor gets <u>lower</u> after-tax rates of return when his tax rate is raised. This need not always be so, however, as the following example shows. In the example presented here, the economy's production is on an extremely steep part of the production frontier. When taxes on the low-tax investor are raised, inputs fall. The resulting rise in i outweighs the increase in t_{p2}, so that the low-tax investor's after-tax rate of return rises. Note that the interest rates in the example are extremely high; we have not succeeded in concocting a "normal" example where this phenomenon occurs.

$\bar{f} = 2000$
$\omega_{10} = 6000$
$\omega_{11} = 6000$
$\omega_{20} = 1000$
$\omega_{21} = 3910$

The production function we use in this example is $f(z) = z + z^{1/1.1}$. The utility functions are the power utility functions used in the previous examples; however, in this example we assume that the high-tax consumer has $\gamma = 4$, whereas the low-tax consumer has $\gamma = 2$. Government consumption at Date 0 is 6000. The corporate tax rate used is 0.5.

Solutions to the model are given by (see also Figure III):

t_{p1}	0.7975	0.7904	0.7833	0.7693	0.7624	0.7554	0.7486	0.7417
t_{p2}	0	0.05	0.1	0.2	0.25	0.3	0.35	0.4
D	49.474	47.845	46.056	42.050	39.857	37.533	35.149	32.656
i	34.597	37.782	41.355	49.915	55.049	60.761	67.501	75.052
r	17.298	18.891	20.677	24.957	27.524	30.380	33.750	37.526
B	214.73	207.36	199.71	183.70	175.40	167.28	158.39	149.74
$i(1-t_{p2})$	34.597	35.892	37.220	39.932	41.287	42.533	43.875	45.031
z	8.4-15	3.2E-15	1.2E-15	1.5E-16	5.1E-17	1.7E-17	5.4E-18	1.7E-18
f(z)	1.7-13	7.0E-14	2.8E-14	4.3E-15	1.6E-15	5.9E-16	2.1E-16	7.1E-17
c_{10}	2049.4	2097.8	2146.0	2242.0	2289.8	2337.5	2385.1	2432.6
c_{11}	4238.8	4430.3	4630.7	5060.7	5292.0	5539.0	5791.1	6060.7
c_{20}	950.52	902.15	853.94	757.94	710.14	662.46	614.85	567.34
c_{21}	5671.1	5479.6	5279.2	4849.2	4617.9	4370.9	4118.8	3849.2

4. Example 4: "Crowding Out"

In the next two examples we illustrate solutions to the model in which government and corporate borrowing move in opposite directions (crowding out) and in the same direction (crowding in). In this example we illustrate the former phenomenon.

We use the following parameters:

$\bar{f} = 2000$
$\omega_{10} = 6000$
$\omega_{11} = 2000$
$\omega_{20} = 950$
$\omega_{21} = 1000$

The example employs the power utility functions of the previous examples, with $\gamma = 4$ for the high-tax consumer and $\gamma = 2$ for the low-tax consumer. The production function employed is $f(z) = z + z^{1/1.5}$. Government consumption at Date 0 is 5700. The corporate tax rate is 0.5.

Solutions to the model are (see also Figure IV):

t_{p1}	0.6028	0.5910	0.5792	0.5556	0.5439	0.5321	0.5203	0.5086
t_{p2}	0	0.05	0.1	0.2	0.25	0.3	0.35	0.4
D	23.293	20.475	17.777	12.770	10.476	8.3337	6.3509	4.5377
i	0.2322	0.2336	0.2350	0.2377	0.2390	0.2402	0.2413	0.2424
r	0.1161	0.1168	0.1175	0.1188	0.1195	0.1201	0.1206	0.1212
B	1177.4	1199.0	1220.6	1264.1	1285.8	1307.6	1329.5	1351.5
z	189.23	185.78	182.48	176.38	173.58	170.97	168.53	166.28
f(z)	222.19	218.33	214.65	207.83	204.70	201.77	199.04	196.52
c_{10}	2134.0	2182.1	2230.2	2326.3	2374.3	2422.3	2470.3	2518.2
c_{11}	2193.4	2243.3	2293.1	2392.6	2442.3	2492.0	2541.6	2591.3
c_{20}	926.70	882.02	837.22	747.22	702.02	656.66	611.14	565.46
c_{21}	1028.7	975.02	921.53	815.19	762.35	709.73	657.34	605.19

5. Example 5: "Crowding In"

In this example we give an example where government and corporate borrowing move together. Parameters of the example are:

$\bar{f} = 0$
$\omega_{10} = 20000$
$\omega_{11} = 3000$
$\omega_{20} = 1000$
$\omega_{21} = 1000$

The example uses a production function $f(z) = z + z^{1.6}$, and the same utility functions as those employed in the previous example. Government consumption at Date 0 is 8000. The corporate tax rate varies from 0.10 to 0.34, and the low tax rate is set equal to zero.

The model's solutions are (see also Figure V):

t_{p1}	0.34870	0.34861	0.348526	0.34843	0.34825	0.34816	0.34807	0.34798
t_c	0.1	0.13	0.16	0.19	0.25	0.28	0.31	0.34
D	7.21543	7.45664	7.71453	7.99090	8.60763	8.95312	9.32751	9.73457
i	0.02972	0.03074	0.031842	0.03302	0.03566	0.03714	0.03875	0.04052
r	0.02675	0.02675	0.02675	0.02675	0.02674	0.02674	0.02674	0.02674
B	1472.02	1607.73	1743.45	1879.20	2150.75	2286.57	2422.41	2558.28
z	4461.96	4462.20	4462.46	4462.75	4463.37	4463.72	4464.11	4464.52

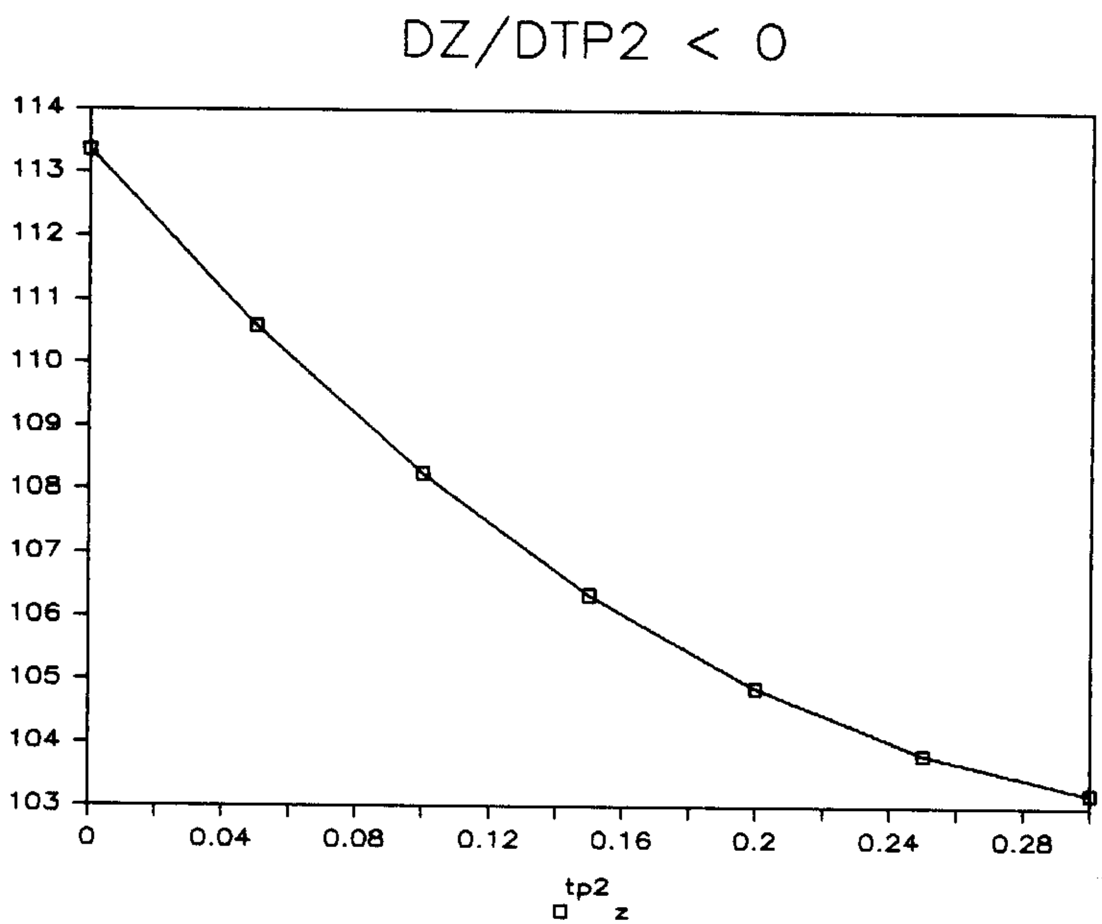
DZ/DTP2 < 0
114
113
112
111
110
109
108
107
106
105
104
103
0
0.04
0.08
0.12
0.16
0.2
0.24
0.28
tp2
z

EXHIBIT I

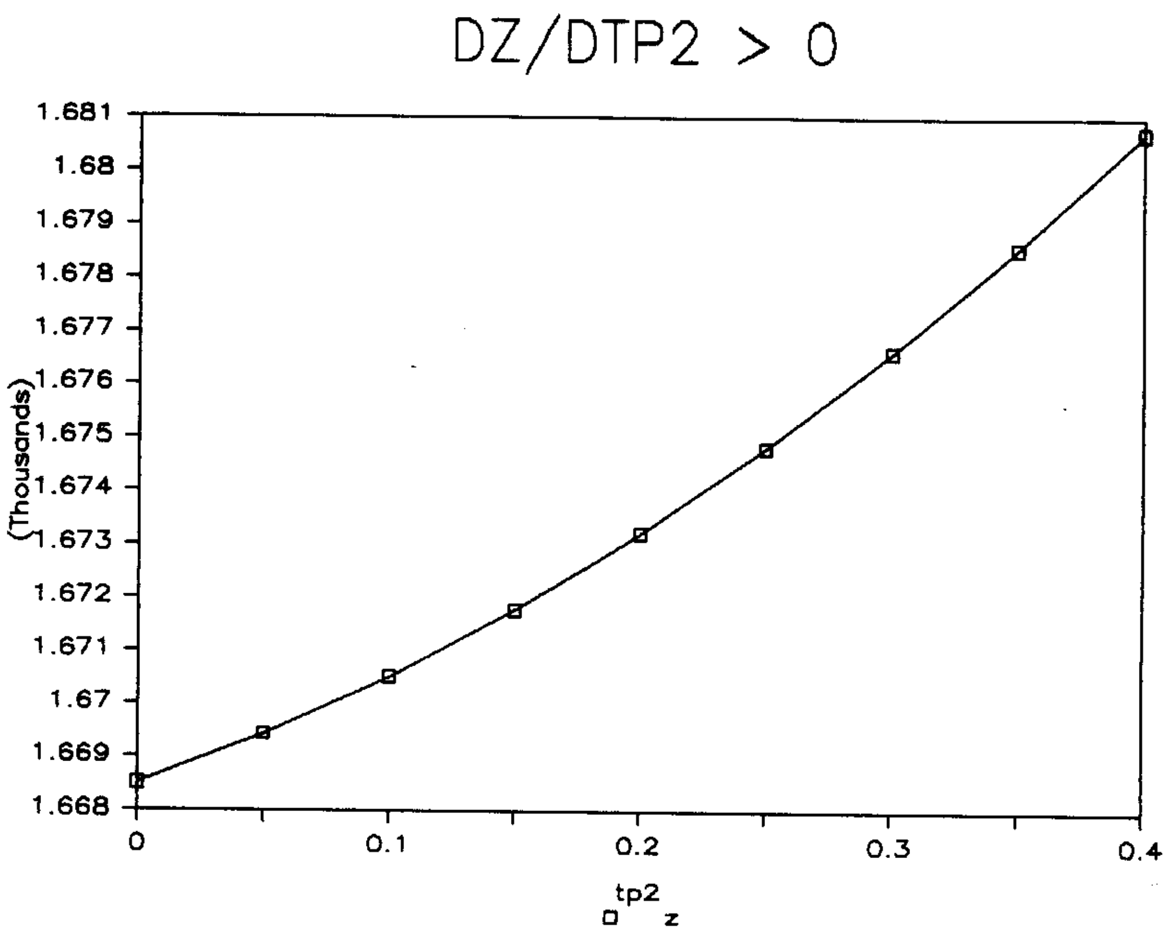
DZ/DTP2 > 0
(Thousands)
1.681
1.68
1.679
1.678
1.677
1.676
1.675
1.674
1.673
1.672
1.671
1.67
1.669
1.668
0
0.1
0.2
0.3
0.4
tp2
z

EXHIBIT II

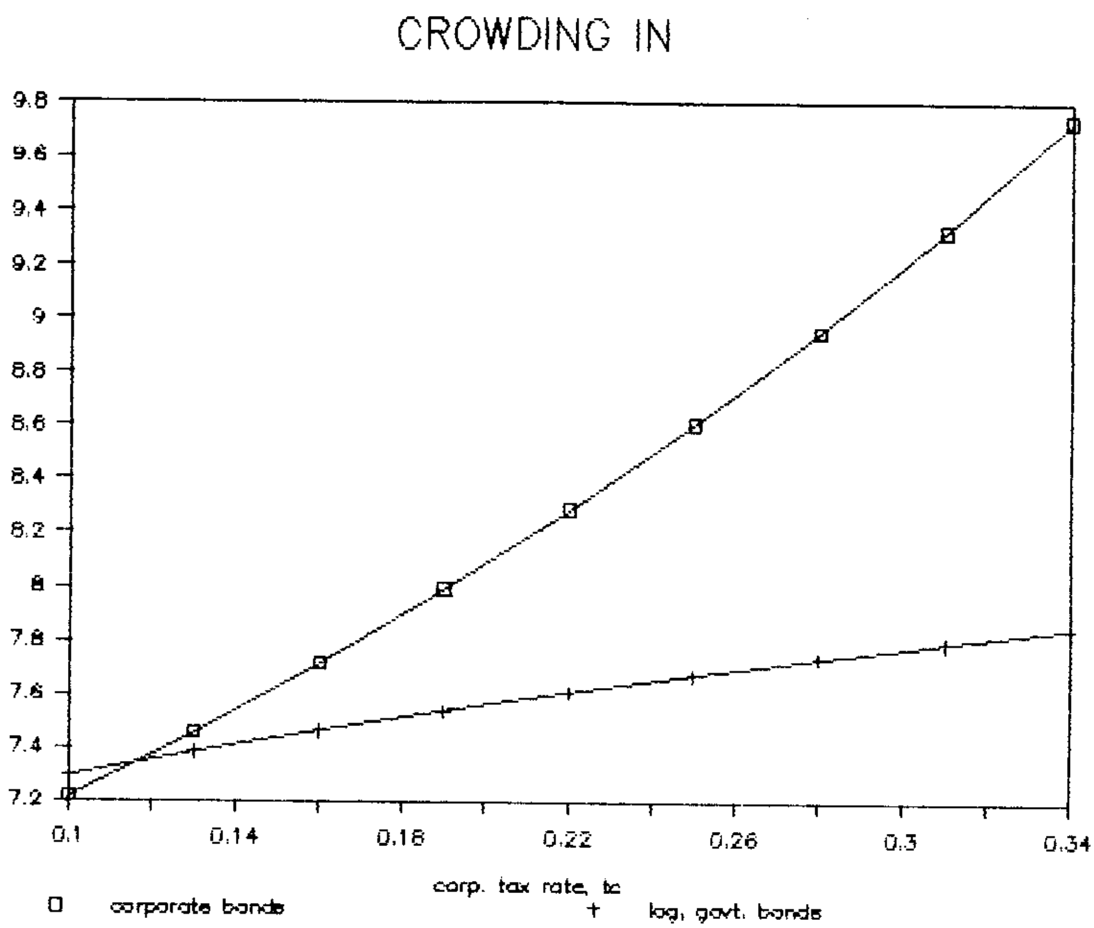
CROWDING IN
9.8
9.6
9.4
9.2
9
8.8
8.6
8.4
8.2
8
7.8
7.6
7.4
7.2
0.1
0.14
0.18
0.22
0.26
0.3
0.34
corp. tax rate, tc
corporate bonds
log. govt. bonds

EXHIBIT V

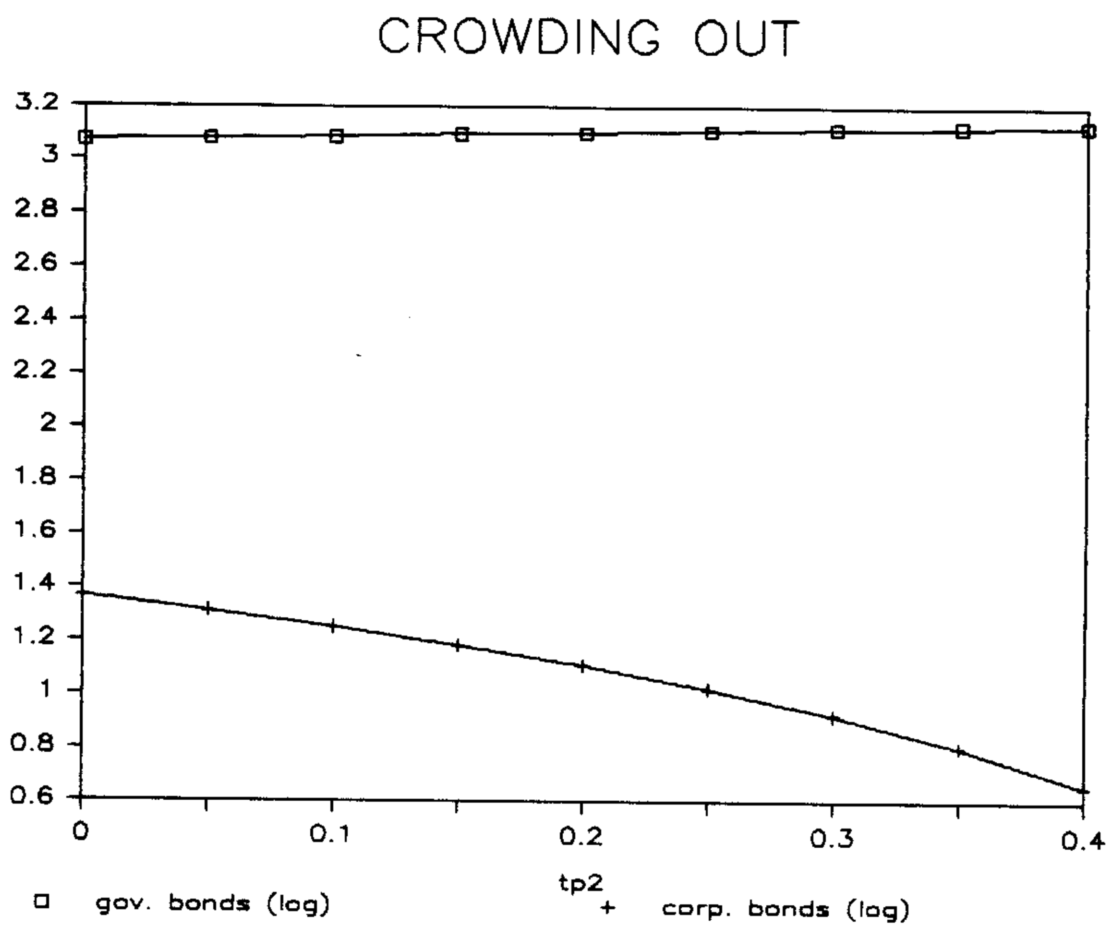
CROWDING OUT
3.2
3
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0
0.1
0.2
0.3
0.4
tp2
gov. bonds (log)
corp. bonds (log)

EXHIBIT IV

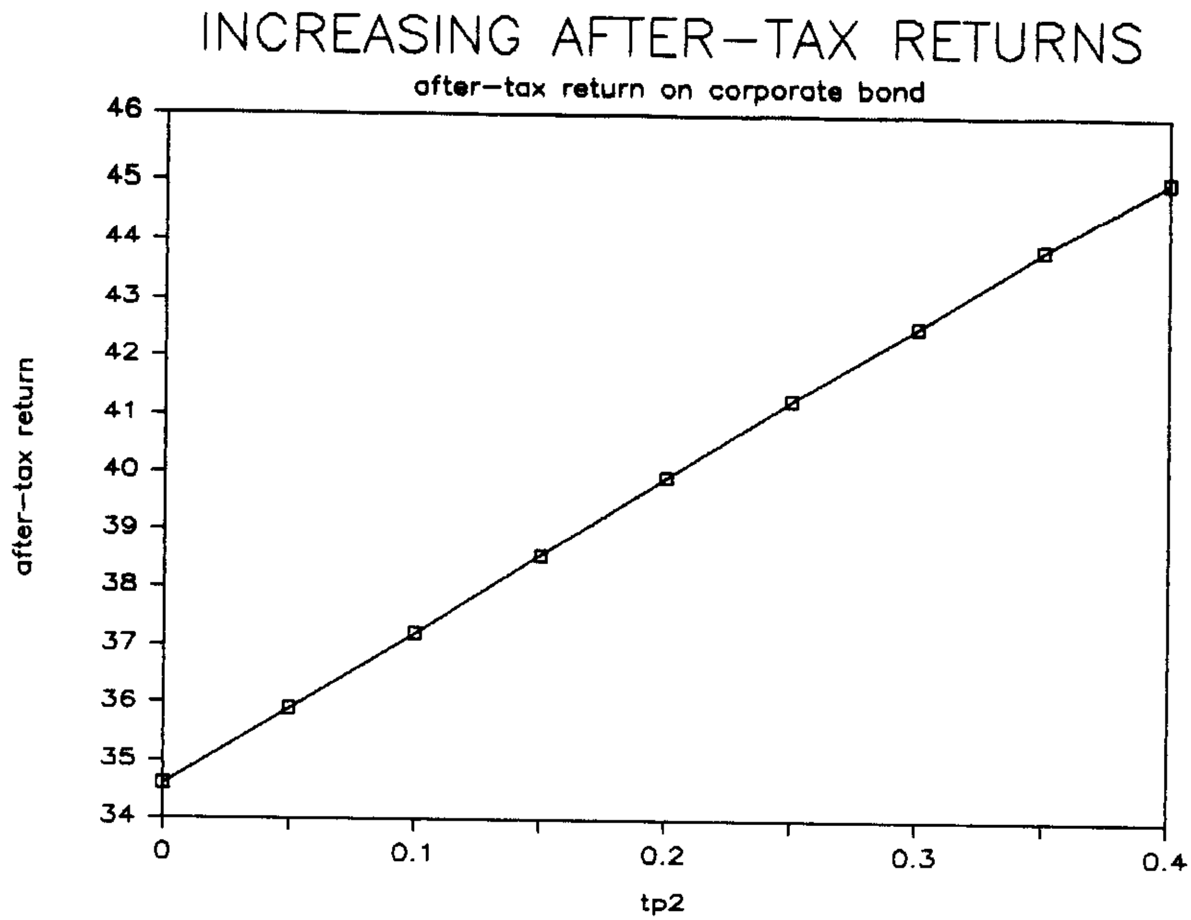
INCREASING AFTER-TAX RETURNS
after-tax return on corporate bond
46
45
44
43
42
41
40
39
38
37
36
35
34
after-tax return
0
0.1
0.2
0.3
0.4
tp2

EXHIBIT III

FOOTNOTES

[1]A number of papers, Taggart (1980, 1981), Gordon and Malkiel (1981), and Hakansson (1983), have recognized that the absence of a government sector in the Miller model is problematic, particularly when considering the effects of changes in the model's parameters. Some of these issues have been studied in a partial equilibrium framework by McDonald (1983). In McDonald's paper production is taken as given and the government budget constraints are not endogenized. We model explicitly both these aspects of the economy's general equilibrium. This results in the endogenization of the interest rates in the economy, and enables us to consider the interaction of government debt and the income tax rates. The resulting model is different in both the insights produced and the policy implications.

[2]Our results would not be affected by the addition of taxable government bonds.

[3]One can think of the endowments representing labor income, which we take to be fixed.

[4]We could add to the model government consumption at Date 1; this would not affect any of the results.

[5]Strictly speaking, the substitution of the government budget constraints into the consumer budget constraints should be on a per-capita basis, to reflect the fact that we are considering representative consumers. Doing this would increase the notational complexity of the presentation, with no change in the results.

[6]In a different framework from that employed here, an earlier literature concentrates on the equivalence between financing government consumption with debt and with taxes (generally lump-sum). This literature does not allow for the market segmentation which results from differential income taxation or for

corporate financial decisions. See Barro (1974), Chan (1983) and Benninga and Protopapadakis (1983).

[7]We can always guarantee that the first-order condition for the high-tax consumer hold with equality if we assume that $f'(z) \to \infty$ as $z \to 0$.

[8]The general derivation of the comparative statics of this section assumes that equation (9) holds with equality. However, in succeeding sections we shall also consider cases in which $D = 0$.

[9]Indeed, most of empirical studies found no evidence of such a link. For a recent survey of empirical research in this area, see The Economic Outlook, Congressional Budget Office, February 1984, Appendix A, pp. 99-102.

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Taggart, R. A. (1980). "Taxes and Corporate Capital Structure in an Incomplete Market," Journal of Finance 35, 645-59.

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Warren, A. C. and Auerbach, A. J. (1982). "Transferability of Tax Incentives and the Fiction of Safe-Harbor Leasing," Harvard Law Review 95, 1752-86.