FINANCIAL INNOVATION IN AN INCOMPLETE MARKET: AN EMPIRICAL STUDY OF STRIPPED GOVERNMENT SECURITIES

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The unbundling of coupon bonds into pure discount bonds made feasible patterns of dated nominal claims that were not previously attainable. Differences between the market values of coupon bonds and the spanning portfolios of pure discount bonds are consistent with a non-tax related segmentation of the bond market. In most periods, long-maturity coupon bonds selling at a premium could have been profitably unbundled. A comparison of pure discount bond prices with estimates of the respective reservation prices for investors in coupon bonds indicates that long-term time-contingent claims were generally more highly valued once unbundled. The high initial profit from bond stripping and its subsequent decline are consistent with long-run competitive supply adjustments by investment banks. Volatility differences of actual and reservation prices for pure discount bonds are insignificant.

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1. Introduction

This study examines the role of a major security innovation in an incomplete capital market, namely an increase in the set of feasible portfolios brought about by the creation of intermediate and long-term pure discount bonds. Although pure discount bonds play a prominent role in financial theory, until the middle of 1982, default-free pure discount bonds did not exist for maturities over one year. Since August 1982, a number of investment banks have been unbundling Treasury securities and issuing trust deposit receipts [1] that give the holder the right to a portion of a coupon payment or of the corpus of a Treasury security that is held in the trust. Between August 1982 and March 1984, close to $31 billion (face value) of such pure discount bonds have been sold. [2] Of the $31 billion, Merrill Lynch was responsible for about $11 billion ("TIGRS"), and Salomon Brothers, for about $14 billion ("CATS"). The remaining $7 billion included other trade names such as LIONS and COUGARS issued by individual firms, and, since January 1984, generic certificates named Treasury

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1 While, prior to August 1982, the "stripping" of bonds (i.e., the separate sales of coupons and corpus) was possible because Treasury securities were "physicals", this practice was extremely limited and discouraged by all involved Federal agencies.

2 Note that the $31 billion in face value of pure discount bonds correspond to only $8 billion in face value of Treasury securities, because the face value of Treasury securities does not include the summation of all the coupon payments.

Receipts (TR's) issued by a syndicate of investment firms.[3] A secondary market for trade-name certificates was typically maintained by the investment bank or syndicate responsible for the specific unbundling. During the period covered by this study, the annual volume traded in the secondary market was about twice the amount of outstandings. The interest for stripped bonds was viewed as increasing the demand for Treasury securities and induced the Treasury to issue bonds with separately registered coupons, making it unnecessary for investment banks to depend on depository trusts. Since the spring of 1985, it has become possible to strip newly auctioned long Treasury securities by requesting the Federal Reserve Bank of New York to identify separately their cash flows (i.e. through different CUSIP numbers). Unbundling has generally been profitable for investment bankers who have sold separately the components of stripped Treasury securities; i.e. corpus and coupons, at higher prices than as unstripped Treasury securities.

The introduction of stripped securities is a prominent example of security innovation in response to an existing set of government securities that was not sufficiently tailored to meet investors' preferences. The intent of this paper is to provide insights into this recent case of security innovation in an incomplete capital market. This paper examines the characteristics of bonds that could have been profitably stripped and

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[3] In 1984, the same technique has also been applied to bonds issued by the Inter-American Development Bank and by the International Bank for Reconstruction and Development.

the relative contributions of different maturity ranges of pure discount bonds to those profits.

The interest income on pure discount bonds is imputed each year as if interest were paid at a rate equal to the yield to maturity at the bond's purchase date and the interest received were reinvested at the same rate. In contrast with Treasury securities, where a major proportion of private holders are taxable, over the period of our study tax-exempt institutions held almost 95% of all pure discount bonds, while the bulk of the remainder is held in IRA's and in trust accounts for individuals in very low tax brackets (e.g., children). This ownership pattern suggests that over the period of our study taxes should have little or no influence on the relative pricing of different pure discount bonds.[4] However, the unfavorable tax treatment of pure discount bonds and their ownership patterns suggest the possibility of a segmentation of the secondary market for Treasury coupon bonds and the secondary market for pure discount bonds.

When there are restrictions on short-sales, pure discount bonds cannot be created as linear combinations of Treasury coupon bonds. Furthermore, due to the relatively unfavorable tax treatment of pure

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[4] In the more recent Treasury strips, those created from the corpus of a coupon bond are popular with individual Japanese investors because their return is treated as a capital gain, which is tax free.

discount bonds based on constant-yield amortization, it is not possible for a taxable investor to rebundle a group of pure discount bonds into the coupon bond from which they were stripped. This study examines the relationship between the reservation prices of pure discount bonds obtained from Treasury bonds selling at par or at a premium, and the prices of pure discount bonds, and compares their respective volatilities. The bond valuation model uses only Treasury coupon bonds selling at par or at a premium because the tax treatment of these bonds is unfavorable relative to Treasury coupon bonds selling at a discount and, like stripped securities, they would be expected to appeal primarily to tax-exempt investors. In fact, bonds at a premium are typically those bond issues which are unbundled by investment bankers to create pure discount bonds. The use of only par-and-premium bonds to estimate the reservation prices of pure discount bonds is consistent with the existence of tax-related investment clienteles of the type discussed by Schaefer (1982a).

2. Reservation Prices of Pure Discount Bonds Implicit in the Relative Pricing of Coupon Bonds

In the United States, the interest on coupon bonds is taxed as ordinary income while capital appreciation is generally taxed at preferential rates. A coupon bond selling at a discount in the secondary market may be viewed as a complex security consisting of a bundle of pure-dated claims on the semi-annual interest coupons and pure-dated claims on capital gain income and non-taxable repayment of principal at maturity. That is, the j-th coupon bond has a constant interest payment C_j at each coupon date over its life. It also has a capital gain of $(100-P_j)$ and a non-taxable repayment of P_j at maturity. When a bond is selling at a premium in the secondary market, the investor is able to offset a portion of the tax liability on the interest payment with a straight line amortization of the premium. Thus, at each coupon date, there is in effect an interest income payment of $(C_j-(P_j-100)/n)$ and a non-taxable payment of $(P_j-100)/n$. In addition, it has a non-taxable repayment of 100 at maturity.

The stripping of coupon bonds by investment bankers is not just a simple repackaging of complex securities into simpler, pure discount securities. The tax treatment of pure discount bonds is independent from the characteristics of the bonds from which they were stripped. For pure discount bonds, taxable income is imputed each year based on the yield to maturity at the time the bond is purchased. Thus, a pure discount bond is taxed as if its holder earned interest each year at a rate equal to the yield to maturity at the time the bond was purchased, and the interest income were reinvested. Thus, the imputed interest on the j-th pure discount bond with the maturity of M days is determined by applying an interest rate of $(100/P_j)^{365/M}-1$ each year over the life of the pure discount bond to the sum of the initial investment P_j and of the cumulative imputed interest from prior years. [5]

For a rational tax-exempt investor, the classification of cash-flows as principal repayment, interest and/or capital gain is irrelevant. Ignoring liquidity considerations, a package of pure discount

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[5] For simplicity, the above discussion makes use of yields to maturity calculated on a bond yield equivalent basis which simply multiplies the semi-annual bond equivalent yield by 2. See equation (A1) of Appendix I.

bonds whose before-tax cash-flows are identical to those of a given coupon bond would be a perfect substitute for that coupon bond. Therefore, for rational tax-exempt investors to hold a given coupon bond, the price of that bond cannot exceed the market value of the package of pure discount bonds with identical before-tax cash-flows. Thus, a coupon bond selling at a price exceeding the market value of the pre-tax equivalent package of pure discount bonds would be _prima facie_ evidence that this bond is not held in the portfolios of rational tax-exempt investors. However, institutional restrictions on the short-sale of bonds would prevent tax-exempt investors from making arbitrage profits. It should be noted that it would not necessarily be irrational for a tax-exempt investor to hold some pure discount bonds even if the value of the repackaging of these with other appropriate pure discount bonds (which it does not hold) to reconstruct an existing coupon bond would exceed the market value of that bond.

Schaefer (1982a) has shown that differences in investor tax rates and short-selling restrictions could induce tax-related clientele effects in the holding of government bonds. In contrast to Schaefer's segmented market model, Litzenberger and Rolfo (1984) demonstrated that a locus of income and capital gain tax rates may be consistent with the observed prices of government bonds. However, their theoretical analysis of the locus does not apply to U.S. government bonds selling at a premium in the secondary market because the amortization of the premium may be deducted from interest payment to determine taxable income. An extension of their analysis to premium bonds indicates the existence of a unique income tax rate that is implicit in the relative pricing of these bonds (Kanemasu, Litzenberger and Rolfo (1985)). This suggests the possibility of a tax-related segmentation of markets for bonds selling at a premium and bonds selling at a discount. Investment bankers have stripped only par-and-premium bonds. Therefore, the tax rate implicit in the relative pricing of par-and-premium bonds is used in estimating the reservation prices of investors in par-and-premium bonds for the pure discount bonds that were obtained by unbundling these securities.

In market equilibrium under restricted short-sales, the marginal rate of substitution between present consumption and investment in the j-th bond for an individual k, m^k_j, would be identical for all investors holding the j-th premium bond. That is, in portfolio equilibrium, these investors' marginal rates of substitution would be equal to the price of that premium bond:

$$P_j = m_j^k = [C_j\,(1 - \tau_i^k(t)) + \tau_i^k(t)\,(P_j - 100)/n] \sum_{t=T_1}^{T_n} m^k(t) + 100\, m^k(T_n) \quad \text{for any k and j} \qquad (1)$$

where

P_j = the price of the j-th fixed-income security,

n = the number of cash flows,

$T_1, \ldots, T_n$ = the sequence of cash flow dates on the j-th bond,

C_j = the interest payment at the end of each period on the j-th bond,

$\tau_i^k(t)$ = interest income tax rate for period t, for an individual k,

$m^k(t)$ = marginal rate of substitution between present consumption and non-taxable wealth in period t, for an individual k.

Under restricted short-sales, marginal rates of substitution between present consumption and non-taxable wealth in period t, $m^k(t)$, are not necessarily the same for all holders of the j-th premium bond. This study estimates the income tax rate and the marginal rate of substitution that would be consistent with a representative investor holding all par-and-premium bonds. These parameters for the representative investor are used to derive price estimates for pure discount bonds. These price estimates should be interpreted as reservation prices at which the representative investor would just be willing to hold small amounts of the respective pure discount bonds in his portfolio. A systematic difference between these estimated reservation prices and the actual prices of pure discount bonds would be evidence of segmentation between the market for par-and-premium bonds and the market for pure discount bonds. However, the cause of this segmentation would appear to be unrelated to tax considerations since major holders of par-and-premium bonds and pure discount bonds are tax-exempt investors.

3. Characteristics of Treasury Securities that Could Have Been Profitably Stripped

Table I lists Treasury securities known to have been stripped. Each of these securities was selling at a premium for most of the period covered by this study. Note that the tax treatment of coupon bonds selling at a premium is unfavorable relative to coupon bonds selling at a discount in the secondary market. Taxable investors would *cet. par.* bid a higher price for a dollar of capital gains than for a dollar of interest. Since the tax treatment of stripped securities is independent of the tax treatment of the securities from which they were unbundled, premium bonds would be anticipated to have a higher value-added through stripping. The value-added through stripping is measured as the ratio of the market value of a package of pure discount bonds (that gives the same pre-tax time-contingent payoffs as a given coupon bond) and the market price of the coupon bond less unity.

TABLE I

Non-Inclusive List of Treasury Securities Known to Have Been Stripped [a]

Coupon	Maturity	Coupon	Maturity
15-7/8%	9/1985	12-5/8%	5/1995
16-1/8%	11/1986	11-3/4%	2/2001
14%	5/1987	13-1/8%	5/2001
14%	7/1988	13-3/8%	8/2001
15-3/8%	10/1988	15-3/4%	11/2001
14-5/8%	1/1989	14-1/4%	2/2002
14-3/8%	4/1989	11-5/8%	11/2002
14-1/2%	7/1989	11-1/8%	8/2003
14-1/2%	5/1991	11-7/8%	11/2003
14-7/8%	8/1991	12-3/4%	11/2005-10
14-1/4%	11/1991	13-7/8%	5/2006-11
14-5/8%	2/1992	14%	11/2006-11
13-3/4%	5/1992	12%	8/2008-13

a This non-inclusive list was obtained from interviews with bond dealers.

Table II presents some summary statistics on the bonds that could have been profitably stripped. Bonds with maturity less than five years are excluded. The first and second columns give the percentage of bonds selling at par or at a premium and the percentage of bonds selling at a discount which could have been profitably stripped, respectively (ignoring

TABLE II

Summary of the Ranking of Treasury Securities in Terms of Value Added if Stripped[a]

Month	Bonds With Value Added if Stripped (%)		Total Number of Bonds		% of Stripped Bonds Ranked in First 20	Maximum Value Added[b]	Cross-Sectional Rank Correlation of Value Added With	
	Par and Premium	Discount	Par and Premium	Discount			Duration	Premium
09/82	4.8	0.0	21	17	100.0	0.11	0.15	0.88
10/82	81.5	0.0	27	12	100.0	4.20	-0.07	0.96
11/82	88.5	7.7	26	13	100.0	4.87	0.26	0.90
12/82	78.6	0.0	28	12	100.0	4.73	0.23	0.91
01/83	68.0	0.0	25	14	100.0	4.01	0.22	0.90
02/83	66.6	0.0	30	11	100.0	3.84	0.21	0.86
03/83	71.0	0.0	31	11	100.0	3.77	0.17	0.89
04/83	65.6	0.0	32	10	94.1	4.21	0.40	0.83
05/83	35.7	7.1	28	14	100.0	1.19	0.23	0.83
06/83	25.0	0.0	28	15	100.0	0.39	0.21	0.86
07/83	0.0	0.0	22	20	100.0	-0.45	-0.19	0.91
08/83	0.0	0.0	21	23	93.8	-0.35	-0.14	0.91
09/83	66.7	13.6	24	22	100.0	2.03	0.26	0.73
10/83	45.5	13.0	22	23	100.0	1.30	.32	0.67
11/83	10.5	16.7	19	24	93.7	0.99	0.20	0.70
12/83	57.9	16.7	19	24	93.7	1.85	0.21	0.69
01/84	19.2	10.0	26	20	100.0	0.55	0.29	0.69
02/84	0.0	0.0	21	24	100.0	-0.13	0.11	0.55

[a] Securities with maturity less than 5 years are excluded.

[b] The highest value-added-if-stripped among the bonds in the cross-sectional data.

the associated legal, administrative and marketing costs). The third and fourth columns give the total number of coupon bonds selling at par or at a premium and the total number of bonds selling at a discount, respectively. Note that, over this time period, the percentage of par-and-premium bonds with positive value-added through stripping is substantially larger than the corresponding percentage for discount bonds. Nonetheless, there were months where some long-maturity bond issues that were selling at a discount from face value could have been profitably stripped. On the other hand, there were three months during which no bond could have been profitably stripped. The fifth column gives the percentage of Treasury securities known to have been stripped that ranked in the top twenty in terms of the measure of value-added through stripping used in this study. For every month, over 90% of these Treasury securities known to have been stripped ranked in the top twenty in terms of their value-added through stripping. Unfortunately, the particular months in which each issue was stripped is not publicly available information. Nevertheless, the information summarized in column five is consistent with the hypothesis that the bonds that were stripped were among those having the highest value-added. The sixth column displays the percentage value-added for the Treasury coupon bond issue most attractive for stripping. Note that there are five months during which the investment bankers' gross margin exceeds 4% of the market value of the most attractive Treasury security. However, there are three months when even the highest value-added is negative. The seventh and eighth columns give the rank-order correlations between value-added through stripping, and duration and percentage premium, respectively.

The rank-order correlation of value-added with percentage premium is significantly positive in every month. The simple correlation between value-added through stripping and percentage premium for the pooled time-series and cross-sectional sample is 0.75. In contrast, the correlations with duration, while positive except in three months, are generally insignificant, and the simple correlation for the pooled sample is only 0.04. Thus, there does not appear to be a simple linear relationship between potential profitability through stripping and duration. However, the multiple correlation coefficient for a pooled regression of value-added on the percentage premium and duration is 0.80 and the t-values are 29.1, and 9.8, respectively. Thus, long maturity bonds selling at a premium tend to be the most profitable candidates for stripping.

The value-added for bonds known to have been stripped during the period covered by this study are presented in Table III along with their respective premiums. Note that the potential profits from stripping generally declined over the period, due to competitive pressures. Profits earned by investment banks through the competitive financial innovations are similar to profits earned by non-financial firms through competitive technological innovations. A successful innovation results in high initial profits that are eventually reduced by competitive pressures. The high initial profits allow firms to earn on average an expected return from innovation that provides them with a normal competitive profit. Thus, the high initial profit from bond stripping and its subsequent decline is consistent with competitive supply adjustments by financial intermediaries.

TABLE III

HISTORY OF STRIPPED BONDS IN TERMS OF VALUE ADDED CREATED BY STRIPPING [a]

Stripped Bonds Coupon	Maturity	9/82	10/82	11/82	12/82	1/83	2/83	3/83	4/83	5/83	6/83	7/83	8/83	9/83	10/83	11/83	12/83	1/84	2/84
14	7/88	-	2.32 (11.40)	1.11 (12.98)	1.09 (13.15)	-	0.25 (13.95)	0.61 (13.57)	0.16 (14.25)	-	-	-	-	-	-	-	-	-	-
15 3/8	10/88	-	3.14 (16.83)	2.18 (18.02)	1.87 (18.47)	0.36 (18.77)	0.49 (19.81)	0.86 (19.37)	0.48 (20.03)	-	-	-	-	0.08 (14.32)	-	-	-	-	-
14 5/8	1/89	-	2.74 (14.46)	1.90 (15.70)	1.81 (15.80)	0.38 (15.96)	0.68 (16.94)	1.11 (16.41)	0.79 (16.94)	-	-	-	-	0.23 (11.64)	-	-	-	-	-
14 3/8	4/89	-	2.91 (13.74)	1.88 (14.97)	1.60 (15.42)	1.49 (15.03)	0.51 (16.35)	0.96 (15.75)	0.58 (16.44)	-	-	-	-	0.09 (11.14)	-	-	-	-	-
14 1/2	7/89	-	2.79 (14.78)	1.96 (15.85)	1.70 (16.42)	0.54 (16.02)	0.66 (17.22)	1.00 (16.72)	0.70 (17.25)	-	-	-	-	0.38 (11.64)	-	-	-	-	-
14 1/2	5/91	-	2.80 (17.06)	2.19 (18.40)	2.13 (18.48)	1.16 (17.75)	1.22 (19.29)	1.38 (18.66)	1.40 (19.63)	0.26 (18.84)	-	-	-	0.34 (13.64)	0.15 (13.24)	-	0.12 (12.86)	-	-
14 7/8	8/91	-	3.05 (19.24)	2.36 (20.53)	2.04 (20.89)	1.24 (19.97)	1.60 (21.41)	1.48 (21.06)	1.60 (21.91)	0.21 (21.31)	0.26 (19.17)	-	-	0.28 (15.92)	0.17 (15.36)	-	0.37 (14.71)	-	-
14 1/4	11/91	-	2.88 (16.25)	2.28 (17.63)	2.10 (17.86)	1.10 (17.09)	1.38 (18.45)	1.39 (18.00)	1.48 (18.94)	0.18 (18.28)	-	-	-	0.38 (12.83)	0.21 (12.40)	-	0.22 (11.99)	-	-
Federal Funds		10.21	9.55	8.56	8.54	8.52	8.39	8.80	8.65	8.70	9.18	9.49	9.32	9.29	9.34	9.22	9.25	9.49	9.66
T. Bonds 20 yrs.		11.95	10.77	10.52	10.71	11.01	10.71	10.84	10.62	10.90	11.12	11.55	11.71	11.78	11.90	11.82	12.03	11.76	12.12

[a] Stripped bonds with the maturity longer than 5 years are presented. Numbers in parentheses are the premium levels. The last two rows show the interest rate series. The averages of the weeks (ending on Friday) containing the quotation dates are shown.

TABLE III (continued)

HISTORY OF STRIPPED BONDS IN TERMS OF VALUE ADDED CREATED BY STRIPPING

Stripped Bonds Coupon	Maturity	9/82	10/82	11/82	12/82	1/83	2/83	3/83	4/83	5/83	6/83	7/83	8/83	9/83	10/83	11/83	12/83	1/84	2/84
14 5/8	2/92	-	3.20 (18.37)	2.31 (19.94)	2.14 (20.30)	0.98 (19.44)	1.42 (20.91)	1.37 (20.47)	1.54 (21.50)	0.10 (20.72)	0.13 (18.36)	-	-	0.29 (15.07)	0.13 (14.59)	-	0.32 (13.96)	-	-
13 3/4	5/92	-	2.60 (14.31)	1.98 (15.72)	2.15 (15.93)	0.99 (14.78)	0.77 (16.80)	1.07 (16.01)	1.44 (17.01)	-	-	-	-	0.10 (10.77)	0.23 (10.00)	-	0.13 (9.74)	-	-
12 5/8	5/95	-	0.66 (11.95)	2.61 (11.67)	2.11 (11.72)	0.61 (10.95)	0.86 (12.18)	0.81 (12.08)	1.44 (13.14)	-	-	-	-	- (6.52)	0.07 -	-	-	-	-
11 3/4	2/01	-	2.11 (6.51)	3.10 (7.84)	3.07 (7.01)	2.91 (4.40)	2.97 (6.22)	2.95 (6.08)	3.48 (7.67)	1.07 (5.90)	0.24 (4.06)	-	-	1.40 (0.40)	1.14 (1.01)	0.28 (-0.18)	1.16 (-1.47)	-	-
13 1/8	5/01	-	4.17 (14.88)	4.71 (17.05)	4.42 (16.53)	3.69 (14.29)	3.58 (16.18)	3.57 (16.01)	4.01 (17.83)	1.19 (16.45)	0.38 (14.40)	-	-	1.63 (10.21)	1.30 (8.79)	0.23 (10.02)	1.06 (8.65)	-	-
13 3/8	8/01	-	4.11 (17.10)	4.73 (18.93)	4.11 (18.25)	3.94 (15.89)	3.90 (18.05)	3.65 (18.14)	4.28 (19.76)	1.23 (18.42)	0.44 (16.33)	-	-	1.63 (12.18)	1.03 (11.00)	-	1.15 (10.40)	0.09 (12.20)	-
15 3/4	11/01	0.11 (27.89)	4.19 (35.31)	4.85 (37.59)	3.91 (37.86)	3.30 (35.12)	3.66 (36.66)	3.52 (36.63)	3.68 (39.06)	0.57 (37.96)	0.39 (34.70)	- -	- -	1.55 (29.86)	0.86 (28.71)	-	0.94 (28.05)	-	-
14 1/4	2/02	-	4.20 (24.03)	4.91 (25.86)	4.73 (25.12)	4.01 (22.76)	3.72 (25.36)	3.83 (25.01)	4.16 (27.07)	1.14 (25.63)	0.39 (23.36)	-	-	1.66 9.83)	1.00 (17.66)	-	1.03 (17.06)		-
11 5/8	11/02	n.a.	2.46 (5.58)	3.52 (7.12)	3.75 (6.07)	3.25 (3.68)	2.90 (5.72)	3.17 (5.30)	3.92 (6.77)	1.19 (5.12)	0.55 (2.99)	-	-	2.03 (1.04)	1.12 (1.82)	0.51 (-1.25)	1.64 (-2.78)	0.34 (-0.92)	-
11 1/8	8/03	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	-	-	-	1.48 (4.10)	1.27 (5.60)	0.94 (-5.57)	1.91 (-6.68)	0.53 (-4.82)	-
11 7/8	11/03	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	1.65 (1.40)	1.48 (0.20)	0.83 (0.44)	1.71 (-0.88)	0.41 (1.02)	0.05 (-1.48)
Federal Fun^s		10.21	9.55	8.56	8.54	8.52	8.39	8.80	8.65	8.70	9.18	9.49	9.32	9.29	9.34	9.22	9.25	9.49	9.66
T. Bonds 20 yrs.		11.95	10.77	10.52	10.71	11.01	10.71	10.84	10.62	10.90	11.12	11.55	11.71	11.78	11.90	11.82	12.03	11.76	12.12

Table IV illustrates examples of bonds with long maturities that could have been profitably stripped even though they were selling at a discount. The most extreme example is the 11-1/8% 8/03 issue that sold at a 6.7% discount from face value in December 1983 but nevertheless had a potential gross margin from stripping of 1.9%. All cases of non-callable government bond issues with maturities over five years that had positive value-added in excess of 10 basis points (one tenth of one percent) when selling at a discount from face value are presented in Table IV. Note that each of these Treasury securities had maturities greater than twenty years. This suggests that the value-added from long-term pure discount bond that could be stripped from the coupon bond exceeded the value of having a portion of the return on the coupon bond taxed as a capital gain.

To provide additional insights into the determinants of the profits from bond stripping, two additional explanatory variables are introduced. The first variable is the ratio between the estimated reservation price of the bond and the actual price. The estimated reservation price is obtained each month using a maximum likelihood procedure for fitting a cubic spline to all the par-and-premium bonds. The maximum likelihood estimation procedure is developed in Kanemasu, Litzenberger and Rolfo (1985)and is discussed in Section 4. Thus, this variable may be viewed as the predicted value-added through stripping if the marginal holders of par-and-premium bonds and marginal holders of the stripped securities are the same. Even though the estimated tax rate of investors holding par-and-premium bonds is zero, this variable reflects tax considerations since actual prices of coupon bonds selling at a discount would be anticipated to be lower than those predicted by a de facto pre-tax model of bond prices. Alternatively, this variable reflects the percentage

TABLE IV

Coupon Bonds Selling at a Discount with Positive Value Added-if-Stripped [a]

Month	Coupon	Maturity	Price	Value-Added-if-Stripped
1/83	10 3/4	2/2003	97.94	2.12
9/83	11 1/8	8/2003	95.90	1.48
	10 3/4	2/2003	93.22	1.26
	10 3/4	5/2003	93.22	1.25
10/83	11 1/8	8/2003	94.40	1.27
	10 3/8	5/2003	91.71	1.12
	10 3/4	2/2003	91.74	1.07
11/83	11 1/8	8/2003	94.63	0.94
	10 3/4	5/2003	92.03	0.74
	10 3/4	2/2003	92.01	0.71
12/83	11 1/8	8/2003	93.31	1.91
	10 3/4	5/2003	90.72	1.75
	11 7/8	11/2003	99.12	1.71
	10 3/4	2/2003	90.72	1.69
	11 5/8	11/2002	97.22	1.64
01/84	11 1/8	8/2003	95.18	0.53
	10 3/4	5/2003	92.46	0.46
	10 3/4	2/2003	92.46	0.45

[a] Discount bonds with value-added less than 0.1 are not included in the table. Also, only those with prices lower than 98 are shown.

deviation of coupon bond prices from those fitted by the econometric model. This variable is denoted as "mispricing" for expository purposes. The simple correlation between the "mispricing" variable (the predicted value-added) and the measured value-added is 0.70. About half of the variation in the actual value-added remains unaccountable by the predicted value-added, suggesting a market segmentation between stripped securities and premium bonds.

The next explanatory variable considered is the change in the slope of the yield curve relative to the previous month. The slope of the yield curve is measured as the difference between the Treasury 20-year constant maturity yield and the Federal Fund rate. This time series variable is introduced to examine the relationship between the profitability of stripping in different periods and the immediately preceding shifts in the shape of the yield curve. In particular, the wide time series fluctuations in the profitability of stripping evidenced in Table III may be related to shifts in the relative prices of long and short-term securities. When the long-term interest rate increases (holding the short-term rate constant), the resulting drop in long-term bond prices could result in an increase in the value-added of the long-term bonds if stripped bonds were held by a different clientele whose marginal rate of substitution remains relatively unchanged. In a non-segmented capital market where a subset of rational investors hold a market portfolio of all coupon and stripped bonds, the term structure shift variable would be unrelated to times series fluctuations in the value-added through stripping. The simple correlation between value-added and the term structure shift variable is 0.23. This correlation is consistent with a segmentation between these markets.

For any month, the values of three of the four variables (duration, percentage premium, and mispricing) differ among securities while the value of the term structure shift is the same for all securities. The results of ordinary least-square regressions indicate a non-negligible degree of residual serial correlation as measured by the first-order serial correlations computed on the series of residuals ranked in chronological order for each security. The estimation includes only those securities with time-to-maturity longer than five years and price quotations available throughout the period. There are considerable variations in the residual first-order serial correlations and residual variances among securities. The ordinary least-square procedure is modified to account for the individual error correlation and variance. Assuming a first-order autoregressive process over time for the errors corresponding to each Treasury security, each observation is replaced by the difference between this observation and the product of the preceding value of the same variable for the same security (when ranked in chronological order) and the first-order autoregressive coefficient, this difference being in turn divided by the residual variance corresponding to the underlying security.[6] The regression based on the transformed variables constitutes a generalized least-square estimation. The equation estimated using this

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[6] Except for the first month (t=1) at which the observations for the j-th security are transformed simply by multiplying them by $(1 - \hat{\phi}_j^2)^{1/2}/\sigma_{aj}$ where $\hat{\phi}_j$ and σ_{aj} are the auto-regressive parameter and the residual standard deviation for the j-th security, respectively.

procedure over the four derived independent variables (duration, percentage premium, mispricing, and term structure shift) is given by:

$$y_{jt} = \underset{}{0.9920} + \underset{(7.36)}{0.00405\, x_{j1t}} + \underset{(13.42)}{0.01012\, x_{j2t}} + \underset{(7.65)}{0.00523\, x_{j3t}} + \underset{(9.26)}{0.00363\, x_{j4t}} \quad (2)$$

where x's are the normalized versions of the original variables using the sample means and the standard deviations computed over the entire sample, and the numbers in parentheses are the t-ratios. The t-value for each of the explanatory variables is above 7.0; i.e. 13.42 for the percentage premium, 7.36 for duration, 7.65 for the mispricing variable, and 9.26 for term structure shifts. Thus, the higher the premium, the longer the duration, the greater the mispricing relative to the econometric model, and the greater the increase in the slope of the yield curve relative to the prior month, the more profitable it is to strip a coupon bond.

The results presented above indicate that in periods when it is profitable to strip coupon bonds the longer term issues selling at a premium are the most likely candidates for stripping. To provide some insights into the particular maturities that are contributing to the potential profits from stripping, reservation prices for stripped securities are estimated for investors holding non-callable par-and-premium coupon bonds. Section 4 presents a brief discussion of the method used to estimate these reservation prices and discusses the empirical evidence.

4. Estimates of Reservation Prices for Stripped Securities

If there were two premium bonds (or one premium bond and one par bond) maturing at each cash-flow date, the ordinary income tax rate and the marginal rate of substitution of the representative investor (holding all par-and-premium bonds) could be inferred analytically for each date. In actual bond markets, the number of par and premium bonds is considerably less than twice the number of cash-flow dates; therefore, it is not possible to infer analytically a tax rate, $\tau_i(t)$, and a marginal rate of substitution, $m^k(t)$, for each cash-flow date. Following the approach taken by McCulloch (1975) and others, the tax rate is constrained to be an intertemporal constant and the $m^k(t)$'s are constrained using a cubic spline. The specification of the $m^k(t)$'s to be a cubic spline function of time is consistent with the existence of money which implies that the $m^k(t)$'s are a monotone decreasing function of time. The existence of money also constrains $m^k(0)$ to be unity. The continuous first derivative that is implicit in the use of a cubic spline is necessary for the existence of marginal rates of substitution between wealth in adjacent (continuous) periods. The continuous second derivative that is implicit in the use of a cubic spline is necessary for these marginal rates of substitution between adjacent periods to be smooth functions of time. For a more detailed discussion of cubic splines, see Litzenberger and Rolfo (1984).

Each of the imposed restrictions is a potential source of misspecification and the pricing relationship given in equation (1) would not hold, in general, without error. The resulting error term would be attributable to the incorrect specification of a stationary marginal tax rate/or to the functional form between individuals' marginal rates of substitution and time.

In an incomplete bond market, it is possible to reduce the number of parameters to the exact number of bonds having linearly independent payoffs in terms of the two classes of claims. However, such an approach would not necessarily result in the most accurate estimates of the individuals' marginal rates of substitution. For example, any misspecification resulting from constraining the marginal tax rates on interest income to be an intertemporal constant would affect the estimation of parameters of marginal rates of the substitution function. Furthermore, the price data used in the study are based on dealer bid quotes. Under such a procedure, the measurement errors associated with the data would also be reflected in parameter estimates.

An alternative approach is to choose a functional form of the discount function where the number of parameters to be estimated is substantially smaller than the number of bonds having linearly independent payoffs. The marginal tax rate on interest income and the marginal rates of substitution for a representative investor who holds all bonds in positive amounts could then be estimated econometrically. Since the number of parameters of the cubic spline is substantially smaller than the number of existing bonds, the system of equations is over-identified. An error term is added to equation (1), and the maximum likelihood criterion is employed to estimate econometrically the tax rate and the marginal rate of substitution function.

Based on a cubic spline, the $m^k(t)$ are given by:

$$m^k(t) = 1 + b_1 t + c_1 t^2 + d_1 t^3 + \sum_{i=1}^{L} F_{i+1} (t - t_i)^3 D_i(t) \qquad (3)$$

where $t_1, \ldots, t_L$ are non-zero knot points defining L+1 consecutive intervals and $D_i(t)$ is a step function with $D_i(t)=0$ for $t<t_i$ and 1 for $t>t_i$. The discount function is uniquely defined by the L+3 parameters b_1, c_1, d_1, F_2, ..., F_{L+1}. After substituting the right-hand side of equation (3) into equation (1), the estimation problem is reduced to a linear model with L+3 parameters:

$$P - nA - 100 = b_1 \, [A \sum_{t=T_1}^{T_n} t + 100 \, T_n] + c_1 \, [A \sum_{t=T_1}^{T_n} t^2 + 100 \, T_n^2] + d_1 \, [A \sum_{t=T_1}^{T_n} t^3 + 100 \, T_n^3]$$
$$+ \sum_{i=1}^{L} F_{i+1} \, [A \sum_{t=T_1}^{T_n} (t - t_i)^3 \, D_i(t) + 100 \, (T_n - t_i)^3 \, D_i(T_n)] + \varepsilon \qquad (4)$$

where $A = C(1 - \tau_i) + \tau_i \, (P - 100)/n$ and ε represents the error term.

The formalism presented in (4) assumes the simultaneity of the cash-flows and of their tax consequences. However, in contrast with previous work, this paper effectively distinguishes the timing of the tax implication of a coupon payment (or of the amortization of a premium) from the actual cash-flow date. It is assumed that the tax liability associated with a coupon payment is incurred in two installments: 90% on the first estimated tax date following the coupon payment, and 10% on the April 15 tax date of the next calendar year. A similar breakdown is assumed in the case of the amortization of a premium. This more detailed specification is discussed in Appendix I. The above econometric model is estimated using a maximum likelihood approach. The statistical procedures used are discussed in Kanemasu, Litzenberger and Rolfo (1985).

The previously discussed econometric model is used to estimate the reservation prices of pure discount bonds for investors in par-and-premium bonds. These estimates of reservation prices are compared to the reported prices of pure discount bonds in the secondary market as quoted by Merrill Lynch. The analysis encompasses 18 end-of-month dates starting in September 1982 when TIGRS were first issued.

Table V presents for ordinary income tax rates 0, 5, 10, and 15 percent the marginal log-likelihood function (log-likelihood function maximized with respect to parameters other than the income tax rate), the standard deviation of the residuals, and the root mean square forecast error of the residuals (RMSE). Table V shows that a zero income tax level gives the highest value of the maximized log likelihood for all 18 months examined. This is consistent with a capital market where the marginal holders of par-and-premium government bonds are tax-exempt institutions.

Graph I displays the ratios of the estimated reservation prices of pure discount bonds (implicit in the relative pricing of par-and-premium bonds) and observed pure discount bond prices minus unity. Graph I shows that observed pure discount bond prices are lower than the estimated reservation prices for "short" maturities. The range of "short" maturities varies from month to month, but is never under a year. Short-term pure discount bonds are priced lower than Treasury bills and recently auctioned short-term Treasury notes for which they are close but less liquid

TABLE V

Maximized Log-Likehood for Income Tax Rates Implicit in the Prices of Par-and-Premium Bonds [a]

Date	Number of Observations	Maximized Log-Likelihood and Fit Statistics				
September 1982	52	Income Tax (%)	0.00	0.05	0.10	0.15
		Likelihood	19.832*	19.497	19.124	18.710
		Std. Deviation	0.511	0.517	0.524	0.532
		RMSE	0.520	0.526	0.533	0.541
October 1982	61	Income Tax (%)	0.00	0.05	0.10	0.15
		Likelihood	1.133*	0.624	0.089	-0.458
		Std. Deviation	0.953	0.970	0.987	1.003
		RMSE	0.951	0.967	0.985	1.001
November 1982	60	Income Tax (%)	0.00	0.05	0.10	0.15
		Likelihood	24.338*	23.682	22.991	22.264
		Std. Deviation	0.676	0.692	0.709	0.727
		RMSE	0.673	0.689	0.707	0.725
December 1982	61	Income Tax (%)	0.00	0.05	0.10	0.15
		Likelihood	31.343*	30.640	29.865	29.030
		Std. Deviation	0.706	0.723	0.737	0.756
		RMSE	0.706	0.722	0.737	0.755
January 1983	58	Income Tax (%)	0.00	0.05	0.10	0.15
		Likelihood	43.929*	43.263	42.538	41.756
		Std. Deviation	0.588	0.603	0.619	0.636
		RMSE	0.587	0.602	0.618	0.635
February 1983	66	Income Tax (%)	0.00	0.05	0.10	0.15
		Likelihood	23.653*	22.808	21.897	20.941
		Std. Deviation	0.805	0.823	0.842	0.856
		RMSE	0.804	0.823	0.842	0.855
March 1983	58	Income Tax (%)	0.00	0.05	0.10	0.15
		Likelihood	46.793*	45.999	45.061	43.986
		Std. Deviation	0.572	0.588	0.605	0.623
		RMSE	0.571	0.587	0.605	0.623
April 1983	67	Income Tax (%)	0.00	0.05	0.10	0.15
		Likelihood	37.549*	36.247	34.924	33.554
		Std. Deviation	0.744	0.758	0.788	0.819
		RMSE	0.747	0.762	0.792	0.825
May 1983	54	Income Tax (%)	0.00	0.05	0.10	0.15
		Likelihood	55.025*	54.250	53.325	52.260
		Std. Deviation	0.426	0.436	0.448	0.461
		RMSE	0.424	0.435	0.447	0.460

[a] A "*" indicates the highest likelihood.

TABLE V
(Continued)

Date	Number of Observations	Maximized Log-Likelihood and Fit Statistics				
June 1983	48	Income Tax (%)	0.00	0.05	0.10	0.15
		Likelihood	71.024*	70.619	70.021	69.252
		Std. Deviation	0.189	0.203	0.221	0.242
		RMSE	0.189	0.202	0.220	0.240
July 1983	43	Income Tax (%)	0.00	0.05	0.10	0.15
		Likelihood	71.464*	71.433	71.130	70.544
		Std. Deviation	0.141	0.141	0.142	0.145
		RMSE	0.140	0.140	0.141	0.144
August 1983	44	Income Tax (%)	0.00	0.05	0.10	0.15
		Likelihood	52.391*	51.513	50.518	49.418
		Std. Deviation	0.247	0.256	0.266	0.277
		RMSE	0.245	0.254	0.264	0.275
September 1983	41	Income Tax (%)	0.00	0.05	0.10	0.15
		Likelihood	46.425*	45.658	44.766	43.754
		Std. Deviation	0.324	0.338	0.353	0.369
		RMSE	0.327	0.341	0.355	0.372
October 1983	46	Income Tax (%)	0.00	0.05	0.10	0.15
		Likelihood	62.918*	62.520	61.960	61.243
		Std. Deviation	0.257	0.262	0.269	0.275
		RMSE	0.254	0.259	0.266	0.273
November 1983	50	Income Tax (%)	0.00	0.05	0.10	0.15
		Likelihood	75.041	75.088*	74.766	74.071
		Std. Deviation	0.179	0.179	0.182	0.186
		RMSE	0.177	0.178	0.180	0.184
December 1983	45	Income Tax (%)	0.00	0.05	0.10	0.15
		Likelihood	78.664*	78.127	76.497	75.356
		Std. Deviation	0.178	0.177	0.179	0.180
		RMSE	0.176	0.176	0.178	0.179
January 1984	56	Income Tax (%)	0.00	0.05	0.10	0.15
		Likelihood	64.961*	63.482	61.809	59.490
		Std. Deviation	0.214	0.215	0.218	0.224
		RMSE	0.213	0.214	0.217	0.224
February 1984	44	Income Tax (%)	0.00	0.05	0.10	0.15
		Likelihood	71.669*	71.153	70.195	68.911
		Std. Deviation	0.335	0.322	0.309	0.296
		RMSE	0.333	0.321	0.307	0.294

GRAPH I

TR Price Ratios (using the Par and Premium Discount Yield Curve)

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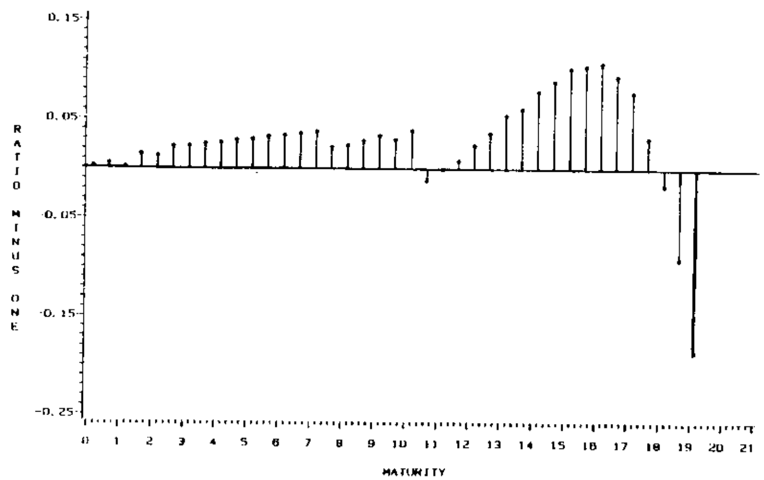

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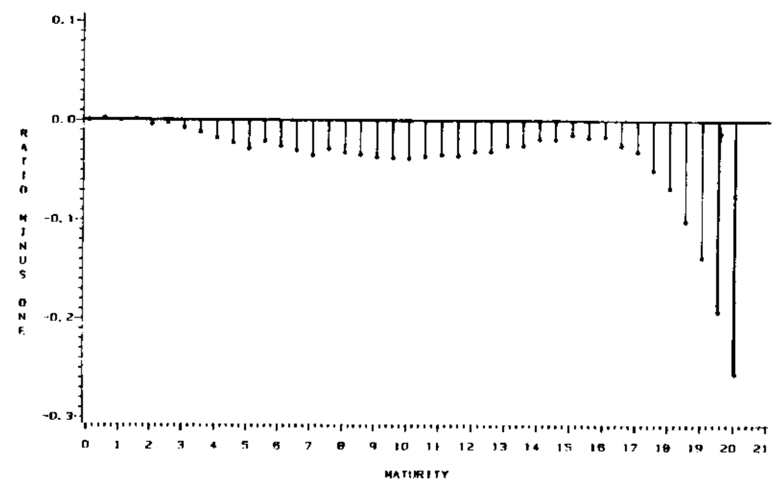

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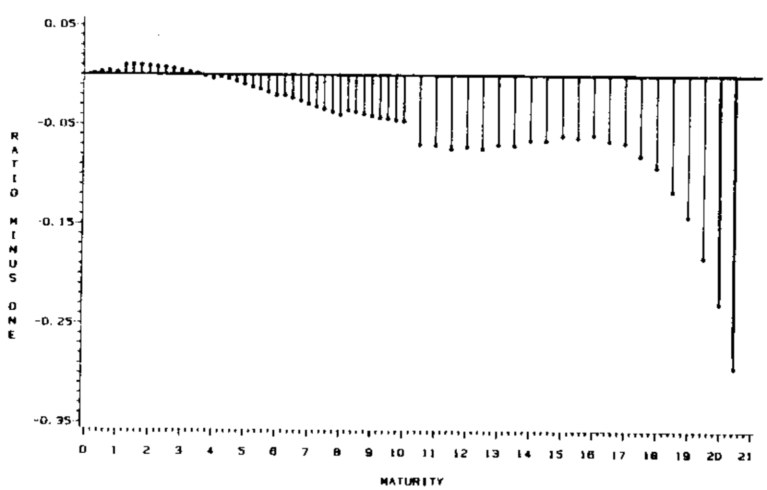

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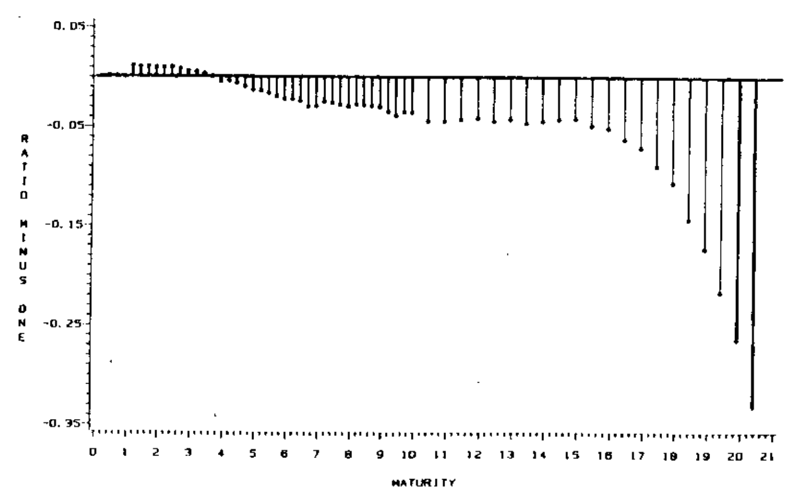

GRAPH I

TR Price Ratios (using the Par and Premium Discount Yield Curve)

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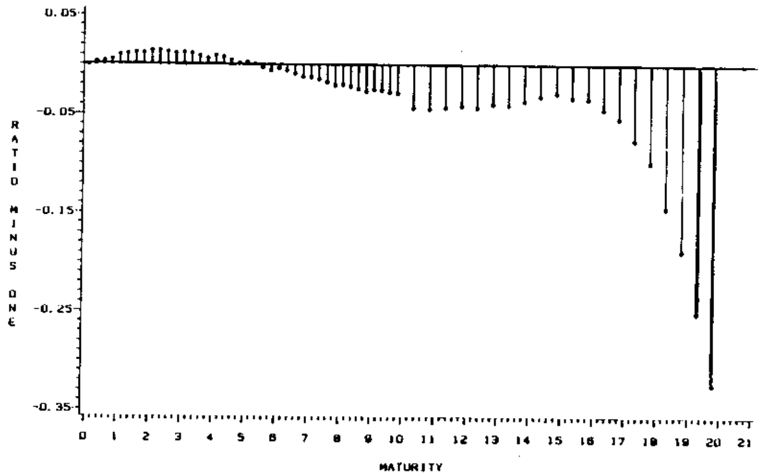

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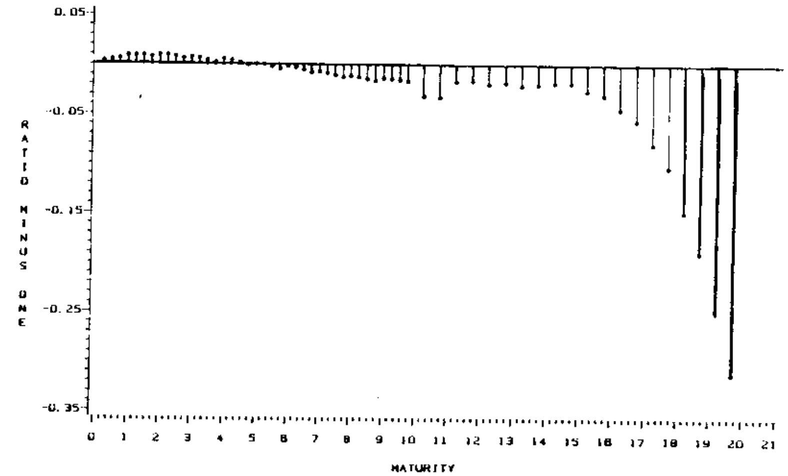

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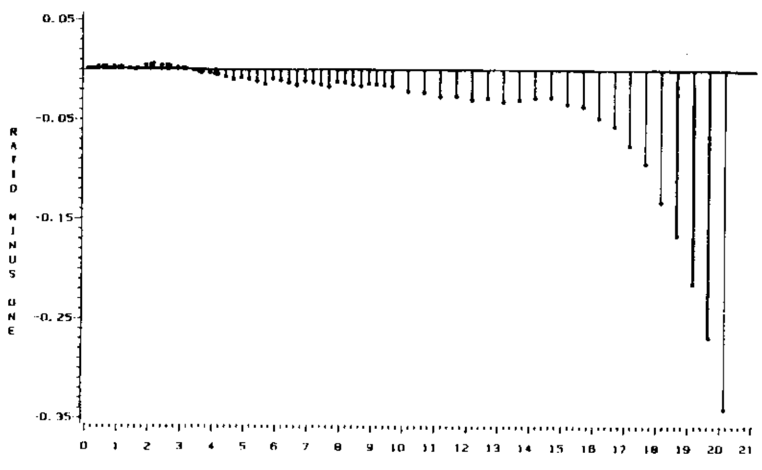

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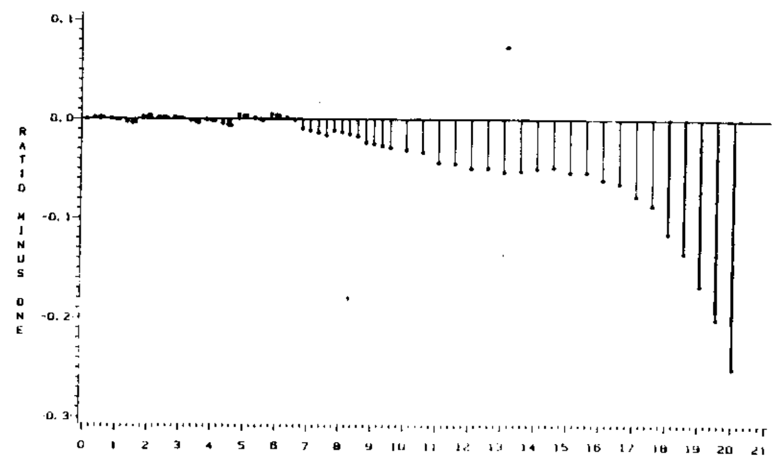

GRAPH I

TR Price Ratios (using the Par and Premium Discount Yield Curve)

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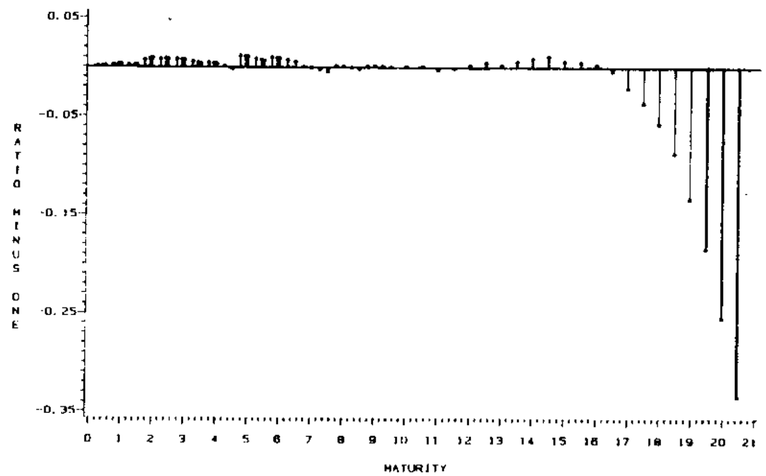

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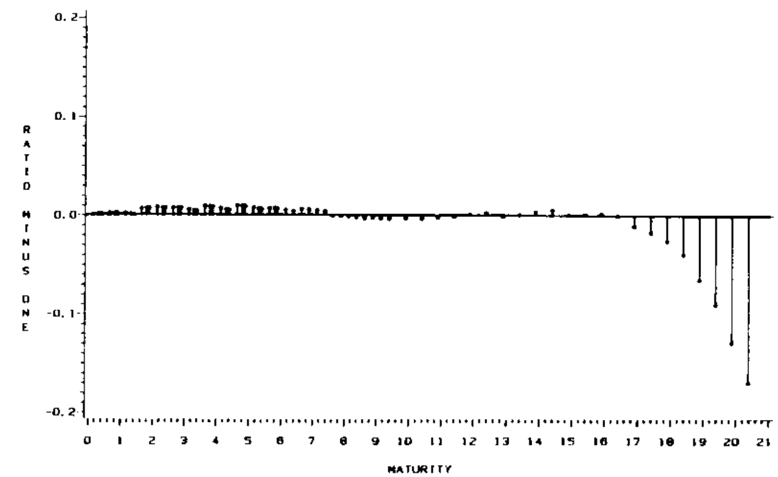

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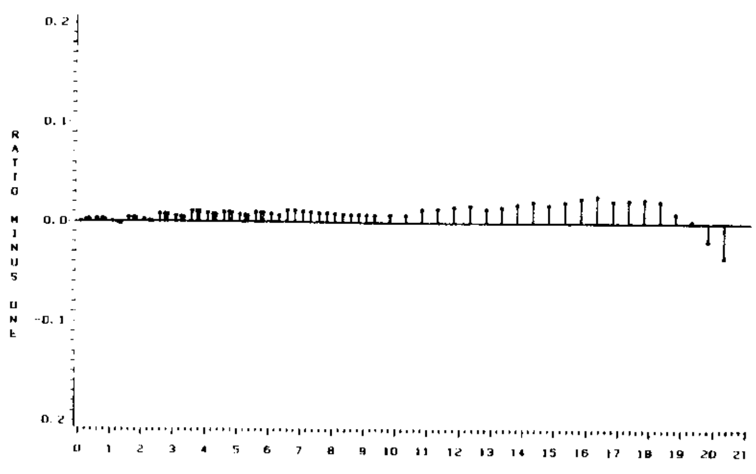

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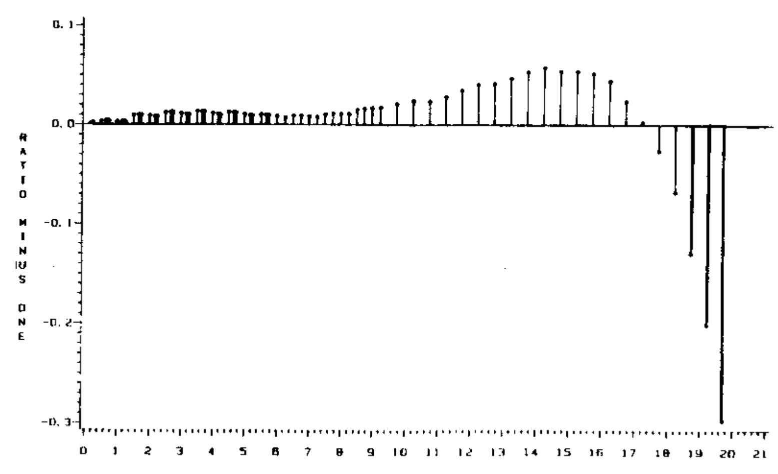

GRAPH I

TR Price Ratios (using the Par and Premium Discount Yield Curve)

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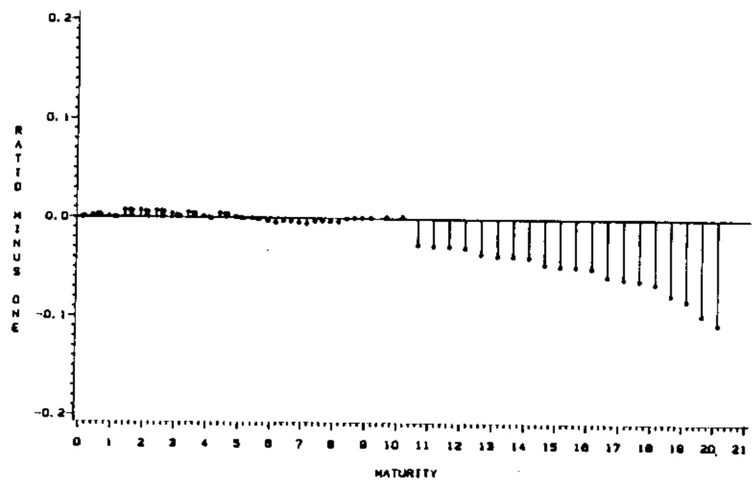

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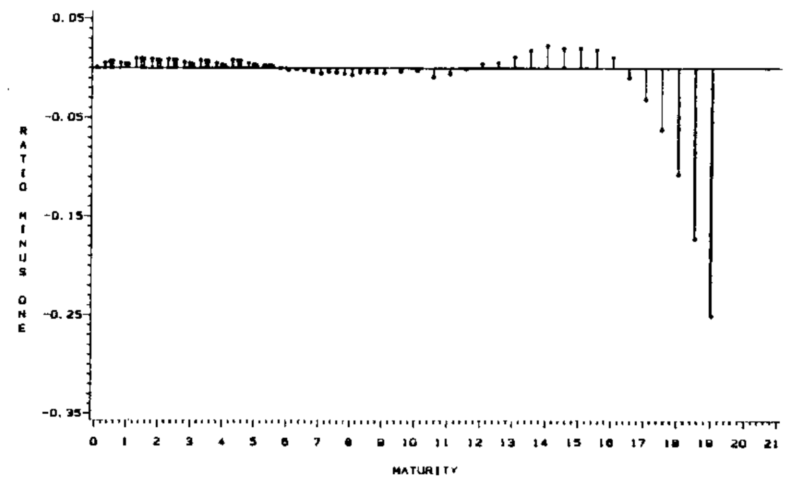

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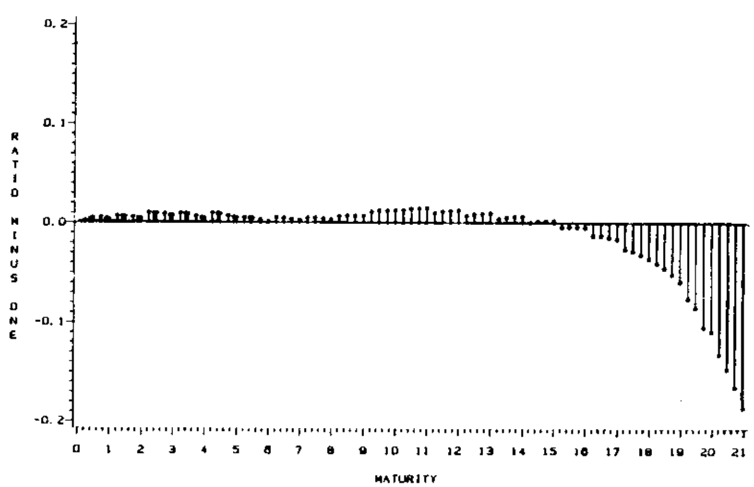

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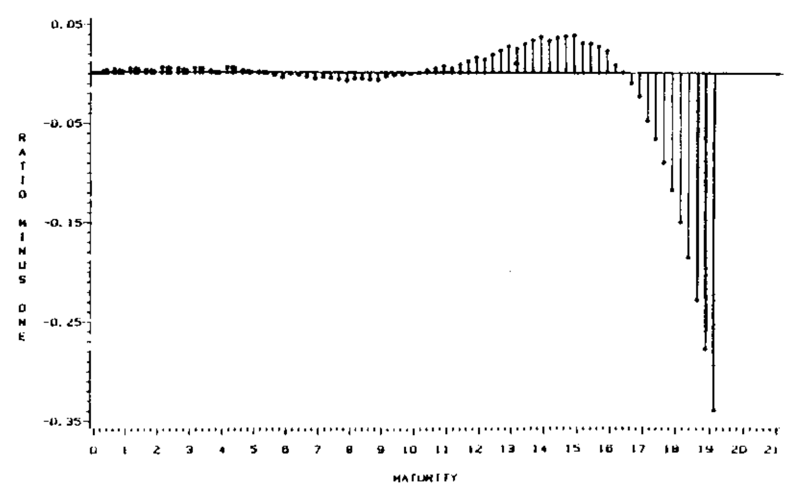

GRAPH I

TR Price Ratios (using the Par and Premium Discount Yield Curve)

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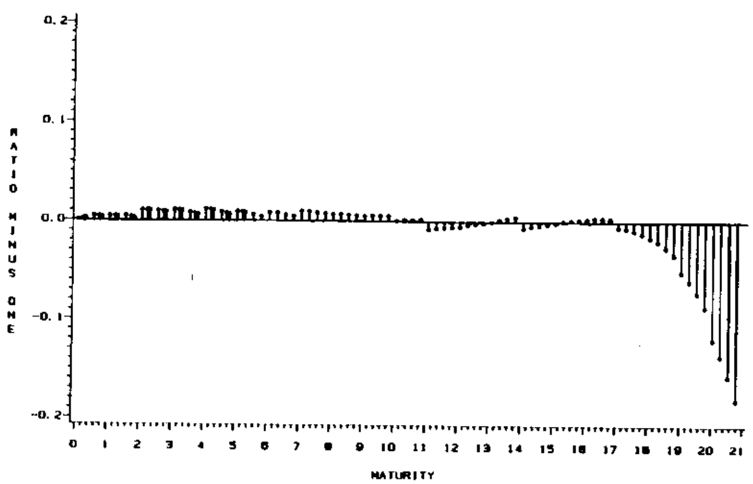

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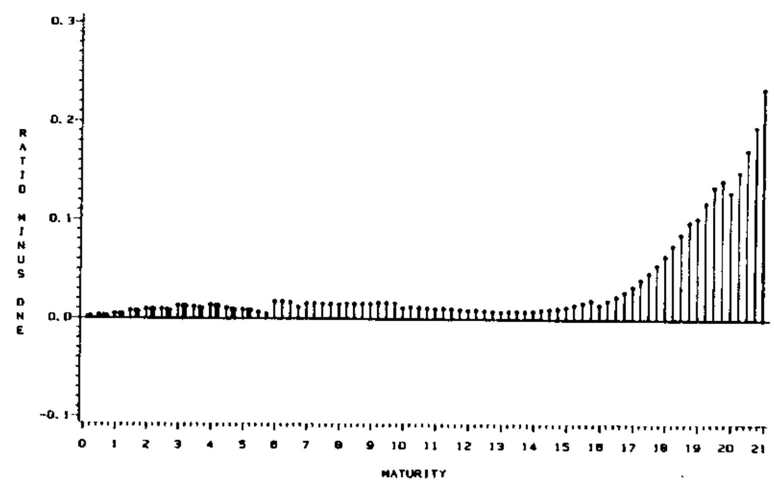

substitutes.[7] For longer maturities, observed pure discount bond prices are frequently above their estimated reservation prices.

However, there are some months when the observed prices are uniformly below the estimated reservation prices for all pure discount bonds (2/84) or for all pure discount bonds except at the highest maturity range (9/82, 7/83, 8/83). Note that, for these months, no Treasury security with maturity of five years and above could have been profitably stripped (see Table II). In contrast, for the 7 months from 10/82 to 4/83 where the potential profitability from stripping was highest, estimated reservation prices exceeded the actual pure discount bond prices for maturities of 2 to 5 years after which the actual pure discount bond prices exceeded the reservation prices.

Recall that there is a strong and consistent positive association between the value-added through stripping and the premium on coupon bonds (see Table II). This raises the possibility that the intertemporal shifts in the relationship between actual and reservation prices for pure discount bonds (see Graph I) is associated with changes in premium and discount resulting from movements in the slope of the term structure.

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[7] In November 1984, the U.S. Treasury indicated its intention to extend the book-entry registration (with an allocation of individual CUSIP's) to the pure discount bonds resulting from the stripping of selected Treasury securities. There is no difference between those pure discount bonds and originally auctioned Treasury securities. The stripping of the selected Treasury securities will not require the opening of depository trusts.

These graphs of the price ratios provide some insight into the relationship between maturity (or duration) and value-added which was earlier noted to be time-dependent. The higher value-added for securities of longer maturities observed for months 10/82 to 4/83 is clearly due to actual prices of pure discount bonds of longer maturities exceeding the corresponding estimated reservation prices of investors in par-and-premium bonds. The intertemporal shift pattern of price ratios seen in Graph I suggests the possibility that the volatility of observed prices of pure discount bonds may be different from the volatility of estimated reservation prices based on par and premium bonds. This question is analyzed in Section 5.

5. Volatility of Pure Discount Bond Prices and Marginal Rates of Substitution of Representative Investors in Coupon Bonds

In a non-segmented bond market, the marginal holders of pure discount bonds would be the same as the marginal holders of coupon bonds. Thus, in market equilibrium in a non-segmented capital market, the prices of pure discount bonds would be equal to the corresponding marginal rates of substitution between present consumption and future wealth of the marginal holders of coupon bonds. Under differential taxation and restricted short sales, Schaefer (1982a) has suggested that the bond market would be segmented into tax clienteles. However, the holdings of both pure discount bonds and par-and-premium coupon bonds are heavily concentrated among tax-exempt and low-tax bracket individuals suggesting that tax considerations would not explain a segmentation between investors in pure discount bonds and investors in par-and-premium bonds. While there is not a sufficient number of par-and-premium bonds to span even the pre-tax payoffs of pure discount bonds, the previous section developed econometric estimates of the reservation prices or marginal rates of substitution between present consumption and investment in pure discount bonds that are implicit in the prices of par-and-premium coupon bonds. These estimates of the marginal rates of substitution of the representative investor in par-and-premium coupon bonds were shown to differ from the observed prices of pure discount bonds. This evidence is consistent with a segmentation between investors in pure discount securities and investors in coupon securities that is attributable to differences in preferences rather than differences in marginal tax rates.

In absence of segmentation, the volatility of a pure discount bond prices would be equal to the volatility of the corresponding marginal rates of substitution of the representative investor in par-and-premium bonds. Differences in these volatilities are <u>prima-facie</u> evidence of segmentation between these markets. Only in the presence of segmentation between these markets can these volatilities differ; however, the direction of such differences cannot be predicted <u>a priori</u>.

The popular financial press has characterized the prices of pure discount bonds as more volatile than the prices of coupon bonds often making comparisons between pure discount and coupon securities with the same maturity. For example, the following is an excerpt from an article of the 6/1/84 Wall Street Journal entitled "Zero-Coupon Bonds' Price Swings Jolt Investors Looking for Security":

> Many investors don't realize that prices of zeros, as they are called, fluctuate much more than Treasury bond prices. According to figures Salomon Brothers gave its customers, the price of 30-year zeros fell

25% in the year ended March 31; returns on conventional 30-year government bonds fell only 3% in the same period.

Such comparisons are misleading because a coupon bond should be viewed as a portfolio of pure discount bonds whose weighted average maturity (duration) is considerably shorter than its maturity, while the maturity and duration of a pure discount bond are identical.

The volatility of the marginal rate of substitution between present consumption and wealth t periods hence for the representative investor in par-and-premium bonds is an economically meaningful benchmark against which the volatility of a t-period pure discount bond price may be compared. Unfortunately, the volatility of the representative investor's marginal rate of substitution cannot be directly measured because the underlying true marginal rates of substitution are not observable. The previous section discussed the econometric estimation of the representative investor's marginal rates of substitution based on assumptions concerning the smoothness of the implied forward rates and the specification of the functional relationship between marginal rates of substitution and maturity as a cubic spline. Assuming that the econometric estimates are unbiased, the representative investor's marginal rates of substitution at a given point in time is equal to the econometric estimate plus an unobservable residual having a zero expectation. That is

$$Y_{ti} = \hat{Y}_{ti} + e_{ti}$$

where: Y_{ti} is the representative investor's marginal rate of substitution between current consumption in the i-th period and wealth t periods hence,

$\hat{Y}_{ti}$ is an unbiased econometric estimate of Y_{ti}, and

e_{ti} is the unobservable residual term which is equal to Y_{ti} (which is unobservable) minus $\hat{Y}_{ti}$.

The volatility of a t-period pure discount bond is measured as the time-series variance of its price changes. Letting P_{ti} denote the price of a t-period pure discount bond at period i, the volatility of that bond over the sample interval i=1 to I is measured as

$$\mathrm{var}(\Delta P_{ti}) = \sum_{i=1}^{I} \frac{(P_{t-1,i+1} - P_{ti})^2}{I} - \left(\sum_{i=1}^{I} (P_{t-1,i+1} - P_{ti}) / I\right)^2 \tag{5}$$

Note that the times-series variance of the changes of the econometric estimates of the representative investor's marginal rate of substitution between current consumption and future wealth t periods hence is a downward biased estimate of the volatility of the true underlying marginal rate of substitution. If the residual terms (e_{ti} for i=1 to I) were observable, the volatility of the representative investor's marginal rate of substitution between present consumption and future wealth t periods hence would be estimated as

$$\mathrm{var}(\Delta Y_{ti}) = \mathrm{var}(\Delta \hat{Y}_{ti}) + \mathrm{var}(\Delta e_{ti}) \tag{6}$$

$$\mathrm{var}(\Delta \hat{Y}_{ti}) = \sum_{i=1}^{I} \frac{(\hat{Y}_{t-1,i+1} - \hat{Y}_{ti})^2}{I} - \left(\sum_{i=1}^{I} (\hat{Y}_{t-1,i+1} - \hat{Y}_{ti}) / I\right)^2$$

$$\mathrm{var}(\Delta e_{ti}) = \sum_{i=1}^{I} \frac{(e_{t-1,i+1} - e_{ti})^2}{I} - \left(\sum_{i=1}^{I} (e_{t-1,i+1} - e_{ti}) / I\right)^2$$

Since the econometric estimates $\hat{Y}_{ti}$'s are observable, the first component of the volatility of the representative investor's marginal rate of substitution $var(\Delta\hat{Y}_{ti})$ may be calculated directly. The expected value of the second component may be expressed in terms of the standard error of the estimates from the cross-sectional estimations. Under the assumption that residuals are serially uncorrelated and that estimates are unbiased:

$$E[var(\Delta e_{ti})] = \sum_{i=1}^{I} (\sigma^2_{t-1,i+1} + \sigma^2_{ti})/I \tag{7}$$

where σ^2_{ti} is the standard error of the estimate of the price of the t period discount bond at month i.

In Table VI below, the volatilities of the actual prices of pure discount bonds are compared to their estimated volatilities based on the estimates of the marginal rates of substitution of representative investors in par and premium bonds. The comparisons are made for 1, 5, 10, 15, and 17-year maturities.

TABLE VI

Volatility of Pure Discount Bond Prices as a Percentage of Estimated Volatility Based on Par-and-Premium Bonds

Maturity (Years)	$var(\Delta P_{ti})/var(\Delta Y_{ti})$ (%) (Adjusted) [a]	$var(\Delta P_{ti})/var(\Delta \hat{Y}_{ti})$ (%) (Unadjusted) [a]
1	130	138
5	276	314
10	220	251
15	96	278
17	104	159

a For estimation error.

The volatilities of the actual discount bond prices are consistently higher than the estimated volatility based on par-and-premium bonds except for the 15-year maturity where it is 4% lower. For the other maturities, the percentage difference varies between 4% for the 17-year maturity discount bond to 176% for the 5-year maturity discount bond. The percentage differences are substantially greater when the variance of the residual terms is ignored. While some of these differences appear quite large, a test of whether they are statistically significant is complicated by the fact that the changes in the actual discount bond prices and the changes in the econometric estimates of the representative investor marginal rates of substitution are not independent. Using odd months to calculate the volatility of the actual disçount bond prices and even months to estimate the volatility based on par-and-premium bonds, only the volatilities for the 1- and 5-year maturities are significantly high at the 5% level. Using even months to calculate the volatility of actual discount bonds and odd months to estimate the volatility based on par-and-premium bonds, the volatilities of the actuals are never significantly higher than their estimated volatilities at the 5% level. In fact, using the even months, the variances of the actuals are below the estimated for the 1-, 5-, 10-, and 17-year maturities, although never significantly.

TABLE VII

Test of Difference Between the Volatility of Pure Discount Bond Prices and their Volatility Based on Par-and-Premium Bonds using an F-Test on Variances of Price Changes

Maturity (Years)	Odd Observations for Actual Market Prices	Even Observations for Actual Market Prices
1	6.35* (A) [a]	2.40 (E)
5	9.00* (A)	3.04 (E)
10	3.02 (A)	1.13 (E)
15	1.15 (A)	1.09 (A)
17	1.48 (A)	1.17 (E)

a An "*" indicates statistical significance at the 5% level. An "(A)" indicates that the larger variance corresponds to actual market prices; it is "(E)" for estimated prices.

These significance tests indicate that, given the high volatility of bond prices in both these segment markets, there does not appear to be a statistical basis for concluding that pure discount bond prices are overly volatile relative to par-and-premium bonds. Furthermore, it appears that the statistically significant differences that do occur are attributable to the high change in the initial month of the sample. If this month were not included in these tests there would have been no statistically significant differences in the actual and estimated volatilities.

The volatility of a coupon bond which is selling at a premium in the secondary market can be directly compared with the volatility of a portfolio of pure discount bonds that provides the same time-contingent pattern of the pre-tax cash-flows. If the prices of the coupon bonds and of the pure discount bonds were observed contemporaneously without measurement error, and if the market for coupon bonds selling at a premium and the market for pure discount bonds were non-segmented, then the monthly rates of price change for the coupon bond and the spanning portfolio of pure discount bonds would be identical. Thus, these rates of price changes would be perfectly correlated and have the same variances. However, if these markets were segmented, like the previous evidence suggests, the marginal investors holding coupon bonds selling at a premium would differ from the marginal investors holding pure discount bonds and restrictions on shortselling could result in a correlation less than unity and different variances.

The variances of the rates of price changes and the correlations between the rates of price changes are shown in Table VIII for all non-callable coupon bonds with maturities of five years or more that sold at a premium over the entire sample period and for the corresponding spanning portfolios of pure discount bonds. With the exception of a single bond issue, the correlations are 0.93 and above. The variance of the rate of price changes for the spanning portfolios of pure discount bonds are consistently above the variances for the corresponding coupon bonds. These differences are too large to be explained by observation errors in a non-segmented bond market. In a non-segmented market, if the measurement errors in price changes for a given coupon bond, e_{ct}, and the corresponding

TABLE VIII

Comparison of the Volatilities of Coupon Bonds Selling at a Premium and of Portfolios of Pure Discount Bonds with Identical Pre-Tax Cash-Flows

Treasury Security				Variance of Price Changes[b]			Estimated Variance of Measurement Errors[b]	
Coupon	Maturity	t-value[a]	Correlation of Price Changes	Spanning Portfolio	Coupon Bond	Common	Spanning Portfolio	Coupon Bond
14.375	4/15/89	1.07	0.82	536	414	315	215	99
14.50	7/15/89	1.27	0.96	1,508	893	1,070	438	-176
13.00	11/15/90	1.56	0.95	1,307	687	855	452	-168
14.50	5/15/91	1.79	0.96	1,381	715	916	465	-201
14.875	8/15/91	1.05	0.95	1,407	828	974	433	-146
14.25	11/15/91	1.78	0.96	1,424	684	910	514	-226
14.625	2/15/92	1.31	0.93	1,448	747	900	548	-153
13.75	5/15/92	2.09	0.96	1,453	673	911	542	-238
12.625	5/15/95	1.46	0.95	1,427	975	1,065	362	- 90
13.125	5/15/01	2.27*	0.96	1,662	732	1,017	645	-285
13.375	8/15/01	1.95	0.93	1,727	928	1,095	632	-167
15.75	11/15/01	2.18*	0.95	1,650	789	1,030	620	-241
14.25	2/15/02	2.01	0.93	1,742	945	1,110	632	-165

a t-test on the paired differences d_{ji}'s between absolute values of the deviations of price changes from mean price changes as defined by equation (8). An "*" denotes significane at 5% level.

b Variance x 10^6 is shown.

measurement errors for the spanning portfolio of pure discount bonds, e_{st}, were uncorrelated between themselves and also uncorrelated with the common price change observed without measurement error, r_t, then the covariance of the observed rates of price changes for the coupon bond and the corresponding spanning portfolio would be an unbiased estimator of the variance of the common rate of price change. That is: if

$E[e_{ct}]=E[e_{st}]=E[e_{ct}\cdot e_{st}]=0$ and $E[e_{ct}\cdot(r_t-E(r_t))]=E[e_{st}\cdot(r_t-E(r_t))]=0$, then

$$\begin{aligned}&E[r_t+e_{ct}-E(r_t+e_{ct})][r_t+e_{st}-E(r_t+e_{st})]\\&=E[r_t+e_{ct}-E(r_t)][r_t+e_{st}-E(r_t)]\\&=E[r_t-E(r_t)]^2\end{aligned}$$

An unbiased estimator of the variance of the measurement error for the rate of price change of the coupon bond would be the variance of the observed rate of price change minus the above discussed estimator of the variance of the common true rate of return. The same approach may be used to obtain an unbiased estimate of the variance of the measurement error for the spanning portfolio of pure discount bonds. However, the high correlations between the coupon-bond rates of price changes and those of the spanning portfolios of pure discount bonds combined with the substantially higher variances of the rates of price changes for the spanning portfolios of pure discount bonds results in the (non-segmented bond market) estimates of the variances of the measurement errors being negative (with a single exception). This strongly suggests that the differences in the variances of the observed rates of price changes are too large to be explained by measurement errors <u>in a non-segmented bond market</u> where the rates of price change were perfectly correlated and the underlying true variances were identical. An alternative explanation is that the covariation between the measurement errors was positive over the sample period, resulting in a small upward bias in the estimate of the variance of the true rate of price change and corresponding downward biased estimates for the variances of the measurement errors. This alternative explanation is unsatisfactory given the fact that these negative estimates of variances of the errors of observation occur for 10 out of 11 bond issues. Consistent with the previous evidence concerning the relationship between the actual prices of pure discount bonds and the corresponding estimates of the reservation prices of investors in par-and-premium coupon bonds, the above evidence supports the existence of a non-tax related segmentation of the market for par-and-premium bonds and the market for pure discount bonds.

The statistical significance of the differences in the volatilities will now be examined without making any assumptions concerning any segmentation or lack of segmentation. As price changes of coupon bonds and of portfolios of pure discount bonds are highly correlated, a test based on a comparison of sample variance cannot be performed. For the j-th coupon bond (j=1 to 13), a paired t-test is applied to the series (d_{ji}) of differences between the absolute values of the deviations of price changes from the mean price changes:

$$d_{ji} = | \hat{R}_{ji} - \hat{R}_j | - | R_{ji} - R_j | \qquad i = 2, \ldots, 18 \tag{8}$$

where R_{ji} is the price change on the i-th coupon bond from time i-1 to i,

$\hat{R}_{ji}$ is the price change on the portfolio of pure discount bonds providing the same pre-tax cash-flows as the j-th coupon bond, R_j and $\hat{R}_j$ are the time series means of R_{ji} and $\hat{R}_{ji}$, respectively.

Results of the t-test are presented in Table VIII. Only two of the eleven t-values are significant at the 5% level. Thus, the direct evidence concerning the relative volatility of coupon bonds and their spanning portfolios is consistent with that evidence comparing the volatility of actual pure discount bond price with the corresponding reservation price of investors in par and premium bonds. The hypothesis that the pure discount bonds are no more volatile than coupon bonds selling at a premium is accepted.

6. Summary and Conclusions

This study examines a major security innovation involving the tailoring of a new type of security to the preferences of investors in an incomplete capital market. The unbundling of Treasury securities into pure discount bonds made feasible patterns of time-contingent nominal claims not spanned by coupon bonds. In the absence of restrictions on short sales and/or transaction costs, the possibility of arbitrage by tax-exempt investors would insure that the market value of a package of stripped securities reproducing the pre-tax dated cash-flows of a coupon bond would equal the market price of that bond. In such a capital market, the existence of a given security, but not its supply, would be critical to the underlying competitive equilibrium since investors would in effect be able to issue any available security. The observed differences between the market values of these equivalent packages of pre-tax nominal claims is *prima facie* evidence of a segmented capital market resulting from differing preferences for time-contingent claims and restrictions on short sales and/or transaction costs. Furthermore, these differences between bundled and unbundled market values exist for par-and-premium bonds, which, like stripped securities, are held primarily by tax-exempt investors. Thus, the observed price differences and the implied market segmentation cannot be explained by the tax-related clienteles discussed by Schaefer (1982a).

This paper analyses the characteristics of non-callable coupon bonds that could have been profitably stripped; i.e., coupon bonds which are priced below the market values of the respective portfolios of stripped securities having identical pre-tax cash-flows. Coupon bonds with the highest value-added when stripped are generally those selling at a premium and with long maturities. Consistent with the unfavorable tax treatment of par-and-premium bonds which suggests that they would appeal primarily to tax-exempt investors, the econometric estimate of the tax rate implicit in the relative pricing of par-and-premium bonds is zero. Since stripped securities are also held primarily by tax-exempt investors, their prices are compared to estimates of the respective reservation prices for investors in par-and-premium bonds. This analysis indicates that very long-term dated nominal claims appear to be valued more highly when unbundled than when sold in a portfolio together with short and intermediate term-claims packaged in the form of a coupon bond. This is consistent with the observation that it is more profitable to strip long-term premium bonds.

While the popular financial press has characterized pure discount bonds as more volatile than coupon bonds, the comparisons are usually made between coupon bonds and discount bonds of the same maturity. Such comparisons are obviously misleading because a coupon bond is in effect a package of pure discount bonds whose value-weighted average maturity (duration) is considerably less than its maturity. The duration of even the longest non-callable bonds are less than ten years. Hence the volatilities of pure discount bonds with maturities over ten years are not directly comparable to the volatilities of coupon bonds. This study compares the volatilities of pure discount bonds with the volatilities of the corresponding reservation prices estimated from par-and-premium bonds. It also compares the volatilities of par-and-premium bonds with the volatilities of portfolios of pure discount bonds providing identical pre-tax cash-flows. The hypothesis that the volatilities are identical cannot be rejected.

APPENDIX I

Pricing of a Pure Discount Bond in a Tax Model

Consider a pure discount bond with M days to maturity, with $M=d_1+365n+d_2$, where d_1 is the number of days remaining in the current calendar year, n is the number of subsequent complete calendar years spanned by the bond, and d_2 is the number of days after which the bond matures in the last year. Let $t(j,i)$, $j=0$ to $n+1$, $i=1$ to 4, be the number of days between settlement date and the "i-th" tax date in year j, where $j=0$ for the initial year, and $n+1$ for the year of maturity. Tax dates are January 15, April 15, June 15, and September 15. The price and the yield to maturity of the pure discount bond are related in the following way:

(A1) $$100 = P\,(1 + y/2)^{2(M/365)}$$

for a par value of 100.

During the initial year (year "0"), the taxable imputed interest per dollar of investment, h(0), is equal to:

(A2) $$h(0) = [(1 + y/2)^{2d_1/365} - 1]$$

Let D(j,i) be the number of days elapsed since the purchase date and the last day of the calendar month preceding the tax date t(j,i). When the last day of the calendar month preceding the tax date t(j,i) precedes the purchase date, D(j,i) = 0. 90% of the tax liability of the initial year is prorated between the tax dates of the initial year and the January 15 tax date of year 1 as a function of the number of days covered by each of the tax date. The remaining 10% is paid on April 15 of year 1, t(1,2). The value per dollar of initial investment of the tax liability associated with the initial year is given by

(A3)
$$\tau_i\, h(0)\, [\, 0.9\, (\, \sum_{i=2}^{4} (D(0,i) - D(0,i-1))\, m(t(0,i)) + (D(1,1) - D(0,4))\, m(t(1,1))\,)\, /\, D(1,1) + 0.1\, m(t(1,2))\,]$$

For the following n complete calendar years, the taxable imputed interest per dollar of initial investment is:

(A4)
$$h(j) = [(1 + y/2)^2 - 1]\, (1 + y/2)^{[2(d_1/365)+2(j-1)]} \qquad 1 \leq j \leq n$$

Ninety percent of the tax liability is prorated on the tax dates of April 15, June 15, and September 15 of year j,t(j,i),i=2 to 4, and on the January tax date of year j+1, t(j+1,1) with the following ratios k(i):k(2)=1/4,k(3)=1/6,k(4)=1/4, and k(1)=1/3. These ratios are proportional to the number of days within the period covered by a tax date. The remaining 10% are paid on the April tax date of year j+1, t(j+1,2). As a result, on the April tax date of every year, a tax payment combining 10% of the tax liability of the preceding year to 22.5% of the tax liability of the current year will be made. Payments on the January 15 tax date correspond to 30% of the tax liability of the previous year. Payments on the June 15 and September 15 tax dates correspond to 15% and 22.5%, respectively, of the tax liabilities of the current year.

The taxable interest imputation factor corresponding to the maturity year is defined as:

(A5)

$$h(n+1) = [(1 + y/2)^{2d_2/365} - 1] \, (1 + y/2)^{2d_1/365+2n}$$

and the tax payment dates are estimated as in the case of a complete calendar year.

The value of the pure discount bond, is equal to the present value of the principal, 100m(M), minus the present value of the tax payments. Dividing by P and replacing 1/P by its expression as a function of y, one obtains:

(A6)

$$\begin{aligned}
&[1 - m(M)\,(1 + y/2)^{2((d_1+d_2)/365+n)}] \,/\, \tau_i \\
&+h(0)\ [\ 0.9\ (\ \sum_{i=2}^{4} (D(0,i) - D(0,i-1))\ m(t(0,i)) + (D(1,1) - D(0,4))\ m(t(1,1))\ )\ /\ D(1,1) \\
&+ 0.1\ m(t(1,2))\ ] \\
&+ \sum_{j=1}^{n+1} h(j)\ [\ 0.9\ (\ \sum_{i=2}^{4} k(i)\ m(t(j,i)) + k(1)\ m(t(j+1,1))\ ) + 0.1\ m(t(j+1,2))\ ] = 0
\end{aligned}$$

As h(.) are functions of y, the yield to maturity, the solution for y needs to be determined numerically.

APPENDIX II

A Brief History of the Market for Pure Discount Bonds

Investors can now purchase receipts giving them the promise of single coupon or corpus payments associated with the unbundling of a Treasury security. Before TEFRA (the Tax Equalization and Fiscal Responsibility Act of 1982), there had been cases of stripping done directly on a Treasury bond in physical form. This practice was frowned upon by Federal agencies and is presently made impossible by the dispositions on Treasury securities, in particular with the book-entry system of Treasury securities inaugurated in July 1983.

The Federal Reserve System Banks did not approve of the stripping of Treasury securities. The SEC viewed the possession of physical coupons as providing a tenuous proof of ownership. Coupons could be lost or mutilated, while ownership without the certificate of principal would be difficult to establish, which would jeopardize the safety of the procedure followed by the Federal Reserve System for the replacement of mutilated or lost coupons (i.e. accepting the bond certificate as proof of ownership). Small investors could be defrauded, and coupons forged. Finally, the Treasury and the IRS felt that stripping unduly reduced investors' tax liabilities. The latter hurdle, the most important, disappeared with TEFRA which clarified the tax treatment of discount instruments. To assuage the other problems, investment banks decided to issue certificates against parts of Treasury securities held in a trust.

Prior to TEFRA, bond stripping could be viewed as a tax-avoidance scheme. The holder of an interest-bearing Treasury security would, for instance, sell a coupon. He would reduce the cost basis of the security by the amount he would receive for the coupon, treating it as a return on principal. The tax liability he would incur would be a capital gains tax (presumably long-term) on the sale price of the coupon, deferred to the maturity of the Treasury security. The purchaser would treat the coupon as a discount instrument, and would recognize the difference between the payment on the coupon and purchase price as income at the time of the coupon payment. TEFRA has clarified the tax question. On a pre-tax basis, pure discount bonds are equivalent to long Treasury Bills because they give rise to only one cash flow payment. However, for a taxable investor, the price appreciation between purchase price and face value is viewed as income, not capital gain.[8] Pure discount bonds created by bond stripping are comparable to zero coupon bonds in that the holder is subject to tax on the accrued interest during the current fiscal year in spite of the fact that no cash flow occurs before the maturity of the bond (if held to maturity). As a result, on an after-tax basis, a pure discount bond can be viewed as a bond with negative after-tax semi-annual cash-flow payments up to maturity.

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[8] That was not so originally for Japanese investors for which it was viewed as capital gain, and therefore not taxed for individuals. Measures taken by Japanese authorities have effectively stopped the sales of pure discount bonds in the Japanese market (pure discount bonds as discussed in this paper are not considered to be securities).

Prior to the Tax Equalization and Fiscal Responsibility Act of 1982, the taxation of the price appreciation of an "original discount bond" assumed a linear amortization. Since the Tax Equalization Act, the accretion is based on a constant-yield, i.e., it is as if the bond accrues on a constant-yield basis (and that the accrued interest is automatically reinvested every six months at the same yield).[9]

From the point of view of the investment bank, the difference between the proceeds of the sales of the pure discount bonds and the purchase price of the corresponding Treasury securities is treated as income and taxed correspondingly. Once Treasury securities are in the trust, there is no additional liability for the investment bank.

An investment bank purchases a substantial amount of Treasury long bonds in a Treasury auction or in the secondary market, then sells to its clients, primarily tax-exempt institutions, receipts giving right to a specific cash flow. The underlying Treasury securities are kept in an irrevocable trust, which survives the institution creating it.

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[9] There is a difference between the tax treatment of pure discount bonds and "original issue discount" bonds (including zero coupon bonds): for the former, the amortization is based on the yield at which the holder has bought the instrument, while for the latter, it is the yield at issuance, independently of the yield at which the instrument may have been bought in the secondary market. This is based on an interpretation of TEFRA. The Treasury has, however, recently requested that the original discount (i.e., at the time of the stripping) be printed on the certificate, which some view as a possible change in the Treasury position.

There are some differences between the various trade-name or generic receipts, in particular, the size of the smallest denominations (for instance coupons are multiples of TIGRS' denominations), or the existence of clipped coupons (i.e. coupons separated from the corpus) in the trust (e.g. for CATS). In addition, prior to 12/31/1982, pure discount bonds (and Treasury securities) were issued in bearer form, while they are issued in registered form since 1/1/1983. This may create a price differential between pure discount bonds of the same maturity (and with the same brand name). Some price differential may also be induced by a special demand for a specific issue based on investors' preference to continue a series or due to covenants limiting the purchase to specific series of pure discount bonds.

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