OPTIMAL PORTFOLIO CHOICE AND THE COLLAPSE OF A FIXED-EXCHANGE RATE REGIME

by

Alessandro Penati & George Pennacchi

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RODNEY L. WHITE CENTER FOR FINANCIAL RESEARCH
The Wharton School
University of Pennsylvania
Philadelphia, PA 19104

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Department of Finance
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I. Introduction

Speculative attacks on Central Banks' foreign official reserves that lead to collapses of fixed exchange rate regimes have been the focus of important research work in international macroeconomics. Although a casual examination of many recent experiences would support the view that exchange crises seem to take most people by surprise, it has been recognized since the original contribution of Krugman (1979), that these crises need not be the result of unpredictable speculative behavior. Rather, they can be the result of the optimizing behavior of rational investors who readjust their portfolios in anticipation of a break down of a fixed exchange rate parity, once they realize that the domestic credit policy of a country is inconsistent with the exchange rate level maintained by the Central Bank.[1]

In the existing models, if economic agents can rebalance their portfolios continuously, there will be a run on the Central Bank when the stock of foreign exchange reserves is exactly equal to the desired decline in money holdings that is caused by the move from the fixed to the floating exchange rate regime. Because the excess supply of money that is created by the collapse of the fixed exchange rate regime is eliminated by the run on the official reserves, the exchange rate need not jump when the collapse occurs, in order to clear the money market. Therefore, with continuous trading in financial assets and no restrictions on capital mobility there is a smooth transition from the fixed to the floating regime.[2] The smooth transition also implies that the Central Bank loses foreign reserves at the same rate it creates domestic credit before the collapse and that domestic interest rates do not diverge from foreign rates until the beginning of the floating.[3]

Although these models capture important elements of the dynamics of a fixed exchange rate collapse, they do not seem to account for the stylized features of exchange rate crises.[4] As we document in the paper, a sudden jump in the value of foreign exchange typically follows the run on Central Bank reserves which causes the exchange rate regime to collapse. In addition, runs on reserves never occur abruptly as they are generally preceded by a period during which official foreign exchange reserves are lost at increasing rates. Finally, the forward premium in foreign exchange markets tends to increase well before the regime collapse, a phenomenon that is known as the 'Peso problem' (Krasker 1980).[5]

In this paper, we extend the existing literature in several important directions by focusing on portfolio choices during an exchange rate crisis within the framework of a small open economy where agents' optimal consumption and portfolio decisions are obtained from a monetary intertemporal capital asset pricing model, along the lines of the work by Merton (1971, 1973), Jones (1982), and Stulz (1984). We maintain the assumption of continuous trading in financial assets and zero restrictions on capital mobility--an arguably reasonable approximation for many contemporary financial markets--but we assume that the Central Bank can surprise the private sector about the course of its monetary policy. We show, first, that the exchange rate may jump when the fixed exchange rate regime collapses, even though investors are allowed to rebalance their portfolios continuously and costlessly. Second, we show that there will always be a 'Peso problem' before the collapse, whether or not the exchange rate jumps. And, third, we show that a run on a Central Bank will always be preceded by a period of reserve losses in excess of domestic credit creation because the private sector accumulates foreign assets in order to hedge against the risk of an exchange rate crisis. We also analyze the macroeconomic dynamics of a country choosing to return to a new fixed rate regime following an exchange collapse and a short temporary period of floating. In particular, we show how our model can account for the sizeable short-term capital outflows and inflows that are typically observed before and after parity realignments inside the European Monetary System.

The plan of the paper is the following. In Section 2 we examine four recent exchange rate collapses to illustrate the typical features of a crisis, for which models must ultimately account. In the following sections the model is developed. We initially consider the case in which the collapse permanently ends the fixed exchange rate regime. As in any dynamic programming problem, first we derive consumers' optimal saving plan, their allocation of wealth and the equilibrium path of the exchange rate when the rate is free to float after the collapse (Section 3). In Section 4, we move backwards in time and look at consumption and portfolio decisions during the fixed exchange rate period and in particular at the dynamics of reserves, interest rates and the exchange rate in the transition from fixed to floating exchange rate regime. In Section 5 we consider a regime collapse that is followed by a return to fixed parity and show how the model can be used to gain insight on the functioning of the European Monetary System. Section 6 contains some concluding remarks.

II. Four Exchange Rate Crises

In this section we examine the empirical evidence from four different exchange rate crises, precisely those affecting the currencies of France, Italy, Mexico and Chile. The limited purpose of the section is to point out the common characteristics of these crises, which occurred both in different countries and at different points in time. We will then show that our model can account for these characteristics.

II.1 The collapse of the French franc, March 1984.

In 1981, France was in the middle of a recession like most industrialized countries: output did not grow, investment at constant prices fell by over 1 percent, employment in the industrial sector declined by nearly 3 percent and the current account deficit was becoming larger partly as a result of oil price increases (Table 1). With concern for the approaching presidential elections, the Government tried to revive the economy by adopting expansionary demand policies. In June 1981, a new socialist Government, which had set employment as a top priority in the campaign program, was elected.

The new Government sought to increase output by increasing aggregate demand even further and, accordingly, stepped up the expansionary demand policies of the previous government, especially through budgetary transfers to households and firms. In order to prevent these policies from aggravating the external accounts, the French Franc was devalued twice vis-á-vis the Deutsche Mark--Germany is France's main trade partner. On October 5, 1981, the Franc was devalued by 8.5% and again by 10% on June 14, 1982. This second devaluation was accompanied by a series of policy measures that purportedly tried to stabilize the Franc-Mark rate: price controls, a system of preannounced wage norms in line with the official inflation target, various incentives to firms engaged in exports or import competition, and a shift to more restrictive monetary policy.

Although the exchange rate was fixed to the Mark for the rest of the year, the policy trend remained very expansionary and, in the last quarter of 1982, the Central Bank induced a sharp expansion in domestic credit which grew at an annual rate of 155%. It was apparent that domestic monetary policy was inconsistent with the level of the exchange rate pegged by the Government. As a result, the spread between the forward and the spot rate began to widen as

Table 1

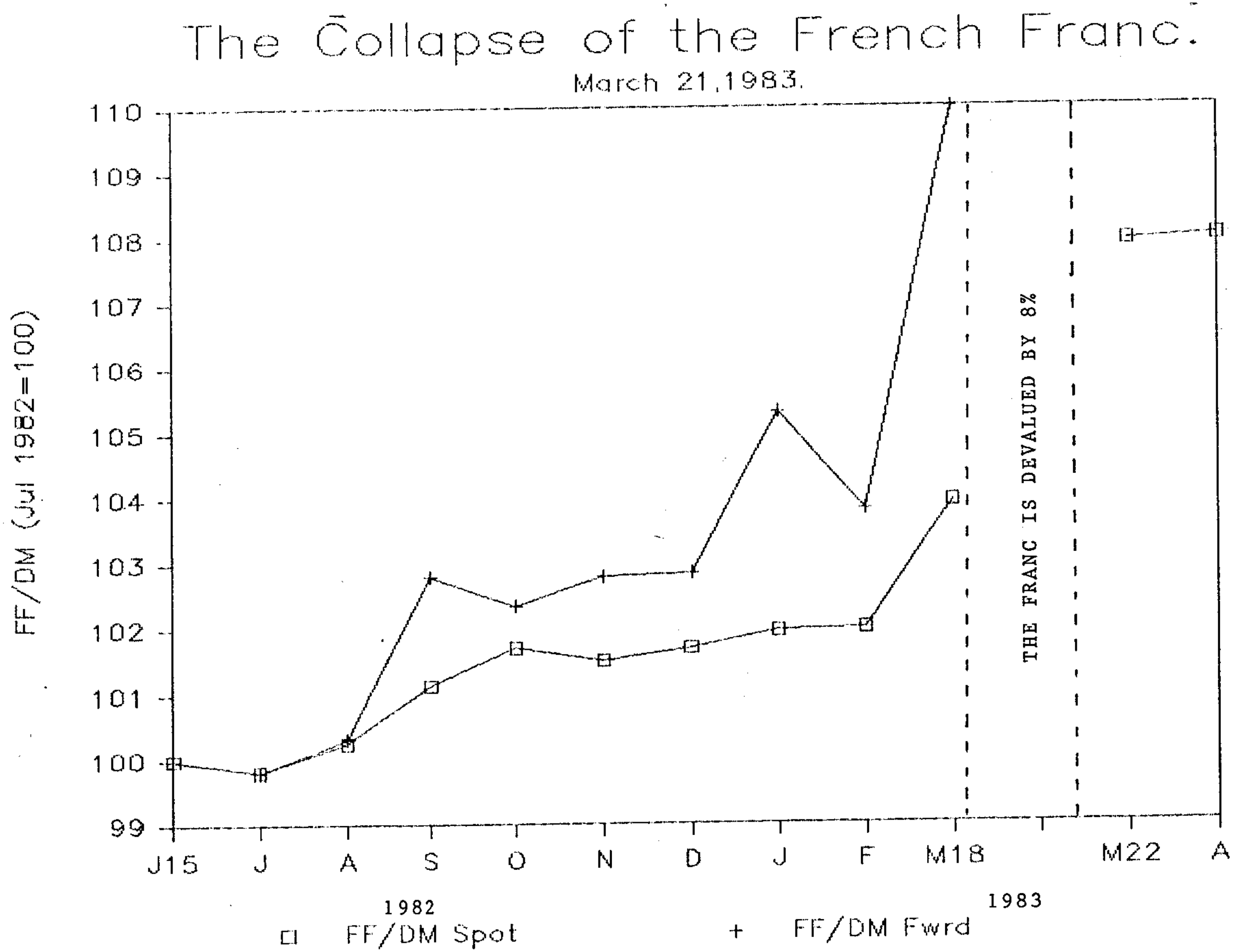
France: Economic Indicators: 1981-1983

	1981				1982				1983
	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1
1) Speculative Capital Flows[1] (billions of FF)	+1.1	-17.7	-7.9	-.03	+4.8	-1.5	+11.3	-7.7	-11.0
2) GDP Growth Rate (1980 prices; percentage change with respect to the same quarter of the previous year)	-1.6	0.29	0.68	1.76	2.49	2.22	1.21	1.30	1.27
3) Industrial Employment (percentage change with respect to the same quarter of the previous year)	-1.8	-2.5	-2.8	-2.8	-2.4	-1.8	-0.8	-1.0	-1.2
5) Central Bank Domestic Credit (percentage change with respect to the previous quarter at annual rate)	-98	+190	+50	-45	+13	+159	-11	+155	-23

Sources: IMF, International Financial Statistics (various issues) and Balance of Payments Statistics Yearbook (1982 and 1983).

[1]Errors and Omissions plus short-term private capital flows.

FIGURE 1


The Collapse of the French Franc.
March 21,1983.
FF/DM (Jul 1982=100)
110
109
108
107
106
105
104
103
102
101
100
99
J15
J
A
S
O
N
D
J
F
M18
M22
A
1982
1983
THE FRANC IS DEVALUED BY 8%
FF/DM Spot
FF/DM Fwrd

early as August 1982, reflecting lack of confidence in the sustainability of the fixed-rate regime, that is, a 'Peso problem' emerged (Figure 1).

To maintain the exchange rate, the Bank of France had to spend an increasing amount of foreign exchange reserves. Data on intervention are not released to the public and the reserve changes recorded in the balance of payments statistics omit the foreign exchange borrowed by various public entities such as state enterprises, which nonetheless was used for intervening in foreign exchange markets. As shown in Sachs and Wyplosz (1986), however, one can estimate the pressure on official reserves by looking at what they term speculative capital flows (errors and omissions plus private short-term capital). From these figures, it appears that the Central Bank started to incur heavy reserve losses in the fall of 1982 (Table 1).

The loss of reserves continued at an accelerating pace until the French authorities were forced to abandon the fixed parity on March 21, 1983, when the Franc was devalued by 8% with respect to the Deutsche Mark. The spread between forward and spot rate peaked on the last trading day before the devaluation (Figure 1). After the collapse of the Franc, the country went back to a fixed rate regime at a level of approximately 3 Francs to the Mark. This time, however, the Government adopted an austerity plan at the time of the devaluation, including a monetary policy consistent with the fixed rate. As a result, the level of the Franc/Mark rate was successfully stabilized during the subsequent three years.

II.2 The collapse of the Italian lira, January 1976.

The Italian Lira was very stable against the major currencies throughout 1974 and 1975. With respect to the Deutsche Mark, the Lira remained practically constant near the 262 level from March 1975 to January 20, 1976, the day before the crisis; with respect to the U.S. dollar, it had remained near the 680 level since August 1975. On January 21, however, the Italian authorities, with only 500 million U.S. dollars left of foreign exchange reserves and faced with the impossibility of arranging new credit lines in the international capital markets, were forced to close the official foreign exchange market. When it reopened, on the first day of March, the Lira had depreciated by over 16 percent with respect to the Deutsche Mark and by nearly 13 percent with respect to the U.S. dollar. From that date until the establishment of the European Monetary System in 1979, Italy, in practice, adopted a freely floating exchange regime.

As in the case of the French Franc, the collapse of the Lira was the result of excessive countercyclical policies that were supposed to speed up the recovery of the economy from a recession. Like most industrial countries, Italy was hit by the 1974 oil shock that caused both a decline in output and a deterioration of the current account. At the beginning the authorities responded by adopting strict demand management policies that brought the current account back in surplus in the second quarter of 1975 (Table 2). These policies, however, deepened the recession as output sharply fell in each of the first three quarters of 1975.

Because of the depth of the recession and the social and political consequences of it--at the time there were fears that the Communist party could gain the relative majority in the approaching elections--the Government decided to switch quickly to very expansionary policies. Public sector borrowing requirement, which averaged 8.8 percent of GDP in 1974, increased to over 13 percent in the first half of 1975 and to 19 percent in the last quarter of that year. During that quarter, the Central Bank chose to monetize government debt aggressively in order to prevent a rise in interest rates and

FIGURE 2

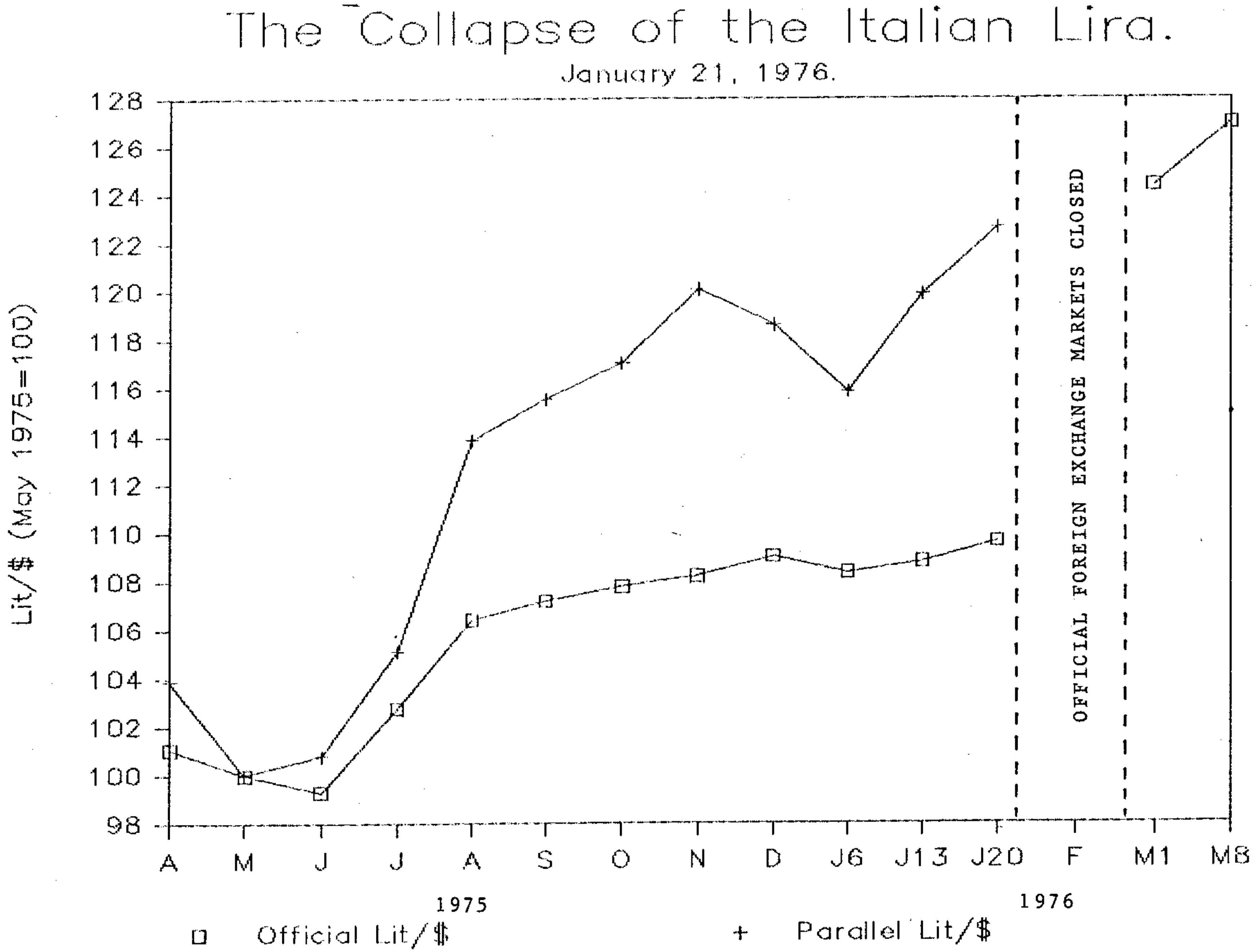

The Collapse of the Italian Lira.
January 21, 1976.
Lit/$ (May 1975=100)
128
126
124
122
120
118
116
114
112
110
108
106
104
102
100
98
A
M
J
J
A
S
O
N
D
J6
J13
J20
F
M1
M8
1975
1976
OFFICIAL FOREIGN EXCHANGE MARKETS CLOSED
Official Lit/$
Parallel Lit/$

Table 2

Italy: Economic Indicators: 1974-1976

	1974				1975				1976
	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q5
1) GDP Growth Rate (1970 prices; percentage change with respect to the same quarter of the previous year).	11.3	7.6	2.4	-2.8	-6.4	-5.8	-3.7	1.1	5.3
2) Public Sector Borrowing Requirement/GDP (percent)	8.9	8.4	9.3	8.6	13.6	13.4	14.0	19.1	9.5
3) Central Bank Credit to the Treasury (percentage change with respect to previous quarter at annual rate)	6.3	9.1	5.8	3.0	0.9	2.5	4.3	12.3	7.1
4) Current Account (billions of Lire)	-1,602	-1,582	-940	-1,087	-328	76	578	-703	-1,123

	1975												1976
	JAN	FEB	MAR	APR	MAY	JUNE	JULY	AUG	SEPT	OCT	NOV	DEC	1/1-1/20
5) Change in Official Net Foreign Assets[1] (billions of lire):	-160	-13	+122	+123	-20	-42	-108	+258	0	+44	-350	-538	-362[2]

Sources: Bank of Italy, Annual Report and Bulletin (various issues); International Monetary Fund, International Financial Statistics (various issues); and Central Institute of Statistics, Monthly Bulletin (various issues).

[1]Change in foreign exchange reserves gross of external official borrowing and of the change in commercial banks net foreign assets position. The reason for lumping together the Central Bank with commercial banks is that banks' foreign exchange operations were heavily regulated--and often manipulated--by the monetary authorities.

[2]Figure taken from a statement by the Governor of the Bank of Italy (Financial Times, 1/22/1976).

the growth rate of its credit to the Treasury jumped to 12 percent from 2 percent in the first half of the year.

The run on Central Bank reserves did not occur abruptly but was preceded by a period of heavy reserve losses that began in November, three months before the crisis. No indication of a 'Peso problem' can be obtained by looking at interest rates and the forward rate because these prices were relatively controlled by the authorities. Clear evidence of a 'Peso problem', however, is provided by the spread between the official lira/dollar rate and the rate prevailing in an unregulated market--the so-called parallel rate.

This market, far from being tolerated, was actually utilized by the monetary authorities to gain information about market expectations and the trading prices were reported in the daily press. It is reasonable to believe that this spread approximated the costs of capital controls and restrictions on the use of foreign exchange that are usually imposed at the time of an exchange rate crisis multiplied by the probability of a crisis. As shown in Figure 2, the spread began to increase in August 1975, five months before the fixed rate collapse, and reached its peak on the eve of it.

II.3 The Collapse of the Mexican peso, February 1982

The various collapses of the Mexican Peso are probably the best documented exchange rate crises. Here we focus on the 28 percent depreciation vis-á-vis the U.S. dollar on February 18, 1982, which ended a nearly five-year long fixed exchange rate regime, because it bears many similarities to the cases of Italy and France as it had its origin in a sharp expansion of domestic nominal demand.

At the beginning of the 1980s, Mexico was enjoying a sustained expansion, mainly because of a boom in its oil industry that accounts for approximately 70 percent of exports. Oil prices steadily increased throughout 1980 and then stabilized at their peak level in 1981. At the same time Mexico was able to boost oil production by over 20 percent (Table 3). Although oil prices began to weaken at the beginning of 1982, production was maintained so that the trade deficit had shrunk since the middle of 1981. The oil industry was not the only one to experience rapid growth as the boom also affected the manufacturing sector: in the first months of 1982, manufacturing production reached the highest level in Mexican history.

During those two years, the Government increased social welfare spending very sharply and, notwithstanding the increase in revenues, the budget deficit doubled as proportion of GDP in 1981 and again more than doubled in 1982, going from 3 percent to 15 percent. The Central Bank monetized the jump in government debt so that its credit to the Government grew at a staggering 132 percent in the first quarter of 1982, the quarter of the exchange rate regime collapse. The excess supply of money fled the country and the Central Bank lost reserves at an increasing rate during the year before the collapse, topping 140 billion Pesos in the last quarter of 1981. Not surprisingly there was a 'Peso problem' indicating that the collapse had been anticipated by at least three months (Figure 3).

II.4 The collapse of the Chilean peso, June 1982.

In the previous three cases, fiscal policy was the ultimate cause of a sudden acceleration in domestic credit. This, however, need not always be the case. In Chile, it was a banking crisis that triggered the Central Bank intervention which, in turn, led to a rapid increase in domestic credit before the fixed rate regime collapse.

From 1979 to 1982, the year of the collapse, Chile continuously experienced budgetary surpluses, even as high as 3 percent of GDP (Table 4). Monetary policy was also tight during most of this period as the country was

FIGURE 3

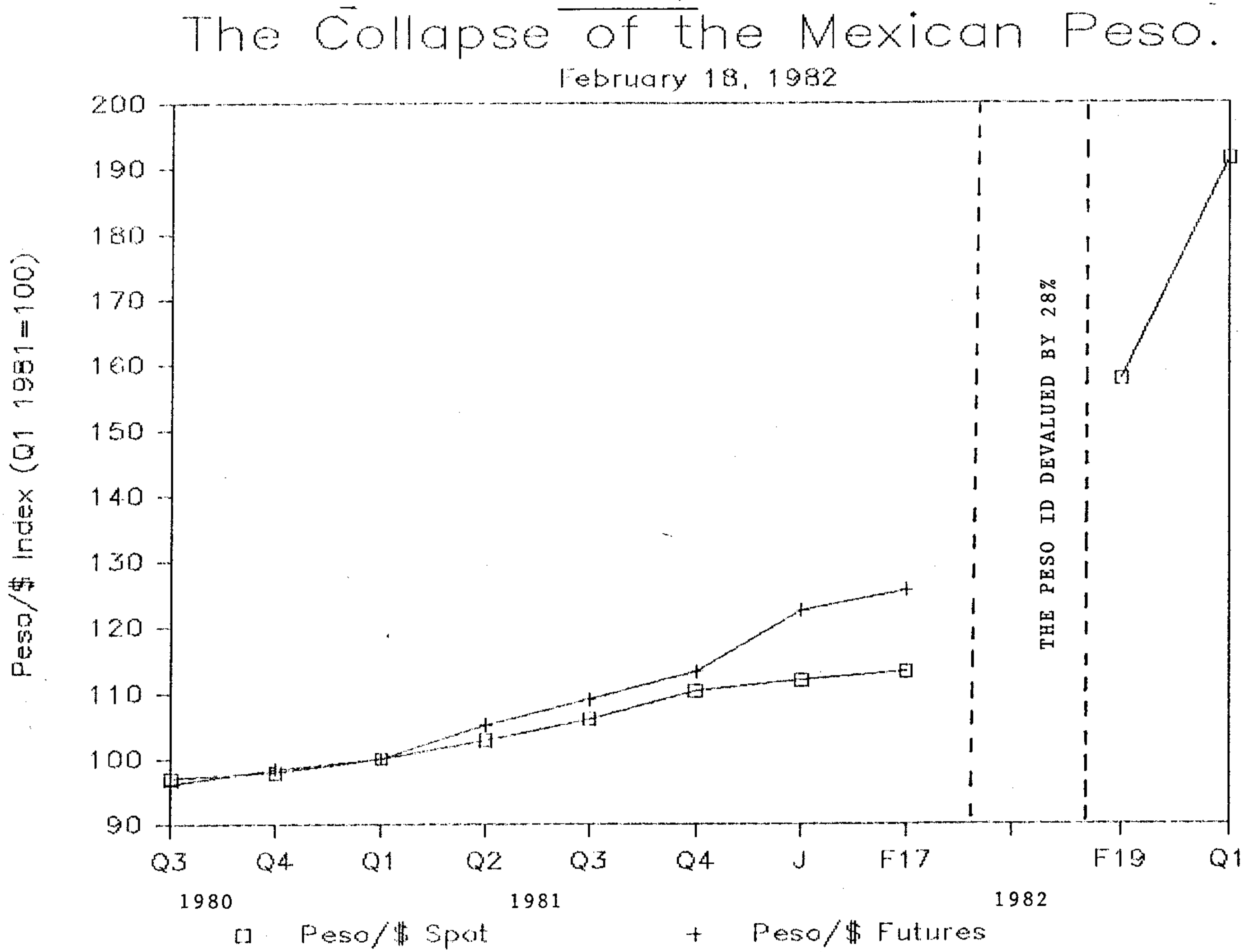

The Collapse of the Mexican Peso.
February 18, 1982
Peso/$ Index (Q1 1981=100)
200
190
180
170
160
150
140
130
120
110
100
90
THE PESO ID DEVALUED BY 28%
Q3
Q4
Q1
Q2
Q3
Q4
J
F17
F19
Q1
1980
1981
1982
Peso/$ Spot
+ Peso/$ Futures

Table 3

Mexico: Economic Indicators: 1980-1982

	1980				1981				1982
	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1
1) Oil Price (1975 = 100)	233	247	267	294	294	294	294	294	294
2) Oil Production (1980 = 100)	94	98	102	106	107	119	114	112	114
3) Manufacturing Production (1980 = 100)	97	99	99	104	104	107	110	108	110
4) Trade Balance (billions of Peso)	4.0	-13.2	-24.5	-29.3	-9.7	-21.5	-40.9	-29.4	-21.8
5) Change in Official Net Foreign Assets[1] (billions of Peso)	-27.8	-4.7	-26.1	-36.3	-26.5	-38.5	-98.9	-139.5	+10.3
6) Central Bank Credit to the Government (percentage change with respect to the previous quarter at annual rate)	17.2	-.5	27.8	76.0	27.3	53.6	43.9	40.8	132.5
	1979	1980	1981	1982					
7) Budget Deficit/GDP (percent)	3.3	3.1	6.7	15.4					

Sources: IMF, International Financial Statistics (various issues), Balance of Payments Statistics Yearbook (1982 and 1983), and Government Finance Statistics Yearbook (1983).

[1] Sum of the change in official reserves and public short- and long-term capital flows.

trying to steadily achieve single digit inflation, down from the 300 percent and above inflation rates of the first part of the 1970s. The central piece of the Government's anti-inflationary plan, however, was an exchange rate policy that, by fixing the external value of the Peso, was expected to "import" the inflation rate of the industrialized world. Accordingly, the Peso was pegged to the U.S. dollar at the 39 level.

At the end of 1981 and in the second quarter of 1982, several bank failures shook the stability of the financial system. There are no unanimous explanations for these failures. It is reasonable to believe, however, that they were related to the difficulties that the manufacturing sector was experiencing as a result of the peso trade-weighted real appreciation which reflected the dollar appreciation with respect to the other major currencies. The Central Bank faced the banking crisis with massive loans to the troubled financial intermediaries. The result was that domestic credit grew at nearly 100 percent in the last quarter of 1981 and in the second quarter of 1982, after declining in absolute value in most of the quarters of the previous two years.

The acceleration in domestic credit coincided with increasing foreign exchange reserve losses by the Central Bank and with a widening of the interest rate differential in favor of Peso assets, which reached its maximum value just before the collapse of the fixed rate regime on June 15, 1982 (Figure 4). After the collapse, Chile adopted a system of preannounced monthly exchange rate devaluations.

The four crises examined in this section clearly present many similarities. First, a jump in the exchange rate (a sudden depreciation) characterizes each of the collapses. Second, the foreign exchange goes at a premium well before the collapse, even though the spot rate is constant; in

FIGURE 4

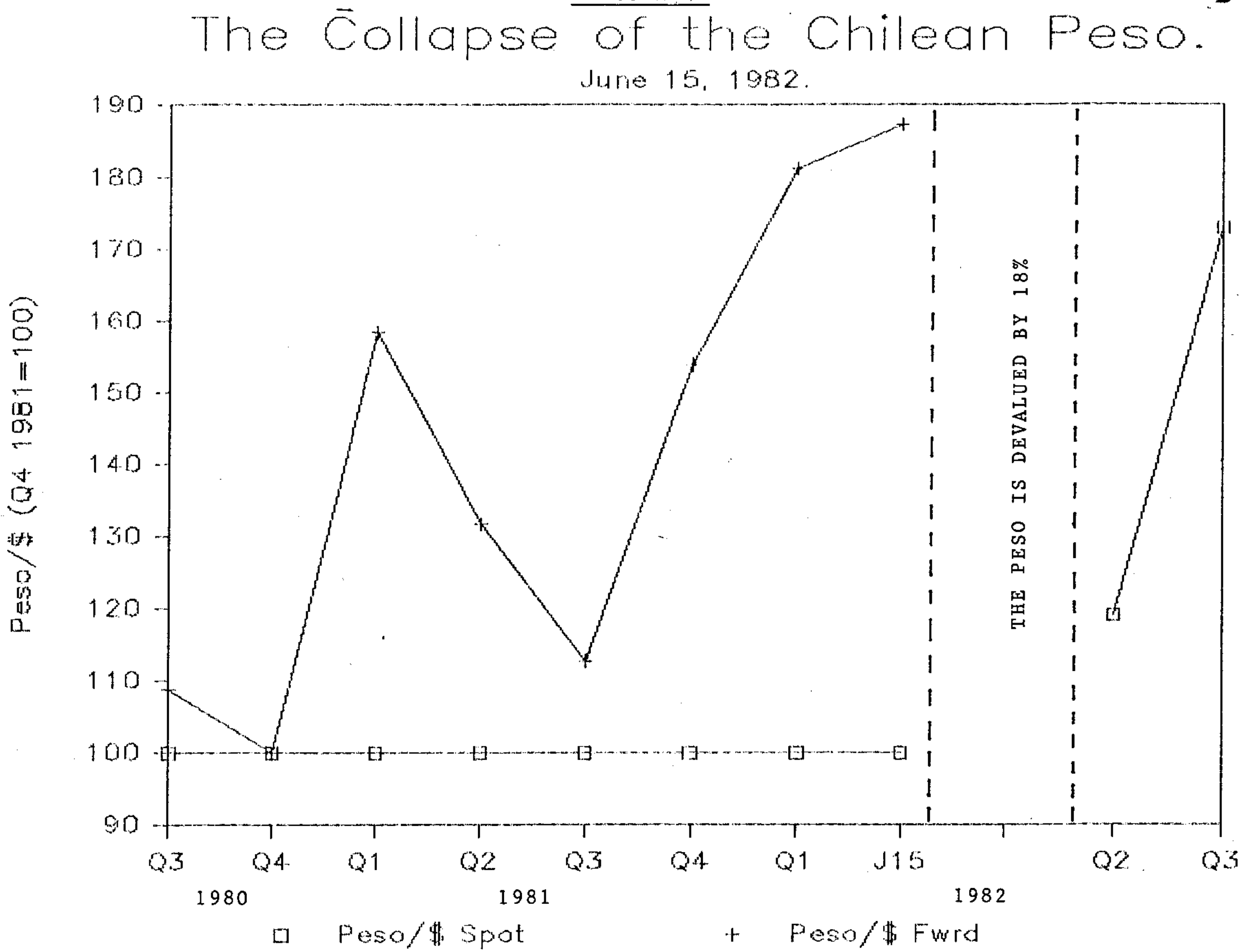

The Collapse of the Chilean Peso.
June 15, 1982.
Peso/$ (Q4 1981=100)
190
180
170
160
150
140
130
120
110
100
90
Q3
Q4
Q1
Q2
Q3
Q4
Q1
J15
Q2
Q3
1980
1981
1982
THE PESO IS DEVALUED BY 18%
Peso/$ Spot
Peso/$ Fwrd

Table 4

Chile: Economic Indicators: 1980-1982

	1980				1981				1982	
	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2
1) Change in Official Foreign Reserves (millions of U.S. dollars)	+448	+206	+395	+135	+65	+60	+137	-172	-255	-216
2) Real Exchange Rate (1979 Q3 = 100)	93.6	94.6	90.4	85.1	79.7	77.5	77.3	78.3	75.5	82.4
3) Central Bank Domestic Credit (percentage change with respect to previous quarter at annual rate)	-23	+4	-4	+12	-110	-33	-16	+117	+4	+96

	1978	1979	1980	1981	1982
4) Fiscal Deficit/GDP (percent)	0.9	-1.7	-0.6	-3.0	-2.3

Sources: IMF, International Financial Statistics (various issues) and Edwards (1985).

other words, one always observes a 'Peso problem'. Third, the countries incur heavy foreign exchange reserve losses, well in excess of domestic credit expansion, for several months before the collapse. Fourth, a period of rapid expansion in domestic credit tends to immediately precede the collapse. And fifth, these features seem to characterize the fixed rate regime collapses regardless of whether the country subsequently adopts a freely floating regime or goes back to a fixed rate.

In what follows, we will develop a model that can account for all these characteristics. While surely other examples of exchange rate crises can be found in which factors other than domestic nominal shocks were the cause, the four exchange rate crises examined above point to the importance of domestic credit expansions. It is the effects of these potential monetary shocks which will be the focus of our theoretical model.

III. The Pure Floating Regime

We begin by developing a model of an exchange rate crisis that brings to an end a fixed rate regime and which is followed by a regime of pure floating. First, we determine the equilibrium path of the exchange rate and the optimal portfolio choices during the pure floating period. We can then determine the demand for real cash balances at the instant after the collapse of the fixed exchange rate regime. Given this demand, in the following section, we are able to determine the excess supply of money that is caused by the collapse and consequently the timing of a run and the conditions that require a jump in the exchange rate. Initially we look at the individual's optimization problem and, then, we aggregate the individual consumption and asset demands to obtain market equilibria.

III.1 The individual's problem.

We consider a small open economy with perfect capital mobility between countries. Because the focus of this paper is on monetary versus real asset portfolio decisions, the production side of this economy is modelled quite simply. Homogeneous physical capital, whether located in the domestic country or abroad, yields a constant riskless real rate of return. Arbitrage in a world of perfect capital mobility insures that this riskless real return, r, is equal across countries. One unit of physical capital can be costlessly transformed into one unit of a consumption good. There are no impediments in trading this capital-consumption good across countries so that purchasing power parity always holds.[6] For simplicity, the foreign price level is set to be a constant, so that;

(1) $$\frac{dS}{S} = \frac{dP}{P}$$

where P is the domestic price level and S is the spot exchange rate (domestic currency price of one unit of foreign currency).

Domestic residents are assumed identical in preferences and choose to divide their portfolio between claims on real capital, foreign currency bonds, domestic currency (nominal) bonds and domestic money.

Foreign currency bonds are held by both domestic residents and the Central Bank in the form of foreign official reserves, which are therefore interest bearing assets. Foreign bonds are also assumed to be risk free in nominal terms when expressed in the foreign currency. Because of the assumptions of PPP and zero inflation abroad, arbitrage will insure that this nominal return on foreign bonds equals r, so that foreign bonds pay a constant real return. Because physical capital, no matter where it is geographically located, yields the same real rate of return as foreign bonds, the two assets are perfect substitutes.

We assume a Ricardian world so that individuals perceive the foreign bonds held by the Central Bank as their own. Later on, we will show that this assumption is indeed consistent with rational expectations. Given this assumption, private real wealth, K, is

$$K = K^D + f \tag{2}$$

where 'K^D' is the stock of physical capital held by domestic residents and 'f' represents the stock of foreign bonds held by both the domestic private sector and the Central Bank.

The only source of uncertainty in the economy is the evolution of domestic credit, which is equal to the money supply during the floating period (the superscript 's' will indicate supply). Money is introduced in the economy through transfers which are unrelated to individuals' holdings of cash balances.[7] Each individual is assumed to receive a transfer that is proportional to his initial stock of capital. We postulate that domestic credit, D, evolves according to the mixed diffusion-jump process

$$\frac{dD}{D} = [\alpha - \lambda(Y - 1)]dt + \sigma dz + dq \ , \qquad Y > 1 \tag{3}$$

where {dq} is a Poisson process with λdt being the probability that q will jump once in dt. If the jump occurs, D goes to YD so that dq is assumed equal to (Y - 1); if there is no jump dq equals zero. The percentage size of the jump, Y, is assumed to be constant where this assumption is made for convenience; a stochastic jump-size would not affect the qualitative results of the model. In the floating rate regime, the supply of money, M^S, follows the same process as domestic credit given in (3).

By modelling monetary policy with a mixed diffusion-jump process we capture the ability of the government to execute a "surprise" injection of domestic credit within a continuous time model. If individuals have less than perfect information concerning current monetary policy, it would seem reasonable to model the government during a fixed exchange rate regime as having the power to increase the supply of money quicker than individuals can react to reduce it through an exchange for foreign reserves at the Central Bank. In more general terms, the money supply-domestic credit process in equation (3) characterizes the notion of a possible change in the monetary policy regime, such as a sudden acceleration in domestic credit. Individuals recognize that this "regime switch" is possible, but its actual date of occurrence is unknown. This characterization of monetary policy is broadly consistent with the empirical evidence presented in the previous section.[8]

In the pure floating rate regime, given the process for the money supply, we conjecture that the price level, and thus the exchange rate, will follow a process of the form:

$$(4) \qquad \frac{dP}{P} = [\mu_\pi - \lambda(Y - 1)]dt + \sigma_\pi dz + dq$$

where a jump in the money supply process implies an equivalent percentage jump in the price level. Later on, we will prove that this is the correct form for the process of 'P' and we will solve for its parameters endogenously.

The rate of return on holdings of real cash balances is simply the percentage change in the inverse of the price level. By applying Ito's lemma to d(1/P)P we have;[9]

(5) $$d(1/P)P = \left[-\mu_{\pi} + \sigma_{\pi}^{2} + \lambda(Y - 1)\right]dt - \sigma_{\pi}dz - dq \ .$$

Because $\lambda(Y - 1)$ is just a constant, it will be denoted by Λ.

Domestic bonds are instantaneously risk-free in nominal terms and are assumed to be in zero net supply, an assumption that turns out to be convenient for expressing our results. Let i denote the instantaneous nominal rate of return of bonds. Given the uncertainty about the inflation rate, the real rate of return on these domestic bonds is $d(B/P)/(B/P)$ where 'B' is the nominal bond price which grows at rate i.

(6) $$d(B/P)/(B/P) = \left[i - \mu_{\pi} + \sigma_{\pi}^{2} + \Lambda\right]dt - \sigma_{\pi}dz - dq \ .$$

Since domestic bonds are in zero net supply, aggregate portfolio wealth, W, is then the sum of private real wealth, K, real cash balances, $m \equiv M/P$, and the present value of the future monetary transfers, m_{τ}, that individuals expect to receive. The sum of m and m_{τ} will be referred to as monetary wealth.

(7) $$W = K + m + m_{\tau}$$

For analytical convenience, we can then think of claims on future monetary transfers as a fourth type of asset that individuals can hold. We conjecture that the rate of return on this asset takes the following form;

(8) $$dm_{\tau}/m_{\tau} = \mu_{\tau}dt + \sigma_{\tau}dz + \eta dq \ .$$

Because all domestic residents have identical preferences and money transfers are proportional to their initial endowments, we will neglect using an individual's superscript for convenience. Let w_k, w_b, w_m, and w_{τ} denote the proportions of each individual's wealth, W, which is held in the form of claims on real capital (physical capital or foreign bonds), domestic bonds, money, and claims on future monetary transfers.[10] By definition these portfolio weights sum to one. Portfolio wealth then follows the process

$$(9) \qquad dW = rw_k Wdt + d(1/P)Pw_m W + d(B/P)/(B/P)w_b W + dm_\tau/m_\tau w_\tau W - Cdt$$

where C is individual consumption of the capital-consumption good.

Subject to this flow budget constraint, the individual's problem is therefore to choose consumption and portfolio holdings to maximize lifetime utility.

$$(10) \qquad \max_{C, w_m, w_b, w_k} J(W, t) = E_t \int_t^\infty e^{-\rho s} U(C, m) ds$$

where ρ is the constant rate of time preference, and utility depends on real money balances as well as consumption.

The optimality equation is[11]

$$(11) \qquad \Phi(C, w_m, w_b, w_\tau) = U(C, m) + J_W\Big[W\big(w_m(-\mu_\pi + \sigma_\pi^2 + \Lambda)$$

$$+ w_b(i - \mu_\pi + \sigma_\pi^2 + \Lambda) + w_\tau \mu_\tau + (1 - w_m - w_b - w_\tau)r\big) - C\Big]$$

$$+ \tfrac{1}{2} J_{WW}\big[W^2\big(\sigma_\pi(w_m + w_b) - \sigma_\tau w_\tau\big)^2 - \rho J$$

$$+ \lambda\big[J\{W - W(Y - 1)(w_m + w_b - \eta w_\tau)\} - J(W)\big] .$$

In order to obtain analytic solutions, we posit the utility function to be logarithmic:

$$(12) \qquad U(C, m) = \ln C + \beta \ln m .$$

Logarithmic utility implies that investors' consumption and portfolio decisions are separable and in particular that the collapse of the exchange rate will not effect consumption plans.[12] Because we want to focus on portfolio decisions, thereby leaving the real sector of the economy in the background, this restriction on the form of the utility function is not unduly restrictive.[13]

Differentiating (11) with respect to C, w_m, and w_b, one obtains;

(13) $$C = 1/J_W$$

(14) $$\beta/w_m + J_W W(\sigma_\pi^2 - \mu_\pi + \Lambda - r) + J_{WW}W^2(\sigma_\pi(w_m + w_b) - \sigma_\tau w_\tau)\sigma_\pi$$
$$- \Lambda W J_W(W[1 - (Y - 1)(w_m + w_b - \eta w_\tau)]) = 0$$

(15) $$J_W W(i + \sigma_\pi^2 - \mu_\pi + \Lambda - r) + J_{WW}W^2(\sigma_\pi(w_m + w_b) - \sigma_\tau w_\tau)\sigma_\pi$$
$$- \Lambda W J_W(W[1 - (Y - 1)(w_m + w_b - \eta w_\tau)]) = 0 .$$

Combining equations (14) and (15) we have:

(16) $$w_m = \beta/J_W W i .$$

Equations (13) and (16) then imply

(17) $$m = \beta C/i .$$

To quantify the equilibrium value of monetary wealth we use an argument put forth by Jones (1982) and Stulz (1984). In this economy consumption can be interpreted as the dividend paid out of capital, K. Since the value of an asset can be equated to the value of its stream of future dividends, the present value of individuals' future consumption streams is simply the value of the current capital stock.

The cost or value of monetary services, im, can also be thought of as the dividend paid out of monetary wealth. However, from equation (17) we know that the cost of monetary services will optimally be set at a proportion, β, of consumption, C, at each point in time. Since the stream of consumption services is worth K, the stream of all future monetary services (monetary wealth) must be worth βK. Hence if individuals optimally choose consumption and money holdings, total wealth is equal to K plus monetary wealth, βK.

$$W = (1 + \beta)K . \tag{18}$$

The value of future monetary transfers can be derived by using (18) and (16):

$$m_\tau = \beta K - m = \beta W/(1 + \beta) - \beta/J_W i . \tag{19}$$

By applying Ito's lemma to (19), we obtain the rate of return on future monetary transfers:

$$\frac{dm_\tau}{m_\tau} = \left[\frac{rJ_W Wi - (1 + \beta)(\mu_\pi + \sigma_\pi^2 - \Lambda)}{J_W Wi - (1 + \beta)}\right]dt + \frac{(1 + \beta)}{J_W Wi - (1 + \beta)}\left[\sigma_\pi dz + dq\right] . \tag{20}$$

Using our results in equations (13), (16), (19), and (20), and imposing the assumption that domestic bonds are in zero supply at an optimum, equation (11) then simplifies to:

$$\begin{aligned} 0 = {} & -\ln J_W + \beta\ln\beta - \beta\ln J_W i - \rho J + J_W\Big\{\frac{\beta(\sigma_\pi^2 - \mu_\pi + \Lambda)}{J_W i} + \frac{r\beta W}{1 + \beta} \\ & - \frac{(-\mu_\pi + \sigma_\pi^2 + \Lambda)\beta}{J_W i} + \left[1 - \frac{\beta}{1 + \beta}\right]rW - \frac{1}{J_W}\Big\} \\ = {} & -\ln J_W + \beta\ln\beta - \beta\ln J_W i - \rho J + rWJ_W - 1 . \end{aligned} \tag{21}$$

Note from equation (21), that when individuals' asset proportions are held at optimal levels, the J_{WW} and jump component terms in J disappear, implying no uncertainty in total wealth. This result makes intuitive sense, as any uncertainty in the rate of return on money is exactly offset by the uncertainty from the value of transfers, since money transfers are the driving force behind price level movements.

The solution to this equation is of the form[14]

(22)
$$J = \frac{(1 + \beta)}{\rho} \ln W + H$$

so that equilibrium consumption and money demand are given by:

(23)
$$C^* = \frac{\rho W}{1 + \beta} = \rho K$$

(24)
$$m^* = \frac{\beta \rho W}{(1 + \beta) i} = \beta \rho K / i \ .$$

From (23) we see that the optimal consumption plan is independent of the money supply process, a consequence of the logarithmic utility assumption.

III.2 Equilibrium interest rates and prices.

The equilibrium nominal interest rate can be obtained by using the first order condition for w_b, (15), and setting $w_b = 0$:

(25)
$$i^* = \mu_\pi - \sigma_\pi^2 + r \ .$$

Thus the nominal interest rate satisfies the stochastic version of the Fisher equation, as shown in Fischer (1975). Through the expected rate of inflation, μ_π, the nominal interest rate depends on the probability and the size of the jumps in the money supply process. Clearly with zero, certain inflation, the nominal and real rates of interest will be equivalent.

Using (23), real wealth, K, evolves according to

(26)
$$dK = (rK - C)dt = (r - \rho)Kdt \ .$$[15]

Since total wealth is a proportion $(1 + \beta)$ of real wealth, it also will grow at rate $(r - \rho)$.

Given the nominal interest rate, we see from (24) that real money demand is a fixed proportion of real wealth; therefore, it will also grow by the rate $(r - \rho)$. Having obtained the dynamics for money supply and demand, we can solve for the parameters of the price level process (4).

(27) $$\frac{d(M^S/P)}{M^S/P} = (r - \rho)dt \ .$$

Using Ito's Lemma once more, (27) becomes

(27') $$\frac{dM^S}{M^S} - \frac{dP}{P} + (\sigma^2 - \sigma\sigma_\pi)dt = (r - \rho)dt \ .$$

From equations (3), (4), and (27'), we have $\sigma_\pi = \sigma$, and $\mu_\pi = \alpha - r + \rho$ is the expected rate of inflation, as well as the expected rate of depreciation of the exchange rate. We now turn to the analysis of this economy when the exchange rate is fixed.

IV. The Fixed-Exchange-Rate Regime

Prior to an exchange rate collapse, when the exchange rate is fixed, the Central Bank continues to expand domestic credit according to the process given by equation (3). During the fixed-rate regime, the money that is created in excess of the amount of money demanded by private residents is taken back to the Central Bank and converted into foreign bonds, thus depleting the Bank's foreign official reserves, R. For a given stock of reserves and the process governing domestic credit (equation 3), the fixed exchange rate regime will be credible if agents know that reserves are sufficient to absorb the excess supply of domestic credit over the next instant in time. During this period no change in the price level will be expected in the immediate future. The demand for money will be then given by equation (24), with i equal to r, because σ_π^2 and μ_π are equal to zero (equation 25).

IV.1 Portfolio choice near a collapse.

When reserves drop below a certain critical level during the fixed-rate regime, however, there will exist a non-zero probability that the regime will collapse over the next instant in time owing to a possible jump in domestic credit. Below we show that the possibility of a collapse, which leads to a depreciation and a jump in the price level, implies a positive level of expected inflation, and hence pushes up the nominal interest rate and reduces money demand.

This critical level of reserves, $\bar{R}(t)$, is given by

$$\bar{R}(t) = m^*(i = r, W(t))\bar{P} - m^*(i = i^*, W(t))\bar{P} + (Y - 1)D(t) \quad (28)$$

where m^* and i^* are given by equations (24) and (25) and $\bar{P}$ is the fixed domestic price level during the fixed rate regime. The right hand side of (28) is the excess supply of money between the fixed regime with nominal interest rate equal to r and the floating regime when a jump in domestic credit occurs. If central bank reserves are exactly equal to $\bar{R}(t)$ when a jump occurs, there will be a smooth transition to the floating rate regime, that is, there are no discrete jumps in P or S since $\bar{R}(t)$ is just sufficient to meet the excess supply of money which individuals want to exchange for foreign bonds at the Central Bank.

If a jump in domestic credit does not occur when reserves are equal to or slightly less than $\bar{R}(t)$, the fixed exchange rate regime will not collapse. However, the probability of a collapse occurring over the next instant in time will now be greater than zero. As actual reserves fall below $\bar{R}(t)$, the occurrence of a jump in domestic credit implies that the resulting excess supply of money exceeds the amount of reserves left at the Central Bank. Such a jump results in an exchange rate collapse in which the exchange rate and the domestic price level also jumps up discretely to clear the money market. Because this jump in the price level leads to a sudden fall in the real value of money balances (as well as domestic nominal bonds), agents will start "hedging" against the possibility of a collapse by accumulating foreign bonds (reducing their money balances) at an increasing rate when reserves are less than $\bar{R}(t)$. This reduction in money demand is brought about by an increase in the nominal interest rate which reflects a non-zero expected inflation, since a jump in domestic credit could occur.

Suppose that there is no jump in domestic credit. The fixed exchange regime will ultimately collapse but there will be a smooth transition to floating, not involving a discrete jump in the exchange rate as in the original model of Krugman (1979). The transition will occur at time T when reserves equal $\hat{R}(t)$ where

$$\hat{R}(t) = m[i(t), W(t)]\bar{P} - m^*[i = i^*, W(t)]\bar{P} < \bar{R}(t) \tag{29}$$

that is, when remaining Central Bank reserves equal the difference between money demand at the interest rate prevailing at time t and money demand at the interest rate of the floating rate regime. Below we will solve for the equilibrium interest rate, i(t).

A run on Central Bank reserves occurs when reserves equal $\hat{R}(t)$ as Central Bank reserves are just sufficient to absorb the excess supply of money created by a collapse. Without a jump in domestic credit, a run would not occur when reserves are above $\hat{R}(t)$ as an instantaneous appreciation of money balances would have to occur to clear the money market, inducing each individual not to take part in the run. A run would also not occur initially at a level of reserves below $\hat{R}(t)$ because in this case reserves would be insufficient to eliminate the excess supply of money and an instantaneous depreciation would be needed to clear the money market. Such a depreciation, however, can be ruled out in a world of rational investors, as investors attempt to avoid the loss in the value of money by reducing their money holdings in exchange for foreign bonds, thus moving back in time the run on Central Bank reserves.

To determine individuals' portfolio choice when reserves, $R(t)$, are between $\bar{R}(t)$ and $\hat{R}(t)$, that is, the situation in which individuals want to "hedge" against a jump in the price level, we need to determine the return on money balances. It is clear that if a jump in domestic credit does not occur over the next instant in time, the exchange rate and price level will remain fixed and the return on money will be zero. If a jump does occur, we must calculate the return on money holdings which depends on the percentage jump in price needed to clear the money market.

If a jump occurs at time t, the real demand for money an instant later, which is the beginning of the floating rate regime and is denoted t^+, will be $m^*[i^*, W(t^+)]$ where m^* and i^* are again given by equations (24) and (25). Nominal money balances at the time of the jump, $M(t)$, will be

$$M(t) = M[i = i(t^-), W(t^-)] + (Y - 1)D(t) - R(t^-) \tag{30}$$

so that the stock of money holdings will be equal to money balances just prior to the jump, at time t^-, plus the jump in domestic credit, $(Y - 1)D(t)$, minus the value of the remaining reserves at the Central Bank just before the jump, $R(t^-)$. The price level will then jump from $\bar{P}$ to $P(t^+)$ at time t so that

$$\frac{M(t)}{P(t^+)} = m^*[i^*, W(t^+)] \ . \tag{31}$$

To determine the stock of reserves just before the collapse, $R(t^-)$, we use the following steps. Let $t = T$ denote the point in time at which $R(t)$ last fell below $\bar{R}(t)$, which is the point in time when individuals begin "hedging" against an exchange rate collapse. Then the stock of reserves at t^- will be equal to the stock of reserves at the beginning of the "hedging" period, $\bar{R}(T)$, plus the accrued interest on the Central Bank's holdings of these bonds, minus the accumulated increase in domestic credit from T to t^-, minus the "hedging" demand for foreign bonds, and plus the increase in real cash balances demanded since T because of the increase in real wealth. Using the notation developed above:

$$(32) \qquad R(t^-) = \bar{R}(T) + \int_T^t rR(s)ds - \int_T^{t-} dD + M(t^-) - m^*(i = r,\ W(T))\bar{P} .$$

Using the definition of $\bar{R}(T)$ in (28), (32) can be re-written:

$$(33) \quad R(t^-) = -m^*(i = i^*,\ W(T))\bar{P} + (Y - 1)D(T) + \int_T^t rR(s)ds - \int_T^{t-} dD + M(t^-) .$$

Substituting (33) into (30) and noting that $\int_T^{t-} dD = D(t^-) - D(T)$ we have:

$$(34) \qquad \frac{M(t)}{\bar{P}} = m^*(i^*,\ W(T)) + \frac{Y(D(t^-) - D(T)) - \int_T^t rR(s)ds}{\bar{P}} .$$

Subtracting (34) from $m^*(i^*,\ W(t^+))$, we obtain the decrease in aggregate real money balances caused by the unexpected price increase at time t. Using the money demand equation (24) and the fact that wealth grows at the rate $(r - \rho)$, we obtain that:

$$(35) \qquad dm = m^*(i^*, W(t)) - \frac{M(t)}{\bar{P}}$$

$$= \frac{\rho\beta}{(1+\beta)i^*}\left[e^{(r-\rho)t} - e^{(r-\rho)T}\right]W_0$$

$$+ \frac{1}{\bar{P}}\left[\int_T^t rR(s)ds - Y\left(D(t^-) - D(T)\right)\right]$$

$$\equiv H\left[t, T, \int_T^t rR(s)ds, D(t^-) - D(T)\right] .$$

From (35), the change in value of real cash balances when a jump occurs consists of the increase in the money demand due to the change in wealth, plus the accumulated interest on foreign bonds at the Central Bank minus Y times the change in domestic credit. Thus the expected rate of return on money during the "hedging" period is

$$(36) \qquad E_t(r_m) \equiv E_t\left(\frac{dm}{m}\right) = \frac{\lambda H(t)}{m(t)} .$$

We can finally derive optimal portfolio and consumption choices during this hedging period. Unlike the floating rate period, the expected rate of return on money, $E_t(r_m)$, is a function of H which is changing over time, depending on the additional state variables $\psi(t) \equiv \int_T^t R(s)ds$ and $D(t)$. Thus while the only source of uncertainty derives from the stochastic process for domestic credit, $D(t)$, the indirect utility function, J, for the fixed rate case will not be Markov in $D(t)$, as it will also depend on past values of $D(t)$ for $t > T$ through the variable $H(\psi(t))$. However, by appropriately expanding the number of state variables to include $\psi(t)$ and $d\psi/dt = R(t)$, we can create a vector Markov process of the form

$$(37) \qquad dX = f(X, t)dt + g(X, t)dz + h(X, t)dq$$

where

$$X(t) \equiv \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} \equiv \begin{vmatrix} D(t) \\ \psi(t) \\ R(t) \end{vmatrix}, \quad \text{and } f(X, t), \quad g(X, t), \quad \text{and } h(X, t)$$

are three dimensional vectors, which do not depend on a given individual's choice of consumption or portfolio holdings. It is straightforward to show that (37) is sufficient to describe the evolution of the rate of return on money holdings. We can then write the Bellman equation for this problem as;

(38)
$$\Phi = U(C, m) + J_W\left[((1 - w_m - w_b - w_\tau)r + w_b i + w_\tau \mu_\tau)W - C\right]$$
$$+ \lambda\left[J\left(W + (w_m + w_b - \eta w_\tau)\frac{H}{m}W\right) - J(W)\right]$$
$$+ \sum_{j=1}^{3} J_{x_j} f_j + \tfrac{1}{2}\sum_{j=1}^{3}\sum_{k=1}^{3} J_{x_j x_k} g_j g_k .$$

Differentiating with respect to C, w_m, and w_b we have[16]

(39)
$$C = 1/J_W$$

(40)
$$\beta/w_m - J_W rW + \lambda J_W\left(W + (w_m + w_b - \eta w_\tau)\frac{H}{m}W\right)\frac{H}{m}W = 0$$

(41)
$$J_W(i - r)W + \lambda J_W\left(W + (w_m + w_b - \eta w_\tau)\frac{H}{m}W\right)\frac{H}{m}W = 0 .$$

Note that with log utility these first order conditions imply that monetary services, i(t)m, continue to be proportional to consumption. Therefore, real, monetary, and total wealth all grow at rate $(r - \rho)$ as in the floating rate regime. Moreover, at an optimum this implies that the jump component term in (38) vanishes so that because by assumption, $w_b = 0$, it must be that $w_m = \eta w_\tau$.

As in the floating rate case, with a logarithmic utility the indirect utility function takes the form:[17]

$$J = a \ln W(t) + H(X, t) . \tag{42}$$

Substituting (39), (40), and (41) into (38), we find that optimal consumption continues to be described by equation (23), while money demand is:

$$m^* = \frac{\beta}{1 + \beta} \rho \frac{W(t)}{i(t)} = \beta\rho \frac{K(t)}{i(t)} \tag{43}$$

Thus with logarithmic utility, consumption decisions are independent of the exchange rate regime while real cash balances demanded change only to the extent that the nominal interest is affected by the probability of a sudden collapse of the fixed exchange rate. Because the interest rate plays a key role in the dynamics of the model, we now focus on its movements during this "hedging" period.

IV.2 Interest rate movements and the 'Peso problem'.

An expression for the nominal interest rate can be found by using equations (41) and (43), plus the assumption that domestic bonds are in zero net supply ($w_b = 0$):

$$i(t) = r \frac{\rho\beta K(t)}{\rho\beta K(t) + \lambda H(t)} . \tag{44}$$

Substituting in for H(t) using equation (35) as well as (30) and (43), the nominal interest rate can also be expressed in terms of the current (endogenous) level of Central Bank reserves;

$$i(t) = \frac{r + \lambda}{\left(1 + \frac{\lambda}{i^*} - \lambda\left[\frac{(Y-1)D(t) - R(t)}{\rho\beta\bar{P}K(t)}\right]\right)} \tag{45}$$

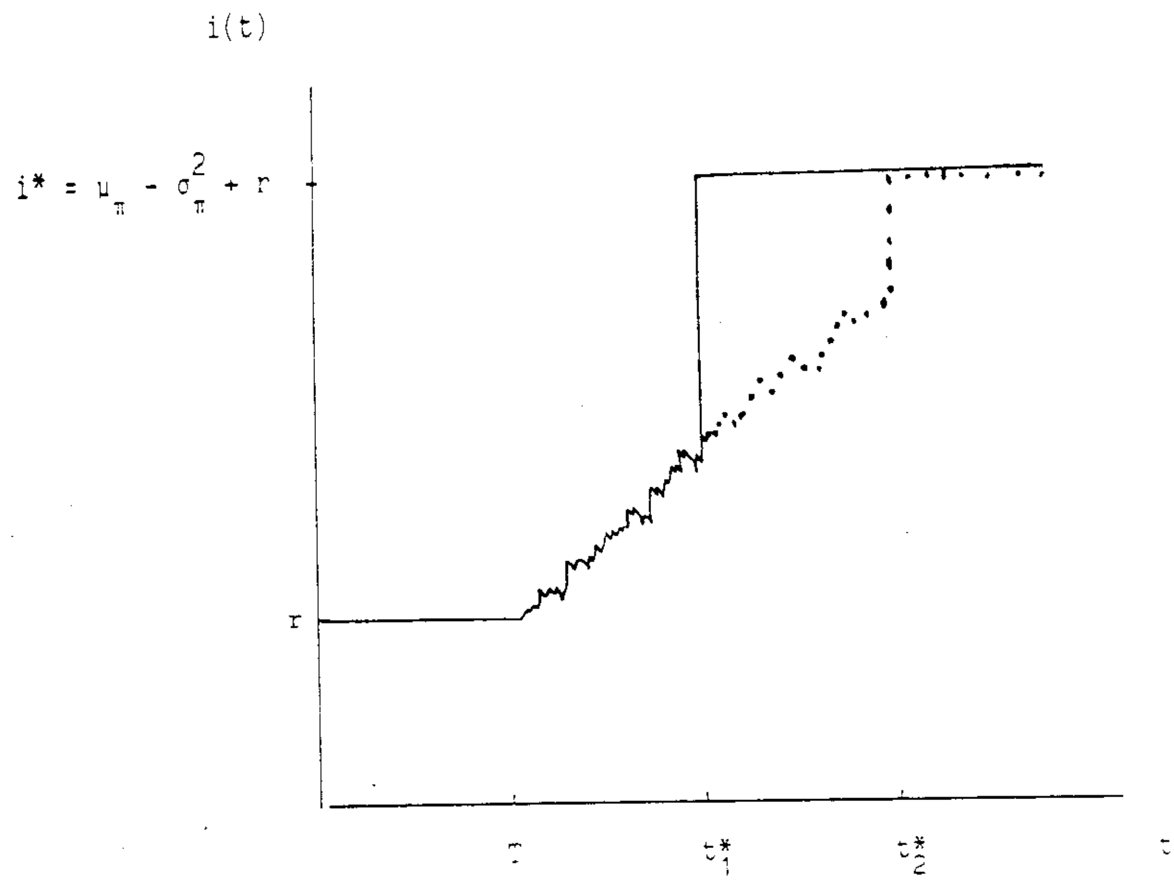
Note that since $H(t) \leq 0$, $i(t)$ is greater than or equal to the risk free rate, and rises as the magnitude of $H(t)$--the loss in the value of money from a jump in domestic credit--increases. From (35), we see that the nominal interest rate, ceteris paribus, is an increasing function of the current level of domestic credit. This rise in the nominal interest rate in anticipation of a possible sudden depreciation--the 'Peso problem'--thus reflects a decrease in money demand, because investors are "hedging" against a possible loss on real balances by converting their domestic currency holdings into foreign exchange reserves.

The interest rate movements during the various phases of an exchange rate crisis are shown in Figure 5. The differential between domestic and foreign interest rates becomes positive at T, the beginning of the "hedging period," where agents can first start suffering a capital loss should a jump occur. If domestic credit, and thus the magnitude of H, continues to increase, the interest rate differential widens. It is important to note that the 'Peso problem' depends only on the possibility of an exchange rate jump and not on its actual occurrence. During this time the forward exchange rate will be consistently above the constant spot rate, without implying market inefficiencies.[18]

In Figure 5 we show the two types of transitions from a fixed to a floating rate regime. The solid line represents the path of the nominal interest rate if a jump in domestic credit occurs at time t_1^*. At this point $i(t)$ jumps to i^*, along with a sudden depreciation of the exchange rate. Alternatively, the dotted line represents the interest rate path for the case in which domestic credit does not jump during the hedging period. Here, at time t_2^*, when reserves equal $\hat{R}$, the interest rate jumps but by a lesser

FIGURE 5

Path of the Nominal Interest Rate
During Exchange Collapse



solid line ———— if jump in domestic credit occurs at t_1^*

dotted line · · · · · if no jump in domestic credit between T and t_2^*

amount, and there is a smooth transition of the exchange rate to the floating regime.

The dynamics of the interest rate explain why a country tends to lose foreign exchange reserves at an increasing rate during the terminal period of a fixed exchange rate regime.[19] Neglecting the deterministic wealth effect on money demand, which occurs when r differs from ρ, and interest accumulated on Central Bank reserves, the decline in reserves prior to T reflects the pace at which domestic credit is created, because money demand is constant. After T, reserves decline at a rate faster than that of credit creation, as agents reduce their demand for real balances in response to the interest rate increase.

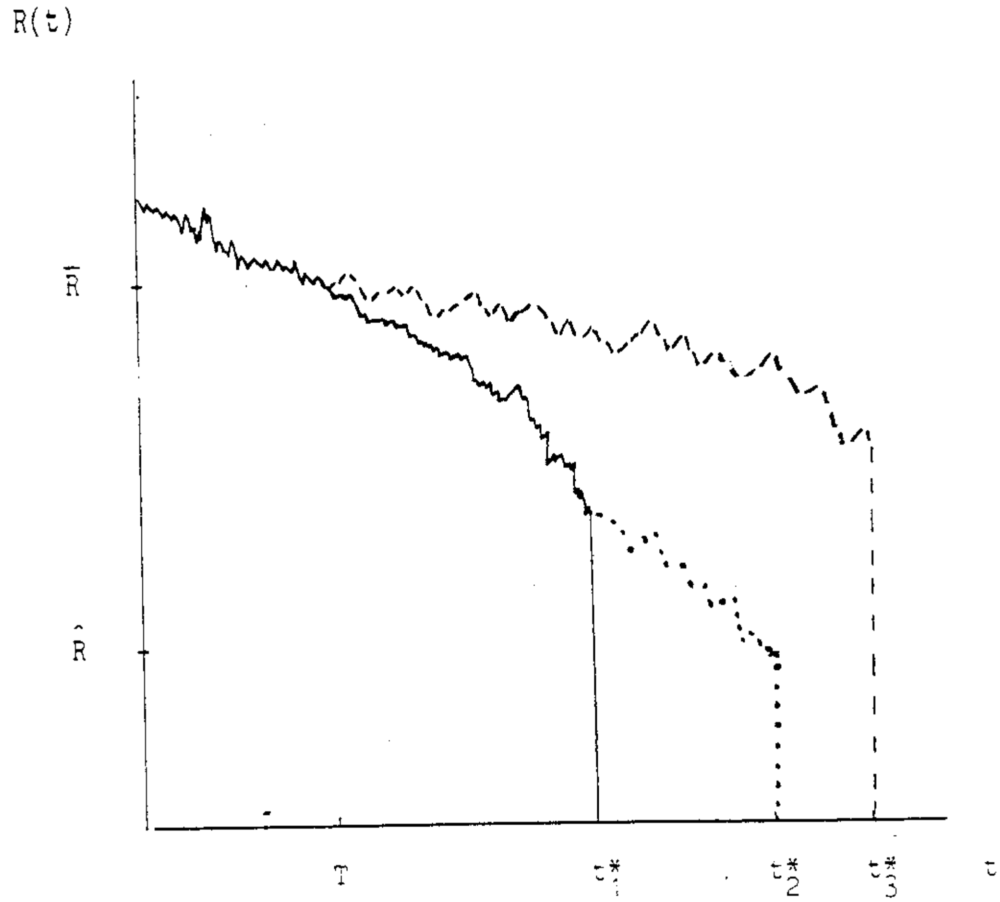
Some possible paths for foreign exchange reserves are presented in Figure 6, the last two of which correspond to the path of the interest rate shown in Figure 5. After time T, the broken-line represents the case $Y = 1$, with no jumps in domestic credit. Here the loss in reserves occurs at approximately the rate of domestic credit growth, with an abrupt run on the Central Bank's reserves at time t_3^* and a smooth transition between exchange rate regimes. By contrast, the solid line after T represents the case of $Y > 1$ in which hedging behavior implies reserve loss at a greater rate than in the no jump case. If a jump occurs at t_1^*, before reserves reach $\hat{R}$, the exchange regime collapses with a sudden depreciation of the exchange rate. Alternatively, if a jump is possible but does not occur before the reserves reach $\hat{R}$ (the dotted line), a smooth transition will occur at $\hat{R}(t_2^*)$, where $\hat{R}$ is lower than the terminal level of reserves in the no jump case.[20]

V. Returning to a fixed rate regime

Our assumption that an exchange rate collapse is followed by an indefinite period of floating exchange rates can be modified to reflect the

FIGURE 6

Path of Central Bank Reserves During Exchange Collapse



broken line ---------- Y = 1 case

solid line ———————— Y > 1 with jump in domestic credit occurring at t_1^*

dotted line · · · · · Y > 1 with no jump in domestic credit between T and t_2^*

practice of some countries which soon return to a fixed exchange rate regime following the breakdown of an initial fixed rate period. We now analyze the transition back to fixed rates after an interim floating period that follows the collapse of the initial fixed rate regime.

There are a variety of ways a country may return to a fixed exchange rate regime. Because after the collapse the Central Bank has zero reserves, one possible way of acquiring reserves, with which to keep the new exchange rate at the new fixed level, is to borrow foreign bonds from the private sector. This borrowing will be financed by current or future taxes proportional to each individual's initial endowment. For instance, suppose the Central Bank acquires an amount of reserves, $R^0(t)$, and then fixes the exchange rate at its current floating level, S^0, with the price level at P^0. By fixing the exchange rate, the Central Bank will experience an inflow of reserves equal to the difference between the quantity of money demanded just after the return to the fixed rate regime and the quantity of money demanded just before the fixing where the nominal interest rate equaled i*. Using these results, along with equations (45) and (43), it is straightforward to show that the equilibrium nominal interest rate immediately after the return to fixed rates is given by:

$$(46) \quad i(t) = \frac{r}{1 - \lambda\left[\frac{(Y-1)D(t) - R^0(t)}{\rho\beta P^0 K(t)}\right.} \qquad \text{if } R^0(t) \le (Y-1)D(t)$$

$$= r \qquad \text{if } R^0(t) > (Y-1)D(t)$$

Thus if the Central Bank's stock of borrowed reserves, $R^0(t)$, exceeds the size of the possible jump in domestic credit, the nominal interest rate will decline to the real rate of interest. Because the Central Bank does not change its monetary policy, the new fixed exchange rate regime will ultimately collapse in the same way as the old one.

Another possible case is when a Central Bank executes a smooth transition to fixed rates by permanently changing monetary policy. Suppose that the process for domestic credit expansion changes from equation (3) to the deterministic growth rate of $(r - \rho)$, after the collapse of the initial regime. Because the new monetary policy is consistent with zero equilibrium inflation, a credible committment to this new policy rule will succeed in fixing the exchange rate at its current level indefinitely. There will then be an initial inflow of reserves to the Central Bank equal to the difference between money demanded at $i(t) = r$ and money demanded at $i(t) = i^*$.[21]

The model can shed light on the short-term capital flows dynamics that accompany most realignments inside the European Monetary System. In Figure 7, we show these flows for France, at the time of the three parity changes of the French Franc. The figure clearly shows how the short-term capital outflows preceding any exchange rate crisis turned into inflows just after the crisis. As the model would predict, these inflows occur because the Central Bank goes back to a fixed rate regime after each crisis and defends the new parity either by borrowing in the international capital markets without changing monetary policy--as France did after the October 1981 and June 1982 devaluations--or by switching to a non-inflationary monetary policy--as in France after the March 1983 devaluation.

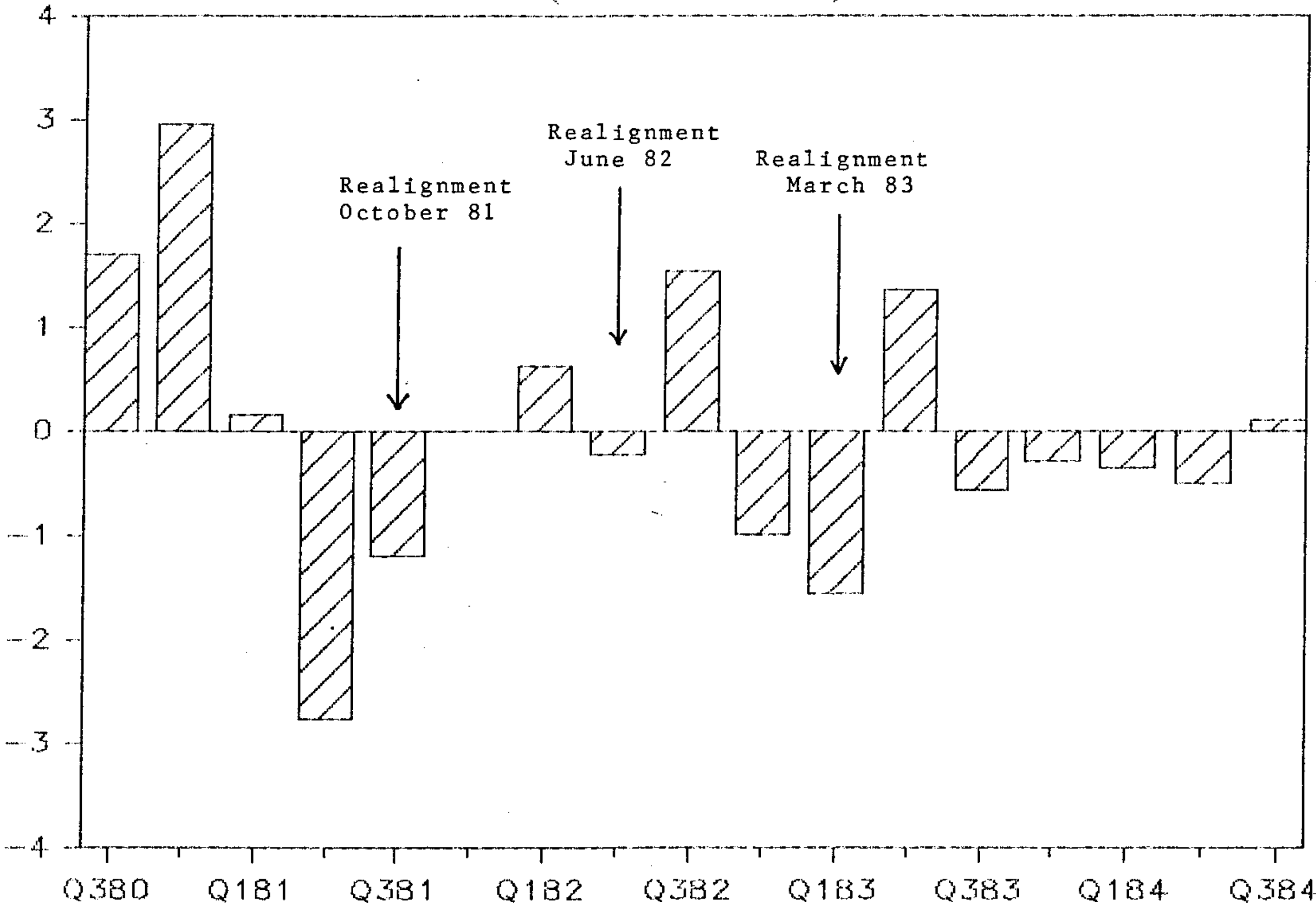
VI. Conclusions

This paper shows how it is possible to use the framework of an intertemporal capital asset pricing model to account for the empirical macroeconomic regularities accompanying the collapse of a fixed exchange rate regime. If the Central Bank has the ability to surprise the private sector

FIGURE 7

FRANCE: Speculative Capital Flows.

(billions of SDRs)



about its monetary policy, agents' portfolio choice and hence the rate at which the Central Bank loses reserves will be affected, whether or not the Central Bank decides to implement this surprise.

The transition from a fixed to a floating exchange rate regime may or may not be accompanied by a sudden depreciation when the government terminates its pegging of the currency. Our analysis suggests, however, that in each instance the termination of a fixed rate regime will be preceded by a period of rising nominal interest rates and forward rates (the peso problem) as well as accelerated losses of Central Bank reserves. The dynamics of an exchange rate crisis that emerges from the model seem to be consistent with the characteristics of some crises observed in the near past.

Footnotes

[1]Flood and Garber (1984), Obstfeld (1984a), Grilli (1986) and Buiter (1986) have extended Krugman's model in several important respects. Obstfeld (1986) has shown that a fixed exchange rate regime may collapse even though the domestic credit policy is consistent with a fixed exchange rate; what is sufficient to have a collapse is that monetary policy validates the new exchange rate regime once a collapse has occurred.

[2]This is true both in a deterministic and in a stochastic model. The important characteristic is whether agents can trade financial assets continuously.

[3]As we discuss in the paper, interest rates will diverge from world rates before the collapse of a fixed exchange rate regime if trading in financial assets takes place at discrete points in time (see for example, Flood and Garber, 1984). We consider, however, that the assumption of discrete trading is an inadequate characterization of modern financial markets.

[4]Blanco and Garber (1986) show that these models have a certain degree of empirical support.

[5]These points can be separately found in Flood and Garber (1984), Obstfeld (1984b) and Collins (1985).

[6]Although PPP does not hold exactly in practice, sharp movements in exchange rates, such as a collapse of a fixed-rate regime, generally have an immediate impact on the general price level. For example, many domestic prices jumped up in Mexico after the February 18, 1982 collapse of the Peso. On February 22, 1982 one can read in the Wall Street Journal ("Mexico Seeking to Hold Peso at 38 to Dollar"): "The inflation problem became apparent over the weekend, as prices on many items surged. Taxi drivers and tour guides began demanding dollars, some grocery prices rose more than 40%, and many real estate sellers automatically doubled asking prices. Anything imported went up by at least the amount of the devaluation."

[7]Stulz (1984) and Jones (1982) introduce money in the economy in a similar way.

[8]An alternative way of modelling the process of domestic credit would be to assume that the level of D is continuous but that the expected rate of growth of D follows a process which includes jumps. This might be an appropriate specification if one were interested in examining the consequences of a stochastically changing inflation tax. Whereas in the model described later in this paper, agents' uncertainty centers on the level of D over the next instant in time, a model incorporating jumps in the growth rate of D would move this uncertainty to the level of real money demand following the collapse of the fixed rate regime. However, the qualitative results of both of these modelling strategies would be fairly similar. We choose to incorporate jumps in the level of D for its analytical simplicity. A jump process for the growth rate in D would likely need to embody a specification insuring stationarity to avoid unbounded equilibrium interest rates. Clearly there are many other ways of modelling monetary policy uncertainty and its

impact on interest and exchange rate determination. See, for example, Lewis (1986), Drazen and Helpman (1986), and Cumby and Van Wijnbergen (1983).

[9]Ito's lemma with a jump process is given in Merton (1976, p. 129).

[10]While our analysis implicitly assumes that claims on future monetary transfers are tradable, this assumption is not necessary for our results since we show that individuals' optimal holdings of these claims equal the amount of their initial endowment.

[11]See Lo (1986) for applications of Ito's lemma with jump processes.

[12]Claessens (1986) has shown how the impact of an exchange rate crisis on consumption and the current account depends on the substitutability or complementarity in utility between money and consumption.

[13]Calvo (1986) analyzes the impact of exchange rate crisis on the current account dynamics when some goods are not perfectly traded.

[14] $H = \frac{\beta}{\rho} \ln(\beta/i) + \frac{r(1+\beta)}{\rho} - \frac{(1+\beta)}{\rho} \ln\left(\frac{1+\beta}{\rho}\right) - \frac{1}{\rho}$.

[15]Without further modelling of the rate of time preference, as done by Obstfeld (1981) for example, or the rate of return on capital as in Sidrauski (1967), the model does not necessarily possess a steady state. This however is not a problem since we are not interested in long run comparative statics but on the short run properties of the model. On this point see Svensson and Razin's (1983) discussion of Obstfeld's model.

[16]Note that the individual's portfolio choice does not affect H/m, that is, the individual views the percentage loss on money due to the exchange rate collapse as exogenous to his (small) actions.

[17]See, for example, Merton (1971), page 402. Here, $a = (1 + \beta)/\rho$.

[18]Flood and Garber (1982) obtain a 'Peso problem' by assuming trading in financial assets at discrete points in time. The interest differential thus reflects the cost of getting caught with the wrong portfolio composition during a period in which trading is not allowed.

[19]This empirical regularity is stressed by Collins (1985) to criticize most exchange rate crisis models in which the run on Central Bank reserves occurs abruptly. In view of the empirical evidence presented in Section 2, this criticism appears to be well founded.

[20]The transition between regimes will always occur later for the $Y = 1$ (no jump) case, i.e., $t_3^* \geq t_2^*$. The reason is that with reserves being lost slower in the no jump case, accumulated interest on the Central Bank's reserves will be larger, thus adding more to the reserve stock.

[21]These foreign bond reserves acquired by the Central Bank are assumed to be transferred to individuals, in proportion to their initial endowments, at some point in the future. This action would justify individuals' Ricardian beliefs.

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APPENDIX

DATA USED FOR THE FIGURES

A) ITALY

		Official Lira/$ Spot	Lira/$ Parallel Market
1975	A	635.3	655.0
	M	628.5	630.4
	J	624.2	635.5
	J	645.9	622.8
	A	668.9	717.7
	S	673.8	728.4
	O	677.5	747.8
	N	680.3	757.1
	D	685.4	747.8
1976	J 6	681.2	730.2
	J 13	683.9	755.8
	J 20	689.2	773.3
	F	Official foreign exchange markets closed on Wednesday, January 21; they are reopened on Monday, March 1.	
	M 1	781.8	
	M 8	798.1	

Note: The exchange rate for a given month refers to the day closest to the middle of that month. Both the official and the parallel rates are the average bid-ask rates quoted by the Italian financial newspaper Il Sole-24 Ore.

B) FRANCE

		Spot FF/DM	Forward FF/DM
1982	J 14	The French Franc is devalued by 10% with respect to the Deutsche Mark on Monday, June 14.	
	J 15	2.78	2.83
	J	2.77	2.83
	A	2.79	2.84
	S	2.81	2.91
	O	2.83	2.90
	N	2.82	2.91
	O	2.83	2.91
1983	J	2.83	2.98
	F	2.83	2.94
	M 18	2.89	3.12
	M 21	The French Franc is devalued by 8% with respect to the Deutsche Mark on Monday, June 21.	
	M 22	3.00	
	A	3.00	

Note: The exchange rate for a given month refers to the first trading day of that month. Both spot and forward rates are noon buying rates quoted by the Federal Reserve Bank of New York; Board of Governors of the Federal Reserve System, Statistical Release H 10.

C) MEXICO

		Spot Peso/$	3-Month Peso/$ Futures
1980	Q4	23.2	24.3
1981	Q1	23.8	24.7
	Q2	24.4	26.0
	Q3	25.2	26.9
	Q4	26.2	28.0
1982	J	26.6	30.3
	F 17	26.9	31.1
	F 18	The Peso is devalued by 28% with respect to the U.S. dollar on Thursday, February 18.	
	F 19	37.5	
	Q1	45.5	

Note: The spot exchange rate for a given quarter or month refers to the last trading day of that quarter or month and is from IMF, International Financial Statistics. The spot rate for February 17 and 19, 1982, are selling rates quoted by the Wall Street Journal. The futures rates are the settle prices on the last trading day of the quarter or of the month for a contract expiring three months later. The futures prices for 1982 refer to the June 1982 contract. The prices are from the CRSP Data File, the University of Chicago.

D) CHILE

		Spot Peso/$	Domestic Nominal Interest Rate (%)	Labor (%)
1980	Q3	39	31.8	12.2
	Q4	39	34.2	16.2
1981	Q1	39	45.0	16.5
	Q2	39	40.1	17.2
	Q3	39	38.8	18.5
	Q4	39	42.5	14.5
1982	Q1	39	48.2	15.6
	A 1-J 14	39	49.9	16.2
	J 15	The Peso is devalued by 18% with respect to the U.S. dollar on Tuesday, June 15		
	Q2	46.5		
	Q3	67.4		

Note: The spot exchange rate for a given quarter refers to the last trading day of that quarter and is from IMF, International Financial Statistics. The interest rate figures are from Edwards (1985).

Table 1

France: Economic Indicators: 1981-1983

	1981				1982		
	Q1	Q2	Q3	Q4	Q1	Q2	Q
1) Speculative Capital Flows[1] (billions of FF)	+1.1	-17.7	-7.9	-.03	+4.8	-1.5	+11
2) GDP Growth Rate (1980 prices; percentage change with respect to the same quarter of the previous year)	-1.6	0.29	0.68	1.76	2.49	2.22	1.2
3) Industrial Employment (percentage change with respect to the same quarter of the previous year)	-1.8	-2.5	-2.8	-2.8	-2.4	-1.8	-0.
5) Central Bank Domestic Credit (percentage change with respect to the previous quarter at annual rate)	-98	+190	+50	-45	+13	+159	-1

Sources: IMF, International Financial Statistics (various issues) and Balance of Payments Statistics Yearbook (198

[1] Errors and Omissions plus short-term private capital flows.

Table 1

France: Economic Indicators: 1981-1983

	1981				1982				1983
	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1
lows[1]	+1.1	-17.7	-7.9	-.03	+4.8	-1.5	+11.3	-7.7	-11.0
prices; n respect to ne previous year)	-1.6	0.29	0.68	1.76	2.49	2.22	1.21	1.30	1.27
(percentage o the same us year)	-1.8	-2.5	-2.8	-2.8	-2.4	-1.8	-0.8	-1.0	-1.2
Credit th respect to the nnual rate)	-98	+190	+50	-45	+13	+159	-11	+155	-23

onal Financial Statistics (various issues) and Balance of Payments Statistics Yearbook (1982 and 1983).

ns plus short-term private capital flows.

Table 2

Italy: Economic Indicators: 1974-1976

	1974				1975		
	Q1	Q2	Q3	Q4	Q1	Q2	Q3
1) GDP Growth Rate (1970 prices; percentage change with respect to the same quarter of the previous year).	11.3	7.6	2.4	-2.8	-6.4	-5.8	-3.
2) Public Sector Borrowing Requirement/GDP (percent)	8.9	8.4	9.3	8.6	13.6	13.4	14.
3) Central Bank Credit to the Treasury (percentage change with respect to previous quarter at annual rate)	6.3	9.1	5.8	3.0	0.9	2.5	4.3
4) Current Account (billions of Lire)	-1,602	-1,582	-940	-1,087	-328	76	578

	1975											
	JAN	FEB	MAR	APR	MAY	JUNE	JULY	AUG	SEPT	OCT	NOV	DEC
5) Change in Official Net Foreign Assets[1] (billions of lire):	-160	-13	+122	+123	-20	-42	-108	+258	0	+44	-350	-538

Sources: Bank of Italy, Annual Report and Bulletin (various issues); International Monetary Fund, International Fi issues); and Central Institute of Statistics, Monthly Bulletin (various issues).

[1]Change in foreign exchange reserves gross of external official borrowing and of the change in commercial bank position. The reason for lumping together the Central Bank with commercial banks is that banks' foreign exchange o regulated--and often manipulated--by the monetary authorities.

[2]Figure taken from a statement by the Governor of the Bank of Italy (Financial Times, 1/22/1976).

Table 2

Italy: Economic Indicators: 1974-1976

	1974				1975				1976
	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q5
prices; respect to the evious year).	11.3	7.6	2.4	-2.8	-6.4	-5.8	-3.7	1.1	5.3
g Requirement/GDP	8.9	8.4	9.3	8.6	13.6	13.4	14.0	19.1	9.5
the Treasury h respect to nual rate)	6.3	9.1	5.8	3.0	0.9	2.5	4.3	12.3	7.1
ons of Lire)	-1,602	-1,582	-940	-1,087	-328	76	578	-703	-1,123

	1975												1976
	JAN	FEB	MAR	APR	MAY	JUNE	JULY	AUG	SEPT	OCT	NOV	DEC	1/1-1/20
	-160	-13	+122	+123	-20	-42	-108	+258	0	+44	-350	-538	-362[2]

Annual Report and Bulletin (various issues); International Monetary Fund, International Financial Statistics (various itute of Statistics, Monthly Bulletin (various issues).

xchange reserves gross of external official borrowing and of the change in commercial banks net foreign assets lumping together the Central Bank with commercial banks is that banks' foreign exchange operations were heavily pulated--by the monetary authorities.

statement by the Governor of the Bank of Italy (Financial Times, 1/22/1976).

Table 3

Mexico: Economic Indicators: 1980-1982

	1980				1981		
	Q1	Q2	Q3	Q4	Q1	Q2	Q3
1) Oil Price (1975 = 100)	233	247	267	294	294	294	294
2) Oil Production (1980 = 100)	94	98	102	106	107	119	114
3) Manufacturing Production (1980 = 100)	97	99	99	104	104	107	110
4) Trade Balance (billions of Peso)	4.0	-13.2	-24.5	-29.3	-9.7	-21.5	-40.
5) Change in Official Net Foreign Assets[1] (billions of Peso)	-27.8	-4.7	-26.1	-36.3	-26.5	-38.5	-98.
6) Central Bank Credit to the Government (percentage change with respect to the previous quarter at annual rate)	17.2	-.5	27.8	76.0	27.3	53.6	43.9
	1979	1980	1981	1982			
7) Budget Deficit/GDP (percent)	3.3	3.1	6.7	15.4			

Sources: IMF, International Financial Statistics (various issues), Balance of Payments Statistics Yearbook (1982 ar
Finance Statistics Yearbook (1983).

[1] Sum of the change in official reserves and public short- and long-term capital flows.

Table 3

Mexico: Economic Indicators: 1980-1982

	1980				1981				1982
	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1
	233	247	267	294	294	294	294	294	294
	94	98	102	106	107	119	114	112	114
on	97	99	99	104	104	107	110	108	110
	4.0	-13.2	-24.5	-29.3	-9.7	-21.5	-40.9	-29.4	-21.8
	-27.8	-4.7	-26.1	-36.3	-26.5	-38.5	-98.9	-139.5	+10.3
o the Government th respect to the nnual rate)	17.2	-.5	27.8	76.0	27.3	53.6	43.9	40.8	132.5
	1979	1980	1981	1982					
	3.3	3.1	6.7	15.4					

onal Financial Statistics (various issues), Balance of Payments Statistics Yearbook (1982 and 1983), and Government tics Yearbook (1983).

in official reserves and public short- and long-term capital flows.

Table 4

Chile: Economic Indicators: 1980-1982

	1980				1981		
	Q1	Q2	Q3	Q4	Q1	Q2	Q3
1) Change in Official Foreign Reserves (millions of U.S. dollars)	+448	+206	+395	+135	+65	+60	+137
2) Real Exchange Rate (1979 Q3 = 100)	93.6	94.6	90.4	85.1	79.7	77.5	77.3
3) Central Bank Domestic Credit (percentage change with respect to previous quarter at annual rate)	-23	+4	-4	+12	-110	-33	-16

	1978	1979	1980
4) Fiscal Deficit/GDP (percent)	0.9	-1.7	-0.6

Sources: IMF, International Financial Statistics (various issues) and Edwards (1985).

Table 4

Chile: Economic Indicators: 1980-1982

	1980				1981				1982	
	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2
eign Reserves ars)	+448	+206	+395	+135	+65	+60	+137	-172	-255	-216
	93.6	94.6	90.4	85.1	79.7	77.5	77.3	78.3	75.5	82.4
Credit h respect to nual rate)	-23	+4	-4	+12	-110	-33	-16	+117	+4	+96
		1978		1979		1980		1981		1982
rcent)		0.9		-1.7		-0.6		-3.0		-2.3

nal Financial Statistics (various issues) and Edwards (1985).