A GAME THEORETIC APPROACH TO THE CAPTIAL STRUCTURE AND INVESTMENT DECISION PROBLEMS OF A LEVERED FIRM

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ABSTRACT

This paper provides a formal model that characterizes conflicts of interest between bondholders and shareholders and that derives the perverse incentive effect of debt financing. A two person non-cooperative game is utilized. The Rational Expectation Nash Equilibrium(RENE) concept is applied to find equilibrium solutions. When the firm issues a large amount of debt, the RENE solution is shown to be Parĕto Inferior and MM Proposition I fails to obtain due to failure of MM Proposition III. This paper provides two sufficient conditions to restore the MM propositions in equilibrium. The use of convertible financial instruments conjectured by Jensen and Meckling(1976) is proved to be insufficient to restore the MM propositions globally in equilibrium. The use of protective covenants similar to the Grossman and Hart's(1982) precommitment or bonding behavior is shown to be sufficient and it is indeed in the best interest of shareholders.

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In their classic paper on the cost of capital, Modigliani and Miller (21; hereafter MM) demonstrated that in a perfect capital market and given some other assumptions such as absence of bankruptcy costs and tax subsidies on the payment of interest the value of the firm is independent of the financial structure (MM Proposition I) and that the firm's investment decision may be separated from its financing decision if the firm acts in the best interest of the shareholders (MM Proposition III). A number of subsequent studies reexamine the validity of the MM Propositions. These studies can be classified into two sets. The first set of studies, in general, tries to find which of the assumptions originally made by MM are important to the validity of MM Proposition I and how relaxation of those assumptions affects MM Proposition I.

For example, Hirshleifer (14), Stiglitz (29, 31), Fama and Miller (8) and Rubinstein (24) show that in addition to perfectly competitive market, the critical assumption is that bonds issued by individuals and firms are free of default risk. However they prove that relaxation of the no bankruptcy probability assumption does not affect the validity of MM Proposition I if one of the following three sufficient conditions hold: (a) a complete Arrow and Debreu security market (Hirshleifer, Stiglitz), (b) separation (Stiglitz, Rubinstein) and (c) me-first-rule (Fama and Miller). The second set of studies tries to break down MM Proposition I indirectly by first negating the MM Proposition III. For example, Stiglitz (30) and Myers (22) demonstrate that the value of the firm is a decreasing function of its debt equity ratio because the real decision is adversely affected by its financial decision. Jensen and Meckling (15) show that due to the principal-agent relationship there exist agency costs associated with both debt financing and equity financing. Among these costs are loss in the firm value due to the management's suboptimal real decision. Therefore MM Proposition I fails to obtain in these studies due to the failure of MM Proposition III. Among these studies, only Jensen and Meckling (15) provide a conjecture that the use of convertible financial instruments may help to restore the MM Propositions in equilibrium. Following their conjecture, a number of recent papers such as Mikkelson, Lo (17) and Green (9) show that the conversion privileges induce the firm to choose its value maximizing investment decision.

The principal objective of this paper is to reexamine the validity of the MM Propositions and the results obtained from subsequent studies, especially Jensen and Meckling's conjecture. Contrary to most of the current finance theory which seems only to spotlight the roles of shareholders and the firm (or management), the rational bondholders in this paper are endowed with their definite role. They try not only to protect their wealth from being expropriated by shareholders but also to influence the firm's real decision indirectly. This paper proposes a formal model that characterizes conflicts of interest between bondholders and shareholders and that derives the perverse incentive effect of debt financing. A two person non-cooperative game is utilized. The rational expectation Nash Equilibrium (RENE) concept is applied to find equilibrium solutions in a simple two period world. When the firm issues a large amount of risky debt the RENE solution is shown to be Pareto Inferior. MM Proposition I fails to obtain due to the failure of MM Proposition III. The loss in the market value of the firm incurred by the failure of MM Proposition III is absorbed by shareholders and our analysis reaffirms the previous results, especially Myers (22). Two sufficient conditions to restore the MM propositions are provided. The first sufficient condition is to eliminate the bondholders' incentive to participate in a game and the second sufficient condition is to get rid of the shareholders' incentive to be biased toward the riskier and suboptimal project, namely, to remove the perverse incentive effect of debt financing. The use of convertible financial instruments conjectured by Jensen and Meckling (15) is proved to be insufficient to restore MM propositions. Instead, the firm's voluntary provision of a binding contract with severe protective covenants to the bondholders satisfies one of two sufficient conditions. Shareholders, indeed, have incentives to support the management's action. The intuition behind the use of protective covenants is similar to the Grossman and Hart's (10) precommitment or bonding behavior of management.

This paper is organized as follows. Section I sets a simple two period model and obtains the same results as in the first set of studies. Section II applies a game theoretic model to demonstrate the failure of the MM propositions and provides two sufficient conditions to restore the MM proposition. Section III proves that the use of protective covenants rather than the use of convertible financial instruments is sufficient to restore MM propositions. Section IV discusses the existence of the optimal capital structure and the debt capacity of the firm. Finally section V concludes the paper.

I. VALUE OF THE FIRM WITH FIXED INVESTMENT

In order to develop some structure for the analysis to follow we make two sets of assumptions. The first set (permanent assumptions) are those which shall be maintained throughout this paper. The second set (temporary assumptions) are made only for expositional purposes and are relaxed as soon as the basic point has been clarified.

Permanent Assumptions:

(P1) Financial markets are competitive and perfect with no transaction costs including taxes and bankruptcy costs.

(P2) The rate of return on a safe asset is $1 + r_f$.

(P3) Investors are risk neutral, have rational expectations and share a symmetric information set.

(P4) The firm's management (or the firm) maximizes the wealth of current shareholders.

(P5) The firm has the option to default (limited liability) and pays no dividend until liquidation.

In this section, we shall discuss the effect of financial policy on the value of the firm with fixed investment under uncertainty in a simple two period model.

We define the following notations.

Define:

S_0 = Market value of equity

B_0 = Market value of non-coupon bearing bonds outstanding

V_0 $\equiv$ market value of the firm

R = Fixed claim in the form of a non-coupon bearing bond B_0

Assume: (T.1) Project Y requires an initial capital outlay I in period 0 and yields a random cash flow y in period 1 where y has a known density function g (y) and a cumulative distribution function G (y) on a compact support (a, b) where a = 0 and $b < \infty$.

The free entry condition in a competitive market dictates that net present value of the project Y is zero.[1] Suppose the firm took project Y. It raises I by issuing debt and equity and hence the cash flow identity is $V_0 \equiv I \equiv S_0 + B_0$. When the firm issues risky bonds worth B_0 with a face value of R_1, the return on the risky bond is

$$B_1 = \begin{cases} R_1 & \text{if} \quad y \geq R_1 \\ y & \text{if} \quad y < R_1 \end{cases} \tag{1}$$

and the return to the shareholders after paying the bondholders is

$$S_1 = \begin{cases} y - R_1 & \text{if} \quad y \geq R_1 \\ 0 & \text{if} \quad y < R_1 \end{cases} \tag{2}$$

We must now determine how R_1, the promised payment or the fixed claim, depends on the value of the bonds B_0. Since individuals are assumed to be risk neutral, all that bondholders require is that they receive the same average return as on a safe asset, $R_f = B_0 (1 + r_f)$.

Thus the expected return on bonds is

$$E (B_1 | R_1) = \int_a^{R_1} yg(y)dy + R_1 \int_{R_1}^{b} g(y)dy = R_f \tag{3}$$

Integrating equation (3) by parts and solving for R_1 gives

$$R_1 = R_f + \int_a^{R_1} G(y)dy \tag{4}$$

Equation (4) can be interpreted in this manner: when the firm issues risky bonds it should compensate the bondholders by the bankruptcy probability premium of $\int_a^{R_1} G(y)dy$,[2] otherwise no investors are willing to hold the bonds.

The equilibrium face value schedule R_1 is a convex function of B_0 and lies above the 45 degree line[3]

because $$dR_1/dB_0 = 1\ /dB_0/dR_1 = (1+r_f)\ /(1-G(R_1)) > 1 \tag{5}$$

and $$d^2R_1/dB_0{}^2 = (1+r_f)g(R_1)\ /(1-G(R_1))^3 > 0 \tag{5}$$

The expected return to the shareholders is

$$E(S_1|R_1) = \int_{R_1}^{b} (y-R_1)g(y)dy = \int_{R_1}^{b} yg(y)dy - R_1 \int_{R_1}^{b} g(y)dy \tag{6}$$

The equilibrium conditions for risk neutral investors are again

$$E(S_1/S_0|R_1) = E(B_1/B_0|R_1) = 1 + r_f \tag{7}$$

Equation (7) can be written

$$E(S_1+B_1/V_0|R_1) = 1 + r_f \tag{8}$$

Substituting equation (3) and (6) into equation (8) gives

$$\frac{1}{V_0}\int_a^b yg(y)dy = E(Y)/V_0 = 1 + r_f \tag{9}$$

It is easily seen that given the parameters $(E(y), r_f)$ there exists an infinite number of (S_0, B_0) combinations which satisfy the equilibrium conditions of equations (7), (8) and (9) and leave the value of the firm unchanged. Thus the existence of bankruptcy probability can not affect MM Proposition I.

II. INTERACTIONS BETWEEN REAL AND FINANCIAL DECISIONS

A. Suboptimality of Investment Decision

A well known value identity is $V_0 \equiv S_0+B_0$. The firm's real decision establishes the value of the firm V_0 by determining the characteristics of the return distribution of the firm. Given its investment decision, the financial decision of the firm, however, determines S_0 and B_0 by imposing the bankruptcy probability on bondholders without affecting characteristics of the return distribution of the firm. Therefore if there were "no external drains" such as taxes and bankruptcy costs, MM Proposition I would always obtain as an identity. The above scenario would be intuitively supportable provided that the firm had already fixed its investment decision or that there were no interactions between the real and financial decisions of the firm, i.e., if MM Proposition III held. In this section we relax the assumption that the firm has fixed its investment decision. Applying game theory we discuss how and why the firm's financial decisions affect its investment decisions and hence the value of the firm.

When the firm issues risky debt, it provides the first claim right on the value of the firm to the bondholders. Shareholders, instead, are given the right to vote, the right to claim residual value of the firm and the option to default, namely, limited liability. The difference in the priority of their claim rights and the limited liability make the bondholders' return function concave to the origin and the shareholders' return function convex to the origin as depicted in figure 1.

Insert Figure 1

The objective function to be maximized by the risk neutral bondholders and shareholders are respectively $E[U \cdot B_1|.,R]$ and $E[U \cdot S_1|.,R]$. Without loss of generality, a unit value is assigned to U so that both shareholders and bondholders maximize their expected return. It is quite natural to expect that any equilibrium solution should have the opposite implication among bondholders and shareholders because the objective functions of both parties are exactly opposite in their shape. Thus conflicts of interest between bondholders and shareholders are inevitable. When there exist conflicts of interest between individuals, game theory can be applied to illustrate the decision making process. Before constructing the game to be played by bondholders and shareholders, we make the following assumption.

Suppose there are two mutually exclusive investment projects Y and Z. Both projects require the same initial investment I.

Then:

(T.2): The return on project Z is distributed as a mean decreasing spread of the random return on project Y. In other words, $\tilde{z} \sim \tilde{y}+\tilde{w}+\tilde{e}$ where "~" means "has the same distribution as", $\tilde{w} \leq 0$ is a non-positive random return and $\tilde{e}$ is an uncorrelated noise term, $E(\tilde{e}|\tilde{y}+\tilde{w}) = 0$.

(T.2) simply asserts that $\tilde{z}$ can be constructed from $\tilde{y}$ by shifting some probability mass downward and adding some uncorrelated noise and figure 2 depicts this relationship. Without loss of generality we will use the dummy variable x for a variable of integration throughout the paper.

Insert Figure 2

Denote G(x) as a cumulative distribution function(CDF) of y and F(x) as a CDF of z. There exists an m such that

$$\begin{aligned} G(x) &< F(x) \qquad \text{if } x < m \\ G(x) &= F(x) \qquad \text{if } x = m \qquad (10) \\ G(x) &> F(x) \qquad \text{if } x > m \end{aligned}$$

$$\text{And } |\text{Area A}| = \int_a^m F(x)-G(x)dx > \int_m^b G(x)-F(x)dx = |\text{Area B}|$$

A special case of (T.2) that is studied in Rothschild and Stiglitz (23) occurs when $\tilde{w} = 0$, i.e., a case of mean preserving spread(MPS). The implications of (T.2) may be expressed as follows.

Implication 1: Project Y is the firm value maximizing project.

Proof: Let $C(x) = E(Y|G) - E(Z|F)$ (11a)

$$\text{Then } C(x) = \int_a^b xg(x)dx - \int_a^b xf(x)dx \qquad (11b)$$

$$= \int_a^b [F(x)-G(x)]dx \qquad (11c)$$

$$= \int_a^m [F(x)-G(x)]dx + \int_m^b [F(x)-G(x)]dx \qquad (11d)$$

$$= \text{Area A} - \text{Area B} > 0 \qquad (11e)$$

And the equilibrium condition dictates $V(Y|G)$ & $V(Z|F)$. **Q.E.D.**

Implication 2: There exists t such that $G(x) < F(x)$ if $x \varepsilon (a, t)$

and $\int_t^b [F(x)-G(x)]dx = 0$ and $\int_t^R [F(x)-G(x)]dx > 0$ if $R \varepsilon (t, b)$.

Proof: Let $C(t) = \int_t^b [F(x)-G(x)]dx$. Then $C(t)$ is a continuous and monotone decreasing function of t which is positive when t = a and negative when t = m. The intermediate value theorem implies the existence of $t \varepsilon (a, m)$ satisfying C = 0. **Q.E.D.**

Implication 3: There exists q such that

$$\int_a^q [F(x)-G(x)]dx + \int_m^b [F(x)-G(x)]dx = 0 .$$

Proof: Obvious.

Let us describe the game to be played by bondholders and shareholders. In game theory, the following two assumptions are often made.

1. **Rationality assumption:** each player is rational in the sense that given two alternatives, he will always choose the one he prefers, i.e., the one with the larger payoff.

2. **Knowledge assumption:** each player has full knowledge of the game, i.e., he knows not only the rules of the game but also the payoff function of the other player (game with perfect information).

These assumptions are compatible with ours. A game consists of:

1. 2 players denoted by S and B which stand for shareholders and bondholders.

2. 2 sets of pure strategies S_s and S_b, S_s for shareholders and S_b for bondholders.

3. Strategic outcome function f: $S_s \times S_b$ --> outcome.

Each player S or B attempts to maximize his expected value of outcome in a situation where his outcome depends not only upon his choice but upon the choice of the other player; in turn, the other's choice is influenced by choice the other thinks he is going to make, for the other too is attempting to maximize a function over which the other does not have full control. The type of game to be played by S and B is a "two person non-cooperative game" and the rational expectation Nash Equilibrium (RENE)[4] concept is applied to find equilibrium solutions.

Under the assumption (T.2), shareholders' strategy set S_s contains two pure strategies a_1 and a_2. a_1 denotes to choose project Y and a_2 is to choose project Z. Bondholders' strategy set also contains two pure strategies. Their strategies are b_1 which is to require R_1 and b_2 which is to require R_2 where R_1 is defined previously and R_2 is the amount of the fixed claim which makes bondholders indifferent between the return on a riskless asset and the return on risky bonds when the firm chooses project Z.

The payoff matrix of the game in normal form is shown in table 1.

Insert Table 1

$[E(S_1|a_1,b_1),\ E(B_1|a_1,b_1)]$ denotes the expected return on shareholders and bondholders respectively when the firm chooses project Y and the bondholders requires R_1 on their bond. The other entries are interpreted similarly. We now state and prove the following lemmas which are useful to establish the main propositions.

LEMMA 1: Given a project, the expected return on bonds, $E(B_1|.,\ R)$, is monotone increasing with the amount of the promised payment R, whereas the expected return on equity, $E(S_1|.,R)$, is a monotone decreasing function of R.

Proof: To prove the lemma, we should be able to show that the function $K(B_1|R) = E(B_1|.,R_2) - E(B_1|.,R_1)$ is monotone increasing in R_2.

We know if $R_1 = R_2$ then $K(B_1|R_2=R_1) = 0$. If $R_2 > R_1$,

$$K(B_1|R) = \int_a^{R_2} x dG(x) + R_2 \int_{R_2}^{b} dG(x) - \int_a^{R_1} x dG(x) - R_1 \int_{R_1}^{b} dG(x) \tag{12a}$$

$$= \int_{R_1}^{R_2} (1 - G(x))dx . \tag{12b}$$

Thus $K(B_1|R)$ is always positive and also monotone decreasingly increasing because $K'(B_1|R_2) = (1-G(R_2) > 0$ and $K'' = -g(R_2) < 0$. The later half part of lemma is easily proved by remembering that $E(y|G) = E(S_1|G,R) + E(B_1|G,R)$.
Q.E.D.

LEMMA 2: The promised payment R_2 under the project Z should be greater than R_1 under the project Y for a given B_0.

Proof: By the definition of R_1 and R_2,

$$E(B_1|G,R_1) = E(B_1|F,R_2) = (1+r_f)B_0 \ .$$

Our proof is complete if we prove $E(B_1|G,R_1) > E(B_1|F,R_1)$.

Let $K(B_1|R_1) = E(B_1|G,R_1) - E(B_1|F,R_1)$

$$\text{Then, } K = \int_a^{R_1} x dG(x) + R_1 \int_{R_1}^{b} dG(x) - \int_a^{R_1} x dF(x) - R_1 \int_{R_1}^{b} dF(X) \quad (13a)$$

$$= \int_a^{R_1} [G(x) - F(x)] dx \quad (13b)$$

Thus $K(B_1|R_1)$ is positive by the equation (11c) in implication 1. Applying LEMMA 1 leads to $R_2 > R_1$. **Q.E.D.**

The following inequalities result from monotonicity of the bondholders'objective function established in lemmas 1 and 2.

$$E(B_1|a_1=Y,b_1=R_1) < E(B_1|a_1=Y,b_2=R_2)$$

and

$$E(B_1|a_2=Z,b_1=R_1) < E(B_1|a_2=Z,b_2=R_2)$$

Therefore the bondholders' dominant strategy is to require R_2, namely, b_2. The shareholders' dominant strategy, however, is totally dependent upon the financial structure of the firm. The next proposition clarifies this point.

PROPOSITION 1: Under the assumptions (P.) & (T.2), the firm with high debt structure (R>t) always chooses the riskier and inferior project Z whereas the firm with low debt structure (R≤t) always chooses the less risky and firm value maximizing project Y. Though both decisions are indeed the self-fulfilling RENE, the former is Pareto Inferior but the latter is Pareto Optimal. Unfortunately, it is inevitable for shareholders to suffer from their wealth transfer to the bondholders in Pareto Optimal equilibrium.

Proof: (1) When the firm is heavily indebted, $R_2 > R_1 \geq t$.

Let $Q(S_1|R_1) = E(S_1|a_2=Z, b_1=R_1) - E(S_1|a_1=Y, b_1=R_1)$.

$$\text{Then, } Q = x(F(x) - G(x))\Big|_{R_1}^{b} - \int_{R_1}^{b} (F(x) - G(x))dx - R_1(F(x) - G(x))\Big|_{R_1}^{b}$$

$$= \int_{R_1}^{b} (G(x) - F(x))dx \geq 0 \qquad (14)$$

by implication 2.

$$\text{And } Q(S_1|R_2) = E(S_1|a_2=Z, b_2=R_2) - E(S_1|a_1=Y, b_2=R_2)$$

$$= \int_{R_2}^{b} (G(x) - F(x))dx > 0 \qquad (15)$$

again by implication 2.

The strategy a_2 strictly dominates a_1. Combined with bondholders optimal strategy b_2, a pair $(a_2=Z,\ b_2=R_2)$ becomes a stable but Pareto Inferior RENE. The Pareto dominant decision $(a_1=Y, b_1=R_1)$[5] can not be in equilibrium because both players have an incentive to break the equilibrium.

(2) When the firm is lightly indebted, $R_1 < R_2 \leq t$ and the same calculations as the above gives

$$E(S_1|a_2=Z,b_1=R_1) - E(S_1|a_1=Y,b_1=R_1) < 0$$

and (16)

$$E(S_1|a_2=Z,b_2=R_2) - E(S_1|a_1=Y,b_2=R_2) \leq 0 .$$

Thus the dominant strategy for shareholders becomes to choose the value maximizing project Y. A pair of decision (a_1=Y, b_2=R_2) constitutes a Pareto Optimal decision in the sense that no other decision can make one of the players better off without making the other worse off. In this equilibrium, the bondholders are paid too much and the shareholders suffer from their wealth transfer to the bondholders. **Q.E.D.**

An introduction of a corollary is in order.

Corollary 1: If there exists a wealth transfer from shareholders to bondholders, the shareholders' loss exactly offsets the bondholders' gain and the value of the firm remains unchanged.

Proof: It is obvious because $[E(S_1|a_1,b_2) - E(S_1|a_1,b_1)] + [E(B_1|a_1,b_2)] - [E(B_1|a_1,b_1)] = E(S_1+B_1|G) - E(S_1+B_1|G) = 0$. **Q.E.D.**

B. Sufficient Conditions to Restore the MM propositions

PROPOSITION 1 demonstrates the failure of MM Proposition III and hence the failure of MM Proposition I even in this simple model with informational symmetry. There are two interesting features in PROPOSITION 1. One is that high debt structure induces the firm to invest in a suboptimal project and the other is that shareholders always regret after the game is over.

The first feature is called "the perverse incentive effect of debt financing." Our simple model vividly illustrates how perversely debt financing affects the firms's investment decision. Let $Q(S_1|R) = E(S_1|G,R) - E(S_1|F,R)$. Then $Q(S_1|R) = \int_R^b [F(x)-G(x)]dx$. $Q(S_1|R)$ describes the shareholders' preference among two mutually exclusive projects Y and Z. A positive $Q(S_1|R)$ means that shareholders prefer the value maximizing project Y to the suboptimal project Z. If Q = 0, they would be indifferent among the two projects. And a negative Q indicates that shareholders prefer the suboptimal project Z. The sign of Q, however, is totally dependent upon R that is determined by the firm's financial structure. Q is a monotone decreasing function of R in an interval (a, m) because $dQ(S_1|R)/dR = G(R) - F(R) < 0$ by implication 1. Thus the firm and hence shareholders are biased toward the suboptimal and riskier project Z if the firm issues risky debt.

We now discuss it more intuitively. When the firm is all equity financed, shareholders share proportionately in the firm's return distribution. They prefer the less risky and firm value maximizing project Y because Q is postive by implication 1. If the firm were able to issue the riskless debt, i.e., the low bound $a > (1+r_f)B_0$, they would again support the project Y regardless of capital structure. Why? Because the safe debt implies a shift of the return distribution of the firm to the lefthand side by a constant of $B_0(1+r_f)$, namely, $\tilde{x}-(1+r_f)B_0$; shareholders still share proportionately in the return distribution. Again $Q > 0$ by implication 1. In this case, the firm's financing decision is separable from its investment decision, viz., MM Proposition III obtains and it is a simple extension of Fisher Separation Theorem to the uncertainty case.

Suppose the firm issued only risky debt. The firm's return distribution is no longer shared proportionately by all claim holders because of difference in their priority of claim rights on the firm value. Furthermore, the limited liability provision truncates the lefthand side of the firm's return distribution for shareholders and the righthand side of the firm's return distribution for bondholders. Therefore the shareholders' return function becomes convex to the origin but the bondholders' return function becomes concave to the origin. Now shareholders maximize their expected return over a truncated return distribution. Or to put it another way, shareholders maximize the expected value of their convex return function over a firm's return distribution.

As the firm issues more debt, the degree of truncation of the shareholders' return distribution or the degree of convexity of their return function becomes severer and the less risky and value maximizing project loses its attractiveness monotonically. When the firm's debt level reaches a critical level where $R=t$, shareholders become indifferent between two projects Y and Z. If the firm issues more debt, $R>t$, shareholders are always biased toward risker and suboptimal project Z. Thus the perverse incentive effect of debt financing is originated from truncation of the shareholders' return distribution or convexity of the shareholders' return function. Obviously bondholders always prefer the less risky and value maximizing project Y because their return function is concave to the origin and their return distribution is truncated from the righthand side.[6]

In the game context, we discuss the second feature that is also very much related with the perverse incentive effect of debt financing. The rational bondholders who share the same information set with shareholders should recognize the possibility of their wealth expropriation by the shareholders if the firm does not fix its investment decision. They surely have an incentive to participate in a game. In order to protect their wealth from expropriation by shareholders, bondholders require R_2 because R_2 gives an equilibrium expected return to bondholders even when the firm chooses the suboptimal prject Z. Shareholders, however, are not able to persuade bondholders to believe that they are going to choose the value maximizing project because the rational bondholders know that shareholders have an incentive to break their promise and to expropriate some part of the bondholders' wealth by investing in the suboptimal and riskier project.

If the firm under high debt structure decided to choose the value maximizing project, shareholders would be even worse off.[7] Therefore the firm under high debt structure has no other choice but to invest in the inferior project. The firm under low debt structure, on the other hand, always choose the value maximizing project because shareholders are still better off under project Y even after considering their wealth transfer to bondholders.[8] Again shareholders are lamenting for their misfortune.[9] It is interesting to see why the game theoretic approach provides a different explanation on the perverse incentive effect. Contrary to the current finance theory that seems to spotlight the roles of shareholders and firms (or managements), bondholders in this paper are endowed with their definite role which is to protect their own wealth from expropriation by shareholders.

Armed with the above discussion, we are now able to provide two sufficient conditions to restore MM propositions I and III. In order to restore MM Propositions I and III, the firm should be made to choose the value maximizing project X under any financial structure. In other words, if MM Proposition III is restored first then MM Proposition I is obtained immediately as we pointed out at the start of this section. The first sufficient condition is to eliminate the bondholders' incentive to participate in a game completely. If the game becomes a single person game, a game against uncertainty, the firm always chooses the value maximizing project. The second sufficient condition is to eliminate the shareholders' incentive to be biased toward the riskier and suboptimal project, namely, to eliminate the perverse incentive effect of debt financing. Though the firm would choose the value maximizing project under the second sufficient condition, it may or may not make the game a single person game. Thus shareholders always prefer the former to the latter because shareholders still suffer from their welath transfer to bondholders in NE if bondholders remain in the game.[10]

There are two ways to satisfy the first sufficient condition. The first way is to make bonds riskless. Since the firm is assumed to be subject to bankruptcy probability, the only viable way to make bonds riskless is to assume unlimited liability, that is by no means an interesting case. The second way that is of interest in this paper is to persuade bondholders to believe that they are guaranteed to receive the same average return as on a safe asset under any game situation. How should the firm persuade the rational bondholders? The firm should voluntarily provide a binding contract with protective covenants to the rational bondholders.

There are also two ways to satisfy the second sufficient condition. The first way is again to issue a riskless bond. But we are not interested in this trivial way. The second way is to use the convertible financial instruments conjectured by Jensen and Meckling (15) and utilized by Mikkelson, Lo (17) and Green (9) to restore the MM propositions. The next section provides some proof that the use of convertible financial instruments satisfied neither of the sufficient conditions globally.

III. THE ROLE OF CONVERTIBLE DEBT AND PROTECTIVE CONVENANTS

Two different prescriptions have been made so far: (i) protective covenants proposed by us and (ii) convertible financial instruments suggested by Jensen and Meckling (1976). The potency of any prescription inherently hinges upon its effectiveness to satisfy either of the sufficient conditions. We now evaluate the potency of the respective prescriptions.

A. Convertible Debt

Suppose the firm issues convertible debt instead of straight debt. The bond can be converted at any time into N share of common stock (the conversion ratio). The face value of the bond divided by N, R_c/N, is called the conversion price. The value of convertible bonds depends on its "bond value" and its "conversion value".. The bond value is what the bond would sell for if it could not be converted. The conversion value is what the bond would sell for if it had to be converted. In period 1, the bondholders would choose to convert if $(\frac{N}{N + E})\, x = vx > R_c$ where E is the number of shares outstanding, R_c is the promised payment to the convertible bondholders and $v \in [0, 1)$. Following Lo (17), we call v the conversion fraction. The return to the convertible bondholders is

$$B_1 = \begin{cases} vx & \text{if} \quad R_c/v \le x \\ R_c & \text{if} \quad R_c \le x < R_c/v \\ x & \text{if} \quad x < R_c \end{cases} \qquad (17)$$

The equilibrium condition again dictates that the expected return on a convertible bond should be equal to the return on a safe asset.

$$E(B_1|G,R_c,v) = E(B_1|G,R_1) = E(B_1|F,R_2) = R_f$$

The above equalities also imply $R_f \leq R_c < R_1 < R_2$ because a convertible bond is a bond plus a call option on the firm stock. Is the convertible bond able to guarantee the equilibrium expected return to the bondholders in any game situation? In other words, is the convertible bond able to change the type of game from a two person non-cooperative game to a single person game, i.e., a game against uncertainty? If yes, the firm would simply choose whichever project pays most to shareholders and it is indeed the firm value maximizing project because the bond value is a constant now and the value of the firm is simply a sum of S_0 and B_0.

The first sufficient condition, in this situation, can be expressed as

$$E(B_1|G,R_c,v) = E(B_1|F,R_c,v) = R_f \tag{18}$$

Then:

LEMMA 3: Under the assumptions (P.) & (T.2), the use of convertible debt does not satisfy the first sufficient condition globally but satisfies it locally by making a proper choice of v if and only if $R_c<q$ where q is defined in implication 3.

Proof: Let $K(B_1|R_c,v) = E(B_1|G,R_c,v) - E(B_1|F,R_c,v)$, then

$$K = \int_a^{R_c} xd[G(x)-F(x)] + \int_{R_c}^{\frac{R_c}{v}} R_c d[G(x)-F(x)] + \int_{\frac{R_c}{v}}^{b} vxd[G(x)-F(x)] \tag{19a}$$

$$= \int_a^{R_c} [F(x) - G(x)]dx + \int_{\frac{R_c}{v}}^{b} v[F(x) - G(x)]dx \tag{19b}$$

Note the first integral in equation (19b) is always positive by (T.2). If the second integral in equation (19b) is made to be equal to the first in absolute value, $K(B_1|R_c,v)$ becomes 0. If $R_c < q$, then $K(B_1|R_c,v) = 0$ by choosing v properly in the neighborhood of R_c/m. But if $R_c \geq q$, then any $v \varepsilon$ [0, 1) makes $K(B_1|R_c,V) > 0$ by implication 3. Therefore the first sufficient condition is not satisfied globally. **Q.E.D.**

We now ask whether the use of convertible debt is able to eliminate the shareholders' incentive to be biased toward the suboptimal project Z. The second sufficient condition can be represented succinctly as

$$E(S_1|G,R_c,v) > E(S_1|F,R_c,v) \tag{20}$$

The return to the shareholders is

$$S_1 = \begin{cases} (1 - v)x & \text{if} \quad R_c/v \le x \\ x - R_c & \text{if} \quad R_c \le x < R_c/v \\ 0 & \text{if} \quad x < R_c \end{cases} \tag{21}$$

Then:

LEMMA 4: Under the assumptions (P.) & (T.2), the use of convertible debt does not satisfy the second sufficient condition globally but satisfies it only locally up to $R_c \le m$.

Proof: Let $Q(S_1|R_c,v) = E(S_1|G,R_c,v) - E(S_1|F,R_c,v)$, then

$$Q(S_1|R_c,v) = [E(Y|G) - E(B_1|G,R_c,v)] - [E(Z|F) - E(B_1|F,R_c,v)]$$

$$= \int_{R_c}^{b} (F(x) - G(x))dx - \int_{\frac{R_c}{v}}^{b} v(F(x) - G(x))dx \ . \tag{22}$$

If $R_c \le t$, $Q(S_1|R_c,v) > 0$ for all $v \varepsilon [0, 1)$

If $t < R_c \le m$, $Q(S_1|R_c,v) \ge 0$ for some $v \varepsilon [0, 1)$.

If $R_c > m$, $Q(S_1|R_c,v) < 0$ for all $v \varepsilon [0, 1)$. **Q.E.D.**

We are in a good position to provide a proposition which describes the effectiveness and the limitations of the use of convertible bonds.

PROPOSITION 2: The use of convertible bond is not sufficient to restore the firm value maximizing decision. However the use of convertible bond raises from t to m the maximum debt level up to which the firm makes the Pareto Optimal decision.

Proof: Applying lemmas 3 and 4 establishes the proposition.

Remark 1: Provision of an intuitive explanation is in order. When the firm issues the convertible debt rather than the straight debt, the return function to the shareholders in equation (21) is a mixture of convex and concave curves whereas the return to the bondholders in equation (17) is a mixture of concave and convex curves as depicted in figure 3. The shareholders would prefer the value maximizing project X provided that the degree of concavity determined by the conversion privilege v is large enough to dominate the degree of convexity or truncation determined by the financial structure R_c. If $R_c > m$, no matter how much conversion privilege the firm provides to bond holders the perverse convexity effect resulting from the lefthand side truncation completely dominates the mitigating concavity effect resulting from the use of convertible financial instruments. Therefore the firm and hence shareholders are biased toward the riskier and suboptimal project and the use of convertible financial instruments can not resolve the perverse incentive effect.

Insert Figure 3

2. Protective Covenants

Suppose the bond contract contains indenture provisions which specifies the protective covenants such that

- Without making fair compensation to the bondholders the firm can not change its financial and investment policy which would decrease the value of the existing debt.
- Failure to comply with the first provision results in firm bankruptcy.
- Upon bankruptcy the claims of the bondholders equal the promised payment.

We further assume that these provisions are costlessly written and enforced. Under the assumption of informational symmetry between the shareholders and the bondholders, the covenants should work perfectly. The bondholders' payoff is now guaranteed by the covenants. The game becomes a single person game because bondholders have no incentive to participate in a game. Then the firm which maximizes the shareholders' wealth shall choose the project that provides the higher return to the shareholders. That project is indeed the value maximizing project because the bond value is fixed by the covenants and the firm value is simply the sum of S_0 and B_0. The preceding analysis may be summarized as follows:

THEOREM 1: Under the assumptions (P.) & (T.2), the protective covenants are sufficient to induce the firm to invest in the value maximizing project.

Proof: See the preceding discussion.

Remark 2: It is too naive to conclude that either shareholders or bondholders should suffer from their wealth expropriation in a symmetric information world. The rational expectation equilibrium concept in informational symmetry should lead one to a fair game result. The intuition behind the use of protective covenants is similar to Grossman and Hart's (10) precommitment or bonding behavior of the management to persuade the principal, the shareholders, that the management will act in the principal's interest. Jensen and Meckling (15) originally provide the example of precommitment or bonding behavior in a different context. They consider a situation where an entrepreneur must decide what share to retain in his firm. Since the entrepreneur is also a manager, his share in the firm will determine to what extent he pursues profits rather than perquisites. Therefore, his shareholding is a example of precommitment in the same way as debt in the Grossman and Hart model.[11]

IV. OPTIMAL CAPITAL STRUCTURE

In section II, we showed that the real decision of the firm and hence the value of the firm were very much affected by its financial decision, viz., the value of the firm decreased with increases in the equilibrium promised payment to the bondholders, R, because the increase in R above t induced the firm to choose suboptimal project Z. We also showed that the loss in market value of the firm, due to the suboptimal real decision caused by the existence of the risky debt, was absorbed by the firm's current shareholders.

In section III, we demonstrated that the shareholders would voluntarily include the indenture provision in their bond contract lest they should suffer from loss of their wealth. Then the firm's investment decision became Parato Optimal in equilibrium.

In this section, we discuss the existence of the optimal capital structure which maximizes the value of the firm and the existence of the maximum debt capacity the firm can service.

A. Optimal Financial Policy and Debt Capacity without Covenants

The next proposition is established based on the result in section III.

PROPOSITION 3: Under the assumptions (P.) & (T.2) and no covenants, the following are true:

- The value of the firm is a stepwise decreasing function of the promised payment (i.e., MM Proposition I holds only for the subinterval of the domain of the value function).

- The optimal financial policy which maximizes the value of the firm is to borrow up to $B_0 = E(B_1|*,R\leq t)/1+r_f$ but the optimal financial policy for shareholders is 100% equity financing.
- The maximum debt capacity the firm can service is less than 100% debt level.

Proof: As shown in PROPOSITION 2, the firm would choose the value maximizing project Y if $R \leq t$ and it would choose the inferior project if $R > t$. The optimal financial policy which maximizes the firm value is to borrow up to the point in which the equilibrium promised payment is less than t. Shareholders, however, suffer from their wealth transfer to the bondholders. Therefore the optimal financial policy for the shareholders is 100% equity financing. In figure 4 we illustrate the value of the firm schedule which is the vertical sum of the equity value curve and the debt value curve ($V_0 = S_0+B_0$).

Insert Figure 4

In order to illustrate the equilibrium bond value schedule, we solve equation (4) for B_0.

$$B_0 = (R_1 - \int_a^{R_1} G(x)dx)/1+r_f \tag{23}$$

This ideal bond value schedule should be a concave function of R and lie below the 45 degree line because $dB_0/dR = (1-G(R))/(1+r_f)$ is positive but less than unit value and $d^2B_0/dR^2 = -g(R)/(1+r_f) < 0$.

When the firm does not fix its investment decision between projects Y and

Z, the maximum amount of debt that the firm can borrow from the rational bondholders, is at most $B_0^* \le V_0(Z|F) < V_0(Y|G) = I$, i.e., the rational bondholders would not lend their money more than the value of the firm under the inferior project Z otherwise they will suffer from their wealth loss when the firm chooses project Z rather than project Y. Even if the firm really wants to invest in project Y which requires initial outlay I it can not raise I by all debt. Thus the maximum achievable debt level is always less than its potential 100% debt level. **Q.E.D.**

B. **Generalization**

Suppose there exists n mutually exclusive projects that fit into assumption (T.2) where n is any positive integer such as $2<n<\infty$. We can show by induction that there exist n-1 t_i such as $t_1 <, - - -, <t_{n-1}<m$ and n such value curves for the firm, for equity and for debt similar to those in figure 4.

Then the value of the firm schedule would be a smooth and monotone decreasing function in R, though the speed of decrease in the firm value is dependent upon the characteristics of the return distributions of those mutually exclusive projects. However the equity value curve should be convex (increasingly decreasing) and the debt value curve be concave (decreasingly increasing) regardless of the characteristics of the return distributions (solid curves).

Insert Figure 5

The shape of the debt value curve in figure 5 is determined by the shape of the firm value curve. For example, if the firm value approaches zero as R approaches b the debt value curve would be a concave down parabola as in Myers (22: pp. 154). But the shape of the equity value curve should be convex always(broken curves). The optimal financial policy for both shareholders and firm value is 100% equity financing and the maximum achievable debt capacity is also less than 100% level as in PROPOSITION 3.

C. Optimal Financial Policy and Debt Capacity with Covenants

As we have shown in the previous section, shareholders would provide covenants to the bondholders in order to persuade the bondholders that they would choose the value maximizing project Y which is in turn in the shareholders' best interest. In that case the value of the firm becomes $V(Y|G)$ and both the bondholders and the shareholders earn the equilibrium expected return $1+r_f$ and therefore MM Propositions I & III obtain again. The firm now has 100% debt capacity. These results are shown in figure 6.

Insert Figure 6

It is obvious from figure 6 that the rational shareholders want to have their value schedule (1) rather than the others because the value schedule (1) clearly dominates the value schedules (2) & (3).

V. CONCLUSION

We have established the irrelevance of financial policy and the separability of the financial decision from the investment decision, namely, MM Propositions I and III, under a fairly general set of conditions applying game theory. The logic backing the MM irrelevance proposition is that investors can create any return opportunities that a firm is able to create by altering its capital structure. Following Stiglitz (30) we implicitly assume a mild segmentation of security markets on purpose to block an application of the above logic and hence to result in failure of MM propositions. Considering this, our results are all the more striking since the MM propositions are restored in equilibrium.

In this paper, we develop a formal model in which the perverse incentive effect of debt financing is derived explicitly. We provide two sufficient conditions to restore the MM propositions and demonstrate that the use of convertible financial instruments is not necessarily able to restore the MM propositions.

The use of game theory in corporation finance is a new approach and deserves further attention. A simple two period model is unfortunately not suited to represent the real world where the firm is a going concern. This fact suggests that infinite replications of the game may give quite different solutions. In that case, the role of reputation may supersede our binding covenants even under the asymmetric information setting.

FOOTNOTES

[1]A positive net present value assumption can be safely incorporated into our model.

[2]It should be clear that $\int_a^{R_1} G(y)dy$ is positive because $0 \leq G(y) \leq 1$ by the definition of CDF and the domain of integration is a subset of the positive real line.

[3]This analysis implies that given a firm's investment decision if shareholders are able to issue the new debt with an equal claim on the asset of the firm, bondholders will always experience their wealth expropriation by shareholders.

[4]Definition of NE can be found in any game theory book. Giving a brief definition, $(s_1, - - -, s_n)$ is a NE if $U_i(f(s_1, - - -, s_n))$ P_i $U_i(f(s_1, - -, s_{i-1}, s_{i*}, s_{i+1}, - -, s_n))$ for all $i=1, - - -, n$ and $s_i \in S_i$ where S_i is a set of strategies available to i. Verbally, every unilateral change can not make him better off.

[5](a_1, b_1) Pareto dominates (a_2, b_2) because the shareholders can be made better off without making the bondholders worse off under (a_1, b_1). That is, $E(S_1|a_1=Y, b_1=R_1) - E(S_1|a_2=Z, B_2=R_2) = [E(Y|G) - E(B_1|a_1=Y, b_1=R_1)] - [E(Z|F) - E(B_1|a_2=Z, b_2=R_2)] = E(Y|F) > 0$ because $E(B_1|a_1=Y, b_1=R_1) = E(B_1|a_2=Z, b_2=R_2)$ by the definitions of R_1 and R_2.

[6]The perverse incentive effect of debt financing is originally demonstrated by utilization of the option pricing model. Merton (19) demonstrates that $\partial S_0/\partial\sigma = -\partial B_0/\partial\sigma > 0$. It is very easy to obtain the same comparative static results from our model. Suppose the firm's random return is normally distributed, i.e., $\tilde{y} \sim N(\mu,\sigma^2)$. The first two moments fully characterize the distribution. And an increase in risk in Rothschild-Stiglitz sense becomes synonymous with an increase in variance in this case. The value of the firm under risk neutrality assumption is $S_0 = E(S_1|G,R)/1+r_f$. For a normal distribution, $S_0 = \{(\mu-R)(1-G(R))+\sigma^2 g(R)\}/1+r_f$.
Then $-\partial B_0/\partial\sigma = \partial S_0/\partial\sigma = g(R)/1+r_f > 0$ via several manipulations. Other comparative static results of Merton are easily obtained from our simple model. Also a simple standardization of the normal random variable and the convolution principle leads our simple model to the discrete time option pricing model of Brennan (4),
$S_0 = (V_0-R(1+r_f)^{-1})N\{[V_0(1+r_f)-R]/\sigma\} + (1+r_f)^{-1}\sigma\, n\{[R-V_0(1+r_f)]/\sigma\}$ under normal distribution. If we assume a lognormal distribution, RNVRs of Brennan and Rubinstein (25) is generated easily from our model. For detail, refer to RYU (27).

[7]See equation (14).

[8]See equation (16).

[9]This result is very similar to Myers (22) though he draws a quite different line of reasoning for it. Myers shows that the firm financed with debt shall pass up valuable investment opportunities which could make positive net contributions to the market value of the firm so that the firm value may

be the decreasing function of risky debt financing. He also shows that the loss in market value is absorbed by the firm's current shareholders and that the optimal strategy is to issue no risky debt.

[10]The monotonicity of the bondholders' objective function dictates bondholders to choose b_2 as their dominant strategy.

[11]Also in Leland and Pyle (16).

REFERENCE

1. H.E. Allen. "Capital Structure and Imperfect Competition in Product Market," mimeo, The Wharton School, University of Pennsylvania, 1984.

2. F. Black and M. Scholes. "The Pricing of Options and Corporate Liabilities," Journal of Political Economy 81 (May-June 1973), 637-659.

3. R. Brealey and S. Myers. "Principles of Corporate Finance," New-York:McGraw-Hill, 1981.

4. M. J. Brennan. "The Pricing of Contingent Claims in Discrete Time Models," Journal of Finance 34 (March 1979), 53-68.

5. H. DeAngelo. "Competition and Unanimity," American Economic Review 71 (March 1981), 18-27.

6. S. Ekern and R. Wilson. "On the Theory of the Firm in an Economy with Incomplete Markets," Bell Journal of Economics and Management Science 5 (Spring 1974) 171-180.

7. E.F. Fama. "The Effects of a Firm's Investment and Financing Decisions on the Welfare of Its Security Holders," American Economic Review 68 (June 1978) 272-284.

8. E.F. Fama and M.H. Miller. "The Theory of Finance," New-York:Holt, Rinehart, and Winston, 1972.

9. R.C. Green. "Investment Incentives, Debt, and Warrents," Journal of Financial Economics 13 (March 1984), 115-136.

10. S. Grossman and O. Hart. "Corporate Financial Structure and Managerial Incentives," in:J. McCall ed., The Economics of Information and Uncertainty, University of Chicago Press, Chicago, IL. 1982.

11. J. Hardar, and W.R. Russell. "Rules for Ordering Uncertain Propects," American Economic Review 49 (March 1969), 25-34.

12. G. Hanoch and H. Levy. "The Efficiency Analysis of Choices Involving Risk," Review of Economic Studies 36 (July 1969), 335-346.

13. R. Haugen and L. Senbet. "Resolving Agency Problems of External Capital through Options," Journal of Finance 36 (1981), 629-647.

14. J. Hirshleifer. "Investment Decision under Uncertainty: Applications of the State Preference Approach," Quarterly Journal of Economics 80 (May 1966), 252-277.

15. M. Jensen and W. Meckling. "Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure," Journal of Financial Economics 3 (October 1976), 305-360.

16. H. Leland and D. Pyle. "Informational Asymmetries, Financial Structure, and Financial Intermediation," Journal of Finance 32 (May 1977), 371-387.

17. A. Lo. "Suboptimal Investment Policies under Bankruptcy Risk and the Use of Convertible Debt," mimeo, Harvard University, MA, 1983.

18. R. Luce and H. Raiffa. "Games and Decisions," New-York, Wiley, 1957.

19. R.C. Merton. "A Rational Theory of Option Pricing," Bell Journal of Economics and Management Science 4 (Spring 1973), 141-183.

20. __________. "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates," Journal of Finance 29 (May 1974), 449-470.

21. F. Modigliani and M. Miller. "The Cost of Capital, Corporation Finance and the Theory of Investment," American Economic Review 48 (June 1958), 261-297.

22. S. Myers. "Determinants of Corporate Borrowing," Journal of Financial Economics 4 (November 1977), 147-176.

23. M. Rothschild and J. Stiglitz. "Increasing Risk: I. A Definition, Journal of Economic Theory 2 (September 1970), 225-243.

24. M. Rubinstein. "A Mean-Variance Synthesis of Corporate Financial Theory," Journal of Finance 28 (March 1973), 167-181.

25. ____________. "The Valuation of Uncertain Income Streams and the Pricing of Options," Bell Journal of Economics and Management Science 7 (Autumn 1976), 407-425.

26. S.W. Ryu. "On the Role of Risk Neutrality in Corporation Finance and its Application," mimeo, The Wharton School, Universtiy of Pennsylvania, PA, 1985 c.

27. C. Smith and J. Warner. "On Financial Contracting: An Analysis of Bond Covenants," Journal of Financial Economics 7 (June 1979), 117-161.

28. J. Stiglitz. "A Re-examination of the Modigliani-Miller Theorem," American Economic Review 59 (December 1969), 784-793.

29. __________. "Some Aspects of the Pure Theory of Corporate Finance: Bankruptcies and Take-Overs," Bell Journal of Economics and Management Science 3 (Autumn 1972), 458-482.

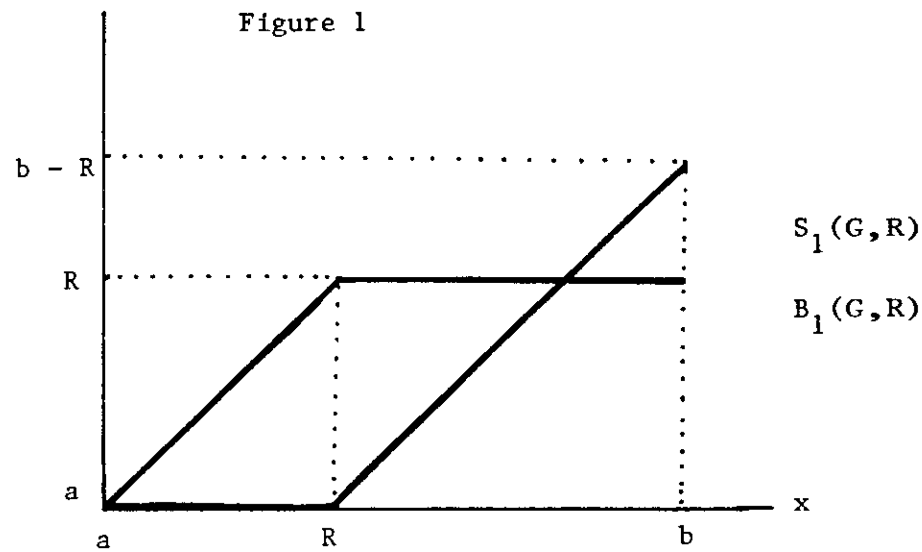
30. __________. "On the Irrelevance of Corporate Financial Policy," American Economic Review 64 (December 1974), 851-866.

Payoff matrix in normal form

S \ B	b_1	b_2
a_1	$E(S_1 \mid a_1=Y,\ b_1=R_1),\ E(B_1 \mid a_1=Y,\ b_1=R_1)$	$E(S_1 \mid a_1=Y,\ b_2=R_2),\ E(B_1 \mid a_1=Y,\ b_2=R_2)$
a_2	$E(S_1 \mid a_2=Z,\ b_1=R_1),\ E(B_1 \mid a_2=Z,\ b_1=R_1)$	$E(S_1 \mid a_2=Z,\ b_2=R_2),\ E(B_1 \mid a_2=Z,\ b_2=R_2)$

Table 1

Figure 1



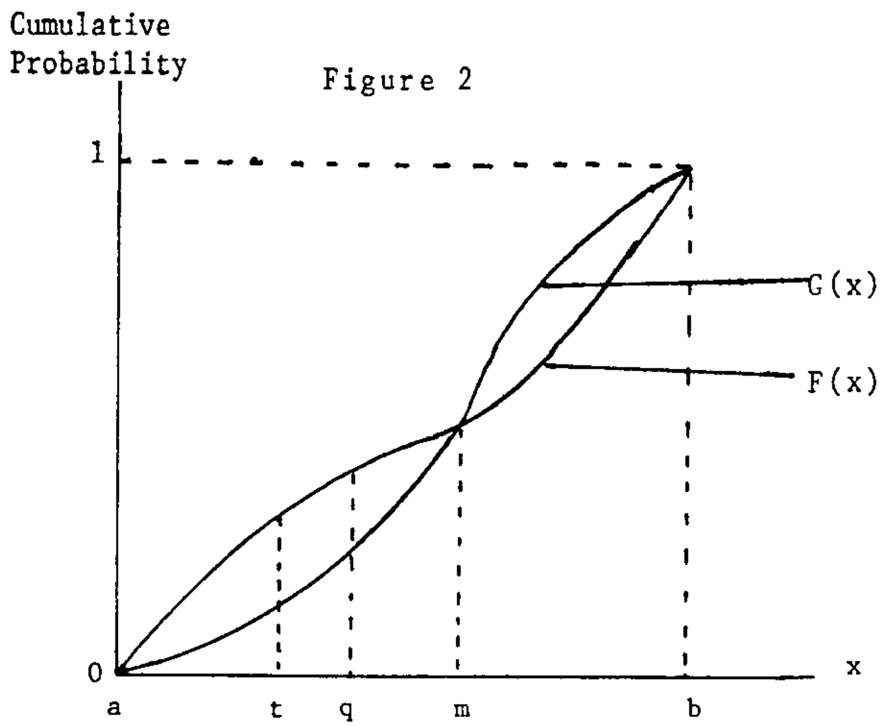


Figure 2

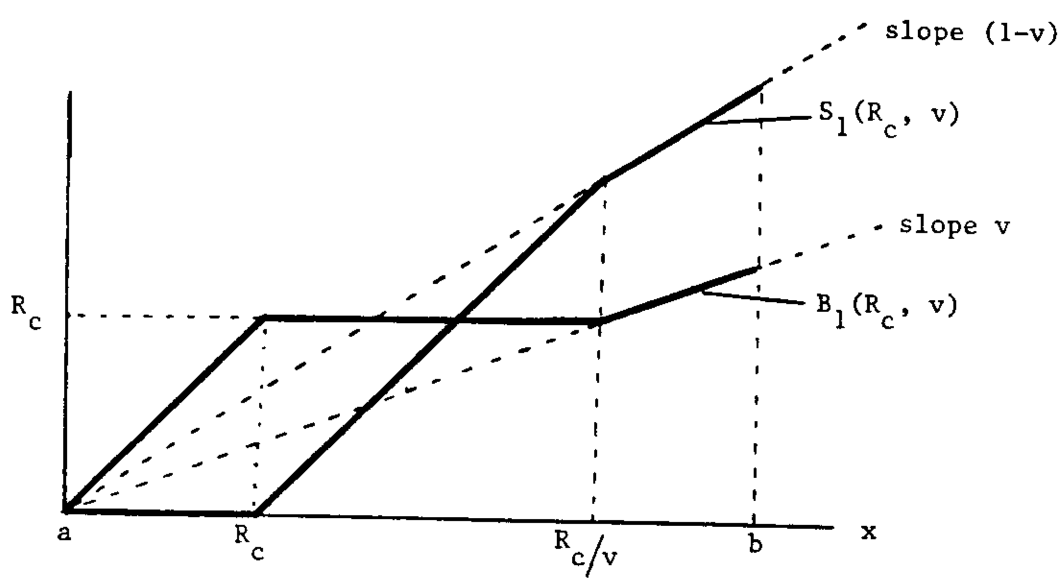


Figure 3

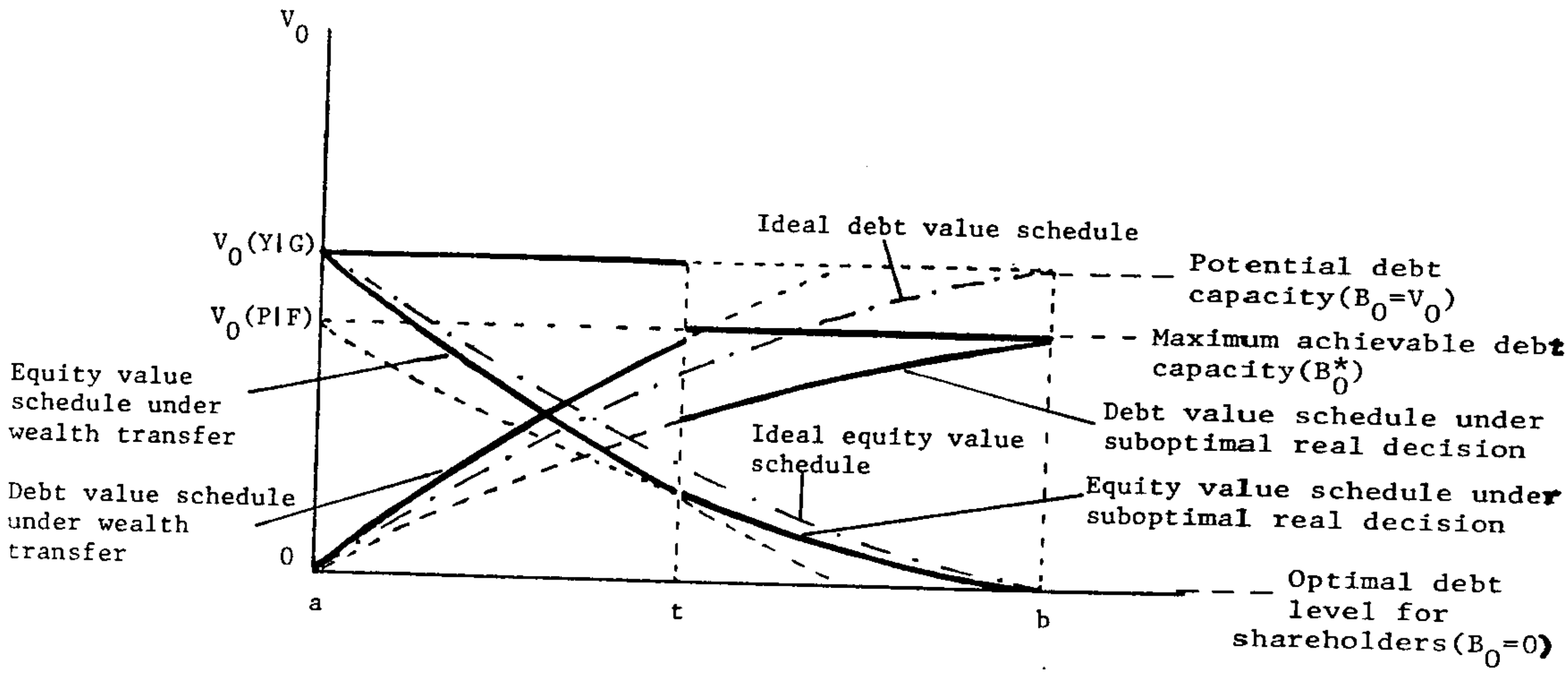

V0
V0(Y|G)
V0(P|F)
0
a
t
b
Ideal debt value schedule
Potential debt capacity(B0=V0)
Maximum achievable debt capacity(B*0)
Debt value schedule under suboptimal real decision
Ideal equity value schedule
Equity value schedule under suboptimal real decision
Optimal debt level for shareholders(B0=0)
Equity value schedule under wealth transfer
Debt value schedule under wealth transfer

Figure 4

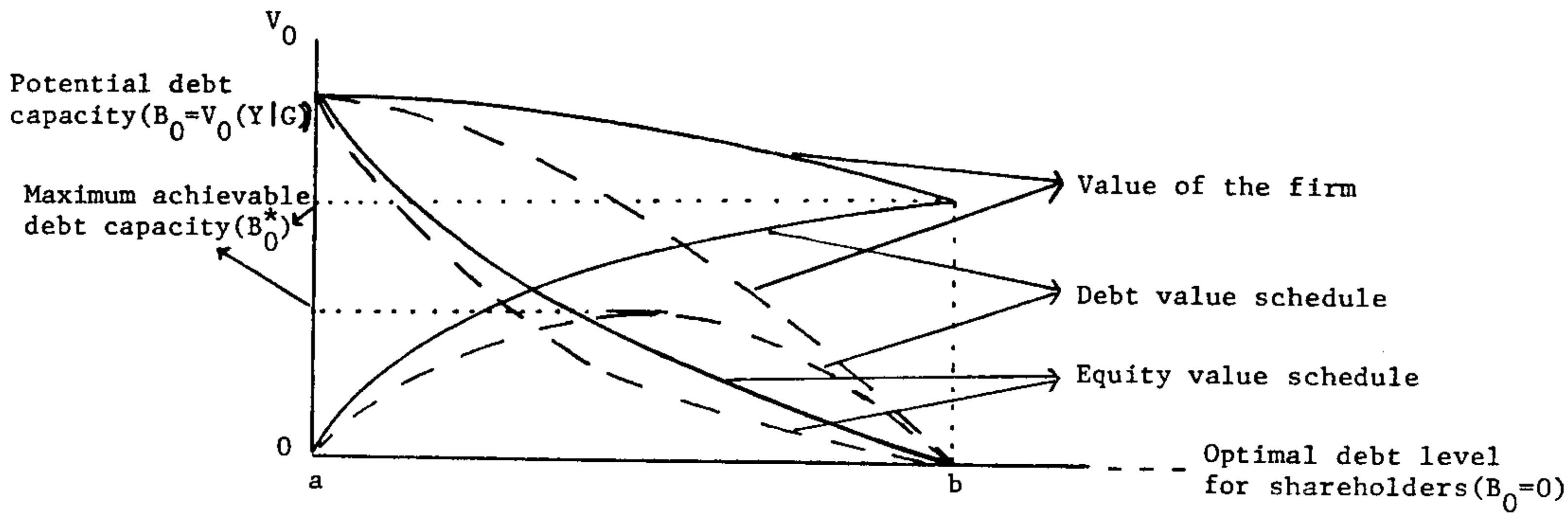


Figure 5

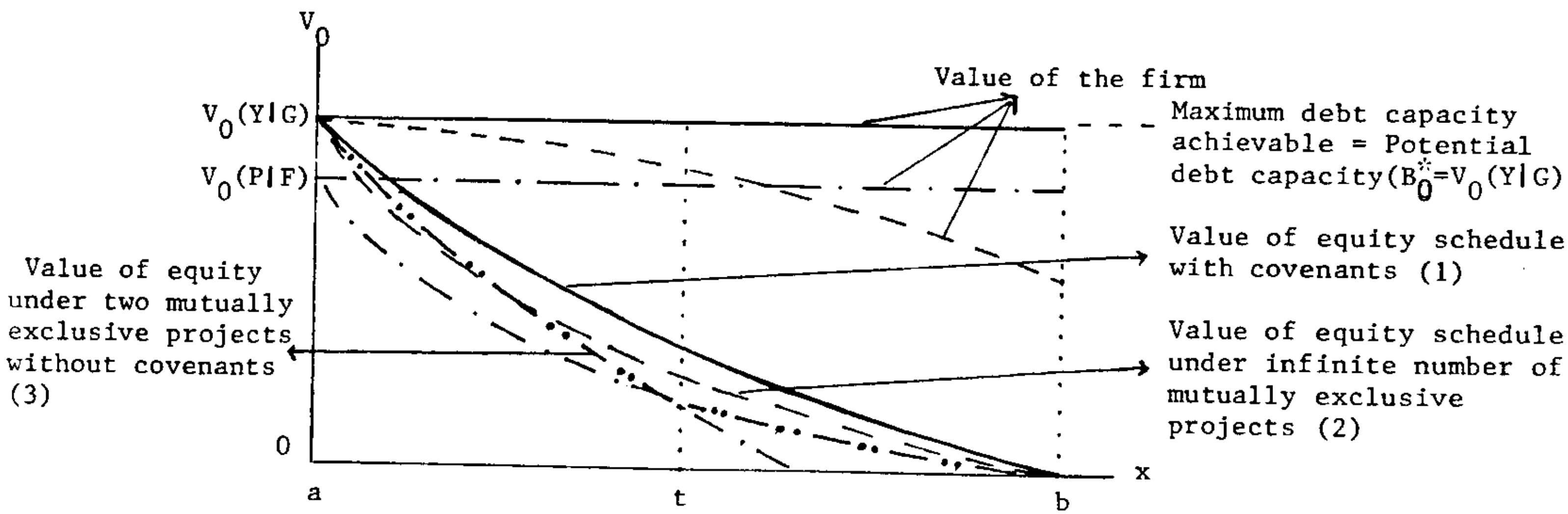


Figure 6