DECISION INTERVALS AND PORTFOLIO STRATEGY

by

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The comments of George Pennachi are greatly appreciated.

I. Introduction

A question that portfolio managers frequently ask is how frequently should the proportions in a portfolio be re-evaluated and rebalanced. If the costs of evaluating a portfolio and trading were zero, the answer would clearly be continuous rebalancing. Since there are such costs, continuous revision is probably not optimal. The purpose of this paper is to explore the relationship between the length of the interval between rebalancing the proportions in a portfolio and the investment strategy of individual investors. The results of this paper are also of relevance in evaluating the assumption of continuous trading implicit in many financial models, such as the option pricing models of Black and Scholes or of Merton.

To simplify the analysis, we shall assume that there are only two available assets and that short sales of either asset are prohibited. Under these assumptions, the investment strategy of an investor can be characterized by how the proportion invested in one of the assets varies with the length of the trading interval. In the process of developing this relationship, the paper will try to provide some intuition into this relationship.

II. Prior Literature

Despite its obvious practical relevance, there has been very little investigation into the relationship between investment strategy and the length of the trading interval. A notable exception is the work of Goldman (1979), who has provided some results on how investment strategy changes as the length of the trading interval approaches infinity. His particular model assumes that there are two risky assets whose returns follow Weiner processes and that the utility functions of investors display constant proportional risk aversion. Goldman further allows the returns of the two risky assets to be correlated cross-sectionally, but not over time. This further assumption of cross-sectional correlation is not critical to Goldman's main results.

As the trading interval approaches infinity, Goldman shows that an investor would, except in one special case, steadily increase the investment in one of the two assets and, as a result, reduce the investment in the other. Interpreting this plunging behavior as an anti-diversification result, Goldman concludes that diversification becomes less relevant as the trading horizon increases and what he terms "the favoritism principle" becomes more important.

Obviously bothered by this result, Fischer and Pennachi (1985) augment Goldman's model with the assumption that the returns of either asset are serially correlated over time. With this additional assumption, they show that this plunging behavior does not necessarily occur.

III. A Binomial Process

For the purposes of this section, consider a two-period or three-date world with dates designated 0, 1 and 2. Further, assume that the investor has a utility function defined over wealth at date 2 and that this utility function displays constant proportional risk aversion. Previous empirical work suggests that the coefficient of relative risk aversion is greater than one.[1]

In this world, we compare the investment strategy of an investor who can trade at date 0 and trade again at date 1 with that of an investor who can trade only at date 0. For subsequent reference, let the first case be termed the "revision" case and the second the "no revision" case.

In this three-date world, there are two assets. The first is a riskfree asset with a return of r_f per period. The second is a risky asset with a

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[1] Cf. Friend and Blume (1975) and Morin and Suarez (1983).

return of r_h or r_ℓ per period with each of the returns equally likely. To avoid stochastic dominance, we shall assume that $r_h > r_f > r_\ell$ and that the expected return of the risky asset is greater than the riskfree return. For subsequent use, let R_i represent the return relative for return i, defined as $1 + r_i$.

A. A Numerical Example

To be concrete, let us for the moment attach specific values to the returns and the coefficient of relative risk aversion. Specifically, let r_f be 0.10, r_ℓ be -0.10 and r_h be 0.50. Let the coefficient of relative risk aversion be 2.0, so that the utility function takes the form $-W^{-1}$, where W is wealth at date 2. Finally, let the investor's wealth at date 0 be one dollar.

The investor who is allowed to revise at date 1 can use dynamic programming to determine the optimal decision at date 0. As shown in Mossin (1968), the recursive function in this problem will be of the same form as a constant proportional risk aversion function with an unchanged coefficient of relative risk aversion. In this binomial model of returns, the optimal proportion to place in the risky asset can be determined analytically and is 0.66726. This number will also be the optimal proportion to put in the risky asset at date 1 due to the assumption of constant relative risk aversion. But, as shown below and not surprisingly, this proportion is not optimal for an investor who is precluded from revising at date 1.

To explore the effect of the length of the trading interval, let us compare the distributions of wealth at date 2 for an investor who invests optimally at date 0 and revises optimally at date 1 to an investor who puts the same proportion at risk initially but does not revise at date 1. The resulting distributions are as follows:

	Wealth (initial proportion) = 0.66726	
Probability	With Revision	Without Revision
0.25	1.86843	1.90395
0.50	1.32118	1.30342
0.25	0.93421	0.94310
E(W)	1.36125	1.36347
σ(W)	0.33272	0.34498

Not unexpectedly, the standard deviation of returns without revising is greater than with revising. Fischer and Pennachi (1985) observe the same increase in standard deviation as the trading interval increases using a Weiner process instead of binomial processes. However, the increase in standard deviation does not fully capture what is happening. Not only is the largest outcome without revision greater than the largest outcome with revision, but also the worst outcome without revision is greater than the worst outcome with revision. Moreover, the arithmetic difference between the largest outcome without revision and that with revision is greater than the difference for the worst outcomes. These increases would, of course, increase expected utility. But the middle outcome without revision is less than the middle outcome with revision. This decreases expected utility and by a greater amount than the increase in expected utility due to the changes in extreme outcomes.

What is happening is extremely complex due to the compounding effects associated with multiperiod returns. The first and second moments no longer capture all the information that is relevant to an investor for evaluating an investment. Obviously, third and higher moments are required. Indeed, the measure of skewness increases from 0.356 in the revision case to 0.506 in the no revision case.

Moreover, these relationships between the probability distribution with and without revision are quite general. Specifically, as long as the proportion invested in the risky asset is not zero or one, it can be shown that the pattern of greater extreme values and lesser middle values holds for any binomial process whose returns satisfy the non-dominance condition given above.[1]

An investor who is not permitted to revise at date 1 has three options: increase the proportion at risk in comparison to the optimal solution for an investor who is allowed to revise, not change the proportion, or decrease the proportion. Increasing the proportion would lead to an increase in the largest outcome and a decrease in the worst outcome. The increase in the largest outcome would increase expected utility, while the decrease in the worst outcome would decrease expected utility. For the specific numbers in this example, the middle outcome would also increase, further increasing expected utility.

Whether the middle outcome increases or decreases depends upon the relationship of the geometric mean of the risky asset and the riskfree asset. If the geometric mean $(R_\ell R_h)^{\frac{1}{2}}$ exceeds R_f, as it does in this example, increasing the proportion at risk will increase the value of the middle outcome. If they are equal, there is no change. If less, the middle outcome decreases. As will be seen in the next section, the relationship of the geometric mean to R_f will play a key role in how an investor with a logarithmic utility function will alter the proportion placed at risk as the trading

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1 Let α be the proportion placed at risk. If the risky return is the same in both time periods, the wealth with revision will be $(\alpha R_f+(1-\alpha)R_i)^2$ and without revision $\alpha R_f^2 + (1 - \alpha)R_i^2$. By convexity, the wealth with revision will exceed the wealth without revision. To show that the wealth without revision exceeds the wealth with revision if the risky return is less than r_f in one period and greater than r_f in the other period, note that $(R_\ell - R_f)(R_h - R_f) < 0$. With some manipulation, this inequality can be rewritten as $[(1-\alpha)R_f + \alpha R_\ell][(1-\alpha)R_f + \alpha R_h] > [(1-\alpha)R_f^2 + \alpha R_\ell R_h]$, proving the statement.

interval increases.

Not changing the proportion at risk would leave the distribution as above. Decreasing the proportion at risk would increase the worst outcome but decrease the largest outcome. The increase in the low outcome increases expected utility, while the decrease in the largest outcome reduces expected utility. Again for the specific numbers in this example, the middle outcome would decrease, reducing expected utility.

In this example, the investor who could not revise would want to increase the proportion at risk at date 0 from 0.66726 to 0.66997.[1] The effect on the outcomes is shown in Table 1. Also in the table is another example which is the same as the above except that the coefficient of relative risk aversion is 8.0. In this second example, the investor would decrease the proportion at risk from what would be optimal if revision were allowed at date 1. It might be noted that the arithmetic changes in optimal proportions from the revision case to the non-revision case are not large. More will be said of this later.

Finally, it is instructive to compare this problem to the one of estimation risk that Klein and Bawa (1976) have examined. Intuitively, it might be thought that, as the trading interval increases, the inability to readjust the portfolio introduces random noise into the final return distribution. Such random noise might be likened to the estimation error that Klein and Bawa analyze. They show that the investor would unambiguously increase the proportion placed in the riskfree asset as such estimation risk increases. However, the model of this paper is quite different from theirs. Klein and Bawa assume that the returns are normally distributed for any trading interval and that estimation risk merely adds some normal noise to the return distribution.

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[1] A binary search of the derivative of the expected utility provides this new optimal proportion.

Thus, estimation risk makes the investor's subjective distribution of returns "fatter tailed." Because of the symmetry of the normal process and the assumed concavity of the utility function, the investor will always decrease the proportion at risk as the trading interval increases. Their assumption of normality does not take into account the compounding effect of returns. It is this compounding effect that makes the model of this paper fundamentally different from that of Klein and Bawa.

B. Some Propositions

To introduce some notation, let α be the proportion that the investor who can revise at date 1 places at risk at date 0. Because of the assumption of constant proportional risk aversion, let also assume that the proportion that this investor would place at risk at date 1 is also α, so that there is no need for a time subscript. Let β be the proportion that the investor who cannot revise at date 1 places at risk at date 0. Let γ be the coefficient of relative risk aversion. Thus, the utility function in terms of wealth at date 2, W_2, is of the form

$$U(W_2) = \begin{cases} \frac{1}{1-\gamma} W_2^{1-\gamma} & , \quad \gamma \neq 1 \text{ and } \gamma > 0 \ . \\ \ln W_2 & , \quad \gamma = 1 \ . \end{cases} \tag{1}$$

If the investor revises at date 1 the proportion in the risky asset to the same α, the distribution of wealth at date 2 is

$$\begin{aligned} W_{\ell h} &= (R_f + \alpha(R_h - R_f))^2 \\ W_{h\ell} &= (R_f + \alpha(R_h - R_f))(R_f + \alpha(R_\ell - R_f)) \\ W_{\ell\ell} &= (R_R + \alpha(R_\ell - R_f))^2 \end{aligned} \tag{2}$$

with respective probabilities of ¼ , ½ and ¼ . The distribution of wealth at date 2 for the investor who cannot revise at date 1 for a given β is

$$\begin{aligned} V_{hh} &= R_F^2 + \beta(R_h^2 - R_F^2) \\ V_{h\ell} &= R_F^2 + \beta(R_h R_\ell - R_F^2) \\ V_{\ell\ell} &= R_F^2 + \beta(R_\ell^2 + R_F^2) \end{aligned} \tag{3}$$

with respective probabilities of ¼ , ½ and ¼ .

The optimal value of α^* is that value that maximizes

$$\tfrac{1}{4}\, U(W_{hh}) + \tfrac{1}{4}\, U(W_{\ell\ell}) + \tfrac{1}{2}\, U(W_{h\ell}) \tag{4}$$

The optimal value of β^* is that value that maximizes

$$\tfrac{1}{4}\, U(V_{hh}) + \tfrac{1}{4}\, U(V_{\ell\ell}) + \tfrac{1}{2}\, U(V_{h\ell}) \quad . \tag{5}$$

All of these maximization problems are subject to the constraint that α or β be in the closed interval 0.0 to 1.0.

The first proposition shows that if an investor who can revise at date 1 would invest everything in either the risky or the riskfree asset, an investor who could not revise at date 1 would choose the same strategy. Thus,

<u>Proposition 1</u>. For any $\gamma > 0$,

a) if $\alpha^* = 0$, then $\beta^* = 0$, or

b) if $\alpha^* = 1$, then $\beta^* = 1$.

<u>Proof:</u> The choice set of the investor who can revise at date 1 includes the choice set of the investor who cannot revise. If the investor who can revise chooses as an optimal strategy one that requires no revision, it follows that the investor who cannot revise would choose the same strategy.

Proposition 1 is not as meaningful as it might first appear.

Under the conditions needed to avoid stochastic dominance, the investor who could revise at date 1 would never want to invest everything in the riskfree asset. Thus,

<u>Proposition 2</u>. For any $\gamma > 0$, if $R_\ell < R_f < R_h$ and $(R_\ell + R_h)/2 > R_f$, then $\alpha^* > 0$.

<u>Proof:</u> If $\gamma \neq 1$, the assumption of constant proportional risk aversion implies that α^* will maximize the one-period problem

$$\tfrac{1}{2}\frac{1}{1-\gamma}\left[R_f + \alpha(R_h - R_f)\right]^{1-\gamma} + \tfrac{1}{2}\frac{1}{1-\gamma}\left[R_f + \alpha(R_\ell - R_f)\right]^{1-\gamma} \quad .$$

Setting the derivative of the above to zero yields

$$\alpha^* = \frac{R_F(1 - A)}{(R_h - R_f) + (R_\ell + R_f)} \quad , \quad \gamma \neq 1 \qquad (6)$$

where

$$A = -\frac{(R_h - R_f)}{(R_\ell - R_f)}^{1/\gamma} \quad .$$

An inspection of (6) discloses that $\alpha^* = 0$ if $A = 1$. $A = 1$ if $(R_h - R_f) = -(R_\ell - R_f)$, but this equality contradicts the assumption that $(R_\ell + R_h)/2 > R_f$.

If $\gamma = 1$, the optimal proportion to place at risk is

$$\alpha^* = -\frac{R_F[(R_h - R_f) + (R_\ell - R_f)]}{2(R_h - R_f)(R_\ell - R_f)} \quad . \qquad (7)$$

The optimal proportion $\alpha^* = 0$ if $R_h - R_f = -(R_\ell - R_f)$, again contradicting the assumption.

Thus, part (a) of Proposition 1 will never occur if asset returns satisfy the no dominance condition. Under some combinations of returns, α^* can be 1.0, so that part (b) of Proposition 1 has economic content.

<u>Proposition 3</u>. If the utility function is logarithmic ($\gamma = 1$), then $\alpha^* \gtreqless \frac{1}{2}$ if and only if

$$R_\ell R_h \gtreqless R_f^2 \quad . \tag{8}$$

<u>Proof:</u> This follows by setting $\alpha^* = \frac{1}{2}$ in (7) to obtain

$$(R_h - R_f) + (R_\ell - R_f) \gtreqless - \frac{(R_h - R_f)(R_\ell - R_f)}{R_f} \quad . \tag{9}$$

Multiplying by R_f and collecting terms yields (8). Since all the steps are reversible, the converse follows.

Proposition 3 relates the geometric mean of the risky asset to the riskfree return. When the two are equal, an investor with a logarithmic utility function who can revise at date 1 will invest an equal amount in the risky and riskfree asset. The next two propositions establish the relationship between the proportion, α^*, that an investor who can revise at date 1 would optimally place at risk with the proportion, β^*, that an investor who can not revise at date 1 would place at risk.

<u>Proposition 4</u>. If the utility function is logarithmic, then $\alpha^* = \frac{1}{2}$ if and only if $\beta^* = \frac{1}{2}$.

<u>Proof:</u> The first part of the proof is to show that $\alpha^* = \frac{1}{2}$ implies $\beta^* = \frac{1}{2}$. The first derivative of the expected utility for the no-revision problem is

$$\frac{1}{4}\,\frac{\partial U(V_{hh})/\partial\beta}{V_{hh}} + \frac{1}{4}\,\frac{\partial U(V_{\ell\ell})/\partial\beta}{V_{\ell\ell}} + \frac{1}{2}\,\frac{\partial U(V_{h\ell})/\partial\beta}{V_{h\ell}} \tag{10}$$

or

$$\frac{1}{4}\,\frac{R_h^2 - R_f^2}{R_f^2 + \beta(R_h^2 - R_f^2)} + \frac{1}{4}\,\frac{R_\ell^2 - R_F^2}{R_F^2 + \beta(R_\ell^2 - R_f^2)} + \frac{1}{2}\,\frac{R_h R_\ell - R_F^2}{R_f^2 + \beta(R_h R_\ell - R_f^2)} \quad .$$

The optimal value of β is that obtained by setting (10) equal to zero. If $\alpha^* = \frac{1}{2}$, $R_h R_\ell = R_f^2$. Thus, the third term of (10) is zero for all values of β. Setting this third term to zero, determining the β for which (10) is zero, and substituting $R_h = R_f^2/R_\ell$ shows that $\beta^* = \frac{1}{2}$.

To prove the converse, note that (10) is zero when $\beta^* = \frac{1}{2}$ and $R_h R_\ell = R_F^2$, the condition for α^* to be $\frac{1}{2}$.

Proposition 5. If the utility function is logarithmic and if $\alpha^* < \frac{1}{2}$, then $\beta^* < \alpha^*$, and if $\alpha^* > \frac{1}{2}$, then $\beta^* > \alpha^*$.

Proof: First consider the case in which $\alpha^* < \frac{1}{2}$. The manner of proof will be to set β to α^* and show that the derivative of the problem without revision is negative, namely that (10) is negative. As a result, $\beta^* < \alpha^*$ since the second derivative of the expected utility with respect to β is negative.

If $\alpha^* < \frac{1}{2}$, Proposition 3 shows that $R_\ell R_h - R_f^2 < 0$. Thus, the third term of (10) is negative. The sum of the first two terms is also negative for $\beta = \alpha^*$. Substituting α^* from (7) into the ratio of the first term of (10) to the second term yields, after considerable simplification,

$$\frac{\dfrac{\partial U(V_{hh})/\partial\beta}{V_{hh}}}{\dfrac{\partial U(V_{\ell\ell})/\partial\beta}{V_{\ell\ell}}} = \frac{(R_\ell - R_f)(R_h + R_f)}{(R_h - R_f)(R_\ell + R_f)} \quad . \tag{11}$$

The right-hand side of (11) is less than one. By Proposition (3),

$$R_\ell R_h - R_f^2 < -(R_\ell R_h - R_f^2) \qquad (12)$$

Adding $R_f(R_\ell - R_h)$ to both sides of (12), we have

$$R_\ell R_h - R_f^2 + R_f(R_\ell - R_h) < -(R_\ell R_h - R_f^2) + R_f(R_\ell - R_h)$$

or

$$(R_\ell - R_f)(R_h + R_f) < -(R_h - R_f)(R_\ell + R_f) \quad .$$

Since $(R_h - R_f)(R_\ell + R_f)$ is positive by the non-dominance criterion, the right-hand side of (11) is less than one. Thus, the sum of the first two terms on the left-hand side of (10) is negative. Since the third term is also negative, the first derivative is negative. Thus, $\beta^* < \alpha^*$.

The other part of the theorem is proved similarly.

Proposition 4 and 5 parallel those developed by Goldman (1979). However, Goldman's theorems are based upon the assumption that a Weiner process generates the returns of the risky asset rather than the binomial process used here. Utilizing a binomial process, counterexamples show that Propositions 4 and 5 do not hold if, for instance, the coefficient of relative risk aversion is two. If the riskfree rate is zero, there are various combinations of r_ℓ and r_h such that the optimal solution with revision is 0.50, but without revision different from 0.50 (Table 2).

That such modest changes in the underlying assumption for the probability distribution of the risky asset from those made by Goldman (1979) result in different theorems suggests that these theorems are not that robust. Perhaps, further work will disclose some characteristics of the relationship between trading interval and investment strategy that are less sensitive to the specific assumpptions. For example, is it the case that the proportion placed at risk is a monotonic function of the length of the trading interval? Fischer and Pennachi (1985) show this to be false if returns are correlated over time, but what if returns are independent? Our conjecture is that, even with independence, the proportion placed at risk is not always a monotonic function.

IV. More Realistic Examples

The previous results utilize a binomial distribution, whereas actual returns are continuously distributed. This section is based upon a log normal distribution of the returns of the risky asset, and thus parallels the asympotic results of Goldman.

Specifically, assume that the risky asset is distributed by a log normal distribution with an expected log annual return of 15 percent and a standard deviation of the log annual return of 25 percent. The corresponding expected return is 19.9 percent, and the standard deviation is 30.4 percent. Also, assume that the log of the riskfree rate is 10 percent.[1] In comparison to the data prepared by Ibbotson and Singuefield, this standard deviation is greater than its historical values.[2] Using a greater than historical value for the standard deviation should help to highlight the relationship between the length of the trading interval and the investment strategy of the investor. After all, if there were no risk, the investor would merely invest in the asset with the greatest return.

Based upon these assumptions, we utilize numerical analysis techniques to calculate the proportion that should optimally be placed in the risky asset as the length of the trading interval increasess. If the coefficient of relative

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1 Cf. Aitchison and Brown (1957).

2 Stocks, Bonds, Bills and Inflation Yearbook (1984).

risk aversion is two, the proportion placed at risk with continuous revision is 0.65. As the trading interval increases to four years, the proportion placed at risk increases to 0.65746. If the coefficient of relative risk aversion is 8, the proportion placed at risk with continuous revision of 0.16250, gradually decreasing to 0.15732 for an interval of four years.

These changes do not appear to be large, suggesting that the investment strategy is not very sensitive to the length of the trading interval. The following analysis shows that these small changes are not inconsequential, but before moving to that analysis, let us compare the results based upon a log normal process to those based upon the binomial process.

As with the binomial process, an investor who cannot revise and who selects the same proportion to invest in the risky asset as the investor who can revise will have a lesser probability of realizing substantial losses and also a greater chance of realizing extremely large gains. Table 4 contains some statistics if date 2 is taken to be one year. Thus, the behavior of the extreme values from the binomial example appears to generalize to the log normal case.

Although the proportion that is placed at risk does not change substantially as the trading interval increases, what is important in evaluating the significance of the changes to an investor is how much the investor would pay to have a shorter trading interval. This is the same type of question that Benninga and Blume (1985) explore in another context of determining the value of adding an additional asset to an incomplete market.

Let $E^{(0)}(U(W_2))$ be the expected utility of an optimal policy with no revision between date 0 and date 2. Let $E^{(1)}(U(W_2))$ be the expected utility of an optimal policy if revision is permitted at date 1. For the same initial wealth, $E^{(1)}$ is equal to or greater than $E^{(2)}$. If the ability to revise is important to an investor, the investor should be willing to pay some fraction of his or her wealth at date 2 in order to be able to revise.

Let π be the fraction of wealth that the investor would pay to be able to revise at date 2. The value of π is determined by equating

$$E^{(2)}\{U[(1 - \pi)W_2]\} = E^{(1)}\{U(W_2)\} \quad . \tag{13}$$

If $\gamma \neq 1$, (13) can be rewritten as

$$E^{(2)}\{\frac{1}{1 - \gamma}[(1 - \pi)W_2]^{1-\gamma}\} = E^{(1)}\{(\frac{1}{1 - \gamma})W_2^{1-\gamma}\}$$

or, solving for π,

$$\pi = 1 - \left[\frac{E^{(1)}\{U(W_2)\}}{E^{(2)}\{U(W_2)\}}\right]^{1/(1-\gamma)} \quad . \tag{14}$$

If $\gamma = 1$, (13) can be rewritten as

$$\pi = 1 - e^{E^{(1)}[U(W_2)] - E^{(2)}[U(W_2)]} \quad .$$

The proportion of wealth that an investor would pay to revise at date 1 depends upon the time interval to date 2 and the coefficient of relative aversion. Not surprisingly, the proportion of wealth that the investor would pay increases as the time interval to date 2 increases and as the coefficient of relative risk aversion increases. Thus, if date 2 is four years from date 0, an investor with a coefficient of relative risk aversion of two would pay 0.069 percent to revise at date 1. In contrast, an investor with a coefficient of relative risk aversion of eight would pay 4.1 percent to revise at date 1.

Table 5 contains, for various time intervals to date 2 and for various coefficients of relative risk aversion, the proportions that an investor would pay to revise at date 1. For some cases, the proportion that an investor would pay is small and for other cases, fairly substantial. Thus, it is a mistake to assume that merely because the proportion placed at risk does not change very much as the trading interval increases, an investor would find these small changes in the proportion placed at risk inconsequential.

V. Conclusion

This paper has shown that as the trading interval changes, the proportion that an investor would place at risk would generally change. The paper explored in some detail the nature of these changes for a binomial process. Then, assuming that returns follow a log normal distribution, the paper shows that the proportion placed at risk does not change very much for revision periods up to four years and for reasonable values of the coefficient of relative risk aversion. Nonetheless, it turns out that these small changes are sometimes of great consequence to an individual investor in that, on occasion, an investor would pay a substantial proportion of his or her wealth to revise more frequently.

Table 1

Distribution of Wealth Under Various Portfolio Strategies
Binomial Process

	With Revision (Optimal Proportion)	Without Revision (Non-Optimal Proportion)	Without Revision (Optimal Proportion)
A. Relative Risk Aversion = 2			
Proportion at Risk	0.66726	0.66726	0.66997
Distribution of Outcomes			
Probability			
0.25	1.86843	1.90395	1.90677
0.50	1.32118	1.30342	1.30380
0.25	0.93421	0.94310	0.94201
$E(W)$	1.36125	1.36347	1.36409
$\sigma(W)$	0.33272	0.34498	0.34638
m_3	0.01311	0.02079	0.02105
B. Relative Risk Aversion = 8			
Proportion at Risk	0.16107	0.16107	0.15879
Distribution of Outcomes			
Probability			
0.25	1.35589	1.37751	1.37515
0.50	1.24336	1.23255	1.23223
0.25	1.14017	1.14557	1.14648
$E(W)$	1.24570	1.24705	1.24652
$\sigma(W)$	0.07631	0.08328	0.08210
$m_3 \times 100$	0.00408	0.02925	0.02802

Table 2

Optimal Values for α^* and β^* for a Coefficient of Relative Risk Aversion of Two for Various Combinations of r_ℓ and r_h with $r_f = 0$

r_ℓ	r_h	α^*	β^*
-0.1	0.12509812	0.5000000	0.500015
-0.2	0.33718815	0.5000000	0.500409
-0.3	0.83333333	0.5000000	0.504890
-0.01	391.99954260	0.5000000	0.501565

Table 3

Proportion Placed At Risk For Various Trading Intervals

Relative Risk Aversion	Revision Interval				
	Continuous	Every Six Months	Once A Year	Once Every Two Years	Once Every Five Years
2	0.65000	0.65105	0.65206	0.65398	0.65746
4	0.32500	0.32384	0.32275	0.32074	0.31729
8	0.16250	0.16112	0.15984	0.15753	0.15732
16	0.08125	0.08030	0.07945	0.07792	0.07543

Table 4

Distribution of Wealth Under Various Portfolio Strategies
Log Normal Process

	With Revision (Optimal Proportion)	Without Revision (Non-Optimal Proportion)	Without Revision (Optimal Proportion)
A. Gamma = 2			
Alpha	0.65105	0.65105	0.65205
Distribution of Outcomes			
Relative Ranges			
< 0.87	0.04701	0.04157	0.04450
.87 - 1.23	0.68368	0.68459	0.68135
> 1.23	0.26931	0.27385	0.27415
E(r)	1.16567	1.16610	1.16620
σ(r)	0.19449	0.19811	0.19841
B. Gamma = 8			
Alpha	0.16112	0.16112	0.15984
Distribution of Outcomes			
Relative Ranges			
< 1.06	0.13929	0.13649	0.13557
1.06 - 1.14	0.61715	0.61270	0.62258
> 1.14	0.24357	0.25081	0.24185
E(r)	1.12001	1.12027	1.12015
σ(r)	0.04672	0.04873	0.04833

Table 5

Percentage of Wealth That An Investor Would Pay
To Be Able To Revise At Date 1

Relative Risk Aversion	Date 2 One Year	Two Years	Four Years
2	0.005	0.019	0.069
4	0.079	0.296	1.049
8	0.327	1.203	4.107
16	0.893	3.242	10.652

REFERENCES

Aitchison, J. & J.A.C. Brown, The Log Normal Distribution, Cambridge: Cambridge University Press, (1957).

Benninga, Simon & Marshall E. Blume, "On the Optimality of Portfolio Insurance," Journal of Finance 40, (1985), 1341-1352.

Fischer, Stanley & George Pennachi, "Serial Correlation of Asset Returns and Optimal Portfolios for the Short and Long Term," Rodney White Center for Financial Research Working Paper #9-85, University of Pennsylvania, (1985).

Friend, I. & M.E. Blume, "The Demand for Risky Assets," American Economic Review 65, (1975), 900-922.

Goldman, Barry M., "Anti-Diversification, or Optimal Programs for Infrequently Revised Portfolios," Journal of Finance 34, (1979), 505-516.

Klein, Roger & Vijay Bawa, "The Effect of Estimation Risk on Optimal Portfolio Choice," Journal of Financial Economics 3, (1976), 215-231.

Morin, R.A. & A.F. Suarez, "Risk Aversion Revisited," Journal of Finance 3, (1983), 1201-1216.

Mossin, J., "Optimal Multiperiod Portfolio Policies," Journal of Business 41, (1968), 215-219.

Stocks, Bonds, Bills and Inflation Yearbook (1926-1983), Chicago: R.G. Ibbotson Associates, Inc., (1984).