STOCK RETURN ANOMALIES AND THE ASSET PRICING TESTS: THE CASE OF THE ARBITRAGE PRICING THEORY

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1. Introduction

Recent research documents a peculiar seasonal pattern of the common stock returns. It is now a very well known phenomenon that stock returns are abnormally high for January. More puzzling is the fact that the smaller firms have proportionally higher returns. See for example Keim (1983), Gultekin and Gultekin (1983). While such a seasonal pattern in stock returns is an interesting phenomenon in itself and has yet to be fully explained, Tinic and West (1984) point out a serious implication of the observed seasonality in stock returns on the tests of the Capital Asset Pricing Model (CAPM).

Tinic and West (1984) report that the risk return relation described by the CAPM also exhibits a similar seasonality in January, a result also reported by Rozeff and Kinney (1976). Most importantly, however, they show that only one month, January, shows a consistently positive and reliable relation between expected return and risk. Once the data for January are drawn from the analysis of the risk-return tradeoff, the estimates of risk premia are no longer reliably different from zero. According to Tinic and West, "January is not simply the month in which overall stock returns have been higher relative to the rest of the year, and when small firms' stocks have outperformed the market as a whole, it is the only month when shareholders have consistently paid for taking on risk."

Tinic and West do not offer an explanation for their findings. Their results, however, still raise a number of difficult and serious questions about the empirical tests of the Capital Asset Pricing Model and ultimately its validity as a viable model explaining the pricing of risky assets.

The purpose of this paper is to investigate the impact of stock return seasonality (or so called anomalies) on the empirical tests of the Arbitrage Pricing Model. A number of researchers argue on both theoretical and empirical grounds that the APT is a good alternative to the CAPM. The empirical tests of the APT thus far, however, have produced ambiguous results and a good deal of controversy. Before a verdict is handed down about the validity of the APT, we need to gather as much empirical evidence as possible. Since the empirical tests of the APT model also involve a two stage methodology as in the case of the CAPM, it is appropriate at this point to investigate to what extent the stock return anomalies affect the previous tests of the APT.

Our paper is organized in the following order: section 2 briefly reviews the APT and reproduces some of the earlier empirical results to be used as a benchmark to compare the findings of this study; section 3 investigates the impact of stock return seasonality on the tests of the APT, and section 4 concludes the paper.

2. The APT Model and Review of Earlier Empirical Tests

This section provides a very brief summary of the APT model and its empirical implications. Further details of the tests we use in this paper can be found in Roll and Ross (RR) (1980), Dhrymes, Friend and Gultekin (DFG) (1984), Dhrymes, Friend, Gultekin and Gultekin (DFGG) (1984, 1985) and Gultekin and Rogalski (1985).

The APT of Ross (1976) starts with the return-generating process for securities that

$$r_{t\cdot} = E_{t\cdot} + f_{t\cdot}B + u_{t\cdot} \ , \qquad (1)$$

where $r_{t\cdot}$ and $E_{t\cdot}$ are m-element row vectors containing the observed and the expected (mean) rates of returns, respectively, for the m securities at time t. $f_{t\cdot}$ is a k-element row vector of common but unobservable factors affecting the security returns, assumed to have zero mean and unit variance. B is a $k \times m$ matrix of risk measures in terms of the sensitivity of securities to the common factors. $u_{t.}$ is the error term idiosyncratic to each security.[1]

Ross shows that if the number of securities (m) is sufficiently large, there exists a (k + 1)-element row vector $c_{t.}$ such that

$$E_{t.} = c_{t.}B^* \ , \quad t = 1, 2, \ldots, T \ , \tag{2}$$

where $B^{*\prime} = [e:B']$ and e is an m-element column of ones. The result in (2) characterizes the no-arbitrage condition of the APT model and thus (1) can be rewritten with any desired degree of approximation

$$r_{t.} = c_{t.}B^* + f_{t.}B + u_{t.} \ , \qquad t = 1, 2, \ldots, T \ . \tag{3}$$

Based on the specification in (3) the empirical tests of the APT model are based upon a two-stage factor analytic approach. In the first stage, one determines the number of factors (k) and estimates the elements of B. The researcher assumes that the sample size T (the number of trading days) is sufficiently large, so that he estimates B, say, by $\tilde{B}$, with negligible sampling error using factor analysis.[2] In the second stage, using $\tilde{B}$ as "independent variables," one estimates a vector $c_{t.}$, whose elements have the interpretation that c_{ti} is the "risk premium" attached to the i^{th} factor, i = 1, 2, . . ., k, while c_{t0} is the risk-free rate (or possibly the return on a zero-beta asset). The regression model used to estimate the vector $c_{t.}$ in the second stage for each t is

$$\tilde{c}'_{t.} = (\tilde{B}^*\tilde{\Psi}^{-1}B^{*\prime})^{-1}\tilde{B}^*\tilde{\Psi}^{-1}r'_{t.} \ , \qquad t = 1, 2, \ldots, T \ . \tag{4}$$

Analogous to the CAPM tests, there are two critical testable hypotheses implied by the APT model. Firstly, the intercept term in (4) is the risk-free (or zero-beta) rate. Secondly, there is a linear relation between the risk measures, B, and the expected returns, implying that the risk premia vector is priced.

In addition to the two hypotheses above, one could also test the APT model against a specific alternative by introducing a relevant explanatory variable in the model shown in (3). The respecified model in this context is

$$r_{t.} = c_{t.}B^{*} + d_{t.}P + f_{t.}B + u_{t.} , \tag{5}$$

where P is a vector of other "extraneous" variable(s). The restriction on empirical evidence implied by (3) is that no other (relevant) economic/financial variables have any effect on the determination of expected rates of return. Therefore the proper null hypothesis is, in this case, that $d_{t.}$ should not be different from zero.

The data used in this paper are the time series of daily stock returns from the Center for Research in Security Prices (CRSP) tapes. The time period is from July 3, 1962 to December 31, 1981. We delete the securities with more than one hundred missing observations. This results in 900 stocks listed in the New York and the American stock exchanges, providing a sample size ranging from 4793 to 4893 daily observations of daily returns per security. These securities are ranked alphabetically and then formed into 30 and 10 groups each containing 30 and 90 securities respectively.

DFG (1984) and DFGG (1984, 1985) demonstrate that the empirical tests of the APT model depend on how one groups the universe of securities and the number of securities. They show that the number of factors increases with the group size. The grouping of securities in earlier research was introduced to get around the computational problems in factor analyzing a large group of securities. DFG and DFGG, for example, show that while a 7-factor is sufficient to describe the return generating process for common stocks, one needs around 17 factors for groups of 90 stocks for the period July 1961-December 1981.[3]

The problems associated with group size in the tests of the APT model have yet to be resolved. Following the earlier research, we use a 7-factor model for groups of 30-stocks and 7 and 17-factor models for groups of 90-stocks in this study. We demonstrate in the next section, however, that the anomalies in stock returns affect the empirical tests of the APT in a way that appears to be independent of the group size.

The second-stage results for the significance tests of risk premia, that is whether they are priced, are presented in Table 1 for the entire period 1962-1981. Panel A shows the chi-square test statistics for the vector of risk premia for groups of 30 stocks each.[4] Row numbers correspond to the 30 groups each arranged alphabetically. Column (1) of Panel A shows that, over the entire period, the risk premium vector is significantly different from zero for only 4 groups (13.3 percent of groups) at the 5% level and for 10 groups (33.3 percent of groups) at the 10% level.

Columns (2) through (5) present results to address the question of whether the use of the factor models exhausts the "explanation" of the factor return process. Following the same methodology as in previous work, we include (total) standard deviation of stock returns and also, separately, the square root of the residual (specific) variance as additional explanatory variables.[5] Columns (2) and (4) show the relevant chi-square values for testing the null hypothesis of zero-risk premia when the factor risk premia vector is re-estimated in conjunction with other extraneous variables. Once (total) standard deviation or residual standard deviation is included, the null hypothesis for the risk premia vector cannot be rejected for any group at the 5% level over the entire period. Both extraneous variables, however, are "priced" at least in 5 groups at 5% level and in 10 groups at 10% level for the same period.

Panels B and C include the summary statistics using groups of 90 stocks for 17 and 7-factor models. Results here are more favorable to the APT when factor loadings are used as explanatory variables alone (Columns (1) in Panels B and C). Risk premia are priced in 5 groups (50 percent of groups) using a 17-factor model and in 9 groups (90 percent of groups) using a 7-factor model at 5% significance level. Corresponding numbers are 6 (60 percent of groups) and 10 (100 percent of groups) at 10% level.

Again, however, once total or residual standard deviation is included as an additional independent variable (Columns (2) and (4) in Panels B and C), risk premia are no longer priced at 5% level, except for two groups in Column 4. Total or residual standard deviations are more often priced; depending on the measure of standard deviation used, 40 to 70 percent of the groups at 5% significance level and 70 to 80 percent of the groups at 10% level (see Columns (3) and (5)).

Based on the partial results presented here and our other more extensive tests, we may conclude that the empirical results are at best mixed for the APT model. We also need to qualify that our results are not the conclusive tests of the APT model because of the problems associated with the grouping procedures and restrictive assumption used here and in most other empirical work.

3. Seasonality in Risk Premiums

The seasonality in stock returns in the U.S. is a well documented phenomenon; Wachtel (1942) may have been the first who investigated the stock market seasonality in the U.S. More recently Rozeff and Kinney (1976) reported seasonal pattern for the U.S., and Gultekin and Gultekin (1983) for a number of other countries.

Table 2 presents means of daily stock returns for calendar months for groups of 30 and 90 stocks. In line with other studies, there is a strong seasonal pattern in daily stock returns. For every group, January returns are consistently five to ten times larger than most other months. November is also a month with larger mean returns.[6] It is interesting to note that the seasonal pattern is uniformly identical for each of the 30 groups. Recall that these groups are formed from an alphabetically ranked universe of NYSE and AMEX stocks. Such a ranking scheme is close to a random ranking of securities and it is thus unlikely that the pattern observed in Table 2 is generated by a few outliers or small firms only.

From the results in Table 2, it appears that the major part of the rewards for holding stocks is systematically realized in a few months of the year and mostly in January. It remains to be seen whether such a seasonal pattern affects the empirical tests of the APT model, or conversely, whether an asset pricing model based on simple covariance measures can explain such a peculiar pattern of daily stock returns.

Results in Table 3 provide partial insight to this question. In Table 3 we summarize month-to-month chi-square values to test the null hypothesis that the risk premia vector is null. Here we estimate the GLS regression model in (4) for each calendar month separately and compute the corresponding test statistics from these month-to-month estimates of risk premia.[7] A casual review of Table 3 reveals that risk premia are <u>always</u>, without exception, significant in January for each of the thirty groups, but rarely priced in other months. There is a strong January seasonal in risk premia estimates for the APT model as in the case of the two stage empirical tests of CAPM reported by Tinic and West (1984, 1985). As their results show, the APT model seems also to hold only in January.

The tests of the APT against a specific alternative for each calendar month are summarized in Panel A of Table 4 for groups of 30 stocks. Again we use total or residual standard deviation as specific alternatives by including them in the GLS model given in equation (4) as additional independent variables separately. Total standard deviation is estimated using both the entire sample of daily observations (second line in row 2) and also using the data from the relevant month only (row 4). Summary results in Table 4 correspond to the month-to-month version of the last two lines in Table 1, and for ease of presentation, they are shown in percentages. The percentages indicate percent of groups with significant test statistics, i.e. rejection of the hypothesis that the risk premia vector is null.

A similar seasonal pattern emerges here as well. Factor loadings and total or residual standard deviation are priced mostly in January. Also consistent with our earlier findings, once the standard deviation measures are introduced, factor loadings are priced in fewer groups. For example, while factor loadings are priced for every group in January, they are priced in 40 to 60 percent of groups once standard deviation is introduced as an additional variable. Interestingly, however, both risk premia and the standard deviation measures are priced rarely in other months.

Panels B and C of Table 4 contain results for groups of 90 stocks using 7- and 17-factor models, respectively. As in the case of groups of 30 stocks, we also observe a similar seasonal pattern both for the 7- and 17-factor models. Row 1 in both Panels shows that in January, risk premia vector is priced for all of the ten groups. Interestingly, February, June and November also have non-zero risk premia vector ranging from 70 to 50 percent for 7-

factor model. Similarly, risk premia vector is priced for half of the groups for February for the 17-factor model, indicating a seasonal pattern at the beginning of the year.

The tests of the APT against specific alternatives also produce similar results. Rows 2 to 3 show the results with total or residual standard deviation as an additional explanatory variable. Standard deviation measures appear to be significant mostly in January and to a lesser extent in February. For other months, however, both risk premia vector and either measure of standard deviation are rarely significant.[8] Furthermore, most interestingly or disturbingly, our results are very robust with respect to the group size. While using group sizes of thirty versus ninety stocks per group produces dramatic changes in the test results of the APT per se (compare Panel A with B or C in Table 1), January anomalies affect the significance tests for all sizes of groups similarly.

3.2. Are The Risk Premia Priced in January Only?

We have by now established the existence of a seasonal pattern in the estimates of risk premia (see footnote 7) using the significance tests. The next logical step is to inquire whether risk premia are priced at all if we exclude the data for months, primarily January, that are responsible for the seasonality in stock returns.

Table 5 provides an answer. Here we exclude January and December returns from the data and compute the significance tests. The decision to exclude December and January returns is based on Keim's (1983) finding that the daily stock returns are the largest during the last week of December and the first two weeks of January.

Two interesting patterns emerge. Firstly, Panel A indicates that when January and December returns are excluded, the risk premia is priced only in one out of 30 groups at the 5% level and 3 groups at the 10% level. Table 1 (column 1) shows the corresponding numbers using returns in all months. They are four and ten. Clearly the results are much weaker without January and December returns. Secondly, as before risk premia are priced in 25 groups at 5% level and in 27 groups at 10% level using returns for January and December alone.[9]

Our findings are similar to those reported by Tinic and West (1984) who investigate the impact of stock return seasonality on the Fama-MacBeth tests of the CAPM. Tinic and West demonstrate that beta as a risk measure is priced in January alone and once January returns are excluded, it is no longer priced. The results in this paper, however, are somewhat more difficult to interpret than the Tinic and West findings for at least three reasons. Firstly, the empirical support for the APT, ignoring the seasonality issues, is weak to start with as shown in Table 1. Secondly, the grouping technique we use poses serious limitations in testing the APT indicated by Dhrymes, Friend and Gultekin (1984). And finally, as a consequence of the grouping procedure, it is difficult to test formally whether reduction in the number of priced groups, say from four to one, when January-December returns are excluded, is statistically significant.[10]

Even in the absence of formal tests, however, the empirical tests appear to be seriously affected by the seasonal pattern in the stock return data. No matter how one interprets the empirical evidence for the APT model in Table 1, it is clear that significant test results are driven by the stock return anomalies around the turn of the year.

Other results in Table 5 provide further support for our observations. Recall that a crucial test of asset pricing models includes introduction of total or residual standard deviation as a specific alternative to covariance based risk measures, such as factor loadings in the APT and beta in the CAPM. These tests are given in columns 2 through 7. Own standard deviations are computed using returns from February through November (column (3)) and from January and December only (column (7)). When January and December returns are excluded (see Panel A), standard deviation measures are less frequently priced in comparison to those results in Table 1. Interestingly, factor loadings are priced as frequently as standard deviation measures in contrast to the results in Table 1 where we use returns in all months. In the months January and February, however, standard deviation measures are priced more frequently than factor loadings (Panel B). But more importantly, the number of groups with priced risk premia goes down from 25 to 2 or 5 when total or residual standard deviation is included as an alternative risk measure. There is also a similar seasonal pattern when we use total or residual standard deviation measures as the sole risk measure (columns 8 through 10). Standard deviation measures are priced in 27 to 29 groups at 5% level in December and January, but only 3 to 6 groups in remaining months, indicating that most of the action takes place around the turn of the year.

In order to gain further insight to these peculiar results, we repeat the cross sectional regressions by excluding returns one month at a time and compute the corresponding test statistics using the returns from the remaining eleven months. These results are summarized in Table 6.

The general pattern of the results gives the impression that there are more groups with significant test statistics when January returns are included (columns February-December). Row 1 contains test statistics when factor loadings are used as risk measures alone. While the relation between factor loadings and returns is not very strong, more groups have priced risk premia when January returns are included.

The seasonal pattern associated with January returns emerges even more distinctly once we introduce total or residual standard deviation as an additional explanatory variable in Rows 2 to 4. Without January returns (see January column in Table 6), standard deviation measures are priced for only 7 percent of groups (2 out of 30 groups). For any other combination of eleven months which includes January returns (columns February through December), standard deviation measures are priced far more frequently: 17 to 43 percent of groups (6 to 14 groups). But most interestingly, factor loadings are not priced at all in the eleven months which include January returns when any measure of standard deviation is added as an alternative risk measure (Rows 2, 3 and 4).

Finally, we present results in Row 5 using standard deviation measures as sole risk measures. Without January returns, sigma measures are priced in 17 to 23 percent of all groups (5 to 7 groups out of 30) while they are priced in 84 to 97 percent (24 to 29 out of 30) of all groups when January returns are included.

Once again these findings indicate a peculiar yet strong seasonal pattern in which most of the significant results are generated by January returns alone. We are able to offer an explanation. Previous research shows that stock returns are abnormally higher in January, particularly for smaller stocks. Furthermore, there is also evidence that those stocks with highly volatile returns tend to produce higher observed returns in January (see Roll (1984)). Consequently, what we find in the two stage tests of the APT and what Tinic and West find for the CAPM is nothing more than the strong association between common stock returns and the corresponding volatility measures at the turn of the year. Once January observations are removed from the data, neither the covariance measures, such as factor loadings in the APT model and the beta in the CAPM, nor standard deviation measures have any explanatory power.[11] Perhaps, for returns in months other than January, there is not much to be explained.

4. Conclusions

Our results show that the two stage empirical tests of the APT model are very sensitive to the anomalies in the stock returns data. There is a strong seasonal pattern also in the estimates of the risk premia from the APT model similar to those reported by Tinic and West (1984, 1985) for the CAPM. The most important implication of the seasonality is that the APT model, like the CAPM, can explain the risk-return relation in January only. These results do not seem to depend on the group size that we use in the empirical tests.

Although we do not have a completely satisfactory explanation for the January seasonal, its effect on empirical research is far more serious than we previously realized. If a crucial test of an asset pricing model is its ability to explain the turn of the year effect, particularly as it relates to size, we are left with a disturbing conclusion. The seasonal pattern in the stock return data is so strong in January that asset pricing models based on covariance measures of risk are not likely to explain the turn of the year effect or size related anomalies. Furthermore, returns are so highly correlated with standard deviations in January that covariance risk measures are bound to fail tests where total or residual standard deviations are used as specific alternatives.

FOOTNOTES

[1]Note that we assume $\{u'_{t.} : t = 1, 2, \ldots\}$ is a sequence of identically independently distributed (i.i.d.) random vectors with $E(u_{t.}) = 0$, $\mathrm{cov}(u'_{t.}, f'_{t.}) = 0$ and $\mathrm{cov}(u'_{t.}) = \Omega$. Regarding the common factors, we specify that $E(f'_{t.}) = 0$ and $\mathrm{cov}(f'_{t.}) = I$. It is a consequence of these assertions that $\{(r_{t.} - E_{t.})' : t = 1, 2, \ldots\}$ is a sequence of i.i.d. random vectors with $E[(r_{t.} - E_{t.})'] = 0$ and $\mathrm{cov}[(r_{t.} - E_{t.})] = B'B + \Omega = \psi$.

[2]This is arguably a strong assumption. Little is known about the sampling properties of factor loadings and, to our knowledge, no one has addressed this question adequately. Dhrymes, Friend and Gultekin (1984) and Dhrymes, Friend, Gultekin and Gultekin (1985) document the effect of changing the sample size on the estimates of factor loadings. They indicate that factor loadings estimates are very sensitive and thus the sampling errors could be high. Thus, the results of APT tests may also be sensitive to sampling errors.

In this paper, however, our objective is not to provide conclusive tests of the APT *per se*, but to demonstrate the sensitivity of previous results to the stock return anomalies. As we shall show in the sequel, the results in this paper are not likely to be affected materially using improved estimates of factor loadings. For further explanations of the tests in this paper, see Dhrymes, Friend and Gultekin (1984) and Dhrymes, Friend, Gultekin and Gultekin (1985a, 1985b).

[3]One can legitimately question the assumption of stationarity using this length of time period. DFGG (1985a, 1985b) use varying lengths of time periods without substantial changes in the results and conclusions for the two stage test of the APT.

[4]Recall from Dhrymes, Friend and Gultekin (1984) that the significance tests for risk premia can only be done jointly for the vector of risk premia but not individually. This is because factor loadings are identifiable only up to left multiplication by an orthogonal matrix. The relevant test statistic for the significance test for the risk premia vector $\tilde{c}^*_{t\cdot}$ for each group is $T\bar{c}^* W^{-1} \bar{c}^{*\prime} \overset{A}{\sim} \chi^2_k$, where $\bar{c}^* = \left(\frac{1}{T}\right) \sum_{t=1}^{T} \tilde{c}^*_{t\cdot}$ and $W = \left(\frac{1}{T}\right) \sum_{t=1}^{T} \left(\tilde{c}^*_{t\cdot} - \bar{c}^*\right)'\left(\tilde{c}^*_{t\cdot} - \bar{c}^*\right)$. The test statistic is chi-square with k degrees of freedom.

[5]Specific variances are contained in the diagonal matrix Ω in $\tilde{\Psi} = \tilde{B}'\tilde{B} + \tilde{\Omega}$.

[6]Note that we do not provide any statistical significance tests for the equality of means in Table 2. Gultekin and Gultekin (1983) provide such tests and report that January returns are statistically greater than most other months. Here, given the magnitudes of the January returns and the standard deviation (not reported here), one can easily surmise that a formal statistical test would confirm our observations.

[7]So far we demonstrated the seasonality in terms of the significance tests for risk premia. One may inquire whether the estimates of risk premia themselves exhibit any seasonal pattern. For this purpose, we run the following regression model for the entire period covered by the data:

$$\tilde{c}_{jt} = \beta_{j1} + \sum_{i=2}^{12} \beta_{ji} D_i + \tilde{e}_{jt}, \qquad j = 1, 2, \ldots, k$$

for a k-factor model. D_2 through D_{12} is a set of dummy variables representing the calendar months February through December. The intercept β_{j1} above measures the mean regression coefficient of the j^{th} risk premium, c_j, in the GLS model in (4) for January. Here, the regression model is essentially an analysis of variance where regression coefficients β_{j2} through β_{j12} measure the mean differences between regression coefficients for January and other months in regression model (4). This analysis is repeated for the intercept term c_0 and for all other risk premia estimates, c_1 through c_k, separately.

We compute an F-ratio to test the hypothesis that the regression coefficients β_2 through β_{12} are all jointly zero. Acceptance of the null hypothesis implies that the means of the risk premia are equal for all months. Using ten groups of 90 stocks, for example, we reject the null hypothesis 20% of the time for risk premia associated with factor one, 50% for factor two, 60% for factor three, 90% for factor four, 80% for factor five and 60% for factors six and seven. Evidently risk premia indeed have a (statistically) strong seasonality.

[8]Recall that February and November are two months where there are more frequent significant test statistics; see Table 4. We repeated the experiments in Table 5 by excluding the January and February and January and November returns from the data. Results are similar to those reported in Table 5. However, the test statistics that correspond to those in Panel B are significant more frequently and those that correspond to Panel A less frequently. We also repeat the experiments in Tables 5 and 6 by reestimating factor loadings after excluding the returns from the data. The results are not affected by this change.

[9]It is interesting to note that both factor loadings and standard deviation measures are always priced when they are used alone in December and January, but they are less frequently priced when these two sets of variables are used as independent variables together. Such results usually indicate a multicollinearity problem. The correlation between standard deviation and factor loadings is not high. The average (absolute) value for correlation

coefficients is .247 and only 12 out of 210 correlations (in a seven-factor model with 30 securities) are reliably different from zero, at 5% level. While there is a possibility of a multicollinearity problem with total standard deviation, residual standard deviations should not suffer from multicollinearity. They are independent of factor loadings by construct.

[10]It is possible to construct binomial tests similar to those reported in Dhrymes, Friend, Gultekin and Gultekin [1985]. The gain from such formal tests is not clear considering the strong assumptions underlying them.

[11]Miller and Scholes (1972) point out that if the returns are not normally distributed, the higher moments of a distribution will be correlated. This result might explain part of our findings in Table 6. Residual variance measure, however, should not be prone to this problem and yet we observe the same pattern for this variable as well. Roll and Ross (1980), Dhrymes, Friend, Gultekin (1984) and Dhrymes, Friend, Gultekin and Gultekin (1984, 1985) are very careful and explicit about the nonnormality of the data and its consequence of the dependency between the mean and the higher moments. They use noncontiguous days to estimate the parameters to overcome this problem. Such experiments did not produce materially different results reported here.

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TABLE 1: TWO STAGE TESTS OF THE ARBITRAGE PRICING THEORY AND TESTS OF SIGNIFICANCE FOR RISK PREMIA AGAINST SPECIFIC ALTERNATIVES. TOTAL STANDARD DEVIATION AND RESIDUAL STANDARD DEVIATIONS ARE USED AS ALTERNATIVES TO THE APT MODEL: SUMMARY STATISTICS--July 3, 1962-December 31, 1981

Group	B χ^2 (a) (1)	B & σ χ^2 (2)	(b) t(σ) (3)	B & Ω χ^2 (4)	(c) t(Ω) (5)	B χ^2 (a) (1)	B & σ χ^2 (2)	(b) t(σ) (3)	B & Ω χ^2 (4)	(c) t(Ω) (5)	B χ^2 (a) (1)	B & σ χ^2 (2)	(b) t(σ) (3)	B & Ω χ^2 (4)	(c) t(Ω) (5)
	A. Groups of 30 stocks each, 7-factor model					B. Groups of 90 stocks each, 17-factor model					C. Groups of 90 stocks each, 7-factor model				
1	6.765	4.532	1.61	5.626	1.60	29.447**	13.391	2.53**	18.410	2.55**	21.230**	8.575	3.16**	8.857	3.13**
2	12.961*	4.856	1.031	7.366	0.90	27.061*	16.828	0.61	20.234	0.60	20.922**	11.167	1.09	15.723**	1.06
3	15.790**	4.682	2.15**	5.953	2.15**	33.765**	18.313	2.23**	19.830	2.22**	27.308**	13.243*	2.55**	14.179**	2.55**
4	9.854	5.046	1.02	5.403	1.03	21.121	13.258	1.93*	12.413	1.77*	17.097**	9.273	1.90*	7.965	1.68*
5	9.757	3.142	0.47	7.222	0.78	27.941**	16.051	1.89*	21.722	1.92*	12.995*	5.596	2.85**	7.081	2.84**
6	6.448	3.669	0.48	4.504	0.29	22.748	14.413	1.07	15.088	1.03	15.841**	8.329	1.55	8.932	1.50
7	13.848*	6.161	0.52	6.328	0.51	33.411**	8.590	1.90*	15.527	1.80*	27.708**	5.285	2.62**	5.752	2.64**
8	14.393**	6.460	1.46	7.749	1.33	24.258	13.751	2.39**	14.729	2.33**	18.564**	9.708	2.72**	8.953	2.65**
9	16.083**	5.603	1.05	7.457	0.94	23.611	13.589	3.54**	13.961	3.51**	18.668**	10.154	3.72**	10.583	3.70**
10	7.934	0.869	1.02	1.952	0.99	28.219**	14.690	1.61	14.421	1.47	21.312**	8.956	1.90*	8.768	1.96**
11	12.717*	6.655	1.59	5.867	1.49										
12	7.325	4.561	1.07	4.393	0.71										
13	6.444	4.038	0.90	6.461	0.91										
14	8.979	3.716	1.87*	8.259	1.88*										
15	5.945	3.178	0.66	3.283	0.62										
16	12.057*	5.870	1.21	5.943	1.20										
17	7.259	0.799	1.73*	1.010	1.77*										
18	6.411	5.789	0.59	6.882	0.68										
19	10.904	2.117	2.05**	2.563	2.05**										
20	19.027**	2.300	1.71*	5.903	1.80*										
21	13.531*	2.693	1.27	3.270	1.16										
22	9.560	3.850	1.65*	4.398	1.56										
23	6.479	1.623	1.08	2.330	0.85										
24	4.346	1.892	2.00**	5.238	2.09**										
25	12.087*	9.472	2.78**	11.551	2.66**										
26	6.421	4.922	1.92*	5.483	1.94*										
27	9.433	2.376	1.27	3.260	1.23										
28	8.445	5.437	0.26	7.334	0.18										
29	11.420	6.905	1.17	6.159	1.25										
30	10.790	3.089	2.25**	3.208	2.19**										
Number of Significant Statistics															
at 10%	10	0	10	0	9	6	0	7	0	7	10	1	8	2	8
at 5%	4	0	5	0	5	5	0	4	0	4	9	0	6	2	7

a χ^2 value is computed by $T\bar{c}^{*}W^{-1}\bar{c}^{*\prime}$ where $\bar{c}^{*} = (\frac{1}{T}) \sum_{t=1}^{T} \tilde{c}^{*}_{t}$ and $W = (\frac{1}{T}) \sum_{t=1}^{T} (\tilde{c}^{*}_{t.} - \bar{c}^{*})'(\tilde{c}^{*}_{t.} - \bar{c}^{*})$. $\tilde{c}_{t.}$'s are from the daily cross-sectional regression estimates using the GLS model in equation (4), $\tilde{c}'_{t.} = (\tilde{B}^{*}\tilde{\psi}^{-1}\tilde{B}^{*\prime})^{-1}\tilde{B}^{*}\tilde{\psi}^{-1}r'_{t.}$, $\tilde{c}_{t.} = (\tilde{c}_{t0}, \tilde{c}^{*}_{t.})$; $B^{*\prime}$ is $[e:B]$, the augmented matrix of factor loadings (B) with unit vector (e). χ^2 tests the null hypothesis that the risk premia vector is null ($\tilde{c}^{*\prime}_{t.} = 0$). It is distributed as chi-square with degrees of freedom equal to the number of factors.

b Standard deviation of each security is included as an additional explanatory variable in B^{*}. χ^2 tests the null hypothesis that the risk premia vector is null. $t(\sigma)$ is the t-ratio for the regression coefficient for standard deviation. t-ratio is given by $\sqrt{T}(\bar{d}/s)$, where $\bar{d} = \sum_{t=1}^{T} \tilde{d}_t$ and $s = [(\frac{1}{T}) \sum_{t=1}^{T} (\tilde{d}_t - \bar{d})^2]^{1/2}$ (see equation (5)).

c Square root of residual (or specific) variance (Ω) is included as an additional explanatory variable. Residual variances correspond to the diagonal elements of $\tilde{\Omega}$ in $\tilde{\psi} = \tilde{B}\tilde{B}' + \tilde{\Omega}$. χ^2 tests the null hypothesis that the risk premia is null and $t(\Omega)$ is the t-ratio for the regression coefficient for residual standard deviation.

The minimum number of daily observations is 4511 and the maximum is 4730.

* and ** indicate the groups for which the null hypothesis is rejected at 10 and 5 percent levels of significance, respectively.

TABLE 2: MEANS FOR MONTH-TO-MONTH DAILY STOCK RETURNS FOR GROUPS OF 30 AND 90 STOCKS EACH
July 3, 1962-December 31, 1981

GRDL	January	February	March	April	May	June	July	August	September	October	November	December
PANEL A: GROUPS OF 30 STOCKS EACH												
1	0.3338**	0.0553	0.0707*	0.0801**	-0.0410	0.0182	0.0603	0.0466	0.0282	-0.0383	0.0789*	0.0747*
2	0.2803**	0.0348	0.0693*	0.0734*	0.0332	-0.0036	0.0531	0.0355	0.0467	-0.0124	0.0895**	0.0438
3	0.3495**	0.0443	0.1055**	0.0799*	-0.0011	0.0361	0.0925**	0.0206	0.0520	0.0025	0.1802**	0.0629
4	0.2262**	0.0454	0.0739*	0.0790*	-0.0022	0.0165	0.0590	0.0501	0.0314	-0.0033	0.1079**	0.0957**
5	0.2982**	0.0692*	0.0872**	0.0583*	0.0266	0.0006	0.0943**	0.0369	0.0409	-0.0625	0.1166**	0.0863**
6	0.2436**	0.0285	0.0701*	0.0709**	-0.0414	0.0010	0.0366	0.0316	0.0011	0.0084	0.0949**	0.0694
7	0.3086**	0.0158	0.0884**	0.0296	-0.0230	0.0209	0.0454	0.0269	0.0693	-0.0114	0.1160**	0.0847*
8	0.3443**	0.0563	0.0734*	0.0862**	-0.0160	-0.0210	0.0904**	0.0199	0.0325	-0.0082	0.1116**	0.1004**
9	0.2876**	0.0676*	0.0496	0.0855**	0.0247	0.0030	0.0362	0.0455	0.0269	-0.0123	0.1233**	0.0836**
10	0.3333**	0.0367	0.0710	0.1014**	-0.0139	0.0306	0.0870*	0.0035	0.0414	0.0008	0.1425**	0.0755*
11	0.2658**	0.0257	0.1186**	0.0531	-0.0328	0.0334	0.0474	0.0470	0.0144	-0.0200	0.0832	0.0967**
12	0.2643**	0.0230	0.0453	0.0619*	-0.0271	0.0050	0.0751*	0.0333	0.0140	-0.0178	0.0914*	0.0628
13	0.2511**	0.0295	0.0541	0.0761**	-0.0025	0.0081	0.0548	0.0544	0.0084	0.0012	0.0811*	0.0737*
14	0.3217**	0.0620	0.0728*	0.0551	0.0233	0.0277	0.0663*	0.0552	0.0460	-0.0171	0.1209**	0.0916**
15	0.1934**	0.0136	0.0596*	0.0416	-0.0292	0.0131	0.0294	0.0599*	0.0087	0.0096	0.0993**	0.0624*
16	0.3310**	0.0485	0.0936**	0.0978**	-0.0145	0.0050	0.0851**	0.0312	0.0352	-0.0124	0.0775*	0.0357
17	0.3592**	0.0390	0.1167**	0.0731	-0.0223	-0.0119	0.0874*	0.0419	0.0687	-0.0184	0.0903	0.0734
18	0.2000**	0.0316	0.0613*	0.1190**	-0.0036	-0.0428	0.0722**	0.0516	0.0055	-0.0043	0.1198**	0.0457
19	0.2707**	0.0417	0.0692*	0.0890**	-0.0004	0.0019	0.0534	0.0343	0.0224	0.0334	0.1130**	0.0471
20	0.3376**	0.0696*	0.0196	0.0683*	0.0273	0.0453	0.0628	0.0922**	0.0297	-0.0036	0.1276**	0.0814*
21	0.2894**	0.0497	0.0578	0.1040**	0.0164	-0.0300	0.0677*	0.0195	0.0150	0.0207	0.1198**	0.0446
22	0.2110**	0.0394	0.0744**	0.0601*	0.0312	0.0211	0.0623	0.0423	0.0042	0.0228	0.1197**	0.1068**
23	0.2652**	0.0338	0.0374	0.0590*	0.0012	0.0093	0.0867**	0.0526*	0.0238	-0.0440	0.0954**	0.0477
24	0.2783**	0.0042	0.1216**	0.1129**	-0.0329	-0.0040	0.0254	0.0583	0.0131	-0.0392	0.1151**	0.0848*
25	0.3039**	0.0565	0.0965**	0.0916**	0.0206	0.0054	0.0345	0.0352	0.0380	0.0226	0.0794*	0.0757*
26	0.1975**	0.0153	0.0415	0.0604*	-0.0012	0.0278	0.0632*	0.0534	0.0418	0.0173	0.1227**	0.0717**
27	0.2814**	0.0538	0.0865*	0.0536	0.0078	0.0159	0.0902*	0.0319	0.0652	0.0020	0.1539**	0.0887*
28	0.2603**	0.0502	0.0700*	0.0656*	-0.0141	0.0012	0.0617	0.0498	0.0163	-0.0385	0.1208**	0.0356
29	0.3219**	0.0320	0.0727*	0.0850**	-0.0184	0.0155	0.0691	0.0458	0.0103	0.0019	0.0692	0.0606
30	0.2955**	0.0524	0.0739*	0.0659	0.0255	0.0048	0.0732*	0.0390	0.0394	0.0035	0.1377**	0.0566
Percent of groups with means different from zero												
10%	100	10	73	77	0	0	47	10	0	0	90	57
5%	100	0	30	43	0	0	20	3	0	0	73	27
PANEL B: GROUPS OF 90 STOCKS EACH												
1	0.3222**	0.0394	0.0832**	0.0810**	0.0027	0.0170	0.0743*	0.0257	0.0399	-0.0154	0.1064**	0.0632*
2	0.2427**	0.0494	0.0784**	0.0641*	0.0043	0.0101	0.0698*	0.0354	0.0353	-0.0207	0.0974**	0.0910**
3	0.3057**	0.0475	0.0634	0.0637*	-0.0117	-0.0061	0.0569	0.0144	0.0566	-0.0273	0.0966**	0.0948**
4	0.2839**	0.0284	0.0660*	0.0770**	-0.0326	0.0104	0.0760*	0.0209	0.0156	-0.0090	0.1026**	0.0826**
5	0.2540**	0.0344	0.0572	0.0573*	-0.0015	0.0157	0.0566	0.0525	0.0231	0.0004	0.1046**	0.0718*
6	0.2899**	0.0347	0.0888**	0.1027**	-0.0035	-0.0222	0.0755	0.0333	0.0382	-0.0220	0.0919**	0.0484
7	0.3028**	0.0556	0.0435	0.0769**	0.0107	0.0019	0.0674**	0.0471	0.0225	0.0295	0.1279**	0.0564
8	0.2494**	0.0278	0.0991**	0.0773**	-0.0072	0.0067	0.0684*	0.0533	0.0169	-0.0281	0.1107**	0.0764**
9	0.2571**	0.0436	0.0830**	0.0604*	0.0113	0.0207	0.0665*	0.0345	0.0488	0.0086	0.1184**	0.0769**
10	0.2999**	0.0608*	0.0710*	0.0688**	-0.0022	0.0235	0.0649*	0.0416	0.0238	-0.0207	0.1109**	0.0589
Percent of groups with means different from zero												
10%	100	10	70	100	0	0	70	0	0	0	100	70
5%	100	0	50	60	0	0	10	0	0	0	100	50
Means for all Groups:												
	0.2835	0.0408	0.0734	0.0746	-0.0033	0.0085	0.0641	0.0415	0.0297	-0.0072	0.1100	0.0707

** and * indicate that means are significant at 5% and 10% level respectively.
Returns are shown in percent.

TABLE 3: SEASONAL PATTERN IN THE TWO STAGE TESTS OF THE ARBITRAGE PRICING THEORY:
CHI-SQUARE SIGNIFICANCE TESTS FOR RISK PREMIA FOR CALENDAR MONTHS USING A 7-FACTOR MODEL FOR 30 GROUPS OF 30 STOCKS EACH.
July 3, 1962-December 31, 1981

Group	January	February	March	April	May	June	July	August	September	October	November	December
1	33.45	5.36	6.82	4.34	4.33	10.72	4.52	3.20	8.34	6.53	5.59	10.78
2	31.10	19.24	8.81	5.59	10.13	8.59	12.02	9.04	8.36	24.11	1.48	2.56
3	59.23	6.97	5.04	5.28	2.23	7.47	3.43	4.47	7.11	10.21	12.04	5.69
4	23.62	16.00	8.51	9.11	7.50	9.93	1.40	4.31	7.45	14.43	7.03	5.00
5	29.47	12.43	7.95	22.83	6.69	5.83	7.98	1.07	1.94	5.86	8.41	4.67
6	24.32	12.76	9.85	13.36	3.28	7.99	1.27	7.33	2.46	6.78	6.98	2.61
7	43.13	8.68	10.33	12.61	5.42	7.69	7.03	6.50	17.18	4.63	8.08	3.76
8	42.22	11.23	4.59	5.43	10.98	4.88	12.79	7.17	9.47	8.97	9.11	5.35
9	20.23	12.46	8.01	6.61	6.49	4.95	4.14	1.98	13.58	3.61	16.00	13.65
10	52.76	6.30	11.36	21.89	5.22	3.77	11.60	5.03	8.11	6.00	4.44	3.61
11	38.24	6.74	8.15	5.11	9.68	6.69	1.85	13.51	4.83	8.41	17.13	4.89
12	39.51	10.93	9.18	6.30	6.14	20.95	5.16	8.00	9.05	15.41	19.68	1.06
13	24.84	1.30	9.18	3.49	3.52	2.48	3.42	2.64	3.58	6.55	10.51	7.88
14	30.77	10.80	4.58	6.10	2.22	2.16	5.53	3.67	4.78	8.51	13.33	4.03
15	26.03	17.87	3.14	11.86	2.79	8.20	8.04	11.05	14.13	8.53	8.26	12.23
16	48.57	6.57	15.93	17.13	9.79	0.64	3.96	3.28	4.69	11.61	7.01	4.60
17	37.86	5.63	4.34	3.77	4.56	4.48	4.02	5.04	2.18	6.03	7.09	5.67
18	19.84	5.53	8.22	14.21	4.32	16.27	4.39	4.55	7.19	9.43	8.22	6.23
19	45.78	7.31	7.95	3.93	6.57	5.97	5.56	7.80	7.99	7.50	11.09	4.29
20	27.65	9.64	7.48	8.25	8.75	6.62	3.61	13.09	6.81	7.19	7.91	4.77
21	37.93	10.77	12.80	17.59	5.87	13.07	1.44	3.94	3.05	2.86	15.50	8.18
22	36.58	4.50	12.64	7.23	6.40	3.77	4.86	4.67	5.40	5.56	5.65	13.11
23	24.80	17.23	1.19	9.53	8.56	8.05	4.29	12.96	4.78	13.08	9.08	5.26
24	30.81	6.27	14.83	7.72	6.10	13.07	3.23	6.96	10.82	14.99	10.76	4.59
25	43.49	4.21	9.77	8.27	9.10	10.19	4.33	3.18	5.66	15.12	14.74	1.32
26	41.96	6.76	5.59	5.67	4.40	14.26	5.77	9.19	7.07	10.91	7.38	7.80
27	26.47	18.73	14.62	3.34	4.86	15.55	3.51	2.09	7.23	14.90	8.62	4.42
28	25.34	8.21	18.35	12.39	3.12	10.14	3.60	4.21	2.11	8.71	6.58	6.95
29	48.86	4.51	10.18	5.79	5.52	11.71	5.56	9.60	13.65	5.98	7.11	6.25
30	27.34	8.29	12.51	3.88	9.62	8.68	1.95	5.44	4.13	5.37	14.95	6.70
Number of significant statistics												
At 10%	30	8	7	8	0	6	2	3	4	7	8	3
At 5%	30	4	4	5	0	4	0	0	2	6	6	0

Factor loadings are the only independent variables; see Table 1 (footnote a) for the regression model.
Chi-square tests the null hypothesis that risk premia vector is null.

The degrees of freedom for Chi-square test is 7 and the critical values are 12.0 and 14.1 at 10 and 5% levels respectively.

TABLE 4: SEASONAL PATTERN IN THE TWO STAGE TESTS OF THE ARBITRAGE PRICING THEORY:
TESTS OF SIGNIFICANCE FOR RISK PREMIA AGAINST TOTAL AND RESIDUAL STANDARD DEVIATIONS AS ALTERNATIVES TO APT MODEL.
SUMMARY STATISTICS FOR CALENDAR MONTHS. PERCENT OF GROUPS WITH SIGNIFICANT TEST STATISTIC.
July 3, 1962 - December 31, 1981

Independent Variables	Test Statistic	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
PANEL A: 7-FACTOR MODEL, 30 GROUPS OF 30 STOCKS EACH													
1. B	Chi-square for B	100	13	13	17	0	13	0	0	7	20	20	0
2. B & Own Sigma	Chi-square for B	50	10	3	10	3	7	0	0	10	3	13	0
(all months)	t-ratio for Sigma	60	17	0	3	3	0	0	0	0	7	0	7
3. B & Res. Sigma	Chi-square for B	60	13	7	13	3	10	0	0	3	7	13	0
	t-ratio for Sigma	57	13	0	3	7	3	0	0	0	0	0	7
4. B & Own Sigma	Chi-square for B	40	7	3	10	0	17	0	0	10	13	13	0
(Single month)	t-ratio for Sigma	80	27	3	10	3	10	0	7	3	7	0	3
PANEL B: 7-FACTOR MODEL, 10 GROUPS OF 90 STOCKS EACH													
1. B	Chi-Square for B	100	70	40	40	0	50	0	0	10	40	50	0
2 B & Own Sigma	Chi-square for B	100	40	20	30	0	50	0	0	10	30	60	0
(all months)	t-ratio for Sigma	90	30	10	0	10	0	0	0	0	0	10	0
3. B & Res. Sigma	Chi-square for B	90	40	30	30	0	40	0	0	10	30	50	0
	t-ratio for Sigma	90	30	10	0	0	0	0	0	0	0	10	0
4. B & Own Sigma	Chi-square for B	100	40	20	40	0	40	0	0	10	30	50	0
(Single month)	t-ratio for Sigma	100	70	10	20	50	20	20	0	10	10	10	0
PANEL C: 17-FACTOR MODEL, 10 GROUPS OF 90 STOCKS EACH													
1. B	Chi-square for B	100	50	10	10	10	20	0	0	10	20	30	0
2. B & Own Sigma	Chi-square for B	100	40	0	10	10	10	0	0	10	0	40	0
(all months)	t-ratio for Sigma	90	50	10	0	0	0	0	0	0	10	10	0
3. B & Res. Sigma	Chi-square for B	100	50	10	10	10	10	0	0	10	0	30	0
	t-ratio for Sigma	80	50	10	0	0	0	0	0	0	10	10	0
4. B & Own Sigma	Chi-square for B	90	40	0	10	10	30	0	0	10	20	20	0
(Single Month)	t-ratio for Sigma	90	60	10	20	40	20	0	10	10	20	0	0

Notes:

1. Factor loadings are the only independent variables. See Table 1 (footnote a) for the regression model. Chi-square tests the null hypothesis that the risk premia vector is null.

2. Total standard deviations of the securities for the sample period are included as additional explanatory variables. Chi-square tests the null hypothesis but the risk premia vector is null. t-ratio is for the regression coefficient for standard deviation.

3. Square root of residual (or specific) variances (Ω) of securities are included as additional explanatory variables. Residual variance corresponds to the diagonal elements of $\tilde{\Omega}$ in $\tilde{\psi} = \tilde{B}\tilde{B}' + \tilde{\Omega}$ in factor analysis. Test statistics correspond to the same tests as in note (2) above.

4. Own standard deviation is estimated using the returns in the corresponding month only. Maximum number of observation for any month is 432 and minimum is 349. Tests are reported for 5% level.

TABLE 5: SEASONAL PATTERN IN THE TWO STAGE TESTS OF THE ARBITRAGE PRICING THEORY:
TESTS OF SIGNIFICANCE FOR RISK PREMIA AGAINST TOTAL AND RESIDUAL STANDARD DEVIATIONS
AS ALTERNATIVES TO THE APT MODEL FOR GROUPS OF 30 STOCKS EACH:
SUMMARY STATISTICS USING RETURNS IN JANUARY AND DECEMBER AND REMAINING MONTHS.
NUMBER OF GROUPS WITH SIGNIFICANT TEST STATISTICS.

GROUP	B[a]	B & own sigma[b]		B & res. sigma[c]		B & own sigma[d]		sigma only[e]		
	χ^2	χ^2	$t(\sigma)$	χ^2	$t(\Omega)$	χ^2	$t(\sigma)$	own $t(\sigma)$	res. $t(\Omega)$	own[d] $t(\sigma)$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Panel A: FEBRUARY THROUGH NOVEMBER										
At 10%	3	4	3	4	3	4	3	7	5	7
At 5%	1	3	2	3	2	4	2	5	3	6
PANEL B: JANUARY & DECEMBER										
At 10%	27	5	13	6	11	3	15	30	27	29
At 5%	25	3	11	5	9	2	12	29	27	29

[a] Factor loadings are the only independent variables. See Table 1 (footnote a) for the regression model. χ^2 tests the null hypothesis that the risk premia vector is null.

[b] Standard deviation of the stocks over the entire sample period are included as additional independent variables. χ^2 tests the null hypothesis that the risk premia vector is null. $t(\sigma)$ is the t-ratio for the regression coefficient for own standard deviation.

[c] Square root of the residual (or specific variance) (Ω) is included as an additional independent variable. Residual variance corresponds to the diagonal elements of $\tilde{\Psi} = \tilde{B}\tilde{B}' + \tilde{\Omega}$ in the factor analysis. χ^2 is to test the hypothesis that the risk premia vector is null. $t(\Omega)$ is the t-ratio for the regression coefficient for residual variance.

[d] Standard deviation is estimated using only the observations from January and February in Panel A and February through November for Panel B.

[e] Total or residual standard deviation is the only independent variable.

TABLE 6: TESTS OF SIGNIFICANCE FOR RISK PREMIA AGAINST
TOTAL AND RESIDUAL STANDARD DEVIATION AS ALTERNATIVES BY EXCLUDING
A MONTH OF STOCK RETURNS. PERCENT OF GROUPS WITH SIGNIFICANT TEST STATISTICS

July 3, 1962 - December 31, 1981

Independent Variables		Test Statistic	Excluded Month Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1.	B	Chi-square for B	7	7	7	17	17	27	17	13	10	43	20	17
2.	B & Own Sigma	Chi-square for B	4	0	0	0	0	0	0	0	0	0	0	0
	(all months)	t-ratio for Sigma	7	17	30	27	23	30	33	23	27	30	43	40
3.	B & Res. Sigma	Chi-square for B	3	0	0	0	0	0	0	0	0	0	0	0
		t-ratio for Sigma	7	20	30	23	23	30	33	20	23	33	43	37
4.	B & Own Sigma	Chi-square for B	7	0	0	0	0	0	0	0	0	0	0	0
	(Single month)	t-ratio for Sigma	10	17	33	30	20	30	38	20	27	37	43	43
5.	Sigma Only	t-ratio	17	87	84	90	97	97	93	87	93	97	97	93
	Residual Sigma Only	t-ratio	17	73	73	83	87	90	87	73	80	90	83	90
	Sigma Only	t-ratio	23	87	84	90	97	97	93	87	93	97	97	93

Notes:

1. Factor loadings are the only independent variables. See Table 1 (footnote a) for the regression model. Chi-square tests the null hypothesis that the risk premia is null.

2. Own standard deviations of the securities for the sample period are included as additional explanatory variables. Chi-square tests the null hypothesis but the risk premia vector is null. t-ratio is for the regression coefficient for standard deviation.

3. Square root of residual (or specific) variances (Ω) of securities are included as additional explanatory variables. Residual variance corresponds to the diagonal elements of $\tilde{\Omega}$ in $\tilde{\psi} = \tilde{B}\tilde{B}' + \tilde{\Omega}$ in factor analysis. Test statistics correspond to the same tests as in note (2) above.

4. Own standard deviation is estimated using the returns in the corresponding eleven months. Regressions are run using observations for 11 months. Months above indicate the excluded observations. Model tested is a 7-factor model on 30 groups of 30 stocks each. Numbers indicate percent of group (out of 30) with significant test statistic, i.e., the null hypothesis that risk premia vector is null is rejected. Tests are reported for 5% level.

TABLE 4: SEASONAL PATTERN IN THE TWO STAGE TESTS OF THE ARBITRAGE PRICING THEORY:
TESTS OF SIGNIFICANCE FOR RISK PREMIA AGAINST TOTAL AND RESIDUAL STANDARD DEVIATIONS AS ALTERNATIVES TO APT MODEL.
SUMMARY STATISTICS FOR CALENDAR MONTHS. PERCENT OF GROUPS WITH SIGNIFICANT TEST STATISTIC.
July 3, 1962 - December 31, 1981

Independent Variables	Test Statistic	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
PANEL A: 7-FACTOR MODEL, 30 GROUPS OF 30 STOCKS EACH													
1. B	Chi-square for B	100	13	13	17	0	13	0	0	7	20	20	0
2. B & Own Sigma (all months)	Chi-square for B	50	10	3	10	3	7	0	0	10	3	13	0
	t-ratio for Sigma	60	17	0	3	3	0	0	0	0	7	0	7
3. B & Res. Sigma	Chi-square for B	60	13	7	13	3	10	0	0	3	7	13	0
	t-ratio for Sigma	57	13	0	3	7	3	0	0	0	0	0	7
4. B & Own Sigma (Single month)	Chi-square for B	40	7	3	10	0	17	0	0	10	13	13	0
	t-ratio for Sigma	80	27	3	10	3	10	0	7	3	7	0	3
PANEL B: 7-FACTOR MODEL, 10 GROUPS OF 90 STOCKS EACH													
1. B	Chi-Square for B	100	70	40	40	0	50	0	0	10	40	50	0
2 B & Own Sigma (all months)	Chi-square for B	100	40	20	30	0	50	0	0	10	30	60	0
	t-ratio for Sigma	90	30	10	0	10	0	0	0	0	0	10	0
3. B & Res. Sigma	Chi-square for B	90	40	30	30	0	40	0	0	10	30	50	0
	t-ratio for Sigma	90	30	10	0	0	0	0	0	0	0	10	0
4. B & Own Sigma (Single month)	Chi-square for B	100	40	20	40	0	40	0	0	10	30	50	0
	t-ratio for Sigma	100	70	10	20	50	20	20	0	10	10	10	0
PANEL C: 17-FACTOR MODEL, 10 GROUPS OF 90 STOCKS EACH													
1. B	Chi-square for B	100	50	10	10	10	20	0	0	10	20	30	0
2. B & Own Sigma (all months)	Chi-square for B	100	40	0	10	10	10	0	0	10	0	40	0
	t-ratio for Sigma	90	50	10	0	0	0	0	0	0	10	10	0
3. B & Res. Sigma	Chi-square for B	100	50	10	10	10	10	0	0	10	0	30	0
	t-ratio for Sigma	80	50	10	0	0	0	0	0	0	10	10	0
4. B & Own Sigma (Single Month)	Chi-square for B	90	40	0	10	10	30	0	0	10	20	20	0
	t-ratio for Sigma	90	60	10	20	40	20	0	10	10	20	0	0

Notes:

1. Factor loadings are the only independent variables. See Table 1 (footnote a) for the regression model. Chi-square tests the null hypothesis that the risk premia vector is null.

2. Total standard deviations of the securities for the sample period are included as additional explanatory variables. Chi-square tests the null hypothesis but the risk premia vector is null. t-ratio is for the regression coefficient for standard deviation.

3. Square root of residual (or specific) variances (Ω) of securities are included as additional explanatory variables. Residual variance corresponds to the diagonal elements of $\tilde{\Omega}$ in $\tilde{\psi} = \tilde{B}\tilde{B}' + \tilde{\Omega}$ in factor analysis. Test statistics correspond to the same tests as in note (2) above.

4. Own standard deviation is estimated using the returns in the corresponding month only. Maximum number of observation for any month is 432 and minimum is 349. Tests are reported for 5% level.

TABLE 2: MEANS FOR MONTH-TO-MONTH DAILY STOCK RETURNS FOR GROUPS OF 30 AND 90 STOCKS EACH
July 3, 1962-December 31, 1981

GRDL	January	February	March	April	May	June	July	August	September	October	November	December
PANEL A: GROUPS OF 30 STOCKS EACH												
1	0.3338**	0.0553	0.0707*	0.0801**	-0.0410	0.0182	0.0603	0.0466	0.0282	-0.0383	0.0789*	0.0747*
2	0.2803**	0.0348	0.0693*	0.0734*	0.0332	-0.0036	0.0531	0.0355	0.0467	-0.0124	0.0895**	0.0438
3	0.3495**	0.0443	0.1055**	0.0799*	-0.0011	0.0361	0.0925**	0.0206	0.0520	0.0025	0.1802**	0.0629
4	0.2262**	0.0454	0.0739*	0.0790*	-0.0022	0.0165	0.0590	0.0501	0.0314	-0.0033	0.1079**	0.0957**
5	0.2982**	0.0692*	0.0872**	0.0583*	0.0266	0.0006	0.0943**	0.0369	0.0409	-0.0625	0.1166**	0.0863**
6	0.2436**	0.0285	0.0701*	0.0709**	-0.0414	0.0010	0.0366	0.0316	0.0011	0.0084	0.0949**	0.0694
7	0.3086**	0.0158	0.0884**	0.0296	-0.0230	0.0209	0.0454	0.0269	0.0693	-0.0114	0.1160**	0.0847*
8	0.3443**	0.0563	0.0734*	0.0862**	-0.0160	-0.0210	0.0904**	0.0199	0.0325	-0.0082	0.1116**	0.1004**
9	0.2876**	0.0676*	0.0496	0.0855**	0.0247	0.0030	0.0362	0.0455	0.0269	-0.0123	0.1233**	0.0836**
10	0.3333**	0.0367	0.0710	0.1014**	-0.0139	0.0306	0.0870*	0.0035	0.0414	0.0008	0.1425**	0.0755*
11	0.2658**	0.0257	0.1186**	0.0531	-0.0328	0.0334	0.0474	0.0470	0.0144	-0.0200	0.0832	0.0967**
12	0.2643**	0.0230	0.0453	0.0619*	-0.0271	0.0050	0.0751*	0.0333	0.0140	-0.0178	0.0914*	0.0628
13	0.2511**	0.0295	0.0541	0.0761**	-0.0025	0.0081	0.0548	0.0544	0.0084	0.0012	0.0811*	0.0737*
14	0.3217**	0.0620	0.0728*	0.0551	0.0233	0.0277	0.0663*	0.0552	0.0460	-0.0171	0.1209**	0.0916**
15	0.1934**	0.0136	0.0596*	0.0416	-0.0292	0.0131	0.0294	0.0599*	0.0087	0.0096	0.0993**	0.0624*
16	0.3310**	0.0485	0.0936**	0.0978**	-0.0145	0.0050	0.0851**	0.0312	0.0352	-0.0124	0.0775*	0.0357
17	0.3592**	0.0390	0.1167**	0.0731	-0.0223	-0.0119	0.0874*	0.0419	0.0687	-0.0184	0.0903	0.0734
18	0.2000**	0.0316	0.0613*	0.1190**	-0.0036	-0.0428	0.0722**	0.0516	0.0055	-0.0043	0.1198**	0.0457
19	0.2707**	0.0417	0.0692*	0.0890**	-0.0004	0.0019	0.0534	0.0343	0.0224	0.0334	0.1130**	0.0471
20	0.3376**	0.0696*	0.0196	0.0683*	0.0273	0.0453	0.0628	0.0922**	0.0297	-0.0036	0.1276**	0.0814*
21	0.2894**	0.0497	0.0578	0.1040**	0.0164	-0.0300	0.0677*	0.0195	0.0150	0.0207	0.1198**	0.0446
22	0.2110**	0.0394	0.0744**	0.0601*	0.0312	0.0211	0.0623	0.0423	0.0042	0.0228	0.1197**	0.1068**
23	0.2652**	0.0338	0.0374	0.0590*	0.0012	0.0093	0.0867**	0.0526*	0.0238	-0.0440	0.0954**	0.0477
24	0.2783**	0.0042	0.1216**	0.1129**	-0.0329	-0.0040	0.0254	0.0583	0.0131	-0.0392	0.1151**	0.0848*
25	0.3039**	0.0565	0.0965**	0.0916**	0.0206	0.0054	0.0345	0.0352	0.0380	0.0226	0.0794*	0.0757*
26	0.1975**	0.0153	0.0415	0.0604*	-0.0012	0.0278	0.0632*	0.0534	0.0418	0.0173	0.1227**	0.0717**
27	0.2814**	0.0538	0.0865*	0.0536	0.0078	0.0159	0.0902*	0.0319	0.0652	0.0020	0.1539**	0.0887*
28	0.2603**	0.0502	0.0700*	0.0656*	-0.0141	0.0012	0.0617	0.0498	0.0163	-0.0385	0.1208**	0.0356
29	0.3219**	0.0320	0.0727*	0.0850**	-0.0184	0.0155	0.0691	0.0458	0.0103	0.0019	0.0692	0.0606
30	0.2955**	0.0524	0.0739*	0.0659	0.0255	0.0048	0.0732*	0.0390	0.0394	0.0035	0.1377**	0.0566
Percent of groups with means different from zero												
10%	100	10	73	77	0	0	47	10	0	0	90	57
5%	100	0	30	43	0	0	20	3	0	0	73	27
PANEL B: GROUPS OF 90 STOCKS EACH												
1	0.3222**	0.0394	0.0832**	0.0810**	0.0027	0.0170	0.0743*	0.0257	0.0399	-0.0154	0.1064**	0.0632*
2	0.2427**	0.0494	0.0784**	0.0641*	0.0043	0.0101	0.0698*	0.0354	0.0353	-0.0207	0.0974**	0.0910**
3	0.3057**	0.0475	0.0634	0.0637*	-0.0117	-0.0061	0.0569	0.0344	0.0566	-0.0273	0.0966**	0.0948**
4	0.2839**	0.0284	0.0660*	0.0770**	-0.0326	0.0104	0.0760*	0.0209	0.0156	-0.0090	0.1026**	0.0826**
5	0.2540**	0.0344	0.0572	0.0573*	-0.0015	0.0157	0.0566	0.0525	0.0231	0.0004	0.1046**	0.0718*
6	0.2899**	0.0347	0.0888**	0.1027**	-0.0035	-0.0222	0.0755	0.0333	0.0382	-0.0220	0.0919**	0.0484
7	0.3028**	0.0556	0.0435	0.0769**	0.0107	0.0019	0.0674**	0.0471	0.0225	0.0295	0.1279**	0.0564
8	0.2494**	0.0278	0.0991**	0.0773**	-0.0072	0.0067	0.0684*	0.0533	0.0169	-0.0281	0.1107**	0.0764**
9	0.2571**	0.0436	0.0830**	0.0604*	0.0113	0.0207	0.0665*	0.0345	0.0488	0.0086	0.1184**	0.0769**
10	0.2999**	0.0608*	0.0710*	0.0688**	-0.0022	0.0235	0.0649*	0.0416	0.0238	-0.0207	0.1109**	0.0589
Percent of groups with means different from zero												
10%	100	10	70	100	0	0	70	0	0	0	100	70
5%	100	0	50	60	0	0	10	0	0	0	100	50
Means for all Groups:												
	0.2835	0.0408	0.0734	0.0746	-0.0033	0.0085	0.0641	0.0415	0.0297	-0.0072	0.1100	0.0707

** and * indicate that means are significant at 5% and 10% level respectively.
Returns are shown in percent.

TABLE 1: TWO STAGE TESTS OF THE ARBITRAGE PRICING THEORY AND TESTS OF SIGNIFICANCE FOR RISK PREMIA AGAINST SPECIFIC ALTERNATIVES.
TOTAL STANDARD DEVIATION AND RESIDUAL STANDARD DEVIATIONS ARE USED AS ALTERNATIVES TO THE APT MODEL: SUMMARY STATISTICS--July 3, 1962-December 31, 1981

Group	A. Groups of 30 stocks each, 7-factor model					B. Groups of 90 stocks each, 17-factor model					C. Groups of 90 stocks each, 7-factor model				
	B[a] χ^2	B & σ χ^2	(b) $t(\sigma)$	B & Ω χ^2	(c) $t(\Omega)$	B (a) χ^2	B & σ χ^2	(b) $t(\sigma)$	B & Ω χ^2	(c) $t(\Omega)$	B (a) χ^2	B & σ χ^2	(b) $t(\sigma)$	B & Ω χ^2	(c) $t(\Omega)$
	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
1	6.765	4.532	1.61	5.626	1.60	29.447**	13.391	2.53**	18.410	2.55**	21.230**	8.575	3.16**	8.857	3.13**
2	12.961*	4.856	1.031	7.366	0.90	27.061*	16.828	0.61	20.234	0.60	20.922**	11.167	1.09	15.723**	1.06
3	15.790**	4.682	2.15**	5.953	2.15**	33.765**	18.313	2.23**	19.830	2.22**	27.308**	13.243*	2.55**	14.179**	2.55**
4	9.854	5.046	1.02	5.403	1.03	21.121	13.258	1.93*	12.413	1.77*	17.097**	9.273	1.90*	7.965	1.68*
5	9.757	3.142	0.47	7.222	0.78	27.941**	16.051	1.89*	21.722	1.92*	12.995*	5.596	2.85**	7.081	2.84**
6	6.448	3.669	0.48	4.504	0.29	22.748	14.413	1.07	15.088	1.03	15.841**	8.329	1.55	8.932	1.50
7	13.848*	6.161	0.52	6.328	0.51	33.411**	8.590	1.90*	15.527	1.80*	27.708**	5.285	2.62**	5.752	2.64**
8	14.393**	6.460	1.46	7.749	1.33	24.258	13.751	2.39**	14.729	2.33**	18.564**	9.708	2.72**	8.953	2.65**
9	16.083**	5.603	1.05	7.457	0.94	23.611	13.589	3.54**	13.961	3.51**	18.668**	10.154	3.72**	10.583	3.70**
10	7.934	0.869	1.02	1.952	0.99	28.219**	14.690	1.61	14.421	1.47	21.312**	8.956	1.90*	8.768	1.96**
11	12.717*	6.655	1.59	5.867	1.49										
12	7.325	4.561	1.07	4.393	0.71										
13	6.444	4.038	0.90	6.461	0.91										
14	8.979	3.716	1.87*	8.259	1.88*										
15	5.945	3.178	0.66	3.283	0.62										
16	12.057*	5.870	1.21	5.943	1.20										
17	7.259	0.799	1.73*	1.010	1.77*										
18	6.411	5.789	0.59	6.882	0.68										
19	10.904	2.117	2.05**	2.563	2.05**										
20	19.027**	2.300	1.71*	5.903	1.80*										
21	13.531*	2.693	1.27	3.270	1.16										
22	9.560	3.850	1.65*	4.398	1.56										
23	6.479	1.623	1.08	2.330	0.85										
24	4.346	1.892	2.00**	5.238	2.09**										
25	12.087*	9.472	2.78**	11.551	2.66**										
26	6.421	4.922	1.92*	5.483	1.94*										
27	9.433	2.376	1.27	3.260	1.23										
28	8.445	5.437	0.26	7.334	0.18										
29	11.420	6.905	1.17	6.159	1.25										
30	10.790	3.089	2.25**	3.208	2.19**										
Number of Significant Statistics															
at 10%	10	0	10	0	9	6	0	7	0	7	10	1	8	2	8
at 5%	4	0	5	0	5	5	0	4	0	4	9	0	6	2	7

a χ^2 value is computed by $T\bar{c}^*W^{-1}\bar{c}^{*\prime}$ where $\bar{c}^* = (\frac{1}{T})\sum_{t=1}^{T}\tilde{c}^*_t$ and $W = (\frac{1}{T})\sum_{t=1}^{T}(\tilde{c}^*_{t.} - \bar{c}^*)'(\tilde{c}^*_{t.} - \bar{c}^*)$. $\tilde{c}_{t.}$'s are from the daily cross-sectional regression estimates using the GLS model in equation (4), $\tilde{c}'_{t.} = (\tilde{B}^*\tilde{\psi}^{-1}\tilde{B}^{*\prime})^{-1}\tilde{B}^*\tilde{\psi}^{-1}r'_{t.}$, $\tilde{c}_{t.} = (\tilde{c}_{t0}, \tilde{c}^*_{t.})$; $B^{*\prime}$ is $[e:B]$, the augmented matrix of factor loadings (B) with unit vector (e). χ^2 tests the null hypothesis that the risk premia vector is null ($\tilde{c}^{*\prime}_{t.} = 0$). It is distributed as chi-square with degrees of freedom equal to the number of factors.

b Standard deviation of each security is included as an additional explanatory variable in B*. χ^2 tests the null hypothesis that the risk premia vector is null. $t(\sigma)$ is the t-ratio for the regression coefficient for standard deviation. t-ratio is given by $\sqrt{T}(\bar{d}/s)$, where $\bar{d} = \sum_{t=1}^{T}\tilde{d}_t$ and $s = [(\frac{1}{T})\sum_{t=1}^{T}(\tilde{d}_t - \bar{d})^2]^{1/2}$ (see equation (5)).

c Square root of residual (or specific) variance (Ω) is included as an additional explanatory variable. Residual variances correspond to the diagonal elements of $\tilde{\Omega}$ in $\tilde{\psi} = \tilde{B}\tilde{B}' + \tilde{\Omega}$. χ^2 tests the null hypothesis that the risk premia is null and $t(\Omega)$ is the t-ratio for the regression coefficient for residual standard deviation.

The minimum number of daily observations is 4511 and the maximum is 4730.

* and ** indicate the groups for which the null hypothesis is rejected at 10 and 5 percent levels of significance, respectively.