Feedback Effects and the Limits to Arbitrage*

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Abstract

This paper identifies a limit to arbitrage that arises from the fact that a firm's fundamental value is endogenous to the act of exploiting the arbitrage opportunity. Trading on private information reveals this information to managers and helps them improve their real decisions, in turn enhancing fundamental value. While this increases the profitability of a long position, it reduces the profitability of a short position – selling on negative information reveals to the manager that firm prospects are poor, causing him to cancel investment projects. Optimal abandonment increases the firm's value and may cause the speculator to realize a loss on her initial sale. Thus, investors may strategically refrain from trading on negative information. This has potentially important real consequences – if negative information is not incorporated into stock prices, negative-NPV projects may not be abandoned, leading to overinvestment.

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1 Introduction

Whether financial markets are informationally efficient is one of the most hotly-contested debates in finance. Proponents of market efficiency argue that profit opportunities in the financial market will lead speculators to trade in a way that eliminates any mispricing. For example, if speculators have negative information about a stock, and this information is not reflected in the price, they will find it profitable to sell the stock. This will push down the price, causing it to reflect speculators' information. However, a sizable literature identifies various limits to arbitrage, which may deter speculators from trading on their information.[1] For example, De Long, Shleifer, Summers, and Waldmann (1990) and Shleifer and Vishny (1997) show that the slow convergence of price to fundamental value may render arbitrage activities too risky. This in turn dissuades trading if the speculator has a short horizon, which may in turn arise from informational asymmetries with her own investors. Other explanations for limited arbitrage rely on market frictions such as short-sales constraints. All of these mechanisms treat the firm's fundamental value as exogenous and rely on market imperfections to explain why speculators will not drive the price towards fundamental value. Thus, as financial markets develop, these limits to arbitrage may weaken.

In this paper, we identify a quite different limit to arbitrage, which does not rely on market imperfections and thus may not attenuate with the development of financial markets. Instead, our mechanism stems from the fact that the *value of the asset being arbitraged is endogenous to the act of exploiting the arbitrage* – it depends on speculators' trading behavior and market prices. The argument is based on the idea that, by trading, speculators cause prices to move, which in turn reveals information to decision makers on the real side of the economy (such as managers, board members, capital providers, employees, customers, and regulators). These decision makers then take actions based on the information revealed in the price, and these actions change the underlying value of the asset. This may make the initial trading unprofitable, deterring it from occurring in the first place.

To fix ideas, consider the following example. Suppose that a firm (acquirer) announces an acquisition of another firm (target). Also assume that some speculators conducted some analysis suggesting that this acquisition will be value-destructive. Traditional theory suggests that these speculators should sell the stock of the acquirer, attempting to profit from (what they believe is) the low underlying value resulting from the upcoming acquisition. However, large-scale selling in the financial market will convey to the acquirer that speculators believe the acquisition is a bad idea. As a result, the acquirer may end up cancelling the acquisition. In turn, cancellation of a bad acquisition will boost the value of the firm, thus causing the speculator to suffer a loss on her short position. Put differently, the acquirer's decision to cancel the acquisition means that the negative information possessed by speculators is no longer relevant, and hence they should not trade on it. Thus, the information ends up not being reflected in the price.

[1] This notion of "arbitrage" is broader than the traditional textbook notion of risk-free arbitrage from trading two identical securities. Here, we use "arbitrage" to refer to investors trading on their private information.

Our mechanism is based on the presence of a feedback effect from the financial market to real economic decisions. A common perception is that managers know more about their own firms than outsiders (e.g. Myers and Majluf (1984)). While this is likely plausible for internal information about the firm in isolation, optimal managerial decisions also depend on external information (such as market demand for a firm's products, or potential synergies with a target) about which outsiders may be more informed. Even for internal information, while the manager is likely more informed than any individual investor, the stock market aggregates information from millions of investors who may collectively know more than the manager (Hayek (1945).) A classic example of how information from the stock market shapes managerial decisions is Coca-Cola's attempted acquisition of Quaker Oats in 2000. On November 20, 2000, the *Wall Street Journal* reported that Coca-Cola was in talks to acquire Quaker Oats. Shortly thereafter, Coca-Cola confirmed such discussions. The market reacted negatively, sending Coca-Cola's shares down almost 8% on November 20th, and more than 2% on November 21st. Coca-Cola management brought the deal to its board on November 21st, and the board rejected the acquisition later that evening. The following day, Coca-Cola's shares rebounded almost 8%. Thus, speculators who had short-sold on the initial merger announcement, based on the belief that the acquisition would destroy value, may have ended up losing money – precisely the effect modeled by this paper. In the same context, Luo (2005) provides large-sample evidence that acquisitions are more likely to be cancelled if the market reacts negatively to them, and that the effect is more pronounced when the acquirer is more likely to have something to learn from the market. More broadly, Chen, Goldstein, and Jiang (2007) show that the sensitivity of investment to price is higher when the price contains more private information not known to managers. Edmans, Goldstein, and Jiang (2011) demonstrate that a firm's market price affects the likelihood that it becomes a takeover target, which may arise because potential acquirers learn from the market price. Moreover, our model can apply to corrective actions undertaken by stakeholders other than the manager, who likely have less information than the manager and may be more reliant on information held by outsiders. Examples include managerial replacement (undertaken by the board, or by shareholders who lobby the board), and a disciplinary takeover (undertaken by an acquirer).

An important aspect of our theory is that it generates asymmetry between trading on negative information and trading on positive information. The feedback effect generates an equilibrium where speculators trade on positive news but do not trade on negative news. Yet, it does not give rise to the opposite equilibrium, where speculators trade on negative news only. The intuition is as follows. When speculators trade on information, they improve the efficiency of the firm's decisions – regardless of the direction of their trade. If the speculator has positive information on a firm's prospects, trading on it will reveal to the manager that investment is profitable. This will in turn cause the firm to invest more, thus increasing its value. If the speculator has negative information, trading on it will reveal to the manager that investment is unprofitable. This will in turn cause the firm to invest less, also increasing its value as contraction is the correct decision. When a speculator buys and takes a long position in a firm, he benefits further from increasing its value via the feedback effect. By contrast, when he sells and takes a short position, he loses from increasing the firm's value via the feedback effect.

Even though the speculator's trading behavior is asymmetric, it is not automatic that the impact on prices is asymmetric. The market maker is fully rational and takes into account the fact that the speculator buys on positive information and does not trade on negative information. Thus, he adjusts his pricing function accordingly. Therefore, it may seem that negative information will be impounded in prices to the same degree as positive information – even though it may lead to a neutral rather than negative order flow, the market maker knows that a neutral order flow can stem from the speculator having negative information but choosing not to trade, and may decrease the price accordingly. By contrast, we show that the asymmetry in trading behavior does translate into asymmetry in price impact, despite the rationality of the market maker. The crux is that the market maker cannot distinguish the case of a speculator who has negative information but chooses to withhold it, from the case in which the speculator is absent (i.e. there is no information). Thus, a neutral order flow does not lead to a large stock price decrease, and so negative information has a smaller effect on prices. Indeed, Hong, Lim, and Stein (2000) show empirically that bad news is incorporated in prices more slowly than good news. They speculate that this arises because it is firm management that possesses value-relevant information, and they will publicize it more enthusiastically for favorable than unfavorable information. Our paper presents a formal model that offers an alternative explanation. Here, key information is held by a firm's investors rather than its managers, who "publicize" it not through public news releases, but by trading on it. They also choose to disseminate good news more readily than bad news, but for different reasons from firm management, i.e., because of the feedback effect.

The asymmetry of our effect may generate important real consequences. Since negative information is not incorporated into prices, it does not influence management decisions. Thus, while positive-NPV projects will be encouraged, some negative-NPV projects will not be canceled even though there is an agent in the economy who knows with certainty that the project is negative-NPV, leading to overinvestment overall. In contrast to standard overinvestment theories which are based on the manager's private benefits (e.g., Jensen (1986), Stulz (1990), Zwiebel (1996)), here the manager is fully aligned with firm value and there are no agency problems. The manager wishes to maximize firm value by learning from prices, but is unable to do so since speculators refrain from revealing their information. Applied to M&A as well as organic investment, the theory may explain why M&A appears to be "excessive" and a large fraction of acquisitions destroy value (see, e.g., Andrade, Mitchell, and Stafford (2001).)

As mentioned above, the primary motivation for our paper is to identify a limit to arbitrage. Different authors have emphasized different factors that lead to limits on arbitrage activities. Campbell and Kyle (1993) focus on fundamental risk, i.e., the risk that the firm's fundamentals will change while the arbitrage strategy is being pursued. In their model, such changes are unrelated to speculators' arbitrage activities. De Long, Shleifer, Summers, and Waldmann (1990) argue that noise-trading risk, i.e., the risk that noise trading will increase the degree of mispricing, may render arbitrage activities unprofitable. Noise trading only affects the asset's market price and not its fundamental value, which is again exogenous to the act of arbitrage. Shleifer and Vishny (1997) show that, even if an arbitrage strategy is sure to converge in the long-run, the possibility that mispricing may widen in the short-term may deter speculators from trading on it. Similarly, Kondor (2009) demonstrates that arbitrageurs may stay out of a trade if they believe that it may become more profitable in the future. Many authors (e.g., Pontiff (1996), Mitchell and Pulvino (2001), and Mitchell, Pulvino, and Stafford (2002)) focus on the transaction costs and holding costs that arbitrageurs have to incur while pursuing an arbitrage strategy. Others (Geczy, Musto, and Reed (2002), and Lamont and Thaler (2003)) discuss the importance of short-sales constraints. While these papers emphasize market frictions as the source of limits to arbitrage, our paper shows that limits to arbitrage arise when the market performs its utmost efficient role: guiding the allocation of real resources. Thus, while limits to arbitrage based on market frictions may attenuate with the development of financial markets, the effect identified by this paper may strengthen – as investors become more sophisticated, managers will learn from them to a greater degree.[2]

Our paper is related to the literature exploring the theoretical implications of the feedback effects from market prices to real decision making. Several papers in this literature have shown that the feedback effect can be harmful for real efficiency. Most closely related is Goldstein and Guembel (2008), who show that it provides an incentive to uninformed speculators to short sell a stock, reducing its value by having a real decision based on false information. Their paper also highlighted an asymmetry between buy-side and sell-side speculation, but it was applied to uninformed trading, whereas here we show that negatively-informed speculators are less likely to trade than positively-informed speculators. Also closely related is the paper by Bond, Goldstein, and Prescott (2010), which discusses the implications of price non-monotonicity due to corrective actions on equilibrium outcomes. As will become clear later, the limit to arbitrage in our paper relies strongly on this price non-monotonicity. Bond, Goldstein, and Prescott (2010), however, did not analyze trading incentives and the effect of non-monotonicity on strategic traders. Other related papers are Dow, Goldstein, and Guembel (2010), and Goldstein, Ozdenoren, and Yuan (2010) who also model complexities arising from the feedback effect. Overall, the point in our paper – that negatively informed speculators will strategically withhold information from the market, because they know that the release of negative information will lead managers to fix the underlying problem – is new in this literature.

This paper proceeds as follows. Section 2 presents the model. Section 3 contains the core of the analysis, demonstrating the asymmetric limit to arbitrage. Section 4 discusses potential

[2]Still, as will become clear in the model description, our mechanism does require a non-zero trading cost in addition to the feedback effect.

applications of the model, and Section 5 concludes. Appendix A contains all proofs not in the main text.

2 The Model

The model has three dates, $t \in \{0, 1, 2\}$, and a firm whose stock is traded in the financial market. The firm's manager needs to take a decision whether to continue or abandon an investment project. The manager's goal is to maximize expected firm value; since there are no agency problems between the manager and the firm, we will use these two terms interchangeably. At $t = 0$, a risk-neutral speculator may be present in the financial market. In this case, she is informed about the state of nature θ that determines the profitability of continuing vs. abandoning the investment project. Trading in the financial market occurs at $t = 1$. In addition to the speculator, two other types of agents participate in the financial market: noise traders whose trades are unrelated to the realization of θ, and a risk neutral market maker. The latter collects the orders from the speculator and the noise traders and sets a price at which he executes the orders out of his inventory. At $t = 2$, the manager takes the decision, which may be affected by the events that took place in the financial market at $t = 1$. Finally, all uncertainty is realized and payoffs are made. We now describe the firm's investment problem and the trading process in more detail.

2.1 The Firm's Decision

Suppose that the firm has an investment project that can be either continued or abandoned at $t = 2$. We denote the firm's decision as $d \in \{i, n\}$, where $d = i$ represents continuing the investment and $d = n$ represents no investment (also referred to as "abandonment" or "correction"). The firm faces uncertainty over the realization of value under each possible action. In particular, there are two possible states $\theta \in \Theta \equiv \{H, L\}$ ("high" and "low"). We denote the value of the firm realized in $t = 2$ as $v = R_\theta^d$, which depends on both the state of nature θ and the manager's action d. We make the following assumptions about firm value:

$$R_H^i > R_H^n \tag{1}$$

$$R_L^n > R_L^i \tag{2}$$

$$R_H^i > R_L^i \tag{3}$$

$$R_L^n > R_H^n. \tag{4}$$

Equations (1) and (2) imply that continuation is optimal in state H, while abandonment is optimal in state L. Equations (3) and (4) imply that if the project is continued, firm value is higher in state H, while if it is abandoned, firm value is higher in state L. Importantly, these assumptions imply non-monotonicity of firm value in the underlying state. As will become clear later, this is a crucial ingredient of our model. The idea is that one state does not dominate the other; instead, the firm must choose the appropriate action for the state. For example, consider the case where continuation implies moving forward with a takeover decision, and abandonment implies keeping the cash for future opportunities. State H corresponds to a state in which current acquisition opportunities dominate future ones, and state L refers to the reverse. Under continuation, the value of the firm is higher in state H, whereas under abandonment, the value of the firm is higher in state L.

The prior probability that the state is $\theta = H$ is $\frac{1}{2}$, which is common knowledge. We use q to denote the posterior probability the manager assigns to the case $\theta = H$. The manager's decision is conditioned on q, which in turn is calculated using information arising from trades in the financial market. Let γ denote the posterior belief that the state is H such that the manager is indifferent between continuation and abandonment, i.e.:

$$\gamma R_H^i + (1-\gamma) R_L^i = \gamma R_H^n + (1-\gamma) R_L^n. \tag{5}$$

The value of γ represents a "cutoff" that determines the manager's action. If and only if his posterior belief q is greater than γ, he will continue the project. In the body of the paper, we focus on the case where $\gamma < \frac{1}{2}$. Since the prior probability that the state is $\theta = H$ is $\frac{1}{2}$, this implies that without further information, the firm would continue the investment. Since firm value is higher in state H under continuation, but lower under abandonment, this assumption is consistent with the interpretation of state H as representing the "high" state with good fundamentals. In essence, this state represents good news for the status quo (continuing with the investment decision), and this is how the reader should interpret the meaning of "good news" or "positive information" throughout the paper. We will use the term "positively-informed speculator" to describe a speculator who observes $\theta = H$, and "negatively-informed speculator" to describe a speculator who observes $\theta = L$. In Appendix B, we consider the opposite case of $\gamma > \frac{1}{2}$ and demonstrate that our results on the asymmetric limit to arbitrage hold in that case as well. Essentially, that case is a mirror image of the case of $\gamma < \frac{1}{2}$.

2.2 Trade in the Financial Market

In $t = 0$, with probability $\lambda < 1$, a speculator arrives in the financial market. Whether the speculator is present or not is unknown to anyone else.[3] If the speculator is present, she sees the state of nature θ with certainty. We will use the term "positively-informed speculator" to describe a speculator who observes $\theta = H$, and "negatively-informed speculator" to describe a speculator who observes $\theta = L$.

Trading in the financial market happens in $t = 1$. Always present at this time is a noise trader, who trades $z = -1$, 0, or 1 with equal probabilities. If the speculator is present,

[3] The assumption that there is uncertainty about whether the speculator is present in the financial market is similar to Chakraborty and Yilmaz (2004).

she makes an endogenous trading choice $s \in \{-1, 0, 1\}$. Following Kyle (1985), orders are submitted simultaneously to a market maker who sets the price and absorbs order flows out of his inventory. The orders are market orders and are not contingent on the price. The competitive market maker sets the price equal to expected asset value, given the information contained in the order flow. The market maker can only observe total order flow $X = s + z$, but not its individual components s and z. Possible order flows are $X \in \{-2, -1, 0, 1, 2\}$ and the pricing function is $p(X) = E(v|X)$, where v is firm value. We assume that trading either -1 or 1 is costly for the speculator and entails paying a fixed cost of κ.

The manager of the firm observes total order flow X, and uses the information in X to form his posterior q, which is then used in the investment decision. Allowing the manager to observe order flow X has the same effect as allowing him to observe the price p as there is a one-to-one correspondence between the price and the order flow, and thus simplifies the analysis without affecting the economic insights of the model. It is also realistic to assume that firm managers have access to information about trading quantities in the financial market (see Dow, Goldstein, and Guembel (2010) for a related discussion on this issue).

2.3 Equilibrium

The equilibrium concept we use is the Perfect Bayesian Nash Equilibrium. Here, it is defined as follows: (i) A trading strategy by the speculator: $S : \Theta \to \Delta\{-1, 0, 1\}$ that maximizes his expected final payoff $s(v - p) - |s|\kappa$, given the price setting rule, the strategy of the manager, and his information about the realization of θ. (ii) An investment strategy by the firm $D : \mathcal{Q} \to \{i, n\}$ (where $\mathcal{Q} = \{-2, -1, 0, 1, 2\}$), that maximizes expected firm value $v = R_\theta^d$ given the information in the order flow and all other strategies. (iii) A price setting strategy by the market maker $p : \mathcal{Q} \to \mathbb{R}$ that allows him to break even in expectation, given the information in the price and all other strategies. Moreover, (iv) the firm and the market maker use Bayes' rule in order to update their beliefs from the order they observe in the financial market. Finally, (v) all agents have rational expectations in the sense that each player's belief about the other players' strategies is correct in equilibrium.

3 An Equilibrium with Asymmetric Limits to Arbitrage

Our main result is that, under some conditions, an equilibrium exists where the speculator buys the security after receiving good news (i.e., after learning that $\theta = H$), but does not trade after receiving bad news (i.e., after learning that $\theta = L$). Hence, there is an asymmetric limit to arbitrage. The underlying source of this equilibrium is the existence of a feedback effect from the trading in the financial market to the investment decision of the firm. We discuss the role of the required conditions in Section 3.1. We also show that there is no equilibrium with the opposite asymmetry, i.e., where the feedback effect leads the speculator to sell but not to buy.

We start by developing the equilibrium with an asymmetric limit to arbitrage. Suppose that the speculator pursues the following strategy:

θ	-1	0	1
H	0	$1-\mu_H$	μ_H
L	μ_L	$1-\mu_L$	0

The table describes the probability of trading -1, 0, or 1 after observing signals H or L. Here, $0 \leq \mu_H,\ \mu_L \leq 1$. Hence, according to this strategy, the positively-informed speculator never sells and the negatively-informed speculator never buys. (We will later show that this is indeed optimal in the equilibrium we derive.) Using Bayes' rule gives the posterior q, the manager's decision d and the price p as follows:

Lemma 1 *Assume that the positively-informed speculator never sells and the negatively-informed speculator never buys. For a given order flow X, the posterior q, the manager's decision d and the price p are given by the following table:*

X	-2	-1	0	1	2
q	0	$\frac{1-\lambda\mu_H}{2-\lambda\mu_H}$	$\frac{1}{2}$	$\frac{1}{2-\lambda\mu_L}$	1
d	n	$?$	i	i	i
p	R_L^n	$?$	$\frac{1}{2}R_H^i+\frac{1}{2}R_L^i$	$\frac{1}{2-\lambda\mu_L}R_H^i+\frac{1-\lambda\mu_L}{2-\lambda\mu_L}R_L^i$	R_H^i

where the question mark ? denotes that the outcome depends on parameter values.

Proof. The posteriors q are calculated from Bayes' rule and given in Appendix A. The manager takes $d=n$ if $q<\gamma$ (where $\gamma<\frac{1}{2}$) and $d=i$ otherwise. The price p is given by $qR_H^d+(1-q)\,R_L^d$. ■

We can use Lemma 1 to derive the optimal trading behavior of the speculator, i.e. the variables μ_H and μ_L. Consider first the positively-informed speculator. If she chooses to buy one unit:

- With probability (w.p.) $\frac{1}{3}$, $X=2$ and she is fully revealed. Thus, trading profits are zero.
- W.p. $\frac{1}{3}$, $X=1$ and she pays $\frac{1}{2-\lambda\mu_L}R_H^i+\frac{1-\lambda\mu_L}{2-\lambda\mu_L}R_L^i$ per share. The fundamental value of each share is R_H^i, and so her profit is $\frac{1-\lambda\mu_L}{2-\lambda\mu_L}\left(R_H^i-R_L^i\right)>0$.
- W.p. $\frac{1}{3}$, $X=0$ and she pays $\frac{1}{2}R_H^i+\frac{1}{2}R_L^i$ for a share which is worth R_H^i, yielding a profit of $\frac{1}{2}\left(R_H^i-R_L^i\right)>0$.

Thus, her expected gross profit is given by:

$$\frac{1}{3}\frac{1-\lambda\mu_L}{2-\lambda\mu_L}\left(R_H^i-R_L^i\right)+\frac{1}{3}\frac{1}{2}\left(R_H^i-R_L^i\right)=\frac{1}{3}\left(R_H^i-R_L^i\right)\left(\frac{1-\lambda\mu_L}{2-\lambda\mu_L}+\frac{1}{2}\right) \qquad (6)$$

Assuming that the trading cost κ is less than the profit in (6), the positively-informed speculator will always buy and so $\mu_H = 1$. This strategy in turn affects the market maker's and manager's posterior upon observing $X = -1$. Since $X = -1$ is inconsistent with the speculator buying, and the speculator always buys if positively informed, the only way the state can be good is if the speculator is absent. Thus, obtain $q(-1) = \frac{1-\lambda}{2-\lambda}$: the posterior is higher the more likely the speculator is to be absent (i.e. the lower λ is). Whether this posterior is sufficiently low to drive the manager to abandon the project depends on whether $\frac{1-\lambda}{2-\lambda}$ is greater or smaller than the critical value γ. We make the following assumption:

Assumption 1 $\frac{1-\lambda}{2-\lambda} < \gamma$.

Assumption 1 means that, when $X = -1$, the manager becomes sufficiently pessimistic and chooses to abandon the project – even though there is a possibility that the negative order flow arises because the speculator is absent (rather than negatively informed) and $\theta = H$. Essentially, this assumption implies that there is enough information in the market (the speculator is sufficiently likely to be present) to create a feedback effect to the firm's investment decision, so that the investment is abandoned after $X = -1$. We make Assumption 1 for the remainder of this section and consider the opposite case of $\frac{1-\lambda}{2-\lambda} > \gamma$ in Section 3.1.1 below.

Then, if $X = -1$, the manager takes the corrective action and Lemma 1 specializes to Lemma 2:

Lemma 2 *Assume that* $\kappa < \frac{1}{3}\left(R_H^i - R_L^i\right)\left(\frac{1-\lambda\mu_L}{2-\lambda\mu_L} + \frac{1}{2}\right)$ *and* $\frac{1-\lambda}{2-\lambda} < \gamma$. *For a given order flow* X, *the posterior* q, *the manager's decision* d *and the price* p *are given by the following table:*

X	-2	-1	0	1	2
q	0	$\frac{1-\lambda}{2-\lambda}$	$\frac{1}{2}$	$\frac{1}{2-\lambda\mu_L}$	1
d	n	n	i	i	i
p	R_L^n	$\frac{1-\lambda}{2-\lambda}R_H^n + \frac{1}{2-\lambda}R_L^n$	$\frac{1}{2}R_H^i + \frac{1}{2}R_L^i$	$\frac{1}{2-\lambda\mu_L}R_H^i + \frac{1-\lambda\mu_L}{2-\lambda\mu_L}R_L^i$	R_H^i

We now consider the negatively-informed speculator. If she chooses to sell one unit:

- W.p. $\frac{1}{3}$, $X = -2$ and she is fully revealed, so trading profits are zero.

- W.p. $\frac{1}{3}$, $X = -1$ and she receives $\frac{1-\lambda}{2-\lambda}R_H^n + \frac{1}{2-\lambda}R_L^n$ for a share which is worth R_L^n, which yields a profit of $\frac{1-\lambda}{2-\lambda}\left(R_H^n - R_L^n\right) < 0$. This profit is negative, even though the speculator is trading in the direction of her information. This loss is the basis for our main result – the asymmetric limit to arbitrage due to the feedback effect – and we will elaborate more on it below.

- W.p. $\frac{1}{3}$, $X = 0$ and she receives $\frac{1}{2}R_H^i + \frac{1}{2}R_L^i$ for a share which is worth R_L^i, which yields a profit of $\frac{1}{2}\left(R_H^i - R_L^i\right) > 0$.

Thus, the speculator's overall profit from selling is

$$\frac{1}{3}\left[\frac{1}{2}\left(R_H^i - R_L^i\right) + \frac{1-\lambda}{2-\lambda}\left(R_H^n - R_L^n\right)\right]. \tag{7}$$

The first (positive) term is the profit if $X = 0$. It represents the "fundamental" effect which is common to all informed trading models where firm value is exogenous to the trading process – the speculator profits from buying on a positive signal and selling on a negative signal. When $X = 0$, order flow is uninformative and so the manager takes the ex-ante optimal decision of continuation. Thus, the order flow does not create any feedback, and so firm value is unaffected. Given that the speculator knows that continuation is a bad decision, she profits from selling the stock of the firm at a price that does not fully reflect her information. The second (negative) term is the loss if $X = -1$. It stems from the "feedback" effect which is unique to this paper and arises because firm value is endogenous to the act of arbitrage: selling causes the manager to take the optimal action and abandon the investment. This causes the value of the security to rise above its price, and leads the speculator to make a loss on her short position.

The complexity in generating this result is that, for a loss to emerge, we need the price that the speculator receives from the market maker to be lower than her fundamental valuation of the share, i.e., a difference in beliefs about the firm's value between the market maker and the speculator. However, it is not obvious that the market maker will set a "wrong" price that is different from the firm's value. The market maker never gets the manager's action wrong, as he is fully rational. The manager's action depends on his posterior, which is in turn a function of his prior and the observed order flow. Since the market maker also knows the prior (because it is common) and also observes the order flow, he perfectly predicts the manager's posterior and thus his action. For example, if $X = -1$, the market maker knows that correction will take place, and so he takes this into account when setting his price. Instead, the difference between the market maker's price and the speculator's valuation arises because, even though both the speculator and market maker agree that abandonment will occur if $X = -1$, they disagree on the value of the firm conditional on abandonment. This happens despite the fact that both the market maker and the speculator are fully rational and thus take into account the fact that the manager will abandon the investment following a total order flow of X=-1. Hence, the loss occurs because the speculator has different information from the market maker regarding the value of the firm conditional on abandonment. The speculator knows that the corrective action will be taken (since $q(-1) < \gamma$), *and* that correction is desirable for firm value (since she knows with certainty that $\theta = L$), and so the fundamental value of the firm is R_L^n. In contrast, the market maker knows the corrective action will be taken (since $q(-1) < \gamma$) but is not certain that correction is desirable for firm value, because she is unsure of the underlying state of nature θ. While the speculator observes θ perfectly, the market maker can only infer it from the order flow X. Order flow $X = -1$ is consistent with the speculator not being present and the noise trader selling 1 share. Hence, it is possible that $\theta = H$, in which case the manager's corrective action is undesirable, leading to firm value of R_H^n. Therefore, the market maker sets a lower price than the fundamental value perceived by the speculator, and so selling will cause the speculator to lose money. Crucial to this reasoning is the assumption that, conditional on abandonment, the value of the firm in the high state R_H^n is below its value in the low state R_L^n. This in turn requires the corporate decision to be a corrective action that improves firm value in the low state, such as optimal divestment or replacement of an underperforming manager. The model does not apply to "amplifying" actions that worsen firm value in the low state, such as employees, suppliers or customers terminating their relationship with a troubled firm.

In summary, selling by the speculator creates a feedback effect – it reveals to the manager that $\theta = L$ with high enough probability and that correction is desirable. Conveying information to the manager improves his decision, changing it from continuation to optimal abandonment. Improved decision-making in turn enhances fundamental firm value, and thus reduces the profitability of taking a short position. Arbitrage is limited because *the value of the asset being arbitraged is endogenous to the act of arbitrage.* Note that there is asymmetry between buy-side and sell-side speculation. We show formally later that there will not be an equilibrium where the feedback effect causes the speculator to sell but not buy. The reason for the asymmetry is that the feedback effect is inherently asymmetric. Trading on information (both buying on good information and selling on bad information) improves price informativeness, regardless of the direction of the trade. This greater price informativeness always improves the manager's decision. This (weakly)[4] augments the profitability of a long position, but reduces the profitability of a short position.

If the expression in (7) is less than κ, the negatively-informed speculator never sells, and so $\mu_L = 0$. Then, the profits from a positively-informed speculator buying (6) become:

$$\frac{1}{3}\left(R_H^i - R_L^i\right).$$

Therefore, a necessary condition for the trading cost to generate an equilibrium with $\mu_H = 1$ and $\mu_L = 0$ is:

$$\frac{1}{3}\left(R_H^i - R_L^i\right) > \kappa > \frac{1}{3}\left[\frac{1}{2}\left(R_H^i - R_L^i\right) + \frac{1-\lambda}{2-\lambda}\left(R_H^n - R_L^n\right)\right]. \tag{8}$$

That is, the trading cost must be sufficiently high that a negatively-informed speculator does not wish to sell, but sufficiently low that a positively-informed speculator does wish to buy. Clearly, since $(R_H^n - R_L^n)$ is negative, the set of possible trading costs that satisfy equation (8) is non-empty. Intuitively, since the feedback effect (weakly) enhances the profitability of

[4] In the core model, continuation is ex ante optimal, and so buying on good information does not change the manager's decision. Thus, we write that the feedback effect only weakly augments the profitability of a long position. In an alternative model in which there are different levels of investment, or continuation is ex ante suboptimal, buying on good information does change the manager's decision and so the feedback effect strictly augments the profitability of a long position.

a long position and reduces the profitability of a short position, the profits from informed buying exceed the profits from informed selling, and so there are a continuum of trading costs in between that will satisfy equation (8). Then, when κ falls in this region, Lemma 2 specializes to Lemma 3 below.

Lemma 3 *Assume that Equation (8) and Assumption (1) hold. For a given order flow X, the posterior q, the manager's decision d and the price p are given by the following table:*

X	-2	-1	0	1	2
q	0	$\frac{1-\lambda}{2-\lambda}$	$\frac{1}{2}$	$\frac{1}{2}$	1
d	n	n	i	i	i
p	R_L^n	$\frac{1-\lambda}{2-\lambda}R_H^n + \frac{1}{2-\lambda}R_L^n$	$\frac{1}{2}R_H^i + \frac{1}{2}R_L^i$	$\frac{1}{2}R_H^i + \frac{1}{2}R_L^i$	R_H^i

Note that, with $\mu_L = 0$, the order flow $X = -2$ is now off the equilibrium path and so we are free to assign any posterior in this case: we are no longer restricted to $q(X = -2) = 0$. Thus, other equilibria potentially exist. The proof of Lemma 3 shows that all other equilibria also involve $\mu_H = 1$ and $\mu_L = 0$.

Finally, to complete the construction of the equilibrium where the positively-informed speculator buys with probability 1 and the negatively-informed speculator does not trade, we only need to rule out the possibility that the positively-informed trader sells and that the negatively-informed trader buys. These are the only possibilities that we did not consider thus far.

Consider the possibility that the positively informed speculator decides to sell:

- W.p. $\frac{1}{3}$, $X = 0$. She receives $\frac{1}{2}R_H^i + \frac{1}{2}R_L^i$ for a share which is worth R_H^i, so she makes a loss of $\frac{1}{2}(R_H^i - R_L^i)$.
- W.p. $\frac{1}{3}$, $X = -1$. She receives $\frac{1-\lambda}{2-\lambda}R_H^n + \frac{1}{2-\lambda}R_L^n$ for a share which is worth R_H^n, so makes a profit of $\frac{1}{2-\lambda}(R_L^n - R_H^n)$.
- W.p. $\frac{1}{3}$, $X = -2$. She receives R_L^n for a share which is worth R_H^n, so makes a profit of $(R_L^n - R_H^n)$.

Thus, the positively-informed speculator can make a profit by selling – because she can manipulate the manager into taking the corrective action, knowing that the corrective action is undesirable because the state is actually good. Since she has a short position, she benefits from the manager taking the incorrect action.[5] Her overall profits are given by:

$$\begin{aligned} &\frac{1}{3}(R_L^n - R_H^n) + \frac{1}{3}\left(\frac{1-\lambda}{2-\lambda}R_H^n + \frac{1}{2-\lambda}R_L^n - R_H^n\right) - \frac{1}{3}\left(\frac{1}{2}(R_H^i - R_L^i)\right) \\ &= \frac{1}{3}\left(\frac{3-\lambda}{2-\lambda}(R_L^n - R_H^n)\right) - \frac{1}{3}\left(\frac{1}{2}(R_H^i - R_L^i)\right). \end{aligned} \quad (9)$$

[5] Goldstein and Guembel (2008) show that an uninformed speculator may have incentives to manipulate the stock price by selling, for similar reasons. In our model, the speculator is always informed.

For the positively-informed speculator to choose buying over selling, her profits must be greater under the former. This requires:

$$\begin{aligned} R_H^i - R_L^i &> \left(\frac{3-\lambda}{2-\lambda} \left(R_L^n - R_H^n \right) \right) - \frac{1}{2} \left(R_H^i - R_L^i \right) \\ \frac{3}{2} \left(R_H^i - R_L^i \right) &> \frac{3-\lambda}{2-\lambda} \left(R_L^n - R_H^n \right). \end{aligned} \tag{10}$$

The first term is the "fundamental" effect, which represents the profits from trading in the direction of one's private information. The second term is the "feedback" effect, which arises because selling manipulates the order flow and causes the manager to take the wrong decision.

We must verify that condition (10) is consistent with Assumption 1. For (10) to hold, we require λ to be not too high, else the market maker views the order flow as more informative, and so the speculator can gain more by manipulating the order flow. For Assumption 1, we require λ to be not too low: the order flow must be sufficiently informative that if $X = -1$, the manager changes his decision from continuation to correction (i.e. there is feedback from the order flow to the manager's action). However, under the following condition, (10) holds for every λ:

$$\frac{3}{2} \left(R_H^i - R_L^i \right) > 2 \left(R_L^n - R_H^n \right). \tag{11}$$

Thus, condition (10) is consistent with Assumption 1.

Now consider the profit for the negatively-informed speculator from buying:

- W.p. $\frac{1}{3}$, $X = 2$. She pays R_H^i for a share which is worth R_L^i, and so makes a loss of $(R_L^i - R_H^i)$.

- W.p. $\frac{1}{3}$, $X = 1$. She pays $\frac{1}{2} R_H^i + \frac{1}{2} R_L^i$ for a share which is worth R_L^i, and so makes a loss of $\frac{1}{2} \left(R_L^i - R_H^i \right)$.

- W.p. $\frac{1}{3}$, $X = 0$. She pays $\frac{1}{2} R_H^i + \frac{1}{2} R_L^i$ for a share which is worth R_L^i, and so makes a loss of $\frac{1}{2} \left(R_L^i - R_H^i \right)$.

In all cases, she makes a loss, and hence she never chooses to buy. It is intuitive that the negatively-informed speculator never wishes to buy. Trading in the opposite direction of one's information causes the manager to make the wrong decision. Thus, it can only be profitable if the speculator establishes a short position. Hence, while the positively-informed speculator may have an incentive to sell, the negatively-informed speculator will never wish to buy.

We therefore have an equilibrium in which the positively-informed speculator always buys, but the negatively-informed speculator never trades. This result is summarized and stated formally in Proposition 1 below:

Proposition 1 *(Asymmetric limits to arbitrage.) Assume that Assumption 1 and equations (8) and (10) hold. There exists an equilibrium in which* $\mu_H = 1$ *and* $\mu_L = 0$*, i.e. the speculator always buys on positive information, but never trades on negative information.*

The source of the asymmetric limit to arbitrage is the feedback effect. Formally, we say that the feedback effect exists when the manager's decision d, and hence firm value, are affected by the order flow X for $X \in \{-1, 0, 1\}$. We only consider the cases of $X \in \{-1, 0, 1\}$ since the speculator's information is fully revealed when $X = -2$ and $X = 2$ and her trading profits are automatically zero; thus, the manager's decision d is irrelevant. In the above equilibrium, we have $d = n$ for $X = -1$ and $d = i$ for $X \in \{0, 1\}$. It is the change in the manager's decision when $X = -1$ that is critical for the negatively-informed speculator to make a loss when she sells. Proposition 2 below states that there is never an equilibrium that contains feedback (i.e. the manager's decision d depends on the order flow for $X \in \{-1, 0, 1\}$) in which we have the opposite result; i.e., one speculator type always sells, and the other speculator type never trades. The proof of the proposition is in Appendix A.

Proposition 2 *For any parameter values, there does not exist an equilibrium in which one speculator type always sells, the other speculator type never trades, and the manager's decision* d *depends on the order flow for* $X \in \{-1, 0, 1\}$*.*

The intuition behind this result should be clear by now. Trading on information improves the firm's fundamental value, which reduces the profitability of a short position but enhances the profitability of a long position. Hence, buying on information is generally more profitable than selling on information, and so the asymmetric equilibrium that we find must feature buying and not selling.

3.1 Discussion

This section discusses the role of our assumptions in creating the asymmetric limit to arbitrage. These assumptions in turn lead to empirical predictions, since they demonstrate the conditions under which the asymmetric limit to arbitrage will exist.

3.1.1 The Role of Assumption 1 ($\frac{1-\lambda}{2-\lambda} < \gamma$)

Consider the case where Assumption 1 does not hold, and so $\frac{1-\lambda}{2-\lambda} > \gamma$. Recall that $\frac{1-\lambda}{2-\lambda}$ is the manager's posterior probability of $\theta = H$ if he observes $X = -1$. With $\frac{1-\lambda}{2-\lambda} > \gamma$, the posterior is sufficiently high that the manager does not take the corrective action if $X = -1$. Then, the trading outcomes in Lemma 2 become:

X	-2	-1	0	1	2
q	0	$\frac{1-\lambda}{2-\lambda}$	$\frac{1}{2}$	$\frac{1}{2-\lambda\mu_L}$	1
d	n	i	i	i	i
p	R_L^n	$\frac{1-\lambda}{2-\lambda}R_H^i + \frac{1}{2-\lambda}R_L^i$	$\frac{1}{2}R_H^i + \frac{1}{2}R_L^i$	$\frac{1}{2-\lambda\mu_L}R_H^i + \frac{1-\lambda\mu_L}{2-\lambda\mu_L}R_L^i$	R_H^i

The conditions for the positively-informed speculator to wish to buy in equilibrium are the same as in the core model. Now, consider the decision of the negatively-informed speculator. If she sells:

- W.p. $\frac{1}{3}$, $X = -2$ and she is fully revealed, so trading profits are zero.
- W.p. $\frac{1}{3}$, $X = -1$ and she receives $\frac{1-\lambda}{2-\lambda} R_H^i + \frac{1}{2-\lambda} R_L^i$ for a share that is worth R_L^i, which yields a profit of $\frac{1-\lambda}{2-\lambda} \left(R_H^i - R_L^i\right)$. Critically, unlike under Assumption 1, the profit is positive. This is because selling does not change the manager's decision: he is still continuing the project. Thus, there is only the "fundamental" effect of trading in the direction of one's private information, and no confounding feedback effect.
- W.p. $\frac{1}{3}$, $X = 0$ and she receives $\frac{1}{2} R_H^i + \frac{1}{2} R_L^i$ for a share that is worth R_L^i, which yields a profit of $\frac{1}{2} \left(R_H^i - R_L^i\right) > 0$.

Overall, the ex-ante profit from selling on negative information is $\frac{1}{3} \left(\frac{1-\lambda}{2-\lambda} + \frac{1}{2}\right) \left(R_H^i - R_L^i\right)$ which is unambiguously positive. Thus, if

$$\kappa < \frac{1}{3} \left(\frac{1-\lambda}{2-\lambda} + \frac{1}{2}\right) \left(R_H^i - R_L^i\right) < \frac{1}{3} \left(R_H^i - R_L^i\right) \tag{12}$$

we have an equilibrium where both $\mu_L = 1$ and $\mu_H = 1$ (recall that the profit from buying on positive information is $\frac{1}{3} \left(R_H^i - R_L^i\right)$): the speculator sells on negative information and buys on positive information, so there is no limit to arbitrage. However, when

$$\frac{1}{3} \left(R_H^i - R_L^i\right) > \kappa > \frac{1}{3} \left(\frac{1-\lambda}{2-\lambda} + \frac{1}{2}\right) \left(R_H^i - R_L^i\right) \tag{13}$$

then there is an equilibrium with $\mu_H = 1$ and $\mu_L = 0$, i.e., an asymmetric limit to arbitrage as in the core model.

It is important to stress the differences from the asymmetric equilibrium in our core model. The asymmetric limit to arbitrage here is not driven by feedback: for $X \in \{-1, 0, 1\}$, the manager's decision is always $d = i$ regardless of the order flow. This is why the negatively-informed speculator makes positive profits from selling if $X = -1$, whereas in the core model she makes a loss in this case. Assumption 1 is necessary in the core model to create feedback and trading losses, since if and only if $\frac{1-\lambda}{2-\lambda} < \gamma$, the posterior upon $X = -1$ is sufficiently low to change the manager's decision from continuation to correction. Instead, the intuition for the asymmetric limit to arbitrage here is as follows. *Given* that the equilibrium involves not selling on negative information, buying is highly profitable. This is because the speculator earns high profits not only if $X = 0$, but also if $X = 1$. Since the speculator does not sell on negative information, $X = 1$ is fully consistent with the speculator not selling and having negative information, and so the market maker sets a low price of $\frac{1}{2} R_H^i + \frac{1}{2} R_L^i$. This allows the speculator to make high profits by selling. Conversely, *given* that the equilibrium involves buying on positive information, selling is less profitable. This is because the speculator only earns high profits if $X = 0$, but not if $X = -1$. Since she always buys on positive information, and $X = -1$ is inconsistent with her buying, it must be that the speculator has negative information (or is absent). Hence the market maker sets a low price, meaning the speculator earns low profits by selling if $X = -1$.

The fact that the asymmetric equilibrium here does not result from feedback and does not involve a path of trading losses for the negatively-informed trader has two important implications. First, the asymmetric equilibrium is less likely to arise here (under no feedback) than in the core model (under feedback). This can be seen easily by noting that the set of transaction costs κ that satisfy condition (13) is a strict subset of that which satisfies condition in(8).

Second, in the core model (under feedback), there is an equilibrium with $\mu_H = 1$ and $\mu_L = 0$, but no equilibrium with $\mu_H = 0$ and $\mu_L = 1$. However, here (under no feedback), there is also an equilibrium in which $\mu_H = 0$ and $\mu_L = 1$. To see this, note that in this equilibrium[6]:

X	-2	-1	0	1	2
q	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2-\lambda}$	1
d	n	i	i	i	i
p	R_L^n	$\frac{1}{2}R_H^i + \frac{1}{2}R_L^i$	$\frac{1}{2}R_H^i + \frac{1}{2}R_L^i$	$\frac{1}{2-\lambda}R_H^i + \frac{1-\lambda}{2-\lambda}R_L^i$	R_H^i

Again, there is no feedback, since $d = i$ for $X \in \{-1, 0, 1\}$. Simple calculations give the positively-informed speculator's profits from buying as $\frac{1}{3}\left(\frac{1}{2} + \frac{1-\lambda}{2-\lambda}\right)(R_H^i - R_L^i)$ and the negatively-informed speculator's profits from selling as $\frac{1}{3}(R_H^i - R_L^i)$, which is higher. The intuition is exactly analogous to the earlier case of $\mu_H = 1$ and $\mu_L = 0$: *given* that the equilibrium involves not trading on positive information and selling on negative information, the speculator does not wish to deviate from this. If (13) holds, then a positively-informed speculator will not buy but a negatively-informed speculator will sell. If also $\kappa > \frac{1}{3}\left[(R_L^n - R_H^n) - (R_H^i - R_L^i)\right]$, then the positively-informed speculator will not sell either, so the equilibrium of $\mu_H = 0$ and $\mu_L = 1$ is sustainable.

3.1.2 Speculator Is Sometimes Absent

In the core model, the speculator is only present with probability $\lambda < 1$. This is necessary for the limit to arbitrage to exist. Recall that a limit to arbitrage requires the market maker to set a price that is different from the firm's fundamental value to the speculator. The latter in turn depends on both the manager's decision, and the firm value given this decision. While the market maker always predicts the manager's decision correctly (since he observes all of the information the manager uses to form his posterior), he must disagree with the speculator on whether the decision is desirable for firm value for the speculator to make a loss. In the core

[6]Note that $X = 2$ is off the equilibrium path so we have freedom to specify any belief. We choose $q = 1$ as this is sufficient to support the equilibrium of $\mu_H = 0$ and $\mu_L = 1$. Regardless of $q\,(X = 2)$, there is no feedback since we have $d = i$ for $X \in \{-1, 0, 1\}$.

model, this disagreement occurs at $X = -1$: both the speculator and market maker know that correction will occur, but the speculator knows with certainty that correction is desirable and values the firm highly, whereas the market maker is not certain that correction is desirable and so sets a low price. It is necessary for $\lambda < 1$ to create this asymmetry. Observing $X = -1$ tells the market maker that the speculator could not have bought. With $\lambda = 1$, the speculator is always present. Since she is always informed, the only way that she could not have bought is if she has negative information. Thus, the market maker knows with certainty that $\theta = L$, and has exactly the same posterior as the speculator ($q = 0$). She sets a price of R_L^n, which is exactly the speculator's valuation and so the speculator does not make a loss. By contrast, with $\lambda < 1$, the absence of a purchasing speculator is consistent with $\theta = H$: it could be that the true state is good, and the low total order flow is because the speculator is not present. Put differently, $\lambda < 1$ creates an information asymmetry between the speculator and the market maker – the speculator knows whether she is present, but the market maker does not. This in turn creates asymmetry in beliefs – the speculator and the market maker attach different valuations to the firm, which creates the limit to arbitrage.

We would achieve the same result by instead assuming that the speculator is always present and informed, but can only trade with probability λ – for example, if with probability $1 - \lambda$ she receives a liquidity shock that prevents her from trading. Thus, for a limit to arbitrage to exist, there must be sufficient uncertainty over whether there is a speculator who can trade – either because there is uncertainty over whether she is present, or there is uncertainty over her ability to trade on her information conditional upon being present.

3.1.3 Speculator's Initial Position is Zero

In our model, the speculator's initial position is zero, so that trading on negative information requires her to take a short position. If the speculator maximizes absolute returns, this is a necessary assumption since, only by taking a short position will she lose by inducing the firm to take an efficient corrective action. If the speculator initially holds several shares in the firm, she may choose to sell some of her shares on negative information – if her trade improves the manager's decision, this will increase the value of her remaining shares.

However, if the speculator maximizes returns relative to other speculators, then our limit to arbitrage may exist even if her initial position is strictly positive. Assume that the speculator is a mutual fund who is benchmarked against the performance of other mutual funds, and that each fund holds 10 shares in the firm. If the speculator sells 6 of her shares and this causes the firm to take an optimal corrective action, this will increase the value of her remaining 4 shares. However, it will benefit her rivals even more, who still have 10 shares in the firm. Even though selling does not require the speculator to take a short position, in which she loses in absolute terms from an improvement in firm value, selling causes her to suffer losses relative to her peer group and this may deter her from trading in the first place. Thus, the limit to arbitrage identified by this paper may exist even in the presence of short-sales constraints.

While short-sales constraints do not deter selling to a non-negative final position, the feedback effect can deter such selling if the speculator maximizes relative performance.

3.2 Stock Returns

Thus far, we have shown that positive news received by the speculator has a different impact on her trades (and thus total order flow) than negative news. However, it is not obvious that this will translate into a differential impact on stock prices. The market maker is fully rational and takes into account the fact that the speculator does not sell on negative information and adjusts his pricing function accordingly. For example, as shown in Lemma 2, the market maker recognizes that $X = 1$ could be consistent with a negatively-informed speculator who chooses not to trade, and so $p(1)$ is no higher than $p(0)$. Thus, even though bad news can lead to a neutral (or mildly positive) order flow rather than a negative order flow, the market maker knows that such an order flow can stem from a negatively-informed and non-trading speculator, and will decrease the price accordingly. Put differently, although negative information does not cause a negative order flow (on average), it can still have a negative price impact. Thus, it may seem that good and bad news should have a symmetric effect on stock prices. This section calculates the stock price impact between $t = 0$ and $t = 1$ of the speculator receiving either positive or negative information and shows that bad news possessed by the speculator has a smaller impact on prices than good news.

Let p^{ante} denote the "ex ante" stock price at $t = 0$, before the state has been realized. With probability $\frac{1}{2}$, the state will be $\theta = L$ and there is no trade, regardless of whether the speculator is present. Thus, order flow is -1, 0 or 1 with equal probability. With probability $\frac{1}{2}$, the state will be $\theta = H$. If the speculator is absent (w.p. $(1-\lambda)$), there is no trade and we again have $X \in \{-1, 0, 1\}$ with equal probability. If the speculator is present, $X \in \{0, 1, 2\}$ with equal probability. Letting $p(X)$ denote the stock price set by the market maker after observing order flow X, we have:

$$\begin{aligned}
p^{ante} &= \frac{\lambda}{2}\left(\frac{1}{3}p(0) + \frac{1}{3}p(1) + \frac{1}{3}p(2)\right) + \left(1 - \frac{\lambda}{2}\right)\left(\frac{1}{3}p(-1) + \frac{1}{3}p(0) + \frac{1}{3}p(1)\right) \\
&= \frac{1}{3}\left(\frac{\lambda}{2}p(2) + p(1) + p(0) + \left(1 - \frac{\lambda}{2}\right)p(-1)\right) \\
&= \frac{1}{6}\left[(1-\lambda)R_H^n + R_L^n + (2+\lambda)R_H^i + 2R_L^i\right] \qquad (14)
\end{aligned}$$

If the speculator is present and receives positive information, she will buy one share and so the expected price becomes:

$$p_H^{spec} = \frac{1}{3}\left(p(2) + p(1) + p(0)\right).$$

The stock return realized when the speculator receives good information is thus given by:

$$\begin{aligned} p_H^{spec} - p^{ante} &= \frac{1}{3}\left(p\left(2\right)+p\left(1\right)+p\left(0\right)\right) - \frac{1}{3}\left(\frac{\lambda}{2}p\left(2\right)+p\left(1\right)+p\left(0\right)+\left(1-\frac{\lambda}{2}\right)p\left(-1\right)\right) \\ &= \frac{1}{3}\left(1-\frac{\lambda}{2}\right)\left(p\left(2\right)-p\left(-1\right)\right) \\ &= \frac{1}{3}\left(1-\frac{\lambda}{2}\right)\left(R_H^i - \frac{1-\lambda}{2-\lambda}R_H^n - \frac{1}{2-\lambda}R_L^n\right) > 0. \end{aligned}$$

Similarly, if the speculator is present and receives negative information, we have:

$$p_L^{spec} = \frac{1}{3}\left(p\left(1\right)+p\left(0\right)+p\left(-1\right)\right)$$

$$\begin{aligned} p_L^{spec} - p^{ante} &= \frac{1}{3}\left(p\left(1\right)+p\left(0\right)+p\left(-1\right)\right) - \frac{1}{3}\left(\frac{\lambda}{2}p\left(2\right)+p\left(1\right)+p\left(0\right)+\left(1-\frac{\lambda}{2}\right)p\left(-1\right)\right) \\ &= \frac{1}{3}\frac{\lambda}{2}\left(p\left(-1\right)-p\left(2\right)\right) < 0. \end{aligned}$$

Note that:

$$\begin{aligned} & abs\left(p_H^{spec} - p^{ante}\right) - abs\left(p_L^{spec} - p^{ante}\right) \\ &= \frac{1}{3}\left(1-\frac{\lambda}{2}\right)\left(p\left(2\right)-p\left(-1\right)\right) - \frac{1}{3}\frac{\lambda}{2}\left(p\left(2\right)-p\left(-1\right)\right) \\ &= \frac{1}{3}\left(1-\lambda\right)\left(p\left(2\right)-p\left(-1\right)\right) > 0. \end{aligned} \tag{15}$$

i.e. the stock price increase upon positive information exceeds the stock price decrease upon negative information. Put differently, positive information is impounded into prices to a greater degree than negative information. Since good and bad news are equally likely, this means that the expected return, conditional on the speculator being present, is positive:

$$\begin{aligned} p^{spec} - p^{ante} &= \frac{1}{2}\left[\frac{1}{3}\left(p\left(2\right)+p\left(1\right)+p\left(0\right)\right) + \frac{1}{3}\left(p\left(1\right)+p\left(0\right)+p\left(-1\right)\right)\right] - p^{ante} \\ &= \frac{1}{3}\frac{1-\lambda}{2}\left(p\left(2\right)-p\left(-1\right)\right) > 0. \end{aligned} \tag{16}$$

This result is stated formally in Proposition 3 below:

Proposition 3 *(Asymmetric effect of positive and negative information.) The price impact of the speculator being present and positively informed is greater in absolute terms than the price impact of the speculator being present and negatively informed. The expected return at $t = 1$, conditional on the speculator being present, is positive.*

Proposition 3 holds even though the market maker is rational and takes into account the fact that the speculator trades asymmetrically when devising his pricing function. Thus, as proven in Appendix A, the expected return unconditional on whether the speculator is present is zero. The market is fully efficient: an uninformed speculator cannot buy the stock at $t = 0$ and expect a positive average return at $t = 1$. Instead, Proposition 3 states that the expected return, *conditional* on the speculator being present, is positive – i.e. good news received by the speculator has a greater impact on the price than bad news received by the speculator. The source of this result is that, even though the market maker is rational, he is unable to distinguish the case of a negatively-informed speculator from that of an absent speculator (i.e. no information), and so negative information has a smaller effect on prices. By contrast, if the speculator is always present, the market maker has no such inference problem and there is no asymmetry. This can be seen by plugging $\lambda = 1$ into equation (15). Just as $\lambda < 1$ was a necessary condition for the limit to arbitrage to exist in the first place, it is a necessary and sufficient condition for bad news to have a lower effect on stock prices than good news.

4 Summary of Implications

This section discusses several implications of our model. The first is that this paper identifies a limit to arbitrage which, in contrast to alternative explanations, is likely to persist over time even as markets evolve and investors become more sophisticated. One existing source of limited arbitrage is market frictions such as short-sales constraints, which will likely diminish with the development of financial markets. A second is that investors in professional money managers make their allocation decisions based on short-run measures of performance, which leads to mutual funds avoiding arbitrage trading that will only converge in the long run (Shleifer and Vishny (1997)). Such behavior can either be irrational over-extrapolation, or rational if they have limited information on the fund manager's quality but instead must infer it imperfectly from short-run performance. Either way, if investor sophistication and information improve over time, this force will also diminish.

By contrast, the limit to arbitrage analyzed by this paper stems from firm value being endogenous to the act of arbitrage. This is a fundamental force that does not rely on market imperfections, investor irrationality or investor limited information, and so may continue to persist over time. (The only market imperfection that our model requires is trading costs, which exist even in developed financial markets). All agents in the model act with full rationality: the market maker takes into account the manager's learning when setting the price, and this in turn affects the speculator's decision to trade; in addition, the market maker knows that the speculator is pursuing an asymmetric trading strategy. If anything, the limit to arbitrage may *increase* with investor sophistication, as this augments the extent to which speculators have value-relevant information which the manager attempts to learn by observing the price.

The second main category of applications stems from the fact that the limit to arbitrage is asymmetric. While the speculator buys on good information, he does not sell on bad information. Even though the market maker takes this into account, Proposition 3 shows that negative information will enter into prices more slowly, as found empirically by Hong, Lim, and Stein (2000). While Hong, Lim, and Stein's results are consistent with the Hong and Stein (1999) model that news travels more slowly in small firms with low analyst coverage, Hong and Stein do not predict an asymmetry between good and bad news.[7] Hong, Lim, and Stein speculate that the asymmetry arises because key information is held by the firm's managers, and they disseminate favorable information more enthusiastically than unfavorable information because they are evaluated according to the stock price. Our theoretical model offers a potential alternative explanation. Key information is held by a firm's investors, who disseminate information not through public news releases, but by trading on it. Their reluctance to disseminate bad news is not because they are evaluated according to the stock price, but due to the limit to arbitrage created by the feedback effect.

Moreover, the feedback effect means that the lack of negative information in prices will have further consequences on real decisions. In particular, if speculators choose not to trade on negative information, then such negative information does not become incorporated into stock prices and fails to influence the manager's behavior. Thus, some negative-NPV projects will not be optimally abandoned, leading to overinvestment – even though there is an agent who knows with certainty that the investment is undesirable, it still takes place. In the model, even if $\theta = L$, we have $d = i$ if the noise trader does not sell. Critically, overinvestment does not occur because the manager is pursuing private benefits, as in the standard theories of Jensen (1986), Stulz (1990) and Zwiebel (1996). In contrast, the manager is fully aligned with firm value and there are no agency problems. The manager wishes to maximize firm value by learning from prices, but is unable to do so since speculators refrain from impounding their information into prices. Note that overinvestment occurs even though the manager is fully aware that the speculator does not trade on negative information and takes this into account.

The above overinvestment result can apply to M&A as well as organic expansion. Luo (2005) shows that managers sometimes use the market reaction to announced M&A deals to guide whether they should cancel the acquisition. While he finds that some transactions are canceled in equilibrium, our model suggests that there are other negative-NPV deals that should optimally be canceled but are not because speculators do not impound their negative views into prices. This may explain why a large proportion of M&A deals destroy value (see, e.g., Andrade, Mitchell and Stafford (2001).)

Our theory also suggests why investor outflows from mutual funds upon poor performance are less pronounced than fund inflows upon good performance, as found by Lynch and Musto (2003). Their explanation is that poor performance will lead to the fund family taking corrective actions, such as replacing the fund manager, removing the incentive to withdraw from a poorly-performing fund. In that paper, investors' fund flows have no direct effect on the fund family's

[7] Note that our paper focuses on the effect of news on stock prices. It does not address the predictability of future returns from past returns, which is another component of the Hong, Lim, and Stein (2000) findings.

decision to undertake a corrective action, which is instead purely based on the fund performance. Our model (applied to a mutual fund setting) suggests that the fund family will learn from investor flows in order to guide their correction decision, assuming that investors have private information on fund manager ability, over and above the publicly observable poor performance measure. Therefore, investors who are evaluated according to relative performance may choose not to withdraw, since doing so will directly affect the family's decision. It is thus most applicable to mutual fund investors with detailed information on management quality, such as funds-of-funds or large institutional investors. In addition to having detailed information, such investors are also likely evaluated according to relative performance.[8]

5 Conclusion

This paper has modeled a limit to arbitrage that stems from the fact that firm value is endogenous to the act of exploiting the arbitrage. We showed that investors may refrain from trading on negative information even in the absence of short-sale constraints, risk aversion or short horizons. Instead, the speculator strategically withholds negative information to avoid it improving the manager's investment decisions and causing her to realize a loss on her short position. Unlike existing limits to arbitrage based on exogenous market frictions, this mechanism may not attenuate with the development of financial markets. The model can potentially explain why negative information is incorporated into prices more slowly than positive information, and why managers may overinvest even in the absence of agency problems.

[8]Mutual funds always trade at net asset value (NAV) regardless of the manager in charge. Thus, we require a redemption cost for the limit to arbitrage to exist (similar to the transaction cost κ in this model), otherwise an investor could withdraw from the fund at NAV, and immediately re-inject the funds at NAV if the manager is replaced. For closed-end funds, which need not trade at NAV, no redemption cost is needed.

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A Proofs

Proof of Lemma 1

Given the speculator's strategy, any $X \in \mathcal{Q}$ is on the path of play. So from Bayes' rule, we have

$$\begin{aligned} q(X) &= \Pr(H|X) \\ &= \frac{\Pr(X|H)}{\Pr(X|H) + \Pr(X|L)}. \end{aligned}$$

We thus have:

$$\begin{aligned} q(-2) &= \frac{0}{0 + \mu_L} \\ &= 0; \\ q(-1) &= \frac{(1-\mu_H)\lambda(1/3) + (1-\lambda)(1/3)}{(1-\mu_H)\lambda(1/3) + (1-\lambda)(1/3) + \lambda(1/3) + (1-\lambda)(1/3)} \\ &= \frac{1-\lambda\mu_H}{2-\lambda\mu_H}; \\ q(0) &= \frac{\lambda\mu_H(1/3) + \lambda(1-\mu_H)(1/3) + (1-\lambda)(1/3)}{\lambda\mu_H(1/3) + \lambda(1-\mu_H)(1/3) + (1-\lambda)(1/3) + \lambda\mu_L(1/3) + \lambda(1-\mu_L)(1/3) + (1-\lambda)(1/3)} \\ &= \frac{1}{2}; \\ q(1) &= \frac{\lambda\mu_H(1/3) + \lambda(1-\mu_H)(1/3) + (1-\lambda)(1/3)}{\lambda\mu_H(1/3) + \lambda(1-\mu_H)(1/3) + (1-\lambda)(1/3) + \lambda(1-\mu_L)(1/3) + (1-\lambda)(1/3)} \\ &= \frac{1}{2-\lambda\mu_L}; \\ q(2) &= \frac{\lambda\mu_H(1/3) + (1-\lambda)(1/3)}{\lambda\mu_H(1/3) + (1-\lambda)(1/3)} \\ &= 1. \end{aligned}$$

Then, sequential rationality implies the table in Lemma 1.

Proof of Lemma 3

With $\mu_L = 0$, we are free to specify any posterior for $X = -2$. Call this posterior $\widehat{q}$. The set of equilibria is given as follows:

X	-2	-1	0	1	2
q	$\widehat{q}$	$\frac{1-\lambda}{2-\lambda}$	$\frac{1}{2}$	$\frac{1}{2}$	1
d	?	n	i	i	i
p	?	$\frac{1-\lambda}{2-\lambda}R_H^n + \frac{1}{2-\lambda}R_L^n$	$\frac{1}{2}R_H^i + \frac{1}{2}R_L^i$	$\frac{1}{2}R_H^i + \frac{1}{2}R_L^i$	R_H^i

There are two cases to consider:

Case 1: $0 < \widehat{q} < \frac{1-\lambda}{2-\lambda}$. We thus have $d(-2) = n$, and so $p(-2) = \widehat{q}R_H^n + (1-\widehat{q})R_L^n$. Since $p(-2)$ is even lower than the core model where $\widehat{q} = 0$, this makes it even less profitable for the negatively-informed speculator to sell and so it remains the case that $\mu_L = 0$. Thus, all of the other equilibria with $\widehat{q} < \frac{1-\lambda}{2-\lambda}$ are outcome-equivalent to the core model since d and μ_L remain the same.

Case 2: $\widehat{q} > \frac{1-\lambda}{2-\lambda}$. We now have $d(-2) = i$ and so $p(-2) = \widehat{q}R_H^i + (1-\widehat{q})R_L^i$. The positively-informed speculator's decision is unchanged. It now becomes automatic that she does not want to sell, because she will not be able to manipulate the manager into taking the wrong decision – the manager will take the correct decision of $d = i$ if $X = -2$. However, the negatively-informed speculator may now want to sell, because she manipulates the manager into taking the wrong decision of $d = i$ if $X = -2$. If she sells:

- W.p. $\frac{1}{3}$, $X = -2$ and she receives $\widehat{q}R_H^i + (1-\widehat{q})R_L^i$ for a share which is worth R_L^i, which yields a profit of $\widehat{q}(R_H^i - R_L^i) > 0$.
- W.p. $\frac{1}{3}$, $X = -1$ and she receives $\frac{1-\lambda}{2-\lambda}R_H^n + \frac{1}{2-\lambda}R_L^n$ for a share which is worth R_L^n, which yields a profit of $\frac{1-\lambda}{2-\lambda}(R_H^n - R_L^n) < 0$.
- W.p. $\frac{1}{3}$, $X = 0$ and she receives $\frac{1}{2}R_H^i + \frac{1}{2}R_L^i$ for a share which is worth R_L^i, which yields a profit of $\frac{1}{2}(R_H^i - R_L^i) > 0$.

Her expected profit is now:

$$\frac{1}{3}\left[\left(\widehat{q}+\frac{1}{2}\right)\left(R_H^i - R_L^i\right) + \frac{1-\lambda}{2-\lambda}\left(R_H^n - R_L^n\right)\right]. \tag{17}$$

If this is less than κ, then we have $\mu_L = 0$ as before, so we continue to have an asymmetric limit to arbitrage. Recall the profit from buying on positive information is $\frac{1}{3}(R_H^i - R_L^i)$, so there is a non-empty set of κ's that is high enough to deter selling on negative information but not buying on positive information.

If (17) is greater than κ, then the negatively-informed speculator will sell. Then, $q(-2) = 0$ and so $\widehat{q} > \frac{1-\lambda}{2-\lambda}$ cannot be an equilibrium. In sum, the only equilibria involve $\mu_L = 0$ and $\mu_H = 1$.

Proof of Proposition 2

We will show that there is no equilibrium in which one speculator type always sells, and the other speculator type buys with probability less than 1 (and otherwise does not trade). There are thus two possible equilibria to consider: one in which the speculator sells if $\theta = L$ and does not always buy if $\theta = H$, and one in which the speculator sells if $\theta = H$ and does not always buy if $\theta = L$.

We consider the first case first. Using the notation in the main text, this involves $\mu_L = 1$ and $\mu_H < 1$. Thus, from Lemma 1, we have $d(X = 1) = d(X = 0) = i$. For a feedback equilibrium, we require $d(X = -1) = n$, and so the result in Lemma 1 becomes:

X	-2	-1	0	1	2
q	0	$\frac{1-\lambda\mu_H}{2-\lambda\mu_H}$	$\frac{1}{2}$	$\frac{1}{2}$	1
d	n	n	i	i	i
p	R_L^n	$\frac{1-\lambda\mu_H}{2-\lambda\mu_H}R_H^n + \frac{1}{2-\lambda\mu_H}R_L^n$	$\frac{1}{2}R_H^i + \frac{1}{2}R_L^i$	$\frac{1}{2}R_H^i + \frac{1}{2}R_L^i$	R_H^i

To show that the negatively-informed speculator does not sell with greater frequency than the positively-informed speculator buys, it is sufficient to show that the profits from buying if $\theta = H$ exceed the profits from selling if $\theta = L$. To do so, we will compare analogous cases.

First, if a positively-informed speculator buys and $X = 2$, she is fully revealed and her profits are zero. Similarly, if a negatively-informed speculator sells and $X = -2$, she is fully revealed and her profits are zero. Second, if a positively-informed speculator buys and $X = 0$, she makes a profit of $\frac{1}{2}(R_H^i - R_L^i)$. Similarly, if a negatively-informed speculator sells and $X = 0$, she makes a profit of $\frac{1}{2}(R_H^i - R_L^i)$. Finally, If a positively-informed speculator buys and $X = 1$, she makes a profit of $\frac{1}{2}(R_H^i - R_L^i)$. If the negatively-informed speculator sells and $X = -1$, she makes a loss of $\frac{1}{2-\lambda\mu_H}(R_L^n - R_H^n)$. Hence, overall, the profit for a positively-informed speculator from buying exceeds the profit for a negatively-informed speculator from selling. Thus, there cannot be an equilibrium in which $\mu_L = 1$ and $\mu_H < 1$.

The second case is when the speculator sells on positive information and buys with probability less than 1 on negative information, i.e. her strategy profile is given by:

θ	-1	0	1
H	μ_H	$1-\mu_H$	0
L	0	$1-\mu_L$	μ_L

and the outcomes of the trading game are:

X	-2	-1	0	1	2
q	1	$\frac{1}{2-\lambda\mu_L}$	$\frac{1}{2}$	$\frac{1-\lambda\mu_H}{2-\lambda\mu_H}$	1
d	i	i	i	?	n
p	R_H^i	$\frac{1}{2-\lambda\mu_L}R_H^i + \frac{1-\lambda\mu_L}{2-\lambda\mu_L}R_L^i$	$\frac{1}{2}R_H^i + \frac{1}{2}R_L^i$	?	R_L^n

Since only a positively-informed speculator sells in equilibrium, selling conveys to the manager that the state is good and leads the manager to invest. The positively-informed speculator knows that the state is good and so values the firm at R_H^i. If $X = -2$, the speculator breaks even but if $X \in \{-1, 0\}$, the market maker does not know that the state is good and so pays a price below R_H^i. Thus, the speculator makes a loss, and so this is not an equilibrium.

Proof that the Unconditional Expected Return is Zero

If $\theta = H$ is realized, the expected price becomes:

$$p_H = (1-\lambda)\left[\frac{1}{3}p(-1) + \frac{1}{3}p(0) + \frac{1}{3}p(1)\right] + \lambda\left(\frac{1}{3}p(0) + \frac{1}{3}p(1) + \frac{1}{3}p(2)\right)$$

and so the short-run return to good news is

$$p_H - p^{ante} = \frac{\lambda}{6}\left[p(2) - p(-1)\right] > 0.$$

If $\theta = L$ is realized, the expected price becomes:

$$\begin{aligned} p(\theta = L) &= \frac{1}{3}\left(p(0) + p(1) + p(2)\right) \\ &= \frac{1}{3}\left(\frac{1-\lambda}{2-\lambda}R_H^n + \frac{1}{2-\lambda}R_L^n\right) + \frac{1}{3}\left(R_H^i + R_L^i\right). \end{aligned}$$

and so the short-run return to bad news is

$$\begin{aligned} p(\theta = L) - p^{ante} &= \frac{\lambda}{6}\left[\left(\frac{1-\lambda}{2-\lambda}R_H^n + \frac{1}{2-\lambda}R_L^n\right) - R_H^i\right] \\ &= \frac{\lambda}{6}\left[p(2) - p(-1)\right] < 0. \end{aligned}$$

Note that $p(\theta = H) - p^{ante} = -\left(p(\theta = L) - p^{ante}\right)$: the negative effect of bad news equals the positive effect of good news. Thus, the unconditional expected return is zero, consistent with market efficiency.

B The Case of $\gamma > \frac{1}{2}$

We now consider the case of $\gamma > \frac{1}{2}$. Now, correction is the ex ante optimal decision. Since, under correction, firm value is now higher under state L, and correction occurs more frequently with $\gamma > \frac{1}{2}$, seeing that $\theta = L$ is effectively good news for the firm. Thus, we now refer to a speculator who observes $\theta = L$ as positively-informed, and one who observes $\theta = H$ as negatively-informed. We first assume that the speculator pursues the following strategy:

θ	-1	0	1
H	μ_H	$1-\mu_H$	0
L	0	$1-\mu_L$	μ_L

i.e. the positively-informed speculator never sells and the negatively-informed speculator never buys. (We will prove this formally later.) Using Bayes' rule yields:

$$\begin{aligned}
q(2) &= \frac{0}{0+\mu_L} \\
&= 0; \\
q(1) &= \frac{(1-\mu_H)\lambda(1/3)+(1-\lambda)(1/3)}{(1-\mu_H)\lambda(1/3)+(1-\lambda)(1/3)+\lambda(1/3)+(1-\lambda)(1/3)} \\
&= \frac{1-\lambda\mu_H}{2-\lambda\mu_H}; \\
q(0) &= \frac{\lambda\mu_H(1/3)+\lambda(1-\mu_H)(1/3)+(1-\lambda)(1/3)}{\lambda\mu_H(1/3)+\lambda(1-\mu_H)(1/3)+(1-\lambda)(1/3)+\lambda\mu_L(1/3)+\lambda(1-\mu_L)(1/3)+(1-\lambda)(1/3)} \\
&= \frac{1}{2}; \\
q(-1) &= \frac{\lambda\mu_H(1/3)+\lambda(1-\mu_H)(1/3)+(1-\lambda)(1/3)}{\lambda\mu_H(1/3)+\lambda(1-\mu_H)(1/3)+(1-\lambda)(1/3)+\lambda(1-\mu_L)(1/3)+(1-\lambda)(1/3)} \\
&= \frac{1}{2-\lambda\mu_L}; \\
q(-2) &= \frac{\lambda\mu_H(1/3)}{\lambda\mu_H(1/3)} \\
&= 1.
\end{aligned}$$

Thus, sequential rationality implies the following table:

X	-2	-1	0	1	2
q	1	$\frac{1}{2-\lambda\mu_L}$	$\frac{1}{2}$	$\frac{1-\lambda\mu_H}{2-\lambda\mu_H}$	0
d	i	?	n	n	n
p	R_H^i	?	$\frac{1}{2}R_H^n+\frac{1}{2}R_L^n$	$\frac{1-\lambda\mu_H}{2-\lambda\mu_H}R_H^n+\frac{1}{2-\lambda\mu_H}R_L^n$	R_L^n

We first consider the positively-informed speculator who observes $\theta = L$. If she chooses to buy one unit:

- W.p. $\frac{1}{3}$, $X = 2$ and she is fully revealed, so trading profits are zero.
- W.p. $\frac{1}{3}$, $X = 1$ and she pays $\frac{1-\lambda\mu_H}{2-\lambda\mu_H}R_H^n + \frac{1}{2-\lambda\mu_H}R_L^n$ for a share which is worth R_L^n, which yields a profit of $\frac{1-\lambda\mu_H}{2-\lambda\mu_H}(R_L^n - R_H^n) > 0$.
- W.p. $\frac{1}{3}$, $X = 0$ and she pays $\frac{1}{2}R_H^n + \frac{1}{2}R_L^n$ for a share which is worth R_L^n, which yields a profit of $\frac{1}{2}(R_L^n - R_H^n) > 0$.

Therefore, a positively-informed speculator will choose to buy and receives an ex-ante profit of $\frac{1}{3}[\frac{1-\lambda\mu_H}{2-\lambda\mu_H}(R_L^n - R_H^n) + \frac{1}{2}(R_L^n - R_H^n)]$ (gross of trading costs).

We assume that $\frac{1}{2-\lambda} > \gamma$, so that observing $X = -1$ causes the manager's decision to chance, i.e. there is a feedback effect. The table becomes:

X	-2	-1	0	1	2
q	1	$\frac{1}{2-\lambda}$	$\frac{1}{2}$	$\frac{1-\lambda\mu_H}{2-\lambda\mu_H}$	0
d	i	i	n	n	n
p	R_H^i	$\frac{1}{2-\lambda}R_H^i + \frac{1-\lambda}{2-\lambda}R_L^i$	$\frac{1}{2}R_H^n + \frac{1}{2}R_L^n$	$\frac{1-\lambda\mu_H}{2-\lambda\mu_H}R_H^n + \frac{1}{2-\lambda\mu_H}R_L^n$	R_L^n

We now consider the negatively-informed speculator who observes $\theta = H$. If she sells,

- W.p. $\frac{1}{3}$, $X = -2$ and she is fully revealed, so trading profits are zero.
- W.p. $\frac{1}{3}$, $X = -1$ and she receives $\frac{1}{2-\lambda}R_H^i + \frac{1-\lambda}{2-\lambda}R_L^i$ for a share which is worth R_H^i, which yields a profit of $\frac{1}{3}\frac{1-\lambda}{2-\lambda}(R_L^i - R_H^i) < 0$.
- W.p. $\frac{1}{3}$, $X = 0$ and she receives $\frac{1}{2}R_H^n + \frac{1}{2}R_L^n$ for a share which is worth R_H^n, which yields a profit of $\frac{1}{3}\frac{1}{2}(R_L^n - R_H^n) > 0$.

Note that the case of $X = -1$ represents a limit to arbitrage similar to the core model. By selling, the negatively-informed speculator changes the decision towards continuation. Continuation is indeed optimal since $\theta = H$, and so the act of arbitrage improves the firm's decision and reduces the profitability of a short position. Thus, the speculator makes a loss. Overall, her ex-ante profit is $\frac{1}{3}\frac{1-\lambda}{2-\lambda}(R_L^i - R_H^i) + \frac{1}{3}\frac{1}{2}(R_L^n - R_H^n)$.

Therefore, if the trading cost κ satisfies the following conditions:

$$\frac{1}{3}(R_L^n - R_H^n) > \kappa > \frac{1}{3}\left[\frac{1-\lambda}{2-\lambda}\left(R_L^i - R_H^i\right) + \frac{1}{2}(R_L^n - R_H^n)\right],$$

then $\mu_H = 0$ and $\mu_L = 1$. Since $R_L^i < R_H^i$, there always exists a set of trading costs κ that satisfies the above inequalities. The table becomes

X	-2	-1	0	1	2
q	1	$\frac{1}{2-\lambda}$	$\frac{1}{2}$	$\frac{1}{2}$	0
d	i	i	n	n	n
p	R_H^i	$\frac{1}{2-\lambda}R_H^i + \frac{1-\lambda}{2-\lambda}R_L^i$	$\frac{1}{2}R_H^n + \frac{1}{2}R_L^n$	$\frac{1}{2}R_H^n + \frac{1}{2}R_L^n$	R_L^n

We now wish to verify that the positively-informed speculator indeed does not wish to sell, and the negatively-informed speculator indeed does not wish to buy. If the negatively-informed speculator buys, then

- W.p. $\frac{1}{3}$, $X = 2$ and she receives a profit of $R_H^n - R_L^n$.
- W.p. $\frac{1}{3}$, $X = 1$ and she receives a profit of $\frac{1}{2}(R_H^n - R_L^n)$.

- W.p. $\frac{1}{3}$, $X = 0$ and she receives a profit of $\frac{1}{2}(R_H^n - R_L^n)$.

Since $R_H^n < R_L^n$, the overall profit is negative, as in the core model. If the positively-informed speculator sells, then

- W.p. $\frac{1}{3}$, $X = -2$ and she receives a profit of $R_H^i - R_L^i > 0$.
- W.p. $\frac{1}{3}$, $X = -1$ and she receives a profit of $\frac{1}{2-\lambda}(R_H^i - R_L^i) > 0$.
- W.p. $\frac{1}{3}$, $X = 0$ and she receives a profit of $\frac{1}{2}(R_H^n - R_L^n) < 0$.

The profit is positive if $X < 0$. As in the core model, by selling on good information, the speculator can dupe the manager into taking the incorrect decision – in this case, continuation even though $\theta = L$. This increases the profitability of a short position. Her overall profit from selling is $\frac{1}{3}[\frac{3-\lambda}{2-\lambda}(R_H^i - R_L^i) + \frac{1}{2}(R_H^n - R_L^n)]$. For her to prefer buying over selling, we must have:

$$\frac{1}{3}[\frac{1}{2}(R_L^n - R_H^n) + \frac{1}{2}(R_L^n - R_H^n)] > \frac{1}{3}[\frac{3-\lambda}{2-\lambda}(R_H^i - R_L^i) + \frac{1}{2}(R_H^n - R_L^n)]$$
$$\frac{1}{2}(R_L^n - R_H^n) > \frac{3-\lambda}{2-\lambda}\left(R_H^i - R_L^i\right).$$

If this is satisfied, we indeed have $\mu_H = 0$ and $\mu_L = 1$, i.e. the negatively-informed speculator does not trade while the positively-informed speculator sells.

As in the core model, the limit to arbitrage requires that $X = -1$ alters the manager's prior sufficiently that his action changes (in this case, from correction to continuation) – i.e. there is feedback from the speculator's trade to the manager's action. Hence, $\frac{1}{2-\lambda} > \gamma$ is a necessary condition. With $\frac{1}{2-\lambda} < \gamma$ we would have $d\,(X = -1) = 1$ and the manager would still abandon the project even if $X = -1$, so there would be no feedback. The no-feedback equilibria and the analogy of the proof of Proposition 2 are similarly mirror images of the core model.