Governance and Comovement Under Common Ownership*

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August 18, 2014

Abstract

This paper studies the corporate governance and asset pricing implications of investors owning blocks in multiple firms. Common wisdom is that multi-firm ownership weakens governance because the blockholder is spread too thinly. We show that this need not be the case. In a single-firm benchmark, the blockholder governs through exit, selling her stake if the firm underperforms. With multiple firms, the blockholder may sell even a value-maximizing firm, to disguise her exit from another underperforming firm as being motivated by a portfolio-wide liquidity shock. This reduces the manager's effort incentives and weakens governance. On the other hand, governance can be stronger, because selling one firm and not the other is a powerful signal of underperformance. Common ownership leads to firms' stock prices being correlated, even if their fundamentals are uncorrelated. We derive empirical predictions for the direction of correlation and for whether governance is stronger or weaker with multiple firms.

Keywords: Blockholders, corporate governance, exit, trading, correlation
JEL Classification: D72, D82, D83, G34

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Most existing theories of blockholder governance consider a single firm. However, in reality, many institutional investors hold blocks in multiple firms.[1] This paper studies the implications of common ownership for corporate governance and asset pricing. In particular, we address two broad questions. First, does holding multiple blocks weaken governance by spreading a blockholder too thinly, as commonly believed? If not, under what conditions can multi-firm ownership improve governance? Second, can common ownership lead to correlation between stocks with independent fundamentals, and if so, in which direction?

In our model, the blockholder governs through "exit", disciplining the manager by selling shares if he shirks. Such sales reduce the value of the manager's equity compensation ex post, thus inducing him to maximize firm value ex ante. We model governance through exit rather than "voice" (intervention) for three reasons. First, McCahery, Sautner, and Starks (2011) report that exit is the main governance mechanism used by institutions.[2] Second, if holding multiple blocks means that the investor holds smaller stakes in each firm, she may lack sufficient control rights to intervene. Third, exit has asset pricing implications, since it involves the blockholder trading the firm's stock.

As a benchmark against which to assess the effects of common ownership, we start with a model in which the blockholder owns shares in a single firm. The manager can take an action (such as shirking, cash flow diversion, or empire building) that yields a fixed private benefit, but reduces firm value by a random amount privately known to him. In equilibrium, the manager shirks if and only if the value loss is below a threshold; a lower threshold corresponds to greater efficiency and thus superior governance. The blockholder privately observes the manager's action, and based on this information, may either sell shares or retain them until firm value is realized. As in Admati and Pfleiderer (2009), her trade is observed by the market maker, but not fully revealing because she may also suffer a liquidity shock that forces her to sell half of her stake (although she may choose to sell more). In equilibrium, the blockholder sells shares, reducing the stock price, if she needs liquidity or the manager shirks. The threat of selling disciplines the manager and lowers the threshold below which he shirks.

[1]See Antón and Polk (2014), Bartram, Griffin, Lim, and Ng (2014), Gao, Moulton, and Ng (2014), Hau and Lai (2013), and Jotikasthira, Lundblad, and Ramadorai (2012)

[2]See Parrino, Sias, and Starks (2003), Bharath, Jayaraman, and Nagar (2013), Edmans, Fang, and Zur (2013), and Duan and Jiao (2014) for further evidence of governance through exit.

The core analysis is a model with two independent firms, where the blockholder owns a stake in each firm. She can satisfy her liquidity need by selling either half of her stake in each firm or her entire stake in one firm. Importantly, the decision to sell is made at the portfolio rather than firm level. As we discuss below, this is a key implication of common ownership.

If both managers work and there is no liquidity shock, the blockholder retains both stakes. If there is a liquidity shock and both managers either work or shirk, she sells half of her stake in each firm ("balanced exit").[3] The interesting case is when one manager shirks and the other works. The blockholder has two options. First, she sells only the underperforming firm ("imbalanced exit"). The disadvantage is that selling only one firm fully reveals that its manager has shirked, and so the blockholder receives a low price for the sold share. Second, the blockholder engages in balanced exit, to disguise her trades as being driven by a liquidity shock. While selling the value-maximizing firm leads to losses, they may be offset by the higher price received from selling the underperforming firm.

The balanced exit strategy is more likely if the liquidity shock is more common, so that selling both shares provides more camouflage. It is also more likely if agency problems are weak (private benefits from shirking are lower, or the manager's stock price concerns are higher so that he is more concerned with exit), because then balanced exit is more likely to result from a liquidity shock than both managers shirking. The market maker thus sets high prices upon observing balanced exit, encouraging the blockholder to choose it.

Our first main result is that the two-firm equilibrium is more efficient than the single-firm benchmark if and only if the blockholder chooses imbalanced exit with sufficient probability. As explained above, the probability of imbalanced exit depends upon the severity of the agency problem. Thus, our model relates the efficiency of multi-firm governance to the underlying characteristics of the portfolio companies.

The intuition is as follows. With one firm, if the manager works, his firm is sold only if the blockholder suffers a liquidity shock. With two firms and balanced exit, if the other manager shirks, then even if the first manager works and the blockholder suffers no shock, his firm is still sold. Thus, the first manager's incentives to work are

[3] Balanced exit may also involve the blockholder selling a different proportion (not necessarily half) of her stake in each firm. We show that the exact number of shares the blockholder sells in each firm, under balanced exit, makes no difference to the manager's strategy or to efficiency.

lower, reducing efficiency. In contrast, an equilibrium with imbalanced exit is more efficient than the single-firm benchmark. This is because the existence of the second firm gives the blockholder another channel to satisfy her liquidity needs, which has two consequences. First, the reward for working is higher. In the single-firm model, if the manager works and the blockholder suffers a liquidity shock, his firm is automatically sold. In the two-firm model, a value-maximizing firm is not sold even if the blockholder suffers a liquidity shock, since she can sell the other firm and will indeed do so if it is underperforming. Second, the punishment for shirking is higher. With one firm, being sold is not a severe punishment, since it is consistent with a liquidity shock and thus leads only to a moderate price decline. With two firms, the blockholder can severely punish an underperforming firm by selling it and retaining the other firm. Selling only one firm is a particularly strong signal that it is underperforming, because if the blockholder had instead suffered a liquidity shock, she would have sold both firms equally. Thus, the price of the sold firm is very low, significantly deterring shirking.

In sum, when the agency problem is strong in a single-firm setting, because private benefits are high or the manager's stock price concerns are low, it is mitigated by common ownership. More generally, while empirical blockholder studies typically analyze the size of the largest blockholder or the number of blockholders, our paper theoretically motivates a new measure of blockholder governance – the number of blocks owned by an investor. Faccio, Marchica, and Mura (2011) and Ekholm and Maury (2014) empirically study a related measure, the concentration of a stock in an investor's portfolio. The former posit that concentration is negatively related with firm value, since a concentrated investor will turn down risky, positive-NPV projects. The latter hypothesize that it is positively related with firm value, since firms will focus their monitoring on their largest holdings. Our theory suggests a different channel through which portfolio concentration can affect firm value. Moreover, the effect of portfolio concentration is nuanced: it is not universally positive or negative, but depends on the severity of the agency problem and the probability of liquidity shocks. This may explain the heterogeneity of cross-ownership structures between firms.

Our second main result is that common ownership leads to correlation between the stock prices of portfolio companies, even with independent fundamentals. On the one hand, liquidity shocks cause the blockholder to sell both firms, driving both prices down. On the other hand, imbalanced exit depresses the price of the sold firm but increases the price of the retained firm. Overall, the correlation is negative if and only if the probability of imbalanced exit is sufficiently high. Hence, the severity of the agency problem affects the direction of correlation, through determining the probability of imbalanced exit. Comovement typically depend on the correlation between random variables, e.g. firms' business conditions. Here, it depends on non-random parameters. In addition, while comovement is an asset pricing concept that typically depends on loadings on macroeconomic factors, here it is affected by corporate finance variables (e.g. the severity of agency problems) as these govern the blockholder's trading strategy.

We also show that common ownership creates strategic interactions between the managers of otherwise unrelated firms. If the blockholder pursues balanced exit, a working manager's firm is sold if the other manager shirks. The greater the likelihood that the other manager shirks, the lower the first manager's incentive to work, and so the managers' actions are strategic complements. With imbalanced exit, the managers' actions can be strategic substitutes. If one manager works and the blockholder suffers a liquidity shock, he is not sold if the other manager shirks, as the blockholder will sell only the other firm. The greater the likelihood that the other manager shirks, the greater the first manager's incentive to work. Overall, regardless of whether we have strategic complements or substitutes, a novel implication is that common ownership can give rise to governance externalities between firms – greater effort by one manager affects the effort incentives of another manager.[4]

In the core analysis, we focus on "ex-post symmetric" equilibria, where the blockholder makes the same trade in each firm whenever the managers take the same action. Indeed, investors (e.g., mutual funds) typically keep portfolio weights fixed unless firm fundamentals change asymmetrically. For example, Coval and Stafford (2007) show that liquidity shocks (due to investor outflows) do not cause mutual funds to sell stakes more selectively than funds that do not experience a shock. For completeness, we relax this focus in an extension, and characterize the broader set of equilibria that emerge. We show that the most efficient two-firm equilibrium is not ex-post symmetric: when both managers work and the blockholder suffers a liquidity shock, she does not automatically sell a half-share in each firm, but chooses imbalanced exit with posi-

[4] Here, governance externalities arise from common ownership. In Acharya and Volpin (2010), Dicks (2012), and Levit and Malenko (2014), they arise from labor market competition.

tive probability. Since imbalanced exit is now consistent with both managers working, the price the blockholder receives for the sold firm is relatively high. As a result, the blockholder has stronger incentives to choose imbalanced exit when one manager shirks. This exerts stronger governance and creates negative price correlations due to the same intuition as the core model. Moreover, under this equilibrium, the two-firm model is always more efficient than the single-firm benchmark. This extension reinforces the core model's insight that common ownership exerts stronger governance if and only if the blockholder chooses imbalanced exit.

This paper builds on a recent theoretical literature on governance through exit. Admati and Pfleiderer (2009) and Edmans (2009) study a single firm and single blockholder, which Goldman and Strobl (2013) and Dasgupta and Piacentino (2014) extend to the case in which the blockholder has career concerns. Levit (2013) analyzes the interaction between exit and communication between investors and managers, in a single-firm, single-blockholder framework. Edmans and Manso (2011) model both exit and voice in a single-firm, multi-blockholder model. To our knowledge, the effectiveness of exit in a multi-firm model has not been previously studied. Away from exit theories, Gervais, Lynch, and Musto (2005) show that mutual fund families can add value by monitoring multiple managers, since firing one manager increases investors' perceived skill of retained managers. This intuition is similar to why imbalanced exit strengthens governance. In addition to the context, there are important differences between the two models. First, our model studies moral hazard while Gervais et al. study adverse selection. Second, Gervais et al. assume that the fund family can commit to a firing policy; under this assumption, the fund family always creates value. Here, we do not assume that commitment is possible. The blockholder will only engage in imbalanced exit if she has incentives to do so; as discussed above, these incentives depend on the nature of the agency problem. Therefore, the combination of these two departures from Gervais et al. generates our first main result. Vayanos and Woolley (2012) study how common ownership leads to positive price correlations. Here, correlations can be negative and common ownership also affects real outcomes.

Empirically, Gao, Moulton, and Ng (2014) document return predictability across economically unrelated stocks with common institutional ownership. Antón and Polk (2014) show that common ownership led to stocks exhibiting excess comovement during the 2003 mutual fund trading scandal, which caused implicated funds to suffer a liquidity shock. Jotikasthira, Lundblad, and Ramadorai (2012) find that liquidity shocks to mutual funds cause comovement between the markets they invest in. Bartram, Griffin, Lim, and Ng (2014) find that a company's stock return is higher when the returns to foreign stocks held by its institutional investors are high. These asset pricing effects can lead to common ownership having corporate finance implications (as in this paper), due to the feedback effect from financial markets to real decisions: Hau and Lai (2013) show that mutual funds exposed to bank stocks suffered large losses, which forced them to sell unrelated stocks, reducing their prices and in turn investment and employment. While the above papers study how the *event* of a liquidity shock (or unusual returns in one sector) can propagate to other sectors through common ownership, we argue that common ownership *per se* affects stock prices, even outside of liquidity events. Moreover, it can also affect governance and managerial behavior: the mere threat of selling one firm and retaining another can induce effort.[5]

Some of the model's implications are both consistent with existing evidence and at the same time yield new predictions. Scholes (1972), Mikkelson and Partch (1985), Holthausen, Leftwich, and Mayers (1990), and Sias, Starks, and Titman (2006) show that sales by large investors reduces the stock price; these declines are permanent and thus likely result from the sale conveying negative information. While consistent with our model, these results are also consistent with any exit theory. However, our model yields an additional prediction that is, to our knowledge, untested – the price impact of blockholder exit depends not only on how many shares she sells in the firm in question, but also whether she sells her stakes in other, otherwise unrelated firms.

This paper is organized as follows. Section 1 presents the model setup. Section 2 presents a benchmark in which the blockholder owns a single firm and Section 3 considers the core multi-firm model. Section 4 analyzes ex-post asymmetric equilibria and Section 5 concludes. Appendix A gives all proofs not in the main text, and the Online Appendix considers supplemental analyses.

[5]Away from return correlation, Lou (2012) empirically analyzes a different consequence of common ownership: strong performance by one fund improves the performance of others holding the same stock, by inducing inflows into the first fund which cause upward price pressure. Matvos and Ostrovsky (2008) and Harford, Jenter, and Li (2011) study how common ownership of two firms affects the likelihood of them merging.

1 The Model

The model consists of three periods and two public and symmetric firms, i and j. A blockholder owns α shares in each firm, which we normalize to one to save on notation. The remaining shares are owned by dispersed shareholders who play no role in the model. The blockholder represents a mutual fund, hedge fund, or other institutional investor who has superior information about firm value and thus can engage in governance through exit.[6]

Each firm is run by a manager. At $t = 1$, manager i takes action $a_i \in \{L, H\}$, which affects the fundamental value of the firm at $t = 3$. If $a_i = H$, then firm value is $\overline{v} > 0$, which is common knowledge. Action $a_i = L$ yields the manager a private benefit $\beta > 0$, where β is common knowledge, but reduces firm value by a random amount $\theta_i \in [0, \overline{v}]$. Overall, firm value at $t = 3$ is given by:

$$v(\theta_i, a_i) = \overline{v} - \theta_i \mathbf{1}_{a_i = L}. \tag{1}$$

The variable θ_i is privately known to manager i (but not manager j) when he chooses his action, and independent of θ_j. The cumulative distribution function of θ_i is given by F, which is continuous and has full support. We assume that

$$\beta \in (0, \overline{v}), \tag{2}$$

which is a necessary condition for $a_i = L$ to reduce total surplus: the manager's private benefit is less than the maximum value erosion. Examples of such actions include shirking, cash flow diversion, perk consumption, and empire building. For simplicity, we will refer to $a_i = L$ as "shirking" and $a_i = H$ as "working". We will abuse language slightly by using the phrase "shirking firm" to refer to a firm run by a manager who has shirked, and "working firm" analogously.

[6] Edmans (2009) shows that, if there are short-sales constraints or non-trivial short-sales costs, a blockholder will endogenously gather more information than small investors, even if she has no superior access to information. Alternatively, the blockholder may be endowed with more information if her large stake gives her improved access to management. Even though Regulation FD prohibits managers from selectively disclosing material information, investors still talk to managers to learn their views on market conditions, strategic choice, etc.

Manager i's objective function is given by:

$$u_{M,i} = v(\theta_i, a_i) + \omega p_i + \beta \mathbf{1}_{a_i=L}, \tag{3}$$

where p_i is firm i's stock price at $t = 2$, set by a market maker as described below. The variable $\omega > 0$ captures the manager's concern for the stock price, which is standard in exit theories and can stem from a number of sources introduced in prior work: takeover threat (Stein (1988)), termination threat (Edmans (2011)), concern for managerial reputation (Narayanan (1985), Scharfstein and Stein (1990)), or the manager expecting to sell his own shares at $t = 2$ (Stein (1989)). If the manager is indifferent between working and shirking, we assume that he works.

The blockholder privately observes $a = (a_i, a_j)$ but is uninformed about $\theta = (\theta_i, \theta_j)$.[7] After doing so, she can trade at $t = 2$ with a competitive risk-neutral market maker who does not observe a or θ. If the blockholder is not hit by a liquidity shock, she is free to choose whether to retain or sell her stake in each firm. With probability ("w.p.") $\delta \in [0, 1)$, she is hit by a liquidity shock which forces her to sell half of her combined holdings (i.e. 1 share in total), although she may choose to sell more. We use $\chi = 1$ ($\chi = 0$) to denote the case where the blockholder is (is not) subject to a liquidity shock. In addition, denote by $s_i(a_i, a_j, \chi) \in \{0, 0.5, 1\}$ her trade in firm i. Since there are two ex-ante identical firms, we allow the blockholder to trade shares in half-units, but the proofs of Sections 2 and 3 in Appendix A allow her to sell any fraction of her portfolio and show that the results continue to hold. As in Admati and Pfleiderer (2009) and Edmans (2009), the blockholder cannot buy additional shares; those papers show that the exit governance mechanism is robust to allowing for purchases. We will abuse language slightly by using the phrase "the manager will be sold" to refer to the stock of the firm run by the manager being sold.

As in Admati and Pfleiderer (2009), the market maker observes the blockholder's trading decisions $s = (s_i, s_j)$, but not whether she has suffered a liquidity shock. He sets the stock price $p_i(s)$ to equal the firm's expected value. Overall, conditional on s,

[7] Our assumption that the blockholder observes (a_i, a_j) but not (θ_i, θ_j) is consistent with her being less informed than the manager. Her information advantage over the market maker (and other investors) arises from observing whether firm value is $\overline{v}$ or less than $\overline{v}$, i.e. when the firm is operating below its potential. Allowing the blockholder to observe (θ_i, θ_j) would lead to substantial complexity as the blockholder's trading decision (and thus the market maker's inference) will now depend on the value destroyed by any shirking, in addition to the act of shirking.

the blockholder's utility is given by:

$$u_B(s) = \sum_i [s_i p_i(s) + (1 - s_i) v(\theta_i, a_i)]. \quad (4)$$

The equilibrium concept we use is Perfect Bayesian Nash Equilibrium. Here, it is defined as follows: (i) A trading strategy by the blockholder that maximizes her expected utility u_B given the price-setting rule of the market maker, the strategy of each manager, and her information on a. (ii) A decision rule by each manager i that maximizes his expected utility $u_{M,i}$ given his information on θ_i, the strategy of manager j, the price-setting rule of the market maker, and the trading strategy of the blockholder. (iii) A price-setting strategy by the market maker that allows him to break even in expectation, given the strategy of the blockholder and managers. Moreover, (iv) the market maker uses Bayes' rule to update his beliefs from the blockholder's trades, and (v) all agents have rational expectations in that each player's belief about the other players' strategies is correct in equilibrium.

We focus on symmetric equilibria, in which the managers follow the same strategy and the market maker uses a symmetric pricing function.[8] Moreover, in the core analysis, we require that the equilibrium is *ex-post symmetric* as defined below.

Definition 1. *An equilibrium is* ex-post symmetric *if on the equilibrium path,* $a_i = a_j \Rightarrow s_i = s_j$ *w.p. 1.*

An equilibrium is ex-post symmetric if, whenever the blockholder observes the same action in both firms, she sells the same amount of shares in each firm. Ex-post symmetry is a stronger requirement than symmetry: for example, choosing $s = (1, 0)$ and $s = (0, 1)$ with equal probability when $a = (H, H)$ is a symmetric strategy that violates ex-post symmetry. Ex-post symmetry is a reasonable selection criterion particularly in the context of financial markets, since investors typically prefer to keep portfolio weights fixed unless fundamentals have changed. Hereafter, unless otherwise noted,

[8]Asymmetric equilibria may exist. We focus on symmetric equilibria since firms are symmetric. Thus, if there is an asymmetric equilibrium, there exists another equilibrium in which firms switch roles (since they are ex-ante identical). These equilibria are unattractive as it is indeterminate which role each firm will play. Relatedly, in Lemma 4 in Appendix A, we show that if the managers follow the same strategy and the market maker follows a symmetric pricing strategy, the blockholder's best response is a symmetric strategy as well.

references to an equilibrium are to an ex-post symmetric equilibrium.[9] In Section 4, we relax ex-post symmetry and discuss the additional equilibria that emerge.

We define efficiency as the maximization of firm value v, rather than total surplus (which includes the private benefit β), since governance is typically focused on shareholder value maximization.

2 Single-Firm Benchmark

As a benchmark against which to compare the effects of multi-firm ownership, we consider a variant of the baseline model in which the blockholder owns two shares in firm i and zero in firm j.[10] We denote the blockholder's trading decision by $s_i \in \{0, 0.5, 1, 1.5, 2\}$. Her utility function is now:

$$u_B(s_i) = sp(s_i) + (2 - s_i)\, v(\theta_i, a_i). \tag{5}$$

The equilibrium is given in Proposition 1 below:

Proposition 1. *(Single-firm benchmark): With a single firm, an equilibrium always exists. There is a unique $\theta_1^* \in (0, \overline{v})$ such that, in any equilibrium, there exist $\underline{s} < 1$ and $\overline{s} \geq 1$ and the following hold:*
(i) If $a_i = H$ and $\chi = 0$, then $s_i = \underline{s}$.
(ii) If $a_i = L$ or $\chi = 1$, then $s_i = \overline{s}$.
(iii) Prices on the equilibrium path satisfy

$$p(s_i) = \begin{cases} \overline{v} & \text{if } s_i = \underline{s} \\ \overline{v} - \frac{F(\theta_1^*)}{F(\theta_1^*) + \delta[1 - F(\theta_1^*)]}\mathbb{E}[\theta_i | \theta_i < \theta^*] & \text{if } s_i = \overline{s}. \end{cases} \tag{6}$$

Prices off the equilibrium path satisfy $p(s_i) = \underline{v}(\theta_1^*) \equiv \overline{v} - \mathbb{E}[\theta_i | \theta_i < \theta_1^*]$.
(iv) The manager shirks if and only if $\theta_i < \theta_1^*$, where θ_1^* is uniquely defined by $\psi_1(\theta_1^*) =$

[9] Note that imposing ex-post symmetry as a selection criterion does not restrict the blockholder's strategy space. The blockholder is allowed to trade asymmetrically: our ex-post symmetric equilibria must ensure that she will not wish to deviate to doing so.

[10] This model also depicts a scenario in which there are two firms, each owned by a different blockholder. Since the firms are unrelated in both fundamentals and ownership, the analysis of each firm can be conducted separately.

θ_1^*, *where*

$$\psi_1(y) = \beta - \omega(1-\delta)\frac{F(y)}{F(y)+\delta(1-F(y))}\mathbb{E}[\theta|\theta<y]. \quad (7)$$

When deciding his action, the manager trades off the private benefit from shirking with both the expected price decline and the firm value erosion from doing so. Part (iv) of Proposition 1 states that this trade-off gives rise to a threshold strategy: he shirks if and only if the value erosion θ_i is sufficiently small – less than a threshold $\psi_1(\theta_1^*)$, which equals the private benefit minus the expected price decline (the subscript 1 refers to a one-firm model). Therefore, ex-ante shareholder value in equilibrium is given by $V(\theta_1^*)$, where

$$V(y) \equiv \overline{v} - \Pr[\theta_i < y]\,\mathbb{E}[\theta_i|\theta_i < y], \quad (8)$$

which is decreasing in θ_1^*. A higher θ_1^* (more shirking) reduces firm value.

From part (iv), the threshold θ_1^* increases with β: when the private benefit from shirking is higher, the manager is less likely to work. Part (iv) also shows that θ_1^* decreases with the manager's stock price concern ω. From parts (i)-(iii), if the manager shirks, the blockholder sells more shares and reduces the stock price.[11] A higher ω makes this price reduction more costly to the manager, and so θ_1^* falls. In addition, θ_1^* is increasing in δ: more frequent liquidity shocks encourage shirking. If the blockholder suffers a liquidity shock, she exits even if the manager has worked, thus reducing his incentives to do so. Put differently, with a liquidity shock, the blockholder's trade is less driven by firm fundamentals, and so the manager has a weaker incentive to improve firm fundamentals by working.

Exit is most powerful when the blockholder has incentives to exit if and only if the manager shirks. Proposition 1 states that, in a single-firm model, this is the case if and only if the blockholder does not suffer a liquidity shock. In Section 3, we show that the ability to make exit decisions at the portfolio level – an implication of common ownership – affects real efficiency by changing the circumstances under which the blockholder finds it optimal to exit in equilibrium.

[11]Note that the manager does not care about how many shares the blockholder sells per se, but only the price impact of the blockholder's decision to sell. Thus, even though the equilibrium may not be unique ($\overline{s}$ and $\underline{s}$ are not uniquely defined), the manager's threshold is unique.

3 Multi-Firm Governance

3.1 Analysis of Equilibria

We now move to the main analysis where the blockholder owns one share in each firm. To simplify exposition, we use the following notation: if $s = (s_i, s_j)$ then $s^T = (s_j, s_i)$. We use blockholder "type" to refer to her liquidity need and/or the actions she has observed. For example, a "$\chi = 1$ and $a = (H, H)$-type" blockholder is one who has experienced a liquidity shock and observed that both managers have worked.

Given our focus on symmetric equilibria, if type $a = (L, H)$ chooses s with positive probability, type $a = (H, L)$ chooses s^T with the same probability. To keep the exposition concise, we omit the description of the strategy of type $a = (H, L)$. Moreover, since symmetry implies $p_i(s) = p_j(s^T)\ \forall\ s$, we omit the subscript i whenever there is no risk of confusion.

We start with a number of useful results that hold for all symmetric equilibria.

Lemma 1. *In any symmetric equilibrium, the following hold:*
(i) There exists $\theta^* \in (0, \overline{v})$ such that manager i chooses $a_i = L$ if and only if $\theta_i < \theta^*$.
(ii) $p(s) \geq \underline{v}(\theta^*)$ for all s.
(iii) If $a = (L, H)$, then $s_i \geq s_j$.
(iv) If $a = (H, H)$ and $\chi = 0$, then $s_i + s_j < 1$.
(v) If $a = (L, L)$, then $s_i + s_j \geq 1$.
(vi) If $s = (1, 1)$ is played with positive probability, then $s = (0.5, 0.5)$ is played w.p. 0.

Part (i) of Lemma 1 states that manager i follows a threshold strategy, as in Section 2. Part (ii) states that the lowest possible price is one that assumes that the manager shirked with certainty. Part (iii) implies a working manager is never sold more than a shirking manager. Part (iv) states that, if both managers work and the blockholder does not suffer a liquidity shock, she sells at most half of her portfolio. Part (v) states that if both managers shirk, the blockholder sells at least as much as if she suffered a liquidity shock, i.e. at least half of her portfolio. Intuitively, doing so allows her to disguise her sale as being motivated by a liquidity shock rather than shirking. Part (vi) says that there is no equilibrium in which both $s = (1, 1)$ and $s = (0.5, 0.5)$ are on the equilibrium path. Such an equilibrium can only be sustained if a blockholder who observes $a = (L, L)$ plays $s = (1, 1)$ and a blockholder who observes $a = (H, H)$

plays $s = (0.5, 0.5)$. However, this implies that $p(0.5, 0.5)$ is high relative to $p(1, 1)$, encouraging a blockholder who observes $a = (L, L)$ to deviate to it.

Lemma 1 applies to all symmetric equilibria, even those that are not ex-post symmetric. Ex-post symmetry, combined with part (iv) of Lemma 1, implies that the blockholder retains both shares if $a = (H, H)$ and $\chi = 0$. Parts (v) and (vi) of Lemma 1 imply that in any ex-post symmetric equilibrium, there is a unique $s_{BE} \in \{(0.5, 0.5), (1, 1)\}$ such that the blockholder chooses s_{BE} if $a = (H, H)$ and $\chi = 1$, or $a = (L, L)$. The abbreviation *"BE"* stands for balanced exit, i.e. the blockholder sells the same positive amount of shares in each firm.

The most interesting case is when $a = (L, H)$. We know from Lemma 1 part (iii) that $s_i \geq s_j$. Thus, the blockholder has up to three options. First, she may choose s such that $s_i = s_j$, i.e., $s = s_{BE}$. By selling the same quantity in both firms, the blockholder can pretend that her sales are motivated by a liquidity shock rather than shirking. On the one hand, she receives a higher price for stock i, since the market maker is not certain that manager i has shirked: $s = s_{BE}$ is consistent with $a = (H, H)$ and $\chi = 1$, and so $p(s_{BE}) > \underline{v}(\theta^*)$. On the other hand, she suffers a loss on her sale of firm j. Its true value is $\overline{v}$, but the price she receives is strictly lower since it incorporates the possibility that the sale was motivated by shirking: $s = s_{BE}$ is also consistent with $a = (L, L)$, and so $p(s_{BE}) < \overline{v}$.

Second, the blockholder could choose $s_i > s_j$, i.e. sell more shares in the shirking firm. Such "imbalanced exit" (denoted *"IE"*) reveals that manager i has shirked and manager j has worked, and so the blockholder receives the lowest price $\underline{v}(\theta^*)$ for firm i, and the highest price $\overline{v}$ for firm j. We use S_{IE} to denote all trades such that $s_i > s_j$. If $s_{IE} \in S_{IE}$, then $p_i(s_{IE}) = \underline{v}(\theta^*)$ and $p_i\left(s_{IE}^T\right) = \overline{v}$. We will refer to manager i as being "sold more" and manager j as being "sold less".

Finally, if in addition to $a = (L, H)$ we have $\chi = 0$, the blockholder may choose $s = (0, 0)$. In Proposition 5 in Appendix B in the Online Appendix, we show that, if there is an equilibrium in which the blockholder chooses $s = (0, 0)$ with a strictly positive probability when $a = (L, H)$, then there is another equilibrium in which she never chooses $s = (0, 0)$ in this case, and this equilibrium is weakly more efficient. Intuitively, by retaining a shirking manager, the blockholder exerts weaker governance, reducing real efficiency. Moreover, $s = (0, 0)$ implies that the blockholder is not exiting even though one manager has shirked, inconsistent with our focus on governance through exit. For these reasons, we focus on equilibria in which the blockholder never retains both firms when $a = (L, H)$.

Let γ be the probability that the blockholder chooses s_{BE} in equilibrium when $a = (L, H)$. Given the discussion above, she chooses $s_{IE} \in S_{IE}$ w.p. $1-\gamma$. For any γ and θ^* we calculate the price functions. If $\gamma < 1$, there is $s_{IE} \in S_{IE}$ (not necessarily unique) on the equilibrium path such that $p_i(s_{IE}) = \underline{v}(\theta^*)$. Moreover, if $s \in \{s_{IE}^T, (0,0)\}$, then $p_i(s) = \overline{v}$. Finally, from Bayes' rule, $p(s_{BE}) = \pi(\theta^*, \gamma) \in (\underline{v}(\theta^*), \overline{v})$, where

$$\pi(y, \gamma) \equiv \overline{v} - \left(\frac{F(y)^2 + F(y)(1 - F(y))\gamma}{F(y)^2 + 2F(y)(1 - F(y))\gamma + (1 - F(y))^2 \delta} \right) \mathbb{E}[\theta_i | \theta_i < y]. \quad (9)$$

Using (9) and the above price functions, Lemma 2 shows how the value of γ in equilibrium depends on the threshold θ^*.

Lemma 2. *Let* $Y(\delta) \equiv F^{-1}\left(\frac{\sqrt{\delta}}{1+\sqrt{\delta}}\right)$. *In any equilibrium,*

$$\gamma \in \Gamma(\theta^*) \equiv \begin{cases} \{1\} & \text{if } \theta^* < Y(\delta) \\ [0,1] & \text{if } \theta^* = Y(\delta) \\ \{0\} & \text{if } \theta^* > Y(\delta). \end{cases} \quad (10)$$

To understand the intuition behind Lemma 2, recall that if type $a = (L, H)$ chooses imbalanced exit, her payoff is $\underline{v}(\theta^*) + \overline{v}$. If she chooses balanced exit, her payoff is

$$2 s_{BE,i} p(s_{BE}) + (1 - s_{BE,i})(\underline{v}(\theta^*) + \overline{v}). \quad (11)$$

Regardless of the value of $s_{BE,i}$, the blockholder prefers balanced exit if and only if

$$p(s_{BE}) \geq \frac{\underline{v}(\theta^*) + \overline{v}}{2}. \quad (12)$$

Based on (9), simple algebra shows that (12) holds if and only if $\theta^* \leq Y(\delta)$.

The intuition is as follows. A fall in θ^* has two effects: managers are less likely to shirk and less value is destroyed by shirking. The second effect increases both the price the blockholder receives upon balanced exit (see (9)) and the price she receives upon imbalanced exit ($\underline{v}(\theta^*)$). The first effect increases $p(s_{BE})$ only. Recall that the price set by the market maker upon observing balanced exit incorporates the possibility of

$a = (H, H)$ and $\chi = 1$, $a = (L, L)$, and $a = (L, H)$. As θ^* decreases, the market maker attaches a higher probability to s_{BE} arising from $a = (H, H)$ and $\chi = 1$. Thus, a fall in θ^* raises $p(s_{BE})$ more than $p(s_{IE})$, and the blockholder prefers s_{BE}. Put differently, when shirking is less likely, balanced exit is less revealing of shirking and more revealing of a liquidity shock. Thus, the price received from balanced exit is higher, encouraging this strategy. We will later discuss how changes in the underlying parameters β, ω, and δ affect whether $\theta^* \lesseqgtr Y(\delta)$ and thus the frequency of balanced exit γ. We will show that this frequency in turn affects the efficiency of the two-firm model compared to the benchmark.

In equilibrium, each manager has rational expectations about the price functions, the blockholder's exit strategy, and the other manager's threshold strategy. Then, manager i shirks ($a_i = L$) if and only if

$$\overline{v} - \theta_i + \beta + \omega \left[\begin{array}{c} F(\theta^*)\, p(s_{BE}) \\ +(1 - F(\theta^*))(\gamma^* p(s_{BE}) + (1-\gamma^*)\, \underline{v}(\theta^*)) \end{array} \right] \tag{13}$$
$$> \overline{v} + \omega \left[\begin{array}{c} F(\theta^*)(\gamma^* p(s_{BE}) + (1-\gamma^*)\,\overline{v}) \\ +(1 - F(\theta^*))(\delta p(s_{BE}) + (1-\delta)\,\overline{v}) \end{array} \right].$$

As stated in Lemma 1, the manager follows a threshold strategy. Note that manager i's decision to shirk depends on how frequently he expects manager j to shirk. This is reflected by the dependence of both the left-hand side ("LHS") and right-hand side ("RHS") of (13) on $F(\theta^*)$, the probability that manager j shirks.

Lemma 3 studies whether the managers' strategies are strategic complements or substitutes. In our context, strategic complements (substitutes) arise if, given the prices and blockholder strategies, the best response (that is, the threshold) of manager i increases (decreases) with the threshold of manager j.

Lemma 3. *Given the prices and the blockholder's selling strategy, the managers' decisions are strategic complements if and only if* $\theta^* < F^{-1}(0.5)$, *where* $F^{-1}(0.5) > Y(\delta)$.

From Lemma 2, if $\theta^* < Y(\delta)$, the blockholder chooses balanced exit when $a = (L, H)$. Thus, the reward for working decreases with the likelihood that the other manager shirks, because if he does so, the working manager is still sold. The incentive to shirk increases, and so the managers' decisions are strategic complements. If $\theta^* > Y(\delta)$, the blockholder chooses imbalanced exit when $a = (L, H)$. Imbalanced exit generates two forces that work in opposite directions. On the one hand, the reward for the working manager increases with the likelihood that the other manager shirks, because if the blockholder suffers a liquidity shock, the working manager is not sold if the other manager shirks. This force tends to lead to strategic substitutes. On the other hand, the punishment for shirking decreases with the likelihood that the other manager shirks: if both managers shirk, the blockholder engages in balanced exit which only leads to a small price decline; if only manager i shirks, he is sold more and suffers the lowest possible price of $\underline{v}(\theta^*)$. This force tends to lead to strategic complements. In the proof of Lemma 3 we show that if θ^* is sufficiently high, that is, $\theta^* > F^{-1}(0.5)$ $(> Y(\delta))$, the former effect dominates and so managers' decisions are strategic substitutes.[12]

Regardless of whether managers' decisions are strategic substitutes or complements, the model highlights a source of governance externalities between firms. A greater tendency for one manager to work affects the effort incentives of the manager of an unrelated firm. While the parameters affecting the severity of the agency problem (β and ω) are deliberately independent, so that the firms are related only through common ownership, the model can be extended to the case in which β and ω are firm-specific. Thus, a strengthening in corporate governance in one firm, that manifests in a reduction in private benefits or an increase in stock price concerns, will create spillovers in another firm. This echoes the literature on corporate governance externalities (e.g., Acharya and Volpin (2010), Dicks (2012), and Levit and Malenko (2014)). In those papers, the externalities arise from firms competing for CEOs or directors in the same labor market; here they arise from common ownership.

The next result characterizes the threshold θ_2^* and the probability γ_2^* that arise in equilibrium (where the subscript 2 refers to the two-firm model).

Proposition 2. *(Ex-post symmetric equilibrium): An ex-post symmetric equilibrium always exists. There exist unique θ_2^* and $\gamma_2^* \in \Gamma(\theta_2^*)$ such that, in any ex-post symmetric equilibrium, there is a unique $s_{BE}^* \in \{(0.5, 0.5), (1, 1)\}$ and the following hold:*

[12] The intuition is as follows. When θ^* is very high, balanced exit is more likely to result from shirking than a liquidity shock, so $p(s_{BE})$ is low. Thus, an increased probability of j shirking only slightly reduces the punishment to i shirking – even though i is more likely to suffer balanced rather than imbalanced exit, the former is a reasonably severe punishment since $p(s_{BE})$ is low. Similarly, when θ^* is very high and $p(s_{BE})$ is low, an increased probability of j shirking significantly increases the reward to i working – if the blockholder suffers a liquidity shock, manager i avoids balanced exit if he works and j shirks, and thus avoids the low price $p(s_{BE})$. Thus, when $\theta > F^{-1}(0.5)$, a greater probability of j shirking significantly increases the reward to i working, and only slightly reduces the punishment to i shirking. Overall, the manager's actions are strategic substitutes.

(i) If $a=(H,H)$ and $\chi=0$, the blockholder chooses $s=(0,0)$ w.p. 1.
(ii) If $a=(H,H)$ and $\chi=1$, or $a=(L,L)$, the blockholder chooses s^*_{BE} w.p. 1.
(iii) If $a=(L,H)$, the blockholder chooses s^*_{BE} w.p. γ^*_2 and $s_{IE}\in S_{IE}$ w.p. $1-\gamma^*_2$, where

$$\gamma^*_2=\max\left\{0,1+\min\left\{0,1-\sqrt{\delta}-\frac{\beta-Y(\delta)}{\omega}\frac{2}{\mathbb{E}\left[\theta_i|\theta_i<Y(\delta)\right]}\right\}\right\}. \tag{14}$$

(iv) Prices on the equilibrium path satisfy:

$$p^*(s)=\begin{cases}\overline{v} & \text{if } s\in S^T_{IE}\cup\{(0,0)\}\\ \pi\left(\theta^*_2,\gamma^*_2\right) & \text{if } s=s^*_{BE}\\ \underline{v}\left(\theta^*_2\right) & \text{if } s\in S_{IE}.\end{cases}$$

Prices off the equilibrium path satisfy: $p^*(s)=\underline{v}\left(\theta^*_2\right)$.
(v) Manager i shirks if and only if $\theta_i<\theta^*_2$, where θ^*_2 is uniquely defined by $\psi_2\left(\theta^*_2,\gamma^*_2\right)=\theta^*_2$, where

$$\psi_2(y,\gamma)=\beta-\omega\left[\begin{array}{c}\left[(1-F(y))(\gamma-\delta)+(1-\gamma)F(y)\right](\overline{v}-\pi(y,\gamma))\\ +(1-\gamma)(1-F(y))(\overline{v}-\underline{v}(y))\end{array}\right]. \tag{15}$$

Except from the expression for γ^*_2, parts (i)-(iv) of Proposition 2 follow from Lemma 1 and the discussion in the text. If both managers work and the blockholder does not suffer a liquidity shock, she retains both shares and both firms are priced at $\overline{v}$. If both managers work and she suffers a liquidity shock, or if both managers shirk, she engages in balanced exit. Regardless of whether balanced exit involves selling half a share or one share in each firm, both firms are priced at $\pi\left(\theta^*_2,\gamma^*_2\right)\in\left(\underline{v},\overline{v}\right)$. If only manager i shirks, she engages in balanced exit w.p. γ^*_2, and otherwise in imbalanced exit, in which case $p\left(s_{IE}\right)=\underline{v}\left(\theta^*_2\right)$ and $p\left(s^T_{IE}\right)=\overline{v}$ (regardless of whether $s_{IE}=(0.5,0)$, $(1,0)$, or $(1,0.5)$). Part (iv) yields the empirical implication that the price impact of blockholder exit depends on her trades in other firms that she owns. Finally, part (v) shows that, as in the single-firm benchmark, θ^*_2 is lower (governance is stronger) when β and δ are lower, and ω is higher.

The equilibrium in Proposition 2 need not be unique. In particular, it is indeterminate whether $s_{IE}=(0.5,0)$, $(1,0)$, or $(1,0.5)$, and while there is always an equilibrium where $s_{BE} = (0.5, 0.5)$, there is also another equilibrium with $s_{BE} = (1, 1)$ if the agency problem is sufficiently weak.[13] However, managers are sensitive only to the price impact of blockholder exit, and not to the number of shares sold per se. Therefore, while multiple ex-post symmetric equilibria exist, all equilibria have the same managerial threshold θ_2^* and probability of balanced exit γ_2^*. The invariance of θ_2^* and γ_2^* means that we do not need to consider these different equilibria separately when studying the model's comparative statics and the efficiency comparison with the benchmark.

We now turn to these comparative statics. We study $\gamma_2^*(\beta, \omega, \delta)$, the frequency of balanced exit. As we will explain later, this frequency has important implications for efficiency and comovement.

Corollary 1. *(i) γ_2^* decreases with β and increases with ω. (ii) There exists $\underline{\delta} > 0$ such that if $\delta < \underline{\delta}$ then $\gamma_2^* = 0$.*

Corollary 1 follows directly from (14) and its intuition is as follows. Recall from Lemma 2 that $\gamma_2^* = 1$ (0) if $\theta_2^* < (>)\ Y(\delta)$. From Proposition 2, the threshold θ_2^* is increasing in β and decreasing in ω, but $Y(\delta)$ in Lemma 2 is independent of these parameters. Thus, a rise in β and a fall in ω make it more likely that $\theta_2^* > Y(\delta)$, and so γ_2^* rises. As per the discussion below Lemma 2, when shirking is more likely, selling is more revealing of shirking and leads to a lower price, reducing the attractiveness of balanced exit. The effect of δ on γ_2^* is more complicated since both θ_2^* and $Y(\delta)$ increase with δ. The blockholder's incentive to engage in balanced exit, if manager i has shirked, arises from the possibility of disguising the sale of firm i as stemming from a liquidity shock. This disguise is more plausible if the frequency of liquidity shocks δ is higher, and so $Y(\delta)$ increases in δ. However, θ_2^* also increases in δ: more frequent shocks encourage shirking. If δ is sufficiently low ($\delta < \underline{\delta}$), the former effect dominates and the blockholder chooses imbalanced exit if $a = (L, H)$, i.e. $\gamma_2^* = 0$.

3.2 Efficiency and Comovement

Having analyzed the equilibria in both the benchmark and the two-firm models, we now compare their efficiency. Theorem 1 gives a condition under which the multiple-

[13] The required condition is $\beta - 0.5\omega\left(1 - \sqrt{\delta}\right)\mathbb{E}\left[\theta_i | \theta_i < Y(\delta)\right] \leq Y(\delta)$; the full analysis is available on request.

firm model is more efficient than the single-firm benchmark ($\theta_2^* < \theta_1^*$), and so holding stakes in multiple firms strengthens governance. It is the first core result of the paper.

Theorem 1. *(Efficiency comparison): A two-firm equilibrium is strictly* more efficient *($\theta_2^* < \theta_1^*$) than the single-firm benchmark if and only if $\gamma_2^*(\beta, \omega, \delta) < \sqrt{\delta}$.*

Theorem 1 states that multi-firm governance is more efficient if and only if balanced exit is sufficiently infrequent. If $\gamma_2^*(\beta, \omega, \delta) > \sqrt{\delta}$, the blockholder engages in balanced exit often. Based on Corollary 1, $\gamma_2^*(\beta, \omega, \delta)$ is high when the agency problem is mild (low β and high ω). Balanced exit leads to weak governance, because even if manager i works and there is no liquidity shock, he will be sold if manager j shirks. Thus, his incentives to work are lower relative to the single-firm benchmark. As a result, $\theta_1^* < \theta_2^*$, i.e. the single-firm benchmark is more efficient.

In contrast, if $\gamma_2^*(\beta, \omega, \delta) < \sqrt{\delta}$, then imbalanced exit is common, which leads to stronger governance for two reasons, both of which arise from the blockholder's ability to choose which firm to sell if she suffers a liquidity shock. First, the reward for working is higher in the two-firm equilibrium. If the blockholder suffers a liquidity shock in the benchmark, the manager is sold even if he works. If the blockholder suffers a liquidity shock in the two-firm model, a working manager is not sold if the other manager shirks, since the blockholder can sell the second firm. Second, the punishment for shirking is higher in the two-firm equilibrium. If the firm is sold in the benchmark, the price is not too low since the sale is consistent with a liquidity shock. If a firm is sold more in the two-firm model, it fully reveals shirking, leading to the lowest possible price of $\underline{v}(\theta^*)$. This is because being sold more is not consistent with a liquidity need, since the blockholder can satisfy such a need by selling both firms. Overall, both forces increase the incentive to work in the two-firm equilibrium, and so $\theta_1^* > \theta_2^*$.

In sum, whether governance is stronger or weaker under common ownership depends on the frequency with which the blockholder engages in balanced exit when one manager shirks, which in turn depends on the severity of the agency problem. Thus, we are able to relate the efficiency of multi-firm governance to the underlying characteristics of the portfolio firms. When agency problems are strong (weak) to begin with, they are mitigated (exacerbated) under common ownership. The model generates the empirical prediction that blockholders are more likely to hold multiple stakes if they own firms with weak governance, but fewer stakes if they own well-governed firms. The efficiency of multi-firm governance also depends on the probability of liquidity shocks. If $\delta < \underline{\delta}$, then $\gamma_2^* = 0$: since it is harder to disguise exit as being motivated by a liquidity shock, the blockholder engages in imbalanced exit and so we have superior governance under common ownership.

Theorem 1 also implies that if the blockholder could commit to imbalanced exit – punishing a shirking manager to the greatest extent possible – then multi-firm governance is always more efficient, and so it is in her interest to make such a commitment.[14] However, the blockholder cannot commit to this strategy, and so it is only credible if she has incentives to execute it ex post. The severe punishment under imbalanced exit arises because the shirking firm receives the lowest possible price – but this also dissuades the blockholder from selling such a firm and may render the strategy not incentive-compatible. Thus, she must have incentives to choose imbalanced exit, which is the case if agency problems are strong, as then she receives a low stock price for both firms upon balanced exit.

Theorem 2 studies the correlation between the stock prices of the two firms, and represents the second main result of the paper.

Theorem 2. *(Price correlations): The equilibrium correlation between the stock price of the two firms is negative if and only if* $\gamma_2^*(\beta, \omega, \delta) < \sqrt{\delta}$.

Theorem 2 delivers the interesting result that, even though firm fundamentals are independent (θ_i and θ_j are uncorrelated), stock prices comove. Typically, comovement arises from exposure to a common factor, such as a macroeconomic variable; here, it arises because the firms share a common blockholder. Moreover, the direction of such comovement is nuanced. Intuitively, it may seem that the correlation should be positive, since a liquidity shock causes the blockholder to sell both stocks equally. Indeed, this force leads to a positive correlation as in the theory of Vayanos and Woolley (2012). However, this is not the only force in the model, and so the overall correlation can be negative. The other important force is the blockholder's decision when one manager

[14] Prior to observing (a_i, a_j) and χ, the expected value of each firm in the blockholder's portfolio equals expected shareholder value,

$$V(\theta_2^*) = \overline{v} - \Pr[\theta_i < \theta_2^*] \, \mathbb{E}[\theta_i | \theta_i < \theta_2^*].$$

Ex post, her profits from trading on information are exactly offset by her losses if she is forced to trade due to a liquidity shock. Thus, ex ante, the blockholder fully internalizes the governance implications of common ownership, and so would wish to commit to engage in imbalanced exit if efficient.

shirks. If she chooses imbalanced exit, this drives down the price of the shirking firm (to the lowest possible value of $\underline{v}$) and drives up the price of the working firm (to the highest possible value of $\overline{v}$). This "tournament" aspect tends the model towards a negative correlation. If the frequency of imbalanced exit is sufficiently strong, i.e. if $\gamma_2^*(\beta, \omega, \delta) < \sqrt{\delta}$, then this force outweighs the first effect and leads to a negative correlation overall. Interestingly, the condition for the price correlation to be negative is exactly the same as the condition for the two-firm equilibrium to be more efficient. This is intuitive, since imbalanced exit both moves stock prices in different directions and exerts strong discipline on a shirking manager.

In sum, whether the correlation is positive or negative depends on the frequency of balanced exit, which in turn depends on the severity of the agency problem. Thus, just like the effectiveness of governance, we are able to relate the direction of correlation to the underlying characteristics of the portfolio firms. When the agency problem is weak (i.e. β is low and ω is high) and the frequency of shocks δ is low, the correlation is negative. Typically, stock price correlations depend on the direction of correlation between random variables (e.g. business conditions) that affect each firm, but not fixed parameters: for example, changing the maximum fundamental value $\overline{v}$ will have no effect on correlation. Here, even though the parameters δ, β, and ω are fixed parameters rather than random variables, they affect the direction of correlation. Similarly, while stock price correlation (an asset pricing concept) typically depends on exposures to macroeconomic variables (another asset pricing concept), here we show that it depends on agency problems (a corporate finance concept), as they endogenously affect the blockholder's trading strategy. Thus, asset pricing studies of correlation should control for common ownership and agency variables.

More generally, while empirical studies of blockholder governance typically analyze the size of the largest blockholder or the number of blockholders, our theory highlights a new measure – the number of blocks owned by an investor – that affects both corporate governance and asset pricing. The direction of the effect of multi-firm ownership is nuanced, and so simply regressing (say) firm performance or comovement on this measure may not lead to clear results. The model provides precise conditions on the situations under which multi-firm ownership will strengthen or weaken governance, or lead to negative or positive correlations.

4 Additional Equilibria

In this section we extend the core model and consider equilibria that are not ex-post symmetric. Our focus on ex-post symmetric equilibria was motivated by the idea that investors typically prefer to keep their portfolio weights constant, unless the fundamentals of their portfolio companies change asymmetrically. However, if we relax this restriction, we may have equilibria in which the blockholder trades asymmetrically even when the managers' actions are the same, i.e. when both managers shirk or both work. In Appendix B, we characterize all the equilibria that emerge when ex-post symmetry is not required. Proposition 3 characterizes the most efficient equilibrium with two firms, and shows that it is generally not ex-post symmetric.[15]

Proposition 3. *(Most efficient equilibrium): There exist* $\upsilon^{**} \in (0,1]$*,* $\theta^{**} \in (0,\overline{v})$ *and* $s_{BE}^{**} \in \{(1,1),(0.5,0.5)\}$ *such that the most efficient equilibrium satisfies the following properties:*

(i) If $a=(H,H)$ *and* $\chi=0$*, the blockholder chooses* $s=(0,0)$ *w.p. 1.*

(ii) If $a=(L,L)$*, the blockholder chooses* s_{BE}^{**} *w.p. 1.*

(iii) If $a=(L,H)$*, the blockholder chooses* $s=(1,0)$ *w.p. 1.*

(iv) If $a=(H,H)$ *and* $\chi=1$*, the blockholder chooses* s_{BE}^{**} *w.p.* υ^{**}*; w.p.* $1-\upsilon^{**}$ *she chooses between* $s=(1,0)$ *and* $s=(0,1)$ *at random.*

(v) Manager i shirks if and only if $\theta_i<\theta^{**}$*, where* θ^{**} *is uniquely defined by* $\psi_{EF}(\theta^{**})=\theta^{**}$*, where*

$$\psi_{EF}(y)=\beta-\omega\frac{1+(1-\delta)(1-F(y))}{1-(1-\delta)(1-F(y))^2}F(y)\,\mathbb{E}[\theta|\theta<y]\,. \tag{16}$$

Similar to the equilibrium in Proposition 2, in Proposition 3 the blockholder retains both firms if $a=(H,H)$ and $\chi=0$, and engages in balanced exit if $a=(L,L)$. The key difference is that in Proposition 2, the blockholder always engages in balanced exit when $a=(H,H)$ and $\chi=1$. In Proposition 3, because she is not restricted to a symmetric trade, she sometimes engages in imbalanced exit in this scenario (see part (iv)). Since a firm that is sold more may still be value-maximizing, its price $p(1,0)$ is relatively high – in fact the proof of Proposition 3 shows that it equals $p(0.5,0.5)$. Since

[15]For simplicity, the analysis in the Online Appendix restricts attention to trades in half units. However, allowing for trades in all quantities does not affect the existence and characterization of the equilibria in Proposition 3.

$p(1,0)$ is high, the blockholder always engages in imbalanced exit when $a = (L, H)$. In contrast, in Proposition 2, if $\theta^* < Y(\delta)$ the blockholder engages in balanced exit, which leads to weaker governance as discussed previously.

If $\theta^* > Y(\delta)$, the blockholder always engages in imbalanced exit in Proposition 2, just as in Proposition 3, and so the above advantage does not apply. Even in this case, the equilibrium in Proposition 3 remains strictly more efficient. The intuition is intricate, and is as follows. That type $a = (H, H)$ and $\chi = 1$ sometimes engages in imbalanced exit leads to two additional differences. First, it reduces the payoff to shirking. When $\theta^* > Y(\delta)$, manager j is likely to shirk, and so if manager i shirks, he is likely to experience balanced exit. In Proposition 2, balanced exit occurs with certainty if $a = (H, H)$ and $\chi = 1$, and so $p(s_{BE})$ is relatively high. In Proposition 3, balanced exit does not occur with certainty if $a = (H, H)$ and $\chi = 1$, because the blockholder sometimes engages in imbalanced exit in this scenario. Thus, $p(s_{BE})$ is relatively low, reducing the payoff to shirking. Second, it increases the payoff to working. In Proposition 2, even if manager i works, he is sold with certainty if manager j also works and $\chi = 1$, because the blockholder engages in balanced exit in this scenario. Since balanced exit also reflects the possibility that the manager shirks, the lower share price reduces the payoff to working. In Proposition 3, with probability $1 - \upsilon^{**}$ the blockholder engages in imbalanced exit in this scenario. Thus, with probability $\frac{1-\upsilon^{**}}{2}$, the working manager is not sold, which fully reveals to the market maker that the manager has worked, and so the firm is priced at $\overline{v}$. This increases the payoff to working.[16] Overall, under the equilibrium in Proposition 3, the manager's payoff to working is higher, and his payoff to shirking is lower, leading to stronger governance.

Theorem 3 compares the most efficient two-firm equilibrium, as described in Proposition 3, with the single-firm benchmark.[17]

Theorem 3. *(Efficiency comparison of most efficient equilibrium): The most efficient two-firm equilibrium is always more efficient than the single-firm benchmark.*

Since the most efficient two-firm equilibrium always involves imbalanced exit when

[16] Note that imbalanced exit also yields a probability $\frac{1-\upsilon^{**}}{2}$ that a working manager is the only firm sold, which never occurs under Proposition 2, and so the overall effect is seemingly unclear. However, when $\theta^* > Y(\delta)$ (shirking destroys significant value), the benefit of being revealed as not having shirked, and being priced at $\overline{v}$, is particularly strong and so dominates this other consideration.

[17] Since both Propositions 2 and 3 continue to hold if trades different from half units are allowed, Theorem 3 also continues to hold in these cases.

$a = (L, H)$, it is more efficient than the single-firm benchmark. The intuition is the same for why the two-firm equilibrium in the core model is more efficient if imbalanced exit is sufficiently frequent.

We finally consider the stock price correlations.

Proposition 4. *(Price correlations in most efficient equilibrium): The correlation between the stock price of the two firms under the most efficient equilibrium is always negative.*

Unlike Theorem 2, here the correlation between the stock prices is always negative. This is intuitive, because imbalanced exit is frequent. Not only does it occur with certainty when $a = (L, H)$ (which was sufficient for the correlation to be negative in the core model) but it also occurs with probability $1 - \upsilon^{**}$ if $a = (H, H)$ and $\chi = 1$.

5 Conclusion

This paper has studied how common ownership affects the effectiveness of corporate governance and leads to stock price correlations between economically unrelated firms. Under common ownership, the blockholder's decision to sell is made at the portfolio level rather than the firm level. Whether governance is stronger under common ownership than a single-firm benchmark depends on the blockholder's trading strategy when one manager shirks and the other works. If she pursues balanced exit (sells both firms), governance is weaker because the manager's effort incentives are lower – even if he works, he will still be sold if the other manager shirks. If she pursues imbalanced exit, governance is stronger because the blockholder has a choice of which firm to sell if she suffers a liquidity shock. This has two implications. First, if a manager works, he is not necessarily sold if the blockholder requires liquidity – she may sell the other firm. Second, if a manager is the only one sold, this is a strong signal that he has shirked rather than that the blockholder has suffered a liquidity shock, since she could have sold a half-share in both firms in the latter case. For both reasons, the link between managerial effort and the blockholder's exit decision is stronger. While the model of multi-firm governance considers two firms, the advantages of being able to make exit decisions at the portfolio level likely hold with three or more firms. It remains the case that a blockholder need not sell a value-maximizing firm if she suffers a liquidity shock, as she can sell another firm in her portfolio. In addition, it remains the case that, if a blockholder sells only one firm and retains the others, this is a powerful signal of underperformance.

We show that the blockholder is more likely to pursue imbalanced exit, and thus governance under common ownership is stronger, when the agency problem is severe (private benefits are high and stock price concerns are low), or liquidity shocks are infrequent. Moreover, by encouraging the blockholder to choose imbalanced exit, severe agency problems lead to the portfolio firms' stock prices being negatively correlated, even though their fundamentals are uncorrelated. Common ownership also leads to strategic interactions between firms, thus giving rise to governance externalities.

Over and above these specific comparative statics, the paper has broader implications for governance and comovement. First, we introduce a new determinant of a blockholder's effectiveness in exerting corporate governance – the number of blocks that she owns – that gives rise to new empirical predictions. Second, allowing a blockholder to own stakes in multiple firms need not weaken governance by spreading the blockholder too thinly, as commonly argued. Third, common ownership between stocks can lead to negative correlation by introducing a "tournament" aspect in the blockholder's exit decision, rather than only the positive correlation commonly believed. Fourth, comovement between stocks is driven not only by asset pricing variables such as loadings on macroeconomic factors, but also corporate finance variables such as common ownership and the severity of agency problems. Fifth, the price impact of blockholder depends not only on how many shares she sells of the firm in question, but also her trades in otherwise unrelated firms.

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A Proofs

We start with a Lemma that justifies our focus on symmetric equilibria (see footnote 8).

Lemma 4. *If the managers follow the same strategy and the market maker follows a symmetric pricing strategy, the blockholder's best response is a symmetric strategy as well.*

Proof of Lemma 4. To show that it is weakly optimal for the blockholder to respond with a symmetric strategy, it is sufficient to show that $\forall\ \hat{a}_i \in \{L, H\}$, $\hat{a}_j \in \{L, H\}$, $s_i \in [0, 1]$, and $s_j \in [0, 1]$:

$$\mathbb{E}\left[u_B\left(s_i, s_j\right)|a = \left(\hat{a}_i, \hat{a}_j\right)\right] = \mathbb{E}\left[u_B\left(s_j, s_i\right)|a = \left(\hat{a}_j, \hat{a}_i\right)\right]. \tag{17}$$

Since both managers follow the same strategy, $\mathbb{E}\left[v\left(\theta_j, a_j\right)|a_j = \hat{a}_j\right] = \mathbb{E}\left[v\left(\theta_j, a_i\right)|a_i = \hat{a}_j\right]$ and $\mathbb{E}\left[v\left(\theta_i, a_i\right)|a_i = \hat{a}_i\right] = \mathbb{E}\left[v\left(\theta_i, a_j\right)|a_j = \hat{a}_i\right]$. Since the market maker follows a symmetric strategy, $p_i\left(s_i, s_j\right) = p_j\left(s_j, s_i\right)$. Thus, (17) holds $\forall\ s \in [0, 1]^2$. □

Proof of Proposition 1. We prove the proposition in several steps. To simplify notation we set $\underline{v} = \underline{v}\left(\theta^*\right)$. First, we argue that if $a_i = L$, then $s_i > 0$ for sure. Suppose that, on the contrary, the blockholder chooses $s = 0$ with positive probability. Since $a_i = L$ implies $v = \underline{v}$, this requires

$$2\underline{v} \geq \max_{s_i > 0} sp\left(s_i\right) + \left(2 - s_i\right)\underline{v}.$$

Since $p\left(s_i\right) \geq \underline{v}\ \forall\ s$, it follows that $p\left(s_i\right) = \underline{v}\ \forall\ s$. However, since $\delta > 0$, there is a strictly positive probability that $s > 0$ when $a_i = H$. This implies $\max_{s>0} p\left(s_i\right) > \underline{v}$, a contradiction. Second, we prove that, if $a_i = L$, then $s_i \geq 1$ for sure. From the previous step we know that $s_i > 0$. Suppose that, on the contrary, the blockholder chooses $s_i' \in (0, 1)$ with positive probability. Therefore, $p\left(s_i'\right) < \overline{v}$. We argue, if that is the case, then whenever $a_i = H$, we must have $s_i \neq s_i'$. Indeed, if $a_i = H$ and $\chi = 0$, the blockholder is strictly better off choosing $s_i = 0$. If $a_i = H$ and $\chi = 1$, the blockholder must choose $s_i \geq 1$ to meet her liquidity needs. Either way, if $a_i = H$, the blockholder never chooses s_i'. Therefore, $p\left(s_i'\right) = \underline{v}$. However, since $\delta > 0$, there is a strictly positive probability that $s_i \geq 1$ when $a_i = H$. Therefore, $\max_{s_i \geq 1} p\left(s_i\right) > \underline{v}$, and whenever $a_i = L$, the blockholder is strictly better off choosing $s_i \geq 1$, a contradiction. Third, we argue that, if $a_i = H$ and $\chi = 0$, then $s_i < 1$. Suppose that, on the contrary, the blockholder chooses $s_i \geq 1$ with positive probability. Let $\bar{s}_i$ be the highest trade that type $a_i = H$ and $\chi = 0$ chooses with positive probability. By revealed preference, since $a_i = H$ implies $v = \overline{v}$, then $p(\bar{s}_i) = \overline{v}$. Therefore, if $a_i = L$ then either the blockholder chooses $\bar{s}_i$ w.p. 0, or $p(\bar{s}_i) < \overline{v}$. Moreover, since $p(\bar{s}_i) = \overline{v}$, then if $a_i = L$ and $\chi = 0$, the blockholder never chooses $s_i < \bar{s}_i$ (she would be strictly better off by instead choosing $s_i = \bar{s}_i$). We conclude that type $a_i = L$ chooses $s_i > \bar{s}_i$ for sure. Therefore, we must have $\bar{s}_i < 2$. Suppose that, if $a_i = L$ and $\chi = 0$, the blockholder chooses $\hat{s}_i \in (\bar{s}_i, 2]$ with positive probability, and so $p(\hat{s}_i) < \overline{v}$. Since $\hat{s}_i > \bar{s}_i$, then by revealed preference,

$$\bar{s}_i p(\bar{s}_i) + (2 - \bar{s}_i)\underline{v} \leq \hat{s}_i p(\hat{s}_i) + (2 - \hat{s}_i)\underline{v} \Leftrightarrow$$
$$\bar{s}_i(\overline{v} - \underline{v}) \leq \hat{s}_i(p(\hat{s}_i) - \underline{v}),$$

since $p(\bar{s}_i) = \overline{v}$. Therefore, we must have $p(\hat{s}_i) > \underline{v}$. Since $\hat{s}_i > \bar{s}_i$, this condition can be met only if the blockholder chooses $\hat{s}_i$ with strictly positive probability when $a_i = H$ and $\chi = 1$. However, since $p(\bar{s}_i) = \overline{v} > p(\hat{s}_i)$ and (by the contradicting assumption) $\bar{s}_i \geq 1$, when $a_i = H$ and $\chi = 1$ the blockholder is strictly better off choosing $\bar{s}_i$ instead of $\hat{s}_i$. Indeed, since $\hat{s}_i > \bar{s}$ and $p(\hat{s}_i) < p(\bar{s}_i)$, then

$$\bar{s}_i p(\bar{s}_i) + (2 - \bar{s}_i)\overline{v} > \hat{s}_i p(\hat{s}_i) + (2 - \hat{s}_i)\overline{v}.$$

Therefore, we must have $p(\hat{s}_i) = \underline{v}$, a contradiction. Fourth, let $\bar{s}_i \geq 1$ be the highest trade that type $a_i = H$ and $\chi = 1$ chooses with positive probability, and note that $p(\bar{s}_i) > \underline{v}$. We argue that, when $a_i = L$, the blockholder chooses $\bar{s}_i$ with positive probability. If this were not the case, then $p(\bar{s}_i) = \overline{v}$ and so choosing $\bar{s}_i$ strictly dominates any $s_i < \bar{s}_i$. In addition, $s_i > \bar{s}_i$ implies $a_i = L$ and thus $p(s_i) = \underline{v}$. It follows that the blockholder has strict incentives to choose $s_i = \bar{s}_i$, a contradiction. We also argue that, when $a_i = L$, the blockholder chooses $s_i > \bar{s}_i$ w.p. 0. If this were not the case, by definition of $\bar{s}_i$ and given the previous steps, $s_i > \bar{s}_i$ implies $a_i = L$ and thus $p(\bar{s}_i) = \underline{v}$. It follows that the blockholder has strict incentives to choose $s_i = \bar{s}_i$, a contradiction. Fifth, we argue, that if $a_i = L$, the blockholder chooses $\bar{s}_i$ w.p. 1, where

$\bar{s}_i$ is the highest trade that type $a_i = H$ and $\chi = 1$ chooses with positive probability. Note that $\bar{s}_i \geq 1$. By revealed preference,

$$\bar{s}_i p\left(\bar{s}_i\right) + \left(2 - \bar{s}_i\right) \overline{v} \geq \hat{s}_i p\left(\hat{s}_i\right) + \left(2 - \hat{s}_i\right) \overline{v}$$

$\forall\ \hat{s}_i \in [1, \bar{s}_i)$. Therefore,

$$\bar{s}_i p\left(\bar{s}_i\right) + \left(2 - \bar{s}_i\right) \underline{v} > \hat{s}_i p\left(\hat{s}_i\right) + \left(2 - \hat{s}_i\right) \underline{v}$$

$\forall\ \hat{s}_i \in [1, \bar{s}_i)$. This means that if $a_i = L$, the blockholder is strictly better off choosing $\bar{s}_i$ over $\hat{s}_i$, as required. Sixth, we argue that if $a_i = H$ and $\chi = 1$, the blockholder chooses $\bar{s}_i$ w.p. 1. By definition, the blockholder does not choose $s_i > \bar{s}_i$ with positive probability. Based on step five, if $a_i = L$, the blockholder does not choose $s_i < \bar{s}_i$ with positive probability. Therefore, if instead the blockholder who observes $a_i = H$ chooses $\hat{s}_i \in [1, \bar{s}_i)$ with positive probability, then $p\left(\hat{s}_i\right) = \overline{v}$. However, since the blockholder chooses $\bar{s}_i$ with positive probability when $a_i = L$, then $p\left(\bar{s}_i\right) < \overline{v}$. For this reason,

$$\begin{aligned} \hat{s}_i p\left(\hat{s}_i\right) + \left(2 - \hat{s}_i\right) \overline{v} &= \overline{v} \\ &> \bar{s}_i p\left(\hat{s}_i\right) + \left(2 - \bar{s}_i\right) \overline{v}, \end{aligned}$$

which contradicts the assumption that the blockholder chooses $\bar{s}_i$ with positive probability when $a_i = H$ and $\chi = 1$. We conclude that, if an equilibrium exists, there is $\bar{s}_i \geq 1$ s.t., if $a_i = L$ or $\chi = 1$, the blockholder chooses $\bar{s}_i$ w.p. 1. Steps 1-6 above conclude parts (i) and (ii). Seventh, we argue that if $a_i = L$, then $s_i = \bar{s}_i \geq 1$ and the price is $p\left(\bar{s}_i\right)$. If $a_i = H$, then w.p. δ we have $\chi = 1$ and so $s_i = \bar{s}_i \geq 1$, in which case the price is $p\left(\bar{s}_i\right)$, and w.p. $(1 - \delta)$ we have $\chi = 0$ and so $s_i = \underline{s}_i < 1$, in which case the price is $p\left(\underline{s}_i\right) = \overline{v}$. Thus, the manager chooses $a_i = L$ if and only if

$$\overline{v} - \theta_i + \omega p\left(\bar{s}_i\right) + \beta > \overline{v} + \omega\left(\delta p\left(\bar{s}_i\right) + (1 - \delta)\overline{v}\right). \tag{18}$$

Therefore, in any equilibrium, there is θ^* such that $a_i = L$ if and only if $\theta_i \leq \theta^*$. Given parts (i) and (ii), the prices on the equilibrium path follow from Bayes' rule, and so are given by the terms in part (iii). Given these prices, (18) is equivalent to $\psi_1\left(\theta^*\right) > \theta_i$. Therefore, an equilibrium must be the fixed point of ψ_1. Since $\beta \in (0, \overline{v})$ (equation

(2)), we have $\psi_1(0) > 0$ and $\psi_1(\overline{v}) < \overline{v}$. The Intermediate Value Theorem ("IVT") implies that a fixed point of $\psi_1(\cdot)$ always exists, and lies in the interval $(0, \beta)$. Since $\psi_1(y)$ is decreasing in y, it has a unique fixed point, we denote it by θ_1^*. □

While Sections 2 and 3 allow the blockholder to trade shares in half-units, the proofs here are more general and allow the blockholder to sell any number of shares.

Proof of Lemma 1. Consider part (i). If manager i chooses $a_i = H$, his utility is $\overline{v} + \omega\mathbb{E}[p_i(s)|a_i = H]$, which is independent of θ_i. If he chooses $a_i = L$, his utility is $\overline{v} - \theta_i + \omega E[p_i(s)|a_i = L] + \beta$, which is decreasing in θ_i. Thus, he chooses $a_i = L$ if and only if

$$\beta - \omega(\mathbb{E}[p_i(s)|a_i = H] - \mathbb{E}[p_i(s)|a_i = L]) \geq \theta_i,$$

i.e. θ_i is below a threshold θ_i^*. We argue that $\theta_i^* \in (0, \overline{v})$. Suppose that, in contrast, $\theta_i^* = 0$ ($\theta_i^* = \overline{v}$). The market maker knows that $a_i = H$ ($a_i = L$) for sure, and so

$$\mathbb{E}[p_i(s)|a_i = H] = \mathbb{E}[p_i(s)|a_i = L].$$

Thus, the market maker does not learn about a_i from s. It follows that the manager has incentives to choose $a_i = L$ whenever $\beta \geq \theta_i$ and $a_i = H$ whenever $\beta < \theta_i$. From (2), $\beta > 0$ and $\beta < \bar{v}$, which yields a contradiction. We conclude if an equilibrium exists, then $\theta_i^* \in (0, \overline{v})$. Part (ii) follows from the observation that, based on the blockholder's information, the lowest valuation that firm i can have is when manager i has shirked with certainty, i.e. $\mathbb{E}[\overline{v} - \theta_i \mathbf{1}_{a_i = L}|a_i = H]$. Since θ_i and θ_j are independent, this term is given by $\underline{v}(\theta^*)$. Consider part (iii). Suppose that, contrary to the lemma, $a = (L, H)$ but $s_i < s_j$. By symmetry, the blockholder could sell s_j shares of firm i and s_i shares of firm j and obtain the same price for the sold shares, and retain more of the higher valued firm (i.e., firm j). Therefore, this strategy must be strictly preferred, contradicting $s_i < s_j$. Consider part (iv). Suppose that, contrary to the lemma, there is $s' = (s'_i, s'_j)$ such that $s'_i + s'_j \geq 1$ and the blockholder chooses s' with positive probability when $a = (H, H)$ and $\chi = 0$. We let s' be the trade with highest $s'_i + s'_j$ with these properties. From symmetry, the blockholder chooses s'^T with the same positive probability. Note that if the blockholder chooses $s = (0, 0)$, her payoff is $2\overline{v}$, the highest possible. Since $s'_i + s'_j > 0$, it must be that $p_i(s'_i, s'_j) = \overline{v}$ if $s'_i > 0$ and $p_i(s'_i, s'_j) = \overline{v}$ if $s'_j > 0$. Otherwise, the blockholder strictly prefers $s = (0, 0)$

over (s_i', s_j') and (s_j', s_i'). Next, we argue that type $a = (L, L)$ has strict incentives to choose s', contradicting the observation that $p_i(s_i', s_j') = \overline{v}$. Suppose on the contrary that type $a = (L, L)$ chooses s' w.p. 0. Therefore, there is s'' such that $s_i'' + s_j'' > 1$ and type $a = (L, L)$ chooses s'' with positive probability (at least when $\chi = 1$). Since type $a = (L, L)$ chooses s'' with positive probability, $p_i\left(s_i'', s_j''\right) < \overline{v}$ and $p_j\left(s_i'', s_j''\right) < \overline{v}$. Therefore, type $a = (H, H)$ never chooses s'' (since she is strictly better off choosing s' and obtaining a payoff of $2\overline{v}$, even if $\chi = 1$). Note that type $a = (L, L)$ weakly prefers s'' over s' if and only if

$$\begin{gathered}\left(2 - s_i'' - s_j''\right)\underline{v} + s_i'' p_i\left(s_i'', s_j''\right) + s_j'' p_j\left(s_i'', s_j''\right) \geq \left(2 - s_j' - s_i'\right)\underline{v} + \left(s_j' + s_i'\right)\overline{v} \Leftrightarrow \\ s_i''\left(p_i\left(s_i'', s_j''\right) - \underline{v}\right) + s_j''\left(p_j\left(s_i'', s_j''\right) - \underline{v}\right) \geq \left(s_j' + s_i'\right)\left(\overline{v} - \underline{v}\right).\end{gathered}$$

There are two cases to consider:

1. Suppose $s_i'' > s_j''$. Based on part (iii), if $s_i'' > s_j''$, then type $a = (H, L)$ never chooses s'' as well. This implies that s'' is chosen either by $a = (L, L)$ or $a = (L, H)$, or both. Either way, $p_i\left(s_i'', s_j''\right) = \underline{v}$. The above inequality becomes

$$s_j''\left(p_j\left(s_i'', s_j''\right) - \underline{v}\right) \geq \left(s_j' + s_i'\right)\left(\overline{v} - \underline{v}\right).$$

   Since $s_j'' \leq 1 \leq s_j' + s_i'$ and $p_j\left(s_i'', s_j''\right) < \overline{v}$, the above inequality never holds, contradicting the assumption that type $a = (L, L)$ chooses s'' with positive probability.

2. Suppose $s_i'' = s_j''$. From symmetry, $p_i\left(s_i'', s_j''\right) = p_j\left(s_i'', s_j''\right)$. Moreover, $p_i\left(s_i'', s_j''\right) > \underline{v}$ if and only if type $a = (H, L)$ chooses s'' with positive probability. Thus, type $a = (L, L)$ weakly prefers s'' over s' only if type $a = (H, L)$ chooses s'' with positive probability.

Type $a = (H, L)$ prefers s'' over s' if and only if

$$\begin{gathered}\left(1 - s_i''\right)\overline{v} + \left(1 - s_j''\right)\underline{v} + s_i'' p_i\left(s_i'', s_j''\right) + s_j'' p_j\left(s_i'', s_j''\right) \geq \left(1 - s_i'\right)\overline{v} + \left(1 - s_j'\right)\underline{v} + \left(s_j' + s_i'\right)\overline{v} \Leftrightarrow \\ 2s_i''\left(p_i\left(s_i'', s_j''\right) - \frac{\underline{v} + \overline{v}}{2}\right) \geq s_j'\left(\overline{v} - \underline{v}\right).\end{gathered}$$

Thus, we must have $p_i\left(s_i'', s_i''\right) \geq \frac{\underline{v}+\overline{v}}{2}$. Let γ be the probability that $a = (H, L)$ and

$a = (L, H)$ chooses s'' (from symmetry, type $a = (L, H)$ chooses s'' with the same positive probability as type $a = (H, L)$). Similarly, let η be the probability that $a = (L, L)$ chooses s''. By assumption, γ and η are strictly positive. According to Bayes' rule, the market maker sets

$$p_i\left(s_i'', s_j''\right) = \frac{\left[\gamma F(\theta^*)(1 - F(\theta^*)) + \eta F(\theta^*)^2\right] \underline{v} + \gamma (1 - F(\theta^*)) F(\theta^*) \overline{v}}{\gamma F(\theta^*)(1 - F(\theta^*)) + \gamma (1 - F(\theta^*)) F(\theta^*) + \eta F(\theta^*)^2},$$

and note that this term is always strictly smaller than $\frac{\underline{v}+\overline{v}}{2}$. We conclude that type $a = (H, L)$ strictly prefers s' over s'', and so chooses s'' w.p. 0. As argued above, this implies that type $a = (L, L)$ plays s'' w.p. 0, a contradiction. We conclude, if $a = (H, H)$ and $\chi = 0$, then $s_i + s_j < 1$. Consider part (v). Suppose that, contrary to the lemma, type $a = (L, L)$ chooses with positive probability s' such that $s_i' + s_j' < 1$. From symmetry, type $a = (L, L)$ also chooses s'^T with the same positive probability. Without loss of generality, suppose $s_i' \geq s_j'$. Since $s_i' + s_j' < 1$, if type $a \neq (L, L)$ chooses s', then it has to be $\chi = 0$. Note that, since type $a = (L, L)$ chooses s', then $p_i\left(s_i', s_j'\right) < \overline{v}$ and $p_j\left(s_i', s_j'\right) < \overline{v}$. Therefore, type $a = (H, H)$ strictly prefers $s = (0, 0)$ over s', and does not choose s' with positive probability. If $p_i\left(s_i', s_j'\right) > \underline{v}$, type $a = (H, L)$ must choose s' with positive probability. However, based on part (iii), this requires $s_i' = s_j'$, and so $p_i\left(s_i', s_j'\right) = p_j\left(s_i', s_j'\right)$. Note that type $a = (H, L)$ can obtain a payoff of $\underline{v} + \overline{v}$ by choosing $s = (0, 0)$. The payoff of type $a = (H, L)$ from choosing s' exceeds $\underline{v} + \overline{v}$ if and only if

$$(1 - s_i')\overline{v} + \left(1 - s_j'\right)\underline{v} + s_i' p_i\left(s_i', s_j'\right) + s_j' p_j\left(s_i', s_j'\right) \geq \underline{v} + \overline{v} \Leftrightarrow p_i\left(s_i', s_j'\right) \geq \frac{\overline{v} + \underline{v}}{2}.$$

However, similar to the argument in the proof of part (iv), we have $p_i\left(s_i', s_j'\right) < \frac{\overline{v}+\underline{v}}{2}$. Therefore, type $a = (H, L)$ must choose s' w.p. 0. This implies $p_i\left(s_i', s_j'\right) = \underline{v}$. If $s_i' = s_j'$, then from symmetry, $p_j\left(s_i', s_j'\right) = \underline{v}$ as well. We argue $p_j\left(s_i', s_j'\right) = \underline{v}$ even if $s_i' > s_j'$. Suppose that, on the contrary, $p_j\left(s_i', s_j'\right) > \underline{v}$ and $s_i' > s_j'$. Then, type $a = (L, H)$ must choose s' with positive probability. Type $a = (L, H)$ can secure a payoff of $\underline{v} + \overline{v}$ by choosing $s = (0, 0)$. The payoff of type $a = (L, H)$ from choosing s' exceeds $\underline{v} + \overline{v}$ if and only if

$$(1 - s_i')\underline{v} + \left(1 - s_j'\right)\overline{v} + s_i' p_i\left(s_i', s_j'\right) + s_j' p_j\left(s_i', s_j'\right) \geq \underline{v} + \overline{v} \Leftrightarrow p_j\left(s_i', s_j'\right) \geq \overline{v}.$$

However, since $p_j\left(s_i', s_j'\right) < \overline{v}$, we have a contradiction. Therefore, $p_j\left(s_i', s_j'\right) = \underline{v}$ even if $s_i' > s_j'$. We conclude that, since $p_i\left(s_i', s_j'\right) = p_j\left(s_i', s_j'\right) = \underline{v}$, by choosing s' type $a = (L, L)$ obtains the lowest possible payoff of $2\underline{v}$. However, since $\delta > 0$, if $a = (H, H)$, the blockholder is forced to choose some s'' such that $s_i'' + s_j'' \geq 1$ with positive probability. Therefore, there exists s'' such that $s_i'' + s_j'' \geq 1$ and $p\left(s''\right) > \underline{v}$. Thus, if type $a = (L, L)$ chooses s'', her payoff strictly exceeds $2\underline{v}$. This contradicts the assumption that, with positive probability, type $a = (L, L)$ chooses s' such that $s_i' + s_j' < 1$. Finally, we consider part (vi). The extension of part (vi) to the case in which the blockholder can sell any number of shares is the statement that there exist no $0 < s_1 < s_2 \leq 1$ such that both (s_1, s_1) and (s_2, s_2) are played with positive probability. We now prove this statement. Suppose that, on the contrary, there exist $0 < s_1 < s_2 \leq 1$ such that both (s_1, s_1) and (s_2, s_2) are played with positive probability. Without loss of generality, let s_1 be the lowest trade and s_2 be the highest trade with these properties. Type $a = (L, L)$ prefers (s_2, s_2) over (s_1, s_1) if and only if

$$\begin{aligned} 2\left(s_1 p\left(s_1, s_1\right) + \left(1 - s_1\right)\underline{v}\right) \leq 2\left(s_2 p\left(s_2, s_2\right) + \left(1 - s_2\right)\underline{v}\right) &\Leftrightarrow \\ \underline{v} &\leq \frac{s_2 p\left(s_2, s_2\right) - s_1 p\left(s_1, s_1\right)}{s_2 - s_1}. \end{aligned}$$

Similarly, types $a = (L, H)$ and $a = (H, L)$ prefer (s_2, s_2) over (s_1, s_1) if and only if

$$\frac{\underline{v} + \overline{v}}{2} \leq \frac{s_2 p\left(s_2, s_2\right) - s_1 p\left(s_1, s_1\right)}{s_2 - s_1},$$

and type $a = (H, H)$ prefers (s_2, s_2) over (s_1, s_1) if and only if

$$\overline{v} \leq \frac{s_2 p\left(s_2, s_2\right) - s_1 p\left(s_1, s_1\right)}{s_2 - s_1}.$$

Therefore, if both (s_1, s_1) and (s_2, s_2) are played with positive probability, we must have

$$\underline{v} \leq \frac{s_2 p\left(s_2, s_2\right) - s_1 p\left(s_1, s_1\right)}{s_2 - s_1} \leq \overline{v}.$$

There are three cases to consider:

1. Suppose
$$\underline{v} \leq \frac{s_2 p\left(s_2, s_2\right) - s_1 p\left(s_1, s_1\right)}{s_2 - s_1} < \frac{\underline{v} + \overline{v}}{2}.$$

Then, type $a = (L, L)$ is the only type who chooses (s_2, s_2) with positive probability. Therefore, $p(s_2, s_2) = \underline{v}$, and by choosing (s_2, s_2), type $a = (L, L)$ obtains a payoff of $2\underline{v}$. However, since $\delta > 0$, if $a = (H, H)$, the blockholder is forced to choose some s'' such that $s''_i + s''_j \geq 1$ with positive probability. Therefore, there is s'' such that $s''_i + s''_j \geq 1$ and $p(s'') > \underline{v}$. Therefore, if type $a = (L, L)$ chooses s'', she obtains a payoff strictly greater than $2\underline{v}$, and so has no incentives to choose $s = (s_2, s_2)$. This contradicts the assumption that (s_2, s_2) is chosen with positive probability.

2. Suppose
$$\frac{\underline{v} + \overline{v}}{2} < \frac{s_2 p(s_2, s_2) - s_1 p(s_1, s_1)}{s_2 - s_1} \leq \overline{v}.$$
Then, (s_1, s_1) is chosen with positive probability only if $a = (H, H)$. For this reason, $p(s_1, s_1) = \overline{v}$. Let γ be the probability that types $a = (L, H)$ and $a = (H, L)$ choose (s_2, s_2), and let η be the probability that type $a = (L, L)$ chooses (s_2, s_2). According to Bayes' rule,
$$p(s_2, s_2) = \frac{\left[\gamma F(\theta^*)(1 - F(\theta^*)) + \eta F(\theta^*)^2\right] \underline{v} + \gamma (1 - F(\theta^*)) F(\theta^*) \overline{v}}{\gamma F(\theta^*)(1 - F(\theta^*)) + \gamma (1 - F(\theta^*)) F(\theta^*) + \eta F(\theta^*)^2} \leq \frac{\underline{v} + \overline{v}}{2}.$$
However, if $p(s_1, s_1) = \overline{v}$ and $p(s_2, s_2) \leq \frac{\underline{v}+\overline{v}}{2}$ then $\frac{s_2 p(s_2,s_2) - s_1 p(s_1,s_1)}{s_2 - s_1} < \frac{\underline{v}+\overline{v}}{2}$, a contradiction.

3. Suppose
$$\frac{\underline{v} + \overline{v}}{2} = \frac{s_2 p(s_2, s_2) - s_1 p(s_1, s_1)}{s_2 - s_1}.$$
Then, type $a = (H, H)$ chooses (s_2, s_2) w.p. 0, and type $a = (L, L)$ chooses (s_1, s_1) w.p. 0. Let γ_2 and γ_1 be the probabilities that types $a = (L, H)$ and $a = (H, L)$ choose (s_2, s_2) and (s_1, s_1), respectively. Also, let η be the probability that type $a = (L, L)$ chooses (s_2, s_2), and let ϕ be the probability that type $a = (H, H)$ chooses (s_1, s_1). By the contradicting assumption, $\eta + \gamma_1 > 0$ and

$\phi + \gamma_2 > 0$. According to Bayes' rule,

$$p(s_1,s_1) = \frac{\gamma_1 F(\theta^*)(1-F(\theta^*))\underline{v} + \left[\gamma_1(1-F(\theta^*))F(\theta^*) + \phi(1-F(\theta^*))^2\right]\overline{v}}{\gamma_1 F(\theta^*)(1-F(\theta^*)) + \gamma_1(1-F(\theta^*))F(\theta^*) + \phi(1-F(\theta^*))^2} \geq \frac{\underline{v}+\overline{v}}{2}$$
$$p(s_2,s_2) = \frac{\left[\gamma_2 F(\theta^*)(1-F(\theta^*)) + \eta F(\theta^*)^2\right]\underline{v} + \gamma_2(1-F(\theta^*))F(\theta^*)\overline{v}}{\gamma_2 F(\theta^*)(1-F(\theta^*)) + \gamma_2(1-F(\theta^*))F(\theta^*) + \eta F(\theta^*)^2} \leq \frac{\underline{v}+\overline{v}}{2}.$$

Note that $p(s_1,s_1) \geq \frac{\underline{v}+\overline{v}}{2}$ and $p(s_2,s_2) \leq \frac{\underline{v}+\overline{v}}{2}$ satisfy $\frac{\underline{v}+\overline{v}}{2} = \frac{s_2 p(s_2,s_2) - s_1 p(s_1,s_1)}{s_2 - s_1}$ only if $p(s_1,s_1) = p(s_2,s_2) = \frac{\underline{v}+\overline{v}}{2}$. This is feasible only if $\eta = \phi = 0$. Therefore, there exist $\left(s_i', s_j'\right)$ and $\left(s_i'', s_j''\right)$ such that type $a = (H,H)$ chooses $\left(s_i', s_j'\right)$ with positive probability, and hence weakly prefers $\left(s_i', s_j'\right)$ over (s_1,s_1), and type $a = (L,L)$ chooses $\left(s_i'', s_j''\right)$ with positive probability, and hence weakly prefers $\left(s_i'', s_j''\right)$ over (s_2,s_2). That is,

$$\begin{aligned}
&2\left(s_1 p(s_1,s_1) + (1-s_1)\overline{v}\right) \leq s_i' p_i\left(s_i', s_j'\right) + s_j' p_j\left(s_j', s_j'\right) + \left(2 - s_i' - s_j'\right)\overline{v} \Leftrightarrow \\
&-s_1\left(\overline{v} - \underline{v}\right) + \left(s_i' + s_j'\right)\overline{v} \leq s_i' p_i\left(s_i', s_j'\right) + s_j' p_j\left(s_j', s_j'\right)
\end{aligned}$$

and

$$\begin{aligned}
&2\left(s_2 p(s_2,s_2) + (1-s_2)\underline{v}\right) \leq s_i'' p_i\left(s_i'', s_j''\right) + s_j'' p_j\left(s_i'', s_j''\right) + \left(2 - s_i'' - s_j''\right)\underline{v} \Leftrightarrow \\
&s_2\left(\overline{v} - \underline{v}\right) + \left(s_i'' + s_j''\right)\underline{v} \leq s_i'' p_i\left(s_i'', s_j''\right) + s_j'' p_j\left(s_i'', s_j''\right)
\end{aligned}$$

Note that the payoff of types $a = (L,H)$ and $a = (H,L)$ from choosing (s_1,s_1) or (s_2,s_2) is $\underline{v} + \overline{v}$. Thus, (s_1,s_1) and (s_2,s_2) are played with positive probability only if both $a = (L,H)$ and $a = (H,L)$ weakly prefer (s_1,s_1) or (s_2,s_2) over $\left(s_i', s_j'\right)$ and $\left(s_i'', s_j''\right)$. This would be the case if and only if

$$\underline{v} + \overline{v} \geq s_i' p_i\left(s_i', s_j'\right) + s_j' p_j\left(s_j', s_j'\right) + \max\left\{\begin{array}{c} \left(1 - s_i'\right)\overline{v} + \left(1 - s_j'\right)\underline{v}, \\ \left(1 - s_i'\right)\underline{v} + \left(1 - s_j'\right)\overline{v} \end{array}\right\} \Leftrightarrow$$

$$\min\left\{s_i'\overline{v} + s_j'\underline{v}, s_i'\underline{v} + s_j'\overline{v}\right\} \geq s_i' p_i\left(s_i', s_j'\right) + s_j' p_j\left(s_j', s_j'\right),$$

and

$$\underline{v}+\overline{v}\geq s_i''p_i\left(s_i'',s_j''\right)+s_j''p_j\left(s_i'',s_j''\right)+\max\left\{\begin{array}{c}\left(1-s_i''\right)\overline{v}+\left(1-s_j''\right)\underline{v},\\ \left(1-s_i''\right)\underline{v}+\left(1-s_j''\right)\overline{v}\end{array}\right\}\Leftrightarrow$$

$$\min\left\{s_i''\overline{v}+s_j''\underline{v},s_i''\underline{v}+s_j''\overline{v}\right\}\geq s_i''p_i\left(s_i'',s_j''\right)+s_j''p_j\left(s_i'',s_j''\right).$$

Combined, we require

$$-s_1\left(\overline{v}-\underline{v}\right)+\left(s_i'+s_j'\right)\overline{v}\leq\min\left\{s_i'\overline{v}+s_j'\underline{v},s_i'\underline{v}+s_j'\overline{v}\right\}\Leftrightarrow\max\left\{s_i',s_j'\right\}\leq s_1,$$

and

$$s_2\left(\overline{v}-\underline{v}\right)+\left(s_i''+s_j''\right)\underline{v}\leq\min\left\{s_i''\overline{v}+s_j''\underline{v},s_i''\underline{v}+s_j''\overline{v}\right\}\Leftrightarrow s_2\leq\min\left\{s_i'',s_j''\right\}.$$

Since s_1 is the lowest trade and s_2 is the highest trade with the properties that satisfy the contradicting assumption, $\max\left\{s_i',s_j'\right\}\leq s_1<s_2\leq\min\left\{s_i'',s_j''\right\}$ implies that $s_i''\neq s_j''$ and $s_i'\neq s_j'$. In particular, it implies that type $a=(H,H)$ never chooses $\left(s_i'',s_j''\right)$ and $\left(s_i'',s_j''\right)^T$ with positive probability (since any s' that type $a=(H,H)$ prefers over (s_1,s_1) must be smaller than (s_1,s_1), while any s'' that type $a=(L,L)$ prefers over (s_2,s_2) must be greater than (s_2,s_2), and $s_1<s_2$). From symmetry, type $a=(L,L)$ chooses with positive probability both $\left(s_i'',s_j''\right)$ and $\left(s_i'',s_j''\right)^T$. Without loss of generality, suppose $s_i''>s_j''$. Based on part (iii), type $a=(H,L)$ never chooses $\left(s_i'',s_j''\right)$. Therefore, $\left(s_i'',s_j''\right)$ is chosen either by $a=(L,L)$ or $a=(L,H)$. Hence, $p_i\left(s_i'',s_j''\right)=\underline{v}$ and $p_j\left(s_i'',s_j''\right)<\overline{v}$. There are two sub-cases to consider:

(a) Suppose type $a=(L,H)$ chooses $\left(s_i'',s_j''\right)$ w.p. 0. Then, $p_j\left(s_i'',s_j''\right)=\underline{v}$. Therefore, the payoff type $a=(L,L)$ from choosing s'' is $2\underline{v}$. However, since $\delta>0$, if $a=(H,H)$, the blockholder is forced to choose some s''' such that $s_i'''+s_j'''\geq 1$ with positive probability. Therefore, there is s''' such that $s_i'''+s_j'''\geq 1$ and $p\left(s'''\right)>\underline{v}$. If type $a=(L,L)$ chooses s''', her payoff is strictly higher than $2\underline{v}$. This contradicts the assumption that type $a=(L,L)$ chooses s'' with positive probability.

(b) Suppose type $a=(L,H)$ chooses $\left(s_i'',s_j''\right)$ with positive probability. Then, she must be indifferent between $\left(s_i'', s_j''\right)$ and (s_2, s_2), since only types $a = (L, H)$ and $a = (H, L)$ choose (s_2, s_2) with positive probability. That is,

$$s_i''\underline{v} + s_j''\overline{v} = s_i''p_i\left(s_i'', s_j''\right) + s_j''p_j\left(s_i'', s_j''\right),$$

Since $p_i\left(s_i'', s_j''\right) = \underline{v}$, the indifference condition above is satisfied if and only if $p_j\left(s_i'', s_j''\right) = \overline{v}$, a contradiction. Either way, we conclude that type $a = (L, L)$ has no incentives to choose s'' with positive probability, a contradiction.

□

Proof of Lemma 3. Note that (13) can be rewritten as

$$\beta - \omega \begin{bmatrix} F(\theta^*)(1-\gamma^*)(\overline{v} - p(s_{BE})) \\ -(1-F(\theta^*))\left[(\gamma^* - \delta)p(s_{BE}) + (1-\gamma^*)\underline{v} - (1-\delta)\overline{v}\right] \end{bmatrix} > \theta_i$$

The derivative of the LHS with respect to $F(\theta^*)$ is proportional to

$$\Delta = (\gamma^* - \delta)\overline{v} + (1 + \delta - 2\gamma^*)p(s_{BE}) - (1-\gamma^*)\underline{v}.$$

If $\gamma^* = 1$ then $\Delta = (1-\delta)(\overline{v} - p(s_{BE})) > 0$. If $\gamma^* \in (0,1)$, then from (12), $p(s_{BE}) = \frac{\underline{v}+\overline{v}}{2}$ and $\Delta = (1-\delta)\frac{\overline{v}-\underline{v}}{2} > 0$. If $\gamma^* = 0$ then $\Delta > 0 \Leftrightarrow p(s_{BE}) > \frac{\underline{v}+\delta\overline{v}}{1+\delta}$. Based on (9), this condition holds if and only if $F(\theta^*) \leq \frac{1}{2}$. Finally, based on Lemma 2, if $\gamma^* = 0$ then $F(\theta^*) \geq \frac{\sqrt{\delta}}{1+\sqrt{\delta}}$, and if $\gamma^* > 0$ then $F(\theta^*) \leq \frac{\sqrt{\delta}}{1+\sqrt{\delta}}$. Since $\frac{\sqrt{\delta}}{1+\sqrt{\delta}} < \frac{1}{2}$, we conclude that $\Delta < 0 \Leftrightarrow F(\theta^*) > \frac{1}{2}$. □

Proof of Proposition 2. Suppose an ex-post symmetric equilibrium exists. Part (i) follows from Lemma 1, part (iv), and ex-post symmetry. Part (ii) follows from Lemma 1, parts (v) and (vi), and ex-post symmetry. Part (iii), except for the explicit term of γ_2^* (which we will prove later), follows from Lemma 1, part (iii), and the discussion in the main text. Part (iv) follows from Lemma 1, parts (i), (ii) and (iii), and the discussion in the main text. We now consider part (v). Given γ^*, manager i chooses $a_i = L$ if and only if (13) holds. Using the explicit expressions for $p(s_{BE})$, (13) becomes $\psi_2(\theta^*, \gamma^*) > \theta_i$, where $\psi_2(y, \gamma)$ is given by (15). If θ_2^* is the threshold in equilibrium, then it must satisfy $\psi_2(\theta_2^*, \gamma_2^*) = \theta_2^*$. Overall, parts (i)-(v) of the proposition are all necessary in any ex-post symmetric equilibrium. We now argue that $\psi_2(y,\gamma) = y$ has a unique solution. Recall that it is necessary that $\gamma^* \in \Gamma(\theta^*)$. Therefore, $\psi_2(y,\gamma^*)$ can be rewritten as

$$\psi_2(y,\gamma^*) = \beta - \omega\mathbb{E}[\theta_i|\theta_i < y] \times \begin{cases} h_1(y) \equiv \frac{(1-\delta)(1-F(y))}{1+\frac{1-F(y)}{F(y)}(F(y)(1-\delta)+\delta)} & \text{if } y < Y(\delta) \\ 1 - \frac{\gamma^*+\sqrt{\delta}}{2} & \text{if } y = Y(\delta) \\ h_0(y) \equiv 1 - \frac{\delta F(y)(1-F(y))}{F(y)^2+(1-F(y))^2\delta} & \text{if } y > Y(\delta), \end{cases} \tag{19}$$

and note that $\delta \in (0,1)$ implies $Y(\delta) \in (0,\overline{v})$. We proceed in several steps. First, $\psi_2(y,\gamma) \leq \beta\ \forall\ \gamma \in [0,1]$ and $y \leq \beta$. From (2), the solution of $\psi_2(\theta^*,\gamma^*) = \theta^*$, if it exists, is in $(0,\overline{v})$. Second, it can be verified that $y < Y(\delta) \Rightarrow \frac{\partial h_1(y)}{\partial y} > 0$ and $y > Y(\delta) \Rightarrow \frac{\partial h_0(y)}{\partial y} > 0$. Moreover, $h_1(y) < h_0(y)\ \forall\ y$. Combining these results, $\psi_2(y,\gamma^*)$ is decreasing in y. Therefore, the solution of $\psi_2(\theta^*,\gamma^*) = \theta^*$, if it exists, is unique. Third, if $\psi_2(Y(\delta),1) \leq Y(\delta)$, then the unique solution satisfies $\gamma^* = 1$. If $Y(\delta) \leq \psi_2(Y(\delta),0)$ then the unique solution satisfies $\gamma^* = 0$. If $\psi_2(Y(\delta),0) < Y(\delta) < \psi_2(Y(\delta),1)$, then $\psi_2(Y(\delta),\gamma)$ is strictly increasing and continuous in γ, and $\psi_2(Y(\delta),\gamma^*) = Y(\delta)$ has a unique solution $\gamma^* = \gamma^{**}$ where

$$\gamma^{**} = 2 - \sqrt{\delta} - \frac{\beta - Y(\delta)}{\omega}\frac{2}{\mathbb{E}[\theta_i|\theta_i < Y(\delta)]} \in (0,1).$$

Overall, a solution for $\psi_2(\theta^*,\gamma^*) = \theta^*$, s.t. $\gamma^* \in \Gamma(\theta^*)$, always exists and is unique. We now verify the explicit expression for γ_2^* in part (iii). Note that if $\psi_2(Y(\delta),1) \leq Y(\delta)$ then $\gamma^{**} \geq 1$, if $Y(\delta) \leq \psi_2(Y(\delta),0)$ then $\gamma^{**} \leq 0$, and if $\psi_2(Y(\delta),0) < Y(\delta) < \psi_2(Y(\delta),1)$ then $\gamma^{**} \in (0,1)$. Therefore, $\gamma^* = \max\{0,\min\{1,\gamma^{**}\}\}$, which yields expression (14). We conclude that, if an ex-post symmetric equilibrium exists, we must have $\theta^* = \theta_2^*$ and $\gamma^* = \gamma_2^*$, where θ_2^* and γ_2^* are unique and satisfy $\psi_2(\theta_2^*,\gamma_2^*) = \theta_2^*$. To show that an ex-post symmetric equilibrium with the properties in the the proposition indeed exists, let $s_{BE} = (0.5,0.5)$ and $s_{IE} = (1,0)$, and the prices be as in part (iv). If the blockholder follows this strategy, then prices indeed are as given by part (iv), and $\theta^* = \theta_2^*$ and $\gamma^* = \gamma_2^*$. Moreover, it is trivial to see from Definition 1 that this equilibrium is ex-post symmetric. To complete the existence proof, we show that the trading strategies are incentive-compatible. There are three cases to consider:

1. Suppose $a = (H,H)$ and $\chi = 0$. If $s = (0,0)$ the blockholder obtains the highest

payoff possible, $2\overline{v}$, and so has weak incentives to choose $s = (0, 0)$. Suppose $a = (L, L)$. If the blockholder chooses s_{BE}, her payoff is $p^*(s_{BE}) + \underline{v}$, where $p^*(s_{BE}) > \underline{v}$. If the blockholder chooses $s \neq s_{BE}$, her payoff is $2\underline{v}$. Therefore, the blockholder is strictly better off choosing s_{BE}.

2. Suppose $a = (H, H)$ and $\chi = 1$. If the blockholder chooses s_{BE}, her payoff is $p^*(s_{BE}) + \overline{v}$. If $s = (1, 1)$, her payoff is $2\underline{v}$, and if $s \in \{(1, 0), (1, 0.5)\}$, her payoff is $\underline{v} + \overline{v}$. Either way, she is strictly better off choosing s_{BE}.

3. Finally, suppose $a = (L, H)$. If the blockholder chooses $s \in \{s_{IE}, (0, 0)\}$, her payoff is $\underline{v} + \overline{v}$, and if she chooses s_{BE}, her payoff is $p^*(s_{BE}) + \frac{\underline{v}+\overline{v}}{2}$. Based on Part (iii) of Lemma 1, the blockholder never chooses s such that $s_i < s_j$. She is better off choosing s_{BE} if and only if $p^*(s_{BE}) \geq \frac{\underline{v}+\overline{v}}{2}$. Since $p^*(s_{BE}) \geq \frac{\underline{v}+\overline{v}}{2} \Leftrightarrow \theta_2^* > Y(\delta)$, from Lemma 2, if $p^*(s_{BE}) > \frac{\underline{v}+\overline{v}}{2}$ then $\gamma_2^* = 1$, and if $p^*(s_{BE}) < \frac{\underline{v}+\overline{v}}{2}$ then $\gamma_2^* = 0$. Therefore, the blockholder's strategy is incentive-compatible.

□

Proof of Theorem 1. We start with several observations. First, based on (7) and (15), lengthy algebra shows that, $\forall\ y$,

$$\psi_2(y, 0) < \psi_1(y) < \psi_2(y, 1). \tag{20}$$

Second, recall $y > Y(\delta) \Rightarrow \frac{\partial \psi_2(y,0)}{\partial y} < 0$, $y < Y(\delta) \Rightarrow \frac{\partial \psi_2(y,1)}{\partial y} < 0$, and $\frac{\partial \psi_1(y)}{\partial y} < 0\ \forall\ y$. Third, lengthy algebra shows that

$$\psi_1(Y(\delta)) < \psi_2(Y(\delta), \gamma) \Leftrightarrow \gamma > \sqrt{\delta}. \tag{21}$$

Fourth, θ_2^* satisfies $\psi_2(\theta_2^*, \gamma_2^*) = \theta_2^*$, where $\gamma_2^* \in \Gamma(\theta_2^*)$, and θ_1^* satisfies $\psi_1(\theta_1^*) = \theta_1^*$. We now argue that argue $\theta_2^* < \theta_1^* \Leftrightarrow \gamma_2^* < \sqrt{\delta}$. We proceed in two steps. First, suppose $\theta_2^* = \theta_1^* = y^*$. Therefore, $\psi_2(y^*, \gamma_2^*) = \psi_1(y^*)$, and based on (10) and (20), $y^* = Y(\delta)$ and $\gamma_2^* \in (0, 1)$. Therefore, $\psi_2(Y(\delta), \gamma_2^*) = \psi_1(Y(\delta))$. Based on (21), $\gamma_2^* = \sqrt{\delta}$. Second, suppose $\theta_2^* > (<)\, \theta_1^*$. Therefore,

$$\psi_2(\theta_2^*, \gamma_2^*) = \theta_2^* > (<)\, \theta_1^* = \psi_1(\theta_1^*)$$

Since $\frac{\partial \psi_1(y)}{\partial y} < 0 \; \forall \; y$, $\psi_1(\theta_1^*) \geq (\leq) \, \psi_1(\theta_2^*)$. Therefore, $\psi_2(\theta_2^*, \gamma_2^*) > (<) \, \psi_1(\theta_2^*)$. Based on (10), (20), (21), either $\gamma_2^* = 1 \, (\gamma_2^* = 0)$, or $\gamma_2^* \in (0,1)$ and $\theta_2^* = Y(\delta)$. In the latter case, $\psi_2(Y(\delta), \gamma_2^*) > (<) \, \psi_1(Y(\delta))$, and based on (21), $\sqrt{\delta} < (>) \, \gamma$. Either way, $\sqrt{\delta} < (>) \, \gamma$. □

Proof of Theorem 2. First note that, if $\gamma_2^* = 1$, then in equilibrium $s_i = s_j$ w.p. 1, and so $p_i = p_j$ w.p. 1. The correlation is equal to one. We turn to $\gamma_2^* < 1$. The correlation is positive if and only if the covariance is positive. The covariance is given by:

$$\sigma_{ij} = \mathbb{E}[p_i \times p_j] - \mathbb{E}[p_i] \times \mathbb{E}[p_j],$$

where $\mathbb{E}[p_i] = \mathbb{E}[p_j] = (1 - F(\theta_2^*)) \, \overline{v} + F(\theta_2^*) \, \underline{v}$. Based on Proposition 2,

$$\begin{aligned}
\mathbb{E}[p_i \times p_j] = & \left[F(\theta_2^*)^2 + 2\gamma_2^* F(\theta_2^*)(1 - F(\theta_2^*)) + (1 - F(\theta_2^*))^2 \delta\right] \times p(s_{BE})^2 \\
& + (1 - \delta)(1 - F(\theta_2^*))^2 \, p(0,0)^2 \\
& + 2(1 - \gamma_2^*) F(\theta_2^*)(1 - F(\theta_2^*)) \, p(s_{IE}) \, p\left(s_{IE}^T\right).
\end{aligned}$$

Recall $p(s_{IE}) = \underline{v}$ and $p\left(s_{IE}^T\right) = \overline{v}$. Thus,

$$\begin{aligned}
\sigma_{ij} = & F(\theta_2^*)^2 \times \left(p(s_{BE})^2 - \underline{v}^2\right) \\
& + 2\gamma_2^* F(\theta_2^*)(1 - F(\theta_2^*)) \times \left(p(s_{BE})^2 - \overline{v} \times \underline{v}\right) \\
& + (1 - F(\theta_2^*))^2 \delta \times \left(p(s_{BE})^2 - \overline{v}^2\right).
\end{aligned}$$

In addition, note that $p(s_{BE}) = (1 - H) \, \overline{v} + H \underline{v}$ where

$$H = \frac{F(\theta_2^*)^2 + \gamma_2^* F(\theta_2^*)(1 - F(\theta_2^*))}{F(\theta_2^*)^2 + 2\gamma_2^* F(\theta_2^*)(1 - F(\theta_2^*)) + (1 - F(\theta_2^*))^2 \delta}$$

Then σ_{ij} can be rewritten as

$$\begin{aligned}
\sigma_{ij} = & (1 - H) F(\theta_2^*)^2 \times (\overline{v} - \underline{v})(p(s_{BE}) + \underline{v}) \\
& + 2\gamma_2^* F(\theta_2^*)(1 - F(\theta_2^*)) \times \left(p(s_{BE})^2 - \overline{v} \times \underline{v}\right) \\
& - H(1 - F(\theta_2^*))^2 \delta \times (\overline{v} - \underline{v})(p(s_{BE}) + \overline{v}).
\end{aligned}$$

Consider two cases:

1. Suppose $\gamma_2^* = 0$. Then σ_{ij} can be rewritten as

$$\begin{aligned}\sigma_{ij} = &(1-H)\, F\left(\theta_2^*\right)^2 \times (\overline{v} - \underline{v})\left(p\left(s_{BE}\right) + \underline{v}\right) \\ &- H\left(1 - F\left(\theta_2^*\right)\right)^2 \delta \times (\overline{v} - \underline{v})\left(p\left(s_{BE}\right) + \overline{v}\right).\end{aligned}$$

Note that $(1-H)\, F\left(\theta_2^*\right)^2 = H\left(1 - F\left(\theta_2^*\right)\right)^2 \delta$. Therefore, $\sigma_{ij} < 0$.

2. Suppose $\gamma_2^* \in (0,1)$. Then, $\theta_2^* = Y(\delta) \Leftrightarrow F\left(\theta_2^*\right) = \frac{\sqrt{\delta}}{1+\sqrt{\delta}}$, $F\left(\theta_2^*\right)^2 = \left(1 - F\left(\theta_2^*\right)\right)^2 \delta$, $H = \frac{1}{2}$ and σ_{ij} can be rewritten as

$$\sigma_{ij} = \frac{1}{2} F\left(\theta_2^*\right)(\overline{v} - \underline{v})^2 \left[\gamma_2^*\left(1 - F\left(\theta_2^*\right)\right) - F\left(\theta_2^*\right)\right].$$

Therefore, $\sigma_{ij} < 0 \Leftrightarrow \gamma_2^* < \frac{F\left(\theta_2^*\right)}{1-F\left(\theta_2^*\right)}$. Since, $F\left(\theta_2^*\right) = \frac{\sqrt{\delta}}{1+\sqrt{\delta}}$ then $\frac{F\left(\theta_2^*\right)}{1-F\left(\theta_2^*\right)} = \sqrt{\delta}$.

□

Proof of Proposition 3. Based on the analysis in Appendix B, Propositions 2, 6, 7, and 8 describe the entire set of equilibria. First, consider the equilibria in Proposition 8. Based on (35), $\psi_{SE}(y,\tau)$ is decreasing in τ. Therefore, among all the equilibria described in Proposition 8, the one with $\tau = 1$ is the most efficient. Note that $\psi_{EF}(y) = \psi_{SE}(y,1)\ \forall\ y$, where $\psi_{EF}(y)$ is given by (16). Second, consider the equilibria in Proposition 6, when they exist. Based on (31), $\psi_{FE}(y,\gamma)$ is increasing with γ. Therefore, among all the equilibria described in Proposition 6, the one with $\gamma = 0$ is the most efficient. Note that $\psi_{EF}(y) = \psi_{FE}(y,0)\ \forall\ y$. Third, consider the equilibria in Proposition 2. Based on the proof of Proposition 2, $\psi_2(y,\gamma)$ given in (15) can be rewritten as:

$$\psi_2\left(y,\gamma^*\right) = \beta - \omega \mathbb{E}\left[\theta_i | \theta_i < y\right] \times \begin{cases} h_1(y) \equiv \frac{(1-\delta)(1-F(y))}{1+\frac{1-F(y)}{F(y)}(F(y)(1-\delta)+\delta)} & \textit{if } y < Y(\delta) \\ 1 - \frac{\gamma^*+\sqrt{\delta}}{2} & \textit{if } y = Y(\delta) \\ h_0(y) \equiv 1 - \frac{\delta F(y)(1-F(y))}{F(y)^2+(1-F(y))^2\delta} & \textit{if } y > Y(\delta). \end{cases} \quad (22)$$

Lengthy algebra shows that $\psi_2\left(y,\gamma^*\right) > \psi_{EF}(y)\ \forall\ y$. Therefore, the equilibrium in Proposition 2 is strictly less efficient than the most efficient equilibrium in Proposition 8. Fourth, consider the equilibria in Proposition 7, when they exist. Based on Proposition 7, an equilibrium must satisfy $\psi_{PE}\left(\theta_{PE}^*, \gamma_{PE}^*\right) = \theta_{PE}^*$, where $\psi_{PE}(y,\gamma)$ is given by

(33). Lengthy algebra shows that $\psi_{PE}(y,\gamma)$ is increasing in γ. Thus, the most efficient equilibrium is one where $\gamma = 0$. Lengthy algebra shows that $\psi_{PE}(y,0) > \psi_{EF}(y)$ $\forall$ y. Therefore, the equilibrium in Proposition 7, if it exists, is strictly less efficient than the most efficient equilibrium in Proposition 8. We conclude that the equilibrium in Proposition 8 with $\tau = 1$ is the most efficient equilibrium. Since $\psi_{SE}(y,1) = \psi_{FE}(y,0)$ $\forall$ y, the equilibrium in Proposition 6 with $\gamma = 0$, when it exists, exhibit the same level of efficiency. The properties of the equilibrium in Proposition 3 are shared by both the equilibrium in Proposition 8 with $\tau = 1$ and the equilibrium in Proposition 6 with $\gamma = 0$. In particular, note that $s_{BE} = (0.5, 0.5)$ and $\upsilon^{**} = \frac{F(\theta^{**})}{2-F(\theta^{**})} > 0$ for the equilibrium in Proposition 8 with $\tau = 1$, and $s_{BE} = (1,1)$ and $\upsilon^{**} = 1 - \frac{(1-F(\theta^{**}))^2\delta - F(\theta^{**})^2}{(1-F(\theta^{**}))\delta} > 0$ for the equilibrium in Proposition 6 with $\gamma = 0$. Finally, note that $\psi_{EF}(y)$ is a decreasing function, and so θ^{**} is the unique fixed point of $\psi_{EF}(y)$. □

Proof of Theorem 3. A direct comparison between (16) and (7) shows that $\psi_{EF}(y) < \psi_1(y)$ $\forall$ y. □

Proof of Proposition 4. Recall the covariance between the stock prices is given by:

$$\sigma_{ij} = \mathbb{E}[p_i \times p_j] - \mathbb{E}[p_i] \times \mathbb{E}[p_j],$$

where, under the most efficient equilibrium, $\mathbb{E}[p_i] = \mathbb{E}[p_j] = (1 - F(\theta^{**}))\overline{\upsilon} + F(\theta^{**})\underline{\upsilon}$. There are two cases to consider:

1. Consider equilibria in Proposition 8 with $\tau = 1$. In this equilibrium, $s_{BE} = (0.5, 0.5)$. Note that $\tau = 1$ implies $\upsilon^{**} = \frac{F(\theta^{**})}{2-F(\theta^{**})} < 1$. Moreover, $\tau = 1$ implies $p(0,0) = p(0,1) = \overline{\upsilon}$ and $p(1,0) = p(0.5, 0.5) \equiv p^{**}$ where

$$p^{**} = \overline{\upsilon} - \frac{F(\theta^{*})(2 - F(\theta^{**}))}{F(\theta^{**})(2 - F(\theta^{**})) + (1 - F(\theta^{**}))^2 \delta}\mathbb{E}[\theta|\theta < \theta^{**}]. \tag{23}$$

Therefore,

$$\begin{aligned}\sigma_{ij} &= F(\theta^{**})^2 p^{**2} + 2F(\theta^{**})(1-F(\theta^{**}))p^{**}\overline{v} \\ &+ (1-\delta)(1-F(\theta^{**}))^2\overline{v}^2 + \delta(1-F(\theta^{**}))^2\left[\upsilon^{**}p^{**2} + (1-\upsilon^{**})p^{**}\overline{v}\right] \\ &- \left[(1-F(\theta^{**}))\overline{v} + F(\theta^{**})\underline{v}\right]^2 \\ &= F(\theta^{**})(p^{**}-\underline{v})\left[F(\theta^{**})(p^{**}+\underline{v}) + 2(1-F(\theta^{**}))\overline{v}\right] \\ &- \delta(1-F(\theta^{**}))^2(p^{**}\upsilon^{**}+\overline{v})(\overline{v}-p^{**})\end{aligned}$$

Substituting for p^{**} and υ^{**} yields

$$\sigma_{ij} = -\frac{F(\theta^{**})^2(1-F(\theta^{**}))^2\delta}{F(\theta^{**})(2-F(\theta^{**})) + (1-F(\theta^{**}))^2\delta}\mathbb{E}\left[\theta|\theta<\theta^{**}\right]^2 < 0$$

2. Consider the equilibrium in Proposition 6 with $\gamma = 0$, when it exists. In this equilibrium, $s_{BE} = (1,1)$. Note that $\gamma = 0$ implies $\upsilon^{**} = F(\theta^{**}) + \frac{F(\theta^{**})^2}{(1-F(\theta^{**}))\delta}$. Moreover, $\gamma = 0$ implies $p(0,0) = p(0,1) = \overline{v}$, $p(1,1) = 0.5(p^{**}+\overline{v})$ and $p(1,0) = p^{**}$ where

$$p^{**} = \overline{v} - \frac{2F(\theta^{**})}{1-(1-\delta)(1-F(\theta^{**}))^2}\mathbb{E}\left[\theta_i|\theta_i<\theta^{**}\right].$$

Therefore,

$$\begin{aligned}\sigma_{ij} &= F(\theta^{**})^2 p(1,1)^2 + 2F(\theta^{**})(1-F(\theta^{**}))p(1,0)\overline{v} \\ &+ (1-\delta)(1-F(\theta^{**}))^2\overline{v}^2 + \delta(1-F(\theta^{**}))^2\left[\upsilon^{**}p(1,1)^2 + (1-\upsilon^{**})p(1,0)\overline{v}\right] \\ &- \left[(1-F(\theta^{**}))\overline{v} + F(\theta^{**})\underline{v}\right]^2.\end{aligned}$$

Substituting for $p(1,1)$, p^{**} and υ^{**} yields

$$\sigma_{ij} = -F(\theta^{**})^2(1-F(\theta^{**}))^4\frac{\delta^2 + \delta\frac{F(\theta^{**})(3-2F(\theta^{**}))}{(1-F(\theta^{**}))^2} + \frac{F(\theta^{**})^2(2-F(\theta^{**}))}{(1-F(\theta^{**}))^3}}{\left[1-(1-\delta)(1-F(\theta^{**}))^2\right]^2}\mathbb{E}\left[\theta_i|\theta_i<\theta^{**}\right]^2 < 0.$$

□

Proposition 5. *(Inefficiency of no-exit equilibrium): Consider an equilibrium in which a blockholder who observes $a = (L,H)$ plays $s = (0,0)$ with positive probability. Then,* *an equilibrium in which this type of blockholder does not play* $s = (0, 0)$ *exists and is weakly more efficient.*

Proof of Proposition 5. Here we show that if an equilibrium in which type $a = (L, H)$ and $\chi = 0$ plays $s = (0, 0)$ with positive probability (denoted ι) exists, then one in which she plays $s = (0, 0)$ w.p. 0 also exists and is more efficient. To prove this, we first argue that allowing for $\iota^* > 0$ does not expand the potential set of equilibria beyond those described in Propositions 2, 6, and 7. Given this result, we then can go case by case and show that in any potential equilibrium θ^*, the manager's relative benefit from shirking decreases as we decrease ι^* and simultaneously increase the probability that type $a = (L, H)$ and $\chi = 0$ plays $s = (1, 0)$. Finally, using IVT arguments, we can show that an equilibrium with $\iota^* = 0$ does exist when one with $\iota^* > 0$ exists, and furthermore that the cutoff rule in the former equilibrium is weakly smaller than the cutoff rule in the latter. Before proceeding, first note that in any equilibrium with $\iota^* > 0$, it must be that $p(1, 0) = \underline{v}$. Otherwise, type $a = (L, H)$ and $\chi = 0$ would strictly prefer $s = (1, 0)$ to $s = (0, 0)$. Furthermore, it must be that $p(0, 1) = \overline{v}$ if $s = (1, 0)$ is on the equilibrium path. To see this, since $p(1, 0) = \underline{v}$, a blockholder observing $a = (L, L)$ would obtain $2\underline{v}$ from choosing $s = (1, 0)$. Clearly she could deviate to an alternative strategy and obtain a higher payoff. Based on part (iii) of Lemma 1,[18] type $a = (L, H)$ never chooses $s = (0, 1)$. Therefore, $s = (0, 1)$ implies $a_i = H$, and so $p(0, 1) = \overline{v}$. Without loss of generality, for the remainder of the proof we assume that even if $s = (0, 1)$ is off-equilibrium, $p(0, 1) = \overline{v}$. We now argue that allowing $\iota^* > 0$ does not introduce additional classes of equilibria beyond those that are described by Propositions 2, 6, and 7.[19] To see this, first note that in any

[18]Note that the arguments to prove Lemma 1 did not assume that $\iota^* = 0$, so they continue to hold when we allow for $\iota^* > 0$.

[19]Note that the conditions $p(1, 0) = \underline{v}(\theta^*)$ and $p(0, 1) = \overline{v}(\theta^*)$ immediately rule out an $\iota^* > 0$ equilibrium of the same form as Proposition 8, except as a special case of Proposition 2. For such an equilibrium to exist, it would require $\mu^* = 1$ and $\tau^* = 1$. However, this then returns us to Proposition 2 with $\gamma_2^* = 0$.

equilibrium with $\iota^* > 0$, we require:

$$2p(1,1) \leq \overline{v} + \underline{v}(\theta^*) \tag{24}$$

$$p(1,0.5) + 0.5p(0.5,1) \leq \frac{1}{2}\overline{v} + \underline{v}(\theta^*) \tag{25}$$

$$p(0.5,0.5) \leq \frac{1}{2}(\overline{v} + \underline{v}(\theta^*)), \tag{26}$$

since any violation of these inequalities would mean that $s = (0,0)$ is strictly dominated for type $a = (L,H)$ and $\chi = 0$, contradicting $\iota^* > 0$. There are three cases to consider:

1. First, consider potential equilibria with $\iota^* > 0$ in which $s = (1,1)$ is on the equilibrium path. Then, it must be that (24) holds with equality. To see this, if $2p(1,1) < \overline{v} + \underline{v}(\theta^*)$, then $a = (L,H)$ and $a = (H,H)$ types strictly prefer $s = (1,0)$ to $s = (1,1)$, so the only type possibly playing $s = (1,1)$ in equilibrium is $a = (L,L)$. However, then $p(1,1) = \underline{v}(\theta^*)$, in which case $s = (1,1)$ is a sub-optimal strategy for $a = (L,L)$. Therefore, since $2p(1,1) = \overline{v} + \underline{v}(\theta^*)$, and since inequalities (25) and (26) also hold, type $a = (L,L)$ strictly prefers $s = (1,1)$ to all other strategies. Let γ^*_{FE} and ζ^* denote the probabilities that type $a = (L,H)$ or $a = (H,H)$ plays $s = (1,1)$, respectively. Then, in such an equilibrium we have:

$$p(1,1) = \overline{v} - \frac{F(\theta^*)^2 + F(\theta^*)(1-F(\theta^*))\gamma^*_{FE}}{F(\theta^*)^2 + (1-F(\theta^*))^2\delta\zeta^* + 2F(\theta^*)(1-F(\theta^*))\gamma^*_{FE}}\mathbb{E}[\theta|\theta < \theta^*].$$

Simple algebra shows that, given this price, $2p(1,1) = \overline{v} + \underline{v}(\theta^*)$ holds if and only if

$$\zeta^* = \left(\frac{F(\theta^*)}{1-F(\theta^*)}\right)^2 \frac{1}{\delta}. \tag{27}$$

This implies that, in any such equilibrium, it must be that $F(\theta^*) \leq \frac{\sqrt{\delta}}{1+\sqrt{\delta}}$, since otherwise $\zeta^* > 1$. Lemma 1 implies that in any such equilibrium, it must be that $s = (0.5, 0.5)$ is off-equilibrium. Furthermore, type $a = (H,H)$ and $\chi = 1$ must play $s = (1,0)$ w.p. 0, since otherwise $p(1,0) > \underline{v}(\theta^*)$. Therefore, the $a = (H,H)$ type must be mixing between $s = (1,0.5)$ or $s = (0.5,1)$, and $s = (1,1)$. Then, since $a = (L,H)$ is the only additional type (besides $a = (H,H)$) potentially playing $s = (1,0.5)$, it would be the case that $p(1,0.5) + 0.5p(0.5,1) > \underline{v}(\theta^*) + 0.5\overline{v}$ unless $\zeta^* = 1$. Therefore, in any equilibrium with $\iota^* > 0$ and $s = (1,1)$ on the equilibrium path, it must be that $\zeta^* = 1$, which implies that $\theta^* = Y(\delta)$ given condition (27). Furthermore, it must be that $a = (L, L)$ plays $s = (1, 1)$ w.p. 1, and $a = (L, H)$ mixes between $s = (1, 1)$, $s = (1, 0)$, $s = (1, 0.5)$, and $s = (0, 0)$. This is simply the type of equilibrium in Proposition 6 except that $a = (L, H)$ plays $s = (0, 0)$ with positive probability.

2. Next, consider potential equilibria with $\iota^* > 0$ and $s = (1, 1)$ off the equilibrium path, but $s = (1, 0.5)$ on the equilibrium path. This implies that (25) holds with equality for the same reason as before. If it were a strict inequality, then only $a = (L, L)$ would potentially have incentives to play $s = (1, 0.5)$, which would mean $p(1, 0.5) = p(0.5, 1) = \underline{v}(\theta^*)$, contradicting type $a = (L, L)$ playing this strategy. There are two cases to consider:

   (a) Consider a candidate equilibrium where $s = (0.5, 0.5)$ is also on the equilibrium path, in which case $p(0.5, 0.5) = \frac{1}{2}(\underline{v} + \overline{v}(\theta^*))$. To see this, if $p(0.5, 0.5) < \frac{1}{2}(\underline{v} + \overline{v}(\theta^*))$, then only type $a = (H, H)$ and $\chi = 1$ will have incentives to play $s = (0.5, 0.5)$. This is because type $a = (L, L)$ would receive a payoff of $p(0.5, 0.5) + \underline{v}(\theta^*)$, which is strictly smaller than $\frac{3}{2}\underline{v} + \frac{1}{2}\overline{v}(\theta^*)$, the payoff from $s = (1, 0.5)$. Furthermore, type $a = (L, H)$ would strictly prefer $s = (0, 0)$ or $s = (1, 0)$, which yield payoff $\underline{v} + \overline{v}(\theta^*)$. This implies that $p(0.5, 0.5) = \overline{v}$ in equilibrium, a contradiction. Therefore, we can rule out such an equilibrium with $p(0.5, 0.5) < \frac{1}{2}(\underline{v} + \overline{v}(\theta^*))$. However, when $p(0.5, 0.5) = \frac{1}{2}(\underline{v} + \overline{v}(\theta^*))$, type $a = (H, H)$ and $\chi = 1$ will not have incentives to play $s = (1, 0.5)$ since she can obtain $\frac{1}{2}\underline{v} + \frac{3}{2}\overline{v}(\theta^*)$ from $s = (0.5, 0.5)$. This implies that only types $a = (L, H)$ and $a = (L, L)$ would potentially play $s = (1, 0.5)$. Then, condition (25) holds with equality only if type $a = (L, L)$ plays $s = (1, 0.5)$ w.p. 0. That is, types $a = (L, L)$ and $(a = (H, H), \chi = 1)$ play $s = (0.5, 0.5)$ w.p. 1, while type $a = (L, H)$ mixes between $s = (0.5, 0.5)$, $s = (1, 0)$, $s = (1, 0.5)$, and $s = (0, 0)$. This is simply a special case of Proposition 2, with $s = (0.5, 0.5)$ on the equilibrium path (rather than $s = (1, 1)$) and with $\iota^* > 0$. Therefore, this again does not expand the potential set of equilibria. Note also that in such an equilibrium, with κ^* denoting the probability that type $a = (L, H)$ plays $s = (0.5, 0.5)$, we have:

$$p(0.5,0.5) = \overline{v} - \frac{F(\theta^*)^2 + F(\theta^*)(1-F(\theta^*))\kappa^*}{F(\theta^*)^2 + 2F(\theta^*)(1-F(\theta^*))\kappa^* + (1-F(\theta^*))^2\delta}.$$

Since $p(0.5,0.5) = \frac{1}{2}(\underline{v} + \overline{v}(\theta^*))$, this can only occur if $\theta^* = Y(\delta)$.

(b) Consider the case with $s = (0.5, 0.5)$ off the equilibrium path but $s = (1, 0.5)$ on it. Then, with $p(1,0.5) + 0.5p(0.5,1) = 0.5\overline{v} + \underline{v}(\theta^*)$, type $a = (L, L)$ strictly prefers $s = (1, 0.5)$ and $s = (0.5, 1)$ to all other strategies. Furthermore, type $a = (H, H)$ and $\chi = 1$ is indifferent between $s = (1, 0.5)$ and $s = (1, 0)$, but as noted this type must play $s = (1, 0)$ w.p. 0 in any equilibrium with $\iota^* > 0$. Type $a = (L, H)$ is indifferent between $s = (1, 0)$, $s = (0, 0)$, and $s = (1, 0.5)$, and so will mix between these. Note that this is akin to the Proposition 7 equilibrium, with $\mu^* = 1$ and $\iota^* > 0$. Also note that the form of μ^* given in Proposition 7 implies that $\mu^* = 1$ only if either $\gamma^*_{PE} = 1$ (which is inconsistent with $\iota^* > 0$), or if $\theta^* = F^{-1}\left(\frac{\sqrt{2\delta}}{1+\sqrt{2\delta}}\right) \equiv Z_{EX}(\delta)$. Therefore, an equilibrium of this form with $\iota^* > 0$ can only exist if $\theta^* = Z_{EX}(\delta)$.

3. Finally, consider potential equilibria with $\iota^* > 0$ and where $s = (0.5, 0.5)$ is played with positive probability, but $s = (1, 0.5)$ and $s = (1, 1)$ are off-equilibrium. Condition (26) implies that $p(0.5,0.5) \leq \frac{1}{2}(\overline{v} + \underline{v}(\theta^*))$. Then types $a = (L, L)$ and $(a = (H, H), \chi = 1)$ have strict incentives to play $s = (0.5, 0.5)$ (when $p(1, 0.5) = p(0.5, 1) = p(1, 1) = \underline{v}$, as they are off-equilibrium), while type $a = (L, H)$ would weakly prefer $s = (0, 0)$ or $s = (1, 0)$ to $s = (0.5, 0.5)$. Denoting γ^*_2 as the probability that $a = (L, H)$ plays $s = (0.5, 0.5)$, this implies:

$$p(0.5,0.5) = \overline{v} - \frac{F(\theta^*)^2 + F(\theta^*)(1-F(\theta^*))\gamma^*_2}{F(\theta^*)^2 + (1-F(\theta^*))^2\delta + 2F(\theta^*)(1-F(\theta^*))\gamma^*_2}\mathbb{E}[\theta|\theta < \theta^*].$$

This is consistent with $p(0.5,0.5) \leq \frac{1}{2}(\overline{v} + \underline{v}(\theta^*))$ only if $\theta^* \geq Y(\delta)$. Furthermore, if $\theta^* > Y(\delta)$, then it must be that $\gamma^*_2 = 0$. This is simply the Proposition 2 equilibrium with $\theta^* \geq Y(\delta)$ and $\iota^* > 0$.

The above analysis shows that allowing for $\iota^* > 0$ does not introduce new classes of equilibria beyond those in Propositions 2, 6, and 7. Now, we consider the manager's objective function for each type of equilibrium with $\iota^* > 0$ and show that it is increasing in ι^* as we simultaneously decrease the probability that $a = (L, H)$ and $\chi = 0$ type plays $s = (1, 0)$. Then, we can argue that an equilibrium with $\iota^* = 0$ exists and is more efficient. We consider each case in turn. In each case, we will denote θ_0^* as the equilibrium cutoff with $\iota^* = 0$, and θ_1^* as the equilibrium cutoff with $\iota^* > 0$.

1. Consider the equilibrium with $s = (1, 1)$ on the equilibrium path. As described in point 1 above, it must be that type $a = (L, L)$ as well as type $a = (H, H)$ and $\chi = 1$ plays $s = (1, 1)$ w.p. 1, while $a = (L, H)$ mix between $s = (1, 0)$, $s = (1, 0.5)$, $s = (1, 1)$, and $s = (0, 0)$. Furthermore, an equilibrium with $\iota^* > 0$ exists only if $\theta^* = Y(\delta)$. Then, it must be that:

$$p(0,1) = p(\frac{1}{2}, 1) = \overline{v}$$
$$p(1,0) = p(1, \frac{1}{2}) = \underline{v}(\theta^*)$$
$$p(0,0) = \overline{v} - \frac{F(\theta^*)(1 - F(\theta^*))\iota^*}{2F(\theta^*)(1 - F(\theta^*))\iota^* + (1 - F(\theta^*))^2} \mathbb{E}[\theta|\theta < \theta^*]$$
$$p(1,1) = \overline{v} - \frac{F(\theta^*)^2 + F(\theta^*)(1 - F(\theta^*))\gamma_{FE}^*}{F(\theta^*)^2 + 2F(\theta^*)(1 - F(\theta^*))\gamma_{FE}^* + \delta(1 - F(\theta^*))^2} \mathbb{E}[\theta|\theta < \theta^*].$$

where γ_{FE}^* is the probability that type $a = (L, H)$ plays $s = (1, 1)$. Then, the manager's condition (derived as usual) is:

$$\psi_{Ineff,DE}(Y(\delta), \iota^*, \gamma_{FE}^*) \equiv \beta + \omega\Big[F(Y(\delta))\Big(p(1,1) - \gamma_{FE}^* p(1,1) - \iota^*(1-\delta)p(0,0) - (1 - \gamma_{FE}^* - \iota^*(1-\delta))\overline{v}\Big) + (1 - F(Y(\delta)))\Big(\gamma_{FE}^* p(1,1) + \iota^*(1-\delta)p(0,0) + (1 - \gamma_{FE}^* - \iota^*(1-\delta))\underline{v} - \delta p(1,1) - (1-\delta)p(0,0)\Big)\Big] = Y(\delta),$$

where the last equality comes from the fact that in any such equilibrium, $\theta^* = Y(\delta)$. Note that $p(0, 0)$ is clearly decreasing in ι^*, while all other prices are invariant to changes in ι^*. Therefore, we have

$$\frac{\partial \psi_{Ineff,DE}(Y(\delta),\iota^*,\gamma^*_{FE})}{\partial \iota^*} = \omega\Bigg[F(Y(\delta))\Bigg((1-\delta)(\overline{v}-p(0,0)-\iota^*\frac{\partial p(0,0)}{\partial \iota^*})\Bigg)+ \\ (1-F(Y(\delta)))\Bigg((1-\delta)(p(0,0)-(1-\iota^*)\frac{\partial p(0,0)}{\partial \iota^*}-\underline{v}(\theta^*))\Bigg)\Bigg]. \tag{28}$$

Note that clearly this is positive, since $\frac{\partial p(0,0)}{\partial \iota^*} < 0$ and $\underline{v} < p(0,0) \leq \overline{v}$. Therefore,

$$Y(\delta) = \psi_{Ineff,DE}(Y(\delta),\iota^*,\gamma^*_{FE}) > \psi_{Ineff,DE}(Y(\delta),0,\gamma^*_{FE}).$$

At $\theta^* = Y(\delta)$, we have $p(0,0) = \overline{v}$ and $p(1,1) = \frac{1}{2}(\overline{v}+\underline{v})$. Lengthy algebra shows that this implies:

$$\gamma^*_{FE} < 2 - \sqrt{\delta} - \frac{\beta - Y(\delta)}{\omega}\frac{2}{\mathbb{E}[\theta|\theta < Y(\delta)]},$$

From equality (29) in Proposition 6, this implies that for this γ^*_{FE}, an equilibrium of the form in Proposition 6 exists. Furthermore, since $\psi_{Ineff,DE}(Y(\delta),0,\gamma^*_{FE}) = \psi_{FE}(Y(\delta),\gamma^*_{FE}) < Y(\delta)$ and $\psi_{FE}(0,\gamma^*_{FE}) = \beta > 0$, the IVT implies that $\theta^*_0 < Y(\delta) = \theta^*_1$. This shows that when a full exit equilibrium with $\iota^* > 0$ exists, one with $\iota^* = 0$ exists and is strictly more efficient.

2. Now, consider an equilibrium with $s = (1, 0.5)$ on the equilibrium path and with $\iota^* > 0$. As described above, there are two cases two consider:

   (a) First, suppose $s = (0.5, 0.5)$ is also on the equilibrium path. Then, the above analysis shows that such an equilibrium would consist of types $a = (L, L)$ and $(a = (H, H), \chi = 1)$ playing $s = (0.5, 0.5)$ w.p. 1, while type $a = (L, H)$ mixes between $s = (0.5, 0.5)$, $s = (1, 0)$, $s = (1, 0.5)$, and $s = (0, 0)$. $\theta^* =$

$Y(\delta)$. In this case, the manager's objective function satisfies:

$$\psi_{Ineff,PE,1}(Y(\delta),\iota^*,\gamma_2^*) \equiv \beta + \omega\Big[F(Y(\delta))\Big(p(0.5,0.5) - \gamma_2^* p(0.5,0.5) - \iota^*(1-\delta)p(0,0) - (1-\gamma_2^*-\iota^*(1-\delta))\overline{v}\Big) + (1-F(Y(\delta)))\Big(\gamma_2^* p(0.5,0.5) + \iota^*(1-\delta)p(0,0) + (1-\gamma_2^*-\iota^*(1-\delta))\underline{v} - \delta p(0.5,0.5) - (1-\delta)p(0,0)\Big)\Big] = Y(\delta),$$

with γ_2^* as the probability that type $a=(L,H)$ plays $s=(0.5,0.5)$. Note that, in this equilibrium we have $p(0.5,0.5) = \frac{1}{2}(\overline{v}+\underline{v}(\theta^*))$, making this identical to the analysis of the full exit equilibrium. Therefore, we again have:

$$Y(\delta) = \psi_{Ineff,PE,1}(Y(\delta),\iota^*,\gamma_2^*) > \psi_{Ineff,PE,1}(Y(\delta),0,\gamma_2^*).$$

Finally, at the equilibrium prices, algebra shows:

$$\psi_{Ineff,PE,1}(Y(\delta),0,\gamma_2^*) = \psi_2(Y(\delta),\gamma_2^*) < Y(\delta).$$

Therefore, using the same analysis as in the proof of Proposition 2, there exists either: (i) a $\gamma_0^* > \gamma_2^*$ such that $\psi_2(Y(\delta),\gamma_0^*) = Y(\delta)$, implying that $\theta_0^* = Y(\delta) = \theta_1^*$, or (ii) a $\theta_0^* < \theta_1^*$ such that $\psi_2(\theta_0^*,1) = \theta_0^*$. In either case, the equilibrium with $\iota^* = 0$ exists and is weakly more efficient than the equilibrium with $\iota^* > 0$.

(b) If $s=(0.5,0.5)$ is off-equilibrium, the above analysis shows that $\theta^* = Z_{EX}(\delta)$, in which case both types $a=(L,L)$ and $(a=(H,H),\chi=1)$ play $s=(1,0.5)$ or $s=(0.5,1)$ at random, and $a=(L,H)$ mixes between $s=(1,0)$, $s=(1,0.5)$, and $s=(0,0)$. Again, letting γ_{PE}^* denote the probability that type $a=(L,H)$ plays $s=(1,0.5)$, the manager's objective function must satisfy:

$$\psi_{Ineff,PE,2}(Z_{EX}(\delta),\iota^*,\gamma^*_{PE}) \equiv \beta+\omega\Big[F(Z_{EX}(\delta))\Big(\frac{1}{2}(p(1,0.5)+p(0.5,1))-\gamma^*_{PE}p(0.5,1)-$$
$$\iota^*(1-\delta)p(0,0)-(1-\gamma^*_{PE}-\iota^*(1-\delta))\overline{v}\Big)+(1-F(Z_{EX}(\delta))\Big(\gamma^*_{PE}p(1,0.5)+$$
$$\iota^*(1-\delta)p(0,0)+(1-\gamma^*_{PE}-\iota^*(1-\delta))\underline{v}-\delta\frac{1}{2}(p(1,0.5)+p(0.5,1))-(1-\delta)p(0,0)\Big)\Big]$$
$$=Z_{EX}(\delta).$$

Now again, since $p(1,0.5)$ is invariant to changes in ι^*, the derivative of this function with respect to ι^* is the same as in equation (28), and therefore it is increasing in ι^*. This yields:

$$Z_{EX}(\delta)=\psi_{Ineff,PE,2}(Z_{EX}(\delta),\iota^*,\gamma^*_{PE})>\psi_{Ineff,PE,2}(Z_{EX}(\delta),0,\gamma^*_{PE}).$$

Lengthy algebra also shows that:

$$Z_{EX}(\delta)>\psi_{Ineff,PE,2}(Z_{EX}(\delta),0,\gamma^*_{PE})=\psi_{PE}(Z_{EX}(\delta),\gamma^*_{PE}).$$

Since $\psi_{PE}(\theta,\gamma^*_{PE})$ is continuous in θ, and since $\psi_{PE}(0,\gamma^*_{PE})>0$, by the IVT there exists a $\theta^*_0<Z_{EX}(\delta)=\theta^*_1$ such that $\psi_{PE}(\theta^*_0,\gamma^*_{PE})=\theta^*_0$. That is, an equilibrium with $\iota^*=0$ exists and is strictly more efficient than the equilibrium with $\iota^*>0$.

3. Finally, consider the $\iota^*>0$ equilibrium with $s=(0.5,0.5)$ on the equilibrium path, but $s=(1,0.5)$ and $s=(1,1)$ off-equilibrium. In this case, types $a=(L,L)$ and $(a=(H,H),\chi=1)$ play $s=(0.5,0.5)$ w.p. 1, and type $a=(L,H)$ mixes between $s=(0,0)$, $s=(1,0)$, and $s=(0.5,0.5)$. Let γ^*_2 denote the probability that $a=(L,H)$ plays $s=(0.5,0.5)$. Then, the manager's objective function is:

$$\psi_{Ineff,BE,1}(\theta^*_1,\iota^*,\gamma^*_2)\equiv\beta+\omega\Big[F(\theta^*_1)\Big(p(0.5,0.5)-\gamma^*_2p(0.5,0.5)-\iota^*(1-\delta)p(0,0)-$$
$$(1-\gamma^*_2-\iota^*(1-\delta))\overline{v}\Big)+(1-F(\theta^*_1))\Big(\gamma^*_2p(0.5,0.5)+\iota^*(1-\delta)p(0,0)+$$
$$(1-\gamma^*_2-\iota^*(1-\delta))\underline{v}-\delta p(0.5,0.5)-(1-\delta)p(0,0)\Big)\Big]=\theta^*_1.$$

Note that since:

$$p(0.5, 0.5) = \overline{v} - \frac{F(\theta_1^*)^2 + F(\theta_1^*)(1 - F(\theta_1^*))\gamma_2^*}{F(\theta_1^*)^2 + 2F(\theta_1^*)(1 - F(\theta_1^*))\gamma_2^* + \delta(1 - F(\theta_1^*))^2}\mathbb{E}[\theta|\theta < \theta_1^*],$$

it is invariant to changes in ι^*, yet again the derivative of $\psi_{Ineff,BE,1}(\theta_1^*, \iota^*, \gamma_2^*)$ is the same as in (28), and so this function is increasing in ι^*. There are two cases to consider:

(a) Suppose that $\theta_1^* = Y(\delta)$. Then,

$$Y(\delta) = \psi_{Ineff,BE,1}(Y(\delta), \iota^*, \gamma_2^*) > \psi_{Ineff,BE,1}(Y(\delta), 0, \gamma_2^*).$$

Furthermore, since in such an equilibrium we have $p(0.5, 0.5) = \frac{1}{2}(\overline{v} + \underline{v}(\theta^*))$, algebra shows that:

$$Y(\delta) > \psi_{Ineff,BE,1}(Y(\delta), 0, \gamma_2^*) = \psi_2(Y(\delta), \gamma_2^*).$$

The proof of Proposition 2 shows that $\psi_2(Y(\delta), \gamma_2^*)$ is increasing in γ_2^*. Therefore, either: (i) there exists a $\gamma_0^* > \gamma_2^*$ such that $\psi_2(Y(\delta), \gamma_0^*) = Y(\delta)$, in which case $\theta_0^* = \theta_1^* = Y(\delta)$, or (ii) $\psi_2(Y(\delta), 1) < Y(\delta)$, in which case the proof of Proposition 2 shows there exists a $\theta_0^* < \theta_1^*$ such that $\psi_2(\theta_0^*, 1) = \theta_0^*$. In either case, the equilibrium with $\iota^* = 0$ exists and is weakly more efficient than the equilibrium with $\iota^* > 0$.

(b) Suppose that $\theta_1^* > Y(\delta)$, implying that $\gamma_2^* = 0$. Then, we have

$$\theta_1^* > \psi_{Ineff,BE,1}(\theta_1^*, 0, 0) = \psi_2(\theta_1^*, 0).$$

Therefore, either: (i) $\psi_2(Y(\delta), 0) \geq Y(\delta)$, which implies that by the IVT there exists a $\theta_0^* \in [Y(\delta), \theta_1^*)$ satisfying $\psi_2(\theta_0^*, 0) = \theta_0^*$, or (ii) $\psi_2(Y(\delta), 0) > Y(\delta)$ which, from the analysis of Proposition 2, implies that there is an equilibrium with $\theta_0^* \leq Y(\delta) < \theta_1^*$. In either case, the equilibrium with $\iota^* = 0$ exists and is strictly more efficient than the equilibrium with $\iota^* > 0$.

This completes the proof that when an equilibrium with $\iota^* > 0$ exists, one with $\iota^* = 0$ also exists and is more efficient. □