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Diversification and its Discontents: Idiosyncratic and Entrepreneurial Risk in the Quest for Social Status

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Abstract

Incorporating preference for social status into a simple model of portfolio choice helps to explain a range of qualitative and quantitative stylized facts about the heterogeneity in asset holdings among U.S. households. I specify preferences for status parsimoniously as a function of a household's wealth relative to aggregate wealth. In the model, investors hold concentrated portfolios, suggesting, in particular, a possible explanation for the apparently small premium for undiversified entrepreneurial risk. Consistent with empirical evidence, the wealthier households own a disproportionate share of risky assets, particularly private equity, and experience more volatile consumption growth. The model is calibrated to match the empirical level of risky asset holdings without generating excessive volatility of consumption growth and cross-sectional wealth mobility.

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1 Introduction

Diversification and risk-sharing are fundamental principles of modern finance and macroeconomics. However, empirical evidence suggests that household portfolios are poorly diversified, with many people reporting substantial holdings of a single stock.[1] For the wealthiest households large shares in closely held businesses constitute a particularly important source of risk[2]. Surprisingly, from a standpoint of portfolio theory, entrepreneurship does not appear to be well compensated, implying that many investors are willing to take poorly rewarded risks despite the availability of superior investment opportunities such as public equity that earns a large risk premium[3].

In the present paper I aim to rationalize these facts by appealing to the human desire for social status as a key driver of risk-taking behavior. Intuitively, if the benefit of "getting ahead of the Joneses" outweighs the danger of falling behind, people might be more likely to engage in risky activities with highly idiosyncratic payoffs, such as entrepreneurship, than standard consumption utility theory would predict. Friedman and Savage (1948) suggest that marginal utility of wealth rises as people move to a higher "social class," potentially generating risk-loving behavior[4]. Cole, Mailath, and Postlewaite (2001) show that relative wealth concerns create a wedge in people's attitudes towards aggregate risk and idiosyncratic risk, leading to under-diversified portfolios. Building on these insights, I incorporate preference for social status into a simple portfolio choice framework in which heterogeneous households can optimally choose their level of exposure to idiosyncratic risk. The main prediction of my model is that some investors optimally do not diversify: they hold portfolios concentrated in idiosyncratic assets that earn a positive average

[1]See Curcuru, Heaton, Lucas, and Moore (2004) for a survey of the evidence on household portfolio choice. Some of the earliest evidence of poorly diversified household portfolios was documented by Blume and Friend (1975). Most recently, Calvet, Campbell, and Sodini (2007) measure the extent of underdiversification using data on portfolio composition of Swedish households.

[2]Heaton and Lucas (2000a) emphasize the importance of entrepreneurial risk for the households that own much of the financial wealth in the economy. Entrepreneurial risk might not be fully diversified due to a trade-off between risk-sharing and incentives: e.g. Bitler, Moskowitz, and Vissing-Jørgensen (2005) find evidence of agency costs affecting entrepreneurs' holdings of business equity.

[3]Moskowitz and Vissing-Jørgensen (2002) find that returns on undiversified entrepreneurial investment are no higher than the average return on publicly traded equity despite the greater risk. They refer to this phenomenon as the "private equity premium puzzle." Hamilton (2000) reaches similar conclusions by analyzing the earnings differentials between self-employment and paid employment. Hall and Woodward (2007) calculate that risk-adjusted returns to venture capital-backed entrepreneurs (but not their investors) are small.

[4]Robson (1992) formalizes their intuition by specifying a utility function that features increasing marginal utility of status measured as percentile of the wealth distribution and studies its implications for attitudes towards risk.

return, such as private equity.

I model social status as an increasing function of individuals' wealth relative to the average wealth level, in the spirit of Duesenberry (1949)[5]. The key feature of status preferences in my model is that wealthier households care more about their social position in relation to consumption than do poorer ones. The notion that at higher levels of income people value the "social esteem" brought on by their wealth more than the consumption of goods and services that this higher wealth can buy goes back at least to Adam Smith (see Smith (1759), p. 70). Despite its intuitive appeal, this form of social status concerns has received relatively little attention in the literature[6]. This property implies that investors' marginal utility of wealth rises when they "get ahead of the Joneses" (i.e. advance their relative wealth position). Consequently, they value a marginal dollar of wealth more highly in bad states of the aggregate economy than in good states, even if their own wealth stays constant. The sensitivity of marginal utility to economy-wide shocks increases aversion to aggregate risk and leads investors to reduce their portfolios' exposure to the public equity market. Conversely, at any level of risk aversion status-conscious investors load more heavily on individual-specific (e.g. entrepreneurial) risk, compared to a non-status seeking investor.

The social status model generates striking predictions for the cross-section of households' asset holdings. Qualitatively, the richer households have a larger fraction of their wealth invested in individual-specific idiosyncratic assets, such as private equity, as well as risky assets generally. The standard deviations of individual portfolio returns as well as consumption growth rates are larger for the households in the the upper half of the distribution. The reason for this heterogeneity is that status has luxury good properties in my model. At higher wealth levels the sensitivity to the relative position, and therefore the aversion to aggregate risk, increases, while overall risk aversion declines. Quantitatively, the model is calibrated to match both the overall levels of risk-taking and the shares of household wealth concentrated in a single risky asset that are observed in the U.S. data. In particular, I match both the low shares of risky assets held by the low wealth households, and the large, highly concentrated equity shares of the very wealthy. The large idiosyncratic component of portfolio return risk is what allows the high levels of risky asset

[5]There is a growing literature documenting the importance of relative wealth or relative income concerns on self-reported well-being - e.g. see Luttmer (2005).

[6]Empirically, the intuition that the importance of status concerns rises with wealth is consistent with the evidence from subjective well-being surveys documented by McBride (2001) and Dynan and Ravina (2007).

holdings (among the richer households) to be consistent with a smooth aggregated consumption growth process.

As both a test and an application of the model, I evaluate its ability to match the empirical dynamics of household wealth. Undiversified idiosyncratic risk manifests itself in the dramatic variation of household wealth both across the population and over time. Empirically, the cross-sectional distribution of asset holdings in the U.S. is extremely concentrated, yet at the same time, there is substantial mobility across wealth percentiles over time (e.g. Hurst, Stafford, and Luoh (1998)). My model is able to account for much of the variability in wealth holdings at the top of the wealth distribution, since the richer households bear most of the idiosyncratic risk that drives wealth dispersion. In the simulated model a third of households in the top one percent of the wealth distribution are displaced over the course of ten years, consistently with the data. I conclude that the dramatic idiosyncratic risk exposure predicted by the model for the wealthiest households is empirically reasonable.

1.1 Social status, portfolio choice, and wealth mobility: related literature

Preferences featuring social externalities have already been applied to understanding the lack of diversification of household portfolios.[7] Much of this literature emphasizes "herding" and "conformism" effects of interpersonal preferences (e.g. DeMarzo, Kaniel, and Kremer (2004) and Gollier (2004)). Shore and White (2002) argue that the external habit formation model is able to explain the apparent tendency of investors to prefer assets local to their community and to avoid foreign assets (the so called "home bias puzzle"). DeMarzo, Kaniel, and Kremer (2004) show that preference for a "local good" can generate undiversified portfolios in equilibrium, with households in each community tilting their portfolios toward community-specific assets. These models, however, are not able to explain large holdings of purely idiosyncratic assets, which is

[7]Other attempts at explaining the apparent lack of diversification include models based on non-expected utility preferences, such as cumulative prospect theory (Barberis and Huang (2005)) and rank-dependent utility (Polkovnichenko (2004)), as well as on model misspecification and learning costs (Uppal and Wang (2002), Van Nieuwerburgh and Veldkamp (2005)). Huberman (2001) and Massa and Simonov (2004) provide evidence of undiversification which, they argue, is consistent with explanations based on "familiarity". Some forms of undiversification are consistent with anticipatory utility and optimism - see Brunnermeier and Parker (2005) and Puri and Robinson (2005). Overconfidence is also cited in explaining entrepreneurial behavior (e.g. Bernardo and Welch (2001)). Among the proposed rational explanations of low average payoffs to entrepreneurship are real options-based models, such as Polkovnichenko (2003) and Miao and Wang (2006). While the illiquidity of private business investments might deepen the private equity premium puzzle (e.g. see Kahl, Liu, and Longstaff (2002)), it can also provide a potentially attractive commitment mechanism for agents with time-inconsistent preferences (e.g. Laibson (1997)).

likely to be an important component of the "private equity premium puzzle," which is the main focus of this paper.

The prediction that allocation to risky assets is increasing in wealth appears consistent with more standard models that feature decreasing relative risk aversion, e.g. due to nonhomothetic utility functions. Motivated by models of this class Saks and Shore (2003) find that college students from wealthier families choose riskier careers, while Yogo (2005) examines the household consumption data from the CEX and finds that richer households have higher consumption volatility than the poorer ones. Wachter and Yogo (2007) rationalize the upward sloping portfolio shares of risky assets within a model with luxury goods consumption. The model in this paper is particularly closely related to that of Carroll (2002), who appeals to a "capitalist spirit" motive for wealth accumulation as a driver of decreasing relative risk aversion. Even with decreasing risk aversion, standard portfolio-theoretic models typically predict that household financial portfolios are well diversified, making it difficult to match both the level and the concentration of risky asset holdings. The distinguishing feature of the *relative* wealth model is that it is able to capture heterogeneity in risk taking and under-diversification simultaneously.

The analysis of social mobility from the portfolio choice perspective connects this paper to the large literature on wealth inequality. Investment gains are potentially an important source of wealth dispersion, especially among the rich households (Quadrini and Rios-Rull (1997)). Most macroeconomic models of the wealth distribution have difficulty producing empirically accurate magnitudes of wealth mobility, as well as as the concentration of wealth at the top of the wealth distribution (e.g. see discussion in Castaneda, Diaz-Gimenez, and Rios-Rull (2003)). Following Aiyagari (1994) and Huggett (1993), standard models consider uninsurable labor income risk as the main driver of cross-sectional wealth dispersion. In fact, in the data consumers appear to be insured relatively well against many exogenous idiosyncratic income shocks, such as a temporary job loss or sickness (Cochrane (1991)). At the same time, however, there is substantial cross-sectional dispersion in wealth accumulated over the life-cycle among households with similar earnings histories, even after controlling for various life-time shocks and heterogeneity in asset allocations (Venti and Wise (1998)). This unexplained heterogeneity in wealth suggests a potential role for idiosyncratic risk exposure of individual portfolios (Campbell (2006)). Quadrini (1999), Cagetti and De Nardi (2006) and Reiter (2004) emphasize the role of entrepreneurs' risk exposure and capital accumulation in driving wealth concentration and social mobility.

2 An economy with relative wealth concerns

2.1 Preferences over consumption and social status

I consider a continuum of households, indexed by $i \in \Omega \subset \mathbb{R}$, with the total mass of 1 under the associated measure μ[8]. The wealth of household i at the beginning of time period t is denoted by W_t^i, and the cumulative distribution of normalized wealth F_t is given by

$$F_t\left(x\right) = \mu\left(i : \frac{W_t^i}{\bar{W}_t} \leqslant x\right), \quad \text{where} \quad \bar{W}_t = \int_\Omega W_t^i d\mu(i). \tag{2.1}$$

Each household/investor has a finite lifetime of T periods in which consumption and portfolio decisions are made, and a terminal period in which the remaining wealth (and status conferred by it) is bequeathed to an heir born in the beginning of the period. Therefore, there are T overlapping generations, with each new generation's wealth being drawn from the distribution of bequests. Households' preferences are separable in consumption and social status, which is defined following Bakshi and Chen (1996) as household wealth scaled by the per capita wealth[9]. Specifically, at time t each investor i aged A_t^i year maximizes

$$E_t\left\{\sum_{s=t}^{\tau} \delta^{s-t}\left[\frac{\left(C_s^i\right)^{1-\gamma}}{1-\gamma} + \eta \bar{W}_s^{1-\gamma}\left(\frac{W_s^i}{\bar{W}_s}\right)\right] + \delta^{\tau+1}\psi B\left(W_{\tau+1}^i, \bar{W}_{\tau+1}\right)\right\}, \tag{2.2}$$

where $\tau = t + (T - A_t^i)$. The first term in the period utility is the standard power utility over consumption C; the second term is the utility derived from social status[10]. The parameter that controls the relative importance of consumption and status is $\eta > 0$. It is multiplied by an average wealth term, $\bar{W}_t^{1-\gamma}$, in order to ensure that the relative importance of status and consumption in individual utility is invariant to changes in aggregate wealth over time. The parameter that

[8] Therefore aggregate wealth equals per capita wealth. In a discrete approximation per capita wealth is defined as $\bar{W}_t = \frac{1}{N}\Sigma_{i=1}^N W_t^i$ for some large N.

[9] In much of the economic literature on status it is often modeled more generally as a household's position (percentile rank) in the cross-sectional distribution of wealth (e.g. Cole, Mailath, and Postlewaite (1992), Robson (1992)). I choose the simpler specification for convenience and parsimony. The use of a separable utility specification for consumption and status is also primarily motivated by its simplicity, and is in line with much of the literature on social status (although in contrast to Bakshi and Chen (1996))

[10] In the case $\gamma = 1$ I assume that the consumption utility is logarithmic.

controls the importance of bequest is $\psi \geq 0$. The bequest utility is specified as a function over terminal wealth B that has the same functional form as the period utility over consumption, including both the absolute and the relative components:

$$B\left(W_{\tau+1}^{i}, \bar{W}_{\tau+1}\right)=\frac{\left(W_{\tau+1}^{i}\right)^{1-\gamma}}{1-\gamma}+\eta \bar{W}_{\tau+1}^{1-\gamma}\left(\frac{W_{\tau+1}^{i}}{\bar{W}_{\tau+1}}\right) \tag{2.3}$$

This specification implies that the bequest motive is of a "warm glow" rather than dynastic nature. The person leaving a bequest cares about its absolute and relative size and not directly about his heirs' utility. This interpretation is consistent with the notion of status-driven wealth accumulation, as it can rationalize bequests to charities and large estates left by people with no heirs (see discussion in Carroll (2000)). At the same time, such a specification of bequest utility could be given an altruistic interpretation in a model where relative wealth concerns are endogenous and the utility of future generations depends directly on their relative wealth position, as in Cole, Mailath, and Postlewaite (1992).

2.2 Technology and market structure: aggregate vs. idiosyncratic risk

I model aggregate and idiosyncratic risk exposures via different assets available to investors, following Heaton and Lucas (2004) who consider entrepreneurs' portfolio choice and capital structure decisions jointly. This is in contrast to much of the existing literature. Portfolio choice models with agent-specific idiosyncratic risk commonly assume that its "amount" is exogenously fixed, usually in the form of a stream of labor income. Conversely, models of entrepreneurial choice (e.g. Cagetti and DeNardi (2000)) usually abstract from the composition of financial portfolios.

A wide variety of investment opportunities provide a choice between aggregate and idiosyncratic risk, which poses a modeling challenge. I limit the set of assets available to the households for the sake of tractability. In the model, every household can invest in three linear technologies with returns given by vector $\mathbf{R}^i = [R^f, R^a, R^i]$ distributed according to probability density φ. These investment opportunities are:

- riskless storage technology with return R^f
- common risky technology ("public equity") with return R^a
- idiosyncratic risky technology ("private equity") with return R^i, which is individual-specific.

The specification of the investment opportunities considered here captures the idea that investors might be able to choose the combination of aggregate and idiosyncratic risk optimally. This type of investment decision is meant to encompass human capital (career choice) as well as entrepreneurial investment. In particular, I allow the return on the individual-specific investment to contain an idiosyncratic component that earns a non-zero average return:

$$R^i - R^f = \alpha^i + \beta^i(R^a - R^f) + \epsilon^i,$$

where $E[\epsilon_{R_i}|R^a] = 0$. With some abuse of terminology, I label this technology "private equity". Market incompleteness (i.e., agents cannot invest in each others' private asset) is important in that it allows idiosyncratic risk to be compensated by positive expected returns ($\alpha^i > 0$) without creating arbitrage opportunities.

2.3 Getting ahead of the Joneses and optimal (un)diversification

Consider a one-period version of the model, in which investors maximize expected utility over end-of period wealth and status $E_t\left[U\left(W^i, \bar{W}\right)\right]$, where

$$U\left(W^i, \bar{W}\right) = \frac{\left(W^i\right)^{1-\gamma}}{1-\gamma} + \eta \bar{W}^{1-\gamma}\left(\frac{W^i}{\bar{W}}\right) \tag{2.4}$$

Under the specification (2.4) above marginal utility of wealth is positive, while the first derivative with respect to aggregate reference wealth is negative:

$$U_W = W^{-\gamma} + \eta \bar{W}^{-\gamma} > 0 \text{ and } U_{\bar{W}} = -\gamma\eta\bar{W}^{-\gamma}\left(\frac{W^i}{\bar{W}}\right) < 0 \text{ since } \gamma, \eta > 0.$$

This is intuitive since, holding individual wealth fixed, an increase in per capita wealth reduces the individual's relative status and utility (such effects are often referred to in the literature as exhibiting "jealousy" - e.g. Dupor and Liu (2003)). The individual's risk preferences are controlled by the partial derivatives of the marginal utility of wealth with respect to the state variables: own wealth and per capita wealth in the economy. The former, denoted by U_{WW}, represents aversion to all wealth gambles. The latter, $U_{W\bar{W}}$ captures the attitude towards gambles that are correlated with aggregate wealth. When $U_{WW} < 0$ the consumer is risk averse and when $U_{WW} > 0$

risk seeking. Similarly, when $U_{W\bar{W}} < 0$ the consumer dislikes aggregate risk (in addition to its contribution to overall wealth risk), and conversely when $U_{W\bar{W}} > 0$ the consumer seeks additional exposure to aggregate risk, relative to a no-status benchmark.

The property that marginal value of wealth is decreasing in aggregate wealth ($U_{W\bar{W}} < 0$) that can be termed "getting ahead of the Joneses" captures the idea that an increase in aggregate wealth, holding individual wealth fixed, *lowers* marginal utility of wealth. This is in contrast to a "keeping up with the Joneses" feature ($U_{W\bar{W}} > 0$) that raises marginal utility when the aggregate reference level is high[11]. The intuition for "getting ahead of the Joneses" is that higher relative status *raises* marginal utility of wealth (Friedman and Savage (1948), Becker, Murphy, and Werning (2005)). Since status is an increasing function of the ratio of own wealth to reference wealth, as defined in (2.1), a decrease in aggregate wealth raises some people's status, making them better off, but also raising their marginal utility of wealth. The latter effect causes them to avoid assets that pay off poorly in such states.

The following example captures the main qualitative feature of the relative status preferences: the different attitudes towards idiosyncratic and aggregate risk and, in particular, the "getting ahead of the Joneses" property. Consider the case with logarithmic utility of consumption ($\gamma = 1$). Agent i's optimization problem is

$$\max E\left[\log\left(W^i\right) + \eta\frac{W^i}{\bar{W}}\right].$$

Then standard first-order conditions yield an Euler equation for asset returns:

$$E\left[\left(R^a - R^i\right)\left(\frac{1}{W^i} + \frac{\eta}{\bar{W}}\right)\right] = 0.$$

In order to simplify exposition I assume in this example that i) the individual-specific asset return is independent of the public stock market return, so that aggregation over households diversifies away all idiosyncratic risk $cov\left(R^i, \bar{W}\right) = 0$, ii) expected returns on both assets are the same, i.e.

[11] The taxonomy of Dupor and Liu (2003), which is defined explicitly with respect to consumption rather than wealth externalities, is applicable here, since theirs is a one-period model where consumption and wealth are essentially the same. In much of the literature that features keeping up with the Joneses, the externality is defined over consumption (e.g. Abel (1990), Gali (1994), Ljungqvist and Uhlig (2000)).

$E\left[R^a - R^i\right] = 0$. Then we have

$$\eta cov\left(R^a, \frac{1}{\bar{W}}\right) + cov\left(R^a, \frac{1}{W^i}\right) - cov\left(R^i, \frac{1}{W^i}\right) = 0.$$

If the common asset R^a is in positive net supply, it is positively correlated with aggregate wealth (in fact, $\bar{W}$ is a linear function of R^a). Thus $cov\left(R^a, \frac{1}{\bar{W}}\right)$ is negative and, therefore,

$$cov\left(R^a, \frac{1}{W^i}\right) > cov\left(R^i, \frac{1}{W^i}\right), \tag{2.5}$$

implying that

$$cov\left(R^a, W^i\right) < cov\left(R^i, W^i\right).$$

This means that a "status-conscious" investor is optimally exposed to more idiosyncratic risk and less exposed to aggregate risk than a neoclassical investor. In particular, if R^a and R^i are identically distributed, standard preferences ($\eta = 0$) imply that $cov\left(R^a, W^i\right) = cov\left(R^i, W^i\right)$ and therefore the weights on two assets are equal. Under status preferences that is no longer the case: since the optimally chosen individual wealth process covaries less with the aggregate return than with the idiosyncratic return, this implies that the weight on the former asset in the household's portfolio is lower than on the latter. As follows from (2.5), the magnitude of the difference in portfolio shares depends on the strength of preference for social status, controlled by parameter η, as well as the covariance between aggregate risk and per capita wealth.

3 Quantitative analysis

In this section I define the individual households' decision problem as well as the equilibrium concept associated with the dynamic version of the model, and describe the computational strategy employed in solving for an approximate equilibrium numerically.

3.1 Dynamic optimization

Each household i aged A_t^i at time t solves the following recursive problem:

$$V(W_t^i, \bar{W}_t, A_t^i; I_t) = \max_{C,\mathbf{a}} \left\{ \frac{\left(C_t^i\right)^{1-\gamma}}{1-\gamma} + \eta \bar{W}_t^{1-\gamma} \frac{W_t^i}{\bar{W}_t} + \delta E\left[V(W_{t+1}^i, \bar{W}_{t+1}, A_{t+1}^i; I_{t+1}) \middle| I_t \right] \right\}, \quad (3.1)$$

subject to the resource constraint

$$W_{t+1}^i = \left(W_t^i - C_t^i\right) \mathbf{a}_t^{i\prime} \mathbf{R}_{t+1}^i,$$

where the vector of portfolio allocations to the three assets is given by $\mathbf{a}_t^i = \left[1 - a_t^i - \tilde{a}_t^i, a_t^i, \tilde{a}_t^i\right]$. The agents cannot influence their current-period status, which is determined by their beginning-of-period wealth endowment. Consequently, standard dynamic programming arguments can be applied to analyzing the problem quantitatively.

It is convenient to restate the problem in a way that exploits scale-independence. Let

$$\tilde{c}_t^i = \frac{C_t^i}{W_t^i}, s_t^i = \frac{W_t^i}{\bar{W}_t}, G_{t+1} = \frac{\bar{W}_{t+1}}{\bar{W}_t}. \quad (3.2)$$

Then the value function (3.1) above can be written as

$$V\left(W_t^i, \bar{W}_t, A_t^i; I_t\right) = \left[v(s_t^i, A_t^i; I_t) + \eta s_t^i\right] \bar{W}_t^{1-\gamma}, \quad (3.3)$$

where the scale-invariant function $v(s_t^i, A_t^i; I_t)$ solves the corresponding recursive problem (see appendix A).

3.2 Equilibrium

Since aggregate (per capita) wealth is a state variable that enters the objective function of households, optimal consumption and investment policies that are solutions to the dynamic programming problem (3.1) generally depend on the wealth distribution F and its evolution over time via the implied law of motion for aggregate wealth growth G. Specifically, we can write

$$\begin{aligned} G_{t+1} = \frac{\bar{W}_{t+1}}{\bar{W}_t} &= E\left[s_t^i\left(1-c_t^i\right)\left(1-a_t^i-\tilde{a}_t^i\right)|I_t\right]R^f \\ &+ E\left[s_t^i\left(1-c_t^i\right)\tilde{a}_t^i\left(\alpha^i+\epsilon_{t+1}^i\right)|I_t\right] \\ &+ E\left[s_t^i\left(1-c_t^i\right)\left(a_t^i+\tilde{a}_t^i\beta^i\right)|I_t\right]R_{t+1}^a, \end{aligned} \tag{3.4}$$

where the expectations are taken with respect to the cross-sectional distribution μ as well as idiosyncratic return realizations, and the time-t information set I_t includes the mapping between individual households and their wealth levels summarized by F_t. This law of motion is exogenous to any individual household. At the same time, the fact that per capita wealth in the next period, $\bar{W}_{t+1}$, depends on the previous period average wealth $\bar{W}_t$ as well as on the consumption and investment choices made by households at time t, imposes additional restrictions on the solution procedure. These restrictions lead to the following notion of equilibrium.

Definition 1. *A status/investment equilibrium consists of*

- *household value functions V and optimal policies $[C, a]$*
- *law of motion for the growth rate of aggregate wealth G_{t+1} as a function of aggregate return R_{t+1}^a, household wealth levels contained in I_t, and households' optimal policies*

The equilibrium is a fixed point of the mapping between the aggregate wealth process that is taken as exogenous by investors and the endogenous evolution of aggregate wealth resulting from individual optimization.

3.3 Numerical solution

The equilibrium notion introduced above implies that the state space, which includes the space of wealth distributions, is potentially infinite-dimensional. Finding such an equilibrium in practice is infeasible. Instead I approximate the dynamics of the endogenous aggregate state variable, similarly to Krussell and Smith (1998).

Note that the law of motion for aggregate wealth (3.4) can be written as

$$G_{t+1} = \xi_0\left(I_t\right) + \xi_1\left(I_t\right)R_{t+1}^a,$$

where $\xi_0(I_t)$ and $\xi_1(I_t)$ are determined in equilibrium and can vary over time with the wealth distribution. In my numerical solution I approximate them with constants by simulating the model forward and projecting the resulting path of aggregate wealth growth on the return realizations. I verify that the resulting law of motion is indeed (approximately) time-invariant. To check that the evolving wealth distribution does not alter the law of motion I condition the projection on lagged values of G and confirm that this does not improve the forecasting power of the linear projection.

The numerical approximation procedure therefore consists of solving the individual optimization problems, simulating future wealth distributions for a large number of periods using the optimal policies, updating the resulting law of motion for aggregate wealth, and repeating the procedure until the law of motion stabilizes. Further details of the computational procedure are provided in appendix B.

3.4 Parametrization

I solve the model for $T = 7$ periods so that each period corresponds to a 10-year investment horizon. Thus, if the youngest agents enter the model at age 20 then the last decision-making period corresponds to the age of 80 years. Table I lists the parameters of the investment opportunity set as well as the benchmark values of preference parameters. The unconditional means of the stock return and the risk-free rate (i.e., 10-year Treasury bond yield) approximately match those in the U.S. data, at annualized values (for corresponding logarithmic returns) of 11 and 5 percent, respectively[12]. The risk-free rate is constant. The equity returns are i.i.d. I assume that the expected excess return on the idiosyncratic asset is equal to the public equity premium, consistent with the findings of Moskowitz and Vissing-Jørgensen (2002). I assume that the standard deviation of the idiosyncratic project/private equity return is three times as high as that of the public equity, which is similar to the volatility of publicly traded individual stocks (see Campbell, Lettau, Malkiel, and Xu (2001)). This implies annualized standard deviations of public and private equity logarithmic returns of 15 and 45 percent, respectively. Heaton and Lucas (2004) and Polkovnichenko (2003) consider similar volatility levels in calibrating entrepreneurial project

[12]This assumption overstates the *real* risk-free rate in the data, however, it allows me to sidestep the tension generated by the equity premium and risk-free rate puzzles in calibrating aggregate portfolio holdings. Since explaining these puzzles is not the focus of this paper, I parameterize the model to make them least pronounced.

hurdle rates.

I assume a discrete two-state distribution for the public equity return, with a high realization being twice as likely as the low realization, which implies values of $R^a = [1.0289^{10}, 1.1483^{10}]$. I let idiosyncratic states follow a lognormal distribution and use Gauss-Hermite quadrature (with 10 nodes along the idiosyncratic dimension) to evaluate expectations.[13] In the benchmark calibration I allow private equity returns to covary positively with public equity by setting $\beta^i = 0.5$. This is qualitatively consistent with the empirical evidence in Heaton and Lucas (2000b) that income streams from proprietary businesses are positively correlated with the stock market return. For the two-state public equity return process I use this beta to restrict the conditional mean of private equity return in each of the aggregate states.

The initial wealth distribution used as a starting point for the iterative procedure is calibrated using the percentiles of the U.S. wealth distribution from the 2001 Survey of Consumer Finances. Table I displays the set of points used to approximate the distribution.

4 Status model vs. data

In this section I evaluate the ability of the social status model to explain quantitative as well as qualitative features of the data. First, I calibrate the model to match the asset holdings and consumption volatility at the aggregate level. I also calibrate the model with standard CRRA preferences to match the same features of the data as a benchmark. I then evaluate both models' predictions for the cross-section of individual portfolio allocations. I show that the social status model does a substantially better job explaining the cross section of household asset holdings than does the standard model matched to the same aggregate quantities. I also evaluate the model's predictions for the individual wealth variability over time as well as discuss its implications for savings behavior and entry into entrepreneurship. The data sources used are: Survey of Consumer Finances (SCF), for information on households' asset holdings and the cross-sectional distribution of wealth, and Panel Study of Income Dynamics (PSID), for the evolution of household wealth over time. The details on the use of these datasets are described in the appendix. The estimates of volatility of average consumption growth are from Malloy, Moskowitz, and Vissing-Jørgensen (2005) for 5-year horizon; the average consumption growth volatility is based on the quarterly

[13] See Judd (1999) for a general discussion of numerical integration.

estimates of Wachter and Yogo (2007); both studies use stockholders' consumption expenditure data for nondurable goods and services from the Consumer Expenditure Survey (CEX).

4.1 Calibrating the model: aggregates

The empirical and simulated moments for the aggregate quantities of interest are displayed in table II. The primary targets of my calibration are two key statistics of the data on individual household portfolio allocations: average holdings of risky assets (specifically, public and private equity) and the degree of portfolio concentration. While prices are interpreted as exogenous technology parameters in the model, I set them in accordance with empirical estimates. I then choose preference parameters - utility curvature γ and status weight η - so as to match closely the two moments of household portfolio holdings[14]. I use data from 2001 Survey of Consumer Finances to estimate the average share of household assets allocated to risky assets (including stocks, mutual funds, corporate bonds, private businesses, etc.) and the average share allocated to "concentrated equity" - the household's largest risky asset holding (such as private business or individual stock). Appendix C describes the data in detail. The model counterparts of these moments are the average share of wealth allocated to risky assets (both public and private equity) and the average share of private equity.

In addition, I use the set of empirical facts about aggregate consumption growth volatility to constrain my calibration. I focus on the consumption growth of stockholders, since in my model all households are marginal in the public equity market. I compare the standard deviations (annualized, in percentage points) of average logarithmic consumption growth generated by the model for a range of parameter values with those from the U.S. data. The reported consumption volatility of stockholders is based on the estimates obtained using micro data from the Consumer Expenditure Survey (CEX) (see Malloy, Moskowitz, and Vissing-Jørgensen (2005) and Wachter and Yogo (2007)). I also report the standard deviation of growth in the logarithm of per capita consumption from NIPA. These comparisons should be viewed with some caution, since the model numbers are based on 10-year periods, whereas consumption data are based on quarterly consumption growth observations (I use estimates for quarterly consumption growth over 5 year periods due to data length limitations). Household-level consumption data are available in the

[14]Piazzesi and Schneider (2007) propose a framework for modeling both asset prices and quantities endogenously in a similar portfolio-choice context.

CEX for less than 25 years, making it impossible to estimate the volatility of consumption growth between 10-year periods. Still, since aggregate consumption process is close to a random walk at the annual frequency, this problem might not be too severe.

Alongside the empirical estimates of consumption volatility the table displays corresponding quantities obtained using simulated data produced by the social status model for the consumption curvature parameter $\gamma = 10$ and status weight $\eta = 1$ - the values chosen to approximately match the empirical quantities. I also report the corresponding model quantities for the case $\eta = 0$ (i.e. standard CRRA preferences with no status concerns) and consumption curvature $\gamma = 8$, which is also chosen so as to best match the target moments.

The social status model can match the average portfolio shares fairly closely. The model slightly understates the average share of risky assets (total equity in total assets), at 25 percent vs. 28 percent in the data. The model matches the degree of portfolio concentration at the aggregate level exactly, reproducing the 18 percent of total equity concentrated in the "single largest asset". Consequently, the model only slightly overstates the average fraction of total assets devoted to private equity, at 6 percent. The model can match these asset quantities without generating counterfactually high volatility of consumption growth: the annualized standard deviation of log average consumption growth is 1.85% in the model versus 1.71% in the data; the volatility of average consumption growth (for stockholders) is greater, at just under 5 percent both in the data and the model.

The standard power utility (CRRA) model calibrated similarly to the status model can also match the above empirical quantities fairly well. The CRRA model with curvature $\gamma = 8$ matches the share of risky assets almost exactly, at 27 percent, but underestimates the degree of portfolio concentration, with 15 percent of risky assets invested in private equity (or 4 percent of total assets). The power utility model matches aggregate consumption volatility almost exactly, and overestimates the volatility of average stockholder consumption growth by a third of a percentage point. Both the status and the CRRA model produce greater *average* consumption growth volatility across households, at 6.6 and 6 percent, respectively. This is due to the fact that some of the consumption volatility is idiosyncratic, especially for the status model.

The fact that both the status model and the CRRA model can match aggregate quantities that I target equally well is not surprising. The average holdings of risky assets and, in particular, concentrated equity across the U.S. households are fairly low. Thus, the under-diversification puzzle does not arise at the aggregate level. In order to see the puzzle one needs to focus on households that are likely to own concentrated assets, in particular, the wealthy.

4.2 Evaluating the model: cross-section

The main challenge for the portfolio choice model is to explain the heterogeneity in asset holdings across households, given the constraint imposed by matching the aggregate quantities. The empirical measures of risk-taking and diversification that I analyze are averages of portfolio shares taken over two subsamples of households, subdivided into wealth percentile groups. The first subsample includes all "stockholders" defined broadly as households who own both directly held equity and equity held through mutual funds or other managed accounts. The second one is "stockholders with concentrated holdings" - a subset of stockholders that report positive holdings of one of the following: directly held individual stocks, private business, investment real estate, and other similar risky assets. As discussed above, my empirical analog of "private equity" in the model is the single largest asset from the above list owned by a household. In addition, I look at total "undiversified" equity, which is the sum of all such concentrated holdings (i.e. all equity, public and private, that is held directly rather than in managed accounts).

In order to evaluate the model's ability to explain portfolio allocation decisions I consider the variation in the portfolio shares across the wealth distribution. The average allocations by wealth quantile obtained from the SCF are summarized in table III. The salient feature of the data is that both the share of risky assets in households's portfolios and the degree of asset concentration in the largest risky asset are increasing in wealth.

Table IV reports the corresponding quantities produced by the calibrated social status as well as for the power utility model. The social status model broadly matches the cross-sectional patterns of risky asset holdings (Panel A). The average allocation among the bottom half of the wealth distribution is around 20 percent in the data (19 for all stockholders and 24 for those with concentrated equity). This is matched almost exactly by the model, at 23 percent. Consistently with the data, the share of risky assets in the social status model is increasing in wealth. At the top 5th percentile of the wealth distribution households in the data invest just over half of their wealth in equities, which is captured by the model. For the highest (top one percent) wealth percentile, the model overshoots the risky asset allocation for stockholders (63 percent in the data), almost matching the average allocation among business owners/concentrated shareholders at around 80 percent. By contrast, the standard power utility model, which features constant portfolio shares across the wealth distribution because of homotheticity, cannot match the heterogeneity in portfolio allocations. The equity share of 27 % predicted by the CRRA model (Panel B) are not too far from the empirical estimates for the bottom 90 percent of the wealth distribution. However, within the top decile of the distribution, the standard model dramatically understates the level of risky asset holdings.

Explaining the cross-section of portfolio concentration is an even greater challenge. The social status model does a good job of matching the average portfolio shares allocated to private equity among *all* stockholders, as well as its increasing profile. The model predicts that on average 12 percent of equity, or 3 percent of total assets, is concentrated in the idiosyncratic asset in the lower deciles of the wealth distribution. This is similar to the average shares in the data, as well as to the predictions of the CRRA model. In the top decile of the distribution, however, the concentration shares increase sharply, up to almost 30 percent of total assets for the richest one percent of households. The CRRA model cannot match this increase. The social status model exhibits a sharp increase in concentration shares over the top wealth percentiles, predicting that the entire risky asset holdings of the top one percent of households are comprised of private equity (in fact, their equity stake is 6 percent short the public stock market). This prediction appears extreme relative to the average empirical shares of the *single largest* concentrated equity holdings displayed in table III. However, if we extend the notion of concentrated equity holdings to include all "undiversified" equity, the difference becomes less dramatic. In the data, for households in the top one percent of the wealth distribution and for those in the next 4 percent, the average shares of total equity holdings that are undiversified are 77 and 54 percent, respectively, corresponding to 51 and 31 percent of total assets. Conditional on households having non-zero holdings of such concentrated equity assets these quantities are even greater, with over 80 percent of equity held by top 1 percent of households in the form of undiversified investments. These quantities are still lower than those predicted by the model for the wealthiest household groups. However, it is difficult to assess the extent to which the model overstates under-diversification of the rich using the SCF data. It is possible that some of the equity positions that I classify as "diversified," such as those held in mutual funds and "managed accounts" are in fact highly exposed to idiosyncratic risk. In particular, it is likely that some of the "managed account" holdings of the very wealthy might include hedge fund and private equity fund investments, which can have large idiosyncratic risk exposure[15].

The ability to match the levels of risky asset holdings and portfolio concentration of the richest households without generating excessive volatility of aggregate consumption growth is a distinctive feature of the social status model. The standard CRRA portfolio model with $\gamma = 8$ calibrated to match the same aggregate quantities cannot match either the heterogeneity in risk taking or the extent of portfolio concentration among the rich. The reason the social status model is able to reconcile the aggregate facts with the evidence on portfolio holdings of the very wealthy is that its prediction of high levels of portfolio concentration in a (largely) idiosyncratic asset for investors with high wealth relative to the average.

In the model, *individual* consumption growth volatility is sharply increasing in wealth along with the volatility of portfolio returns, reaching 20 percent for the top wealth groups (table IV)[16]. Much of this volatility is idiosyncratic, driven by the returns on "private equity." The model's allocations to private equity are empirically plausible in that they generally follow the same increasing pattern as the allocation to undiversified equity holdings in the data, although the predicted magnitudes are higher for the top wealth groups. Note, however, that matching these statistics is hard due in part to the measurement difficulties: unlike the public stock market, households' private equity returns are largely unobservable. Nevertheless, the magnitudes are sufficiently similar to conclude that the model can broadly match the empirical patterns of risk taking and the degree of portfolio concentration simultaneously.

4.3 Understanding portfolio heterogeneity

What drives the heterogeneity in portfolio allocations in the social status model? "Getting ahead of the Joneses" property of status preferences implies a wedge between the relative risk aversion towards any wealth gambles, and relative aversion to risk that is correlated with the per capita

[15]Calvet, Campbell, and Sodini (2007) document that wealthier households appear to hold better-diversified portfolios than poorer ones, but at the same time also invest more aggressively, and as a result are exposed to more idiosyncratic risk.

[16]Wachter and Yogo (2007) report estimates of individual consumption volatility growth by wealth groups that are of similar magnitudes.

wealth. Further, the effect of "getting ahead of the Joneses" increases with relative wealth in a non-linear fashion, simultaneously driving down the risk aversion of the wealthiest investors and thus increasing their optimal exposure to idiosyncratic risk.

In order to illustrate the intuition behind this result, it is useful to consider once again the simplified one-period version of the model in 2.4. Since in the one period case the utility is defined directly over wealth, we can compute the relevant measures of risk aversion. The Arrow-Pratt coefficient of relative risk aversion of agent i is

$$RRA = -\frac{W^i U^i_{WW}}{U^i_W} = \frac{\gamma \left(W^i\right)^{-\gamma}}{\left(W^i\right)^{-\gamma} + \eta \bar{W}^{-\gamma}} = \frac{\gamma}{1 + \eta \left(\frac{W^i_{t+1}}{\bar{W}_{t+1}}\right)^{\gamma}},$$

which is a decreasing function of relative wealth, $\frac{W^i_{t+1}}{\bar{W}_{t+1}}$, and is bounded from above by γ, its limit at zero wealth. It tends to zero as relative wealth grows.

As a way to measure the desire to "get ahead of the Joneses" we can similarly calculate the "relative aversion to aggregate wealth risk" (RAWRA), a quantity analogous to a Merton-type hedging demand that stems from the state-dependence of the utility function. Define

$$RAWRA = -\frac{\bar{W} U^i_{W\bar{W}}}{U^i_W} = \frac{\gamma \eta \bar{W}^{-\gamma}}{\left(W^i\right)^{-\gamma} + \eta \bar{W}^{-\gamma}} = \frac{\gamma \eta}{\left(\frac{W^i_{t+1}}{\bar{W}_{t+1}}\right)^{-\gamma} + \eta},$$

which is an increasing function of relative wealth, with the upper limit equal to γ. The lower limit as relative wealth falls is zero. Thus, the poorest individuals, while most risk averse, are the least averse to aggregate risk. Conversely, the wealthiest individuals are the least averse to pure wealth gambles, but also the most averse to aggregate fluctuations. The degree of divergence in risk attitudes for intermediate values of relative wealth is controlled by the magnitude of η, the status weight. The greater this parameter is, the steeper the decrease in risk aversion and the increase in aversion to aggregate risk as relative wealth goes up. For $\eta = 1$ the two types of risk aversion are of equal magnitudes for the average investor (i.e. at $\frac{W^i_{t+1}}{\bar{W}_{t+1}} = 1$). For $\eta > 1$ the aggregate risk aversion overtakes the RRA coefficient at lower relative wealth levels. Figure 1 plots these two measures of risk aversion - RRA and RAWRA - as functions of relative wealth, $s^i = \frac{W^i}{\bar{W}}$ for the case $\gamma = 10$, $\eta = 1$. The sum of the two measures of risk aversion in this example is constant across wealth levels and equal to γ.

Following the state-variable hedging intuition of Merton (1973), the overall allocation of assets to securities that bear aggregate risk is determined by a combination of overall risk aversion and the "hedging demand" for insurance against fluctuations in per capita wealth. In particular, risk averse individuals with "keeping up with the Joneses" preferences might require less compensation for bearing aggregate wealth risk than for purely idiosyncratic risk (e.g. see Gollier (2004)). Conversely, low risk aversion to pure wealth gambles can be consistent with low allocation to aggregate assets even in the face of a high risk premium. The latter feature of "getting ahead of the Joneses" preferences is consistent with the view of the aggregate equity premium that emphasizes low individual risk aversion towards idiosyncratic gambles (e.g. see discussion in Kocherlakota (1996) and Cochrane (1997)).

The intuition behind the cross-sectional differences in risk attitudes is that status preferences exhibit more curvature with respect to consumption than with respect to (relative) wealth, which implies that the latter is treated by consumers as a luxury. This drives down the risk aversion towards pure wealth gambles at high wealth level. At the same time, since relative wealth position is a "luxury," it is relatively more important to the wealthy, so that the strength of "getting ahead of the Joneses" motive increases with wealth, driving up the aversion to aggregate risk. Carroll (2002) argues that a preference for wealth as a luxury good is key to explaining the heterogeneity in portfolio composition across households, in particular the fact that the rich save more as a fraction of their wealth than the poor and that they hold a much larger share of risky assets (including entrepreneurial ventures) in their portfolios. However, the social status preferences analyzed here are not simply a way of introducing decreasing relative risk aversion. A model that has the latter feature but does not exhibit "getting ahead of the Joneses" might be able to explain the increasing pattern of risky asset holdings, but is unlikely to match the degree of portfolio concentration among the wealthy households. Applying the standard assumption that individuals' marginal utility does not depend on aggregate wealth directly implies that households' optimal portfolios are well diversified and closely resemble the aggregate stock market index. Consequently, the resulting aggregate consumption growth should exhibit greater variability.

4.4 Wealth mobility

Does the social status model imply too much variability in individual consumption and wealth, in particular for the richest households? The model does predict high volatility of portfolio returns and consumption growth for the top one percent of households, at 28 and 20 percent (log, annualized), respectively. Unfortunately, it is impossible to assess directly whether these quantities are empirically reasonable. Data on individuals' portfolio returns is unavailable in the U.S., while consumption data from the CEX lacks sufficiently long panel dimension for estimating individual consumption growth volatility over long horizons. In addition, the CEX does not do a very good job sampling the wealthiest households. Thus, in order to evaluate the model's predictions for the degree of exposure to idiosyncratic risk I look at the cross-sectional dynamics of household wealth using data from the Panel Study of Income Dynamics (PSID). Although this dataset, like the CEX, undersamples the rich households, it has a long enough panel dimension that allows me to estimate changes in household wealth over 10-year periods, which match the horizon in my simulated model.

While it is well known that the distribution of household wealth in the U.S. is extremely wide and highly concentrated, there is also a substantial amount of cross-sectional wealth mobility over time. I estimate 10-year transition probabilities of wealth deciles following Hurst, Stafford, and Luoh (1998) They estimate transition probabilities using the PSID wealth supplements over the period 1984-1994. I update their estimates with data from the 1999 supplement. I adjust the estimated transition rates to limit the influence of measurement error and, most importantly, to remove life-cycle accumulation/decumulation effects that are absent in my model, in order to provide an appropriate benchmark for evaluating the model's predictions. Details of this estimation can be found in the appendix.

Table V displays the probabilities of moving upwards or downwards and staying in the same percentile group conditional on being in a given wealth quantile at the beginning of a ten-year period. The empirical transition matrix displays a substantial degree of mobility, especially in the right tail of the wealth distribution (panel A). Among the households in the top one percent two thirds are staying in the same decile, and one third falling into a lower decile. In the 95th to 99th percentile group, over half of all households fall behind after 10 years. At the same time, the movement between the top and the bottom half of the distribution is very limited, with 98 percent of households in the bottom 50 percent remain there after 10 years. As shown in the table V, within the groups of households that report positive holdings of stocks and private businesses the estimates of transition probabilities are very similar, with a slightly higher mobility in the middle deciles.

The social status model is able to generate patterns of social mobility that very closely mimic those in the data for the top percentiles of the wealth distribution. The quantitative features of the transition distribution for the status model are summarized in table V (panel B) alongside the empirical estimates (displayed in . Note that for the top 1 percent of the distribution the model matches the empirical transition probabilities almost exactly. In contrast to the social status model, the standard power utility model (panel C) produces highly persistent cross-sectional wealth distribution, with persistence probabilities of 95 percent in the top percentile of the distribution (compared to about 67% in the data and under the status model). For lower percentile group the match between the social status model and the data is less close, but the model still outperforms the neoclassical benchmark. Overall, even though the social status model is not designed specifically to explain social mobility, it does a good job of matching the empirical facts for the mobility in the upper end of the wealth distribution. It is therefore likely that the model's predictions for the degree of households' exposure to idiosyncratic investment risk are reasonable.

4.5 Entrepreneurship and concentration

In matching the cross-section predictions of the social status model for degree of portfolio concentration I have so far ignored the fact that a large fraction of households, even among stockholders, has no concentrated holdings. In the context of the model, this might not be surprising if not all investors have access to idiosyncratic investment opportunities that earn a positive abnormal return ("alpha"). Separating households who do own concentrated assets helps to match the model's predictions for the idiosyncratic risk exposure of the wealthiest investors' portfolios. At the same time, conditioning on participation in "private equity" market also reveals that the model dramatically understates the degree of portfolio concentration in the bottom half of the wealth distribution. As documented in table III (panel B), households in the lower half of the distribution that do own idiosyncratic assets on average have between 80 and 90 percent of their total equity concentrated in such investments, which corresponds to 20 percent of their total assets. These concentration shares decline somewhat at higher wealth levels before displaying the sharp increase in the top 5 percent group. In contrast, in the model the poorer households have the lowest concentration shares (3 percent of total assets allocated to private equity).

The reason for the discrepancy is not surprising. In the model I allow households to invest a small fraction of their wealth in private equity. In the data, the concentrated equity stakes, especially among the poorer households, are driven by business owners. Given the potential importance of asymmetric information in the private equity market and in financing of small businesses, incentive considerations should dictate that the entrepreneurs' stakes in their businesses must be large relative to their outside assets. In fact, Bitler, Moskowitz, and Vissing-Jørgensen (2005) show that this prediction is indeed borne out in the data. Still, this does not explain why poorer households choose to become entrepreneurs if doing so requires a potentially dramatic increase in portfolio and consumption risk relative to other investment opportunities. For example, setting the minimum required business-owner's private equity stake to be 20% of total assets, which is consistent with estimates obtained by Bitler, Moskowitz, and Vissing-Jørgensen (2005), would imply that, in the social status model, only the wealthiest 5 percent of households find it optimal to become entrepreneurs. In order to confirm this intuition I solve the model restricting the share of private equity in total assets to be at least 20 percent, or else zero. Table IV (panel C) displays the resulting cross-section of private equity shares. Indeed, they are zero for all households outside of the top decile of the wealth distribution.

One possibility for rationalizing this result with the data is to allow for heterogeneity in investment opportunities among investors. In particular, suppose individuals draw idiosyncratic entrepreneurial projects randomly from a distribution of systematic risk exposures. Then, for all but the very wealthy households, entry into entrepreneurship is driven by the diversification benefit of private equity. For example, suppose some entrepreneurs have access to projects that provide a hedge for aggregate risk in the form of a negative beta with the public equity. Then a concentrated investment in such a project might be optimal even for the poorest investors, for whom the status-seeking motive is very weak. The bottom line of table IV (panel C) shows private equity shares simulated from the model with the minimum concentration constraint of 20 percent and negative systematic risk of private equity: $\beta^i = -0.5$. It is evident that in this case, when private equity is a good hedge against the risk of public equity, even the households in the bottom half of the wealth distribution are willing to invest a fifth of their assets in it. The probability of drawing a project with such a large diversification benefit is likely to be small empirically, however. This is consistent with the huge discrepancy in the rate of participation in the private equity market reported in table III between the richer and the poorer households. Only 8 percent in the bottom half of the wealth distribution own concentrated equity, compared to 83 percent of households in the top one percent of the distribution.

An interesting direction for future research is to calibrate a model with explicit heterogeneity in private equity investment opportunities. One likely prediction is that the nonlinear effect of "getting ahead of the Joneses" on risk preferences might lead to a sharp increase in participation rates at the very top of the wealth distribution, with little variation across lower percentile. Hurst and Lusardi (2004) find that wealth itself does not predict entry into entrepreneurship, except for the top 5 percent of the distribution, and that the liquidity constraints, while potentially important, do not explain entry rates either. At the same time, empirically there is some evidence of a link between concentration of financial portfolios and entrepreneurship: Calvet, Campbell, and Sodini (2007) report that the portfolios of entrepreneurs are on average less diversified than those of non-entrepreneurs. This evidence lends further support to the unified view of household diversification offered in this paper.

4.6 Saving and consumption dynamics

The social status model generates considerable heterogeneity in saving rates. The optimal consumption - wealth ratios reported in table VI show that the richest 10% of the households consume a much smaller function of their wealth than the poorest half, and consequently save more. The youngest households at the bottom of the wealth distribution consume 45 percent of their initial wealth (over a 10-year period), as do power utility households. The richest 10 percent of the young (e.g., 20-year olds) consume only 11 percent. The difference is even more dramatic for the old households: the poorest 25 consume 60 percent of their wealth in the second-to-last period of their lifetime (i.e. at age 80), while the richest 5 percent still consume about 12 percent, thus leaving a disproportionately large amount of wealth for their heirs. This prediction of the model is consistent with the stylized empirical observation that the rich elderly do not dissave as predicted by the standard life-cycle model (e.g. see Dynan, Skinner, and Zeldes (2004)). The intuition for the high saving rate among the very rich is that the future status utility provides additional benefit for saving, above an beyond the desire to smooth consumption over time. This motive is particularly strong for the wealthy, since future status is relatively more important to them. This prediction is typical for models where wealth confers social status: e.g. Cole, Mailath, and Postlewaite (1992) and Corneo and Jeanne (1999) discuss the "oversaving" effects generated by relative wealth concerns.

The differences in consumption-wealth ratios across the wealth distribution are not driven by the bequest motive as such. Rather, they are due to the fact that the marginal utility of wealth is increasing in relative wealth (a consequence of "getting ahead of the Joneses" property). This shifts the importance from consumption towards wealth accumulation as individual's wealth grows (relative to the average). Some of the empirical facts concerning the heterogeneity in saving rates can be explained by other models in which preferences for bequest are non-dynastic and have luxury-good properties (e.g. Carroll (2000), DeNardi (2004)). The social status model possesses this desirable feature even though it was not designed specifically to explain savings behavior.

A limitation of my model that does not allow me to match the saving rates produced by the model to the data quantitatively (rather than qualitatively) is the absence of labor income. I leave out labor income from my model in order to focus attention on the endogenous choice of exposure to idiosyncratic risk, which is driven by relative wealth concerns. In order to expose the model's mechanism most clearly, I avoid encumbering it with another source of idiosyncratic risk. The implications of illiquidity of human capital and its decreasing age profile appear to be major determinants of saving behavior as well as portfolio choice over the life cycle (e.g. see Gomes and Michaelides (2005) and Storesletten, Telmer, and Yaron (2007)). However, it is likely that these effects are muted for the very wealthy, whose investment behavior is the primarily focus of this paper, since for them human capital is likely to constitute a much smaller fraction of total wealth than for an average U.S. household. Undoubtedly, incorporating labor income into the social status model would be important for evaluating its predictions for the entire cross-section of households, and is a promising venue for future research.

5 Discussion and Concluding Remarks

In this paper I address the limited diversification of household portfolios together with the apparent lack of a premium for undiversified entrepreneurial risk by considering the investment choices of individuals who exhibit a preference for social status. The assumption that marginal utility of wealth increases with relative status leads investors to optimally hold undiversified portfolios in equilibrium. This feature of the model suggests that at least some of the empirically observed cross-sectional dispersion in accumulated wealth can be understood using a simple portfolio-based approach that allows the amounts of both aggregate and idiosyncratic risk in the economy to be determined endogenously. Thus it supports the argument of Friedman (1953) who emphasizes the role of individual choice and, in particular, risk preferences in shaping the distribution of income and wealth.

The model also has potential implications for the study of investment and, consequently, economic growth. Standard macroeconomic theory is predicated on the assumption that the demand for diversification leads households to pool and share their idiosyncratic risks. Perfect risk sharing is prevented, however, by the incompleteness of insurance markets due to asymmetric information and limited enforcement of contracts. Such market imperfections impose costs on society in the form of foregone investment opportunities, due to the inability of agents to share idiosyncratic risk of individual projects. Preference for social status can mitigate this problem, since it can lead investors to take on more undiversified idiosyncratic risk than predicted by the standard theory, unleashing greater entrepreneurial investment and spurring economic growth. This intuition is similar to the argument of Robson (1996) that evolutionary forces favor agents who are less averse to idiosyncratic than to aggregate risks, since the former are "diversified" at the macro-level, while the latter are not. I provide an example of how status-generated "overinvestment" in individual-specific projects can be socially optimal in economies with limited risk-sharing in Roussanov (2006). This possibility appears consistent with the evidence of Anderson and Reeb (2003) that companies with concentrated founding-family ownership are less, not more, diversified, than other firms, contrary to the predictions of standard theories, such as Shleifer and Vishny (1986). Corneo and Jeanne (1997) and Corneo and Jeanne (2001) make a related argument that "oversaving" generated by social status concerns can help overcome negative externalities arising from technological spillovers, and therefore lead to optimal economic growth.

A link between preferences with relative status concerns and economic growth could help explain the divergent patterns of entrepreneurship and economic development across countries. Becker, Murphy, and Werning (2005) suggest that in societies in which the distribution of status is exogenously fixed (and thus not necessarily closely tied to relative wealth) one should observe less risk-taking. Rules governing assignment of status in a society can arise endogenously as evolutionary outcomes (e.g. see discussion of "wealth-is-status" vs. "aristocratic" equilibria in Cole, Mailath, and Postlewaite (1992)). Differences in cultural and social norms can potentially be at least as important as differences in economic policies in explaining the variation in the pace of economic growth around the world.

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A Bellman equation with scale-invariance

Proposition 1. *The dynamic program (3.1) is equivalent to*

$$v(s_t^i, A_t^i; I_t) = \max_{\tilde{c},\alpha} \left\{ \frac{\left(\tilde{c}_t^i s_t^i\right)^{1-\gamma}}{1-\gamma} + \delta E_t \left[\left(v_{t+1}^i(s_{t+1}^i, A_{t+1}^i) + \eta s_{t+1}^i \right) G_{t+1}^{1-\gamma} \right] \right\}. \tag{A.1}$$

Proceed by backward induction: start with agents who reach the last period of their life T at time τ :

$$\begin{aligned} V(W_\tau^i, \bar{W}_\tau, T; I_\tau) &= \max_{C,\mathbf{a}} \left\{ \frac{\left(C_\tau^i\right)^{1-\gamma}}{1-\gamma} + \eta \bar{W}_\tau^{1-\gamma} \frac{W_\tau^i}{\bar{W}_\tau} + \delta\psi E\left[B(W_{\tau+1}, \bar{W}_{\tau+1}) \middle| I_\tau \right] \right\} \\ &= \max_{C,\mathbf{a}} \left\{ \begin{array}{c} \frac{\left(C_\tau^i\right)^{1-\gamma}}{1-\gamma} + \eta \bar{W}_\tau^{1-\gamma} \frac{W_\tau^i}{\bar{W}_\tau} \\ \delta\psi E\left[\frac{\left(W_{\tau+1}\right)^{1-\gamma}}{1-\gamma} + \eta \bar{W}_{\tau+1}^{1-\gamma} \frac{W_{\tau+1}^i}{\bar{W}_{\tau+1}} \middle| I_\tau \right] \end{array} \right\} + const \\ &\equiv \bar{W}_\tau^{1-\gamma} \left(\frac{\left(\tilde{c}_\tau^i s_\tau^i\right)^{1-\gamma}}{1-\gamma} + \eta s_\tau^i + \delta\psi E\left[G_{\tau+1}^{1-\gamma} B(s_{\tau+1}^i, 1) \middle| I_\tau \right] \right) \\ &\triangleq \bar{W}_\tau^{1-\gamma} \left[v(s_\tau^i, T; I_\tau) + \eta s_\tau^i \right] \end{aligned}$$

and

$$\begin{aligned} V(W_{\tau-1}^i, \bar{W}_{\tau-1}, T-1; I_{\tau-1}) &= \max_{C,\mathbf{a}} \left\{ \begin{array}{c} \frac{\left(C_\tau^i\right)^{1-\gamma}}{1-\gamma} + \eta \bar{W}_{\tau-1}^{1-\gamma} \frac{W_{\tau-1}^i}{\bar{W}_{\tau-1}} \\ +\delta E\left[V(W_\tau^i, \bar{W}_\tau, T; I_\tau) \middle| I_{\tau-1} \right] \end{array} \right\} \\ &= \max_{C,\mathbf{a}} \left\{ \begin{array}{c} \frac{\left(C_{\tau-1}^i\right)^{1-\gamma}}{1-\gamma} + \eta \bar{W}_{\tau-1}^{1-\gamma} s_{\tau-1}^i \\ +\delta E\left[\bar{W}_\tau^{1-\gamma} \left[v(s_\tau^i, T; I_\tau) + \eta s_\tau^i \right] \middle| I_{\tau-1} \right] \end{array} \right\} + const \\ &\equiv \max_{\tilde{c},\mathbf{a}} \left\{ \begin{array}{c} \frac{\left(\tilde{c}_{\tau-1}^i s_{\tau-1}^i\right)^{1-\gamma}}{1-\gamma} + \eta s_{\tau-1}^i \\ +\delta E\left[G_\tau^{1-\gamma} \left(v(s_\tau^i, T; I_\tau) + \eta s_\tau^i \right) \middle| I_{\tau-1} \right] \end{array} \right\} \times \bar{W}_{\tau-1}^{1-\gamma} \\ &\triangleq \bar{W}_{\tau-1}^{1-\gamma} \left[v_{\tau-1}(s_{\tau-1}^i, T-1; I_{\tau-1}) + \eta s_{\tau-1}^i \right]. \end{aligned}$$

Therefore, for any A_t^i we have

$$\begin{aligned} v(s_t^i, A_t^i; I_t)\bar{W}_t^{1-\gamma} &= \bar{W}_t^{1-\gamma} \times \\ & \max_{\tilde{c},\alpha} \left\{ \frac{\left(\tilde{c}_t^i s_t^i\right)^{1-\gamma}}{1-\gamma} + \delta E_t \left[\left(v(s_{t+1}^i, A_{t+1}^i; I_{t+1}) + \eta s_{t+1}^i \right) G_{t+1}^{1-\gamma} \right] \right\}, \end{aligned}$$

which is equivalent to (A.1)

Corollary 2. *Households' optimal consumption and investment policies do not depend on aggregate wealth.*

B Computational Algorithm

The model is solved by iterating on the following steps:

1. *Maximization* of agents' utility
2. *Simulation* of asset returns and the resulting wealth distribution

Maximization

The normalized Bellman equation (A.1) is solved by backward induction. The continuous space of endogenous state variable (agent-specific relative wealth s_t^i) is discretized using a grid with 60 points (logarithmically spaced, so that the grid is denser in the lower relative wealth region, where most of the agents are). For each age and individual wealth state, optimal consumption and portfolio choices are found using grid search. I use shape-preserving Hermite interpolation for the next period's value function (for the young agents) [17].

Simulation

At each iteration for each age and aggregate state I draw a large number (10000 for each age group) relative wealth levels from the initial wealth distribution and interpolate the optimal consumption and portfolio policies from the solutions found in step 1 using linear interpolation. I then simulate idiosyncratic returns for all of the agents and estimate the resulting "empirical"

[17] Piecewise-cubic Hermite polynomial interpolation (PCHIP) is implemented in the MATLAB curve-fitting toolbox

distribution (EDF) of relative wealth in each of the aggregate states. I iterate this step forward until the simulated EDF is approximately stationary. I update the initial guess for the law of motion of aggregate wealth growth by projecting the resulting series of future average wealth on the simulated sequence of aggregate returns using OLS regression:

$$G_{t+1}^{proj} = \xi_0 + \xi_1 R_{t+1}^a,$$

The updated guess is used in the next iteration to solve the portfolio problem. In order to verify that this information is sufficient for capturing the dynamics of aggregate wealth growth, I condition the projection on one lag of G, i.e. estimate

$$G_{t+1}^{proj} = \xi_0 + \xi_0^G G_t + \left(\xi_1 + \xi_0^G G_t\right) R_{t+1}^a.$$

I confirm that the inclusion of lagged wealth growth does not improve the forecasting ability of the projection by computing mean squared prediction error.

The iterations are repeated until the simulated steady-state EDF and the law of motion converge (state by state). I verify that the resulting optimal policies are invariant to small perturbations around the steady-state distribution to ensure that the solution is consistent with rational expectations.

Even though the equilibrium policies feature more risk taking at higher wealth level, the resulting limiting wealth distribution is not degenerate. This is in part due to the coarse discretization of optimal policies, which implies that the set of agents pursuing the most aggressive policy is non-singleton. Given the large amount of idiosyncratic risk exposure in the portfolios of the very wealthy, there is a sufficient amount of mixing at the top of the distribution so that no single agent dominates. The discretization assumption is not without loss of generality, but is innocuous in the case of my calibration. This is because the optimal allocation to private equity as a share of risky assets is greater than 100 percent for the wealthiest households, which involves short positions in public equity. Thus a discrete approximation to the highest share of private equity can be interpreted simply as a short selling constraint.

C Data description and estimation procedures

Asset holdings: Survey of Consumer Finances (SCF)

I use the 2001 SCF public dataset available from the Federal Reserve Board of Governors. The survey is representative of the U.S. population and is designed to oversample the wealthy households. Each household is represented in the dataset by 5 replicates (implicates) constructed in order to compensate for omitted information about households assets, etc; thus, there are 22210 observations produced from the 4442 households actually surveyed. Weights are provided to allow aggregation to population totals. For a detailed discussion of 2001 SCF see, e.g. Kennickell (2003).

The survey contains detailed information on household demographics, income, and asset holdings. I use the following conventions to define the value of the two main components of household risky assets, "public equity" and "private equity". "Risky assets" are assumed to be comprised of both public equity and private equity (as defined in the appendix), and also to include corporate and foreign bonds (although their exclusion does not alter the results); I also consider the definition that includes owner-occupied housing as one of the risky assets.

Public equity includes directly held stocks plus managed assets such as mutual funds (except money market funds), retirement plans, annuities, trusts, thrifts, etc. For the purposes of calculating the households "public equity" investments the following convention is used in regard to these managed assets: full value if described as mostly invested in stock, 1/2 value if described as split between stocks/bonds or stocks/money market, 1/3 value if split between stocks/bonds/money market, etc.

Private equity includes the estimated market value of the households' stakes in private business(es) and/or farm(s), plus loans from household to the business(es), minus loans from business to household, plus value of personal assets used as collateral; it also includes the market value of investment real estate, as well as other financial assets that are likely to be illiquid and/or undiversified, such as oil/gas/mineral leases or investments; association or exchange membership; futures contracts, stock options, hedge funds; royalties, patents; non-publicly traded stock, stock with restricted trading rights.

I define "largest risky asset" to be the largest of the following: market value of a private business interest; value of an investment real estate property; value of "other risky asset"; value of equity if concentrated in a single stock; average size of a stock holding for households holding individual stocks (total value of stocks divided by the number of stocks); value of owner-occupied housing when the latter is included in the definition of risky assets.

In estimating the cross-sectional distribution of wealth I rank households on their total assets (instead of net worth) since in the model human wealth is potentially a component of total wealth, while in the data it is not. Although net worth and total assets are highly correlated, a number of individuals with high assets (as well as other characteristics correlated with human wealth, such as income and education) also have large debt (especially mortgage debt). This puts them into lower percentiles of net worth than individuals with the same level of assets but less debt and potentially lower human capital. Thus, sorts based on assets should better capture the *total* wealth ranking, although results based on net worth are very similar.

Wealth mobility: Panel Study of Income Dynamics (PSID)

I use the PSID wealth supplements for the years 1984, 1989, 1994 and 1999. In order to obtain estimates of wealth transitions over 10-year periods I track individuals who are heads of households in 3 successive observations that span a 10-year period. This results in a sample of 2608 households. I only include households with positive net worth in all 4 observations, which reduces the sample to 1973. This restriction simplifies estimation of growth rates of wealth across households and over time but does not affect the results otherwise. Further restricting the sample to male-headed households, as is often done in the literature due to the difficulties posed by changing head-of-household status for women who either marry or divorce, does not affect the results.

The measure of wealth is net worth (total assets minus total liabilities). Following Hurst, Stafford, and Luoh (1998) I use the beginning-of-period sampling weights (i.e., those for 1984 and 1989 supplements) to compute averages. I consider households that answer the question whether they own stocks, mutual funds or IRAs (farms/proprietary businesses and real estate other than primary residence) affirmatively in any of the 3 successive observations to be stock-owning (business-owning) in estimating transitions for the 10 year period spanned by those observations.

Transition probabilities are estimated by computing the fraction of households from a given decile that move to a target decile after a 10-year period, and averaging these transition rates over the two overlapping 10-year periods. Wealth mobility can be greatly effected by the life-cycle accumulation (and decumulation) of assets due to the fact that labor income cannot be capitalized in the beginning of working life and instead is converted into financial wealth slowly over time. Since my model abstracts from non-tradeable labor income, using the raw estimated transition probabilities might be misleading. In order to estimate wealth transition probabilities adjusted for the life-cycle effects I use cross-sectional regressions for both time periods to predict growth rates of household wealth:

$$\ln W_{t+10}^i - \ln W_t^i = a_0 + a_w \ln W_t^i + a_z Z_{t+10} + \epsilon_{t+10}^i$$

The life-cycle variables included in the vector of controls Z include a quadratic in age (in order to capture both life-cycle accumulation and decumulation), change in marital status, an change in family size. I use the residuals from these regressions to generate artificial end-of-period wealth observations. I estimate the adjusted transition probabilities using these artificial observations as before. In addition to the life-cycle correction I use artificial observations designed to limit the extent to which measurement error in wealth might bias the estimates of transition rates due to spurious volatility. These observations are obtained by averaging the first and the second pairs of observations: $\hat{W}_{86.5}^i = \frac{1}{2}\left(W_{84}^i + W_{89}^i\right)$, $\hat{W}_{96.5}^i = \frac{1}{2}\left(W_{94}^i + W_{99}^i\right)$. The transition probabilities are computed for the single implied period, from mid-1986 to mid-1996. The life-cycle adjustment is applied to the averaged observations as described above.

Hurst, Stafford, and Luoh (1998) use Shorrock's index as a measure of wealth mobility[18]. For the period 1984-1994 they estimate Shorrock's index of 0.85. In my extended data the raw estimate is 0.83, which falls to 0.71 after adjustments for life-cycle and measurement error. Both the life-cycle adjustment and the averaging procedure reduce the estimates of wealth mobility, albeit not dramatically. Table below shows the estimates of Shorrock's index of mobility for the three groups of households: all positive net worth households, stockholders and business owners:

[18] If N is the number of quantiles and $\mathrm{tr}(P)$ is the trace of the corresponding transition matrix P, then Shorrock's index equals $\frac{N-\mathrm{tr}(P)}{N-1}$.

	raw	adjusted	averaged	averaged and adjusted
all	0.83	0.72	0.79	0.71
stockholders	0.83	0.77	0.80	0.71
business owners	0.84	0.74	0.83	0.74

It is apparent that while the removal of life-cycle variation increases persistence, the measurement-error correction has a smaller impact on the estimates.

Table I: **Calibration**

Technology parameters:

Parameter		**Value**
Risk-free Rate	R^f	5%
Public Equity Risk Premium	$E\left(R^a\right)-R^f$	6%
Public Equity Return Volatility	$\sigma\left(R^a\right)$	15%
Private Equity Risk Premiums	$E\left(R^i\right)-R^f$	6%
Private Equity Return Volatility	$\sigma\left(R^i\right)$	45%
Systematic risk of private equity	β_{R^i,R^a}	0.5
Probability of good aggregate state	$\Pr\left\{R^a>R^f\right\}$	$\frac{2}{3}$

Preference parameters (status benchmark)

Parameter		**Value**
Curvature of Consumption Utility	γ	10
Status Utility Weight	η	1
Subjective Discount Factor	β	0.97^{10}
Bequest Utility Weight	ψ	1

Initial wealth distribution

x	0.005	0.013	0.027	0.053	0.133	0.267	0.533	1.333	2.665	5.330
$F(x)$	0.162	0.187	0.219	0.257	0.328	0.447	0.603	0.819	0.920	0.971

The top panel displays the parameters of asset returns used in calibration, annualized via logarithmic returns. Public equity return and risk-free rate are based on 10-year CRSP value-weighted returns and 10-year Treasury yields, respectively. Private equity return is calibrated to have the same mean return as public equity and standard deviation three times as high. Systematic risk of private equity is captured by its beta (loading) on the public equity. Public equity return is approximated by a discrete process with a "good" and a "bad" state. The middle panel displays the range of preference parameter values used in simulations. The bottom panel contains a discrete approximation of the wealth distribution used to initialize the simulated model: for each relative wealth level x the fraction of households with wealth below this level ($\frac{W^i}{W}\leq x$) is given by $F(x)$. This distribution matches the distribution of households net worth in 2001 SCF (cf. Kennickell (2003)).

Table II: **Asset holdings and consumption growth volatility**

	Data	Status model	CRRA
Equity/total assets	28	25	27
Concentrated/total equity	18	18	15
Conc. equity/total assets	5	6	4
$\sigma\left(\ln\left(\bar{C}_{t+h}/\bar{C}_t\right)\right)$	1.71†	1.85	1.69
$\sigma\left(\frac{1}{N}\sum\ln\left(C^i_{t+h}/C^i_t\right)\right)$	4.96‡	4.85	5.33
$\frac{1}{N}\sum\sigma\left(\ln\left(C^i_{t+h}/C^i_t\right)\right)$	8‡	6.62	6.00

Average portfolio allocations to public and private equity and measures of consumption growth volatility in the U.S. data and in the model.
Data: total (public and private) equity as a share of total assets and the share of assets allocated to concentrated equity (private business or individual stock) for households with positive equity holdings, obtained from 2001 SCF; consumption growth volatility, annualized logarithmic 5-year horizon estimates
† - aggregate consumption from NIPA,
‡ - individual stockholder consumption from CEX.
Status model: moments simulated for the calibrated model with $\gamma = 10, \eta = 1$.
CRRA: : moments simulated for the calibrated power utility ($\eta = 0$) model with $\gamma = 8$.
All quantities are in percentage point units.

Table III: **Portfolio allocation: data**

Panel A: all stockholders

Wealth percentile	Bottom half	50-90	90-95	95-99	Top 1 percent
Equity/total assets, %	19	26	44	52	63
Concentrated/ total equity, %	14	17	20	27	37
Concentrated equity/total assets, %	3	5	10	17	26
Undiversified/total equity, %	19	29	41	54	77
Undiversified equity/total assets, %	4	9	19	31	51

Panel B: stockholders with business or other concentrated equity

Wealth percentile	Bottom half	50-90	90-95	95-99	Top 1 percent
Equity/total assets, %	24	34	48	59	68
Concentrated/total equity, %	90	63	44	44	45
Concentrated equity/total assets, %	20	21	21	28	32
Undiversified/total equity, %	80	70	60	69	84
Undiversified equity/total assets, %	20	24	30	42	58
Participation rate, %	8	30	53	66	83

Panel A: average portfolio shares of households that report owning stocks, mutual funds, and other publicly traded risky assets ("equity").
Panel B: average portfolio shares of households that report having concentrated equity stakes, such as shares of private businesses, individual stocks, investment real estate, etc. Participation rate is the fraction of household that own such assets.
"Concentrated equity" is the largest of: private business, individual stock holding, investment real estate holding, etc. "Undiversified equity" is the sum of all such holdings (i.e. all equity held directly, outside of mutual funds or other managed accounts).

Table IV: **Portfolio allocation: model**

Panel A: status

Wealth percentile	Bottom half	50-90	90-95	95-99	Top 1 percent
Equity/total assets, %	23	23	25	51	80
Private/total equity, %	12	12	35	99	107
Private equity/total assets, %	3	3	10	53	86
Portfolio mean return, %	6	6	6	7	10
Portfolio std. dev., %	5	5	6	18	28
Mean consump. growth, %	1	1	2	5	9
Std. cons. growth, %	5	5	7	11	21

Panel B: CRRA

Wealth percentile	Bottom half	50-90	90-95	95-99	Top 1 percent
Equity/total assets, %	27	27	27	27	27
Private/total equity, %	15	15	15	15	15
Private equity/total assets, %	4	4	4	4	4
Portfolio mean return, %	6	6	6	6	6
Portfolio std. dev., %	6	6	6	6	6
Mean consump. growth, %	2	2	2	2	2
Std. cons. growth, %	6	6	6	6	6

Panel C: status, restricted

Wealth percentile	Bottom half	50-90	90-95	95-99	Top 1 percent
Private equity/total assets, bchmrk.	0	0	8	55	89
Private equity/total assets, altern.	20	20	32	81	92

Panel A: average portfolio shares simulated from the status model with $\gamma = 10$, $\eta = 1$.
Panel B: average portfolio shares simulated for the power utility model with $\gamma = 8$, $\eta = 0$.
Panel C: average portfolio concentration simulated for the status utility model with the share of private equity to total assets restricted to be either zero or at least 20 percent. Benchmark case has $\beta_{R^i,R^a} = 0.5$; the alternative case features negative aggregate risk exposure of private equity, $\beta_{R^i,R^a} = -0.5$.

Table V: **Wealth mobility**

Panel A: data

Wealth quantile	Bottom half	50-90	90-95	95-99	Top 1 percent
Move down	0.00	0.19	0.43	0.52	0.33
Stay	0.89	0.73	0.32	0.44	0.67
Move up	0.11	0.07	0.25	0.04	0.00

Panel B: status model

Wealth quantile	Bottom half	50-90	90-95	95-99	Top 1 percent
Move down	0.00	0.04	0.11	0.18	0.33
Stay	0.98	0.95	0.71	0.71	0.67
Move up	0.02	0.02	0.18	0.11	0.00

Panel C: CRRA

Wealth quantile	Bottom half	50-90	90-95	95-99	Top 1 percent
Move down	0.00	0.04	0.13	0.09	0.05
Stay	0.98	0.94	0.77	0.86	0.95
Move up	0.02	0.02	0.10	0.05	0.00

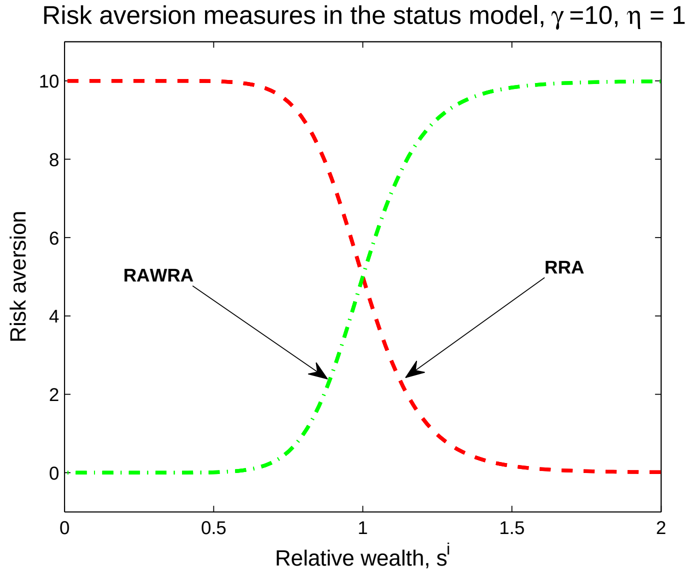
Probabilities of transition between wealth percentile groups over 10-year periods.
Panel A: transition probabilities estimated using the PSID wealth supplement data for 1984-1999.
Panel B: transition probabilities simulated in the status model with $\gamma = 10$, $\eta = 1$
Panel C: transition probabilities simulated in the CRRA model with $\gamma = 8$, $\eta = 0$

Table VI: **Consumption as a share of wealth, per 10-year period, by age**

Wealth percentile	0-25	25-50	50-75	75-90	90-100
20 years old	45	43	31	20	11
50 years old	45	45	34	20	12
80 years old	60	58	40	25	12

Consumption as a share of beginning-of-period wealth simulated from the social status model with $\gamma = 10$, $\eta = 1$.

Figure 1: **Risk aversion measures**



Coefficients of relative risk aversion (RRA) and relative aversion to aggregate wealth risk (RAWRA) as a function of relative wealth, $s^i = \frac{W^i}{\bar{W}}$, in a one-period version of the social status model with $\gamma = 10$ and $\eta = 1$.

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