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# Governance Through Exit and Voice: A Theory of Multiple Blockholders<sup>\*</sup>

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#### Abstract

Traditional blockholder theories advocate concentrating outside equity with a single large shareholder, to provide strong incentives to undertake value-enhancing interventions (engage in "voice"). However, most firms in reality are held by multiple small blockholders. This paper shows that, while such a structure generates free-rider problems that hinder voice, the same coordination difficulties strengthen a second governance mechanism: disciplining the manager through trading (engaging in "exit"). Since multiple blockholders cannot co-ordinate to limit their trades and maximize combined trading profits, competition among them impounds more information into prices. This makes the threat of disciplinary exit more credible, thus inducing higher managerial effort. The optimal blockholder structure depends on the relative effectiveness of manager and blockholder effort, the complementarities in their outputs, liquidity, monitoring costs, and the manager's contract.

KEYWORDS: Multiple blockholders, corporate governance, market efficiency, exit, voice, free-rider problem, Wall Street Rule, voting with your feet

JEL CLASSIFICATION: D82, G14, G32

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# 1 Introduction

Corporate governance can have substantial effects on firm value. Through ensuring that managers act in shareholders' interest, it can minimize the agency costs arising from the separation of ownership and control (Berle and Means (1932)). In turn, traditional theories argue that concentrated ownership is critical for effective governance, since only large investors have incentives to monitor the manager and, if necessary, intervene to correct value-destructive actions.

However, most firms in reality have multiple small blockholders (see, e.g., Zwiebel (1995), Barca and Becht (2001), Faccio and Lang (2002), Maury and Pajuste (2005), Holderness (2007), and Gregoric et al. (2008)). Such a structure appears to be suboptimal, as splitting equity between numerous shareholders leads to a free-rider problem: each investor individually has insufficient incentives to bear the cost of monitoring, and shareholders cannot co-ordinate to share this cost.

Are most firms indeed poorly governed, and should the government design policies to encourage more concentrated stakes, consistent with existing models? This paper demonstrates that multiple blockholders may in fact be an optimal shareholding structure. While splitting a block reduces the effectiveness of blockholder direct intervention ("voice"), we show that it increases the power of a second governance mechanism: "exit".<sup>1</sup> By trading on private information, blockholders move the stock price towards fundamental value, and thus cause it to more closely reflect the effort exerted by the manager to enhance firm value. Blockholders can punish a shirking manager ex post by following the "Wall Street Rule" of "voting with their feet" and selling, which drives down the stock price. However, such a mechanism only elicits effort ex ante if it is dynamically consistent. Once effort has been exerted, blockholders cannot change the manager's action and are only concerned with maximizing their trading profits. A single blockholder will strategically limit her order to reduce the revelation of her private information. This optimizes her profit but also lowers the extent to which prices reflect fundamental value, and thus effort. By contrast, multiple blockholders trade aggressively to compete for profits, as in a Cournot oligopoly. Total quantities (here, trading volumes) are higher than under monopoly, leading to more information being impounded in prices. Multiple blockholders thus serve as a commitment device to reward or punish the manager ex post for his actions.

The co-ordination problems and externalities created by splitting a block play oppos-

<sup>&</sup>lt;sup>1</sup>Prior papers on blockholder trading focus on the "Wall Street Rule" (the possibility of blockholder exit), rather than additional purchases. For example, Hirshman's (1970) book is titled "Exit, Voice, and Loyalty", and the models of Admati and Pfleiderer (2007) and Edmans (2007) only analyze block disposal, not enhancement. Although the blockholder can buy as well as sell in this paper, we use the term "exit" to describe the blockholder's influence on managerial decisions through her trading (in either direction), to be consistent with prior literature.

ing roles in "voice" and "exit." For "voice", the externalities are positive: intervention improves the value of other shareholders' stakes, but this effect is not internalized by the individual blockholder. Since co-ordination problems lead to positive externalities being ignored, there is "too little" intervention with multiple blockholders. For "exit", the externalities are negative. Higher trading volumes reveal more information to the market maker, leading to a less attractive price for other informed traders. Since blockholders optimize their individual trading profits and ignore these externalities, they trade "too much" from the viewpoint of maximizing combined profits. However, firm value does not depend on trading profits; total surplus is also unaffected as such profits are a mere transfer from liquidity traders to blockholders. Instead, "too much" trading is beneficial as it increases price informativeness and induces effort ex ante.<sup>2</sup>

While a number of empirical papers use the ownership stake of the largest shareholder as a measure of both investor informedness and corporate governance, an implication of our paper is that moderately-sized shareholders can also play a significant role. Thus, it may be important to consider the total ownership of all shareholders who plausibly have private information, as well as their number.

We derive an interior solution for the optimal number of blockholders that maximizes firm value. This optimum arises from a trade-off between voice and exit: fewer blocks maximize intervention incentives, but more blocks increase trading. Therefore, the efficient number of blockholders is increasing in the effectiveness of managerial effort and decreasing in the value created by direct blockholder intervention. If blockholders are passive and non-interventionist, as is the case for most mutual funds, a large number is optimal. By contrast, if investors contribute significantly to the firm's operations (as often occurs with early-stage firms), concentrated ownership is efficient. The optimal number is also increasing in the manager's concerns for the short-term stock price, since this augments the feedback from stock price informativeness to ex ante managerial effort decisions.

In the core model, blockholders are automatically informed about firm value. We extend the model to allow for costly information acquisition. In equilibrium some blockholders may decide to stay uninformed, because their trading profits are insufficient to justify gathering information. Since uninformed blockholders do not engage in exit, and reduce intervention through the dilution of ownership, they unambiguously reduce firm value. Thus, the optimal number of blockholders is bounded above, to ensure that competition in trading is sufficiently low that trading profits are adequate to motivate information acquisition. If net trading profits increase, this bound is weakened and so the optimal number of blockholders rises. This in turn occurs if market liquidity and the blockholders' informational advantage increase, and monitoring costs fall.

 $<sup>^{2}</sup>$ The 2007 hedge fund crisis is a real-life example of the substantial price changes that result from multiple investors trading in the same direction.

An additional extension analyzes complementarities between blockholder and manager outputs, departing from the core case of perfect substitutes. One case is negative complementarities, where firm value depends on the higher of the output levels of the two parties rather than the combined output level. This may occur if the blockholders correct managerial shirking: firm value can be high even if the manager does not work, as long as the blockholders exert effort. Since only the higher output level matters, the optimum is determined entirely by the more effective action, and ignores trade-off considerations with the less effective action. The optimal number of blockholders is therefore either very low (if voice is relatively effective) or very high (if managerial effort is relatively effective).

An opposite case is perfect positive complementarities, where firm value depends on the minimum output level. Since managerial effort is only productive if it is accompanied by high blockholder effort (and vice versa), the optimal number of blockholders thus balances the output levels of the manager and blockholders. The effect of effort productivity changes direction: the optimal number is now decreasing in the effectiveness of managerial effort and increasing in the effectiveness of blockholder effort. If managerial effort is ineffective, a high number of blockholders is necessary to "boost" managerial output so that it is at a similar level to blockholder output.

Finally, we show that the firm value optimum may differ from the socially optimal number of blockholders that maximizes total surplus (firm value net of effort costs), and the private optimum that would be chosen by the blockholders to maximize their combined net payoffs. However, the comparative statics with respect to the effectiveness of manager and blockholder effort are the same for all three optima.

This paper is organized as follows. Section 2 is a brief literature review. Section 3 presents the model and analyzes the effect of blockholder structure on both "voice" and "exit". Section 4 derives the optimal number of blockholders and generates comparative static predictions. Section 5 extends the model to analyze costly information acquisition, complementarities and differences in the manager's contract, and Section 6 concludes.

# 2 Literature Review

The vast majority of blockholder models involve the large shareholder adding value through direct intervention, or "voice" as termed by Hirshman (1970). This can involve proposing profitable investment projects and business strategies, or overturning an inefficient managerial action. In Shleifer and Vishny (1986), Admati, Pfleiderer, and Zechner (1994), Maug (1998), Kahn and Winton (1998) and Mello and Repullo (2004), a larger block is unambiguously more desirable as it addresses the free-rider problem and maximizes the blockholder's incentives to intervene. Burkart, Gromb and Panunzi (1997) note that, although beneficial ex post, blockholder intervention may be undesirable ex

ante as it discourages managerial initiative. The optimal block size is therefore finite.<sup>3</sup> While Burkart et al. consider a single blockholder, Pagano and Roell (1998) point out that if this finite optimum is lower than the total amount of external financing required, the entrepreneur will need to raise funds from additional shareholders. Although this leads to a multiple blockholder structure, the extra blockholders play an entirely passive role: they are merely a "budget-breaker" to provide the remaining funds. Replacing the additional blockholders by creditors or dispersed shareholders would have the same effect. In this paper, all blockholders play an active role. Bolton and von Thadden (1998) and Faure-Grimaud and Gromb (2004) achieve a finite optimum through a different channel, as too large a block reduces stock market liquidity. Again, they only advocate one blockholder.<sup>4</sup> In addition, the present paper holds the total equity held by blockholders (and thus the free float) fixed, and focuses on the division of this block between one or multiple blockholders. Liquidity is thus unaffected by the splitting of a block.

Two recent papers by Admati and Pfleiderer (2007) and Edmans (2007) analyze an alternative mechanism through which blockholders can add value: "exit". Informed trading causes prices to more accurately reflect fundamental value, in turn inducing the manager to undertake actions that enhance value. Both models consider a single blockholder and do not feature "voice". To our knowledge, this is the first theory that analyzes both governance mechanisms of exit and voice, and the tradeoffs between them.<sup>5</sup> Parrino, Sias and Starks (2003) and Chen, Harford and Li (2007) provide empirical evidence supporting the notion that institutions are better informed and use their superior information to vote with their feet. Sias, Starks and Titman (2006) shows that such trading has a causal effect on stock price; similarly, Scholes (1972) and Mikkelson and Partch (1985) show that the negative stock price reaction to secondary block distributions is due to information, rather than the sudden increase in supply or a reduction

 $<sup>^{3}</sup>$ An alternative explanation is that, while a single concentrated block would be first-best in an unconstrained world, multiple small blockholders are a second-best optimum in the presence of blockholder wealth constraints and risk aversion (e.g. Winton (1993)). While these frictions are plausible explanations for small managerial stakes, they are likely weaker for outside shareholders. In particular, institutional investors can hold sizable stakes since they have substantial capital and are held by thousands of individual shareholders, who can diversify away any idiosyncratic risk associated with high exposure to one particular corporation.

<sup>&</sup>lt;sup>4</sup>Bolton and von Thadden (1998) do mention that their model might be extended to incorporate more than one non-atomistic shareholder, but suspect that it is "dominated either by full dispersion or by a [single blockholder] structure." Hence they do not derive multiple blockholders as being optimal.

<sup>&</sup>lt;sup>5</sup>Maug (1998, 2002), Kahn and Winton (1998), Mello and Repullo (2004), and Brav and Mathews (2007) allow the blockholder to sell her stake instead of intervening. However, exit does not exert governance on the manager in these models, as there is no feedback from the stock price to firm value: governance is only through voice. Duan (2007) empirically studies the choice between exit and voice (through voting).

in expected blockholder monitoring. Lowenstein (1988) argues that governance through exit is particularly important and common among U.S. investors, especially since they often face legal and institutional hurdles to intervention (see, e.g., Black (1990) and Becht et al. (2007)). Smith and Swan (2008) test whether institutional trading is successful at restraining executive compensation. They find that multiple moderately-sized investors with frequent trading have greatest effect; institutional concentration only matters insofar as it affects trading activity.

Most existing theories of multiple blockholders focus on their extraction of private benefits and the resulting control contests; in our paper, blockholders add value through exerting effort or increasing stock price informativeness. Zwiebel (1995) shows that multiple blockholdings can arise when shareholders compete for the private benefits of control by forming coalitions. The final shareholding structure represents the outcome of a power struggle, whereas in our paper the number of blockholders is optimally chosen to maximize firm value. In Bennedsen and Wolfenzon (2000), multiple blockholders can be optimal as they compete with the manager to divert cash flow, reducing the amount of stealing that the manager is able to undertake. Gomes and Novaes (2006) note that blockholders can either monitor and intervene, or extract private benefits. They derive the optimal shareholding structure in the presence of these two forces. In Müller and Wärneryd (2001), multiple outside owners extract private benefits, reducing the amount of resources over which inside managers can bargain, and thus lessening internal conflict. Maury and Pajuste (2005) consider blockholders who can either steal or monitor; again, multiple blocks arise out of a desire to form a controlling coalition and divert cash flows.

Like us, Noe (2002) models multiple blockholders as enhancing firm value rather than extracting private benefits. Owing to the free-rider problem, each blockholder has an insufficient stake to justify intervention, but supplements her return through trading profits. Unlike in the current model, trading profits do not arise from private information on firm value, but because the blockholder knows whether or not she has intervened. Moreover, there is no feedback from stock prices to ex ante managerial effort, so blockholder exit does not exert governance.

Finally, Bolton and Scharfstein (1996) also demonstrate that free-rider problems among investors can improve firm value. A multiple creditor structure can dominate a single lender, since the resulting co-ordination problems hinder efficient renegotiation in default. This deters the manager from strategically defaulting, and thus makes creditors more willing to lend. In our paper, the benefits of co-ordination problems manifest through informed trading and the effect on stock prices.

Action Stage	Trading Stage
1. Manager takes unobservable action $a$ at cost $a$	1. Blockholder <i>i</i> submits order flow $x_i(\tilde{v})$ .
2. Blockholder takes observable action $b_i$ at cost $b_i$ .	2. Liquidity traders submit order flow $\varepsilon$ .
3. Blockholder <i>i</i> observes firm value $\tilde{v} = \phi_a \log a + \phi_b \log \sum_i b_i + \tilde{\eta}$	3. Market maker observes total order flow $y = \sum_{i} x_{i} + \varepsilon$ and sets price $p = E[\tilde{v} \tilde{y}]$ .

Figure 1: Timeline of the model

# **3** Model and Analysis

In this section, we introduce a model in which blockholders can exert governance either by direct intervention or by trading shares. The model consists of a game between the manager, a market maker and the I blockholders of the firm. The game has two stages, and the timeline is given in Figure 1.

In the first stage, the manager and blockholders take actions that affect firm value. Firm value is given by

$$\tilde{v} = \phi_a \log a + \phi_b \log \sum_i b_i + \tilde{\eta},\tag{1}$$

where a represents the action taken by the manager,  $b_i$  represent the action taken by blockholder *i*, and  $\tilde{\eta}$  is normally distributed noise with mean zero and variance  $\sigma_{\eta}^2$ . The manager incurs personal cost *a* when taking action *a*, while each blockholder *i* incurs personal cost  $b_i$  when taking action  $b_i$ . The manager's action is broadly defined to encompass any decision that benefits firm value but is personally costly, such as exerting effort or forgoing private benefits and pet projects. Similarly, the blockholder's action can involve effort that directly helps the firm (advising the manager), effort that indirectly helps the firm (deterring managerial rent extraction) or choosing not to take private benefits.<sup>6</sup> The parameter  $\phi_a$  ( $\phi_b$ ) measures the productivity of manager (blockholder) effort per unit cost. We use the term "effort" to refer to *a* and  $b_i$  and "output" to refer to  $\phi_a \log a$  and  $\phi_b \log \sum_i b_i$ , i.e. effort scaled by its productivity.

In the core model, the manager's and blockholders' actions are perfect substitutes, i.e. have independent effects on firm value. In other words, the productivity of the manager's effort does not depend on the level of blockholder effort, and vice-versa. This

<sup>&</sup>lt;sup>6</sup>See Barclay and Holderness (1991) for a description of the private benefits that blockholders can extract.

appears to be a plausible specification for a large fraction of firms, and for the manager and blockholder actions: for example, extraction of private benefits by blockholders is detrimental to firm value regardless of the manager's effort. However, in some situations, there may be positive or negative complementarities between the manager's and blockholders' actions. These are analyzed in Section 5.2.

Firm value depends on the logarithm of the combined blockholder effort level, and the action has a linear cost to each blockholder. We choose this functional form to ensure that adding blockholders does not change the available technology. The common assumption of a quadratic cost (and a linear, rather than logarithmic effect of  $b_i$  on  $\tilde{v}$ ) is inappropriate here: with a convex cost function, the blockholders' technology would improve if there are multiple small blockholders (since each would be operating at the low marginal cost part of the curve). A single blockholder would be able to reduce monitoring costs by dividing herself up into multiple small "units", and increase total effort. Instead, the linear costs form means that there are no mechanical reduction in monitoring costs from splitting a block.

Action a is privately observed by the manager, while actions  $b_i$  are publicly observed. The assumption that a is privately observable is standard, and necessary for a moral hazard problem to exist in the first place. Without hidden actions, effort would be contractible. By contrast, the assumption that  $b_i$  is observable is made purely for tractability. The impact of blockholder numbers on free-rider problems in intervention, and competition in trading, is not affected by this assumption.<sup>7</sup>

We normalize the number of shares outstanding to 1. The manager holds  $\alpha$  shares of the firm, and each blockholder holds  $\beta/I$  shares. In our model, the total fraction held by outside blockholders is a constant  $\beta$  and the analysis focuses exclusively on the optimal number of investors among which it is distributed. This separates our paper from previous literature that analyzes the efficient level  $\beta$  of concentrated ownership (e.g. Shleifer and Vishny (1986), Burkart, Gromb and Panunzi (1997), Maug (1998), Bolton and von Thadden (1998), Edmans (2007)), in particular its effect on liquidity. In our paper, free float is fixed at  $1 - \alpha - \beta$  and plays no role.

In the second stage of the game, the blockholders, noise traders, and a market maker trade the firm's equity. As in Admati and Pfleiderer (2007), each blockholder is assumed to observe firm value  $\tilde{v}$  perfectly, while noise traders are uninformed. The blockholders' superior information can be motivated by a number of underlying assumptions. They may have greater access to information than atomistic outsiders by virtue of their large stakes: given their voting power, management will be more willing to meet with them. Blockholders may be more informed even if they only have access to public information.

<sup>&</sup>lt;sup>7</sup>If  $b_i$  is unobserved, the analysis becomes less transparent. As in Maug (1998, 2002) and Kahn and Winton (1998), each blockholder's equilibrium strategy will be to randomize between intervention and non-intervention, and the market maker's pricing rule will reflect this.

Even if information is freely available, analyzing the implications of this information for firm value is costly.<sup>8</sup> Blockholders have stronger incentives to engage in costly information analysis if there are short-sales constraints (or any non-trivial short-sales costs). Information is more useful to them, since they can sell more if it turns out to be negative – hence they have a greater incentive to acquire it in the first place (Edmans (2007)). The results are qualitatively unchanged if each blockholder obtains an imperfect signal of  $\tilde{v}$ : we only require that blockholders have superior information to atomistic investors.

After observing  $\tilde{v}$ , each blockholder submits a market order  $x_i(\tilde{v})$ . Noise traders submit market orders with a normally distributed net quantity  $\tilde{\epsilon}$ , with mean zero and variance  $\sigma_{\varepsilon}^2$ . We use the term "liquidity" to refer to the standard deviation of noise trader demand,  $\sigma_{\varepsilon}$ . After observing total order flow  $\tilde{y} = \sum_i \tilde{x}_i + \tilde{\epsilon}$ , the market maker determines the price  $\tilde{p}$  and trades the quantity necessary to clear the market. Due to perfect competition, the market maker sets  $\tilde{p}$  so that he earns zero profits, i.e. the price equals expected firm value given the order flow.

The manager is risk-neutral and his objective is to maximize the market value of his shares less the cost of effort, i.e.  $\alpha \tilde{p} - a$ .<sup>9</sup> Each blockholder's objective is to maximize her trading profits, plus the fundamental value of her shares, less her cost of effort.

We solve for the equilibrium of the game by backward induction.

### 3.1 The Trading Stage

To proceed by backward induction, we take the decisions a of the manager and  $b_i$  of the blockholders as given. The trading stage of the game is similar to the speculative trading model of Kyle (1985) and its extensions to multiple informed investors.<sup>10</sup>

**Proposition 1** (Trading Equilibrium): The unique linear equilibrium of the trading stage is symmetric and has the form:

$$x_i(\tilde{v}) = \gamma(\tilde{v} - \phi_a \log a - \phi_b \log \sum_i b_i) \quad \forall i$$
(2)

$$p(\tilde{y}) = \phi_a \log a + \phi_b \log \sum_i b_i + \lambda \tilde{y}, \tag{3}$$

<sup>&</sup>lt;sup>8</sup>For example, equity research analysts and mutual funds use only public information to form their own financial projections and valuations. Since a wide range of information is potentially value-relevant, such analysis requires substantial time and expertise.

<sup>&</sup>lt;sup>9</sup>In Section 5.3 we extend the model to allow the manager to be compensated according to both the stock price and fundamental value.

<sup>&</sup>lt;sup>10</sup>See, for example, Kyle (1984), Admati and Pfleiderer (1988), Holden and Subrahmanyam (1992) and Foster and Viswanathan (1993).

where

$$\lambda = \frac{\sqrt{I}}{I+1} \frac{\sigma_{\eta}}{\sigma_{\epsilon}} \tag{4}$$

$$\gamma = \frac{1}{\sqrt{I}} \frac{\sigma_{\epsilon}}{\sigma_{\eta}}.$$
(5)

**Proof** If the market maker uses a linear pricing rule of the form  $p(y) = \mu + \lambda y$  then the *i*th blockholder maximizes:

$$E[(\tilde{v} - \mu - \lambda y)x_i | \tilde{v} = v] = (v - \mu - \lambda \sum_{j \neq i} x_j)x_i - \lambda x_i^2.$$

This maximization problem yields

$$x_i(v) = \frac{1}{\lambda} [v - \mu - \lambda \sum_j x_j(v)] \quad \forall i.$$

The strategies of the blockholders are then symmetric and we thus have

$$x_i(v) = \frac{1}{(I+1)\lambda}(v-\mu) \quad \forall i.$$

The market maker takes the blockholders' strategy as given and sets

$$p(y) = E[\tilde{v}|y]. \tag{6}$$

Using the normality of  $\tilde{v}$  and  $\tilde{y}$  yields

$$\lambda = \frac{\sqrt{I}}{I+1} \frac{\sigma_{\eta}}{\sigma_{\epsilon}},$$
$$\mu = \phi_a \log a + \phi_b \log \sum_i b_i.$$

From this we obtain:

$$x_i(v) = \frac{1}{\sqrt{I}} \frac{\sigma_{\epsilon}}{\sigma_{\eta}} (v - \phi_a \log a - \phi_b \log \sum_i b_i) \quad \forall i,$$
$$p(y) = \phi_a \log a + \phi_b \log \sum_i b_i + \frac{\sqrt{I}}{I + 1} \frac{\sigma_{\eta}}{\sigma_{\epsilon}} y,$$

Q.E.D. ■

From Proposition 1, each blockholder's trading profits are

$$\frac{1}{\sqrt{I}(I+1)}\sigma_{\eta}\sigma_{\epsilon}.$$
(7)

Trading profits are increasing in  $\sigma_{\eta}$  and  $\sigma_{\epsilon}$ , as  $\sigma_{\eta}$  reflects the blockholders' informational advantage and  $\sigma_{\epsilon}$  represents their ability to profit from information by trading with liquidity investors. In addition, aggregate blockholder trading profits are decreasing in the number I of blockholders. This is because multiple blockholders compete as in a Cournot oligopoly. Each blockholder chooses her trading volume to maximize individual profits. A higher volume reveals more information and makes the price less attractive to all informed traders, but she ignores this negative externality and so trades in excess of the level that would maximize combined blockholder profits.

While greater trading volumes reduce aggregate profits, they also impound more information into prices. The following two propositions introduce three measures of price informativeness, show that these measures of price informativeness are equivalent, and show that price informativeness increases with the number I of blockholders.

**Proposition 2** The following three measures of price informativeness are equivalent:

- 1.  $(\operatorname{Var}(\tilde{v}) \operatorname{Var}(\tilde{v}|\tilde{p})) / \operatorname{Var}(\tilde{v}).$
- 2.  $\operatorname{Corr}(\tilde{v}, \tilde{p})^2$ .
- 3. E  $\left[\frac{d\tilde{p}}{dv}\right]$

**Proof** Using the formula for the conditional variance of a bivariate normal distribution

$$\operatorname{Var}(\tilde{v}|\tilde{p}) = (1 - \operatorname{Corr}(\tilde{v}, \tilde{p})^2) \operatorname{Var}(\tilde{v})$$

we immediately have that the first two measures of price informativeness are equivalent. We then only need to show that the last two measures are equivalent. Since, in equilibrium, the price is a linear function of  $\tilde{v}$  and  $\tilde{\epsilon}$ ,

$$\mathbf{E}\left[\frac{d\tilde{p}}{dv}\right] = \frac{\mathrm{Cov}(\tilde{v},\tilde{p})}{\mathrm{Var}(\tilde{v})}.$$

From the law of iterated expectations and (6),

$$\operatorname{Var}(\tilde{p}) = \operatorname{Cov}(\tilde{v}, \tilde{p}).$$

Therefore,

$$\operatorname{Corr}(\tilde{v}, \tilde{p})^2 = \operatorname{E}\left[\frac{d\tilde{p}}{dv}\right],$$

as we wanted to show.  $\blacksquare$ 

The next proposition calculates price informativeness in the equilibrium derived in Proposition 1.

**Proposition 3** Price informativeness, as defined by any of the above measures, is equal to I/(I+1).

**Proof** The result follows from equations (2), (3), (4), and (5).

The first of the three measures is the standard measure of price informativeness used in the market microstructure literature. It represents the proportion of the variance of  $\tilde{v}$  that is explained by prices, and was previously shown to be increasing in the number of informed traders by Kyle (1984). The second of the three measures is the squared coefficient of correlation between  $\tilde{v}$  and  $\tilde{p}$ , which depicts the strength of the linear relation between  $\tilde{v}$  and  $\tilde{p}$ . We introduce a third measure of price informativeness,  $E\left[\frac{d\hat{p}}{dx}\right]$ . This illustrates the extent to which changes in fundamental value manifest in the current stock price, and is thus particularly relevant for our corporate finance setting. It captures the incentives for an agent compensated according to the stock price to improve fundamental value, and thus will later be used to derive the manager's optimal action. We show that this new measure of price informativeness is identical to the two alternative measures, and is thus also increasing in I. In the extreme, as I approaches infinity, prices become fully informative. On the other hand, in the monopolistic Kyle model (I = 1), the blockholder fully internalizes the effect of a higher trading volume on profits. She limits her order, thus leading to a price informativeness of  $\frac{1}{2}$ : prices reveal only one-half of the insider's private information. Empirically, this suggests that price informativeness may depend not only on total institutional ownership, but also on the number of sizable shareholders.<sup>11</sup>

The positive link between the number of blockholders and price informativeness does not arise because a greater number of informed agents mechanically leads to an increase in the amount of information in the market. Indeed, a single blockholder already has a perfect signal of fundamental value; since she faces no trading constraints, she could theoretically impound this entire information into prices. Changing I has no effect on the information held by shareholders. The result arises instead from competition in trading.

We also note that liquidity  $\sigma_{\varepsilon}$  has no effect on price informativeness. From equation (5), greater noise trading allows blockholders to trade more aggressively. This increase in informed trading exactly counterbalances the effect of increased noise and leaves price informativeness unchanged.

<sup>&</sup>lt;sup>11</sup>Oehmke (2007) shows that competition between prime brokers in liquidating collateral reduces sale proceeds and may encourage hedge funds to concentrate collateral with a single broker. Agarwal (2007) and Boehmer and Kelley (2007) find empirically that competition among institutional traders increases price informativeness.

### 3.2 The Action Stage

We now solve for the optimal actions of the manager and the blockholders in the first stage.

**Proposition 4** (Optimal Actions): The manager's optimal action is

$$a = \phi_a \alpha \left(\frac{I}{I+1}\right) \tag{8}$$

and the optimal action of each blockholder is

$$b_i = \phi_b \beta \left(\frac{1}{I}\right)^2. \tag{9}$$

**Proof** The manager maximizes the market value of his shares, less the cost of effort:

$$E\left[\alpha \widetilde{p}-a\right].\tag{10}$$

When setting the price  $\tilde{p}$ , the market maker takes the equilibrium action a of the manager as given. Therefore, the manager's action affects the price only through its influence on  $\tilde{v}$ , and consequently on the order flow submitted by the informed blockholders. Using the chain rule, the first order condition that determines the manager's action is given by:

$$\alpha \left( \mathbf{E} \left[ \frac{d\tilde{p}}{dv} \right] \right) \left( \frac{\phi_a}{a} \right) - 1 = 0.$$
(11)

From Proposition 2, the manager's optimal action is therefore

$$a = \alpha \left(\frac{I}{I+1}\right) \phi_a.$$

Each blockholder maximizes her trading profits, plus the fundamental value of her shares, less her cost of effort. From (7), we know that the blockholder's trading profits do not depend on the action she has taken in the first stage.<sup>12</sup> Therefore, blockholder i simply chooses  $b_i$  to maximize the fundamental value of her shares, less her cost of effort:

$$E\left[\left(\frac{\beta}{I}\right)\widetilde{v} - b_i\right].\tag{12}$$

<sup>&</sup>lt;sup>12</sup>This is because the blockholder's action is publicly observable. Informed trading profits depend on the blockholder's relative information advantage, and this is unaffected by a publicly observable variable. See Kahn and Winton (1998) for a model where the blockholder's action is unobservable.

The optimal action of blockholder i is

$$b_i = \phi_b \beta \left(\frac{1}{I}\right)^2,\tag{13}$$

and the proof is complete.  $\blacksquare$ 

The manager's action a is the product of three variables: the effectiveness of effort  $\phi_a$ , his equity stake  $\alpha$ , and price informativeness  $\frac{I}{I+1}$ . It is thus increasing in the number I of blockholders as this augments price informativeness. The intuition is as follows. Greater price informativeness (a higher  $E\left[\frac{d\tilde{p}}{dv}\right]$ ) implies that the stock price more closely reflects the firm's fundamental value, and consequently the manager's effort. Therefore, the manager is more willing to bear the cost of working. In effect, blockholder trading rewards managerial effort ex post, therefore inducing it ex ante. The dynamic consistency of this reward mechanism depends on the number of blockholders. Critically, trading occurs after the manager has taken his action, at which point the action cannot be undone and shareholders are concerned only with maximizing their trading profits. A single blockholder optimizes her profits by limiting her order, at the expense of price informativeness. Therefore, the promise of rewarding effort by bidding up the price to fundamental value is not credible. By contrast, multiple blockholders trade aggressively, augmenting price informativeness, and thus constitute a commitment device to reward the manager ex post for his actions. While such aggressive trading is motivated purely by the private desire to maximize individual profits in the presence of competition, it has a social benefit by eliciting effort ex ante.<sup>13</sup>

In sum, multiple blockholders lead to greater trading volumes. This both reduces aggregate profits and impounds more information into prices. Since firm value is increasing in price informativeness (as it induces effort ex ante) and independent of trading profits (which are a pure transfer from atomistic shareholders to blockholders), higher trading volumes lead overall to an increase in firm value. While a number of empirical papers use the percentage ownership of the largest shareholder as a measure for corporate governance, this result suggests that moderately-sized shareholders can also play an important role. (In most other models, additional blockholders extract private benefits rather than exerting governance). Hence, the total ownership of sizable shareholders, and their number, may be more relevant measures.

As in earlier models, combined blockholder effort  $\sum_i b_i$  of the blockholders is decreasing in I, owing to the free-rider problem. Therefore, there is a trade-off between the intervention and trading effects.

<sup>&</sup>lt;sup>13</sup>Fishman and Hagerty (1995) also show that introducing additional informed traders is a commitment to trading more aggressively. They use this result to show that, if there are multiple informed agents, a specific informed agent will sell her information to other traders, rather than only exploiting it herself.

### 4 The Optimal Number of Blockholders

In this section we derive the optimal number of blockholders. We start by deriving the optimal number that maximizes firm value, and later discuss the social optimum (that maximizes total surplus, taking into account the costs borne by the manager and blockholder) and the private optimum (that maximizes the total payoff to blockholders).

**Proposition 5** (Firm Value Optimum): The number  $I^*$  of blockholders that maximizes firm value is:

$$I^* = \max\left[1, \frac{\phi_a - \phi_b}{\phi_b}\right].$$
(14)

**Proof** Using the results of Proposition 4, we can write the expected value of the firm as

$$E[\widetilde{v}] = \phi_a \log \left[ \phi_a \alpha \left( \frac{I}{I+1} \right) \right] + \phi_b \log \left[ \phi_b \beta \left( \frac{1}{I} \right) \right].$$
(15)

We need to maximize the above expression with respect to I. The first order condition is given by:

$$\frac{\phi_a - \phi_b - \phi_b I}{I + I^2} = 0.$$
 (16)

 $\hat{I} = (\phi_a - \phi_b)/\phi_b$  satisfies the first order condition. Since the left hand side of (16) is positive for  $I < \hat{I}$  and negative for  $I > \hat{I}$ ,  $I^*$  is indeed a maximum.

The optimal number of blockholders solves the trade-off between the positive effect of more blockholders on managerial effort, and the negative effect on blockholder intervention. The optimum is therefore increasing in  $\phi_a$ , the productivity of the manager's effort, and declining in  $\phi_b$ , the productivity of blockholder intervention.

The magnitude of  $\phi_b$  depends on the nature of blockholders' expertise. Using the terminology of Dow and Gorton (1997), if blockholders have forward-looking ("prospective") information about optimal future investment decisions or strategic choices, direct intervention is particularly valuable and  $\phi_b$  is high. For example, venture capital financiers are typically expert in managing start-up businesses and their effort directly affects the firm's prospects; indeed, venture capital typically features a small number of highly concentrated shareholders and they retain large stakes even after the firm goes public and the "exit" governance mechanism becomes available. On the other hand, if blockholders do not have specialist expertise in how to manage the company but instead are skilled at gathering backward-looking ("retrospective") information to evaluate the effect of past decisions on firm value, their primary contribution is to impound the effects of prior managerial effort into the stock price. In such a case,  $\phi_b$  is low and  $I^*$  is high. As firms mature, active venture capitalist investors are typically replaced by passive institutional shareholders, and the number of blockholders usually increases.

Another determinant of  $\phi_b$  is blockholders' control rights and thus ability to intervene (holding constant the size of their individual stakes).<sup>14</sup> Black (1990) and Becht et al. (2007) note that U.S. shareholders face substantial and institutional hurdles to intervention, compared to their foreign counterparts. This reduces  $\phi_b$ , thus increasing  $I^*$ , and is consistent with the fact that firms in the U.S. typically have smaller and more numerous blockholders compared to overseas.

Given our broad definition of managerial effort (to encompass any action that increases firm value but is personally costly to the manager), a high  $\phi_a$  can result from a number of underlying factors. Since *a* can measure (the negative of) managerial rent extraction,  $\phi_a$  will be high if there are significant agency problems, since the firm value gains from solving such problems are large. Agency problems will be large if the firm has high free cash flow, or there are weak alternative governance mechanisms (e.g. captured boards or low leverage). However, poor governance also raises the scope for blockholder value added through direct intervention to solve such problems, and thus  $\phi_b$ .<sup>15</sup> In sum, the potency of other governance mechanisms affects the scope for blockholders to add value through both "voice" and "exit", rather than the trade-off between them, and so has an ambiguous effect on  $I^*$ .

The literal interpretation of a as effort may allow identification of determinants of  $\phi_a$  that do not also affect  $\phi_b$ , such as managerial talent. This may be directly measured using managerial characteristics (such as education, experience or past performance) or proxied using firm size (see, e.g., Gabaix and Landier (2008)). Overall, we caveat that, while the model generates clear, closed-form predictions for the dependence of  $I^*$  on  $\phi_a$  and  $\phi_b$ , empirical testing of the model is non-trivial since these parameters cannot be measured directly. The key challenge for empiricists is to identify accurate proxies for these variables.

While Proposition 5 is concerned with maximizing firm value, the social optimum maximizes total surplus, which also takes into account the costs of the manager's and blockholders' actions. Informed trading profits do not affect total surplus, since they are a transfer from liquidity traders to the blockholders. In theory, the social optimum would be chosen by a hypothetical social planner. If the noise traders are the firm's atomistic shareholders (as in Kahn and Winton (1998) and Bolton and von Thadden (1998)), it will also be chosen by the initial owner when taking the firm public, since IPO proceeds will equal total surplus. The owner will have to compensate the blockholders (in the form of a lower issue price) for their expected intervention costs, and the manager for his effort in the form of a higher wage. However, trading profits have no effect on

 $<sup>^{14}</sup>$ In reality, control rights will also be increasing in the size of each blockholder's individual stake. This will reinforce the negative effect of I on intervention currently in this paper.

<sup>&</sup>lt;sup>15</sup>In Section 5.2, we consider complementarities between blockholder and manager efforts, where the productivity of the blockholders' action depends on the level of managerial shirking.

IPO proceeds: while blockholders will pay a premium in expectation of trading gains, small shareholders will demand discounts to offset their future trading losses.

**Proposition 6** (Social Optimum): Let  $\tilde{I}$  represent the unique positive solution to

$$\frac{\phi_a}{I(I+1)} - \frac{\phi_b}{I} - \frac{\phi_a \alpha}{(I+1)^2} + \frac{\phi_b \beta}{I^2} = 0.$$
 (17)

The number  $I_{soc}^*$  of blockholders that maximizes total surplus is

$$I_{soc}^* = \max\left[1, \tilde{I}\right].$$
<sup>(18)</sup>

which may be higher or lower than  $I^*$ . Moreover,  $I^*_{soc}$  is increasing in  $\phi_a$  and  $\beta$ , and decreasing in  $\phi_b$  and  $\alpha$ .

**Proof** Total surplus is given by:

$$\phi_a \log \left[ \phi_a \alpha \left( \frac{I}{I+1} \right) \right] + \phi_b \log \left[ \phi_b \beta \left( \frac{1}{I} \right) \right] - \phi_a \alpha \left( \frac{I}{I+1} \right) - \phi_b \beta \frac{1}{I}.$$
(19)

Taking first-order conditions yields equation (17). The Appendix proves that there is a unique positive solution and that it maximizes (19).  $\blacksquare$ 

Compared to equation (15), equation (19) contains two additional terms. Increasing the number of blockholders raises the cost of managerial effort, but reduces the combined cost of blockholder monitoring. The social optimum may thus be higher or lower than the number that maximizes firm value. If  $\beta$  rises, total blockholder costs  $\phi_b \beta_{\overline{I}}^1$  enter more prominently in the social welfare function, and so  $I_{soc}^*$  rises to reduce these costs through exacerbating the free-rider problem. Conversely, a rise in  $\alpha$  increases the importance of the manager's costs and thus lowers  $I_{soc}^*$ . The comparative statics with respect to  $\phi_a$ and  $\phi_b$  are the same as in Proposition 5.

Finally, we analyze the privately optimal division of  $\beta$  that would maximize blockholders' combined payoffs. In other words, we ask the question: if blockholders in aggregate hold  $\beta\%$  of the firm, do they have incentives to split or combine stakes to achieve the number that maximizes either firm value or total surplus?

**Proposition 7** (Private Optimum): Let  $\hat{I}$  represent the unique positive solution to

$$\beta \left[ \frac{\phi_a}{I(I+1)} - \frac{\phi_b}{I} + \frac{\phi_b}{I^2} \right] - \frac{(I-1)}{2\sqrt{I}(I+1)^2} \sigma_\eta \sigma_\varepsilon = 0.$$
(20)

The number  $I_{priv}^*$  of blockholders that maximizes total blockholders' payoff is

$$I_{priv}^* = \max\left[1, \hat{I}\right].$$
(21)

which may be higher or lower than  $I^*$ , and higher or lower than  $I^*_{soc}$ . Moreover,  $I^*_{priv}$  is increasing in  $\phi_a$  and  $\beta$ , and decreasing in  $\phi_b$  and  $\sigma_\eta \sigma_{\varepsilon}$ .

**Proof** Total blockholders' payoff is given by:

$$\beta \left\{ \phi_a \log \left[ \phi_a \alpha \left( \frac{I}{I+1} \right) \right] + \phi_b \log \left[ \phi_b \beta \frac{1}{I} \right] \right\} - \phi_b \beta \frac{1}{I} + \frac{\sqrt{I}}{I+1} \sigma_\eta \sigma_\varepsilon.$$
(22)

Taking first-order conditions yields equation (20). The Appendix proves that there is a unique positive solution and that it maximizes (22).  $\blacksquare$ 

The blockholders' objective function differs from firm value in three ways. They only enjoy  $\beta\%$  of any increase in firm value; bear the costs of intervention; and are concerned with informed trading profits. Increasing *I* above *I*<sup>\*</sup> therefore has an ambiguous effect: it reduces the combined costs of intervention, but also reduces combined trading profits through exacerbating competition. Therefore, as with the social optimum, the private optimum may be higher or lower than the number that maximizes firm value. As with the social optimum, an increase in  $\beta$  augments blockholders' monitoring costs and  $I_{priv}^*$ . If  $\sigma_\eta \sigma_{\varepsilon}$  rises, trading profits become more important and so shareholders combine blocks to reduce competition.

The blockholders' objective function also differs from the social optimum in three ways. In addition to being concerned with informed trading profits and only  $\beta\%$  of firm value, they also ignore the cost of managerial effort. Again, the sum of these three effects is ambiguous. Increasing I above  $I_{soc}^*$  would both reduce total blockholder costs and total trading profits.

The comparative statics with respect to  $\phi_a$  and  $\phi_b$  are the same as in Propositions 5 and 6. Thus, the empirical predictions described earlier continue to hold even if blockholders can trade away from the structure chosen by the initial owner to maximize IPO proceeds.

### 5 Extensions

This section extends the main model in several directions and generate additional empirical implications.

### 5.1 Costly Information Acquisition

In the core model, the blockholders are endowed with private information about firm value  $\tilde{v}$ . In this subsection, we assume that blockholders are initially uninformed but can acquire perfect information about firm value  $\tilde{v}$  by paying a cost c in the first stage of the game. Blockholders that do not pay this cost will remain uninformed in the second stage.

To solve this modified version of the model, we again use backwards induction.

**Proposition 8** (Equilibrium With Costly Information): Let J be the number of blockholders that acquire information in the first stage of the game. Then in the unique linear equilibrium of the trading stage, the I - J uninformed blockholders do not trade. The Jinformed blockholders submit demands as in (2) and the market maker sets the price as in (3) with

$$\lambda = \frac{\sqrt{J}}{J+1} \frac{\sigma_{\eta}}{\sigma_{\epsilon}} \tag{23}$$

$$\gamma = \frac{1}{\sqrt{J}} \frac{\sigma_{\epsilon}}{\sigma_{\eta}}.$$
(24)

In the first stage of the game, the manager's optimal action is

$$a = \phi_a \alpha \left(\frac{J}{J+1}\right) \tag{25}$$

and the optimal action of each blockholder is

$$b_i = \phi_b \beta \left(\frac{1}{I}\right)^2. \tag{26}$$

The number J of blockholders that acquire information is

$$J = \min\{I, n\},\$$

where n is such that

$$\frac{1}{\sqrt{n(n+1)}}\sigma_{\eta}\sigma_{\epsilon} = c$$

**Proof** See Appendix.

Proposition 8 shows that when the number of blockholders I is sufficiently large (greater than n), some blockholders choose not to acquire information. If all blockholders became informed, competition in trading is sufficiently fierce that individual trading profits are insufficient to recoup the monitoring cost c. Hence, in equilibrium, some blockholders remain uninformed and do not participate in the trading stage of the game, earning zero trading profits.

We now analyze the optimal number of blockholders that maximizes firm value. We first observe that it is never optimal to have I greater than n. If I > n, then from Proposition 8 we know that some blockholders will not acquire information in equilibrium. Uninformed blockholders do not trade and thus have no effect on governance through exit. Moreover, they dilute ownership and reduce incentives to engage in voice. Uninformed blockholders are thus unambiguously detrimental to firm value, and so the optimum involves no such blockholders. This leads to the next proposition.

**Proposition 9** (Firm Value Optimum With Costly Information): The optimal number  $I_{costly}^*$  of blockholders that maximizes firm value with costly information acquisition is equal to

$$I_{costly}^* = \max\left[1, \min\left(\frac{\phi_a - \phi_b}{\phi_b}, n\right)\right].$$
 (27)

**Proof** See Appendix.

**Corollary 1** If  $\min\left(\frac{\phi_a-\phi_b}{\phi_b},n\right) > 1$  and  $n < \frac{\phi_a-\phi_b}{\phi_b}$ , firm value is increasing in  $\sigma_\eta$  and  $\sigma_{\varepsilon}$  and decreasing in c. If  $n \geq \frac{\phi_a-\phi_b}{\phi_b}$ , firm value is independent of  $\sigma_\eta$ ,  $\sigma_{\varepsilon}$  and c.

The optimal number  $I_{costly}^*$  of blockholders with costly information acquisition is weakly increasing in  $\sigma_{\eta}$  and  $\sigma_{\epsilon}$  and weakly decreasing in c. The intuition is as follows. If  $1 < n < \frac{\phi_a - \phi_b}{\phi_b}$ , then the optimum with costless information acquisition  $I^*$  is so large that competition in trading reduces individual informed trading profits below the cost of monitoring. Some blockholders thus choose to remain uninformed, and their existence reduces firm value. The optimum is therefore n, the maximum number of blockholders under which competition is sufficiently low that trading profits are adequately high for all blockholders to become informed. A fall in the cost of information acquisition c, an increase in the informational advantage  $\sigma_{\eta}$ , and a rise in liquidity  $\sigma_{\varepsilon}$  all lead to an increase in net informed trading profits. Higher net trading profits in turn raise n, as they allow greater competition in trading to be sustained before net profits become negative. This in turn increases the optimal number of blockholders  $I_{costly}^*$  towards  $I^*$ , and thus raises firm value.

These comparative statics reinforce the results of Section 4 on the dependence of the optimum on blockholder type. Institutional investors typically have limited "prospective" information, and thus a low  $\phi_b$ . By contrast, they are skilled at gathering "retrospective" information on the firm's current value, and thus have a low c. Together, these comparative statics imply that a large number of blockholders are optimal when the firm's principal shareholders are passive institutions.

By contrast, if  $n > \frac{\phi_a - \phi_b}{\phi_b} > 1$ , net trading profits are sufficiently high that all blockholders become informed. The analysis is as in the core model of Section 4, where the optimum depends only on the effectiveness of manager and blockholder effort. The constraint that the number of blockholders is sufficiently low information acquisition is not binding. Changes in net trading profits, and thus changes in  $\sigma_{\eta}$ ,  $\sigma_{\varepsilon}$  and c, have no effect on the optimal number of blockholders or firm value.

### 5.2 Complementarities

In the core model, the manager's and blockholders' actions are perfect substitutes, with independent effects on firm value. This appears to be a reasonable assumption for most firms; indeed, in most existing blockholder papers, the blockholder's value added does not depend on managerial effort (e.g. Shleifer and Vishny (1986), Maug (1998, 2002), Kahn and Winton (1998), Faure-Grimaud and Gromb (2004)). However, in specific circumstances, there may be complementarities between the manager's and blockholders' efforts. If complementarities are positive (negative), the marginal productivity of one party's action is increasing (decreasing) in the effort level of the other party. This subsection extends the core model to these cases.

Positive complementarities arise if manager and blockholder outputs are mutually interdependent. For example, venture capital investors often have particular expertise in devising an effective strategy, which is then executed by the manager. Both strategy formulation and implementation are necessary for the firm to become successful, and so venture capital models typically feature positive complementarities.

In the above example, blockholders are "allies" of the manager, providing him with specialist advice. The opposite case of negative complementarities arises if blockholders are "adversaries" of the manager, preventing rent extraction. Thus, they most productive if managerial effort is low (i.e. private benefit consumption is high). This case is most likely in mature firms, where the optimal strategy is often clear to the manager. Inefficiencies arise not because the manager is unaware of the correct course of action and needs blockholders' advice, but because he has private incentives to depart from the efficient action. For example, managers of "cash cows" often know that they should return excess cash to shareholders, but may choose instead to spend it on pet projects. Where effort is taken to mean "working" rather than "forgoing private benefits", complementarities may also be negative in mature firms. In companies, there is often a limited set of value-enhancing actions that can be taken, so blockholder and manager efforts would be duplicative.

One common way to model complementarities is to use a constant elasticity of substitution production function, e.g.  $\tilde{v} = [(\phi_a \log a)^{\rho} + (\phi_b \log \sum_i b_i)^{\rho}]^{1/\rho} + \tilde{\eta}$ . Unfortunately, such a production function is intractable with the logarithmic functional form that was necessary for the tractability of the core model. We therefore employ a second common method: we model perfect positive (negative) complementarities by specifying that firm value depends only on the minimum (maximum) output level of the manager and blockholders. We start with perfect negative complementarities.

$$\widetilde{v} = \max\left[\phi_a \log a, \phi_b \sum_i b_i\right] + \widetilde{\eta}.$$
(28)

The optimal actions can no longer be derived independently. The manager's optimal action depends on his conjecture  $\hat{b}_i$  for the blockholders' actions. Blockholder *i*'s optimal action depends on her conjecture for the manager's effort ( $\hat{a}$ ) and for the actions of the other blockholders ( $\hat{b}_j$ ,  $j \neq i$ ). We use the Nash equilibrium solution concept, where each party chooses the optimal action given his/her conjectures, and all conjectures are

correct.

**Proposition 10** (Negative Complementarities): The manager's optimal action is

$$a = \begin{cases} \phi_a \alpha \frac{I}{I+1} & \text{if } \alpha \frac{I}{I+1} \left( \phi_a \log \left[ \phi_a \alpha \frac{I}{I+1} \right] - \phi_b \log \sum_i \widehat{b}_i \right) \ge a \\ 0 & \text{if } \alpha \frac{I}{I+1} \left( \phi_a \log \left[ \phi_a \alpha \frac{I}{I+1} \right] - \phi_b \log \sum_i \widehat{b}_i \right) < a. \end{cases}$$
(29)

Similarly, blockholder i's effort level is:

$$b_{i} = \begin{cases} \phi_{b}\beta\left(\frac{1}{I}\right)^{2} & \text{if } \frac{\beta}{I}\left(\phi_{b}\log\left(\phi_{b}\beta\frac{1}{I}\right)^{2} - \left[\phi_{a}\log\widehat{a} - \phi_{b}\log\sum_{j\neq i}\widehat{b}_{j}\right]\right) \ge b_{i} \\ 0 & \text{if } \frac{\beta}{I}\left(\phi_{b}\log\left(\phi_{b}\beta\frac{1}{I}\right)^{2} - \left[\phi_{a}\log\widehat{a} - \phi_{b}\log\sum_{j\neq i}\widehat{b}_{j}\right]\right) < b_{i} \end{cases}$$
(30)

The number of blockholders  $I^*$  that maximizes firm value is

$$I^* = \begin{cases} \infty & \text{if } \phi_a \log \left( \phi_a \alpha \right) \ge \phi_b \log \left( \phi_b \beta \right) \\ 1 & \text{if } \phi_a \log \left( \phi_a \alpha \right) < \phi_b \log \left( \phi_b \beta \right) \end{cases}$$
(31)

#### **Proof** See Appendix.

In the core model of perfect substitutes, firm value depends on both manager and blockholder efforts. Since the optimal shareholder structure must trade-off both,  $I^*$  is typically an interior solution. Here, firm value depends only on the maximum output level and there are no trade-off concerns. If managerial effort is relatively productive,  $I^*$  should be chosen exclusively to maximize the potency of exit and completely ignores voice; thus the optimal number of blockholders is infinite. By contrast, if blockholder effort is relatively productive,  $I^*$  is at its minimum value of 1.

In the core model,  $I^*$  is always smoothly increasing in  $\phi_a$ . Here,  $\phi_a$  has a discontinuous effect. If  $\phi_a \log(\phi_a \alpha) < \phi_b \log(\phi_b \beta)$ ,  $I^*$  is independent of  $\phi_a$ . A small increase in  $\phi_a$  has zero effect on  $I^*$ : since blockholder effort is still relatively more productive,  $I^*$  continues to be exclusively determined by voice, irrespective of the productivity of managerial effort. However, when  $\phi_a$  crosses the threshold that allows  $\phi_a \log(\phi_a \alpha) \ge \phi_b \log(\phi_b \beta)$  to be satisfied,  $I^*$  jumps from 1 to  $\infty$ . For  $\phi_a \log(\phi_a \alpha) \ge \phi_b \log(\phi_b \beta)$ ,  $I^*$  is already exclusively determined by exit considerations, and so further increases in managerial productivity have no effect on  $I^*$ . Similarly, changes in  $\phi_b$  have either a zero or infinite effect on  $I^*$ .

In sum, negative complementarities lead to more extreme results than the core model. The optimal number of blockholders is a corner solution. Moreover,  $\phi_a$  and  $\phi_b$  have the same directional effect as in the core model, but their impact is more discontinuous. The opposite case is perfect positive complementarities. We analyze a Leontief production function where firm value depends only on the minimum output level of the manager and blockholders, i.e.

$$\widetilde{v} = \min\left[\phi_a \log a, \phi_b \sum_i b_i\right] + \widetilde{\eta}.$$
(32)

Again, each party's optimal action depends on his/her conjectures, and we apply the Nash equilibrium solution concept.

**Proposition 11** (Positive Complementarities): The manager's optimal action is

$$a = \min\left(\phi_a \alpha\left(\frac{I}{I+1}\right), \exp\left(\frac{\phi_b}{\phi_a}\log\sum_i \widehat{b}_i\right)\right).$$
(33)

Similarly, blockholder i's effort level is:

$$b_{i} = \begin{cases} \phi_{b}\beta\left(\frac{1}{I}\right)^{2} & if \phi_{a}\log\widehat{a} \geq \phi_{b}\log\left[\phi_{b}\beta\left(\frac{1}{I}\right)^{2}\right] + \phi_{b}\log\sum_{j\neq i}\widehat{b}_{j} \\ \exp\left(\frac{\phi_{a}}{\phi_{b}}\log\widehat{a} - \log\sum_{j\neq i}\widehat{b}_{j}\right) & . \quad (34) \\ & if \phi_{b}\log\sum_{j\neq i}\widehat{b}_{j} \leq \phi_{a}\log\widehat{a} < \phi_{b}\log\left[\phi_{b}\beta\left(\frac{1}{I}\right)^{2}\right] + \phi_{b}\log\sum_{j\neq i}\widehat{b}_{j} \\ 0 & if \phi_{a}\log\widehat{a} < \phi_{b}\log\sum_{j\neq i}\widehat{b}_{j} \end{cases}$$

Let  $\underline{I}$  represent the unique positive solution to

$$\frac{I^2}{I+1} = \frac{\phi_b \beta}{\phi_a \alpha} \exp\left(\phi_b - \phi_a\right). \tag{35}$$

The number I<sup>\*</sup> of blockholders that maximizes firm value is

$$I^* = \max\left[1, \underline{I}\right].$$

 $I^*$  is increasing in  $\phi_b$  and  $\beta$ , and decreasing in  $\phi_a$  and  $\alpha$ .

**Proof** See Appendix.

As with the core case, the optimal number of blockholders  $I^*$  is typically an interior solution, i.e. involves multiple, but finite, blockholders. However, the comparative statics with respect to  $\phi_a$  and  $\phi_b$  are *opposite* to the core case. In the core case,  $I^*$  is increasing in  $\phi_a$ . If managerial effort becomes more productive, it becomes increasingly important in the trade-off between exit and voice, and so  $I^*$  rises to enhance exit. With perfect positive complementarities, the optimal number of blockholders must balance the levels of manager and blockholder outputs. If  $\phi_a$  rises, the effectiveness of managerial effort means that it is not necessary to "boost" it via a high I. Instead, I should be used to enhance blockholder effort so that it becomes sufficiently high to complement the manager's effort. This involves reducing I.

If  $\phi_a$  is significantly greater than  $\phi_b$ ,  $I^*$  is lower under positive complementarities than in the core case of perfect substitutes. It is plausible to assume that the manager is able to add significantly greater value than blockholders, given his close proximity to firm operations. This may explain the concentrated blockholder structure in early-stage firms, where complementarities are high. Moreover, in such firms, the manager often has a significant equity stake (high  $\alpha$ ) which gives him strong incentives to exert effort. From equation (35),  $I^*$  should be low to ensure blockholder effort is also high. Indeed, when venture capital-financed firms go public, the initial investor typically retains a large stake after the IPO, rather than dissipating her stake to new investors.

### 5.3 General Compensation Contract

In the core model, the manager's payoff stems from the market value of his shares,  $\alpha \tilde{p}$ . In a more general setting, the manager can be compensated according to the fundamental value  $\tilde{v}$  as well as the market value  $\tilde{p}$ , for instance using stock with a long vesting period. We thus generalize the manager's objective function to

$$E\left[\alpha\left(\omega p + (1-\omega)v\right) - a\right].$$

 $\omega > 0$  is a standard assumption in the literature, which can be motivated by a number of underlying factors. These include takeover threat (Stein (1988)), concern for managerial reputation (Narayanan (1985), Scharfstein and Stein (1990)), or the manager expecting to sell his shares for  $\tilde{p}$  before  $\tilde{v}$  is realized, e.g. to finance consumption (Stein (1989)).<sup>16</sup> The core model has  $\omega = 1$ .

Proceeding as in the main model, we have

$$a = \alpha \phi_a \left[ 1 - \frac{\omega}{I+1} \right].$$

Effort is therefore decreasing in  $\omega$ , because the interim stock price only partially reflects the effect of effort on long-run fundamental value. As in the core model, effort rises with *I*. The positive effect of *I* is particularly strong if  $\omega$  is high, i.e. reputational concerns or takeover threats are strong, or the manager's stock has short vesting periods.

Firm value is given by:

$$E[v] = \phi_a \ln \left[ \phi_a \alpha \left[ 1 - \frac{\omega}{I+1} \right] \right] + \phi_b \ln \left[ \phi_b \beta \frac{1}{I} \right]$$

<sup>&</sup>lt;sup>16</sup>Kole (1997) shows that vesting periods are short in practice, perhaps because long vesting periods would subject the manager to excessive risk. Even if vesting periods are long, the manager will still care about  $\tilde{p}$  as it affects the terms at which the firm can raise equity (Stein (1996)).

and so the optimal number of blockholders solves is defined implicitly by:

$$\frac{\phi_a \omega}{(I+1-\omega)(I+1)} - \frac{\phi_b}{I} = 0.$$

As before, the optimal number of blockholders represents a trade-off between the positive effect of greater blockholders on "exit", and the negative effect on "voice." Since

$$\frac{\partial^2 E[v]}{\partial I \partial \omega} = \frac{(I+2-\omega)(I+1)\phi_a}{(I+1-\omega)^2} > 0,$$

the optimal number of blockholders  $I^*$  increases with the manager's short-term concerns  $\omega.^{17}$ 

In our model, governance through exit is effective even though the manager's contract is exogenous. It could be even more powerful if the manager is risk-averse and the contract is endogenous, as in Holmstrom and Tirole (1993) and Calcagno and Heider (2007). A more informative (less noisy) stock price means that an incentive contract based on  $\tilde{p}$  imposes less risk on the manager. Hence the manager can be given a more highly-powered contract, which induces greater effort.

### 6 Conclusion

Why are so many firms held by multiple blockholders when such a shareholding structure generates free-rider problems in monitoring? This paper offers a potential explanation. The same co-ordination issues that hinder intervention increase blockholders' effectiveness in exerting governance through an alternative governance mechanism: exit. Multiple blockholders act competitively in their trading behavior, impounding a greater level of information in the stock price. This in turn induces higher managerial effort, particularly if the manager has high stock price concerns.

The optimal number of blockholders that maximizes firm value depends on the relative productivity of managerial and blockholder effort. If outputs are perfect substitutes, the optimum is decreasing in the effectiveness of blockholder intervention and increasing in the potency of managerial effort. It is therefore high if blockholders are mutual funds that gather retrospective rather than prospective information. If there are negative complementarities, changes in productivity can have discontinuous effects, switching the optimal number of blockholders from a very low to a very high level (or vice-versa). However, if complementarities are sufficiently positive, the productivity parameters have

<sup>&</sup>lt;sup>17</sup>From Topkis (1989), it is sufficient to show that the objective function is supermodular in I and the parameter of interest for the comparative statics. Since our objective function is differentiable, we can simply look at the cross-partial derivative of the objective function.

opposite effects on the optimal shareholder structure. If managerial effort is unproductive, many blockholders are necessary to augment it to a sufficient level to complement blockholder effort.

The paper suggests a number of potential avenues for future research. On the empirical side, the model generates a number of empirical predictions for the determinants of blockholder structure. Testing such predictions is non-trivial, since some of the key explanatory variables (productivity of effort) may be difficult to measure precisely. A quite separate empirical implication is that the number of sizable shareholders, or their total ownership, may be a more relevant measure than the ownership of the single largest shareholder, for both governance and investor informedness. On the theoretical side, the paper has assumed symmetric blockholders and the analysis has focused on their optimal number. It would be interesting to extend the analysis to asymmetric blockholders; a potential complementary analysis would be to examine the optimal distribution of shares between a fixed number of blockholders.

Moreover, the model suggests a new way of thinking about the effect of multiple blockholders on firm value. While previous research has focused on free-rider problems in intervention or control contests, this paper suggests that they can be conceptualized as informed traders who affect firm value through influencing prices. Therefore, future corporate finance research could import more complex effects analyzed in asset pricing models of many informed traders, and study how these impact blockholders' effectiveness in exerting governance. The present paper assumes a single trading period, but in reality there may be multiple periods in which information may arrive and blockholders may trade. Trading profits, and thus incentives to acquire costly information, then depend not only on the quality of information but its timeliness. A blockholder who receives information late may find that the price has already moved unfavorably. In addition, in the present paper, blockholders only trade on information. If blockholders are subject to liquidity shocks (as in Faure-Grimaud and Gromb (2004)), the addition of multiple trading rounds may give incentives for other blockholders to "front-run" and sell in advance of an anticipated forced liquidation. This may increase the potency of governance through exit, but reduce incentives to engage in interventions with long-run benefits.

# A Appendix

#### **Proof of Proposition 6 (Social Optimum)**

Putting equation (17) under a common denominator yields

$$\frac{\phi_a I \left(I+1\right) - \phi_b I \left(I+1\right)^2 - \phi_a \alpha I^2 + \phi_b \beta \left(I+1\right)^2}{I^2 \left(I+1\right)^2} = 0.$$
(36)

The equation is thus a cubic, and has at most three roots. The function is discontinuous at I = -1 and approaches  $-\infty$  either side of I = -1 (since the  $-\frac{\phi_a \alpha}{(I+1)^2}$  term dominates). It is also discontinuous at I = 0 and approaches  $+\infty$  either side of I = 0 (since the  $\frac{\phi_b \beta}{I^2}$  term dominates). It is continuous everywhere else.

As  $I \to -\infty$ , the  $-\frac{\phi_b}{I}$  term in equation (17) dominates, and so the function asymptotes the x-axis from above. Since it approaches  $-\infty$  as I rises to -1, and is continuous between  $I = -\infty$  and I = -1, there must be one root between these two points. Similarly, since the function tends to  $+\infty$  as I rises from just above -1 to just below 0, and is continuous between these two points, there must be a second root within this interval. As  $I \to +\infty$ , the  $-\frac{\phi_b}{I}$  term in equation (17) again dominates, and so the function asymptotes the x-axis from below. Since the function tends to  $+\infty$  as I approaches 0 from above, and is continuous between I = 0 and  $I = +\infty$ , there must be a third root  $(\tilde{I})$  between these two points. There can only be one positive root, since there are two negative roots and at most three roots in total. The positive root is a local maximum, since the gradient is negative for  $I < \tilde{I}$  and positive for  $I > \tilde{I}$ .

The comparative statics results follow immediately from taking the cross-partial derivatives of equation (19) with respect to I and  $\alpha$ ,  $\beta$ ,  $\phi_a$  and  $\phi_b$ . (See footnote 17 for further detail on the sufficiency of the cross-partials).

### Proof of Proposition 7 (Private Optimum)

Owing to the  $\sqrt{I}$  term in the denominator of the final term in equation (20), the only real roots are positive. As I tends to 0 from above, the function tends to  $+\infty$  since the  $\frac{\phi_b\beta}{I^2}$  term dominates. As  $I \to +\infty$ , the  $-\frac{\phi_b}{I}$  term dominates the function asymptotes the x-axis from below. There is thus at least one positive root.

The second derivative is given by:

$$\beta \left[ -\frac{\phi_a(2I+1)}{I^2 \left(I+1\right)^2} + \frac{\phi_b}{I^2} \right] - \frac{2\phi_b\beta}{I^3} - \frac{2\sqrt{I}(I+1)^2 - (I-1)\left[2\sqrt{I}(2I+2) + \frac{1}{\sqrt{I}}(I+1)^2\right]}{4I(I+1)^4} \sigma_\eta \sigma_\varepsilon.$$
(37)

As I approaches 0 from above, the second derivative becomes highly negative, as the  $-\frac{2\phi_b\beta}{I^3}$  term dominates. As I rises, the second derivative becomes less negative and eventually becomes positive as the  $\beta \frac{\phi_b}{I^2}$  term increasingly dominates. Since this term

continues to dominate as I approaches  $+\infty$ , the second derivative never becomes negative again. Hence there is only one positive root,  $\hat{I}$ . The positive root is a local maximum, since the gradient is negative for  $I < \hat{I}$  and positive for  $I > \hat{I}$ .

The comparative statics results follow from taking the cross-partial derivatives of the objective function. The cross-partial with respect to I and  $\beta$  is  $\frac{\phi_a}{I(I+1)} - \frac{\phi_b}{I} + \frac{\phi_b}{I^2}$ , which is positive from equation (20). The other cross-partial derivatives can be immediately signed.

### Proof of Proposition 8 (Equilibrium With Costly Information)

The only difference from the previous analysis is that in the action stage of the game, blockholder i now simultaneously chooses her action  $b_i$  and whether to become informed.

We proceed by backwards induction. Let J be the number of blockholders that acquire information in the action stage of the game and fix a and  $b_i$ . In the trading stage of the game, uninformed blockholders cannot expect to make trading profits and thus do not trade. Therefore, only the J informed blockholders trade at this stage and the equilibrium is similar to the one derived in Proposition 1.

Now in the action stage of the game, the manager must choose an action a. Using the same arguments as in Proposition 4, the manager's optimal action is

$$a = \phi_a \alpha \left(\frac{J}{J+1}\right). \tag{38}$$

Blockholders must choose actions  $b_i$  and whether to become informed. These decisions can be taken independently since informed trading profits are independent of  $b_i$  (which is public), and the choice of  $b_i$  depends only on the number of shares owned by blockholder *i*. The optimal action of each blockholder is thus

$$b_i = \phi_b \beta \left(\frac{1}{I}\right)^2 \tag{39}$$

From equation (7), we know that if there are I informed blockholders, then each blockholder's trading profits are given by:

$$\frac{1}{\sqrt{I}(I+1)}\sigma_{\eta}\sigma_{\epsilon}.$$

A blockholder will acquire information if and only if her trading profits are higher than c. This gives the number J of blockholders that decide to become informed in equilibrium.

### Proof of Proposition 9 (Firm Value Optimum With Costly Information)

Let n and J(I) be as given in Proposition 8. Using the results of Proposition 4, we can write the expected value of the firm as

$$E[\widetilde{v}] = \phi_a \log\left[\phi_a \alpha\left(\frac{J(I)}{J(I)+1}\right)\right] + \phi_b \log\left[\phi_b \beta\left(\frac{1}{I}\right)\right].$$
(40)

We wish to maximize the above expression with respect to I. Since J(I) = n for  $I \ge n$ , it is never optimal to increase I beyond n since it reduces the second term in the firm value while keeping the first term constant. Therefore,  $I_{costly}^* \le n$ .

However, when  $I \leq n$ , J(I) = I and the problem is the same as in Proposition 5. From (14) we obtain the desired result.

#### Proof of Proposition 10 (Negative Complementarities)

Deriving  $\widetilde{p}$  as in the main model and solving the manager's objective function, he will choose either  $a = \phi_a \alpha \frac{I}{I+1}$  or a = 0. If  $\phi_a \log \left[\phi_a \alpha \frac{I}{I+1}\right] < \phi_b \log \sum_i \widehat{b}_i$ ,  $a = \phi_a \alpha \frac{I}{I+1}$  will have no effect on  $\widetilde{p}$  and so the manager will choose a = 0. Even if  $\phi_a \log \left[\phi_a \alpha \frac{I}{I+1}\right] \ge \phi_b \log \sum_i \widehat{b}_i$ , it is not automatic that the manager will exert effort. Exerting effort increases  $\widetilde{p}$  not by  $\frac{I}{I+1}\phi_a \log \left[\phi_a \alpha \frac{I}{I+1}\right]$ , as in the core model, but by only

$$\frac{I}{I+1} \left( \phi_a \log \left[ \phi_a \alpha \frac{I}{I+1} \right] - \phi_b \log \sum_i \widehat{b}_i \right)$$

because blockholder effort "supports" firm value even if a = 0. Hence the manager choose  $a = \phi_a \alpha \frac{I}{I+1}$  only if

$$\alpha \frac{I}{I+1} \left( \phi_a \log \left[ \phi_a \alpha \frac{I}{I+1} \right] - \phi_b \log \sum_i \widehat{b}_i \right) \ge a.$$

and thus the optimal a is as given in equation (29). Blockholder *i*'s effort level is derived similarly.

There are two possible candidates for a Nash equilibrium:

$$\begin{cases} a = 0, b_i = \phi_b \beta \left(\frac{1}{I}\right)^2 \\ a = \phi_a \alpha \frac{I}{I+1}, b_i = 0 \end{cases}$$

Firm value is thus either  $\phi_a \log \left[\phi_a \alpha \frac{I}{I+1}\right]$  or  $\phi_b \log \left[\phi_b \beta \frac{1}{I}\right]$ . The former is monotonically increasing in I, and maximized at  $\phi_a \log (\phi_a \alpha)$  for  $I = \infty$ . The latter is monotonically decreasing in I, and maximized at  $\phi_b \log (\phi_b \beta)$  for I = 1. Thus  $I^*$  is as given in equation (31).

#### Proof of Proposition 11 (Positive Complementarities)

We proceed as in the main model, and note that the manager will not exert effort above the level for which

$$\phi_a \log a = \phi_b \log \sum_i \widehat{b}_i,$$

i.e.

$$a = \exp\left(\frac{\phi_b}{\phi_a}\log\sum_i \widehat{b}_i\right).$$

This derives the optimal a as given in equation (33). Similarly, blockholder i will not exert effort above the level for which

$$\phi_b \log b_i = \phi_a \log \widehat{a} - \phi_b \log \sum_{j \neq i} \widehat{b}_j,$$

i.e.

$$b_i = \exp\left(\frac{\phi_a}{\phi_b}\log\widehat{a} - \log\sum_{j\neq i}\widehat{b}_j\right)$$

A Nash equilibrium requires the following three conditions to hold:

$$\phi_b \log Ib_i = \phi_a \log a.$$
$$a \le \phi_a \alpha \left(\frac{I}{I+1}\right)$$
$$b_i \le \phi_b \beta \left(\frac{1}{I}\right)^2.$$

If the first condition was violated, then the party producing the higher output would unambiguously gain by reducing effort. The two inequality conditions represent the maximum levels of effort that the manager and blockholders will exert, given the marginal cost of effort.

Out of the continuum of potential Nash equilibria, we seek the one that maximizes firm value. Since firm value is increasing in both a and  $b_i$ , it is clear that at least one incentive compatibility constraint will bind. If neither constraint binds, then all parties are exerting suboptimal effort. We could raise the effort levels of all parties while maintaining the equality condition and violating neither constraint.

We now show that, in fact, *both* constraints will bind. Consider the case where  $b_i = \phi_b \beta \left(\frac{1}{I}\right)^2$ . (Starting with  $a = \phi_a \alpha \left(\frac{I}{I+1}\right)$  leads to the same result). Then we have

$$\phi_b \log \left[ \phi_b \beta \left( \frac{1}{I} \right) \right] = \phi_a \log a$$
$$a = \exp \left( \frac{\phi_b}{\phi_a} \log \left[ \phi_b \beta \left( \frac{1}{I} \right) \right] \right)$$

Recall that we also require  $a \leq \phi_a \alpha \left(\frac{I}{I+1}\right)$ . Hence firm value is optimized by solving:

$$\max_{I} \exp\left(\frac{\phi_b}{\phi_a} \log\left[\phi_b \beta\left(\frac{1}{I}\right)\right]\right) \text{ s.t. } \exp\left(\frac{\phi_b}{\phi_a} \log\left[\phi_b \beta\left(\frac{1}{I}\right)\right]\right) \le \phi_a \alpha\left(\frac{I}{I+1}\right).$$

The constraint will bind, and so we obtain

$$\phi_a \log \left[ \phi_a \alpha \left( \frac{I}{I+1} \right) \right] = \phi_b \log \left[ \phi_b \beta \left( \frac{1}{I} \right) \right]. \tag{41}$$

The firm value optimum setting I to ensure all parties exert their "full" effort levels. The intuition is as follows. Consider a Nash equilibrium where the blockholders are exerting their full effort, and the manager is not.  $b_i$  is thus constrained by I via the equation  $b_i = \phi_b \beta \left(\frac{1}{I}\right)^2$ , and so firm value rises if I is reduced to relax this constraint and allow  $b_i$  to rise. Unlike in the core model, we do not have the side-effect that reducing Idecreases a. I only determines the upper bound to a, not its level. Since  $a < \phi_a \alpha \left(\frac{I}{I+1}\right)$ , the upper bound is not a constraint anyway. Instead, instead of increasing, a will rise to accompany the increase in  $b_i$  and ensure that  $\phi_b \log I b_i = \phi_a \log a$  still holds.

From equation (41), the optimal number of blockholders is determined implicitly by:

$$\frac{I^2}{I+1} = \frac{\phi_b\beta}{\phi_a\alpha} \exp\left(\phi_b - \phi_a\right) = Z.$$

Using the quadratic formula, the unique positive solution is

$$\underline{I} = \frac{Z + \sqrt{Z^2 + 4Z}}{2},$$

which is increasing in  $\phi_b$  and  $\beta$ , and decreasing in  $\phi_a$  and  $\alpha$ .

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