Optimal Capital Income Taxation

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Abstract

In an economy with identical infinitely-lived households that obtain utility from leisure as well as consumption, Chamley (1986) and Judd (1985) have shown that the optimal tax system to pay for an exogenous stream of government purchases involves a zero tax rate on capital in the long run, with tax revenue collected by a distortionary tax on labor income. Extending the results of Hall and Jorgenson (1971) to general equilibrium, I show that if purchasers of capital are permitted to deduct capital expenditures from taxable capital income, then a constant tax rate on capital income is non-distortionary. Importantly, even though this specification of the capital income tax imposes a zero *effective* tax rate on capital, the capital income tax can collect substantial revenue. Provided that government purchases do not exceed gross capital income less gross investment, the optimal tax system will consist of a positive tax rate on capital income and a zero tax rate on labor income–just the opposite of the results of Chamley and Judd.

The optimal way for a government to collect revenue to pay for its purchases of goods and services is to levy lump-sum taxes. However, lump-sum taxes generally are not available, so some form of economic activity, such as labor income, capital income, cigarette purchases, etc., must be taxed. Because the taxation of economic activities is distortionary, a basic problem of public finance is how to use such taxes to collect revenue in the least distortionary way. A classic problem of this sort analyzes the optimal use of taxes on labor income and capital income to finance an exogenous stream of government purchases in a Ramsey framework with a representative infinitely-lived household. The celebrated result of Chamley (1986) and Judd (1985) is that in the long run, the optimal tax rate on capital income is zero.

The Chamley-Judd result might be particularly puzzling to readers of an older literature on the conditions for the neutrality of capital income taxation. The older literature focused on the capital investment decision of a single firm, and did not embed the firm in a general equilibrium model. Hall and Jorgenson (1971) showed that for a firm that cannot deduct its cost of financing (for example, under U.S. tax law, a firm financed entirely by equity), a tax on capital income that provides for immediate expensing of capital expenditures will be neutral with respect to capital; that is, it will have no effect on a firm's optimal capital accumulation. Tax codes generally allow purchasers of capital to reduce their calculated taxable income by some amount to reflect the cost of acquiring capital. This reduction in taxable income is usually implemented through a schedule of depreciation allowances, which may or may not be accelerated relative to the economic depreciation of the capital asset. The most accelerated version of depreciation allowances is immediate expensing, which leads to tax rate neutrality, as described above. A second neutrality result applies to the case in which a firm can deduct its cost of financing, as would be the case, under U.S. tax law, for a firm financed entirely by debt. In this case, Hall and Jorgenson

show, and an earlier result of Samuelson (1964) implies, that allowing firms to deduct true economic depreciation will lead to tax rate neutrality with respect to capital investment.

The existence of neutral forms of capital income taxation, i.e., forms of capital income taxation that do not affect the capital investment decision of a firm, suggests that these forms of capital taxation may provide the elusive lump-sum tax that can allow the government to finance its expenditures without distortions. To explore whether such capital income tax schemes can provide non-distortionary sources of revenue, two major questions need to be addressed. First, does the neutrality of a capital income tax scheme in the context of a single firm's decision carry over to a general equilibrium framework? Second, can a capital income tax that is neutral in general equilibrium collect a nontrivial amount of revenue? The answer to the first question is different for the two neutral tax schemes mentioned above. Specifically, for a firm that cannot deduct the cost of financing, a capital income tax system that specifies a constant tax rate and includes immediate expensing is neutral in general equilibrium as well as in the context of a single firm. However, the neutrality of allowing firms that can deduct financing costs also to deduct economic depreciation does not carry over to general equilibrium. Therefore, the body of this paper focuses on the case with immediate expensing; the Appendix examines the case with economic depreciation. I will show that for immediate expensing the answer to the second question is also positive: a capital income tax with immediate expensing can collect a substantial amount of revenue. Because a constant capital income tax rate with immediate expensing does not affect the accumulation of capital optimally chosen by purchasers of capital, and can collect substantial revenue, it can be used to finance government spending in a non-distortionary manner. Provided that the amount of government spending does not exceed gross capital income less gross investment,

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there is no need to use distortionary labor income taxation.

The finding that a capital income tax with immediate expensing can collect a nontrivial amount of revenue in a non-distortionary manner turns the Chamley-Judd result on its head. Instead of setting the capital income tax rate equal to zero and using a distortionary labor income tax to collect revenue as in Chamley-Judd, the results in this paper indicate that the optimal configuration of taxes is to set the labor income tax rate equal to zero and to use a constant tax rate on capital income, combined with immediate expensing, to collect revenue. The optimal capital income tax scheme I present here leads to a higher level of utility of the representative household than does the Chamley-Judd prescription because the tax system presented here is non-distortionary, while Chamley-Judd requires the use of distortionary taxes.

The optimal tax scheme I present here holds in every period, not just in the long run. The zero capital income tax rate prescribed by Chamley and Judd holds only in the long run. Chamley also derives the optimal tax rate on capital income at every point in time for the special case in which utility is separable over time, additively separable in consumption and leisure, isoelastic in consumption, and linear in leisure. In this special case, he shows that the optimal tax rate on capital income is initially 100% and remains equal to 100% until some point in time at which it abruptly jumps to zero, and remains zero forever. However, I show that in a tax system that includes immediate expensing, the optimal tax rate is constant over time.

Because the optimal tax policy in this paper stands in sharp contrast to the celebrated zero tax rate on capital income derived by Chamley and Judd for the long run, it is important to explain the source of the difference in the results. The difference is due entirely to the treatment of capital expenditures in calculating capital income. In actual tax codes, depreciation allowances permit firms to amortize the cost of purchasing capital over time, and the present value of depreciation allowances

— Hall and Jorgenson's (1967) famous "z"— is generally between zero and one.[1] Chamley has (implicitly) chosen to set z equal to zero. Judd specifies depreciation allowances equal to economic depreciation, so the implied value of z in his model is between zero and one. As mentioned earlier, in the context of a single firm that can deduct the cost of financing, economic depreciation will make the capital income tax neutral with respect to capital. However, as I demonstrate in the Appendix, this result does not carry over to general equilibrium, so the capital income tax analyzed by Judd is distortionary in a general equilibrium framework.

To illustrate the importance of the value of z, it is simplest to consider the class of capital income tax policies characterized by a constant capital income tax rate, τ^K, and a constant present value of depreciation deductions, z. The search for an optimal capital income tax policy within this class of policies can be described as a search for the optimal values of τ^K and z. Chamley and Judd each chose values of z without considering the optimal value of z. However, in this context, the effective tax rate on capital is (see Section 7) $\frac{1-z}{1-z\tau^K}\tau^K$, so if $0 \leq z < 1$, and $\tau^K > 0$, the effective tax rate on capital is positive; that is, the capital income tax is distortionary. To achieve a zero effective tax rate with $z < 1$–and thus avoid distortions–Chamley and Judd each set $\tau^K = 0$, which destroys the ability to collect tax revenue from capital income. In this paper, I introduce immediate expensing, which implies $z = 1$, so that the effective tax rate on capital is zero, even with a positive capital income tax rate, τ^K. As I discuss in Section 6.1, this tax scheme is equivalent to a one-time seizure of capital. Surprisingly, however, even if the initial capital stock available for seizure is very small, the non-distortionary capital income tax with immediate expensing can collect a sizable amount of revenue along balanced growth paths.[2]

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[1] A more formal definition of z and a brief discussion are presented in Section 7.

[2] Lucas (1990), in discussing the results of Chamley and Judd, points out that a proportional tax on gross capital income combined with an appropriate investment tax credit is non-distortionary, but he does not address the question of how much revenue such a scheme can collect.

The allocation that would prevail under lump-sum taxation can also be attained in a competitive economy with a constant tax rate on the consumption good combined with a subsidy to labor at the same rate. The constant tax rate on the consumption good does not distort the intertemporal marginal rate of substitution. To avoid an intratemporal distortion between the consumption good and contemporaneous leisure, leisure must be taxed at the same rate as the consumption good. That is, labor supply must be subsidized. Although a tax on the consumption good accompanied by a subsidy to labor can achieve the same allocation as a capital income tax with immediate expensing, I will focus mostly on the capital income tax because my results concerning the optimal capital income tax differ so sharply from the well-known Chamley-Judd result.

I develop a general equilibrium model with a capital income tax and immediate expensing in the first three sections of the paper. Section 1 provides a brief description of firms, which carry out production in the economy. Section 2 describes the government's purchases and its means of financing these purchases with taxes and debt. The household's decision problem is described in Section 3, which also derives a characterization of the equilibrium allocation of goods and leisure, and the evolution of rates of return. Section 4 describes the competitive equilibrium when the capital income tax is replaced by lump-sum taxes. This equilibrium represents the first-best allocation, given the exogenous path of government purchases. It serves as a benchmark to show that a capital income tax with immediate expensing will achieve the first-best equilibrium, which I do in Section 5. Then in Section 6, I calculate the amount of tax revenue that can be collected with a capital income tax that includes immediate expensing. I derive a simple expression for capital income tax revenue in a steady state, as well an expression that holds along balanced growth paths. In Section 7, I derive the effective tax rate on capital. I show that the Chamley-Judd

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prescription for optimal tax policy in the long run and my prescription are both characterized by a zero effective tax rate on capital. However, a zero effective tax rate on capital does not mean that capital income tax revenue is zero. Indeed, my formulation of the capital income tax can collect a potentially substantial amount of revenue, though the capital income tax collects zero revenue in the long run in the Chamley-Judd formulation. In Section 8, I demonstrate that a constant tax rate on the consumption good combined with a subsidy to labor at the same rate can achieve the same allocation as can be achieved by a constant capital income tax rate with immediate expensing. I also derive the tax rate on the consumption good that will, in a closed economy, collect the same amount of tax revenue as a given capital income tax rate, and show that the tax rate on the consumption good is higher than the equivalent tax rate on capital income. Concluding remarks are presented in Section 9.

1 Firms

Consider a closed economy in which production is carried out by competitive firms that rent the services of capital and labor in competitive markets. Labor is supplied by a continuum of identical infinitely-lived households. I normalize the measure of households to be one. Each household works H_t hours in period t. The production function is

$$Y_t = F(K_t, L_t), \tag{1}$$

where Y_t is output, K_t is the capital stock, and $L_t \equiv A_t H_t$ is the amount of effective hours of labor input, where A_t is an index of labor-augmenting technical progress that evolves deterministically over time. The production function $F(K_t, L_t)$ is linearly homogeneous in K_t and L_t, with $F_K > 0$, $F_{KK} < 0$, $F_L > 0$, and $F_{LL} < 0$.

The equilibrium wage rate per effective hour of labor is

$$w_t = F_L\left(K_t, L_t\right). \tag{2}$$

Since an hour of labor by a household generates A_t effective hours of labor, the wage per hour of labor is $w_t A_t$.

The gross rental earned by a unit of capital in period t is

$$r_t = F_K\left(K_t, L_t\right). \tag{3}$$

The capital stock depreciates at a constant proportional rate δ, so the evolution of capital over time is given by

$$K_{t+1} = \left(1 - \delta\right) K_t + I_t, \tag{4}$$

where I_t is gross investment in period t.

2   Government

The government purchases and consumes G_t units of output in period t. The value of G_t evolves exogenously and deterministically over time. The government finances its purchases of output by levying taxes on labor income and capital income and by issuing bonds. Specifically, the government levies a tax at rate τ_t^L on labor income in period t and a tax at rate τ_t^K on capital income in period t. Most actual tax systems compute taxable capital income by deducting some allowance for the cost of purchasing capital. Here I adopt a particularly simple form of depreciation allowance. I allow purchasers of capital to immediately expense capital expenditures. Therefore, taxable capital income in period t is $K_t F_K\left(K_t, L_t\right) - I_t$. Capital income tax revenue

in period t equals taxable capital income multiplied by the capital income tax rate, τ_t^K. Therefore, the tax revenue collected from the capital income tax in period t, T_t^K, is

$$T_t^K = \tau_t^K \left[K_t F_K \left(K_t, L_t \right) - I_t \right].$$ (5)

If investment, I_t, exceeds gross capital income, $K_t F_K \left(K_t, L_t \right) - I_t$, then taxable capital income is negative, and the owners of capital receive a payment from the government. Alternatively, but equivalently, if taxable capital income is negative, the owners of capital could receive a tax credit that accrues interest and can be used to pay future capital income tax liabilities. In the long run, however, the dynamic efficiency of the aggregate economy implies that gross capital income exceeds investment, so that taxable capital income is positive.

Let T_t be total tax revenue from the labor income tax and the capital income tax in period t. Using equation (4) to substitute $K_{t+1} - \left(1 - \delta \right) K_t$ for gross investment in equation (5) yields

$$T_t = \tau_t^L w_t A_t H_t + \tau_t^K \left[K_t F_K \left(K_t, A_t H_t \right) + \left(1 - \delta \right) K_t - K_{t+1} \right].$$ (6)

Let B_t be the value of one-period government bonds issued at the end of period $t-1$. In period t, these bonds pay a gross interest rate R_t so that the gross payment to bondholders, including interest and repayment of principal, is $R_t B_t$. Therefore, the government's budget constraint can be written as

$$G_t + R_t B_t = T_t + B_{t+1},$$ (7)

where the left hand side of equation (7) is the government's expenditure on goods plus the interest and repayment of principal on outstanding bonds, and the right hand side of equation (7) is the sum of tax revenue and the funds raised by selling

new bonds in period t. Substituting equation (6) into equation (7) yields

$$G_t + R_t B_t = \tau_t^L w_t A_t H_t + \tau_t^K \left[K_t F_K \left(K_t, A_t H_t \right) + (1 - \delta) K_t - K_{t+1} \right] + B_{t+1}. \quad (8)$$

3   Households

The representative household supplies labor, consumes goods and leisure, holds government bonds and the economy's capital stock, and pays taxes. The household rents the capital stock to firms during period t at a rental price of r_t per unit of capital, and pays taxes on capital income, net of the immediate expensing of capital expenditures.

The representative household maximizes the infinite-horizon utility function

$$\sum_{t=0}^{\infty} \beta^t u \left(C_t, l_t \right), \quad (9)$$

where C_t is the household's consumption of goods in period t and l_t is the household's leisure in period t. I assume that the household is endowed with one hour of time per period, so $l_t = 1 - H_t$. I also assume that $\beta < 1$, $u_c > 0$, $u_{cc} < 0$, $u_l > 0$, and $u_{ll} < 0$. The utility function in equation (9) does not depend on the level of government purchases. Strictly speaking, this exclusion of G_t from the utility function (and from the production function) means that government purchases are purely wasteful. More generally, the inclusion of G_t in the utility function would have no effect on household decisions, provided that the utility function is additively separable in G_t and (C_t, l_t).

The budget constraint of the household is

$$\begin{aligned} C_t &+ \left[K_{t+1} - (1 - \delta) K_t \right] + B_{t+1} \quad (10) \\ &= \left(1 - \tau_t^L \right) w_t A_t H_t + \left(1 - \tau_t^K \right) r_t K_t + \tau_t^K \left[K_{t+1} - (1 - \delta) K_t \right] + R_t B_t. \end{aligned}$$

The left hand side of the budget constraint in equation (10) contains the household's expenditure on consumption in period t, C_t, the expenditure on new capital goods in period t, $I_t = K_{t+1} - (1 - \delta) K_t$, and the purchase of new one-period government bonds in period t, B_{t+1}. The four terms on the right hand side of the budget constraint in equation (10) are, respectively, the household's after-tax wage income, $\left(1 - \tau_t^L\right) w_t A_t H_t$, the household's after-tax capital income (before taking account of the expensing of investment), $\left(1 - \tau_t^K\right) r_t K_t$, the value of the reduction in period t taxes resulting from the expensing of capital expenditures, $\tau_t^K \left[K_{t+1} - (1 - \delta) K_t\right]$, and the interest and principal received from maturing one-period bonds in period t.

The household chooses the sequences of consumption, C_t, hours of work, H_t, capital, K_{t+1}, and bonds, B_{t+1}, to maximize utility in equation (9) subject to the budget constraint in equation (10).[3] The lagrangian for this problem is

$$
\begin{aligned}
L = & \sum_{t=0}^{\infty} \beta^t u\left(C_t, 1 - H_t\right) \\
& + \beta^t \lambda_t \left\{ \begin{array}{c} \left(1 - \tau_t^L\right) w_t A_t H_t + \left(1 - \tau_t^K\right) r_t K_t + \tau_t^K \left[K_{t+1} - (1 - \delta) K_t\right] \\ -C_t - K_{t+1} + (1 - \delta) K_t - B_{t+1} + R_t B_t \end{array} \right\}.
\end{aligned}
\tag{11}
$$

The first-order conditions are

$$
(C_t): \qquad u_C\left(C_t, 1 - H_t\right) = \lambda_t
\tag{12}
$$

$$
(H_t): \qquad u_l\left(C_t, 1 - H_t\right) = \lambda_t \left(1 - \tau_t^L\right) w_t A_t
\tag{13}
$$

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[3] The consumer's maximization is also subject to a no-Ponzi condition, $\lim_{j \to \infty} \left(\prod_{i=1}^{j} R_{t+i}^{-1}\right) B_{t+j} = 0$, which rules out the possibility that the consumer borrows and rolls over debt forever.

$$(K_{t+1}): \qquad \left(1 - \tau_t^K\right) \lambda_t = \left(1 - \tau_{t+1}^K\right) \beta \lambda_{t+1} \left(r_{t+1} + 1 - \delta\right) \tag{14}$$

$$(B_{t+1}): \qquad \lambda_t = \beta R_{t+1} \lambda_{t+1}. \tag{15}$$

The next step is to eliminate the lagrange multiplier λ_t from the system of four equations (12) - (15) and to use the expressions for w_t and r_t in equations (2) and (3) to obtain the following three equations

$$(H_t): \qquad u_l\left(C_t, 1 - H_t\right) = \left(1 - \tau_t^L\right) A_t \left(F_L\left(K_t, A_t H_t\right)\right) u_C\left(C_t, 1 - H_t\right) \tag{16}$$

$$(K_{t+1}): \qquad \left[\beta \frac{u_C\left(C_{t+1}, 1 - H_{t+1}\right)}{u_C\left(C_t, 1 - H_t\right)}\right] \left[\frac{1 - \tau_{t+1}^K}{1 - \tau_t^K}\left(F_K\left(K_{t+1}, A_{t+1} H_{t+1}\right) + 1 - \delta\right)\right] = 1 \tag{17}$$

$$(B_{t+1}): \qquad \left[\beta \frac{u_C\left(C_{t+1}, 1 - H_{t+1}\right)}{u_C\left(C_t, 1 - H_t\right)}\right] R_{t+1} = 1. \tag{18}$$

Equation (16) equates the loss in utility from working an additional hour, and thus reducing leisure by an hour, in period t to the increase in utility that can be achieved by working an additional hour, earning an additional after-tax income of $\left(1 - \tau_t^L\right) A_t F_L\left(K_t, A_t H_t\right)$, and using this income to increase consumption in period t. Equations (17) and (18) are both illustrations of the standard intertemporal optimization condition that requires the product of the intertemporal marginal rate of substitution and the gross rate of return on an asset to be equal to one. In both equations, the intertemporal marginal rate of substitution is given by the term in the first set of square brackets on the left hand side. In the case of capital, the gross rate of return is the term in the second set of square brackets on the left hand side of

equation (17). In the presence of immediate expensing of capital expenditures, the effective price of capital in period t is $1 - \tau_t^K$. The after-tax payoff in period $t + 1$ to a unit of capital purchased in period t is the after-tax marginal product of capital, $\left(1 - \tau_{t+1}^K\right) F_K\left(K_{t+1}, A_{t+1} H_{t+1}\right)$, plus the value of the remaining fraction $1 - \delta$ of the unit of capital, which has an after-tax price of $1 - \tau_{t+1}^K$ in period $t + 1$. Thus, the gross rate of return on capital is the ratio of the after-tax payoff in period $t + 1$ accruing to a unit of capital purchased in period t to the effective purchase price of capital in period t. In the case of bonds, the gross rate of return is simply R_{t+1}, so equation (18) shows that the product of the intertemporal marginal rate of substitution and the gross rate of return on bonds equals one.

4   Allocation with Lump-sum Taxes

If the government can levy lump-sum taxes, then it can achieve the optimal allocation by setting the tax rates on labor income and capital income equal to zero. With $\tau_t^K = \tau_t^L = 0$, the household's first-order conditions in equations (16) - (18) become

$$(H_t): \quad u_l\left(C_t, 1 - H_t\right) = A_t\left(F_L\left(K_t, A_t H_t\right)\right) u_C\left(C_t, 1 - H_t\right) \tag{19}$$

$$(K_{t+1}): \quad \left[\beta \frac{u_C\left(C_{t+1}, 1 - H_{t+1}\right)}{u_C\left(C_t, 1 - H_t\right)}\right]\left[F_K\left(K_{t+1}, A_{t+1} H_{t+1}\right) + 1 - \delta\right] = 1 \tag{20}$$

$$(B_{t+1}): \quad \left[\beta \frac{u_C\left(C_{t+1}, 1 - H_{t+1}\right)}{u_C\left(C_t, 1 - H_t\right)}\right] R_{t+1} = 1. \tag{21}$$

Equations (19) - (21) characterize the optimal allocation of C_t, H_t, K_{t+1}, and B_{t+1} that would prevail under lump-sum taxes.

5   Taxes that Satisfy Optimality Conditions

Now return to the case in which there are no lump-sum taxes.   The government has available only taxes on labor income and on capital income, as described earlier. Let $\tau_t^L = 0$ for all t, and let the capital income tax rate τ_t^K be constant for all t. Observe that with a zero tax on labor income and a constant tax rate on capital income, with immediate expensing of capital expenditures, the household's first-order conditions in equations (16) - (18) are identical to the first-order conditions describing the optimal allocation in equations (19) - (21). This equivalence reflects the fact that with full expensing, the capital income tax is neutral, i.e., that it does not distort the optimal choice of the capital stock.   Notice that the replication of the optimal allocation with a constant capital income tax rate is not just a steady-state result. It holds for arbitrary (deterministic) paths of labor-augmenting technical progress, A_t, and arbitrary (deterministic) paths of government purchases, G_t, provided that the capital income tax can raise sufficient revenue to pay for government purchases.

Lucas (1990, p. 300) pointed out that "a tax on capital income combined with an investment tax credit can imitate a capital levy perfectly" and thus is non-distortionary.   Though he did not state the magnitude of the appropriate invest-ment tax credit, inspection of equation (5) reveals that the investment tax credit rate must equal the capital income tax rate, τ_t^K, for the capital income tax to be non-distortionary.   That is, if gross capital income is taxed at rate τ_t^K, and if there is an investment tax credit at rate ν_t in period t, capital income tax revenue in period t is $T_t^K = \tau_t^K K_t F_K (K_t, L_t) - \nu_t I_t$. If the investment tax credit rate, ν_t, is set equal to the capital income tax rate, τ_t^K, this expression for capital income tax revenue is identical to equation (5), and the capital income tax is non-distortionary.

Now that we have shown that a constant capital income tax rate combined with immediate expensing (or the equivalent investment tax credit) is non-distortionary,

we turn to the second major, and less well explored, question in this paper: Can a non-distortionary tax on capital income collect enough revenue to pay for government purchases, without having to resort to additional (distortionary) taxes?

6 How Much Revenue Can be Collected by a Non-distortionary Tax on Capital Income?

In this section, I will analyze the amount of revenue that can be collected by a constant tax rate on capital income with immediate expensing of capital expenditures. I begin by calculating capital income tax revenue in an arbitrary period t. Next, to illustrate the size of capital income tax revenue, I will focus on balanced growth paths. First, I will analyze the special case with constant A_t and G_t, so that there is a steady state. Then I will analyze balanced growth paths with non-negative growth rates. The reason for analyzing the two cases separately is that the utility function must satisfy some additional restrictions in order for a balanced growth path with positive growth to exist. The steady state does not require additional restrictions on the utility function.

6.1 Capital Income Tax Revenue in an Arbitrary Period

To calculate taxable capital income in the presence of a constant tax rate, set the left hand side of equation (17) equal to the left hand side of equation (18), and set $\tau_{t+1}^K = \tau_t^K$ to obtain

$$F_K (K_t, A_t H_t) = R_t - 1 + \delta. \tag{22}$$

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Define $\gamma_{t+1} \equiv \frac{K_{t+1}}{K_t}$ to be the gross growth rate of the capital stock from period t to period $t+1$. Substituting $\gamma_{t+1}K_t$ for K_{t+1} in equation (4) and rearranging yields

$$I_t = \left(\gamma_{t+1} - 1 + \delta\right) K_t. \tag{23}$$

Define X_t to be taxable capital income, with immediate expensing, in period t, so

$$X_t \equiv K_t F_K \left(K_t, A_t H_t\right) - I_t. \tag{24}$$

Substituting equations (22) and (23) into equation (24) yields

$$X_t = \left(R_t - \gamma_{t+1}\right) K_t. \tag{25}$$

To express taxable capital income as a share of output, Y_t, multiply both sides of equation (22) by $\frac{K_t}{Y_t}$, define $\eta_t \equiv \frac{K_t F_K(K_t, A_t H_t)}{Y_t}$ as the capital share in income in period t, and rearrange to obtain

$$K_t = \frac{\eta_t}{R_t - 1 + \delta} Y_t. \tag{26}$$

Substitute equation (26) into equation (25) to obtain

$$X_t = \frac{R_t - \gamma_{t+1}}{R_t - 1 + \delta} \eta_t Y_t. \tag{27}$$

Equation (27) shows the value of taxable capital income in an arbitrary period t. Let P_t be the present value of taxable capital income from period t onward, discounted back to period t,

$$P_t \equiv X_t + \sum_{j=1}^{\infty} \left(\prod_{i=1}^{j} R_{t+i}^{-1}\right) X_{t+j}. \tag{28}$$

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It can be shown that[4],[5]

$$P_t = R_t K_t. \tag{29}$$

Equation (29) implies that if capital income is taxed at rate τ^K from period t onward, the present value of capital income tax revenue is $\tau^K R_t K_t$. Thus, in terms of its effects on tax revenue, this policy is equivalent to a one-time government seizure of a fraction $\tau^K R_t$ of the capital stock in period t. In other contexts, governments that can seize capital once may face a temptation to seize capital again. This is the nature of the classic time-consistency problem. However, with the capital income tax analyzed here, there is no time-consistency problem, provided that the capital income tax–or the equivalent one-time capital levy–can collect sufficient revenue to fund government purchases. The capital income tax in this economy is non-distortionary and the economy achieves the first-best allocation, so there is no incentive for the government to try to set a lower tax rate initially to entice additional capital accumulation only to seize it later.

For an economy that is in a very early stage of development with a very low level of the capital stock, K_t, equation (29) might appear to imply that the opportunity to finance government purchases with the capital income tax described here is very limited. However, it is important to recognize that a very low value of K_t implies

that $R_t = F_K(K_t, A_t H_t) + 1 - \delta$ is very high. The high value of R_t implies that P_t can be substantially larger than K_t. In addition, the high value of R_t means that future taxable capital income is discounted at a very high rate, which would make the value of P_t appear small relative to the future flows of taxable capital income. A more appropriate way to gauge the size of taxable capital income and the possibility of financing government purchases with a capital income tax is to compare flows over long periods of time, such as in a steady state or along a balanced growth path. I now turn to these cases.

6.2 Steady State

Suppose that the index of labor-augmenting technical progress, A_t, and government purchases, G_t, are both constant and that the economy is in a steady state with constant capital, K, investment, $I = \delta K$, consumption, C, hours, H, and tax rate τ^K. Because the capital stock is constant, $\gamma_{t+j} \equiv \frac{K_{t+j}}{K_{t+j-1}} = 1$, for $j = 0, 1, 2, \dots$. In this case, equation (18) implies

$$R_t = \beta^{-1} \equiv 1 + \rho, \tag{30}$$

where $\rho > 0$ is defined to be the rate of time preference. Substituting equation (30) into equation (25) and setting $\gamma_{t+1} = 1$ yields steady-state taxable capital income (where I have omitted the time subscripts because these variables are constant in a steady state)

$$X = \rho K. \tag{31}$$

Multiplying steady-state taxable capital income in equation (31) by the tax rate on capital income, τ^K, yields

$$T^K = \tau^K \rho K. \tag{32}$$

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Taxable capital income in the steady state is the product of the rate of time preference, ρ, and the capital stock, K. Because the constant tax rate τ^K is not distortionary, tax revenue is proportional to τ^K for $0 \leq \tau^K < 1$. That is, there is no Laffer curve for τ^K. By setting τ^K arbitrarily close to one, the government can collect capital income tax revenue that is arbitrarily close to taxable capital income, which is ρK in the steady state.

The steady-state capital-output ratio is obtained by substituting equation (30) into equation (26), which yields

$$K_t = \frac{\eta}{\rho + \delta} Y_t. \tag{33}$$

To relate steady-state tax revenue to steady-state output, substitute equation (33) into equation (32) to obtain

$$T^K = \tau^K \frac{\rho}{\rho + \delta} \eta Y. \tag{34}$$

The share of taxable capital income in total income, $\frac{X_t}{Y_t}$, is $\frac{\rho}{\rho + \delta} \eta$. As an illustration, suppose that the rate of time preference is $\rho = 0.01$, the depreciation rate is $\delta = 0.08$, and the capital share is $\eta = 0.33$. In this case, taxable capital income is 3.67% of total income. In the next subsection, I will show that along a balanced growth path with a positive growth rate, the share of taxable capital income, $\frac{X_t}{Y_t}$, can be substantially higher.

6.3 Balanced Growth Path

In this subsection, I consider balanced growth paths along which the capital stock, consumption, and effective hours of work all grow at the same constant gross rate $\gamma \geq 1$, which is the exogenous growth rate of the index of labor-augmenting technical progress, A_t. Specifically, $\frac{K_{t+1}}{K_t} = \frac{C_{t+1}}{C_t} = \frac{A_{t+1}}{A_t} = \gamma$. In order for the economy to

be able to attain such a balanced growth path, I assume that the utility function $u(C_t, l_t)$ has a constant elasticity with respect to C_t and is multiplicatively separable in C_t and l_t. Specifically,

$$u(C_t, l_t) = \frac{1}{1-\alpha} C_t^{1-\alpha} v(l_t) \tag{35}$$

where $\alpha > 0$ is the inverse of the intertemporal elasticity of substitution, $v() > 0$, $v'()$ has the same sign as $1 - \alpha$, and $v''()$ has the opposite sign of $1 - \alpha$. I assume that $\beta\gamma^{1-\alpha} < 1$, so that along a balanced growth path the present value of the stream of current and future utility in equation (9) is finite.

Using the utility function in equation (35), along with $\tau_t^L = 0$ and $\tau_t^K = \tau^K$ for all t, the household's first-order conditions in equations (16) - (18) can be written as

$$(H_t): \quad \frac{1}{1-\alpha} C_t v'(1 - H_t) = A_t (F_L(K_t, A_t H_t)) v(1 - H_t) \tag{36}$$

$$(K_{t+1}): \quad \left[\beta \frac{C_{t+1}^{-\alpha} v(1 - H_{t+1})}{C_t^{-\alpha} v(1 - H_t)}\right] [F_K(K_{t+1}, A_{t+1} H_{t+1}) + 1 - \delta] = 1 \tag{37}$$

$$(B_{t+1}): \quad \left[\beta \frac{C_{t+1}^{-\alpha} v(1 - H_{t+1})}{C_t^{-\alpha} v(1 - H_t)}\right] R_{t+1} = 1. \tag{38}$$

Along a balanced growth path, A_t, K_t and C_t all grow at gross rate γ, while hours per worker, H_t, the marginal products $F_L(K_t, A_t H_t)$ and $F_K(K_t, A_t H_t)$, and the capital share in income, η_t, are constant. Therefore, equation (38) implies

$$R_t = \beta^{-1} \gamma^\alpha. \tag{39}$$

Substituting equation (39) into equation (25) yields taxable capital income as a func-

$\frac{X_t}{Y_t}$ = **Share of Taxable Capital Income**				
		Gross growth rate, γ		
		1.00	1.01	1.02
Inverse	2	0.037	0.061	0.078
of	3	0.037	0.084	0.113
Intertemporal	4	0.037	0.103	0.140
Elasticity of	5	0.037	0.120	0.161
Substitution,	8	0.037	0.159	0.205
α	10	0.037	0.178	0.224
capital income share: $\eta = 0.33$				
rate of time preference: $\rho = 0.01$				
depreciation rate: $\delta = 0.08$				

Table 1: Taxable Capital Income as a Share of Total Income

tion of the capital stock along a balanced growth path,

$$X_t = \left(\beta^{-1}\gamma^\alpha - \gamma\right) K_t > 0, \tag{40}$$

where the inequality on the right hand side follows from the assumption that $\beta\gamma^{1-\alpha} < 1$. Substituting equation (39) into equation (27) shows taxable capital income as a fraction of output along a balanced growth path,

$$X_t = \frac{\beta^{-1}\gamma^\alpha - \gamma}{\beta^{-1}\gamma^\alpha - 1 + \delta}\eta Y_t. \tag{41}$$

Multiplying taxable capital income in equation (41) by the tax rate τ^K yields capital income tax revenue along a balanced growth path

$$T_t^K = \tau^K \frac{\beta^{-1}\gamma^\alpha - \gamma}{\beta^{-1}\gamma^\alpha - 1 + \delta}\eta Y_t. \tag{42}$$

Note that when $\gamma = 1$, so that the balanced growth path is a steady state, equation (42) becomes identical to equation (34).

Table 1 shows the share of taxable capital income, X_t, in total income, Y_t, for

various values of α, the inverse of the intertemporal elasticity of substitution, and γ, the exogenous growth rate of A_t. The calculations in this table are based on a capital income share $\eta = 0.33$, a rate of time preference $\rho = 0.01$, and a depreciation rate $\delta = 0.08$. The first column of results in Table 1 shows that in a steady state, i.e., with $\gamma = 1$, taxable capital income is 3.7% of total income, as shown in subsection 6.2, regardless of the value of α. Table 1 shows that with modest growth and a modest value of α, the share of taxable capital income in total income can be substantially higher. For instance, with growth of one percent per year, i.e., $\gamma = 1.01$, and $\alpha = 5$, taxable capital income is 12.0% of total income, which is more than triple the value in the absence of growth. With growth of two percent per year and $\alpha = 5$, taxable capital income is 16.1% of total income.

7 Effective Tax Rate on Capital

The taxation of capital income in actual tax codes generally depends on an array of tax parameters including the tax rate on taxable capital income, the specification of depreciation allowances used to compute taxable capital income, and possibly also an investment tax credit rate and the extent to which financing costs are deductible. The concept of the *effective tax rate on capital* provides a scalar measure of the degree to which all of the relevant aspects of the tax code together affect a firm's optimal capital stock. In this section, I derive the effective tax rate on capital in the special case in which there is no investment tax credit and in which financing costs are not deductible by purchasers of capital. I will then use the effective tax rate on capital in this case to show the relationship between the results of this paper and the findings of Chamley (1986) and Judd (1985).

As a prelude to calculating the effective tax rate on capital, I will briefly review the calculation of the present value of depreciation deductions, z, introduced by Hall

and Jorgenson (1967). Consider a unit of capital that is purchased in period t for a price of \$1, and let $D(a) \geq 0$ be the depreciation allowance in period $t + a$ when the unit of capital has age $a \geq 0$. If nominal cash flows are discounted at rate i, then $z = \sum_{a=0}^{\infty} (1 + i)^{-a} D(a)$. If $\sum_{a=0}^{\infty} D(a) = 1$ and $i > 0$, then $z \leq 1$, with strict inequality if $D(0) < 1$. With immediate expensing, $D(0) = 1$ and $D(a) = 0$, for $a = 1, 2, 3...$, so $z = 1$.

Consider a firm that pays a capital income tax at constant rate τ^K on taxable income, which is calculated as gross capital income minus a specified depreciation allowance. Let M_t be the present value of the stream of pre-tax marginal products of capital accruing to the undepreciated portion of a unit of capital purchased in period t. Suppose that M_t is a decreasing function of $(1 - \delta) K_t + I_t = K_{t+1}$. Let χ_t be the price of acquiring a unit of capital in period t, and let $z\chi_t$ be the present value of depreciation deductions over the life of the capital good. The optimal value of investment in period t, I_t, satisfies

$$\left(1 - \tau^K\right) M_t = \left(1 - \tau^K z\right) \chi_t, \tag{43}$$

where the left hand side of equation (43) is the present value of the stream of after-tax marginal products of capital accruing to the undepreciated portion of a unit of capital acquired in period t, and the right hand side of equation (43) is the cost of acquiring a unit of capital in period t, net of the present value of the depreciation tax shield associated with capital.

The effective tax rate on capital, $\widehat{\tau}$, is the value of the tax rate on gross capital income, i.e., capital income without deducting any allowance for depreciation, from period t onward such that the optimal capital stock is the same as implied by equation

(43). Therefore, the effective tax rate satisfies

$$(1 - \widehat{\tau}) \, \widehat{M}_t = \widehat{\chi}_t, \tag{44}$$

where $\widehat{M}_t$ is the present value of the stream of pre-tax marginal products of capital accruing to the undepreciated portion of a unit of capital purchased in period t, and $\widehat{\chi}_t$ is the marginal cost of investment in period t when gross capital income is taxed at rate $\widehat{\tau}$. The value of $\widehat{\tau}$ is chosen so that the path of the capital stock under the gross capital income tax at rate $\widehat{\tau}$ is identical to the path of the capital stock associated with equation (43) when capital income, net of depreciation allowances, is taxed at rate τ^K. Since the financing cost is not deductible in either case, the discount rate is the same in both cases, so $\widehat{M}_t = M_t$ and $\widehat{\chi} = \chi_t$. Therefore, dividing each side of equation (44) by the corresponding side of equation (43) yields

$$1 - \widehat{\tau} = \frac{1 - \tau^K}{1 - \tau^K z}. \tag{45}$$

Equation (45) can be rearranged to obtain the following expression for the effective tax rate on capital

$$\widehat{\tau} = \frac{1 - z}{1 - \tau^K z} \tau^K. \tag{46}$$

In the prescription for optimal tax policy that I have described in this paper, as well as in the Chamley-Judd prescription for the long run, the effective tax rate on capital is zero. However, my prescription and the Chamley-Judd prescription obtain zero effective tax rates in different ways that have fundamentally different implications for the amount of revenue collected by the optimal capital income tax. The Chamley-Judd prescription sets τ^K equal to zero, which according to equation (46), achieves a zero effective tax rate on capital. However, with $\tau^K = 0$, the capital income tax does not collect any revenue, so it becomes necessary to use distortionary

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labor income taxation to collect revenue. My prescription for the optimal taxation of capital income, which includes immediate expensing, implies $z = 1$. Equation (46) shows that with $z = 1$ the effective tax rate on capital is zero for any non-negative tax rate τ^K less than 1. Thus, unlike the Chamley-Judd prescription, my prescription attains a zero effective tax rate on capital, while retaining the ability to collect revenue using the capital income tax, by setting τ^K greater than zero. As I have shown in Section 6, a substantial amount of capital income tax revenue can be collected with this prescription. If the capital income tax can collect enough revenue to pay for government purchases, there is no need to use distortionary labor income taxation.

8 Consumption Goods Tax with a Labor Subsidy

In this section, I illustrate an alternative tax system that can achieve the same allocation of consumption, leisure, and capital that can be achieved by lump-sum taxes. This alternative system combines a tax on consumption goods, levied at a constant rate over time, with a subsidy to labor. A consumption goods tax levied at a constant rate over time does not distort intertemporal margins, so it would not distort capital accumulation. If labor supply were perfectly inelastic, i.e., if leisure were not in the utility function, a constant tax rate on consumption goods would not affect the equilibrium allocation. However, when utility depends on leisure, a tax on the consumption good reduces the price of leisure relative to the taxed consumption good and thus effectively subsidizes leisure. To counteract this effect, leisure must also be taxed, which effectively subsidizes labor, to replicate the allocation with lump-sum taxes.

To examine the effects of a tax on the consumption good combined with a labor subsidy, I modify the model presented in Sections 1 - 3 by eliminating the capital

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income tax and replacing it with a tax on consumption. Specifically, letting $\tau_t^C > -1$ be the tax rate on the consumption good in period t,[6] total tax revenue in period t is

$$T_t = \tau_t^L w_t A_t H_t + \tau_t^C C_t. \tag{47}$$

With the capital income tax replaced by a tax on the consumption good, the budget constraint of the household in equation (10) is modified to

$$\left(1 + \tau_t^C\right) C_t + \left[K_{t+1} - (1 - \delta) K_t\right] + B_{t+1} = \left(1 - \tau_t^L\right) w_t A_t H_t + r_t K_t + R_t B_t, \tag{48}$$

so the lagrangian for the household's problem is

$$
\begin{aligned}
L &= \sum_{t=0}^{\infty} \beta^t u \left(C_t, 1 - H_t\right) \tag{49} \\
&\quad + \beta^t \lambda_t \left\{ \left(1 - \tau_t^L\right) w_t A_t H_t + r_t K_t - \left(1 + \tau_t^C\right) C_t - K_{t+1} + (1 - \delta) K_t - B_{t+1} + R_t B_t \right\}.
\end{aligned}
$$

Differentiating the lagrangian with respect to C_t, H_t, K_{t+1}, and B_{t+1}, setting the derivatives equal to zero, and then reducing the system of four first-order conditions to a system of three equations by eliminating λ_t and using the expressions for the wage rate and the rental rate in equations (2) and (3) yields

$$(H_t): \qquad u_l \left(C_t, 1 - H_t\right) = \frac{1 - \tau_t^L}{1 + \tau_t^C} A_t \left(F_L \left(K_t, A_t H_t\right)\right) u_C \left(C_t, 1 - H_t\right) \tag{50}$$

$$(K_{t+1}): \qquad \beta \frac{u_C \left(C_{t+1}, 1 - H_{t+1}\right)}{u_C \left(C_t, 1 - H_t\right)} \frac{1 + \tau_t^C}{1 + \tau_{t+1}^C} \left[F_K \left(K_{t+1}, A_{t+1} H_{t+1}\right) + 1 - \delta\right] = 1 \tag{51}$$

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[6]The tax rate on the consumption good can be larger than 100%, but (to keep the subsidized price of the consumption good positive) any subsidy to the consumption good must be smaller than 100%.

$$ (B_{t+1}): \qquad \beta \frac{u_C\left(C_{t+1}, 1-H_{t+1}\right)}{u_C\left(C_t, 1-H_t\right)} \frac{1+\tau_t^C}{1+\tau_{t+1}^C} R_{t+1} = 1. \tag{52} $$

If the consumption tax rate is constant over time, the ratio $\frac{1+\tau_t^C}{1+\tau_{t+1}^C}$ in equations (51) and (52) equals one, so these equations are identical to equations (20) and (21), respectively, for an economy with lump-sum taxation. However, a tax on the consumption good alone will not fully reproduce the allocation in an economy with lump-sum taxes because the tax on the consumption good distorts the labor-leisure choice, as is evident in equation (50). If labor income is subsidized to the same extent that consumption is taxed, specifically, if $\tau_t^L = -\tau_t^C$, then the ratio $\frac{1-\tau_t^L}{1+\tau_t^C}$ in equation (50) equals one, and this equation is identical to equation (19) for the economy with lump-sum taxes. Therefore, the allocation that would prevail under lump-sum taxes can be achieved by taxing the consumption good at a constant rate τ^C while subsidizing labor with a negative labor income tax rate $\tau^L = -\tau^C$.

To calculate the amount of tax revenue that can be collected with $\tau^L = -\tau^C$, substitute $Y_t - I_t - G_t$ for C_t (since the economy is a closed economy) in equation (47), and use $\tau^L = -\tau^C$ to obtain

$$ T_t = \tau^C \left(Y_t - I_t - G_t - w_t A_t H_t \right). \tag{53} $$

Use the facts that $Y_t = K_t F_K\left(K_t, A_t H_t\right) + A_t H_t F_L\left(K_t, A_t H_t\right)$ and $w_t = F_L\left(K_t, A_t H_t\right)$, to rewrite equation (53) as

$$ T_t = \tau^C \left[K_t F_K\left(K_t, A_t H_t\right) - I_t - G_t \right]. \tag{54} $$

Use the expression for taxable capital income with immediate expensing, X_t, in equa-

tion (24) to rewrite equation (54) as

$$T_t = \tau^C \left(X_t - G_t \right). \tag{55}$$

Henceforth, I will confine attention to cases in which $\tau^C > 0$ and the tax revenue in equation (55) is positive, so I assume that $G_t < X_t$. Thus, as shown formally in equation (56) below, to collect a given amount of tax revenue in any period in which $G_t > 0$, τ^C would be higher than the capital income tax rate τ^K that would lead to the same optimal allocation, even though (as in the United States), consumption can be far larger than capital income net of investment expenditure. The reason that τ^C exceeds τ^K is that in order for the consumption tax system to be optimal it must include a subsidy to labor income, which essentially gives back much of the revenue collected by the tax on the consumption good.

To derive the relation between the value of τ^C in an optimal consumption tax system and the value of τ^K in an optimal capital income tax system that would collect the same amount of revenue, I will now focus on balanced growth paths for which G_t and B_t both grow at the gross rate γ, which is the growth rate of A_t, K_t, C_t, and Y_t. I have shown that the optimal allocation can be attained either by taxing capital income, with immediate expensing, at rate τ^K or by taxing the consumption good at rate τ^C and subsidizing labor income at rate τ^C. Equating $\tau^K X_t$, the tax revenue collected by the capital income tax and $\tau^C \left(X_t - G_t \right)$, the tax revenue collected by the consumption goods tax and labor income subsidy in equation (55), yields

$$\tau^C = \frac{X_t}{X_t - G_t} \tau^K \geq \tau^K. \tag{56}$$

Thus, as discussed above, if $G_t > 0$, the consumption tax rate τ^C must exceed the capital income tax rate τ^K that collects the same total revenue.

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In order for the amount of government bonds to grow at rate γ, the amount of tax revenue, T_t, collected in period t must satisfy

$$T_t = G_t + R_t B_t - B_{t+1} = G_t + (R - \gamma) B_t, \tag{57}$$

where the second equality follows from the fact that γ_{t+1} and R_{t+1} are constant along balanced growth paths. Under the capital income tax with immediate expensing, the amount of tax revenue, $\tau^K X_t$, must equal the right hand side of equation (57), which implies that

$$G_t = \tau^K X_t - (R - \gamma) B_t. \tag{58}$$

Substituting equation (58) into equation (56) and simplifying yields

$$\tau^C = \frac{\tau^K}{1 - \tau^K + (R - \gamma) \frac{B_t}{X_t}}. \tag{59}$$

Along a balanced growth path with no government debt outstanding, equation (59) implies

$$\tau^C = \frac{\tau^K}{1 - \tau^K}. \tag{60}$$

Equation (60) indicates that the amount by which τ^C exceeds τ^K can be substantial. For instance, if $\tau^K = 50\%$, then an optimal consumption tax system would have to set $\tau^C = 100\%$ to collect the same amount of revenue.

The tax rate τ^C is levied directly on the consumption good, represented by C_t. However, the consumption bundle in period t consists of both leisure and the consumption good, so a true consumption tax would tax leisure at rate τ^C as well. Taxing leisure at rate τ^C is the same as subsidizing labor income at rate τ^C. In the absence of any tax (positive or negative) on labor income, a household can increase its consumption of leisure in period t by reducing its labor by one hour and

foregoing the hourly wage $A_t w_t$. However, if labor income is subsidized at rate τ^C, so that the after-tax hourly wage is $\left(1 + \tau^C\right) w_t A_t$, then the after-tax price of leisure is $\left(1 + \tau^C\right) w_t A_t$. Thus, a subsidy to labor income at rate τ^C is a tax on leisure at rate τ^C.

I have shown that a constant tax rate on the entire consumption bundle, which can be implemented by a constant tax rate on the consumption good together with a constant subsidy to labor income at the same rate, is equivalent to the optimal capital income tax with immediate expensing. It is important to note, however, that some tax proposals that purport to be consumption taxes are actually taxes on consumption goods that fail to tax leisure. For instance, the Hall-Rabushka (1995) proposal for a "flat tax" is described by its originators as a consumption tax. Hall and Rabushka proposed to implement their consumption goods tax by taxing both capital income and labor income. They define taxable capital income to be the same as I have discussed in this paper. That is, they propose immediate expensing of capital expenditures. However, their proposed tax system also levies a tax on labor, and thus is distortionary if leisure enters the utility function in a nontrivial way.

To summarize, if the optimal tax system is to be implemented by taxing income, then taxable capital income should be calculated with immediate expensing and labor income should not be taxed at all. Hall and Rabushka specify taxation of capital income that includes immediate expensing, but their proposed taxation of labor income is distortionary. Alternatively, if the optimal tax system is to be implemented as a consumption tax, it must tax the entire consumption bundle by taxing the consumption good and taxing leisure by subsidizing labor income. Hall and Rabushka propose to tax only the consumption good, and thus their proposed system is distortionary.

9 Concluding Remarks

A classic problem in public finance is to determine how the government can collect revenue in the least distortionary way possible, given that lump-sum taxes are not available. In a general equilibrium Ramsey framework with infinitely-lived households, Chamley (1986) and Judd (1985) have shown that in the long run it is optimal for the tax rate on capital to be zero. In their analyses, the government would collect tax revenue in the long run by taxing labor income. In this paper, I have re-examined the conditions, previously derived without general equilibrium considerations, under which the tax rate on capital income does not affect the optimal capital stock of a firm. The possibility that a capital income tax can be levied in a way that does not affect the capital investment decision is tantalizing because it may provide the government with a non-distortionary means to collect revenue. To examine this possibility, I addressed two questions. First, does the invariance of the capital investment choice to the tax rate on capital income, under appropriate assumptions about the deductibility of capital expenditures, carry over to a general equilibrium framework? Second, if the answer to the first question is affirmative, can a capital income tax that is non-distortionary collect a substantial amount of revenue?

In response to the first question, I have shown that if purchasers of capital are permitted to immediately expense capital expenditures, then in a Ramsey framework with a representative infinitely-lived household, the accumulation of capital is invariant to a tax rate on capital income that is constant over time. That is, a constant capital income tax rate with immediate expensing is a non-distortionary tax in general equilibrium, as well as in the context of a single firm that cannot deduct its financing costs. For firms that can deduct financing costs, the situation is different. For such firms, economic depreciation leads to the invariance of capital accumulation to the tax rate on capital in the context of a single firm; however, as shown in the Appendix,

this invariance does not carry over to general equilibrium because the after-tax cost of financing–not the pre-tax cost of financing–is invariant to the tax rate in general equilibrium. Thus, I focused the body of the paper on the case without deductibility of financing costs (as for an all-equity firm in the United States) and with immediate expensing so that the capital income tax is non-distortionary.

Since the answer to the first question is affirmative with immediate expensing, I then addressed the second question for the case of immediate expensing. Taxable capital income with immediate expensing is equal to capital income (revenue less wages) minus investment expenditures. Abel, Mankiw, Summers, and Zeckhauser (1989) have shown that in a dynamically efficient economy, capital income is larger than investment, which implies that, even with immediate expensing, there is a positive amount of capital income to be taxed. Since capital accumulation is unaffected by the level of the constant tax rate on capital income, provided there is immediate expensing, there is no "Laffer curve" for the capital income tax rate. That is, the amount of tax revenue collected from the capital income tax is proportional to the tax rate. The amount of revenue that can be collected is arbitrarily close to the amount of taxable capital income in the economy. I derive expressions for the share of taxable capital income in total income, both in a steady state and along a balanced growth both. Illustrative calculations suggest that this amount could be substantial. If one looks at U.S. data to judge the size of taxable capital income with immediate expensing, one could start with gross capital income of about 1/3 of GDP and capital investment expenditures of about 1/6 of GDP, so that taxable capital income is about 1/6 of GDP. Over the ten-year period ending in 2005, Federal government purchases of goods and services were about 6.5% of GDP–an amount that could be financed by a capital income tax rate of 39%. While some may regard this value of the capital income tax rate as somewhat high, it is important to note that the labor income tax

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rate would be zero and that capital expenditures would be immediately expensed.

Appendix

Economic Depreciation with Deductibility of Financing Costs

Hall and Jorgenson (1971) have shown, and the results of Samuelson (1964) imply, that in the context of a single firm that takes its pre-tax cost of financing as fixed and can deduct its cost of financing, the tax rate on capital income does not affect optimal capital accumulation if the firm is allowed to deduct economic depreciation from gross capital income when it computes taxable capital income. Thus, it might seem that economic depreciation and deductibility of financing costs provides another opportunity for non-distortionary taxation of capital income. However, unlike in the case of immediate expensing without deductibility of financing costs, the neutrality result for a single firm does not carry over to a general equilibrium framework. To illustrate why this neutrality result does not carry over to general equilibrium, I begin by illustrating the neutrality result for a single firm that takes the pre-tax cost of financing as fixed.

Consider an infinitely-lived firm with revenue, net of wages, $f(k_t)$ in period t, where k_t is the capital stock used by the firm in period t. Suppose that capital depreciates at rate δ per period, so that $k_{t+1} - (1 - \delta) k_t$ is gross investment in period t. Suppose that the tax rate on capital income in period t is τ_t^K and that the firm is allowed to deduct economic depreciation from net revenue in computing taxable capital income. The pre-tax cost of financing is θ per period, and suppose that the firm can deduct the cost of financing so that its after-tax cost of financing in period t is $\left(1 - \tau_t^K\right)\theta$. In this case, given k_0, the firm chooses the path of the capital stock to maximize

$$\sum_{t=1}^{\infty} \left(\prod_{j=1}^{t} \frac{1}{1 + \left(1 - \tau_t^K\right)\theta} \right) \left[\left(1 - \tau_t^K\right) f(k_t) + \tau_t^K \delta k_t - \left(k_{t+1} - (1 - \delta) k_t\right) \right], \quad \text{(A.1)}$$

where the three terms in square brackets on the right hand side are, respectively, after-tax net revenue $\left(1 - \tau_t^K\right) f\left(k_t\right)$, the value of the tax saving due to the depreciation deduction $\tau_t^K \delta k_t$, and the cost of gross investment $\left(k_{t+1} - \left(1 - \delta\right) k_t\right)$ in period t. Differentiating the right hand side of equation (A.1) with respect to k_{t+1} and setting the derivative equal to zero yields

$$\left(\frac{1}{1 + \left(1 - \tau_{t+1}^K\right)\theta}\right) \left[\left(1 - \tau_{t+1}^K\right) f'\left(k_{t+1}\right) + \tau_{t+1}^K \delta + 1 - \delta\right] = 1. \qquad \text{(A.2)}$$

Multiplying both sides of equation (A.2) by $1 + \left(1 - \tau_{t+1}^K\right)\theta$ and simplifying yields

$$\left(1 - \tau_{t+1}^K\right) \left[f'\left(k_{t+1}\right) - \delta - \theta\right] = 0. \qquad \text{(A.3)}$$

Equation (A.3) illustrates the invariance of the firm's optimal capital stock with respect to the capital income tax rate. For any value the capital income tax rate, τ_t^K, such that $0 \leq \tau_t^K < 1$, the optimal capital stock satisfies $f'\left(k_t\right) = \delta + \theta$. This invariance depends importantly on the fact that the pre-tax cost of financing, θ, is invariant to the tax rate τ_t^K. However, in general equilibrium, the pre-tax cost of financing will not remain invariant to τ_t^K, as I will now show.

Consider the general equilibrium model introduced in Sections 1 - 3, but change the tax treatment of capital income. Specifically, suppose that instead of immediate expensing, the owners of capital are allowed to deduct an amount equal to economic depreciation δK_t. I will write the depreciation deduction in period t as $D_t K_t$ to include also the alternative depreciation allowance schedule that I introduce at the end of this Appendix. With this specification of the tax treatment of capital income,

the lagrangian for the household in equation (11) becomes

$$L = \sum_{t=0}^{\infty} \beta^t u\left(C_t, 1 - H_t\right) \tag{A.4}$$

$$+ \beta^t \lambda_t \left\{ \begin{array}{c} \left(1 - \tau_t^L\right) w_t A_t H_t + \left(1 - \tau_t^K\right) r_t K_t + \tau_t^K D_t K_t \\ -C_t - K_{t+1} + (1 - \delta) K_t - B_{t+1} + R_t B_t \end{array} \right\}.$$

Differentiating the lagrangian with respect to C_t, H_t, K_{t+1}, and B_{t+1}, setting the derivatives equal to zero, and then reducing the system of four equations to a system of three equations by eliminating λ_t yields

$$(H_t): \qquad u_l\left(C_t, 1 - H_t\right) = \left(1 - \tau_t^L\right) w_t A_t u_C\left(C_t, 1 - H_t\right) \tag{A.5}$$

$$(K_{t+1}): \qquad \beta \frac{u_C\left(C_{t+1}, 1 - H_{t+1}\right)}{u_C\left(C_t, 1 - H_t\right)} \left[\left(1 - \tau_{t+1}^K\right) F_K\left(K_{t+1}, A_{t+1} H_{t+1}\right) + \tau_{t+1}^K D_{t+1} + 1 - \delta\right] = 1 \tag{A.6}$$

$$(B_{t+1}): \qquad \beta \frac{u_C\left(C_{t+1}, 1 - H_{t+1}\right)}{u_C\left(C_t, 1 - H_t\right)} R_{t+1} = 1. \tag{A.7}$$

If the allocation C_t, H_t, and K_{t+1}, $t = 0, 1, 2, ...$, is to be invariant to the tax rate on capital income, then the left hand side of equation (A.7) indicates that R_{t+1} must be invariant to the tax rate on capital income. The gross rate of return R_{t+1} is an *after-tax* gross rate of return. The invariance of economic depreciation in the context of a single firm is predicated on the invariance of the *pre-tax* cost of financing to the tax rate on capital income, so that the after-tax cost of financing is a decreasing function of the tax rate. Because the after-tax cost of financing is invariant to the tax rate in general equilibrium, economic depreciation does not lead to neutrality with respect to the tax rate on capital income, as I will show below.

Setting the left hand sides of equations (A.6) and (A.7) equal to each other yields

$$\left(1 - \tau_{t+1}^K\right) F_K\left(K_{t+1}, A_{t+1}H_{t+1}\right) + \tau_{t+1}^K D_{t+1} + 1 - \delta = R_{t+1}. \qquad \text{(A.8)}$$

In the case of economic depreciation, $D_t = \delta$, so equation (A.8) can be rewritten as

$$F_K\left(K_{t+1}, A_{t+1}H_{t+1}\right) - \delta = \frac{R_{t+1} - 1}{1 - \tau_{t+1}^K}. \qquad \text{(A.9)}$$

If the allocation C_t, H_t, and K_{t+1}, $t = 0, 1, 2, ...$, is to be invariant to the tax rate on capital income, then the left hand side of equation (A.9) must be invariant to the tax rate on capital income, and, as discussed above, R_{t+1} must be invariant to the tax rate on capital income. However, unless $R_{t+1} = 1$, the right hand side of equation (A.9) depends on τ_{t+1}^K, which would be inconsistent with the left hand side being invariant to τ_{t+1}^K. Therefore, economic depreciation will not, in general, make the capital income tax a non-distortionary tax in general equilibrium.[7]

Economic depreciation does not make capital accumulation invariant to the tax rate on capital income in general equilibrium because the after-tax cost of financing is invariant to the tax rate. In the context of a single firm, when the after-tax cost of financing is invariant to the tax rate on capital income, neutrality of the tax rate requires that the present value of depreciation allowances, z, must equal one. I will show that it is possible to augment economic depreciation to achieve $z = 1$, and thus to achieve neutrality; however, under this alternative neutral tax scheme, taxable capital income is identically zero.

Suppose that the depreciation factor, D_t, that is introduced in equation (A.4) is

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[7]If it were to turn out that $R_{t+1} = 1$, then equation (A.9) implies that taxable capital income, $F_K\left(K_{t+1}, A_{t+1}H_{t+1}\right)K_{t+1} - \delta K_{t+1}$, would be zero. Therefore, even if it turned out that $R_{t+1} = 1$, so that economic depreciation would lead to tax rate neutrality, the capital income tax would not be able to collect any revenue.

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specified to be

$$D_t = R_t - 1 + \delta, \qquad (A.10)$$

where R_t is the after-tax gross rate of return on bonds in equation (A.7). A unit of capital purchased in period t increases the capital stock in period $t+j$, $j = 1, 2, 3, ...$, by $(1-\delta)^{j-1}$ units, so the present value of depreciation deductions accruing to a unit of capital purchased in period t is $z_t = \sum_{j=1}^{\infty} \left(\prod_{i=1}^{j} R_{t+i}^{-1} \right) (1-\delta)^{j-1} D_{t+j}$. It can be shown that with the specification of depreciation allowances in equation (A.10), $z_t = 1.$[8] To see that the depreciation allowance specified in equation (A.10) is neutral, substitute $R_t - 1 + \delta$ for D_t in equation (A.8) to obtain

$$\left(1 - \tau_{t+1}^K\right) \left[F_K \left(K_{t+1}, A_{t+1}H_{t+1}\right) + 1 - \delta\right] = \left(1 - \tau_{t+1}^K\right) R_{t+1}, \qquad (A.11)$$

which implies

$$F_K \left(K_{t+1}, A_{t+1}H_{t+1}\right) + 1 - \delta = R_{t+1} \qquad (A.12)$$

regardless of the value of τ_{t+1}^K. The amount of taxable income, with the depreciation allowance specification in equation (A.10) is

$$F_K \left(K_t, A_tH_t\right) K_t - D_t K_t = \left[F_K \left(K_t, A_tH_t\right) - \left(R_t - 1 + \delta\right)\right] K_t = 0, \qquad (A.13)$$

where the second equality follows from equation (A.12). Thus, the neutral tax scheme in equation (A.10) will generate precisely zero taxable capital income in each period.

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[8] $z_t = \sum_{j=1}^{\infty} \left(\prod_{i=1}^{j} R_{t+i}^{-1} \right) (1-\delta)^{j-1} \left(R_{t+j} - 1 + \delta\right) = \sum_{j=1}^{\infty} \left(\prod_{i=1}^{j-1} \frac{1-\delta}{R_{t+i}} \right) \left(1 - \frac{1-\delta}{R_{t+j}}\right)$. Let $x_0 = 1$ and $x_{t+j} \equiv \frac{1-\delta}{R_{t+j}}$, for $j = 1, 2, 3, ...$ Therefore, $z_t = \sum_{j=1}^{\infty} \left(\prod_{i=1}^{j-1} x_{t+i} \right) (1 - x_{t+j})$. Applying the Lemma in footnote 5 yields $z_t = 1$.

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