The Benefits of Volume-Conditional Order-Crossing[*]

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Abstract

We assess the role and viability of an order-crossing or market-clearing mechanism that is automatically triggered only when a minimum number of shares can be crossed. Such a mechanism is naturally more attractive to traders who do not require much immediacy for their trades, as liquidity is cheaper in this market than in a continuous-auction market. The volume condition that we propose is crucial to the effectiveness with which this market complements the continuous-auction market in two important ways. First, when appropriately set, the volume condition endogenously adjusts the probability that market-clearing is triggered and so keeps impatient traders and highly informed traders away. Second, because market-clearing with a large volume condition reduces the effects of adverse selection in this market, patient traders are more willing to place orders in it. As we show, these effects often combine into a Pareto-dominating equilibrium when the continuous-auction market and the crossing mechanism with the right volume condition are both open.

1 Introduction

Traders have diverse motives to trade, and their demands for immediacy can also vary widely. While some traders require immediate but potentially expensive liquidity, other traders can afford to be more patient. Pooling the trades of all traders into a single continuous-auction mechanism often forces some traders — those who do not value immediacy as much as others — to reconsider their participation in markets. For example, liquidity may be temporarily too expensive for a position to be taken or liquidated, or an informationally-motivated trade may not be worth the cost of the liquidity that it necessitates. In fact, as shown by Glosten (1989) and Bhattacharya and Spiegel (1991), market breakdowns can occur when the provision of liquidity is not sufficient to overcome severe information asymmetries.

In this paper, we argue that the addition of a specially-designed market-clearing mechanism remedies this situation, partially at least. As we show, this additional market can improve the efficiency of an existing continuous-auction market of the type studied in Kyle (1985) and in Glosten and Milgrom (1985), and in the process make all market participants better off. The augmenting market invites buy and sell orders that are crossed at a single price that is the average of the bid and offer prices in the continuous-auction market; it attracts additional traders without reducing the welfare of those who would have participated in its absence. Although the augmenting market shares many features with Electronic Communication Networks (ECNs) and crossing networks, its clearing mechanism is different: it is designed so that it clears only when orders submitted to it (specifically, the minimum of the orders to buy and those to sell) exceed a threshold volume condition.

The condition on volume in the augmenting market serves as an attractive feature to those traders who find the price of immediacy in the continuous auction market unacceptably high; at the same time it serves to deter those traders whose demand for immediacy (motivated by diverse reasons) is so high that the possibility that accumulated orders in the augmenting market may fall short of the volume threshold is unacceptable (as their order may not be filled, leaving them with a risky position or with a foregone profit opportunity). It is well-known (see Admati and Pfleiderer, 1988) that information-motivated traders would like to trade in a pool with uninformed investors. The threshold volume condition, however, serves to deter informed traders because they will all tend — by virtue of the fact that the information they receive will be positively correlated across members in their group — to assemble on one side of the market, hence increasing the likelihood that an individual informed trader's order will not be filled without affecting the likelihood that market-clearing is triggered.

Markets for financial securities and contracts have proliferated, but the newer markets have adopted mechanisms (like crossing networks and electronic limit order books) to compete with

older inertial markets and with each other for order flow.[1] Much of this competition is directed at attracting traders with diverse motives and demands for immediacy.[2] For example, crossing networks such as ITG's POSIT attempt to cross orders at the midpoint of the bid-offer spread at various times throughout the day, and rely on other trading venues for price discovery. As in our proposed order-crossing mechanism, orders in these crossing networks are anonymous and clear with less than 100% probability.[3] As the competition across these networks intensifies, however, it has been recently argued by Mehta (2007) that fragmentation is likely to eventually cause liquidity problems that these very systems are trying to solve.[4]

The augmenting market in our model is immune to such problems as it is organized and facilitated by the same exchange that serves to operate the continuous-auction mechanism. That way, this bicameral exchange increases its revenues, by charging a fee for those orders that are crossed when the threshold condition is satisfied in the augmented venue, thereby permitting it to lower the bid-offer spread (the cost of immediacy) to those traders who incur this spread in their continuous-auction venue. A lowered spread cost will in general lead to more information gathering and efficient pricing. Interestingly, the Nasdaq and the New York Stock Exchange (NYSE) have each recently proposed their own version of scheduled intraday crosses (Nasdaq's new Intraday Cross and NYSE's newly acquired MatchPoint) aimed at offering more crossing options to their traders and at competing with these third-party crossing platforms. Our results — derived independently of these evolutionary developments in the order-crossing-mechanism landscape — imply that a volume condition imposed on these proposed crossing mechanisms can play a beneficial and vital role.

We have organized the rest of the paper as follows. In section 2, we provide an overview of the model and the proposed augmenting market, and we place them in the context of related research. The technical development of the model begins in section 3, where we first describe the continuous-auction securities market and its participants, who are all restricted to be uninformed and trading with hedging motives. The equilibrium for the continuous-auction market (the only trading venue available to traders at this stage) is characterized here. The second trading venue, the volume-conditional market, is introduced in section 4. The equilibrium in the presence of the additional trading venue is derived, and its welfare properties are then compared to those of the continuous-auction market equilibrium. The possibility that some informed traders are also present in the economy is introduced in section 5. This section reassesses the equilibrium and welfare analysis of the previous section, and discusses how the price discovery process is affected by the presence

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[1]For example, see Fink, Fink and Weston (2006) for information about the recent growth in ECNs.

[2]Easley, Kiefer and O'Hara (1996) term the competition for uninformed order flow as "cream-skimming."

[3]This is also true of ECNs which, according to Mehta (2007), execute less than 10% of their order flow.

[4]The effects of fragmentation in various contexts are also discussed and analyzed by Hamilton (1979), Mendelson (1987) and Madhavan (1995), among others.

of a volume-conditional market alongside the continuous-auction market. An alternative market structure, in which a monopolist specialist is able to capture some rents, is considered in section 6. A discussion of our results and possible extensions of our model are presented in section 7, which also offers concluding remarks. All proofs are contained in the appendix.

2   An Overview

2.1   Description of the Model

The intuition behind the paper's results is best understood using a simplified version of the model, built around Demsetz's (1968) characterization of securities transactions. Suppose that all traders in the economy use the securities market for (uninformed) risk-sharing or hedging purposes, but that these traders differ in terms of their demand for immediacy. In particular, traders are either impatient or patient, i.e., they generate high or low value for immediacy in hedging their risky positions in the market. A specialist stands ready to provide liquidity to the market by posting a price at which he is willing to buy shares and a price at which he is willing to sell them. In an effort to recoup his opportunity cost of making a market or to maximize profits, the specialist purposely leaves some distance, the bid-ask spread, between these quoted prices. In this continuous-auction market, this spread represents the cost of liquidity: even though traders do not possess any payoff-relevant information, some are willing to pay this cost in return for rapid execution. However, the traders who do not require much immediacy prefer to avoid this cost by postponing their trades or by not trading at all. That is, the price of liquidity in the continuous-auction market effectively relegates these traders to the sidelines and, as a result, the economy fails to realize the economic surplus created by their latent hedging motives. The introduction of a volume-conditional market-clearing mechanism, operating jointly with the continuous auction, allows the economy to re-capture some of this surplus.

To see how this mechanism works, suppose that we allow traders to choose between one of two venues for trading: the continuous auction described above, or the proposed crossing market, which we refer to as a *volume-conditional market*. In this market every participant's order is crossed randomly with another's at the continuous auction's mid-quote price with some probability. The market's participants agree to pay a transaction fee (if their order is filled) to the market-maker, who can use this alternative venue to generate more revenues. If the probability of the order being filled is close to one, the problems of the continuous auction are transferred to or shared with this new market: the two mechanisms become essentially identical and traders simply converge to the cheaper venue and leave the other unused. If the probability is zero, trading can only take place in the continuous auction, as before.

However, as we increase the probability of market-clearing in the alternative trading venue above zero and set its fee to a value that makes transactions cheaper than in the continuous-auction market, it eventually starts attracting the more patient hedgers, but leaves the more desperate hedgers in the continuous-auction market. Indeed, the cheaper transactions offered by the conditional market do not always take place, and so traders who require immediate execution do not see a benefit to switching their trading venue. Patient traders who previously chose not to incur the effective spread cost of liquidity in the continuous-auction market view this new venue as an improvement: even though their orders are not always filled quickly, this situation is better than not ever having them filled at all. In addition, if the revenues generated from the conditional market lead the market-maker to reduce his quoted spread in the continuous market then even impatient traders benefit from the presence of the new venue. So even a small increase in the market-clearing probability of this conditional market improves the welfare of some (or all) traders without reducing that of anyone. The Pareto improvement is initially modest, but it increases as the market-clearing probability increases. The welfare gains that the conditional market can generate are maximized when the market-clearing probability is as large as possible, but not large enough to attract impatient hedgers to the conditional market.

Clearly, probabilistic market-clearing (with say a coin toss) for every order that is placed by a market participant in the conditional market is unrealistic. For one thing, it fails to ensure that buy and sell orders are matched. The alternative market-clearing mechanism that we propose in this paper, one that is volume-conditional, takes care of this problem but preserves the probabilistic aspect of market-clearing. The idea is to have the alternative trading venue clear if and only if a sufficient number of orders can be crossed at the same time. This makes market-clearing probabilistic, as traders do not know the likelihood that the volume condition will be met at the time they place their order, for that depends on whether enough traders show up on both sides of the market. For instance, a trader may find himself with an unhedged position when the price of the security shifts on him. As before, if traders have a lot to lose from the smaller probability of market-clearing of the conditional market, they will tend to prefer the immediacy of the more costly continuous-auction venue.

Volume-conditional market-clearing provides another benefit that a (coin toss-driven) probabilistic market-clearing does not: when payoff-relevant information asymmetries exist across traders, it protects uninformed traders from the adverse selection costs that the potential presence of informed traders brings about. Indeed, although uninformed traders will tend to split evenly on the buy and sell sides of the market (as their motive for trading is not correlated with the asset's eventual payoff), informed orders will tend to gather on one side of the market (if the information gathered by informed traders is positive, they will tend to send buy orders to the market). It follows

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that uninformed orders — but not orders from informed agents — increase the likelihood that the volume condition of the conditional market is eventually met. In fact, because informed orders tend to create an imbalance between the number of buy and sell orders, they serve to reduce the probability that any order will get crossed when the conditional market clears. As a result, uninformed traders with unfilled orders are not only stuck with unhedged positions, but they also tend to hold positions that the informed traders' information devalues (e.g., they tend to be short when the information is positive). For a given number of informed orders, a larger volume condition ensures that a large enough fraction of uninformed orders will on average get crossed when market-clearing is triggered. This reduces adverse selection and increases the propensity of uninformed traders to place orders in the conditional market in the first place. Again, some of the extra surplus that this generates for the market-maker may also end up subsidizing the continuous market.

Although the analysis is presented for a volume-conditional market complementing a continuous-auction market, our point is more general. Indeed, as long as the existing trading venues do not offer the possibility for a trader to hide his order and to condition it on the participation of many other traders on both (the buy and sell) sides of the market, some traders with low needs for immediacy will stay away from these trading venues. The volume-conditional market allows the exchange to recapture some of this unrealized economic surplus. For example, the volume-conditional market could also complement a pure limit-order market. As in the market we propose, limit orders allow traders with lesser immediacy needs to reduce their effective transaction costs. However, unlike the volume-conditional market, a limit-order market does not give traders a chance to condition their participation on the activity level of the market and the likely characteristics of the trading crowd that is associated with this activity. In terms of modelling, our point could be made equally well in a rational expectations model of the type studied by Grossman and Stiglitz (1980): if traders are uncertain not only about the value of the risky assets they trade but also about the composition of the trading crowd, they would benefit from conditioning their orders not only on the market-clearing price but also on the number of shares that are exchanged in equilibrium. In other words, the properties of market-clearing and order-crossing are different when fewer or more traders exchange shares.

2.2   Related Research

By studying how trading is affected by the rules for trade execution or organizational design, our paper fits into a long line of market microstructure literature. Whereas early work in this area concentrates on how traders and market-makers interact in one trading venue,[5] later work

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[5]Important papers include Garman (1976), Stoll (1978), Amihud and Mendelson (1980), Ho and Stoll (1981), Kyle (1985), Glosten and Milgrom (1985), and Easley and O'Hara (1987).

considers the impact that adding other competing or complementary trading venues can have on the equilibrium. For example, Seppi (1990) and Grossman (1992) study the impact of adding an upstairs market to an exchange, Chowdhry and Nanda (1991), Glosten (1994), and Parlour and Seppi (2003) study the competition between co-existing exchanges, while Hendershott and Mendelson (2000) study the impact that crossing networks are likely to have on dealer markets. Common to these papers is the adoption of assumptions and model inputs that mimic the structures and rules of existing exchanges. Our analysis is more positive in nature as our model studies a volume-conditional crossing mechanism that, to our knowledge, has yet to receive consideration by existing exchanges. As our analysis shows, such a mechanism could be a worthwhile addition to the operations of exchanges.[6]

In practice, alternative trading mechanisms, like ECNs and crossing networks, have begun to offer market participants the possibility of having their orders automatically matched with those of other participants. The volume-conditional market that we propose shares many of the features of these alternative trading mechanisms. For instance, all are attempts to provide cheaper and more efficient order-crossing platforms for investors who do not face the higher demands for immediacy that are felt by some more impatient traders. Still, two aspects of our volume-conditional trading venue differentiate it from these other trading mechanisms. First, our analysis shows that this trading venue need not compete with the continuous auction process of the exchange; it simply complements it, and in doing so, may even make the participants of the continuous auction better off. Second, the volume condition allows this proposed trading venue to clear precisely when it is most advantageous to traders. So, whereas ECNs may not be well-suited to the crossing of large trades and volume (Stoll, 2006), a volume-conditional market thrives in such conditions.

Our modelling technique is closest to that of Hendershott and Mendelson (2000) (HM henceforth) in that both papers study the equilibrium that prevails when two trading venues, one with intermediaries making a market and one that crosses orders automatically, operate alongside each other. However, there are many differences as well. First, HM model the market-making activity as a dealership market (like the Nasdaq) whereas we model it as a specialist system (like the NYSE). Second, when measuring the performance of the various trading setups they consider, HM use the utility of uninformed traders exclusively, thus ignoring the welfare of informed traders.

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[6]Of course, exchange design issues are not limited to the addition of trading mechanisms or the interactions between them. Many other dimensions of trading mechanisms can be considered. These dimensions include whether the market should operate continuously or as a series of batch auctions (Mendelson, 1987; Madhavan, 1992); whether the market should operate as a dealership market (Biais, Foucault and Salanié, 1998; Viswanathan and Wang, 2002); whether automated trading systems should be used (Domowitz, 1990; Glosten, 1994; Parlour and Seppi, 2003; Stoll, 2006); the role of the limit-order book (Rock, 1996; Seppi, 1997; Foucault, Kadan and Kandel, 2005); the role and impact of after-hours trading (Barclay and Hendershott, 2003, 2004), and the role of anonymous trading and market transparency (Forster and George, 1992; Biais, 1993; Pagano and Röell, 1996). Madhavan's (2000) survey of the market microstructure discusses other dimensions of exchange design.

Our model shows that introducing a conditional market with the right volume condition can make all traders, not just the uninformed ones, at least weakly better off. In this respect, the volume-conditional market represents a clear welfare-improving direction for exchange design, as advocated by Glosten (1998). Finally, our model shows that the volume condition can be crucial in attracting the right types of traders to the crossing market and deterring those traders who are better served by the continuous-auction venue. Ultimately, volume-conditional trading makes the exchange more efficient and its customers better off. Interestingly, the models do generate some results that are similar. For instance, like HM, we find that the (volume-conditional) crossing market is more appealing to uninformed traders who do not need extreme immediacy.

The volume-conditional market that we study is similar to Mendelson's (1982, 1987) clearing house which requires a certain number of orders before market-clearing is allowed to take place.[7] Our model differs from his along several dimensions. First the traders in our model choose whether or not to trade and in which venue to do so. In particular, they have the option to trade in a continuous-auction venue which offers more immediacy than the volume-conditional market. Second, because every trader in our model is characterized by a utility function, we are able to measure the welfare effects of introducing a volume-conditional market. Third, our model accounts for the presence of asymmetric information, which is absent in Mendelson's (1982) model. Finally, although Mendelson (1987) extends his earlier results on the role of the clearing house to various multi-market settings, he does not consider the case in which the clearing house and a specialist market co-exist.

3 The Continuous-Auction Market

3.1 Setup

Consider the market for a risky security whose end-of-period payoff $\tilde{v}$ is either $+1$ or -1 with equal probability.[8] Trading in this market occurs once at the beginning of the period and is done one share at a time by the traders in the economy. Every trader is assumed to be infinitesimally small relative to the total trader mass, which is given by a random variable $\tilde{m}$ uniformly distributed on $[0,1]$. For now, we assume that all traders are "hedgers" — they are uninformed and trade for hedging reasons only — and we maintain this assumption until section 5, when we introduce informed traders.

These traders differ in terms of the urgency of their hedging motive (their "type"), which is

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[7]In the main version of Mendelson's (1982) model, market-clearing takes place at regular intervals regardless of the number of orders submitted. The "alternative market design" considered in section 5 of his paper is the one that introduces the condition that a given number of orders must accumulate before trading takes place.

[8]Without loss of generality, we center the distribution of $\tilde{v}$ around zero, so that the buy and sell sides of the market can be analyzed with the same notation.

denoted by $\tilde{r}_j$ for trader j. For convenience, we assume that $\tilde{r}_j$ is equal to $r_{\mathrm{H}} \in (0,1]$ or to $r_{\mathrm{L}} \in (0, r_{\mathrm{H}})$ with probabilities λ and $1 - \lambda$ respectively, and that it is independent from $\tilde{r}_k$ for any $k \neq j$. Because the number of traders is infinite, this implies that a fraction λ of the traders in the economy will be of type r_{H} and a fraction $1 - \lambda$ will be of type r_{L}. In addition to observing their own type $\tilde{r}_j$ before trading, each uninformed trader j gets an independent wealth shock $\tilde{w}_j$. For each trader this wealth shock is $+1$ or -1 with equal probability, and is assumed to correspond to their initial position in the asset. The traders' type measures their inability to cope with their wealth shocks; a trader who does not hedge his position by selling (if $\tilde{w}_j = +1$) or buying (if $\tilde{w}_j = -1$) a share of the asset suffers a cost of $\tilde{r}_j$ in addition to the end-of-period payoff of his position. This cost can be interpreted as the level of risk aversion of the trader or, equally well, as the urgency with which he needs to trade. For example, the urgency can arise from the reputation cost suffered by a broker who fails to quickly fill an order placed by one of his clients; or from the loss of return relative to a performance benchmark that is felt by a pension portfolio manager who is expected to fully invest new cash infusions. For these reasons, we refer to the trader of type r_{L} as the *patient* hedger and to the trader of type r_{H} as the *impatient* hedger. Let us denote the quantity that uninformed trader j chooses to trade by $y_j \in \{-1, 0, +1\}$. Assuming that shares are transacted at a price of φ, this trader's end-of-period utility is given by $\tilde{u}_j = (\tilde{w}_j + y_j)\tilde{v} - \tilde{r}_j(\tilde{w}_j + y_j)^2 - y_j \varphi$ and, upon incurring the shock at the outset, he seeks to maximize[9]

$$\mathrm{E}\big[\tilde{u}_j \mid \tilde{r}_j, \tilde{w}_j\big] = -\tilde{r}_j(\tilde{w}_j + y_j)^2 - y_j \varphi.$$

For now, these hedgers can only trade in a continuous-auction venue, which we refer to as the *continuous market*. We assume that a market-maker stands ready to fill sell and buy orders at his quoted bid and ask prices respectively. As in Glosten and Milgrom (1985), we assume that market-making is competitive, but we also assume that the market-maker must recoup fixed costs of κ in order to break even. In equilibrium therefore, the market-maker sets bid and ask prices, b and a, so that his expected revenues are exactly equal to κ. Because of the symmetry of the asset's payoff (around zero) and that of all the model's assumptions, it is clear that the bid price quoted by the market-maker will satisfy $b = -a$. This means that we can concentrate the analysis on finding the ask price a that makes the market-maker break even on purchase orders. Finally, because our model only includes one round of trading, we assume that all sell (and buy) orders reach the market-maker simultaneously and are all cleared at the same bid (ask) price.

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[9]As evidenced by the utility function written in this way, our framework has the potential to handle more general distributions of $\tilde{w}_j$ and other (possibly fractional) trade sizes. However, because our current set of assumptions already effectively captures the idea that more impatient/risk-averse traders are more desperate to hedge their positions, increasing the level of generality in this way only comes at a cost of tractability. In section 7, we discuss how and why these alternative assumptions would generate similar results.

3.2 Equilibrium

To derive the equilibrium of this model, let us suppose that the market-maker quotes an ask price of $a \in [0,1]$ (and a corresponding bid price of $-a$). If a trader of type $\tilde{r}_j = r$ with a negative wealth shock $\tilde{w}_j = -1$ chooses not to trade, his expected utility is equal to $\mathrm{E}\big[\tilde{u}_j \mid \tilde{r}_j = r, \tilde{w}_j = -1\big] = -r$. If instead he chooses to hedge this shock by buying a share of the stock at a price of a, his expected utility is $\mathrm{E}\big[\tilde{u}_j \mid \tilde{r}_j = r, \tilde{w}_j = -1\big] = -a$. So only the hedgers of type $\tilde{r}_j \geq a$ trade. This means that, if $a > r_{\mathrm{H}}$, no trader ever trades and so the market-maker cannot recoup κ to break even. Thus the market-maker must quote $a \in [0, r_{\mathrm{H}}]$. Both types of hedgers will choose to trade if $a \in [0, r_{\mathrm{L}}]$, whereas only the impatient hedgers will choose to trade if $a \in (r_{\mathrm{L}}, r_{\mathrm{H}}]$. In what follows, we assume that $r_{\mathrm{L}} < 2\kappa$. This amounts to assuming that the market-maker cannot recoup κ by quoting a price that has everyone trade, since for any $a \in [0, r_{\mathrm{L}}]$, his expected profits are given by

$$\mathrm{E}\big[\tilde{\pi}_{\mathrm{MM}}\big] = a\mathrm{E}[\tilde{m}] - \kappa = \frac{a}{2} - \kappa \leq \frac{r_{\mathrm{L}}}{2} - \kappa < 0. \tag{1}$$

This assumption ensures that there are always some traders who find liquidity too expensive and so forego trading in the continuous-auction market.[10] As a result, the market-maker must be able to recoup κ by quoting an ask price $a \in (r_{\mathrm{L}}, r_{\mathrm{H}}]$. A sufficient assumption for this to be possible is that $r_{\mathrm{H}}\lambda > 2\kappa$. Indeed, because only a fraction λ of the traders show up to trade, we have

$$\mathrm{E}\big[\tilde{\pi}_{\mathrm{MM}}\big] = a\lambda\mathrm{E}[\tilde{m}] - \kappa = \frac{a\lambda}{2} - \kappa, \tag{2}$$

which is greater than zero at $a = r_{\mathrm{H}}$. This leads to the following equilibrium.

Proposition 3.1 (Continuous-Market Equilibrium) *The equilibrium ask price is given by*

$$a = \frac{2\kappa}{\lambda}, \tag{3}$$

and only impatient hedgers ($\tilde{r}_j = r_{\mathrm{H}}$) participate in the market.

In this equilibrium, the market-maker offsets his fixed costs by charging a larger ask price to fewer (impatient) traders, capturing the intuition originally developed by Demsetz (1968). Of course, standard economic theory tells us that equilibria in which some traders are prevented from trading leave some economic surplus unrealized. In such scenarios welfare can potentially be improved by appropriately reorganizing the market to allow these traders to re-enter markets. Proposition 3.1 represents our most basic such scenario, and the question we seek to answer is: how can we modify the market mechanism so that some of the unrealized surplus can be captured? From the point of view of an exchange that benefits from trading (by charging a per share trading

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[10]This would always be the case if the support for $\tilde{r}_j$ were the entire $[0,1]$ interval, but the analysis would also be less tractable.

fee for example), this question is particularly important. In what follows, we introduce a simple market that allows for the lost economic surplus resulting from the endogenous exclusion of some traders to be recaptured, at least in part. As we demonstrate, this market does not require any modification to the continuous-auction market; instead, it comes in the form of a second trading venue that operates in parallel with the continuous market.

4 The Volume-Conditional Market

In this section, we introduce an alternative trading venue for traders to place their orders. This venue combines the features of various existing trading mechanisms, like ECNs and automated trading systems, but adds a feature that sets it apart from them, namely a volume condition. We refer to this other trading venue as the volume-conditional market or the *conditional market* for short. Alternatively, because this trading venue accumulates orders before it attempts to cross them, we also refer to it as the order-crossing market or, more concisely, the *crossing market*.

4.1 Introducing the Conditional Market

As mentioned in the introduction, the Nasdaq and the NYSE are in the process of creating order-crossing platforms to complement their existing trading venues. Like the conditional market that we propose, these trading systems are to be operated by the exchanges, will sporadically cross orders throughout the day, and are targeted at traders with low immediacy needs. The volume condition of the market that we suggest in this paper is the one feature that distinguishes it from these trading systems. Before we show how this one feature could improve the efficiency with which exchanges implement their own crossing networks, let us describe a potential implementation of the proposed order-crossing mechanism. Buy and sell orders are aggregated in this market until enough orders are present — i.e., until a minimum volume condition is met — at which point the largest possible number of orders are crossed at the mid-quote price prevailing in the continuous market at that time. When submitting their orders to the crossing network traders do not know the state of the order book; they understand that there is a risk that their order will not clear for a while. Because the mid-quote price adjusts with the information that percolates the continuous market, traders are protected against price shifts that they cannot or do not wish to monitor.[11],[12]

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[11]Of course, this also means that this crossing network free-rides from the price discovery process of the continuous market, a feature of crossing networks that has been discussed by O'Hara (2004) and Stoll (2006), among others. As we show later, when appropriately chosen, the volume condition will ensure that the crossing network will not harm and might even improve price discovery.

[12]The fact that the price of the volume-conditional market adjusts also sets this trading venue apart from a limit-order market in which orders must be constantly monitored by market participants if they want to avoid market conditions to adversely shift on them. In section 7, we discuss how this and other differences between a limit-order market and a volume-conditional market make their co-existence possible and beneficial.

ψ	# buy orders	# sell orders	# orders crossed	Exchange receives
10	5	8	0	0
10	15	8	0	0
10	10	12	10	$20c$
10	15	12	12	$24c$
10	15	18	15	$30c$

Table 1: This table shows how the volume-conditional market operates. When the number of buy orders and the number of sell orders both exceed ψ, order-crossing is triggered. In that event and in that event only, the largest possible number of orders are crossed, and the exchange collects c for each of these crossed orders.

Our model of the crossing network is more stylized than the above description; it captures the main features and economic benefits of this market in a one-period setting that keeps the analysis tractable. In particular, we assume that the conditional market clears if the total number of buy orders and sell orders placed in that market both exceed ψ, the volume condition for the market. To capture the idea that the conditional market's price adjusts with that in the continuous market, we assume that, upon the volume condition being reached, the orders are crossed at a price of zero, which in the model is always the mid-quote price. If one side (say the buy side) in this market contains more orders than the other then we assume that the appropriate fraction of the buy orders are randomly chosen and cleared against all the sells. This captures the idea that traders do not know where their orders are in the conditional market's queue and so, assuming time priority, realize that their orders, if placed on the heavier side of the book, may require the conditional market to clear more than once before they can be crossed. Although every trade between a buyer and a seller of the risky security takes place at a price of zero, this transaction is not free for either. In particular, we assume that every buyer or seller whose order gets crossed with that of another trader is charged a pre-announced order-crossing fee of c by the exchange or market-maker in charge of the trading venue. Table 1 illustrates the effective operation of the volume-conditional market with an example.

As our model shows, the volume condition that is chosen for the market is crucial. If ψ is set to a large value, the conditional market is unlikely to ever clear, and so the conditional market becomes an unattractive venue. If ψ is set to a small value, then the conditional market becomes an attractive alternative to the continuous market and may even become the dominant trading venue, as traders are almost sure that it will always clear. As our analysis will show, setting ψ to an intermediate value allows trader types to be sorted and can generate Pareto improvements over the situation in which only the continuous market operates. Similarly, the order-crossing fee

c must be set strategically for the conditional market to be useful. Indeed, the conditional market is unattractive when c is large, and too attractive when c is low.

4.2 The Conditional Market as a Stand-Alone Trading Venue

To illustrate how the conditional market functions, we start by modelling it in isolation. That is, let us first assume that shares can only be traded in a conditional market and that trading in a continuous auction is — for this section — prohibited.[13] We assume that the market-maker now quotes two quantities, the volume condition (ψ) and the order-crossing fee (c), instead of bid and ask prices. Knowing the volume condition, traders know that their orders only get crossed if there are at least ψ orders on each side of the market. Knowing the order-crossing fee, they know that they will have to pay the market-maker c if their own order gets crossed.

Clearly, traders only place orders in this market if the cost of the shock they seek to hedge is larger than the cost of hedging it. That is, only traders of type $\tilde{r}_j$ above or equal to c place orders. Thus the number of active traders, $\tilde{m}_A$, is given by

$$\tilde{m}_A = \begin{cases} \tilde{m} & \text{if } c \in [0, r_L] \\ \lambda \tilde{m} & \text{if } c \in (r_L, r_H] \\ 0 & \text{if } c \in (r_H, 1]. \end{cases} \tag{4}$$

Also, because the wealth shocks of traders are uncorrelated and because there is an infinite number of hedgers in the economy, half of them end up placing buy orders and half end up placing sell orders. The volume condition determines if trades actually occur: the market clears only if $\frac{\tilde{m}_A}{2} \geq \psi$.

Lemma 4.1 *Suppose that only the conditional market operates. Suppose also that the market clears if a volume condition $\psi \geq 0$ is met, and that each trader pays c when his order gets crossed. The probability that the conditional market clears (i.e., that the volume condition is met) is then* $\max\{1 - 2\psi, 0\}$ *if $c \in [0, r_L]$,* $\max\left\{1 - \frac{2\psi}{\lambda}, 0\right\}$ *if $c \in (r_L, r_H]$, and zero otherwise.*

Because the number of buy orders and the number of sell orders are always equal, all hedgers who place orders in this conditional market will get their order crossed with probability one when the market clears. This perfect matching of buy and sell orders is an artifact of the assumption that the economy includes an infinite number of hedgers (and no informed trader). With a finite number of hedgers (which would complicate the analysis greatly), some traders may not see their order get filled even when the market clears; this happens when their order ends up on the more

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[13]Technically speaking, our assumption that price discovery takes place in the continuous market is not possible here. This is immaterial as we treat the conditional market as a stand-alone trading venue only for exposition purposes. Our ultimate objective is to show how the conditional market can complement the continuous market when the two operate in parallel. In fact, our analysis will show that the continuous market does strictly better than the conditional market as a stand-alone trading venue.

populated side of the market. For example, if N_B buy orders and $N_\text{S} < N_\text{B}$ sell orders end up being placed, only N_S of the N_B buy orders get filled (as long as N_S exceeds ψ, of course). In this event, to capture time priority and the idea that hedgers do not know the state of the order book when they place their orders, our model of the conditional market would assume that each buy order gets crossed with probability $\frac{N_\text{S}}{N_\text{B}}$. Although such complications do not enter the analysis when the economy consists of an infinite number of hedgers, they will become relevant when we add (even an infinite number of) informed speculators to the model in section 5. Indeed, because the trading motive of informed traders is correlated across traders,[14] these traders will tend to place orders on the same side of the market. Thus not only is it the case that the presence of informed speculators does not increase the probability that the conditional market clears but, it is also the case that it reduces the probability that each trader gets his order crossed when the conditional market does clear. We return to this issue in section 5; for now, given that the conditional market always contains the same number of buy and sell orders from hedgers alone, we have the following result.

Proposition 4.1 *In the conditional market with hedgers submitting orders, as long as the volume condition satisfies*

$$0 \le \psi \le \frac{1}{2}\sqrt{\lambda^2 - \frac{2\kappa\lambda}{r_\text{H}}}, \tag{5}$$

there is an equilibrium in which the market-maker charges

$$c = \frac{2\kappa\lambda}{\lambda^2 - 4\psi^2}. \tag{6}$$

In this equilibrium, only impatient hedgers ($\tilde{r}_j = r_\text{H}$) place orders. These traders are worse off than when the continuous market operates alone unless ψ is set equal to zero. In this latter case, the conditional market is equivalent to the continuous market and impatient hedgers are equally well-off.

The prospects of the volume-conditional market that we propose are not great if this market is to operate alone. Indeed, as before, the market-maker breaks even and the patient hedgers do not participate in markets. Unless the volume condition is removed (i.e., unless ψ is set to zero), the conditional market makes patient hedgers worse off. In short, the stand-alone conditional market is Pareto-dominated by the continuous market. This is intuitive. As in the continuous market, the reduction in the transaction cost c necessary to attract the patient hedgers to the conditional market is too small for the market-maker to recoup his fixed cost κ. Since the conditional market does not always clear when $\psi > 0$, the impatient hedgers do not trade as often, and so the market-maker must charge them more every time they do. Both effects make these traders worse off.

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[14]This has to be the case if their information is valuable and is therefore correlated with $\tilde{v}$.

As we show next, the value of the conditional market does not come from its stand-alone properties; indeed, it can be used to complement another market, like the continuous market, by attracting only those traders whose demand for immediacy is low and for whom existing markets are too expensive.

4.3   The Conditional Market as a Complementary Trading Venue

We now assume that both the continuous market and the conditional market are available at the same time. That is, we now permit traders to choose to trade in one of the two trading venues, both operated by the same exchange.[15] As the previous analysis shows, only welfare losses can result from a situation where all trading is diverted from the continuous market to the conditional market. It follows that welfare improvements over the continuous market operating alone are only possible if some trading takes place in each market.

The market-maker must now choose three quantities: a, c and ψ. We can rule out $\psi = 0$ as the conditional market then becomes identical to the continuous market, as shown in Proposition 4.1. Because the conditional market with $\psi > 0$ does not clear with probability one, the only way to attract some traders to it is to charge them an order-crossing fee c that is less than the costs they face in the continuous market. Otherwise, traders would prefer a sure transaction for a lesser price to a probabilistic one with a larger price.

Lemma 4.2 *Any equilibrium in which both venues are used must have the impatient hedgers use the continuous market and the patient hedgers use the conditional market.*

As the proof of this result shows, making the continuous market attractive to the patient traders by reducing the ask price also makes this market attractive to the impatient traders who value immediacy even more. As our next result shows, however, it can be the case that patient hedgers like to place orders in the conditional market while the impatient hedgers prefer to continue using the continuous markets for their trades. The key for this result to obtain is that the volume condition be set to a large enough value. That is, a conditional market that clears too often becomes too attractive to impatient traders.

Proposition 4.2 *An equilibrium in which impatient hedgers use the continuous market and the patient hedgers use the conditional market always exists. In this equilibrium, c can be set to any*

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[15]The assumption that only one venue can be used by any one hedger is completely harmless. Indeed, hedgers who trade in the continuous market would never place an order in the conditional market at the same time because this would put them back into an unhedged situation and lower their utility. Section 7 discusses how allowing for fractional orders might result in some traders using both venues for their trades; despite the accompanying loss of tractability the inferences and intuition of the model will remain largely unaffected.

value in $[0, r_L]$, ψ *must satisfy*

$$0 < \frac{(r_H - c)\lambda - \sqrt{(r_H - c)^2\lambda^2 - 4(1 - \lambda)(2\kappa - c)c}}{4c} \le \psi \le \frac{1 - \lambda}{2}, \qquad (7)$$

and

$$a = \frac{1}{\lambda}\left[2\kappa - \frac{(1 - \lambda)^2 - 4\psi^2}{1 - \lambda}c\right]. \qquad (8)$$

The conditional market then clears with probability $1 - \frac{2\psi}{1-\lambda} < 1$.

The proof of Proposition 4.2 shows that the equilibrium ask price a prevailing in the continuous market is greater than the cost of transacting in the conditional market, i.e., $a > c$. Conditional on a transaction of one share of the risky security taking place, it is cheaper to get that transaction done in the conditional market. The reason why this conditional market is not attractive to everyone is because it clears only a fraction of the time. For an impatient trader who requires immediacy, the assurance that his order gets filled immediately with probability one is worth the extra cost. At the same time, this makes it a possible trading venue for patient hedgers who have nothing to lose by placing orders there; for them, the worst-case scenario is that the market does not clear, which leaves them in the same situation as before. When the market clears, however, they are better off as $c \le r_L$. In fact, the following proposition shows that all traders are made better off by the presence of a volume-conditional market.

Proposition 4.3 *As long as $\psi < \frac{1-\lambda}{2}$ and $c \in (0, r_L)$, the equilibrium in Proposition 4.2 makes all hedgers strictly better off than when the continuous market operates alone. Furthermore, for a given $c \in (0, r_L)$, all hedgers are made strictly better off as ψ decreases from $\frac{1-\lambda}{2}$ to its lower bound in (7).*

This result clearly implies that, for a fixed $c \in (0, r_L)$, fixing ψ equal to its lowest possible value in (7) represents a Pareto-dominating equilibrium, that is, any other choice of ψ makes *every trader* worse off. In essence, ψ must be high enough so that the volume condition is constraining enough for the impatient traders. Indeed, a higher volume condition makes the conditional market less likely to clear and keeps the demand for immediacy in the continuous market. When the volume condition is too high, however, the conditional market does not clear often, and so the gain in welfare for traders who participate in this market are modest. Lowering the volume condition increases these welfare gains. At the same time, because the conditional market clears more often, the market-maker's revenues from the conditional market increase as he receives c from the patient hedgers more often. These revenues and the assumed competitive nature of market-making lead him to lower the ask price of the continuous market.[16] Effectively therefore, the improvement in

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[16]As we will see in section 6, this result holds even when the market-maker extracts some rents from his role in the market.

the patient hedgers' welfare subsidizes the continuous market's operations. Again, this subsidy is only possible with an appropriately chosen volume condition in the conditional market.

Changing c can never make all hedgers better off. For example, an increase in c within its $(0, r_\mathrm{L})$ range makes the patient traders strictly worse off, as they have to pay a higher transaction cost when their orders get crossed in the conditional market. Because the market-maker then receives more revenues from the conditional market (as every patient hedger's order-placing strategy remains the same), this allows him to further reduce the ask price in the continuous market. This can be seen from the fact that a is decreasing in c in (8). If traders do not know their type before markets open however, their ex ante expected utility is a weighted average of the expected utility of an impatient hedger trading in the continuous market and the expected utility of a patient hedger placing his order in the conditional market, that is,

$$\mathrm{E}\big[\tilde{u}_j\big] = \lambda\mathrm{E}\big[\tilde{u}_j \mid \tilde{r} = r_\mathrm{H}\big] + (1 - \lambda)\mathrm{E}\big[\tilde{u}_j \mid \tilde{r} = r_\mathrm{L}\big].$$

This would correspond to a situation in which the need for immediacy by each hedger varies with time and/or with every transaction. In this case, there will be a $c \in [0, r_\mathrm{L}]$ that maximizes the ex ante expected utility of every trader.[17]

The equilibrium in which $c = 0$ merits further discussion. When transacting in the conditional market is free, the ask price prevailing in the continuous market $a = \frac{2\kappa}{\lambda}$ is the same whether this market operates alone or in parallel with the conditional market. This still does not make the conditional market attractive to all traders. As long as its volume condition is appropriately set,[18] the impatient hedgers still prefer the immediacy of the continuous market to the positive probability of getting stuck with an unhedged position if they were to place an order in the conditional market. The patient traders on the other hand are clearly better off than when the continuous market operates alone, as they now get to hedge their positions for free with a positive probability. The opening of the conditional market does not affect the market-maker: he still collects the same revenues from impatient traders in the continuous market. From his perspective therefore, the conditional market could be operated by a third party, as long as they chose c and ψ in a way such that the conditional market did not affect the continuous market's trading crowd.

To summarize, the addition of a volume-conditional market operating alongside a continuous-auction market can make all hedgers better off as long as the volume condition is appropriately set. In this case, the volume condition serves to make market-clearing in the crossing market a probabilistic event and reduces its attractiveness to hedgers who require high immediacy for their trades.

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[17] Because finding the optimal value for c does not yield any additional insight, we do not perform these calculations here. Our calculations indicate that interior values of c (i.e., $0 < c < r_\mathrm{L}$) will often be optimal.

[18] It is easy to verify that (7) reduces to $0 < \frac{(1-\lambda)\kappa}{\lambda r_\mathrm{H}} \leq \psi \leq \frac{1-\lambda}{2}$ as $c \to 0$.

5 Informed Traders

5.1 Introducing Informed Traders

Let us now assume that, in addition to agents who trade for hedging reasons, the economy is populated with a (known) mass $n > 0$ of agents who trade for speculative reasons.[19] We refer to these agents as *informed traders*. Before trading, informed traders observe a common signal $\tilde{s}$ that is equal to $\tilde{v}$ with probability $\frac{1+p}{2}$ and equal to $-\tilde{v}$ otherwise, where $p \in (0,1]$.[20] It is easily shown that $\mathrm{E}[\tilde{v} \mid \tilde{s}] = \tilde{s}p$ so that a positive (respectively, negative) signal makes traders update their views about the stock's payoff to a posterior that is closer to $+1$ (to -1), and the higher the precision p the closer is the update to the signal. Using this information, informed traders seek to maximize their expected profits from trading. Thus if an informed trader i trades quantity $x_i \in \{-1, 0, +1\}$ shares at a price of φ, his profits are given by $\tilde{\pi}_i = x_i(\tilde{v} - \varphi)$ and, upon receiving his information, he seeks to maximize

$$\mathrm{E}[\tilde{\pi}_i \mid \tilde{s}] = x_i(\tilde{s}p - \varphi).$$

As before, let us start with a characterization of the equilibrium when the continuous market operates alone. In addition to his fixed market-making costs of κ, the market-maker must now also contend with the possibility of losing money to informed traders. More precisely, the equilibrium ask price must be such that the expected revenues that the market-maker makes at the expense of uninformed hedgers exactly offset his fixed market-making costs as well as the losses he incurs when trading against informed traders. This is the tradeoff originally discussed by Bagehot (1971).

Proposition 5.1 *Suppose that the continuous market operates alone. If $p < \frac{2\kappa}{\lambda}$, the equilibrium ask price in this market is given by*

$$a = \frac{2\kappa}{\lambda}, \tag{9}$$

and only impatient hedgers ($\tilde{r}_j = r_\mathrm{H}$) participate in the market. If instead $p \geq \frac{2\kappa}{\lambda}$, an equilibrium exists if and only if

$$2n(p - r_\mathrm{H}) \leq r_\mathrm{H}\lambda - 2\kappa. \tag{10}$$

In that case, the equilibrium ask price is given by

$$a = \frac{2(\kappa + np)}{\lambda + 2n}, \tag{11}$$

and impatient hedgers ($\tilde{r}_j = r_\mathrm{H}$) and informed traders participate in the market.

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[19]We could make their mass n a random variable like the mass $\tilde{m}$ of hedgers but, as our derivations will make clear, only the mean of this random variable would matter.

[20]Although our analysis is simplified by the assumption that the information of each trader is perfectly correlated with that of every other trader, this assumption is not important for our results to obtain. More precisely, the simple fact that every informed trader's information is correlated with $\tilde{v}$ is all that is needed.

When the information precision of informed traders is low, their valuation of the risky security conditional on their information is not extreme enough for it to warrant a trade in our original hedgers-only equilibrium of Proposition 3.1. For example, suppose that an informed trader observes $\tilde{s} = +1$ and that $p < \frac{2\kappa}{\lambda}$. This trader's conditional valuation of the risky security, $\mathrm{E}[\tilde{v} \,|\, \tilde{s} = +1] = p$, is not large enough for him to pay $a = \frac{2\kappa}{\lambda}$ to acquire it. As a result, the equilibrium of Proposition 3.1 is left intact as informed traders, just like the patient hedgers, are effectively relegated to the sidelines by electing not to trade.

When the precision of their information is high enough, informed traders would like to use the continuous market to buy (sell) the risky security when it is underpriced (overpriced). For any ask price below p, the market-maker then loses $p - a$ to each of the n traders. In some cases, these losses are so large that the market-maker cannot possibly recoup his losses and his fixed costs for any $a \in [0, 1]$; that is, the market breaks down, as in Glosten (1989) and Bhattacharya and Spiegel (1991). Condition (10) shows that this happens when $p > r_{\mathrm{H}}$ and n is sufficiently large. Otherwise, the market-maker can simply increase the ask price from that in Proposition 3.1 to a value that allows him to collect sufficiently more revenues from every trade with impatient hedgers to offset the losses he now incurs against informed traders. In what follows, we assume that (10) is satisfied so that market breakdowns, which are outside this paper's scope, can always be avoided.

5.2 The Conditional Market with Informed Traders

We are now in a position to reintroduce a conditional market which is to operate alongside the continuous market.[21] As in section 4, our results will show that in equilibrium the addition of this second trading venue can improve the welfare of every agent in the economy, as long as its volume condition is appropriately set. Interestingly, the volume condition of the conditional market plays an additional role when the economy is populated with some informed traders. In addition to making the conditional market less attractive to some agents as before, the volume condition also makes the conditional market more attractive to other agents. More precisely, the volume condition reduces the amount of adverse selection faced by uninformed traders in the conditional market.

To see how adverse selection comes to affect the conditional market and to better understand this new role played by the volume condition, let us first assume that only patient hedgers and informed traders use the conditional market once it is introduced.[22] At first, it is tempting to conclude that the patient hedgers are not affected by the presence of informed traders once c is fixed: as before, their position is either hedged (for a cost of c) or unhedged (which comes with a

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[21]Our previous result that the conditional market operating alone can never better the continuous market still holds, and so we do not go through this part of the analysis again.

[22]Of course, this means that the impatient hedgers still trade in the continuous market. We derive precise conditions for this to be an equilibrium below.

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utility cost of r_{L}). This conclusion is incorrect however. Because of the informed traders' presence, it is now sometimes the case that some hedgers' orders are not crossed even though the market clears. Indeed, the conditional market crosses as many orders as it can when it clears; if the number of buy orders (say) exceeds the number of sell orders, then some buy orders will remain unfilled even after the market clears. More than that, every hedger's motive for trading is uncorrelated with that of every other hedger (i.e., $\tilde{w}_j$ is uncorrelated with $\tilde{w}_k$ for any $j \neq k$), while every informed trader's motive for trading is correlated (through $\tilde{v}$) with that of every other informed trader.[23] Therefore it is always the case that more orders are placed on the buy side if $\tilde{s} = +1$ (or on the sell side if $\tilde{s} = -1$). This also implies that a hedger whose buy order does not get crossed while the conditional market clears must absorb two costs: r_{L} because his short position ($\tilde{w}_j = -1$) is not hedged, and p because he is short a share whose conditional value (i.e., conditional on the fact that his buy order did not clear when the conditional market cleared) is $\mathrm{E}\big[\tilde{v} \mid \tilde{s} = +1\big] = p$.

To better illustrate this, let us suppose that the volume condition is set to a value such that the market clears when the economy includes $\tilde{m} = m$ hedgers (and that $c < r_{\mathrm{L}}$ as, otherwise, the patient hedgers would never place orders in this market). Because only a fraction $1 - \lambda$ of hedgers are patient (and use the conditional market) and because each hedger's motive for trading is an independent wealth shock (which is positive or negative with equal probabilities), there are $\frac{m(1-\lambda)}{2}$ patient hedgers who place buy orders and the same number of patient hedgers who place sell orders. Suppose further that the informed traders' signal is positive ($\tilde{s} = +1$) and that the precision p of their information is greater than the order-crossing fee c of the conditional market. This implies that the entire mass n of informed traders finds it worthwhile to place buy orders. When the market clears, each of the $\frac{m(1-\lambda)}{2}$ sell orders is crossed with one of the $\frac{m(1-\lambda)}{2} + n$ buy orders. That is, all $\frac{m(1-\lambda)}{2}$ sell orders get crossed, while only a fraction $\frac{m(1-\lambda)/2}{m(1-\lambda)/2+n} = \frac{m(1-\lambda)}{m(1-\lambda)+2n}$ of buy orders get crossed.

Thus orders on the lighter side of the market are crossed with probability one when the market clears. However, because traders do not know the state of the conditional market's order book when they place their order (i.e., they don't know how many orders have priority over theirs), every order on the heavier side of the market is crossed with probability

$$\phi \equiv \frac{m(1 - \lambda)}{m(1 - \lambda) + 2n}. \tag{12}$$

A patient hedger's order will end up on the lighter or heavier sides of the market with equal probabilities. Conditional on the market clearing therefore, every patient hedger's expected utility

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[23]The correlation is perfect in our model but, as mentioned before, it only needs to be greater than zero for our results to obtain.

is given by

$$
\begin{aligned}
\mathrm{E}\big[\tilde{u}_j \mid \tilde{r}_j = r_{\mathrm{L}}\big] &= \frac{1}{2}(-c) + \frac{1}{2}\Big[\phi(-c) + (1-\phi)(-r_{\mathrm{L}} - p)\Big] \\
&= \frac{1+\phi}{2}(-c) + \frac{1-\phi}{2}(-r_{\mathrm{L}} - p),
\end{aligned}
\tag{13}
$$

where we have used the fact that a hedger whose order is unfilled suffers a cost of r_{L} for not hedging as well as an adverse selection cost of p. Because patient hedgers have the option not to trade (and realize a sure utility loss of r_{L}), they benefit from market-clearing if and only if (13) exceeds $-r_{\mathrm{L}}$. Because $-r_{\mathrm{L}} - p < -r_{\mathrm{L}} < -c$, this will be the case when ϕ (the probability that an order on the heavier side of the conditional market gets crossed when this market clears) is close to one, that is, when n is small and/or m is large. This is intuitive: for a given precision p, adverse selection in the conditional market is minimal when this market is populated with only a small relative number of informed traders. As n gets larger relative to m however, the likelihood that a hedger's order goes unfilled as the market clears increases; unless the informed traders' informational advantage (p) is small relative to the patient hedgers' potential gain from trade ($r_{\mathrm{L}} - c$), patient hedgers then prefer the conditional market not to clear for this value of $\tilde{m} = m$. This leads to the following result about the potential viability of the conditional market.

Lemma 5.1 *There is no equilibrium in which patient hedgers and informed traders use the conditional market unless*

$$
n\big[p - (r_{\mathrm{L}} - c)\big] < (1-\lambda)(r_{\mathrm{L}} - c).
\tag{14}
$$

When such an equilibrium does exist, market-clearing in the conditional market improves the welfare of patient hedgers with probability one if and only if the volume condition satisfies

$$
\frac{n\big[p - (r_{\mathrm{L}} - c)\big]}{2(r_{\mathrm{L}} - c)} < \psi < \frac{1-\lambda}{2}.
\tag{15}
$$

Condition (14) is the analogue of condition (10), which prevents market breakdowns when the continuous market operates alone. That is, when the conditional market attracts patient hedgers and informed traders, (14) is necessary for it to be viable. Clearly, this will happen when the number of informed traders (n) and the precision of their information (p) are small enough. Condition (15) ensures that the patient hedgers are better off every time the conditional market clears. Although this condition is sufficient to make these traders use the conditional market in the first place, it is not necessary. Indeed, the patient traders will use the conditional market if it makes them better off *on average*, and not necessarily if they are better off for every realization of $\tilde{m}$ that triggers market-clearing (i.e., for every $\tilde{m} \geq \frac{2\psi}{1-\lambda}$). Still, the point remains that a larger volume condition reduces the adverse selection problem of hedgers in the conditional market by reducing the average

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relative imbalance between buy and sell orders. This makes the conditional market more attractive to patient hedgers, whose presence is necessary for this market to clear.[24] This is yet another role played by the volume condition in the trading venue proposed in this paper.

5.3 Equilibrium, Welfare and Price Discovery

The effect of adding a conditional market on the equilibrium of Proposition 5.1 depends on whether the informed traders use or are excluded from the continuous market when it operates by itself, that is, it depends on whether $p \geq \frac{2\kappa}{\lambda}$ or $p < \frac{2\kappa}{\lambda}$. In the former case, adding the conditional market has a similar effect as in Proposition 4.2: with a high-enough volume condition, impatient hedgers and informed traders do not find this new trading venue attractive and keep using the continuous market for their trades; patient hedgers can therefore use the conditional market to hedge their wealth shocks at least a fraction of the time; in turn, the revenues that the market-maker generates from the conditional market allow him to reduce the ask price in the continuous market. When $p < \frac{2\kappa}{\lambda}$, the conditional market must accommodate both patient hedgers and informed traders. As the analysis of section 5.2 shows, the conditional market allows these traders to coexist and trade (with some probability) as long as the adverse selection brought about by the informed traders is not too severe. Again, the volume condition of the conditional market plays a crucial role: it ensures that the adverse selection cost that the patient hedgers must absorb is low enough that they find it worthwhile to use the conditional market. Also, the extra revenues from the conditional market allow the market-maker to reduce the continuous market's ask price.

Proposition 5.2 *As long as*

(i) $p \geq \frac{2\kappa}{\lambda}$, or

(ii) $p < \frac{2\kappa}{\lambda}$ and (14) holds,

there always exists an equilibrium in which the presence of the conditional market makes every trader in the economy at least as well-off and some traders (strictly) better off than when the continuous market operates alone.

As in section 4.3, the volume condition keeps the impatient hedgers in the continuous market. With informed traders in the economy, this condition does more. It can be used to ensure that there is a sufficient number of patient hedgers in the conditional market when this market clears. Without the volume condition, patient hedgers anticipate the presence of informed traders in the

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[24]Notice that the presence of informed traders in the conditional market does *not* facilitate market clearing. Indeed, without them, there is an equal number $\frac{\tilde{m}(1-\lambda)}{2}$ of patient hedgers on both sides of the market, and the market clears if and only if $\frac{\tilde{m}(1-\lambda)}{2} \geq \psi$, as before.

conditional market, and this makes them more reluctant to use this alternative venue. Since welfare can only increase through the use of the alternative trading venue, the volume condition is crucial.

Interestingly, the fact that informed traders who feel inhibited from trading in the stand-alone continuous market can now trade in the conditional market implies that the information they have can percolate through the market. Indeed, after the conditional market clears, the fact that some traders can infer the informed traders' information (the patient hedgers whose orders are not crossed, in our model) means that these traders become informed. In an extension of our model that would accommodate multiple rounds of trading, these traders become informed traders for later rounds of trading, and their trades can be used by all agents, including the market-maker, to update their posteriors about the eventual payoff of the risky asset. In short, not only is it the case that the volume-conditional market can benefit all traders, but we conjecture that such a market can potentially accelerate price discovery from the simple fact that it facilitates more informed trades overall.

6   The Monopolist Specialist

So far, our model assumes that market-making is competitive so that in equilibrium the market-maker exactly recoups his fixed costs and breaks even. In this section, we replace this assumption by one in which the market-maker can extract some rents from his market-making function. More specifically, in the spirit of Glosten (1989), we assume that the market-maker is a monopolist specialist who maximizes expected profits. To keep the analysis simple, we go back to the framework of section 4, in which the trading crowd consists only of patient and impatient hedgers and excludes informed speculators.

When the continuous market operates in isolation, the equilibrium is straightforward to derive. Indeed, because he faces no competition, the specialist only has to consider two ask prices: r_L and r_H. The former is the highest prices that attracts all traders to the market, whereas the latter is the highest prices that attracts only the impatient hedgers to the market. Because $r_\mathrm{L} < 2\kappa < r_\mathrm{H}\lambda$, it follows that $a = r_\mathrm{H}$ is the profit-maximizing quote.[25] The specialist's optimized expected profits are then $\mathrm{E}\!\left[\tilde{\pi}_{\mathrm{MM}}\right] = r_\mathrm{H}\lambda \mathrm{E}[\tilde{m}] - \kappa = \frac{r_\mathrm{H}\lambda}{2} - \kappa > 0$.

As mentioned before, the conditional market that we propose in this paper is meant to complement the continuous-auction market. To economize on space, we do not consider the conditional market in isolation, as we did in section 4.2, and proceed directly to the case in which the condi-

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[25]Note that with the monopolist specialist, we could remove the assumption that the specialist must at least recoup his fixed costs of κ. Indeed, the monopolist specialist can always realize at least as much expected profits as the competitive market-maker and, as such, κ never affects the equilibrium. In particular, it would be sufficient to assume that $r_\mathrm{L} < r_\mathrm{H}\lambda$ and that $2\kappa < r_\mathrm{H}\lambda$ for this section's results to go through.

tional market and the continuous market operate in parallel.[26] For the same reason as before, the presence of the conditional market can only enhance the stand-alone continuous market if some traders use it in equilibrium. Also, the result of Lemma 4.2 still holds: if patient hedgers choose the continuous market for their trades in equilibrium, it will be the case that impatient hedgers choose the same market for their trades as well. Thus, in order to increase his expected profits, the monopolist must attract impatient hedgers to the continuous market and patient hedgers to the conditional market.

Proposition 6.1 *If the number of impatient hedgers is small relative to the number of patient hedgers, that is, if*

$$\lambda < \frac{2r_{\mathrm{L}}}{r_{\mathrm{H}} + r_{\mathrm{L}}}, \tag{16}$$

the monopolist specialist increases his expected profits by operating a volume-conditional trading venue alongside the continuous-auction venue. In equilibrium, the specialist then sets $c = r_{\mathrm{L}}$,

$$a = r_{\mathrm{L}} + \frac{\lambda}{1 - \lambda} \frac{(r_{\mathrm{H}} - r_{\mathrm{L}})^2}{2r_{\mathrm{L}}}, \tag{17}$$

and

$$\psi = \frac{\lambda(r_{\mathrm{H}} - r_{\mathrm{L}})}{4r_{\mathrm{L}}}. \tag{18}$$

If (16) is not satisfied, the monopolist specialist does not benefit from the presence of a conditional trading venue.

When (16) is satisfied, the conditional market attracts all the patient hedgers and it can easily be verified that market-clearing occurs with probability $1 - \frac{\lambda(r_{\mathrm{H}} - r_{\mathrm{L}})}{2(1-\lambda)r_{\mathrm{L}}} > 0$. Also, as shown in the proof of Proposition 6.1, the equilibrium ask price a in (17) is smaller than r_{H}, the ask price when the continuous market operates alone. Thus, as with competitive market-making, it is the case that the specialist receives some revenues from the conditional market and makes pricing more attractive in the continuous market, but for a different reason. As a monopolist, the specialist feels no pressure to use the revenues of one market to subsidize the other in an effort to break even. However, because the separation of trader types between the two trading venues is necessary to increase the available economic surplus, he does feel the need to keep the impatient traders in the continuous market after the conditional market is introduced. Also, as long as the impatient traders keep trading in the continuous market, the revenues that the specialist generates from the patient traders in the conditional market increase as the market-clearing frequency of this market increases, that is, as the volume condition is loosened. This also increases the specialist's profits.

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[26]The results for the conditional market operating alone are similar to those of section 4.2 in that the continuous market always dominates.

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If the volume condition is relaxed too much impatient hedgers are eventually attracted to the conditional market, eliminating all its benefits. For any ask price a therefore, it must be the case that the volume condition is set to a value that makes impatient traders indifferent between the two trading venues. Hence any decrease in the volume condition in an effort to increase conditional-market revenues must be accompanied by a decrease in the continuous market's ask price and a corresponding decrease in revenues for this market. As long as the gain of the former exceeds the loss of the latter, both the volume condition and the ask price are reduced, making the conditional market probabilistic and a smaller than when the continuous market operates alone (i.e., smaller than r_H). This happens when λ is small enough as prescribed by (16). Otherwise, the specialist prefers to set the continuous market's ask price as large as possible, making sure that this price keeps the impatient trading (i.e., $a = r_H$), and to ignore the conditional venue altogether.

Interestingly, even though the monopolist specialist seeks to capture as much of the traders' surplus as possible, it is still the case that the presence of the conditional market Pareto-improves the equilibrium of the stand-alone continuous market. In this case, the specialist is better off because his expected profits are larger; the impatient traders are better off because prices in the continuous market are more favorable ($a < r_H$); the patient traders are equally well-off because, although they get to hedge their positions with a positive probability, they do so for an order-crossing fee that is equal to the cost they incur for not hedging (i.e., $c = r_L$). This is summarized in the following proposition.[27]

Proposition 6.2 *As long as (16) is satisfied, the equilibrium in Proposition 6.1 makes the specialist and impatient traders strictly better off than when the continuous market operates alone. Patient traders are equally well-off with and without a conditional market.*

7 Conclusion

The spectacular growth in transactions and in volume in financial markets, together with advances in technology, have accompanied the proliferation of alternative and competing venues for trading. Many of these new platforms are attempts to compete for order flow by offering lower transactions costs or more efficient execution. Judging from empirical work on ECNs (e.g., Huang, 2002; Barclay, Hendershott and McCormick, 2003), and crossing networks or third-market broker-dealers (e.g., Chordia and Subrahmanyam, 1995; Battalio, 1997), they have succeeded. The conditional market that we propose in this paper has some of the features of these market-clearing mechanisms. For example, like them, it relies exclusively on the presence of traders for its liquidity, and market-

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[27]The proof of this proposition is omitted, as it follows directly from the results of Proposition 6.1 and this paragraph's discussion.

clearing can be probabilistic. However, unlike them, the volume-conditional trading venue is not meant to compete with the existing continuous-auction market; it is meant to complement it. In that sense, the conditional market is similar to upstairs markets, which have also been documented to attract lesser-informed order flow by facilitating the search for cheap liquidity.[28] It is also similar to the intraday crossing markets that the Nasdaq and the NYSE are starting to implement in order to attract more patient traders who do not require immediacy for their trades.

The volume condition is the feature that distinguishes the proposed trading venue from all existing venues. As we have shown, when the conditional market operates alongside a continuous market, the volume condition adds an important degree of freedom to the operations of the exchange and improves its ability to cope with order flow in two important ways. First, it allows the exchange to endogenously screen traders who are willing to pay for immediacy and traders who are not. As a result, all traders get charged a better price for using the exchange's services and the equilibrium with the conditional market Pareto-dominates that of the stand-alone continuous-auction market.

Second, even if the traders who place orders on the conditional venue are asymmetrically informed, it allows the specialist to effectively control how much of this asymmetric information affects uninformed traders. Indeed, by making sure that a sufficiently large number of orders can be crossed when the market clears, the specialist can make sure that most uninformed traders will see their orders crossed when the market clears, thereby reducing the negative effect that informed orders have on the crossing probability of all orders. This makes it more appealing for uninformed traders to use this conditional market in the first place, and in the process can further the benefits of this market. In sum, the search for liquidity is endogenous in that the volume condition makes market-clearing probabilistic and likely to occur when liquidity providers are present.

Although we have considered both a competitive and a monopolistic market-making scenario in our paper, there are several extensions that could be considered. One such extension could incorporate the possibility for traders to place limit orders. As argued by Handa and Schwartz (1996) and by Foucault (1999), limit orders will tend to be used more heavily by traders who can afford to be patient. As such, we can expect the limit-order market to partially offset the benefits of the volume-conditional market that we propose. Because the limit-order book is visible to market participants however, traders can precisely assess when and how order-crossing is triggered in this market, unlike in the volume-conditional market. Also, limit orders cannot be conditioned on the size of the active trading crowd. Thus the volume-conditional market should still improve the trading prospects of the more patient and less informed traders in the economy. Indeed, by placing their orders in this market, these traders face a smaller risk of being taken advantage of by informed and

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[28]See for example Madhavan and Cheng (1997), Smith, Alasdair, Turnbull and White (2001), Booth, Lin, Martikainen and Tse (2002), and Bessembinder and Venkataraman (2004).

skilled traders than if they were to "advertise" their presence and motive with a visible limit order. The volume-conditional market also gives these traders the ability to participate in markets only when the characteristics of the trading crowd are likely to equate with favorable trading conditions.

Another extension of the model could incorporate the possibility that endowment shocks and trade sizes are different from zero or one and the possibility for any trader to use both the continuous and volume-conditional markets simultaneously. Although solving the model with such an extension is unlikely to produce closed-form solutions, it is possible to anticipate the likely outcome of the model. Indeed, hedgers with larger endowment shocks would feel less patient than their equally risk-averse counterparts who experience smaller shocks. The distinction between trader types would then operate through the realization of every hedger j's endowment shock $\tilde{w}_j$ rather than through (or in addition to) their risk aversion $\tilde{r}_j$. Clearly, hedgers with large shocks would feel compelled to use, at least partially, the continuous market as its immediacy, even though costly, allows them to shed the most pressing portion of their risk. Once this risk layer is removed, these traders resemble low-shock traders and start acting like them. In particular, a volume-conditional market that offers cheap liquidity but less immediacy becomes attractive to them. In short, the most impatient hedgers will split their orders between the two markets while patient hedgers will be better served by a more extensive use of the volume-conditional market.

Finally, a natural extension of the model is to a dynamic setting. This is technically a difficult step, but the recently proposed trading platforms at the Nasdaq and the NYSE lead us to conjecture that convergence to continuous trading complemented with mechanisms that facilitate batched order-crossing is likely. More specifically, the coexisting market mechanisms will closely resemble a continuous-auction venue where the better-informed and the impatient uninformed traders meet, and a periodic crossing mechanism which satisfies the demands of the more patient uninformed traders as well as those of traders with useful but not precise information, who then trade at a very low cost. As we show in this paper, allowing the participants in these complementary crossing mechanisms to condition their orders on volume can only enhance the efficiency of these mechanisms.

Appendix A

Proof of Proposition 3.1

From the discussion preceding the proposition, we know that the equilibrium ask price a must solve

$$\mathrm{E}\big[\tilde{\pi}_{\mathrm{MM}}\big] = \frac{a\lambda}{2} - \kappa = 0,$$

leading to (3). Because $2\kappa > r_{\mathrm{L}}$ and $2\kappa < r_{\mathrm{H}}\lambda$, this ask price is greater than r_{L} and less than r_{H}, and so only impatient hedgers choose to trade. ∎

Proof of Lemma 4.1

We know from the discussion preceding the Lemma that the market clears if $\frac{\tilde{m}_{\mathrm{A}}}{2} \geq \psi$. Using (4), the market clears if $\frac{\tilde{m}}{2} \geq \psi$ when $c \in [0, r_{\mathrm{L}}]$, it clears if $\frac{\lambda\tilde{m}}{2} \geq \psi$ when $c \in (r_{\mathrm{L}}, r_{\mathrm{H}}]$, and it cannot clear otherwise. We can then use the fact that $\tilde{m}$ is uniformly distributed on $[0, 1]$ to obtain the probability that the market clears. ∎

Proof of Proposition 4.1

The market-maker cannot recoup κ if he quotes $c > r_{\mathrm{H}}$, as no trader ever places an order. If $c \leq r_{\mathrm{L}}$, then all traders place orders. However, even if the market always clears (i.e., is ψ is set equal to zero), the market-maker can never recoup κ for the same reason that he could not in the continuous market (see equation (1)). Thus the market-maker can only hope to recoup κ by quoting $c \in (r_{\mathrm{L}}, r_{\mathrm{H}}]$. In this case, only impatient traders place orders and, as derived in Lemma 4.1, the market clears with probability $\max\left\{1 - \frac{2\psi}{\lambda}, 0\right\}$. This implies that ψ must not exceed $\frac{\lambda}{2}$ for the market to ever clear and for the market-maker to have any hope of recouping κ. In this case, we have $\Pr\{\text{market clears}\} = \Pr\{\tilde{m} \geq \hat{m}\} = 1 - \hat{m}$, where $\hat{m} \equiv \frac{2\psi}{\lambda}$. Also, when the market clears, the total number of (buy and sell) orders is on average

$$\lambda\mathrm{E}\big[\tilde{m} \mid \tilde{m} > \hat{m}\big] = \lambda\frac{1 + \hat{m}}{2},$$

and so the expected profits of the market-maker are equal to

$$\mathrm{E}\big[\tilde{\pi}_{\mathrm{MM}}\big] = (1 - \hat{m})\lambda\frac{1 + \hat{m}}{2}c - \kappa = \frac{1 - \hat{m}^2}{2}\lambda c - \kappa = \frac{1 - \left(\frac{2\psi}{\lambda}\right)^2}{2}\lambda c - \kappa.$$

This last quantity is equal to zero if c is given by (6). Finally, we need to verify that, as conjectured, $c > r_{\mathrm{L}}$ and $c \leq r_{\mathrm{H}}$. The former inequality is implied by the fact that $c \geq \frac{2\kappa}{\lambda} > \frac{r_{\mathrm{L}}}{\lambda} > r_{\mathrm{L}}$. The latter inequality holds as long as (5) is satisfied.

The last part of the proposition, that impatient hedgers are worse off than before, comes from the fact that unless $\psi = 0$, $c > \frac{2\kappa}{\lambda} = a$, that is, c is greater than the ask price a in Proposition 3.1. This, combined with the fact that the market does not always clear when $\psi > 0$ implies that these traders are better off. When $\psi = 0$, the conditional market always clears and $c = \frac{2\kappa}{\lambda} = a$, that is, the two markets are exactly equivalent. ∎

Proof of Lemma 4.2

Given that we have only two hedger types, if both trading venues are to be used, it must be the case that one type uses one venue and the other type uses the other venue. Suppose that, in equilibrium, the patient hedgers use the continuous market and that the impatient hedgers use the conditional market. This implies that $a \le r_{\mathrm{L}}$, as the patient traders would not pay a to hedge their wealth shocks if it exceeded their cost for not hedging.

If we denote the probability that the conditional market clears by $1 - \hat{m}$, where $\hat{m}$ is as defined in the proof of Proposition 4.1, the expected utility of a hedger of type $\tilde{r}_j = r \in \{r_{\mathrm{L}}, r_{\mathrm{H}}\}$ who places an order in the conditional market is

$$\mathrm{E}\big[\tilde{u}_j \mid \tilde{r}_j = r\big] = (1 - \hat{m})(-c) + \hat{m}(-r).$$

For the patient (impatient) hedgers to use the continuous (conditional) market for their trades, it must be that deviating and using the conditional (continuous) market is not worthwhile. That is, we must have

$$-a \ge (1 - \hat{m})(-c) + \hat{m}(-r_{\mathrm{L}})$$

and

$$-a \le (1 - \hat{m})(-c) + \hat{m}(-r_{\mathrm{H}}).$$

Since $r_{\mathrm{L}} < r_{\mathrm{H}}$, these two inequalities together imply

$$-a \ge (1 - \hat{m})(-c) + \hat{m}(-r_{\mathrm{L}}) > (1 - \hat{m})(-c) + \hat{m}(-r_{\mathrm{H}}) \ge -a,$$

clearly a contradiction. Thus it must be the case that any equilibrium in which both trading venues are used must have the impatient (patient) hedgers use the continuous (conditional) market for their trades. ∎

Proof of Proposition 4.2

Given Lemma 4.2, any equilibrium in which both venues are used must at least have $c \le r_{\mathrm{L}}$ in order to attract the patient hedgers to the conditional market. So let us pick any $c \in [0, r_{\mathrm{L}}]$. Also, any equilibrium in which both venues are used will have the $(1 - \lambda)\tilde{m}$ patient hedgers use

the conditional market, with half of these traders placing buy orders and half placing sell orders. Thus, given a volume condition of ψ, the conditional market clears with probability

$$\Pr\left\{\frac{(1-\lambda)\tilde{m}}{2} \geq \psi\right\} = 1 - \frac{2\psi}{1-\lambda} \equiv 1 - \hat{m}, \tag{A.1}$$

When it does clear, the total number of traders (number of shares being exchanged) in this market is given by

$$(1-\lambda)\mathrm{E}\big[\tilde{m} \mid \tilde{m} \geq \hat{m}\big] = (1-\lambda)\frac{1+\hat{m}}{2}. \tag{A.2}$$

Thus, under the conjectured equilibrium, the market-maker collects a from an average of $\lambda\mathrm{E}\big[\tilde{m}\big] = \frac{\lambda}{2}$ impatient traders in the continuous market, and c with probability $1 - \hat{m}$ from an average of $(1-\lambda)\frac{1+\hat{m}}{2}$ patient traders in the conditional market. His expected profits are therefore given by

$$\mathrm{E}\big[\tilde{\pi}_{\mathrm{MM}}\big] = a\frac{\lambda}{2} + c(1-\hat{m})(1-\lambda)\frac{1+\hat{m}}{2} - \kappa. \tag{A.3}$$

In equilibrium, he breaks even with

$$a = \frac{2\kappa - (1-\hat{m}^2)(1-\lambda)c}{\lambda}, \tag{A.4}$$

which reduces to (8) after $\hat{m}$ is replaced by $\frac{2\psi}{1-\lambda}$.

We need to verify that patient traders do not prefer trading in the continuous market to trading in the conditional market. A sufficient condition for this is that $a > r_{\mathrm{L}}$. Since $2\kappa > r_{\mathrm{L}}$ and $c \leq r_{\mathrm{L}}$, we have

$$a = \frac{2\kappa - (1-\hat{m}^2)(1-\lambda)c}{\lambda} > \frac{r_{\mathrm{L}} - (1-\hat{m}^2)(1-\lambda)r_{\mathrm{L}}}{\lambda} \geq \frac{r_{\mathrm{L}} - (1-\lambda)r_{\mathrm{L}}}{\lambda} = r_{\mathrm{L}},$$

and so this condition holds (and this establishes that $a > c$ as well). We also need to verify that the impatient traders find it worthwhile to use the continuous market (i.e., $a \leq r_{\mathrm{H}}$), and prefer that market to the conditional market. Since $2\kappa < r_{\mathrm{H}}\lambda$, we have

$$a = \frac{2\kappa - (1-\hat{m}^2)(1-\lambda)c}{\lambda} < \frac{2\kappa}{\lambda} < r_{\mathrm{H}},$$

so indeed the impatient hedgers benefit from trading in the continuous market. Their expected utility from doing so is $-a$. The expected utility of an impatient trader who deviates from the equilibrium and instead places an order in the conditional market, which clears with probability $1 - \hat{m}$, is

$$\mathrm{E}\big[\tilde{u}_j \mid \tilde{r}_j = r_{\mathrm{H}}\big] = (1-\hat{m})(-c) + \hat{m}(-r_{\mathrm{H}}).$$

Thus impatient traders do not deviate as long as this last quantity does not exceed $-a$, that is, as long as $\hat{m} \geq \frac{a-c}{r_{\mathrm{H}}-c}$. To see how this condition reduces to (7), let us first replace a by its expression

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in (A.4), so that the condition becomes

$$\hat{m} \geq \frac{\frac{2\kappa - (1 - \hat{m}^2)(1 - \lambda)c}{\lambda} - c}{r_{\mathrm{H}} - c}$$

which, after reorganizing and simplifying, is equivalent to

$$(1 - \lambda)c\hat{m}^2 - (r_{\mathrm{H}} - c)\lambda\hat{m} + (2\kappa - c) \leq 0. \tag{A.5}$$

Because $2\kappa > r_{\mathrm{L}} \geq c$ this quadratic expression in $\hat{m}$ is positive at $\hat{m} = 0$. Also, the same quadratic expression is negative at $\hat{m} = 1$ (the maximum possible value for $\hat{m}$, which explains why we must have $\psi \leq \frac{1-\lambda}{2}$) since, using $2\kappa < r_{\mathrm{H}}\lambda$, we have

$$(1 - \lambda)c - (r_{\mathrm{H}} - c)\lambda + (2\kappa - c) = 2\kappa - r_{\mathrm{H}}\lambda < 0.$$

Therefore, (A.5) is satisfied when $\hat{m}$ is greater than or equal to the smaller of the quadratic expression's two roots, that is, when

$$\hat{m} \geq \frac{r_{\mathrm{H}} - c - \sqrt{(r_{\mathrm{H}} - c)^2\lambda^2 - 4(1 - \lambda)(2\kappa - c)c}}{2(1 - \lambda)c}.$$

Finally, we can replace replace $\hat{m}$ by $\frac{2\psi}{1-\lambda}$ to get (7). ∎

Proof of Proposition 4.3

The fact that the patient hedgers are better off is obvious, as they never trade in the equilibrium of the stand-alone continuous market derived in Proposition 3.1. When the continuous market is complemented with a conditional market, the same hedgers place orders in the conditional market and see them crossed with a positive probability as long as $\psi < \frac{1-\lambda}{2}$. When this only costs them $c < r_{\mathrm{L}}$, they are strictly better off. Also, as ψ decreases, the probability that the conditional market clears, $1 - \frac{2\psi}{1-\lambda}$, increases. If the cost of transacting is kept fixed at c, this makes the patient hedgers strictly better off.

The impatient hedgers always trade in the continuous market, whether this market operates alone or it is complemented with the conditional market. As such, their welfare is inversely related to the ask price in the continuous market. When $\psi = \frac{1-\lambda}{2}$, the ask price in (8) reduces to $a = \frac{2\kappa}{\lambda}$, which is also the ask price prevailing in the continuous market operating alone, as shown in (3). Thus the impatient hedgers are then equally well-off. However, from (8), it is easy to see that, as long as $c > 0$, decreasing ψ decreases a, making the impatient hedgers strictly better off. ∎

Proof of Proposition 5.1

If an informed trader chooses to trade in the continuous market after observing $\tilde{s} = +1$, his expected profits are equal to $\mathrm{E}\big[\tilde{\pi}_i \mid \tilde{s} = +1\big] = p - a$. Since traders always have the option not to trade, informed traders trade if and only if their common precision p is greater than a. Let us first assume that $p < \frac{2\kappa}{\lambda}$. Then the equilibrium of Proposition 3.1, with $a = \frac{2\kappa}{\lambda}$, still holds. Indeed, because $a > p$, the informed traders choose not to trade in this market. We also know from Proposition 3.1 that the market-maker can recoup his fixed costs of κ by quoting this ask price. Since lowering his ask price can only reduce the market-maker's profits (through lower revenues and potentially more adverse selection if a dips below p), this is the lowest possible a that has him break even.

If instead $p > \frac{2\kappa}{\lambda}$, the equilibrium ask price cannot be $a = \frac{2\kappa}{\lambda}$, as the market-maker breaks even on his trades with impatient hedgers but, since p is then larger than a, he loses $p - a$ to each of the n informed traders who now find it profitable to trade on their information. The market-maker must therefore set a to a value greater than $\frac{2\kappa}{\lambda}$ and his expected profits are then

$$\mathrm{E}\big[\tilde{\pi}_{\mathrm{MM}}\big] = \frac{a\lambda}{2} - (p - a)n - \kappa. \tag{A.6}$$

In equilibrium, the market-maker breaks even and so a must make this last quantity equal to zero. This yields (11). Of course, the conjectured equilibrium only holds if the informed traders and impatient hedgers choose to trade, that is, if $a \leq p$ and $a \leq r_{\mathrm{H}}$ with the value of a just derived. Straightforward manipulations show that the first of these two inequalities is equivalent to $p > \frac{2\kappa}{\lambda}$, which is true in this second part of the proposition. The second inequality reduces to (10). When it does not hold, the ask price required to break even no longer attracts the impatient traders and so no ask price in $[0, 1]$ can produce an equilibrium. ∎

Proof of Lemma 5.1

Let us first consider the second part of the result, that is, suppose that in equilibrium the patient hedgers and informed traders place orders in the conditional market. As mentioned in the paragraph preceding this lemma, a patient trader is better off when the market clears as long as (13) is greater than $-r_{\mathrm{L}}$ for all $\tilde{m} = m$ that trigger market-clearing. It is easy to show that this condition is equivalent to $\phi > \frac{p - (r_{\mathrm{L}} - c)}{p + (r_{\mathrm{L}} - c)}$ which, after replacing ϕ by $\frac{m(1-\lambda)}{m(1-\lambda)+2n}$ as defined in (12), reduces to

$$m > \frac{n\big[p - (r_{\mathrm{L}} - c)\big]}{(1 - \lambda)(r_{\mathrm{L}} - c)}$$

or, equivalently, to

$$\frac{m(1 - \lambda)}{2} > \frac{n\big[p - (r_{\mathrm{L}} - c)\big]}{2(r_{\mathrm{L}} - c)}.$$

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This condition is implied by (15). To see this, suppose that ψ is set equal to a value in that range. The number of patient hedgers in the conditional market when this market clears is then

$$\frac{\tilde{m}(1-\lambda)}{2} > \psi > \frac{n\big[p-(r_{\mathrm{L}}-c)\big]}{2(r_{\mathrm{L}}-c)}.$$

The second inequality in (15) simply ensures that the market clears with positive probability, as in (7). Finally, condition (14) ensures that the interval in (15) in nonempty. ∎

Proof of Proposition 5.2

(i) Suppose first that $p \geq \frac{2\kappa}{\lambda}$, so that both impatient hedgers and informed traders participate in the continuous market if it operates alone, as shown in Proposition 5.1. Let us pick any $c \in [0, r_{\mathrm{L}}]$. Let us conjecture that impatient hedgers and informed traders stay in the continuous market after the conditional market is added, and that patient hedgers start placing orders in the conditional market. As in the proof of Proposition 4.2, the probability that the conditional market clears under this conjecture is $1 - \hat{m}$ where $\hat{m} = \frac{2\psi}{1-\lambda}$. Also, when the conditional market does clear, the average number of hedgers in the economy is $\mathrm{E}\big[\tilde{m} \mid \tilde{m} \geq \hat{m}\big] = \frac{1+\hat{m}}{2}$, so that the average number of patient hedgers in the conditional market is $\frac{1+\hat{m}}{2}(1-\lambda)$. The specialist's expected profits are therefore given by (A.6) plus the revenues he generates from the conditional market, that is,

$$\mathrm{E}\big[\tilde{\pi}_{\mathrm{MM}}\big] = \frac{a\lambda}{2} - (p-a)n + (1-\hat{m})\frac{1+\hat{m}}{2}(1-\lambda)c - \kappa.$$

In equilibrium, this quantity must be equal to zero, and so

$$a = \frac{2(np+\kappa) - (1-\hat{m}^2)(1-\lambda)c}{2n+\lambda}. \tag{A.7}$$

We need to ensure that all trader types trade in the venue conjectured for the equilibrium. Patient hedgers do not deviate from trading in the conditional market as $c \leq r_{\mathrm{L}}$ and

$$a \overset{(c \leq r_{\mathrm{L}} < 2\kappa)}{>} \frac{2(np+\kappa) - (1-\hat{m}^2)(1-\lambda)2\kappa}{2n+\lambda} \overset{(\hat{m} \geq 0)}{\geq} \frac{2(np+\kappa) - (1-\lambda)2\kappa}{2n+\lambda} = \frac{2np+2\lambda\kappa}{2n+\lambda}$$

$$\overset{(r_{\mathrm{L}} < 2\kappa)}{>} \frac{2np+\lambda r_{\mathrm{L}}}{2n+\lambda} \overset{(p \geq \frac{2\kappa}{\lambda} > r_{\mathrm{L}})}{>} \frac{(2n+\lambda)r_{\mathrm{L}}}{2n+\lambda} = r_{\mathrm{L}}.$$

As in the proof of Proposition 4.2, impatient hedgers do not deviate from trading in the continuous market as long as $a \leq r_{\mathrm{H}}$ and $\hat{m} \geq \frac{a-c}{r_{\mathrm{H}}-c}$. The first of these conditions is satisfied as

$$a \overset{(10)}{\leq} \frac{r_{\mathrm{H}}(2n+\lambda) - (1-\hat{m}^2)(1-\lambda)c}{2n+\lambda} = r_{\mathrm{H}} - \frac{(1-\hat{m}^2)(1-\lambda)c}{2n+\lambda} < r_{\mathrm{H}}.$$

After replacing a by its expression in (A.7), the second condition reduces to

$$(1-\lambda)c\hat{m}^2 - (2n+\lambda)(r_{\mathrm{H}}-c)\hat{m} + 2n(p-c) + (2\kappa-c) \leq 0.$$

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Because $r_\mathrm{H} > c$, $p > c$ and $2\kappa > c$, it is straightforward to show that this quadratic expression is positive and downward-sloping at $\hat{m} = 0$, negative at $\hat{m} = 1$, and so negative for $\hat{m}$ large enough (larger than $\hat{m}_\mathrm{H}$, say). Finally, for informed traders not to deviate from the conjectured equilibrium, they must find the continuous market more profitable than not trading and more profitable than the conditional market. The first of these conditions is satisfied since

$$a \overset{(p \geq \frac{2\kappa}{\lambda})}{\leq} \frac{2np + \lambda p - (1 - \hat{m}^2)(1 - \lambda)c}{2n + \lambda} = p - \frac{(1 - \hat{m}^2)(1 - \lambda)c}{2n + \lambda} < p.$$

Each informed trader's expected profits are $p - a$ in the continuous market and

$$\mathrm{E}\big[\tilde{\pi}_i\big] = (1 - \hat{m})(p - c) + \hat{m}(0) = (1 - \hat{m})(p - c)$$

in the conditional market. As such, informed traders do not deviate from the conjectured equilibrium if $\hat{m} \geq \frac{a-c}{p-c}$. After replacing a by its expression in (A.7), this condition reduces to

$$(1 - \lambda)c\hat{m}^2 - (2n + \lambda)(p - c)\hat{m} + 2n(p - c) + (2\kappa - c) \leq 0.$$

Because $p > c$ and $2\kappa > c$, it is straightforward to show that this quadratic expression is positive and downward-sloping at $\hat{m} = 0$, negative at $\hat{m} = 1$, and so negative for $\hat{m}$ large enough (larger than $\hat{m}_\mathrm{I}$, say). Therefore, as long as $\hat{m} = \max\{\hat{m}_\mathrm{H}, \hat{m}_\mathrm{I}\} \in (0, 1)$, impatient hedgers and informed traders trade in the continuous market while patient hedgers use the conditional market, and so the volume condition can be set to $\psi = \frac{1-\lambda}{2}\hat{m}$. Patient hedgers are clearly better off since they did not trade before the conditional market opened, as shown in Proposition 5.1. The other traders are better off if the ask price in the continuous market is now lower than it was in Proposition 5.1. A simple comparison of (A.7) and (11) shows that this is the case for any $\hat{m} \in (0, 1)$.

(ii) Suppose now that $p < \frac{2\kappa}{\lambda}$. If $p < r_\mathrm{L}$, then we can set $c \in (p, r_\mathrm{L}]$ and use the results of Propositions 4.2 and 4.3, as the informed traders are precluded from trading. Indeed, because $p < c$ and $p < a = \frac{2\kappa}{\lambda}$, it is never profitable for informed traders to trade based on their information. If instead $r_\mathrm{L} \leq p < \frac{2\kappa}{\lambda}$, then pick any $c \in [0, r_\mathrm{L}]$. We look for an equilibrium in which impatient hedgers trade in the continuous market, while patient hedgers and informed traders use the conditional market. Since the informed traders all place orders on the same side of the market, they do not affect the probability with which the conditional market clears. Under this conjectured equilibrium therefore, the probability that the conditional market clears is again $1 - \hat{m}$ where $\hat{m} = \frac{2\psi}{1-\lambda}$. Also, when it clears, this market will on average cross $\frac{1+\hat{m}}{2}\frac{1-\lambda}{2}$ buy orders with the same number of sell orders, and so the specialist can expect total revenues of $(1 - \hat{m})\frac{1+\hat{m}}{2}(1 - \lambda)c$ from this market. The market-maker's expected profits are therefore as in (A.3) and the equilibrium ask price is the same as (A.4). Impatient hedgers prefer using the continuous market to not trading since

$$a \overset{(r_\mathrm{H}\lambda > 2\kappa)}{<} \frac{r_\mathrm{H}\lambda - (1 - \hat{m}^2)(1 - \lambda)c}{\lambda} = r_\mathrm{H} - \frac{(1 - \hat{m}^2)(1 - \lambda)c}{\lambda} < r_\mathrm{H}.$$

Also, because their expected utility of deviating from the conjectured equilibrium by placing an order in the conditional market is

$$\mathrm{E}\big[\tilde{u}_j \mid \tilde{r}_j = r_{\mathrm{H}}\big] = (1 - \hat{m})(-c) + \hat{m}(-r_{\mathrm{H}})$$

and $r_{\mathrm{H}} > r_{\mathrm{L}} \geq c$, they prefer using the continuous market when $\hat{m}$ is close enough to one (larger than $\hat{m}_{\mathrm{H}}$, say). Because $p \geq r_{\mathrm{L}}$, if it is the case that $a > p$, then we will know that neither the informed traders nor the patient hedgers would consider using the continuous market. Notice from (A.4) that a is smaller than $\frac{2\kappa}{\lambda}$ and can be made arbitrarily close to it by setting $\hat{m}$ to a large enough value. Thus it is the case that $a > p$ as long as $\hat{m} \geq \hat{m}_I$ for some $\hat{m}_I \in (0, 1)$. Finally, we know from Lemma 5.1 that patient traders can coexist in the conditional market as long as (15) is satisfied, or equivalently, as long as $\hat{m} = \frac{2\psi}{1-\lambda} \geq \frac{n[p-(r_{\mathrm{L}}-c)]}{(1-\lambda)(r_{\mathrm{L}}-c)} \equiv \hat{m}_{\mathrm{L}}$. Therefore, as long as $\hat{m} = \max\{\hat{m}_{\mathrm{H}}, \hat{m}_I, \hat{m}_{\mathrm{L}}\} \in (0, 1)$, impatient hedgers trade in the continuous market while patient hedgers and informed traders use the conditional market, and so the volume condition can be set to $\psi = \frac{1-\lambda}{2}\hat{m}$. Patient hedgers and informed traders are clearly better off since they did not trade before the conditional market opened, as shown in Proposition 5.1. Impatient hedgers are also better off as the ask price in the continuous market is now lower than $\frac{2\kappa}{\lambda}$, the ask price that prevailed in the equilibrium of Proposition 5.1. ∎

Proof of Proposition 6.1

To attract the patient hedgers to the conditional market, the specialist must set c to a value not exceeding r_{L}. Let us pick any $c \in [0, r_{\mathrm{L}}]$. As in the proof of Proposition 4.2, if only the patient hedgers use the conditional market, this market clears with probability $1 - \hat{m}$, where $\hat{m} = \frac{2\psi}{1-\lambda}$ (see (A.1)). The equilibrium must have the impatient hedgers trade in the continuous market, so that it must be the case that $\hat{m} \geq \frac{a-c}{r_{\mathrm{H}}-c}$ as in the proof of Proposition 4.2. Increasing $\hat{m}$ above $\frac{a-c}{r_{\mathrm{H}}-c}$ reduces the probability that the conditional market clears and does not affect how a is set in the continuous market or the equilibrium strategies of traders. As such, doing so only serves to reduce the revenues that the specialist can expect to collect from the conditional market. Thus, for any equilibrium value of a and c, it will be the case that $\hat{m}$ is set equal to $\frac{a-c}{r_{\mathrm{H}}-c} \equiv \hat{m}_{a,c}$.

The specialist sets a and c in order to maximize his expected profits, which are given by (A.3) with $\hat{m}$ replaced by $\hat{m}_{a,c}$, that is,

$$\mathrm{E}\big[\tilde{\pi}_{\mathrm{MM}}\big] = a\frac{\lambda}{2} + c(1-\lambda)\frac{1 - \hat{m}_{a,c}^2}{2} - \kappa. \tag{A.8}$$

In equilibrium, a must not exceed r_{H} (for the impatient traders to use the continuous market), and so it is easy to verify that $\hat{m}_{a,c}$ is decreasing in c. In turn, this implies that $\mathrm{E}\big[\tilde{\pi}_{\mathrm{MM}}\big]$ is increasing

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in c for any a. As such, we know that the specialist will set c to its highest possible value, that is, $c = r_{\mathrm{L}}$.

As a result, the specialist's problem amounts to maximizing (A.8) with respect to a when $c = r_{\mathrm{L}}$. Differentiation with respect to a yields the first-order condition for this problem,[29]

$$\frac{\lambda}{2} - r_{\mathrm{L}}(1 - \lambda)\hat{m}_{a,r_{\mathrm{L}}} \frac{1}{r_{\mathrm{H}} - r_{\mathrm{L}}} = 0 \qquad (\mathrm{A.9})$$

which, after solving for a, yields (17). Using this value for a along with $c = r_{\mathrm{L}}$, we find that $\hat{m}_{a,c}$ reduces to $\frac{\lambda(r_{\mathrm{H}} - r_{\mathrm{L}})}{2(1 - \lambda)r_{\mathrm{L}}}$, so that $\psi = \frac{(1 - \lambda)\hat{m}_{a,c}}{2}$ reduces to (18). Clearly, as conjectured, the ask price in (17) is larger than r_{L}, so that the patient hedgers are not attracted by the continuous market. As such, it only remains to verify that $a \leq r_{\mathrm{H}}$ so that indeed impatient traders choose to trade in the continuous market. Straightforward manipulations show that this is the case when (16) is satisfied. Otherwise, the left-hand side of (A.9) is strictly positive for all value of a below r_{H}, and so the specialist would like to increase a as much as possible without chasing the impatient hedgers away by setting $a = r_{\mathrm{H}}$. With this value for a and with $c = r_{\mathrm{L}}$, we have $\hat{m}_{a,c} = 1$. Thus the conditional market never clears, and its presence has no effect on the specialist's expected profits. ∎

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[29]The second-order condition is easy to verify.

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