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Abstract

We present a method for identifying and estimating the gains from trade in limit order markets and provide new empirical evidence that the limit order market is a good market design. The gains from trade in our model arise because traders have different valuations for the stock. We use observations on the traders' order submissions and the execution and cancellation histories of the traders' order submissions to estimate the distribution of traders' unobserved valuations for the stock. We use the parameter estimates for our model to compute the current gains from trade in the limit order market and the gains from trade that the traders would attain in a perfectly liquid market.

Keywords: Limit Order Markets; Gains from Trade; Discrete Choice; Allocative Efficiency
JEL codes: G10, C35, D61

1 Introduction

The majority of the world's stock exchanges operate some form of a limit order market. A feature of a good market design is that it enables the traders to realize most of the gains from trade. We develop a method for identifying and estimating the gains from trade in a limit order market. We use the maximum gains from trade, which we define as the gains from trade that the traders would attain in a perfectly liquid market, as a benchmark against which we measure the efficiency of the limit order market. We apply our method to a sample from one limit order market, estimating the gains from trade in the limit order market to be approximately 90% of the maximum gains from trade. Our results provide new empirical evidence that the limit order market is a good market design.

A large number of experimental studies document that the gains from trade in the double auction are close to the maximum gains from trade. See, for example, Cason and Friedman (1996) or the survey by Holt (1995). Our results show that the limit order market—a market design similar to the double auction—is also remarkably efficient in field data. To our knowledge, our empirical estimates of the gains from trade are the first such estimates using field data from a limit order market.

Our model is an extension of the model of traders' optimal order submission in limit order markets in Hollifield, Miller, and Sandås (2003). Goettler, Parlour, and Rajan (2004) apply numerical techniques to compute the equilibrium in a similar model. In Hollifield, Miller, and Sandås (2003) traders' optimal order submissions depend on traders' valuations for the stock and the trade-offs between execution probabilities, picking-off risks, and order prices for alternative order submissions. We extend that model to continuous time and include an order execution cost. Extending the model to continuous time allows us to deal with the selection problem that arises when some traders find it optimal not to submit an order.

Our model captures a key feature of limit order markets—the traders' ability to choose whether to submit market or limit orders. Our model shares this feature with theoretical models of the traders' order submissions in limit order markets in Foucault (1999), Foucault, Kadan, and Kandel (2003), and Parlour (1998). The additional flexibility of our model makes it suitable for empirical work. For example, the traders in our model may choose from multiple limit order prices, unlike the traders in Parlour (1998). The traders' limit orders in our model may last for multiple periods, unlike the limit orders in Foucault (1999). Limit orders in our model may be cancelled, unlike the limit orders in Foucault, Kadan, and Kandel (2003).

Researchers have also developed theoretical models that abstract from the choice between market and limit orders to understand the popularity of the limit order market. Glosten (1994) shows that in a competitive environment the limit order market provides enough liquidity to discourage entry by other competing market designs. Sandås (2001) tests and rejects the restrictions implied by one version of the Glosten (1994) model with discrete prices, and a time priority rule as in Seppi (1997). Biais, Martimort, and Rochet (2003) study imperfect competition among a finite number of traders submitting limit order schedules, showing that the limit order book of Glosten (1994) results when the number of limit order submitters becomes large. Glosten (2003) relaxes the assumption of perfect competition, showing that with imperfect competition, the limit order market is an optimal market design considering the gains from trade of both traders who submit market orders and traders who submit limit orders.

Our model differs from Biais, Martimort, and Rochet (2003), Glosten (1994, 2003) and Seppi (1997) because we allow traders to choose between market and limit orders and because we model the dynamics of the individual order submissions. We do not allow endogenous market order quantities or asymmetric information, but we do allow for limit orders to face picking-off risk. As in Glosten (2003), we consider the gains from trade accruing to both traders who submit market orders and traders who submit limit orders. Our empirical evidence on the efficiency of the limit order market complements Glosten's (2003) theoretical results on the efficiency of the limit order market.

Many studies document that the empirical frequency of limit and market order submissions changes with market conditions. Using a sample from the Paris Bourse, Biais, Hillion, and Spatt (1995) show that traders are more likely to submit limit orders in markets with wide spreads or thin limit order books. Similar findings are reported by Griffiths et al. (2000) for the Toronto Stock Exchange, and Ranaldo (2004) for the Swiss Stock Exchange. Using a sample from the New York Stock Exchange, Harris and Hasbrouck (1996) show that the traders are more likely to submit limit orders when the expected payoffs from submitting limit orders increase. We extend the literature by using the empirical variation in the frequency of limit and market order submissions to empirically link the traders' order submissions to the their valuations. Without an empirical link between the traders' valuations and their order submissions it is not possible to estimate the gains from trade.

Our sample contains the traders' order submissions and the execution and cancellation histories of the traders' order submissions. In our model, the traders' gains from trade depend on their valuations for the stock. We apply a discrete choice model linking the traders' observable order submissions and the expected payoffs from the order submissions that the traders could make to the traders' valuations. We use the discrete choice model to estimate the distribution of the traders' valuations, the expected payoffs from alternative order submissions, and the traders' optimal order submission strategy. We use the resulting estimates to compute the gains from trade in the limit order market and the maximum gains from trade. The maximum gains from trade provide an upper bound on the gains from trade in any mechanism and a natural benchmark against which to measure the efficiency of a market design. In our sample, the gains from trade in the limit order market are approximately 90% of the maximum gains from trade. In this respect, the limit order market is a good market design—traders in the limit order market we study realize many of the gains from trade.

2 Model

Our model captures several key features of trading in a limit order market. Any trader can submit market and limit orders, and all traders face the same order submission and order execution rules — there are no designated market makers or other traders with special quoting obligations or trading privileges. The market is transparent — all traders observe the limit order book and general market conditions when making their order submission decisions. All trades involve a limit order being executed by a market order, with limit orders executed according to strict price and time priority.

2.1 Model Structure

The model is set in continuous time. Traders arrive sequentially and differ in their valuations for the stock. The finite dimensional vector x_t denotes the exogenous state variables that determine the conditional trader arrival rate and the conditional distribution of the traders' valuations. The exogenous state variables follow a stationary Markov process.

The probability that a trader arrives between t and $t + dt$ is

$$\Pr\left(\text{Trader arrives in } [t, t + dt) \,|\, x_t\right) = \lambda(x_t; t)dt, \tag{1}$$

with equation (1) interpreted as

$$\lim_{\Delta t \downarrow 0} \frac{\Pr\left(\text{Trader arrives in } [t, t+\Delta t) \middle|\, x_t\right)}{\Delta t} = \lambda(x_t; t). \tag{2}$$

The trader is risk neutral with valuation for the stock of v_t. We decompose v_t into

$$v_t = y_t + u_t. \tag{3}$$

The random variable y_t is the common value of the stock at t; y_t may be interpreted as the traders' time t common expectation of the liquidation value of the stock. Innovations in the common value are drawn from a stationary process, with possibly time-varying conditional moments.

The random variable u_t is the trader's private value for the stock. Different traders have different private values, creating the opportunity for gains from trade. The private value is an independent random variable, drawn from the conditional distribution

$$\Pr\left(u_t \leq u \middle|\, x_t\right) \equiv G\left(u \middle|\, x_t\right). \tag{4}$$

Once a trader arrives at the market, his private value remains fixed while he has an order outstanding.

The conditional trader arrival rates, the conditional distributions of the innovations in the common value, and the conditional distributions of the private values can all depend on the exogenous state variables. The exogenous state variables therefore determine the intensity of trader arrivals, the distribution of changes in the stock's common value, and the aggregate willingness of traders to pay a price away from the common value in order to obtain immediate order execution. An example of a state variable that we have in mind is lagged common value volatility. For example, following a period of high common value volatility the intensity of trader arrival, the future common value volatility, and the traders' aggregate willingness to pay a price away from the common value for immediate order execution may all change.

A trader who arrives at t has a single opportunity to submit either a market order or a limit order for q shares. We normalize q to one unit. We can allow the order quantities to vary exogenously across traders in our model — we present the case of unit quantity to reduce notation. In our empirical work, we condition on the observed order quantities. By assuming that each trader has a single order submission opportunity, we abstract from a trader's endogenous future cancellation and resubmission decisions.

The trader's order submission at t depends on the common value, y_t, his private value, u_t, and his information. The trader's information is captured by the exogenous state variables, x_t, and a finite dimensional vector of endogenous state variables, w_t. Let $z_t \equiv (x_t, w_t)$ denote the state vector that represents the trader's information. The state vector z_t follows a stationary Markov process.

The exogenous state variables x_t predict the future trader arrival rates, the distribution of innovations in the common value, and the distribution of future traders' valuations; the exogenous state variables may therefore predict the execution probabilities and picking-off risks of a new order submission.

The endogenous state vector w_t includes information about the current limit order book and past order submission activity. For example, the current bid-ask spread is likely to predict the execution probability for limit orders and so the bid-ask spread is an element of w_t. Similarly, a limit order submitted with few limit orders in the book is likely to have a different execution probability and picking-off risk than a limit order submitted with many limit orders in the book. The endogenous state vector w_t therefore includes the current limit order book.

The endogenous state vector w_t also includes information that is useful in predicting the distribution of cancellations for the orders in the book. Limit orders are executed according to price and time priority. As a consequence, the execution probability of a newly submitted limit order depends on the cancellation probabilities of the existing limit orders in the book. Suppose that the conditional probability that a limit order is cancelled depends on how long the limit order has been in the book. In this case, the average age of limit orders in the book helps predict the probability that current limit orders in the book are cancelled in the future. For example, past order submission activity is correlated with the age of the unexecuted orders in the book, and so past order submission activity is useful to a trader in predicting the execution probabilities of a new limit order submission.

The decision indicators $d_{t,s}^{sell} \in \{0,1\}$ for $s = 0, 1, \ldots, S$, and $d_{t,b}^{buy} \in \{0,1\}$ for $b = 0, 1, \ldots, B$ denote the trader's order submission at t, where s and b index the finite set of available order submissions: $S < \infty$ and $B < \infty$. Let $p_{t,s}^{sell}$ denote the sell price associated with $d_{t,s}^{sell}$, and let $p_{t,b}^{buy}$ denote the buy price associated with $d_{t,b}^{buy}$. If the trader submits a sell market order, then the order price is the best bid quote, $p_{t,0}^{sell}$, and $d_{t,0}^{sell} = 1$. If the trader submits a sell limit order at the price $p_{t,s}^{sell}$, s ticks above the current best bid quote, then $d_{t,s}^{sell} = 1$. Similar definitions apply to the buy side. If the trader does not submit any order at time t, then $d_{t,s}^{sell} = 0$ for all s, and $d_{t,b}^{buy} = 0$ for all b.

A limit order is either executed or cancelled. We define two latent random times for each order: the latent cancellation time, $t + \tau_{cancel}$, and the latent execution time, $t + \tau_{execute}$. The order is executed at $t+\tau_{execute}$ if $\tau_{execute} \leq \tau_{cancel}$ and the order is cancelled at $t+\tau_{cancel}$ if $\tau_{execute} > \tau_{cancel}$. Orders do not last longer than $T < \infty$; the random variable τ_{cancel} is bounded above by $T < \infty$. The distributions of latent times describe the uncertainty about the limit order's outcome.

There is an order submission cost of $c_o \geq 0$ for all types of order submissions. There is an order execution cost of $c_e \geq 0$: the trader pays a cost of c_e when the order executes. The costs, c_o and c_e, do not depend on the trader's valuation, nor on the trader's order submission at t. One interpretation of c_e is that it represents the commission on the trade. With $c_e = 0$ the payoff from order submissions at t are the same as in Hollifield, Miller, and Sandås (2003).

Suppose that a trader with valuation $v_t = y_t + u_t$ submits a buy limit order b ticks below the ask quote at price $p_{t,b}^{buy}$: $d_{t,b}^{buy} = 1$. The conditional distribution of the latent cancellation time depends on the state vector, z_t, and on the order submission itself, but it does not depend on the trader's private value. Conditional on z_t, the latent cancellation time is independent of all other random variables in the model. One interpretation of the conditional independence assumption is that traders find it too costly to continuously monitor their limit orders. The probability distribution of the latent cancellation time is:

$$\Pr\left(t + \tau_{cancel} \leq t + \tau \,\middle|\, z_t, d_{t,b}^{buy} = 1\right) = F_{cancel}\left(\tau \,\middle|\, z_t, d_{t,b}^{buy} = 1\right). \tag{5}$$

The conditional distribution of the latent execution time depends on the state vector, z_t, and on the order submission, $d_{t,b}^{buy} = 1$, but not on the trader's private value. The probability distribution of the latent execution time is

$$\Pr\left(t + \tau_{execute} \leq t + \tau \,\middle|\, z_t, d_{t,b}^{buy} = 1\right) = F_{execute}\left(\tau \,\middle|\, z_t, d_{t,b}^{buy} = 1\right). \tag{6}$$

The execution time depends on the trader's order submission, future order cancellations, and the arrival of future traders and their order submissions. The execution time therefore depends on how future traders behave given their valuations and the order books and information they face — the distribution of the latent execution times depends on future traders' order submissions.

Define the indicator function for order execution:

$$I_t(\tau_{execute} \leq \tau_{cancel}) = \begin{cases} 1, & \text{if } t + \tau_{execute} \leq t + \tau_{cancel}, \\ 0, & \text{otherwise.} \end{cases} \tag{7}$$

The realized utility from submitting a buy order at price $p_{t,b}^{buy}$ is

$$\begin{aligned} & I_t\left(\tau_{execute} \leq \tau_{cancel}\right)\left(y_{t+\tau_{execute}} + u_t - p_{t,b}^{buy} - c_e\right) - c_o \\ & \quad = I_t\left(\tau_{execute} \leq \tau_{cancel}\right)\left(y_t + u_t - p_{t,b}^{buy} - c_e\right) + I_t\left(\tau_{execute} \leq \tau_{cancel}\right)\left(y_{t+\tau_{execute}} - y_t\right) - c_o. \end{aligned} \tag{8}$$

The first term on the first line is the indicator for execution multiplied by the payoff at execution and the second term is the order submission cost.

Define

$$\psi_b^{buy}(z_t) \equiv E\left[I_t\left(\tau_{execute} \leq \tau_{cancel}\right) \middle| z_t, d_{t,b}^{buy} = 1\right] \tag{9}$$

as the execution probability for the order. For a market order, the execution probability is one.

An order may execute when there is a change in the stock's common value; we call the expected loss from such executions the picking-off risk. Define

$$\xi_b^{buy}(z_t) \equiv E\left[I_t\left(\tau_{execute} \leq \tau_{cancel}\right)\left(y_{t+\tau_{execute}} - y_t\right) \middle| z_t, d_{t,b}^{buy} = 1\right] \tag{10}$$

as the picking-off risk for the order. Since a market order executes immediately, the picking-off risk for a market order is zero. Using the law of iterated expectations, the picking-off risk simplifies to

$$\xi_b^{buy}(z_t) = E\left[\left(y_{t+\tau_{execute}} - y_t\right) \middle| I_t\left(\tau_{execute} \leq \tau_{cancel}\right) = 1, z_t, d_{t,b}^{buy} = 1\right] \psi_b^{buy}(z_t). \tag{11}$$

The picking-off risk is the expected change in the common value between the time of the order submission and the time of the order execution conditional on execution, multiplied by the probability that the order executes.

The conditional distribution of the latent cancellation times, the conditional distribution of the latent execution times, and the expected change in the common value conditional on execution all depend on the state vector, z_t. As a consequence, the execution probabilities and picking-off risks also depend on the state vector.

The trader's expected utility from submitting a buy order at price $p_{t,b}^{buy}$ is the expected value of equation (8), conditional on the trader's information, which, using the definitions of the execution probability and picking-off risk, is

$$U_b^{buy}(y_t + u_t; z_t) = \psi_b^{buy}(z_t)\left(y_t + u_t - p_{t,b}^{buy} - c_e\right) + \xi_b^{buy}(z_t) - c_o. \tag{12}$$

Similarly, the expected utility of submitting a sell order at $p_{t,s}^{sell}$ is

$$U_s^{sell}(y_t + u_t; z_t) = \psi_s^{sell}(z_t)\left(p_{t,s}^{sell} - y_t - u_t - c_e\right) - \xi_s^{sell}(z_t) - c_o. \tag{13}$$

The trader's order submission strategy maximizes his expected utility,

$$\max_{\{d_{t,s}^{sell}\},\{d_{t,b}^{buy}\}} \sum_{s=0}^{S} d_{t,s}^{sell} U_s^{sell}(y_t + u_t; z_t) + \sum_{b=0}^{B} d_{t,b}^{buy} U_b^{buy}(y_t + u_t; z_t), \tag{14}$$

subject to:

$$\sum_{s=0}^{S} d_{t,s}^{sell} + \sum_{b=0}^{B} d_{t,b}^{buy} \leq 1. \tag{15}$$

Equation (15) is the constraint that at most one submission is made at t.

2.2 Optimal Order Submission Strategies

Let $\{d_s^{sell*}(y_t + u_t; z_t),\ d_b^{buy*}(y_t + u_t; z_t)\}$ be the optimal order submission strategy, describing the trader's optimal order submission as a function of the trader's valuation and the state vector z_t.

Hollifield, Miller, and Sandås (2003) show that the optimal order submission strategy has a monotonicity property. Traders with high private values submit buy orders with high execution probabilities. Traders with low private values submit sell orders with high execution probabilities. Traders with intermediate private values either submit no order, or submit buy or sell limit orders with low execution probabilities.

The optimal order submission strategy is represented in terms of threshold valuations. We can partition the set of valuations into intervals. All traders whose valuations lie within the same interval submit the same order. In order to characterize the intervals, we define a set of threshold valuations that mark the boundaries of the intervals. We determine a trader's optimal order submission simply by identifying which interval the trader's valuation falls in.

Define the threshold valuation $\theta_{b,b'}^{buy}(z_t)$ as the valuation of a trader who is indifferent between submitting a buy order at price $p_{t,b}^{buy}$ and a buy order at price $p_{t,b'}^{buy}$

$$\theta_{b,b'}^{buy}(z_t) = p_{t,b}^{buy} + c_e + \frac{\left(p_{t,b}^{buy} - p_{t,b'}^{buy}\right)\psi_{b'}^{buy}(z_t) + \left(\xi_{b'}^{buy}(z_t) - \xi_{b}^{buy}(z_t)\right)}{\psi_{b}^{buy}(z_t) - \psi_{b'}^{buy}(z_t)}. \tag{16}$$

The threshold valuation for indifference between a buy order at price $p_{t,b}^{buy}$ and not submitting an order is

$$\theta_{b,no}^{buy}(z_t) = p_{t,b}^{buy} + c_e - \frac{\xi_{b}^{buy}(z_t) - c_o}{\psi_{b}^{buy}(z_t)}. \tag{17}$$

The threshold valuation for indifference between a sell order at price $p_{t,s}^{sell}$ and a sell order at price $p_{t,s'}^{sell}$ is

$$\theta_{s,s'}^{sell}(z_t) = p_{t,s}^{sell} - c_e - \frac{\left(p_{t,s'}^{sell} - p_{t,s}^{sell}\right)\psi_{s'}^{sell}(z_t) + \left(\xi_{s}^{sell}(z_t) - \xi_{s'}^{sell}(z_t)\right)}{\psi_{s}^{sell}(z_t) - \psi_{s'}^{sell}(z_t)}. \tag{18}$$

The threshold valuation for indifference between a sell order at price $p_{t,s}^{sell}$ and not submitting any order is

$$\theta_{s,no}^{sell}(z_t) = p_{t,s}^{sell} - c_e - \frac{\xi_{s}^{sell}(z_t) + c_o}{\psi_{s}^{sell}(z_t)}. \tag{19}$$

The threshold valuation for indifference between a sell order at price $p_{t,s}^{sell}$ and a buy order at price $p_{t,b}^{buy}$ is

$$\theta_{s,b}(z_t) = \frac{\left(p_{t,b}^{buy}\psi_{b}^{buy}(z_t) + p_{t,s}^{sell}\psi_{s}^{sell}(z_t)\right) + c_e\left(\psi_{b}^{buy}(z_t) - \psi_{s}^{sell}(z_t)\right) - \left(\xi_{b}^{buy}(z_t) + \xi_{s}^{sell}(z_t)\right)}{\psi_{s}^{sell}(z_t) + \psi_{b}^{buy}(z_t)}. \tag{20}$$

It may be the case that some order submissions are not optimal for any trader. Let $\mathcal{S}^*(z_t) = \{s_0(z_t), s_1(z_t), s_2(z_t), \ldots, s_S(z_t)\}$ index the set of sell orders that are optimal for some trader at state z_t sorted by their execution probabilities: $1 \geq \psi_{s_0(z_t)}^{sell}(z_t) > \psi_{s_1(z_t)}^{sell}(z_t) > \ldots > \psi_{s_S(z_t)}^{sell}(z_t)$. Define a sell limit order submitted at price $p_{t,s_S(z_t)}^{sell}$ as the marginal sell order. We assume that a sell market order is optimal for traders with some private values and that some sell limit order is optimal for traders with different private values; $\mathcal{S}^*(z_t)$ has at least two elements. Similarly, let $\mathcal{B}^*(z_t)$ index the set of buy orders that are optimal for some trader in state z_t, also sorted by execution probabilities and define a buy limit order submitted at $p_{t,b_B(z_t)}^{buy}$ as the marginal buy order.

A trader with a valuation lower than the threshold between a marginal buy order and no order subm ission receives a lower expected payoff from submitting any buy order than from submitting no order. A trader with a valuation greater than the threshold between a marginal sell order and no order submission receives a lower expected payoff from submitting any sell order than from submitting no order. If $\theta^{sell}_{s_S(z_t),no}(z_t) \leq \theta^{buy}_{b_B(z_t),no}(z_t)$, then a trader with a valuation between $\theta^{sell}_{s_S(z_t),no}(z_t)$ and $\theta^{buy}_{b_B(z_t),no}(z_t)$ submits no order. If $\theta^{buy}_{b_B(z_t),no}(z_t) \leq \theta^{sell}_{s_S(z_t),no}(z_t)$, then $\theta^{buy}_{b_B(z_t),no}(z_t) \leq \theta_{s_S(z_t),b_B(z_t)}(z_t) \leq \theta^{sell}_{s_S(z_t),no}(z_t)$, and a trader with any possible valuation submits some order. We therefore define the marginal thresholds for sellers and buyers as

$$\begin{aligned}
\theta^{buy}_{marginal}(z_t) &= \max\left(\theta_{s_S(z_t),b_B(z_t)}(z_t), \theta^{buy}_{b_B(z_t),no}(z_t)\right), \\
\theta^{sell}_{marginal}(z_t) &= \min\left(\theta_{s_S(z_t),b_B(z_t)}(z_t), \theta^{sell}_{s_S(z_t),no}(z_t)\right). \qquad (21)
\end{aligned}$$

Using the definition of the thresholds, the sell side of the optimal order submission strategy is

$$d^{sell*}_{s}(y_t + u_t; z_t) = 0, \text{for } s \notin \mathcal{S}^*(z_t), \qquad (22)$$

$$d^{sell*}_{0}(y_t + u_t; z_t) = \begin{cases} 1, & \text{if } -\infty \leq y_t + u_t < \theta^{sell}_{s_0(z_t),s_1(z_t)}(z_t), \\ 0, & \text{else,} \end{cases} \qquad (23)$$

$$d^{sell*}_{s_i(z_t)}(y_t + u_t; z_t) = \begin{cases} 1, & \text{if } s_i(z_t) \notin \{0, s_S(z_t)\} \text{ and} \\ & \theta^{sell}_{s_{i-1}(z_t),s_i(z_t)}(z_t) \leq y_t + u_t < \theta^{sell}_{s_i(z_t),s_{i+1}(z_t)}(z_t), \\ 0, & \text{else,} \end{cases} \qquad (24)$$

$$d^{sell*}_{s_S(z_t)}(y_t + u_t; z_t) = \begin{cases} 1, & \text{if } \theta^{sell}_{s_{S-1}(z_t),s_S(z_t)}(z_t) \leq y_t + u_t < \theta^{sell}_{marginal}(z_t), \\ 0, & \text{else,} \end{cases} \qquad (25)$$

with the buy side defined similarly.

2.3 The Gains from Trade

Each trade involves either a sell limit order executing with a buy market order or a sell market order executing with a buy limit order. The gains from a trade are the sum of the traders' realized utilities from the trade. Using equation (8), the gains from trade for a trade at $t+\tau$ between a sell market order submitted at $t+\tau$ by a trader with valuation $u^{sell}_{t+\tau}$ and a buy limit order submitted at t by a trader with valuation u^{buy}_{t} are:

$$\begin{aligned}
&\left(p^{sell}_{t+\tau,s} - y_{t+\tau} - u^{sell}_{t+\tau} - c_e - c_o\right) + \left(y_{t+\tau} + u^{buy}_{t} - p^{buy}_{t,b} - c_e - c_o\right) \\
&\qquad = \left(-u^{sell}_{t+\tau} - c_e - c_o\right) + \left(u^{buy}_{t} - c_e - c_o\right). \qquad (26)
\end{aligned}$$

The second line follows because $p^{sell}_{t+\tau,s} = p^{buy}_{t,b}$, since the sell market order executes with the buy limit order. The gains from trade for a trade between a sell limit order and a buy market order are computed similarly.

The gains from the trade do not depend on the price because the price is a transfer between the buyer and the seller. The gains from trade do not depend on the common value because they depend on the difference between the traders' valuations at the time of the trade. The buyer's contribution to the gains from trade is $u_t^{buy} - c_e - c_o$ and the seller's contribution to the gains from trade is $-u_{t+\tau}^{sell} - c_e - c_o$. If a trader submits an order that does not execute, he contributes $-c_o$ to the gains from trade.

In the example above we considered the gains from trade for one possible outcome for the buy limit order submitted by the trader at t. For our purposes it is useful to consider the ex ante gains from trade in a given state before the trader's valuation is drawn. Using the distribution of the traders' valuations for the stock and the optimal order submission strategy we compute expectations over the traders' valuations, optimal order submissions, and the outcomes of their order submissions. We define the current gains from trade as the expected contribution to the gains from trade in state z_t. Using the execution probabilities, the traders' optimal order submission strategy, and the distribution of the traders' valuations, the expected contribution to the gains from trade for a trader arriving at state z_t is

$$\text{Current gains}\,(z_t) = E\left[\begin{array}{c} \sum_{s=0}^{S} d_s^{sell*}(y_t + u_t; z_t)\left(\psi_s^{sell}(z_t)\,(-u_t - c_e) - c_o\right) \\ +\sum_{b=0}^{B} d_b^{buy*}(y_t + u_t; z_t)\left(\psi_b^{buy}(z_t)\,(u_t - c_e) - c_o\right) \end{array}\middle|\, z_t\right]. \tag{27}$$

The current common value, limit and market order prices, and the state vector z_t enter through their effects on the traders' optimal order submission strategy.

The maximum gains from trade are determined by finding the post-trade allocation of the stock among the traders that results in the maximum expected gains from trade. The maximum gains from trade may not be achievable by any mechanism because of the inherent frictions caused by traders arriving sequentially with trading opportunities that last for a finite period of time. Incentive compatibility issues will typically further reduce the gains from trade attainable in any feasible mechanism. The main advantage of the maximum gains from trade are that they are easy to compute and provide a useful upper bound on the gains from trade in any feasible mechanism.

To describe a stock allocation, define the sell indicator function

$$I^{sell}(u_t; x_t) = \begin{cases} 1, & \text{if a trader with private value } u_t \text{ sells the stock in state } x_t, \\ 0, & \text{else,} \end{cases} \tag{28}$$

and define the buy indicator function $I^{buy}(u_t; x_t)$ similarly.

Using the sell and buy indicators, the allocation that maximizes the gains from trade solves:

$$\max_{\{I^{sell}(u_t;x_t), I^{buy}(u_t;x_t)\}} E\left[I^{sell}(u_t; x_t)(-u_t - c_e - c_o) + I^{buy}(u_t; x_t)(u_t - c_e - c_o) \middle| x_t \right], \tag{29}$$

subject to:

$$I^{sell}(u_t; x_t) + I^{buy}(u_t; x_t) \leq 1, \text{ for all } u_t, \tag{30}$$

$$E\left[I^{sell}(u_t; x_t) | x_t \right] = E\left[I^{buy}(u_t; x_t) | x_t \right]. \tag{31}$$

Equation (30) is the constraint that each trader has a single opportunity to trade. Equation (31) is the market clearing condition.

The optimal allocation and the maximum gains from trade with a symmetric distribution for the private values with median zero are reported in the next lemma.

Lemma 1 *Suppose that the private values are drawn from the continuous, symmetric distribution $G(\cdot|x_t)$ with median zero. The allocation that solves (29) subject to (30) and (31) is*

$$I^{sell*}(u_t; x_t) = \begin{cases} 1, & \text{for } u_t \leq -c_e - c_o \\ 0, & \text{else,} \end{cases}, \quad I^{buy*}(u_t; x_t) = \begin{cases} 1, & \text{for } u_t \geq c_e + c_o \\ 0, & \text{else.} \end{cases} \tag{32}$$

The maximum gains from trade are:

$$\text{Maximum gains } (x_t) = E\left[I^{sell*}(u_t; x_t)\left(-u_t - c_e - c_o\right) + I^{buy*}(u_t; x_t)\left(u_t - c_e - c_o\right) \middle| x_t \right]. \tag{33}$$

The proof is given in Appendix A. The proof also derives the optimal allocation in the cases where the distribution of valuations is not symmetric and the case where the median is not zero.

By construction the current gains from trade in the limit order market are less than or equal to the maximum gains from trade. The current gains may be lower because limit orders face execution risk and the traders' private incentives may lead them to make order submissions that are differnt thatn the ones that would lead to the social optimum. We decompose the differences between the maximum and current gains into four sources: no execution, no submission, wrong direction, and crowding out.

No execution is the expected loss from traders who buy or sell the stock in the optimal allocation but whose buy or sell orders do not execute in the limit order market:

$$\text{No execution } (z_t) = E\left[\begin{array}{l} I^{sell*}(u_t;x_t)\sum_{s=0}^{S} d_s^{sell*}(y_t+u_t;z_t)(1-\psi_s^{sell}(z_t))(-u_t-c_e) \\ +I^{buy*}(u_t;x_t)\sum_{b=0}^{B} d_b^{buy*}(y_t+u_t;z_t)(1-\psi_b^{buy}(z_t))(u_t-c_e) \end{array} \middle| z_t \right]. \quad (34)$$

Losses from no execution arise because it is sometimes individually optimal for traders with valuations that differ from the common value by more than $c_e + c_o$ to submit limit orders that may fail to execute.

No submission is the expected loss from traders who buy or sell the stock in the optimal allocation but do not submit an order in the limit order market:

$$\text{No submission } (z_t) = E\left[\begin{array}{l} \left(1-\sum_{s=0}^{S} d_s^{sell*}(y_t+u_t;z_t)-\sum_{b=0}^{B} d_b^{buy*}(y_t+u_t;z_t)\right) \\ \left(I^{sell*}(u_t;x_t)(-u_t-c_e-c_o)+I^{buy*}(u_t;x_t)(u_t-c_e-c_o)\right) \end{array} \middle| z_t \right]. \quad (35)$$

Losses for no submission arise because it is sometimes individually optimal for traders with valuations that differ from the common value by more than $c_e + c_o$ to not submit any order.

Although a trader's order submission is individually optimal by construction, it need not lead to a positive contribution to the gains from trade. For example, suppose a sell market order and a buy limit order transact at price $p_{t,0}^{sell}$. If the seller's valuation $u_t^{sell} > 0$, the seller makes a negative contribution to the gains from trade because $-u_t^{sell} - c_e - c_o < 0$. Nevertheless, the trade can be individually optimal for the seller if $p_{t,0}^{sell} - y_t > c_e + c_o$. In this example, a trader with a positive private value—and hence no particular need to sell—may end up selling the security because the limit order book provides an opportunity to sell at a high enough price. The seller transacts with a buy limit order submitted by a previous trader with a high valuation; the common value or trader's private value or both were high. Depending on the seller's private value his trade contributes either to the wrong direction losses or to the crowding out losses.

Wrong direction is the expected loss from traders who buy or sell the stock in the optimal allocation but submit an order to trade in the wrong direction in the limit order market:

$$\text{Wrong direction } (z_t)$$
$$= E\left[\begin{array}{l} I^{sell*}(u_t;x_t)\sum_{b=0}^{B} d_b^{buy*}(y_t+u_t;z_t)\left(-u_t-c_e+\psi_b^{buy}(z_t)(-u_t+c_e)\right) \\ +I^{buy*}(u_t;x_t)\sum_{b=0}^{B} d_b^{sell*}(y_t+u_t;z_t)\left(u_t-c_e+\psi_s^{sell}(z_t)(u_t+c_e)\right) \end{array} \middle| z_t \right]. \quad (36)$$

Crowding out is the expected loss from traders who do not trade in the optimal allocation but submit buy or sell orders in the limit order market:

$$\text{Crowding out }(z_t) = E\left[\left.\begin{array}{l}\left(1 - I^{sell*}(u_t; x_t) - I^{buy*}(u_t; x_t)\right) \\ \left(\sum_{s=0}^{S} d_s^{sell*}(y_t + u_t; z_t)\left(\psi_s^{sell}(z_t)(u_t + c_e) + c_o\right)\right. \\ \left. + \sum_{b=0}^{B} d_b^{buy*}(y_t + u_t; z_t)\left(\psi_b^{buy}(z_t)(-u_t + c_e) + c_o\right)\right)\end{array}\right| z_t\right]. \tag{37}$$

3 Empirical Results

We use a two-step method to estimate the parameters of the model. In the first step, we use the execution and cancellation histories of the order submissions to estimate the execution probabilities and picking-off risks. In the second step, the private value distributions, arrival rates of the traders and costs are estimated by maximizing the conditional log-likelihood function for limit and market order arrival times. We use the estimated parameters to form estimates of the current and maximum gains from trade.

3.1 Description of the Vancouver Stock Exchange and our Sample

Our sample is from the audit tapes of the Vancouver Stock Exchange. The Vancouver Stock Exchange's market design is a limit order market similar to the Paris Bourse, the Stockholm Stock Exchange, and the Toronto Stock Exchange.[1] Forty-five exchange member firms act as brokers, submitting orders for outside traders, and act as dealers, submitting orders on their own account. The are no designated market makers.

The market is open from 6:30 a.m. to 1:30 p.m. Pacific time. Limit orders in the order book are matched with incoming market orders to produce trades, giving priority to limit orders according to the order price and then the time of submission. Order prices must be multiples of a tick size. The tick size varies between one cent for prices below \$3.00, five cents for prices between \$3.00 and \$4.99, and twelve and a half cents for prices at \$5.00 and above. Orders sizes must be multiples of a fixed size which varies between 100 and 1000 shares.

Member firms can also submit hidden limit orders where a fraction of the order size is not visible on the limit order book. The hidden fraction of the order retains its price priority, but loses its time priority. In our sample, few hidden orders are submitted—the assumption of no hidden limit orders in our model is a reasonable approximation for our sample.

[1] In 1999, after the end of our sample, the Vancouver Stock Exchange was involved in an amalgamation of Canadian equity trading and became a part of the Canadian Venture Exchange, which in turn was recently renamed the TSX Venture Exchange. The TSX Venture Exchange is also a limit order market.

Our sample contains a record for every trade, cancellation, or change in the status of an order, and the limit order book at the open of each day. Each record includes the time of the original order submission, but not the member firms' identification codes nor whether or not a member firm submitted an order as a broker or dealer. Combining the records with the limit order book at the open of each day we reconstruct order histories for each order submission, including the initial order submission and every future order execution or cancellation, and the corresponding order books. For less than one percent of the orders there are inconsistencies between the inferred order histories and the trading rules. We drop such orders from our sample.

Our sample goes from May 1990 to November 1993 for three stocks in the mining industry. Table 1 reports the name and ticker symbol of the three stocks. The table reports the total number of order submissions, the percentage of buy and sell market and limit orders submitted in our sample, and the average and standard deviation of the time between order submissions.

3.2 Construction of the Variables

We use a centered moving average of the mid-quotes over a twenty-minute window to proxy for the stock's unobserved common value. Our proxy is reasonable because most of the time the best quotes should straddle the common value. We use a centered moving average to reduce the impact of mechanical shifts in the mid-quote caused by individual order submissions or cancellations.

Table 2 reports our choice for the state vector $z_t = (x_t, w_t)$. The table reports the names of the variables, a brief description of them, and their sample means and standard deviations. In the theoretical model, the exogenous state variables x_t predict the trader arrival rates, the distribution of innovations to the common value, and the conditional distributions of the traders' private values. Our choice of exogenous state variables are reported in the top panel of Table 2. We observe the exogenous state variables at a daily frequency.

We chose exogenous state variables that are likely to be correlated with the traders' desire to change their portfolios and correlated with innovations in the stocks' common value. The exogenous state variables we use are Toronto Stock Exchange (TSX) market index volatility, TSX mining volatility, interest rate volatility, exchange rate volatility, and stock volatility. Traders may be more likely to want to change their portfolio as a result of changes in the market index, interest rates, or the stock price. For example, a change in the market index may lead more traders to wish to change their portfolios and may also change the traders' willingness to pay more to buy immediately or receive less to sell immediately. Such effects are captured by changes in the trader arrival rate and the distribution of private values.

The bottom panel in Table 2 reports our choice of endogenous state variables w_t. Ideally w_t would include the entire limit order book and any other variables known at t that are useful for predicting the outcomes of order submissions at t. It is not practical to use the entire limit order book in our estimation; we must balance the number of variables against the sample size.

The bid-ask spread and measures of depth close to the quotes and away from the quotes directly measure the state of the limit order book. Close depth is the number of shares outstanding at the current best quotes and far depth is the cumulative number of shares outstanding up to and including the marginal limit order. The depth measures on the same side of the book often contain very similar information and in our first-step estimation we therefore include only one depth measure for each side of the market. We use the depth in front of the order as well as the close depth on the other side of the book to predict the execution probabilities and picking-off risks. Execution probabilities depend on book variables through the length of the order queues at different prices. Execution probabilities also depend on book variables indirectly through the book's effect on the current and future order submissions and cancellations.

Past order submission activity is useful to predict the latent execution and cancellation times for new order submissions because the average age of existing limit orders in the book can influence the probability that the existing limit orders are cancelled. We use the number of recent trades and lagged durations to measure past order submission activity. Holding everything else equal, larger orders are likely to have lower execution probabilities and also face higher picking-off risk. The distance between the current mid-quote and our proxy for the common value is included because holding everything else equal, a buy order at two ticks below the common value is less likely to be executed than a buy order one tick below the common value.

We include six hourly dummy variables to capture any deterministic time-of-day patterns in execution probabilities and picking-off risks. Deterministic time-of-day patterns may arise because of deadline effects associated with market closure. For example, some traders cancel unexecuted limit orders at the close of the market. Such behavior introduces time-of-day patterns in the timing of cancellations—orders submitted early are less likely to remain outstanding at the time of the market close than orders submitted later.

We assume that some traders always find it optimal to submit one tick buy and sell limit orders. In the theoretical model, the marginal sell limit order is defined as the highest priced sell limit order that any trader would optimally submit at z_t. The marginal buy limit order defined similarly. Empirically, we set the marginal prices depending on the level of the common value. For each decile of the common value, we define a cut-off sell price as the price such that at least 95% of the sell order prices are below that cut-off sell price. The marginal sell order is defined as the lowest priced sell order above the cut-off price. The marginal buy order is defined similarly. In order to have enough observations we include orders at the marginal price as well as orders submitted at one or two ticks away depending on whether the marginal price falls on one tick or between two ticks. We purposely drop a small fraction of the order submissions to avoid a situation where the orders submitted at extreme prices—which may represent order entry mistakes—would determine the marginal order prices.

By ignoring limit orders submitted outside the price range defined by the marginal prices, we ignore the expected payoffs received by traders whose private values would lead them to submit limit orders outside that price range. Omitting some payoffs may lead to a downward bias in our estimates of the current gains from trade, but should not affect our estimates of the maximum gains from trade.

In the theoretical model, order quantity is normalized to one unit — all orders are either fully executed or cancelled. In our sample, different order submissions have different order quantities; partial executions may occur. Empirically, we handle partial executions by assuming an order was an execution if at least 50% of the order size is executed, otherwise we treat the order as a cancellation. Table 3 reports the average percentage of the submitted limit order quantity that is executed within 48 hours, conditional on that percentage being at least 50% or less than 50%. Less than 1% of the order executions occur more than 48 hours from the time of the order submission. Orders that are executed more than 50% have average execution percentages close to 100%, and orders that execute less than 50% have execution percentages close to 0%. Our assumption for the partial execution is a reasonable approximation in our sample.

In the theoretical model, both execution and cancellation times are random. Our assumption of random execution and cancellation times is justified in our sample. The second panel of Table 3 reports the distribution of the time to execution for all limit orders in our sample. The second panel of Table 3 shows that time to execution is random. The third panel of Table 3 shows that the time to cancellation is random; orders are not cancelled a fixed number of minutes after submission nor are they all cancelled at the end of the trading day.

3.3 Estimates of the Execution Probabilities and Picking-Off Risks

We assume that the traders have rational expectations — their beliefs about the execution probabilities and picking-off risks are consistent with the empirical execution and cancellation histories.

The execution probabilities are determined by the distributions of the latent times to cancellation and execution in equations (5) and (6). The picking-off risks are determined by the execution probabilities and the expected change in the common value conditional on the order executing. We use our sample to estimate the distribution of the latent times and the expected change in the common value conditional on the order executing. The resulting estimates are used to compute estimates of execution probabilities and picking-off risks. The estimates of the execution probabilities and picking-off risks are used to characterize the traders' expectations and the optimal order submission strategy.

Our formulation of independent latent execution and cancellation times is a competing risks model. Lancaster (1990) provides a description of the competing risks model. We use the distribution of the latent execution and cancellation times to compute the execution probabilities. Our approach extends Cho and Nelling (2000) who compute execution probabilities for limit orders based on the parameter estimates for the distribution of the time to execution, assuming that all orders are cancelled at the end of the day.

We parameterize the conditional distributions of the cancellation times as Weibull:

$$F_{cancel}\left(\tau \left| z_t, d_{t,b}^{buy} = 1\right.\right) = 1 - \exp\left(-\exp\left({z_t}'\gamma_b^{buy}\right)\tau^{\alpha_b^{buy}}\right), \tag{38}$$

with z_t the state vector. The hazard rate is defined as the probability that the cancellation time occurs between $t+\tau$ and $t+\tau+d\tau$, conditional on the cancellation time being greater than $t+\tau$. For the Weibull model, the hazard rate is

$$\Pr\left(\tau_{cancel} \in [\tau, \tau + d\tau) \left| \tau_{cancel} \geq \tau, z_t, d_{t,b}^{buy} = 1\right.\right) = \exp\left(z_t'\gamma_b^{buy}\right)\alpha_b^{buy}\tau^{\alpha_b^{buy}-1}d\tau. \tag{39}$$

The parameter vector γ_b^{buy} measures the effect of the state vector on the hazard rate. If a variable has a positive parameter, then an increase in that variable increases the hazard rate. The parameter α_t^{buy} is the Weibull shape parameter. If $\alpha_t^{buy} = 1$, the hazard rate does not depend on τ. If $\alpha_t^{buy} < 1$, the hazard rate is decreasing in τ. If $\alpha_t^{buy} > 1$, the hazard rate is increasing in τ.

Table 4 reports the results for the cancellation time distributions. The models are estimated for one tick and marginal limit orders and are estimated by maximum likelihood. We treat orders that last longer than two days as censored observations.

The parameter estimates for the Weibull shape parameters are all less than one, with an average value of 0.56. The cancellation hazard rates are decreasing in the time that the order is in the book. The age of an order predicts the conditional probability of cancellation. Past activity is correlated with the age of unfilled orders in the book. Past activity therefore can predict the cancellation rates of existing orders in the book, consistent with the assumption in the theoretical model.

We also parameterize the conditional distributions of the time to execution as Weibull:

$$F_{execute}\left(\tau \,\middle|\, z_t, d_{t,b}^{buy} = 1\right) = 1 - \exp\left(-\exp\left({z_t}'\kappa_b^{buy}\right)\tau^{\beta_b^{buy}}\right). \tag{40}$$

The conditional distributions of the time to execution depend on the trader arrival rates, the order cancellation distributions, and future traders' order submissions. A disadvantage of the parametric model is that it imposes auxiliary restrictions on the conditional distributions. An alternative, which does not impose such auxiliary assumptions, is to use non-parametric methods as in Hollifield, Miller, and Sandås (2003). An advantage of the parametric model is that we can use a larger state vector than with a non-parametric method. We use the parametric model because it allows us to approximate the large information set available to the traders.

Table 5 reports the results for the execution time distributions. The models are estimated for one tick and marginal limit orders and are estimated by maximum likelihood. We treat orders that last longer than two days as censored observations.

The parameter estimates for the Weibull shape parameters are all less than one, with an average value of 0.65. The execution hazard rates are decreasing in the time that the order is in the book. The Weibull shape parameter is lower for the cancellation times than for the execution times; the probability that a limit order is cancelled rather executed decreases with the time the order is in the book.

Table 4 and Table 5 also report chi-squared tests for the null hypotheses that the conditional distributions of the execution and cancellation times do not depend on the state vector z_t. We reject the null hypothesis for all order submissions and stocks.

We use the parameter estimates from the conditional distributions of the execution times and cancellation times to forecast the execution probabilities for buy and sell one tick and marginal limit orders at every order submission. We compute the probability that the order executes within two days: $T = 2$ days. Details of the computations of the execution probabilities are reported in Appendix B.

For BHO, the average execution probability for marginal sell limit orders is approximately 16%, for one tick sell limit orders 61%, for marginal buy limit orders 13% and for one tick buy limit orders 63%. The estimates for the other stocks are similar.

From equation (11), the picking-off risk is equal to the product of the expected change in the common value conditional on an execution and the execution probability. We parameterize the expected change conditional on an execution as a linear function:

$$E\left[(y_{t+\tau_{execute}} - y_t)\,\Big|\, I_t(\tau_{execute} \leq \tau_{cancel}) = 1, z_t, d_{t,b}^{buy} = 1\right] = {z_t}'\Lambda_b^{buy}, \tag{41}$$

with z_t the state vector. The expectation of the change in the common value conditional on execution is determined by the trader arrival rates, the cancellation time distributions, and the future traders' order submissions. As in the case of the distribution of execution times, we use a parametric model rather than a non-parametric model to allow for a large state vector.

We estimate the model in equation (41) for buy and sell one tick and marginal limit orders that execute in our sample using ordinary least squares. Table 6 reports the estimates. The table also reports F-tests for the null hypothesis that the expected change in the common value conditional on the order executing does not depend on the state vector z_t. We reject the null hypothesis for all order submissions and stocks.

We use the parameter estimates to forecast the expected change in the common value, conditional on the limit order executing for buy and sell one tick and marginal limit orders at every order submission. At the mean values of the state vector, the expected change in the common value is approximately zero for one tick limit orders, minus four cents for marginal buy limit orders, and four cents for marginal sell limit orders.

We form estimates of the picking-off risk by substituting our estimates of the expected change in the common value conditional on execution and the execution probabilities into equation (11). At the mean values of the state vector, the picking-off risk is close to zero for one tick limit orders, and approximately one cent for marginal limit orders.

3.4 Estimates of the Arrival Rates, Private Value Distributions and Costs

We estimate the remaining parameters of the model by maximizing the conditional log-likelihood function for the timing of market and limit orders. We form the log-likelihood function for sell market orders, sell limit orders between one tick and the marginal sell order, buy limit orders between one tick and the marginal buy order, and buy market orders. The grouping is consistent with the theoretical model and leads to consistent estimators of the remaining parameters. To form the conditional log-likelihood function, we use the optimal order submission strategy, the trader arrival rates, and the distributions of the trader's private values to compute the probabilities of observing a limit order or a market order. The conditional log-likelihood function is reported in Appendix C.

The conditional probability of a buy market order between t and $t + dt$ is the probability that a trader who arrives finds it optimal to submit a buy market order times the probability that a trader arrives. Using the conditional distribution of the private values, $G(u|z_t)$, the trader arrival rate, and the assumption that the one tick buy limit order is an optimal order submission for some trader:

$$\begin{aligned}\Pr\left(\text{Buy market order in } [t, t+dt) \middle| z_t\right) &= \Pr\left(y_t + u_t \geq \theta_{0,1}^{buy}(z_t) \middle| z_t\right) \lambda(x_t; t)dt \\ &= \left[1 - G\left(\theta_{0,1}^{buy}(z_t) - y_t \middle| x_t\right)\right] \lambda(x_t; t)dt. \quad (42)\end{aligned}$$

Similarly, the probability of a buy limit order is

$$\begin{aligned}&\Pr\left(\text{Buy limit order in } [t, t+dt) \middle| z_t\right) \\ &\quad = \left[G\left(\theta_{0,1}^{buy}(z_t) - y_t \middle| x_t\right) - G\left(\theta_{marginal}^{buy}(z_t) - y_t \middle| x_t\right)\right] \lambda(x_t; t)dt. \quad (43)\end{aligned}$$

The probability of a sell market order and a sell limit order are computed similarly.

We may observe no orders submitted between t and $t+dt$ for two reasons. Either a trader does not arrive, or a trader arrives and chooses not to submit any order,

$$\begin{aligned}&\Pr\left(\text{No order submission in } [t, t+dt) \middle| z_t\right) \\ &\quad = 1 - \lambda(x_t;t)dt + \left[G\left(\theta^{buy}_{marginal}(z_t) - y_t \middle| x_t\right) - G\left(\theta^{sell}_{marginal}(z_t) - y_t \middle| x_t\right)\right]\lambda(x_t;t)dt. \end{aligned} \quad (44)$$

Let t_i be the time of the i^{th} order arrival. We use a Weibull parametrization for the trader arrival rate. Suppose that the last order submission was at time t_{i-1}. The arrival rate is

$$\lambda(x_{t_i};t)dt \quad = \quad \exp({x_{t_i}}'\delta)\eta\,(t - t_{i-1})^{\eta-1}\,dt, \quad (45)$$

with x_{t_i} denoting the exogenous state variables immediately after the last order submission. The private value distribution is parameterized as a mixture of two normal distributions with standard deviations depending on the common value and exogenous state variables,

$$G(u|x_t) = \rho\Phi\left(\frac{u}{y_t\sigma_1 \exp\left({x_t}'\Gamma\right)}\right) + (1-\rho)\Phi\left(\frac{u}{y_t\sigma_2 \exp\left({x_t}'\Gamma\right)}\right), \quad (46)$$

where Φ denotes the standard normal cumulative distribution function: $\sigma_1 \neq \sigma_2$, $0 < \rho < 1$, and x_t denotes the exogenous state variables. Since the standard deviation is proportional to the common value, the private values are normalized as percentages of the common value.

Table 7 reports the conditional maximum likelihood estimates of parameters of the arrival rates and traders' private values and c_e, with standard errors reported in parentheses. The likelihood function is relatively flat with respect to the order submission cost, for positive order submission costs. We therefore did not estimate the order submission cost, c_o, but set it equal to 0.1% of the common value.

The first row of the top panel reports estimates of the Weibull shape parameter, η. The parameter estimate is less than one for all stocks: the longer the time since the last order submission, the lower the conditional probability that a new trader will arrive. The second through seventh rows of the top panel report estimates of the parameters on the exogenous state variables. The exogenous state variables are standardized by dividing by their sample standard deviations.

The estimated parameters on many of the exogenous state variables are positive. The arrival rate of traders increases following periods of high TSX market volatility. The parameters on stock volatility are positive and larger in magnitude than the parameters on the other exogenous state variables. The relative magnitudes of the parameters suggest that stock specific shocks are more important than market wide shocks for the trader arrival rates. Higher stock volatility predicts an increase in the trader arrival rates.

The second panel reports parameter estimates for the distributions of the private values. For all stocks, the private values distribution is a mixture of two normal distributions, with approximately 85% weight on a distribution with a standard deviation of approximately 5% and a 15% weight on a distribution with a standard deviation of approximately 54%. The point estimates of the order execution costs, c_e, are reported in the third panel, and are between 1% and 2% of the common value.

The estimated parameters on many of the exogenous state variables are statistically significant. A change in stock volatility has the largest effect on the distributions of the private values. The parameters on stock volatility are all negative — when stock volatility is high, a higher fraction of the traders that arrive have private values close to zero.

Table 8 reports the expected utilities for traders with six different private values across three different market conditions: a low liquidity state with a wide spread and low depth, a high liquidity state with a narrow spread and high depth, and a moving market state where the common value is above the mid-quote. For each private value, we report the expected utility from submitting a market order, a one tick limit order, a marginal limit order, or no order at all. The private values are 1.25%, 2.5%, or 5% higher or lower than the common value; the corresponding private values measured in cents are reported in the second row. The reported expected utility is a lower bound on the trader's expected utility since we do not compute the expected utility for limit orders between one tick from the quotes and marginal limit orders. The maximum utility is indicated for each private value and market condition with a box.

In the low liquidity state, a trader with a private value equal to 2.5% or -2.5% optimally submits a marginal limit order, and a trader with private value equal to 5% or -5% optimally submits a one tick limit order. In the high liquidity state, a trader with a private value equal to 1.25% or -1.25% optimally submits no order in BHO and ERR, and optimally submits a limit order in WEM. A trader with a private value equal to 2.5% or -2.5% optimally submits limit orders for BHO and ERR and submits market orders for WEM. A trader with a private value equal to 5% or -5% optimally demands liquidity by submitting a market order for all stocks.

In the moving market state, the optimal order strategies are asymmetric. With a high common value relative to the mid-quote, the optimal strategy is to submit market orders in ERR and WEM for traders with all three positive private values. Such market order submissions pick off some sell limit orders. For BHO, traders with private values equal to 1.25% and 2.5% submit one tick limit orders rather than market orders.

The expected utility calculations reported in Table 8 show that traders' optimal order submission strategy is state dependent: for a given private value, the optimal order submission changes with the state vector.

Table 9 reports summary statistics for the estimated private values distributions and the optimal order submission strategies for five intervals for the private value. The first row in each panel report the mean proportion of traders in each private value interval. The next rows report the mean and standard deviations of the fitted order submission probabilities for a sell market order, a sell limit order, no order, a buy limit order, and a buy market order.

Traders with extreme private values typically choose market or limit orders and rarely choose not to submit an order. Traders with intermediate private values submit limit orders most frequently but also use market orders and sometimes choose not to submit orders. Traders with private values close to zero almost always submit limit orders when they choose to submit an order, but often choose not to submit an order.

The standard deviations of the order submission probabilities reported in Table 9 indicate that for all five private value intervals the state dependence in the optimal order submission strategy is economically significant—traders' optimal order submissions change quite frequently as the traders' information set and the limit order book changes.

3.5 Estimates of the Gains from Trade

Substituting the mixture of normal distributions assumption for the private values distribution in equation (46) into the maximum gains from trade in equation (33), the maximum gains from trade as a percentage of the common value is

$$\text{Maximum gains }(x_{t_i}) = \rho\left(2\sigma_1 \exp\left(x_{t_i}'\Gamma\right)\phi\left(\frac{c_e+c_o}{\sigma_1\exp\left(x_{t_i}'\Gamma\right)}\right) - (c_e+c_o)2\Phi\left(\frac{-c_e-c_o}{\exp\sigma_1\left(x_{t_i}'\Gamma\right)}\right)\right)$$
$$+(1-\rho)\left(2\sigma_2 \exp\left(x_{t_i}'\Gamma\right)\phi\left(\frac{c_e+c_o}{\sigma_2\exp\left(x_{t_i}'\Gamma\right)}\right) - (c_e+c_o)2\Phi\left(\frac{-c_e-c_o}{\sigma_2\exp\left(x_{t_i}'\Gamma\right)}\right)\right). \quad (47)$$

Here ϕ is the standard normal density function, and Φ the standard normal cumulative distribution function. The derivation of equation (47) is reported in Appendix D, along with a description of how we compute the current gains and the losses from no execution, no submission, wrong direction, and crowding out

The current gains from trade in the limit order market depend on the threshold valuations and the execution probabilities for limit orders. We estimate the conditional execution probabilities for the marginal and the one tick buy and sell limit orders. If the execution probabilities for any buy limit order in a given state are at least as high as the execution probability for the marginal buy limit order, then using the execution probability for the marginal buy limit order for all buy limit orders in that state provides a lower bound on the current gains from trade on the buy side. An analogous argument applies to the sell side. Similarly, using the execution probabilities for one tick limit orders for all limit order execution probabilities provides an upper bound on the gains from trade. We report the lower and upper bounds for the current gains from trade. In addition, we report the current gains from trade computed using the average of the one tick and marginal buy or sell execution probabilities for all buy or sell limit orders in a given state. We refer to the latter as the average current gains from trade.

We take expectations across states x_{t_i} and report the unconditional expected current and maximum gains from trade. For brevity we refer to the expected gains simply as the current gains from trade and the maximum gains from trade. Our model specification also allows us to compute conditional gains from trade, for example, conditional on the level of lagged common value volatility. To save space we only report the unconditional gains from trade estimates here.

We observe an order submission when a trader arrives and submits an order. But some traders who arrive may find it optimal to submit no order. Simply computing the expected gains and losses by averaging the estimated gains over the observed order submissions in our sample therefore would lead to biased estimates because of trader self-selection. In order to deal with the selection bias, our estimates of the gains from trade are obtained by computing a weighted average across states, where the weights are inversely proportional to the probability that a trader who arrives at state z_{t_i} submits an order:

$$\begin{aligned}&\text{Prob}\left(\text{Trader who arrives at time } t \text{ submits an order} \mid z_t\right)\\&\qquad = G\left(\theta^{sell}_{marginal}(z_t) - y_t \middle| x_t\right) + 1 - G\left(\theta^{buy}_{marginal}(z_t) - y_t \middle| x_t\right). \quad (48)\end{aligned}$$

The top panel of Table 10 reports estimates of the maximum and the current gains from trade. The gains are reported as a percentage of the common value. The first row reports the maximum gains from trade. In the next three rows we report estimates of the lower and upper bounds for the current gains from trade and the average current gains from trade. In rows five through seven we report the difference between the maximum and the current gains from trade and in the last three rows of the top panel we report the current gains from trade as a percentage of the maximum gains from trade. Details of the computations are provided in Appendix D.

Across the three stocks, the average of the maximum gains is approximately 8.2% of the common values and the average of the lower bound on the current gains is around 7.4% of the common values. The maximum benefit of increasing the allocative efficiency of the current market design—the limit order market—is less than 1% of the stocks' common values. The current gains from trade are approximately 90% of the maximum gains from trade. We interpret the magnitudes as evidence that in our sample, the limit order market allows the traders to trade to a relatively efficient stock allocation.

From Table 1, approximately 60% of the order submissions in our sample are limit orders. The average fitted execution probability for limit orders is approximately 40% across the three stocks, implying that approximately 36% of the order submissions result in no execution. If we ignored the valuations of the traders, the order submissions and fitted execution probabilities imply that the current gains from trade is approximately 64% of the maximum gains from trade. But we estimate the current gains from trade to be approximately 90% of the maximum gains from trade. We estimate higher current gains from trade than 64% because the traders who contribute the most to the gains from trade endogenously submit market orders and thereby avoid the risk of no execution altogether whereas traders who contribute less to the gains from trade endogenously bear most of the risk of no execution. For example, from Table 9, approximately 80% of the traders with valuations more than 5% away from the common value submit market orders whereas approximately 65% of the traders with valuation between 2.5% and 5% away from the common value submit limit orders.

In our model, losses do not only arise from unexecuted order submissions. Losses also arise from no submission, wrong direction, and crowding out. The bottom panel of the table reports the decomposition of the average losses as percentages of the total losses. No execution is, however, the most important sources of losses accounting for between 70% and 75% of the losses.

No submission accounts for between 1% and 4% of the losses. Using the order submission cost of 0.1% of the common value and the order execution cost of approximately 2%, all traders with privates values farther than 2.1% from the common value should submit orders that execute. From Table 9, less than 3% of the traders with valuations greater than 2.5% away from the common value do not submit an order.

Wrong direction accounts for no more than 1.1% of the losses. The result suggests that situations with extremely stale limit orders in which it optimal for traders with high private values to sell and traders with low private values to buy are relatively rare.

Crowding out accounts for between 22% and 29% of the losses. Using the order submission cost of 0.1% of the common value and the trading cost of approximately 2%, all traders with private values less than 2.1% away from the common value should not submit orders. From Table 9, approximately 64% of the traders with valuations less than 2.5% away from the common value submit an order losses from crowding out are smaller for two reasons. First, the cut-off of 2.5% includes some traders who should trade. Second, all traders who do cause crowding out losses have valuations that are less extreme than the valuations of traders who should trade and therefore the losses per trader are smaller than what the benchmark implies.

4 Conclusions

In a perfectly liquid market in which the traders can realize the maximum gains from trade, the market design provides incentives to traders with high private values to buy the stock, traders with low private values to sell the stock, and traders with intermediate private values — traders with no strong need to trade — to abstain from trading. We compute the expected current and maximum gains from trade using a sample of order submissions and execution and cancellation histories from a limit order market. The current gains are approximately 90% of the maximum gains from trade — the limit order market allows the traders to realize many of the gains from trade.

Close to three-quarters of the difference between the current and maximum gains from trade arise from non execution. No execution arises because the limit order market provides incentives for traders to submit orders with high execution risk. Almost one-quarter of the difference between the current and maximum gains comes from crowding out. Crowding out occurs when traders with no strong need to trade find profitable trading opportunities.

A key feature of our model is that we base our computations of the gains from trade on estimates of the traders' optimal order submission strategy in the limit order market. Our estimates condition on the endogeneity of traders' order submissions. Such endogeneity is consistent with empirical evidence that the composition of the order flow changes as the limit order book and market conditions change. Traders with more extreme private values tend to submit orders with low execution risk and low picking-off risk and traders with more moderate private values tend to submit limit orders with higher execution risk and higher picking-off risk. Our estimates are also consistent with the traders' ability to switch between market orders and limit orders when the relative payoffs of market and limit orders change. Both effects are important empirically.

Since our empirical approach builds directly on the theoretical model, our parameter estimates can be used in numerical solutions for the equilibrium and to study the efficiency under alternative trading rules. Goettler, Parlour, and Rajan (2003) use numerical methods to solve the equilibrium to a model similar to ours. They determine the impact of a change in the tick size for a model using a normal distribution for the private values distribution with a standard deviation of $\$\frac{1}{4}$, and zero costs. They report the current gains as \$0.1693 in the $\$\frac{1}{8}$ tick regime and \$0.1728 in the $\$\frac{1}{16}$ tick regime. Plugging into equation (47), the maximum gains from trade are \$0.1995; the current gains are 84.9% of the maximum gains in the $\$\frac{1}{8}$ tick regime and 86.6% in the $\$\frac{1}{16}$ tick regime.

Our model is based on a one-shot order submission problem with no endogenous order cancellations and order resubmissions. Much work on the efficiency of double-auctions argues that double auctions are relatively efficient because the traders can continue to resubmit bids until many of the gains from trade are exhausted. It would be interesting to extend our model and empirical approach to allow for the possibility of endogenous cancellations and resubmissions. Allowing for endogenous cancellations and order resubmissions may increase our estimates of the efficiency of the limit order market.

Our model also takes the order submission quantity as exogenous. But a trader is likely to choose how many shares to buy or sell depending on his marginal valuations for additional shares. Studying the robustness of our empirical findings to quantity-dependent valuations and endogenous quantity choice is a useful direction for future research.

Some limit order markets have added designated market makers who have an obligation to submit limit orders in less actively traded stocks. The designated market makers typically pay lower order submission fees than other traders, but the market makers face the same rules as all other traders submitting orders. In order to evaluate whether adding the designated market makers improve the efficiency of the limit order market, we need to be able to measure the gains from trade with and without designated market makers. Methods similar to ours could be used to measure the gains from trade with and without designated market makers. We leave such computations for future work.

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A Proof of Lemma 1

The objective function, equation (29), is increasing in the private value u for the traders who buy the stock in the optimal allocation — if a trader with private value u buys the stock in the optimal allocation, then a trader with valuation $u' > u$ also buys the stock in the optimal allocation. A similar result holds for traders who sell the stock in the optimal allocation. Therefore, there exists two numbers, $L(x_t)$ and $H(x_t)$, such that the optimal allocation is:

$$I^{sell*}(u_t; x_t) = \begin{cases} 1, & u_t \leq L(x_t), \\ 0, & \text{else} \end{cases}, \; I^{buy*}(u_t; x_t) = \begin{cases} 1, & u_t \geq H(x_t), \\ 0, & \text{else} \end{cases}. \tag{A1}$$

Let $P(x_t)$ be a solution to

$$G(P(x_t) - c_e - c_o | x_t) = 1 - G(P(x_t) + c_e + c_o | x_t). \tag{A2}$$

As $P(x_t)$ goes to $-\infty$, the left-hand side goes to zero and the right-hand side goes to one, and as $P(x_t)$ goes to ∞, the left-hand side goes to one and the right-hand side goes to zero. Both the left-hand side and the right-hand side are monotonic functions of $P(x_t)$. By the continuity of the distribution, both the left-hand side and the right-hand side are continuous functions, and so a solution always exists to equation (A2). If the distribution is symmetric, $P(x_t)$ is the median of the distribution.

We now show that the cut-offs

$$L(x_t) = P(x_t) - c_e - c_o, \;\; H(x_t) = P(x_t) + c_e + c_o \tag{A3}$$

characterize the optimal allocation. By construction, the market-clearing condition is satisfied at the allocation.

Suppose that $\hat{L} > P(x_t) - c_e - c_o$ and $\hat{H}$ characterize an alternative allocation. The market clearing condition for the alternative allocation is

$$G(\hat{L} | x_t) = 1 - G(\hat{H} | x_t). \tag{A4}$$

Equations (A2) and (A4) imply $\hat{H} < P(x_t) + c_e + c_o$ and

$$\int_{P(x_t) - c_e - c_o}^{\hat{L}} g(u) du - \int_{\hat{H}}^{P(x_t) + c_e + c_o} g(u) du = 0. \tag{A5}$$

Let $I(u_t \leq L)$ be an indicator function for $u_t \leq L$ with $I(u_t \geq H)$ defined similarly.

The difference in the expected gains from trade between the two allocations is

$$
\begin{aligned}
&E\left[I(u_t \le \hat{L})\,(-u_t - c_e - c_o) + I(u_t \ge \hat{H})\,(u_t - c_e - c_o)\middle|\, x_t\right] \\
&\quad - E\left[I(u_t \le P(x_t) - c_e - c_o)\,(-u_t - c_e - c_o) + I(u_t \ge P(x_t) + c_e + c_o)\,(u_t - c_e - c_o)\right|\, x_t] \\
&\qquad = \int_{P(x_t)-c_e-c_o}^{\hat{L}} (-u - c_e - c_o) g(u) du + \int_{\hat{H}}^{P(x_t)+c_e+c_o} (u - c_e - c_o) g(u) du \\
&< \int_{P(x_t)-c_e-c_o}^{\hat{L}} (-(P(x_t) - c_e - c_o) - c_e - c_o) g(u) du + \int_{\hat{H}}^{P(x_t)+c_e+c_o} ((P(x_t) + c_e + c_o) - c_e - c_o) g(u) du \\
&\qquad = P(x)\left(-\int_{P(x_t)-c_e-c_o}^{\hat{L}} g(u) du + \int_{\hat{H}}^{P(x_t)+c_e+c_o} g(u) du\right) = 0, \quad \text{(A6)}
\end{aligned}
$$

where the inequality follows from the monotonicity in u of both integrals.

Similar logic holds for alternative allocations with $\hat{L} < P(x_t) - c_e - c_o$. ■

B Computing execution probabilities

Suppose a buy limit order at price $p_{t,b}^{buy}$ is submitted at t. Assume that the latent cancellation and execution times have the distributions given by equations (38) and (40) in the text. The execution probability is

$$
\begin{aligned}
\psi_b^{buy}(z_t) &= E\left[I_t\left(\tau_{execute} \le \tau_{cancel}\right)\middle| z_t, d_{t,b}^{buy} = 1\right] \\
&= \int_0^T \left(1 - F_{cancel}\left(\tau \middle| z_t, d_{t,b}^{buy} = 1\right)\right) dF_{execute}\left(\tau \middle| z_t, d_{t,b}^{buy} = 1\right) \\
&= \int_0^T \exp\left(-\exp(z_t'\gamma_b^{buy})\tau^{\alpha_b^{buy}}\right) \exp(z_t'\kappa_b^{buy})\beta_b^{buy}\tau^{\beta_b^{buy}-1} \exp\left(-\exp(z_t'\kappa_b^{buy})\tau^{\beta_b^{buy}-1}\right) d\tau. \quad \text{(B1)}
\end{aligned}
$$

The second line follows from the independence of the latent cancellation and execution times, and the assumption that the latent cancellation time is bounded by $t + T$ with probability one. We compute equation (B1) numerically with T equal to two trading days, or 48,600 seconds.

C The conditional likelihood function

Let t_i denote the time of the i^{th} order submission and I the total number of order submissions. Conditioning on the common value, order quantity, x_t and z_t, the conditional log-likelihood is

$$\begin{aligned}\sum_{i=1}^{I}\Bigg\{ & d_{t_i,0}^{sell}\ln\left(G\left(\theta_{0,1}^{sell}(z_{t_i})-y_{t_i}\middle| x_{t_i}\right)\lambda(x_{t_i};t)\right)\\ &+\left(\sum_{s=1}^{S(z_t)} d_{t_i,s}^{sell}\right)\ln\left(\left[G\left(\theta_{marginal}^{sell}(z_{t_i})-y_{t_i}| x_{t_i}\right)-G\left(\theta_{0,1}^{sell}(z_{t_i})-y_{t_i}\middle| x_{t_i}\right)\right]\lambda(x_{t_i};t)\right)\\ &\qquad + d_{t_i,0}^{buy}\ln\left(1-G\left(\theta_{0,1}^{buy}(z_{t_i})-y_{t_i}\middle| x_{t_i}\right)\lambda(x_{t_i};t)\right)\\ &+\left(\sum_{b=1}^{B(z_t)} d_{t_i,b}^{buy}\right)\ln\left(\left[G\left(\theta_{0,1}^{buy}(z_{t_i})-y_{t_i}\middle| x_{t_i}\right)-G\left(\theta_{marginal}^{buy}(z_{t_i})-y_{t_i}\middle| x_{t_i}\right)\right]\lambda(x_{t_i};t)\right)\\ &-\int_{t_{i-1}}^{t_i}\left[1-G\left(\theta_{marginal}^{buy}(z_t)-y_t\middle| x_{t_i}\right)+G\left(\theta_{marginal}^{sell}(z_t)-y_t\middle| x_{t_i}\right)\right]\lambda(x_{t_i};t)dt\Bigg\}. \end{aligned} \quad \text{(C1)}$$

The first line is the contribution from the instantaneous probability of a sell market order at time t_i; the second line is the contribution from the instantaneous probability of a sell limit order; the third line is the contribution from the instantaneous probability of a buy market order; the fourth line is the contribution from the instantaneous probability of a buy limit order; and the final line is the integrated hazard rate.

In our estimation, we assume that the common value, y_t, and state vector, z_t, only change when an order is submitted. Using the assumption, the integrated hazard rate is

$$\begin{aligned}&\int_{t_{i-1}}^{t_i}\left[1-G\left(\theta_{marginal}^{buy}(z_t)-y_t\middle| x_{t_i}\right)+G\left(\theta_{marginal}^{sell}(z_t)-y_t\middle| x_{t_i}\right)\right]\lambda(x_{t_i};t)dt\\ &\quad=\left[1-G\left(\theta_{marginal}^{buy}(z_{t_i})-y_{t_i}\middle| x_{t_i}\right)+G\left(\theta_{marginal}^{sell}(z_{t_i})-y_{t_i}\middle| x_{t_i}\right)\right]\int_{t_{i-1}}^{t_i}\lambda(x_{t_i};t)dt. \end{aligned}\quad \text{(C2)}$$

D Formulas for the Gains to Trade

Suppose that the random variable e is distributed as a mixture of normals, with weight ρ_1 on a normal distribution with mean zero and standard deviation σ_1 and weight $1-\rho_1$ on a normal distribution with mean zero and standard deviation σ_2. Let $a \le b$ be two constants, and $I(a \le e \le b)$ be an indicator function. Then,

$$E\left[I(a\le e\le b)e\right] = \rho_1\left(\sigma_1\phi\left(\frac{b}{\sigma_1}\right)-\sigma_1\phi\left(\frac{a}{\sigma_1}\right)\right)+(1-\rho_1)\left(\sigma_2\phi\left(\frac{b}{\sigma_2}\right)-\sigma_2\phi\left(\frac{a}{\sigma_2}\right)\right), \quad \text{(D1)}$$

and

$$E\left[I(a \leq e \leq b)\right] = \rho_1 \left(\Phi\left(\frac{b}{\sigma_1}\right) - \Phi\left(\frac{a}{\sigma_1}\right)\right) + (1 - \rho_1)\left(\Phi\left(\frac{b}{\sigma_2}\right) - \Phi\left(\frac{a}{\sigma_2}\right)\right), \quad \text{(D2)}$$

with ϕ the standard normal density function with Φ the standard normal cumulative distribution function.

The private value is distributed as a mixture of normals, with standard deviations $\sigma_1(x_t) = y_t\sigma_1 \exp(x_t'\Gamma)$ and $\sigma_2(x_t) = y_t\sigma_2 \exp(x_t'\Gamma)$.

The formula for the maximum gains from trade in equation (47) in the text results applying equations (D1) and (D2) to equations (32) and (33), using the parameterization that the order execution costs and the order submission costs are both proportional to the common value: order execution cost $= y_t c_e$ and order submission cost $= y_t c_o$.

In order to compute the current gains from trade and the losses from no execution, no submission, and crowding out we need to compute expectations over different intervals defined by combinations of threshold valuations and the cut-offs that define the optimal allocation.

Substituting the optimal order submission strategy for the sell side in equations (22)-(25) and the corresponding equations for the buy side into equation (27), the current gains are

$$\begin{aligned}
\text{Current gains}\,(z_t) = {} & E\left[I\left(\infty \leq u_t - y_t \leq \theta_{0,1}^{sell}(z_t)\right)(-u_t - c_e - c_o)\middle|\, z_t\right] \\
& + \sum_{i=1}^{S(z_t)-1} E\left[I\left(\theta_{i-1,i}^{sell}(z_t) \leq u_t - y_t \leq \theta_{i,i+1}^{sell}(z_t)\right)\left(\psi_i^{sell}(z_t)(-u_t - c_e) - c_o\right)\middle|\, z_t\right] \\
& + E\left[I\left(\theta_{S(z_t)-1,S(z_t)}^{sell}(z_t) \leq u_t - y_t \leq \theta_{marginal}^{sell}(z_t)\right)\left(\psi_{S(z_t)}^{sell}(z_t)(-u_t - c_e) - c_o\right)\middle|\, z_t\right] \\
& + E\left[I\left(\theta_{0,1}^{buy}(z_t) \leq u_t - y_t \leq \infty\right)(u_t - c_e - c_o)\middle|\, z_t\right] \\
& + \sum_{i=1}^{B(z_t)-1} E\left[I\left(\theta_{i,i+1}^{buy}(z_t) \leq u_t - y_t \leq \theta_{i-1,i}^{buy}(z_t)\right)\left(\psi_i^{buy}(z_t)(u_t - c_e) - c_o\right)\middle|\, z_t\right] \\
& + E\left[I\left(\theta_{B(z_t)-1,B(z_t)}^{buy}(z_t) \leq u_t - y_t \leq \theta_{marginal}^{buy}\right)\left(\psi_{B(z_t)}^{buy}(z_t)(u_t - c_e) - c_o\right)\middle|\, z_t\right]. \quad \text{(D3)}
\end{aligned}$$

The lower bound for the current gains from trade is obtained by using the execution probabilities for the marginal sell limit orders for all the sell limit order execution probabilities and the execution probabilities for the marginal buy limit order for all buy limit order execution probabilities:

$$
\begin{aligned}
\text{Current gains}\,(z_t) \geq\; & E\left[I\left(\infty \leq u_t - y_t \leq \theta^{sell}_{0,1}(z_t)\right)(-u_t - c_e - c_o)\Big|\, z_t\right] \\
& + E\left[I\left(\theta^{sell}_{0,1}(z_t) \leq u_t - y_t \leq \theta^{sell}_{marginal}(z_t)\right)\left(\psi^{sell}_{S(z_t)}(z_t)\,(-u_t - c_e) - c_o\right)\Big|\, z_t\right] \\
& + E\left[I\left(\theta^{buy}_{0,1}(z_t) \leq u_t - y_t \leq \infty\right)(u_t - c_e - c_o)\Big|\, z_t\right] \\
& + E\left[I\left(\theta^{buy}_{marginal} \leq u_t - y_t \leq \theta^{buy}_{0,1}(z_t)\right)\left(\psi^{buy}_{B(z_t)}(z_t)\,(u_t - c_e) - c_o\right)\Big|\, z_t\right]. \qquad \text{(D4)}
\end{aligned}
$$

The upper bound is obtained by using the execution probabilities for one tick sell and buy limit orders for all sell and but execution probabilities in equation (D3). The average gains from trade are obtained by using the average execution probabilities for the one tick buy and marginal buy limit orders for all buy limit order execution probabilities into equation (D3). The sell limit order executions are defined similarly.

Using equations (D1) and (D2), we can compute a closed form for each term in equation (D4). For example,

$$
\begin{aligned}
& E\left[I\left(\theta^{sell}_{0,1}(z_t) \leq u_t - y_t \leq \theta^{sell}_{marginal}(z_t)\right)\left(\psi^{sell}_{S(z_t)}(z_t)\,(-u_t - c_e) - c_o\right)\Big|\, z_t\right] \\
&= y_t \psi^{sell}_{S(z_t)}(z_t)\Bigg\{ -\rho\left(\sigma_1 \exp(x_t'\Gamma)\phi\left(\frac{\theta^{sell}_{marginal}(z_t)}{\sigma_1 \exp(x_t'\Gamma)}\right) - \sigma_1 \exp(x_t'\Gamma)\phi\left(\frac{\theta^{sell}_{0,1}(z_t)}{\sigma_1 \exp(x_t'\Gamma)}\right)\right) \\
&\quad - (1-\rho)\left(\sigma_2 \exp(x_t'\Gamma)\phi\left(\frac{\theta^{sell}_{marginal}(z_t)}{\sigma_2 \exp(x_t'\Gamma)}\right) - \sigma_2 \exp(x_t'\Gamma)\phi\left(\frac{\theta^{sell}_{0,1}(z_t)}{\sigma_2 \exp(x_t'\Gamma)}\right)\right) \\
&\quad - c_e\left(\rho\left(\Phi\left(\frac{\theta^{sell}_{marginal}(z_t)}{\sigma_1 \exp(x_t'\Gamma)}\right) - \Phi\left(\frac{\theta^{sell}_{0,1}(z_t)}{\sigma_1 \exp(x_t'\Gamma)}\right)\right)\right. \\
&\quad \left. + (1-\rho)\left(\Phi\left(\frac{\theta^{sell}_{marginal}(z_t)}{\sigma_2 \exp(x_t'\Gamma)}\right) - \Phi\left(\frac{\theta^{sell}_{0,1}(z_t)}{\sigma_2 \exp(x_t'\Gamma)}\right)\right)\right)\Bigg\} \\
&\quad - y_t c_o\Bigg\{\rho\left(\Phi\left(\frac{\theta^{sell}_{marginal}(z_t)}{\sigma_1 \exp(x_t'\Gamma)}\right) - \Phi\left(\frac{\theta^{sell}_{0,1}(z_t)}{\sigma_1 \exp(x_t'\Gamma)}\right)\right) \\
&\quad + (1-\rho)\left(\Phi\left(\frac{\theta^{sell}_{marginal}(z_t)}{\sigma_2 \exp(x_t'\Gamma)}\right) - \Phi\left(\frac{\theta^{sell}_{0,1}(z_t)}{\sigma_2 \exp(x_t'\Gamma)}\right)\right)\Bigg\} \qquad \text{(D5)}
\end{aligned}
$$

with the other terms and the upper bound computed similarly.

The losses from no execution, no submission, wrong direction, and crowding out are computed similarly.

Table 1: **Order Submissions**

Stock Ticker	Barkhor Resources BHO	Eurus Resources ERR	War Eagle Mining Co. WEM
Number of order submissions	55, 444	56, 599	47, 578
Percent of submissions that are:			
Sell limit orders	31.7	31.5	31.3
Sell market orders	21.6	23.7	19.9
Buy limit orders	28.5	27.9	32.0
Buy market orders	18.2	16.9	16.8
Time between order submissions in seconds			
Average	247.0	297.9	396.6
Standard deviation	842.4	983.7	1089.4

The sample period is May 1, 1990, to November 30, 1993.

Table 2: **The State Vector,** z_t

Name	Description	BHO Mean	BHO Std.Dev.	ERR Mean	ERR Std.Dev.	WEM Mean	WEM Std.Dev.
	Exogenous state variables: x_t						
TSX market volatility	Absolute value of the lagged close-to-close return on the TSX market index	1.23	1.00	1.20	1.00	1.16	1.00
TSX mining volatility	Absolute value of the lagged close-to-close return on the TSX mining index	1.16	1.00	1.16	1.00	1.14	1.00
Interest rate volatility	Absolute value of the lagged change in the overnight interest rate	0.87	1.00	1.00	1.00	0.99	1.00
Exchange rate volatility	Absolute value of the lagged change in the Canadian/US exchange rate	1.19	1.00	1.14	1.00	1.06	1.00
Stock volatility	Absolute value of the lagged open-to-open stock return	0.77	1.00	0.73	1.00	0.93	1.00
	Endogenous state variables: w_t						
Spread	Bid-ask spread	0.02	0.02	0.05	0.06	0.04	0.03
Close ask depth	Logarithm of ask depth at best ask	1.74	0.95	0.94	0.79	1.02	0.75
Far ask depth	Logarithm of cumulative ask depth up to the marginal sell order	4.04	0.83	2.60	0.81	2.83	0.69
Close bid depth	Logarithm of bid depth at best bid	1.75	1.00	1.04	0.85	1.08	0.83
Far bid depth	Logarithm of cumulative bid depth down to the marginal buy order	4.06	0.78	2.82	0.85	3.14	0.77
Order quantity	Logarithm of the number of shares in the order	1.49	0.83	0.72	0.90	0.84	0.83
Recent trades	Number of trades in the last 10 minutes	8.61	10.88	5.06	6.75	4.02	5.68
Lagged duration	Sum of last 10 durations of order book changes, divided by 1,000	3.10	5.30	3.16	4.96	3.93	5.20
Mid-quote volatility	Volatility of the mid-quote over the last ten minutes	1.71	0.07	1.71	0.09	1.70	0.05
Distance to mid-quote	Percentage deviation between mid-quote and moving average mid-quote	0.00	0.02	0.00	0.04	0.00	0.02
	Time dummies						
First hour	dummy variable for 6:30-7:30						
Second hour	dummy variable for 7:30-8:30						
Third hour	dummy variable for 8:30-9:30						
Fourth hour	dummy variable for 9:30-10:30						
Fifth hour	dummy variable for 10:30-11:30						
Sixth hour	dummy variable for 11:30-12:30						

The table describes the construction of the state vector, $z_t = (x_t, w_t)$ with x_t the exogenous state variables and w_t the endogenous state variables. We also report the means and standard deviations of all the variables.

Table 3: **Order Executions and Cancellations**

	BHO	ERR	WEM
Conditional order execution percentage			
At least 50% executes	97.34	96.97	96.58
	(0.07)	(0.07)	(0.09)
Less than 50% executes	1.40	1.43	1.21
	(0.04)	(0.04)	(0.04)
Distribution of time to execution			
Percent of orders executed:			
Less than 5 minutes after order submission	46.67	42.77	37.29
5 to 15 minutes after order submission	15.92	17.97	17.24
15 minutes to 1 hour after order submission	19.59	21.92	22.60
1 to 3 hours after order submission	10.73	11.46	14.07
More than 3 hours after order submission	7.09	5.88	8.80
Distribution of time to cancellation			
Percent of orders cancelled:			
Less than 5 minutes after order submission	31.79	27.57	23.49
5 to 15 minutes after order submission	13.08	13.81	11.78
15 minutes to 1 hour after order submission	16.42	18.17	16.64
1 to 3 hours after order submission	14.30	15.45	17.70
More than 3 hours after order submission	24.41	25.00	30.39
Percent of orders cancelled:			
Less than 5 minutes before market close	18.77	21.16	26.32
5 to 15 minutes before market close	2.84	2.51	2.80
15 minutes to 1 hour before market close	9.15	8.68	9.46
1 to 3 hours before market close	20.80	20.31	20.35
More than 3 hours before market close	40.78	41.12	33.05
After the first day market close	7.66	6.22	8.02

The top panel reports for all limit orders the average percentage of the submitted order quantity that is executed within 48 hours of order submission, conditional on that percentage being at least 50% or less than 50%. Standard errors are reported in parentheses. The second panel reports, for all limit orders that eventually execute, the percent that execute within five different time intervals. For orders that have several partial executions, the time of execution is weighted by the quantity executed to determine an average execution time. The third panel reports, for all orders that are eventually cancelled, the percent that cancel within five time intervals relative to the time of the order submission, the percent that cancel within five time intervals before the market close on the day of order submission, and the percent that cancel the next day or later.

Table 4: **Weibull Model for Cancellation Times**

	BHO				ERR				WEM			
	Sell orders		Buy orders		Sell orders		Buy orders		Sell orders		Buy orders	
Variable	Marg.	1 Tick	Marg.	1 Tick	Marg.	1 Tick	Marg.	1 Tick	Marg.	1 Tick	Marg.	1 Tick
Shape	0.63	0.51	0.62	0.52	0.62	0.50	0.59	0.54	0.62	0.49	0.65	0.47
	(0.02)	(0.02)	(0.02)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)	(0.01)	(0.01)
Constant	−5.08	−1.72	−6.03	−6.93	−6.64	−5.60	−4.23	−2.76	−4.51	1.11	−3.55	−9.61
	(0.89)	(0.85)	(0.63)	(0.73)	(0.98)	(1.21)	(0.66)	(0.81)	(0.91)	(1.76)	(1.09)	(1.96)
TSX market volatility	0.01	0.02	−0.01	−0.00	0.05	0.00	0.04	0.05	−0.01	0.01	0.01	0.03
	(0.03)	(0.03)	(0.03)	(0.03)	(0.02)	(0.03)	(0.03)	(0.04)	(0.03)	(0.04)	(0.03)	(0.04)
TSX mining volatility	0.06	0.03	0.01	0.00	−0.00	0.01	−0.05	−0.12	0.01	0.07	0.00	−0.01
	(0.03)	(0.02)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.04)	(0.03)	(0.04)	(0.03)	(0.04)
Interest rate volatility	0.00	0.01	0.02	−0.03	0.05	0.05	−0.00	0.05	0.03	−0.00	−0.03	0.00
	(0.03)	(0.02)	(0.03)	(0.02)	(0.02)	(0.03)	(0.03)	(0.05)	(0.03)	(0.04)	(0.03)	(0.04)
Exchange rate volatility	−0.02	0.02	−0.06	−0.05	−0.03	−0.04	−0.02	−0.08	−0.07	−0.01	−0.06	−0.03
	(0.03)	(0.02)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.04)	(0.03)	(0.04)	(0.03)	(0.05)
Stock volatility	0.00	0.07	0.07	0.09	−0.00	−0.06	−0.03	0.01	0.01	0.00	0.04	0.07
	(0.03)	(0.02)	(0.03)	(0.03)	(0.02)	(0.04)	(0.03)	(0.03)	(0.03)	(0.04)	(0.03)	(0.04)
Spread	−0.54	−9.19	3.46	2.20	2.21	−5.59	2.71	0.21	3.54	3.64	2.11	−7.16
	(1.48)	(4.21)	(1.69)	(4.33)	(0.71)	(2.22)	(0.63)	(2.70)	(1.08)	(3.02)	(0.83)	(3.95)
Close ask depth	—	−0.01	0.01	0.05	—	0.00	−0.05	−0.05	—	−0.08	−0.02	0.04
	—	(0.03)	(0.04)	(0.03)	—	(0.04)	(0.03)	(0.04)	—	(0.05)	(0.04)	(0.05)
Far ask depth	−0.15	—	—	—	−0.06	—	—	—	−0.15	—	—	—
	(0.04)	—	—	—	(0.03)	—	—	—	(0.04)	—	—	—
Close bid depth	−0.01	0.00	—	−0.01	−0.01	0.02	—	−0.02	−0.05	0.08	—	−0.08
	(0.03)	(0.02)	—	(0.03)	(0.03)	(0.03)	—	(0.04)	(0.04)	(0.04)	—	(0.04)
Far bid depth	—	—	−0.08	—	—	—	−0.07	—	—	—	−0.09	—
	—	—	(0.04)	—	—	—	(0.03)	—	—	—	(0.04)	—
Order quantity	0.04	−0.05	0.03	−0.12	0.14	−0.03	0.01	−0.02	0.06	−0.06	−0.05	−0.10
	(0.04)	(0.03)	(0.04)	(0.03)	(0.03)	(0.03)	(0.03)	(0.04)	(0.04)	(0.05)	(0.03)	(0.04)
Recent trades	0.01	0.03	0.02	0.03	0.02	0.01	0.03	0.04	0.04	0.04	0.03	0.04
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Lagged duration	−0.11	−0.02	−0.08	−0.05	−0.06	−0.01	−0.06	−0.04	−0.05	−0.03	−0.05	−0.03
	(0.01)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Mid-quote volatility	0.10	−1.09	0.71	1.99	1.04	1.53	−0.08	−0.24	−0.17	−2.74	−0.95	3.84
	(0.49)	(0.50)	(0.36)	(0.43)	(0.56)	(0.71)	(0.38)	(0.46)	(0.52)	(1.03)	(0.64)	(1.15)
Distance to mid-quote	0.22	21.25	−2.83	−11.92	9.41	25.91	−9.10	−31.00	7.16	31.78	−5.01	−12.41
	(2.08)	(2.19)	(1.73)	(2.12)	(2.93)	(5.05)	(2.47)	(6.37)	(2.54)	(3.91)	(2.73)	(5.57)
First hour	−0.67	−1.12	−1.04	−1.01	−1.07	−0.82	−1.09	−0.72	−1.02	−0.96	−1.03	−0.62
	(0.11)	(0.09)	(0.13)	(0.09)	(0.10)	(0.11)	(0.10)	(0.13)	(0.11)	(0.14)	(0.11)	(0.16)
Second hour	−0.66	−0.96	−0.87	−1.11	−0.97	−1.03	−1.02	−0.81	−1.01	−0.80	−0.69	−1.07
	(0.12)	(0.08)	(0.13)	(0.09)	(0.10)	(0.11)	(0.10)	(0.13)	(0.11)	(0.13)	(0.10)	(0.15)
Third hour	−0.37	−0.90	−0.76	−1.00	−0.91	−0.97	−0.94	−1.09	−0.73	−0.87	−0.71	−0.68
	(0.12)	(0.08)	(0.14)	(0.09)	(0.10)	(0.12)	(0.10)	(0.14)	(0.11)	(0.13)	(0.11)	(0.12)
Fourth hour	−0.35	−0.86	−0.77	−0.77	−0.61	−0.82	−0.77	−1.02	−0.74	−0.73	−0.48	−0.55
	(0.12)	(0.09)	(0.14)	(0.08)	(0.10)	(0.11)	(0.11)	(0.13)	(0.11)	(0.13)	(0.11)	(0.12)
Fifth hour	−0.05	−0.64	−0.43	−0.89	−0.77	−0.63	−0.64	−0.77	−0.50	−0.56	−0.33	−0.85
	(0.12)	(0.08)	(0.14)	(0.08)	(0.10)	(0.11)	(0.11)	(0.13)	(0.12)	(0.13)	(0.12)	(0.13)
Sixth hour	0.13	−0.46	−0.25	−0.65	−0.38	−0.49	−0.37	−0.77	−0.33	−0.41	−0.12	−0.49
	(0.12)	(0.08)	(0.15)	(0.09)	(0.10)	(0.11)	(0.11)	(0.13)	(0.11)	(0.12)	(0.11)	(0.12)
χ^2_{19}	290.01	576.14	247.37	648.10	258.47	211.25	304.15	218.13	273.52	188.44	273.48	201.66
P-Value	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
# Obs.	1,748	4,498	1,353	4,105	2,458	2,368	2,138	1,732	1,679	1,793	1,792	1,689

Parameter estimates with asymptotic standard errors in parentheses for a Weibull model for the cancellation times. The χ^2-test is for the null that the state vector z_t does not affect the conditional distribution with p-values in parentheses.

Table 5: **Weibull Model for Execution Times**

	BHO				ERR				WEM			
	Sell orders		Buy orders		Sell orders		Buy orders		Sell orders		Buy orders	
Variable	Marg.	1 Tick	Marg.	1 Tick	Marg.	1 Tick	Marg.	1 Tick	Marg.	1 Tick	Marg.	1 Tick
Shape	0.66	0.58	0.70	0.61	0.65	0.54	0.82	0.64	0.64	0.58	0.79	0.59
	(0.03)	(0.01)	(0.04)	(0.01)	(0.02)	(0.01)	(0.03)	(0.02)	(0.03)	(0.01)	(0.04)	(0.01)
Constant	−15.69	−10.42	−4.31	2.04	−12.97	−10.35	−3.86	3.93	−5.93	−26.41	1.22	12.91
	(1.51)	(0.42)	(1.85)	(0.47)	(0.89)	(0.58)	(1.23)	(1.86)	(1.60)	(1.94)	(2.01)	(2.01)
TSX market volatility	0.07	−0.00	0.05	0.02	0.18	0.03	0.00	−0.03	−0.04	0.04	0.07	0.00
	(0.05)	(0.02)	(0.08)	(0.02)	(0.04)	(0.03)	(0.05)	(0.03)	(0.06)	(0.04)	(0.06)	(0.04)
TSX mining volatility	−0.03	−0.06	0.01	−0.04	−0.16	0.01	0.01	−0.02	−0.26	−0.04	0.10	0.05
	(0.07)	(0.02)	(0.08)	(0.02)	(0.05)	(0.03)	(0.05)	(0.03)	(0.07)	(0.04)	(0.06)	(0.04)
Interest rate volatility	−0.03	−0.02	0.06	0.02	−0.09	−0.08	0.08	−0.02	0.02	0.01	0.06	0.09
	(0.07)	(0.02)	(0.09)	(0.02)	(0.05)	(0.04)	(0.04)	(0.05)	(0.05)	(0.04)	(0.06)	(0.03)
Exchange rate volatility	0.10	0.02	−0.38	−0.04	−0.10	−0.07	−0.07	−0.01	−0.01	−0.05	−0.14	0.03
	(0.06)	(0.02)	(0.10)	(0.02)	(0.05)	(0.04)	(0.05)	(0.04)	(0.06)	(0.05)	(0.07)	(0.04)
Stock volatility	0.15	0.10	0.09	0.06	0.01	0.00	0.07	0.03	−0.05	0.10	0.15	0.10
	(0.06)	(0.02)	(0.08)	(0.02)	(0.04)	(0.04)	(0.03)	(0.03)	(0.06)	(0.03)	(0.06)	(0.04)
Spread	−1.53	56.84	16.29	68.73	3.46	22.51	3.65	25.82	3.98	19.35	3.22	35.60
	(2.69)	(2.36)	(2.78)	(2.67)	(1.29)	(1.57)	(1.20)	(1.81)	(2.13)	(1.43)	(1.68)	(2.53)
Close ask depth	—	−0.06	0.00	0.20	—	−0.28	0.11	0.07	—	−0.02	0.02	0.05
	—	(0.02)	(0.10)	(0.02)	—	(0.04)	(0.06)	(0.04)	—	(0.04)	(0.08)	(0.05)
Far ask depth	−0.28	—	—	—	−0.21	—	—	—	−0.43	—	—	—
	(0.08)	—	—	—	(0.06)	—	—	—	(0.08)	—	—	—
Close bid depth	−0.23	0.14	—	−0.08	0.09	0.20	—	−0.08	0.02	0.17	—	−0.20
	(0.07)	(0.02)	—	(0.02)	(0.05)	(0.03)	—	(0.04)	(0.07)	(0.04)	—	(0.04)
Far bid depth	—	—	0.05	—	—	—	−0.26	—	—	—	−0.49	—
	—	—	(0.10)	—	—	—	(0.06)	—	—	—	(0.07)	—
Order quantity	−0.19	−0.22	−0.50	−0.29	−0.12	−0.02	−0.25	−0.19	−0.20	−0.13	−0.30	−0.20
	(0.08)	(0.03)	(0.10)	(0.03)	(0.05)	(0.03)	(0.06)	(0.04)	(0.08)	(0.05)	(0.07)	(0.04)
Recent trades	0.03	0.04	0.03	0.04	0.01	0.02	0.06	0.01	0.04	0.04	0.06	0.04
	(0.01)	(0.00)	(0.01)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Lagged duration	−0.14	−0.08	−0.07	−0.06	−0.15	−0.09	−0.10	−0.08	−0.08	−0.06	−0.07	−0.04
	(0.03)	(0.01)	(0.03)	(0.01)	(0.02)	(0.02)	(0.02)	(0.01)	(0.02)	(0.01)	(0.02)	(0.01)
Mid-quote volatility	5.61	3.33	−2.17	−4.31	4.05	3.68	−2.53	−4.96	0.28	12.89	−5.37	−10.44
	(0.80)	(0.24)	(1.08)	(0.27)	(0.47)	(0.32)	(0.70)	(1.08)	(0.91)	(1.14)	(1.18)	(1.18)
Distance to mid-quote	26.97	38.51	−21.66	−45.17	36.36	49.49	−45.87	−101.78	25.43	42.53	−31.16	−81.30
	(2.76)	(1.83)	(3.45)	(1.98)	(4.65)	(4.64)	(4.46)	(6.04)	(2.62)	(3.72)	(5.15)	(5.88)
First hour	−0.68	−0.57	0.08	−0.21	−0.55	−0.35	−0.29	0.06	−0.92	−0.81	0.05	−0.20
	(0.27)	(0.09)	(0.39)	(0.09)	(0.20)	(0.12)	(0.24)	(0.13)	(0.22)	(0.15)	(0.31)	(0.16)
Second hour	−0.20	−0.63	−0.23	−0.11	−0.55	−0.48	−0.20	−0.17	−1.03	−0.78	0.02	−0.10
	(0.27)	(0.09)	(0.39)	(0.09)	(0.20)	(0.12)	(0.24)	(0.13)	(0.22)	(0.14)	(0.32)	(0.14)
Third hour	−0.02	−0.44	−0.51	−0.26	−0.42	−0.34	−0.29	−0.13	−0.48	−0.48	0.39	−0.27
	(0.27)	(0.09)	(0.43)	(0.09)	(0.20)	(0.13)	(0.25)	(0.13)	(0.22)	(0.13)	(0.32)	(0.13)
Fourth hour	0.06	−0.41	−0.28	−0.19	−0.10	−0.11	−0.12	−0.33	−0.06	−0.44	−0.20	−0.21
	(0.28)	(0.09)	(0.44)	(0.09)	(0.20)	(0.12)	(0.25)	(0.13)	(0.21)	(0.13)	(0.34)	(0.13)
Fifth hour	0.28	−0.34	−0.46	−0.17	−0.11	−0.11	−0.10	−0.22	−0.14	−0.46	0.63	−0.19
	(0.28)	(0.09)	(0.48)	(0.09)	(0.20)	(0.13)	(0.26)	(0.14)	(0.24)	(0.14)	(0.33)	(0.14)
Sixth hour	0.40	−0.21	0.00	0.01	−0.19	−0.23	−0.17	−0.05	−0.08	−0.13	0.45	−0.07
	(0.29)	(0.09)	(0.49)	(0.09)	(0.22)	(0.13)	(0.28)	(0.13)	(0.22)	(0.13)	(0.33)	(0.13)
χ^2_{19}	302.61	1731.08	188.96	1710.49	290.14	531.01	403.44	539.69	218.20	501.50	244.62	493.20
P-Value	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
# Obs.	1,748	4,498	1,353	4,105	2,458	2,368	2,138	1,732	1,679	1,793	1,792	1,689

Parameter estimates with asymptotic standard errors in parentheses for a Weibull model for the execution time distribution. The χ^2-test is for the null that the state vector z_t does not affect the conditional distribution, with p-values in parentheses.

Table 6: **Regression Model for Common Value Changes**

	BHO				ERR				WEM			
	Sell orders		Buy orders		Sell orders		Buy orders		Sell orders		Buy orders	
Variable	Marg.	1 Tick	Marg.	1 Tick	Marg.	1 Tick	Marg.	1 Tick	Marg.	1 Tick	Marg.	1 Tick
Constant	7.30	−2.83	−6.66	−2.65	21.70	0.72	10.64	−1.21	3.21	−6.78	−11.83	−9.24
	(2.73)	(0.63)	(4.87)	(0.50)	(6.64)	(1.22)	(5.06)	(1.18)	(4.79)	(1.83)	(7.35)	(1.88)
TSX market	−0.05	−0.01	0.10	0.01	−0.14	0.07	0.21	0.03	0.28	0.15	0.10	0.03
volatility	(0.10)	(0.01)	(0.15)	(0.01)	(0.11)	(0.04)	(0.10)	(0.03)	(0.13)	(0.05)	(0.10)	(0.02)
TSX mining	0.36	0.01	−0.02	−0.01	−0.29	−0.06	−0.23	0.00	−0.10	−0.02	−0.05	−0.05
volatility	(0.12)	(0.01)	(0.16)	(0.01)	(0.13)	(0.03)	(0.09)	(0.03)	(0.14)	(0.03)	(0.14)	(0.03)
Interest rate	−0.25	−0.02	0.25	0.01	−0.10	0.05	−0.01	−0.03	0.07	0.00	−0.11	−0.03
volatility	(0.12)	(0.01)	(0.11)	(0.01)	(0.10)	(0.04)	(0.09)	(0.04)	(0.14)	(0.03)	(0.17)	(0.03)
Exchange rate	−0.02	0.01	−0.15	0.02	−0.02	−0.05	−0.07	−0.02	0.18	0.05	−0.04	0.01
volatility	(0.12)	(0.01)	(0.17)	(0.01)	(0.11)	(0.04)	(0.10)	(0.03)	(0.12)	(0.03)	(0.12)	(0.02)
Stock	−0.10	0.00	−0.26	−0.01	−0.06	0.00	0.01	−0.02	0.07	−0.06	−0.27	−0.01
volatility	(0.15)	(0.01)	(0.14)	(0.01)	(0.08)	(0.03)	(0.06)	(0.02)	(0.12)	(0.03)	(0.13)	(0.03)
Spread	3.66	−10.14	3.60	6.23	−11.32	−13.70	9.08	16.85	−15.30	−12.39	16.58	5.71
	(6.27)	(2.35)	(6.27)	(1.67)	(3.26)	(2.24)	(3.85)	(2.30)	(4.63)	(3.24)	(3.78)	(2.97)
Close ask	—	0.01	−0.08	−0.01	—	0.12	−0.12	0.05	—	0.04	0.03	−0.02
depth	—	(0.02)	(0.20)	(0.01)	—	(0.05)	(0.15)	(0.04)	—	(0.04)	(0.20)	(0.03)
Far ask	0.28	—	—	—	−0.03	—	—	—	0.31	—	—	—
depth	(0.16)	—	—	—	(0.14)	—	—	—	(0.14)	—	—	—
Close bid	−0.10	0.00	—	0.01	−0.13	−0.17	—	−0.06	−0.30	−0.12	—	0.01
depth	(0.12)	(0.01)	—	(0.01)	(0.12)	(0.04)	—	(0.04)	(0.14)	(0.04)	—	(0.03)
Far bid	—	—	−0.26	—	—	—	0.23	—	—	—	−0.07	—
depth	—	—	(0.19)	—	—	—	(0.16)	—	—	—	(0.17)	—
Order quantity	0.01	−0.03	0.21	0.00	−0.18	0.00	0.21	0.15	−0.17	−0.05	0.19	0.01
	(0.15)	(0.01)	(0.23)	(0.02)	(0.13)	(0.04)	(0.13)	(0.04)	(0.17)	(0.05)	(0.18)	(0.03)
Recent	0.00	0.00	0.02	0.00	0.00	−0.00	0.00	0.00	−0.03	−0.00	0.07	0.00
trades	(0.00)	(0.01)	(0.02)	(0.01)	(0.03)	(0.01)	(0.03)	(0.01)	(0.02)	(0.01)	(0.02)	(0.01)
Lagged	−0.06	0.00	0.16	0.00	−0.03	−0.02	0.05	−0.01	−0.03	−0.00	0.02	0.01
duration	(0.03)	(0.00)	(0.03)	(0.00)	(0.05)	(0.01)	(0.04)	(0.01)	(0.02)	(0.01)	(0.03)	(0.01)
Mid-quote	−2.34	1.79	1.53	1.47	−8.85	−0.09	−9.08	0.62	0.10	4.24	4.85	5.42
volatility	(1.42)	(0.34)	(3.00)	(0.30)	(3.78)	(0.71)	(3.18)	(0.70)	(2.80)	(1.07)	(4.44)	(1.11)
Distance to	−57.05	−7.50	−42.05	−5.31	−128.71	−33.59	−149.04	−45.03	−54.86	−20.18	−55.17	−11.24
midquote	(6.72)	(1.32)	(12.23)	(0.95)	(16.58)	(5.50)	(18.28)	(5.59)	(14.63)	(5.80)	(22.80)	(4.55)
First hour	−0.62	−0.12	0.81	−0.03	−0.84	−0.12	−0.21	−0.26	0.56	−0.02	−1.17	−0.13
	(0.70)	(0.04)	(1.01)	(0.04)	(0.60)	(0.16)	(0.50)	(0.14)	(0.47)	(0.16)	(0.54)	(0.12)
Second hour	0.20	−0.03	0.77	−0.01	−0.72	−0.31	−0.05	−0.33	0.72	−0.08	−1.04	−0.06
	(0.66)	(0.05)	(1.01)	(0.04)	(0.60)	(0.15)	(0.50)	(0.13)	(0.47)	(0.14)	(0.55)	(0.09)
Third hour	−0.05	0.03	0.87	0.04	−0.55	0.01	−0.51	−0.10	0.77	−0.24	−0.63	−0.06
	(0.66)	(0.04)	(0.98)	(0.04)	(0.57)	(0.15)	(0.52)	(0.13)	(0.42)	(0.13)	(0.50)	(0.09)
Fourth hour	−0.63	−0.02	0.89	0.01	−0.59	0.17	−0.41	−0.10	0.54	−0.21	0.15	0.08
	(0.64)	(0.04)	(1.10)	(0.04)	(0.61)	(0.16)	(0.51)	(0.12)	(0.44)	(0.14)	(0.58)	(0.07)
Fifth hour	−0.06	−0.00	0.47	0.01	−0.36	−0.36	−0.41	−0.30	−0.31	−0.03	0.07	0.02
	(0.66)	(0.04)	(1.02)	(0.04)	(0.59)	(0.15)	(0.52)	(0.12)	(0.46)	(0.14)	(0.55)	(0.08)
Sixth hour	−0.56	0.01	0.08	0.06	−0.44	0.12	−0.52	−0.11	0.20	−0.26	−0.37	−0.03
	(0.69)	(0.04)	(1.04)	(0.04)	(0.60)	(0.15)	(0.69)	(0.13)	(0.42)	(0.12)	(0.64)	(0.07)
R^2	0.33	0.13	0.44	0.09	0.28	0.14	0.40	0.20	0.35	0.20	0.29	0.09
F-test	7.64	12.45	8.64	6.90	5.84	6.21	6.58	8.66	6.77	4.08	6.39	3.45
P-value	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
# Obs.	293	2,388	170	2,261	565	1,194	484	955	327	892	281	858

Parameter estimates with heteroscedasticity adjusted asymptotic standard errors in parentheses for the regressions used to predict changes in the common value condition on order execution. The F-tests is for the null that the state vector z_t does not affect the conditional expectation, with p-values in parentheses.

Table 7: **Conditional Maximum Likelihood Estimates**

	BHO	ERR	WEM
Trader arrival rate			
Shape	0.45	0.47	0.45
	(0.00)	(0.00)	(0.00)
Constant	−2.15	−2.08	−2.55
	(0.02)	(0.02)	(0.02)
TSX market volatility	0.03	0.02	0.03
	(0.01)	(0.01)	(0.01)
TSX mining volatility	0.03	−0.00	0.01
	(0.01)	(0.01)	(0.01)
Interest rate volatility	−0.11	−0.00	0.07
	(0.01)	(0.01)	(0.01)
Exchange rate volatility	0.11	0.06	−0.04
	(0.01)	(0.01)	(0.01)
Stock volatility	0.27	0.15	0.19
	(0.01)	(0.01)	(0.01)
Private value distribution			
Mixing probability, ρ	0.85	0.89	0.84
	(0.04)	(0.03)	(0.05)
σ_1	0.06	0.04	0.04
	(0.00)	(0.00)	(0.00)
$\sigma_2 - \sigma_1$	0.47	0.68	0.48
	(0.04)	(0.06)	(0.05)
Time-varying variance			
TSX market volatility	−0.00	0.03	−0.05
	(0.01)	(0.01)	(0.01)
TSX mining volatility	−0.03	−0.02	−0.05
	(0.01)	(0.01)	(0.01)
Interest rate volatility	0.07	−0.05	−0.03
	(0.01)	(0.01)	(0.01)
Exchange rate volatility	−0.04	−0.02	0.03
	(0.01)	(0.01)	(0.01)
Stock volatility	−0.10	−0.14	−0.12
	(0.01)	(0.01)	(0.01)
Order execution cost, c_e	0.02	0.02	0.01
	(0.00)	(0.00)	(0.00)
χ^2_5: Constant arrival rate	3871.95	680.94	985.28
	(0.00)	(0.00)	(0.00)
χ^2_5: Constant private value variance	434.55	814.08	369.70
	(0.00)	(0.00)	(0.00)

The table reports estimates of the trader arrival rate, the private value distribution and the trading opportunity cost. Asymptotic standard errors are reported in parentheses. The table also reports chi-squared tests for the null hypothesis of a constant trader arrival rate and a constant private value variance, with p-values in parentheses.

Table 8: **Expected Utility by Trader Valuation**

	BHO Private value						ERR Private value						WEM Private value					
Percent	-5.00	-2.50	-1.25	1.25	2.50	5.00	-5.00	-2.50	-1.25	1.25	2.50	5.00	-5.00	-2.50	-1.25	1.25	2.50	5.00
Cents	-4.25	-2.12	-1.06	1.06	2.12	4.25	-9.68	-4.84	-2.42	2.44	4.84	9.68	-6.75	-3.38	-1.69	1.69	3.38	6.75
Low liquidity state: wide spread, low depth																		
Buy market	-6.31	-4.19	-3.13	-1.00	0.06	2.18	-13.65	-8.81	-6.39	-1.55	0.87	5.71	-9.94	-6.57	-4.88	-1.51	0.18	3.55
Buy 1 tick	-4.33	-2.45	-1.51	0.38	1.32	$\boxed{3.20}$	-8.99	-5.08	-3.13	0.77	2.73	$\boxed{6.63}$	-7.12	-4.26	-2.84	0.02	1.45	$\boxed{4.31}$
Buy marginal	1.31	1.54	1.65	$\boxed{1.88}$	$\boxed{2.00}$	2.22	3.46	3.93	4.16	$\boxed{4.63}$	$\boxed{4.86}$	5.33	0.97	1.27	1.42	$\boxed{1.72}$	$\boxed{1.87}$	2.17
No order	1.79	1.79	1.79	1.79	1.79	1.79	4.61	4.61	4.61	4.61	4.61	4.61	1.54	1.54	1.54	1.54	1.54	1.54
Sell marginal	2.47	$\boxed{2.04}$	$\boxed{1.83}$	1.40	1.18	0.75	5.72	$\boxed{5.02}$	$\boxed{4.66}$	3.96	3.61	2.90	2.67	$\boxed{2.11}$	$\boxed{1.83}$	1.28	1.00	0.44
Sell 1 tick	$\boxed{3.16}$	1.28	0.35	-1.52	-2.46	-4.33	$\boxed{6.17}$	2.05	0.00	-4.12	-6.17	-10.29	$\boxed{3.64}$	1.64	0.64	-1.36	-2.36	-4.36
Sell market	2.18	0.06	-1.00	-3.13	-4.19	-6.31	4.56	-0.28	-2.70	-7.54	-9.96	-14.80	3.56	0.18	-1.51	-4.88	-6.57	-9.95
High liquidity state: narrow spread, high depth																		
Buy marginal	-4.36	-2.24	-1.18	0.95	2.01	$\boxed{4.13}$	-11.70	-6.86	-4.44	0.40	2.82	$\boxed{7.66}$	-7.23	-3.86	-2.17	1.20	$\boxed{2.89}$	$\boxed{6.26}$
Buy 1 tick	-0.27	0.55	0.95	1.77	$\boxed{2.17}$	2.99	1.28	2.35	2.89	3.97	4.51	5.58	-0.91	0.11	0.62	$\boxed{1.64}$	2.15	3.17
Buy marginal	1.38	1.54	1.61	1.77	1.84	2.00	3.37	3.84	4.07	4.54	$\boxed{4.78}$	5.25	1.11	1.28	1.37	1.55	1.64	1.81
No order	1.79	1.79	$\boxed{1.79}$	$\boxed{1.79}$	1.79	1.79	4.61	4.61	$\boxed{4.61}$	$\boxed{4.61}$	4.61	4.61	1.54	1.54	1.54	1.54	1.54	1.54
Sell marginal	2.12	1.88	1.76	1.51	1.39	1.15	5.40	$\boxed{4.80}$	4.50	3.91	3.61	3.02	2.28	1.85	1.63	1.20	0.99	0.55
Sell 1 tick	2.81	$\boxed{2.05}$	1.67	0.91	0.53	-0.23	5.23	4.48	4.10	3.35	2.97	2.22	3.94	2.48	$\boxed{1.75}$	0.29	-0.44	-1.90
Sell market	$\boxed{4.13}$	2.01	0.95	-1.18	-2.24	-4.36	$\boxed{8.81}$	3.97	1.55	-3.29	-5.71	-10.55	$\boxed{6.27}$	$\boxed{2.89}$	1.20	-2.17	-3.86	-7.23
Moving market state: common value proxy higher than the mid-quote																		
Buy market	-3.69	-1.52	-0.44	1.73	2.81	$\boxed{4.98}$	-5.12	-0.08	2.44	$\boxed{7.48}$	$\boxed{10.00}$	$\boxed{15.04}$	-6.04	-2.60	-0.87	$\boxed{2.57}$	$\boxed{4.29}$	$\boxed{7.73}$
Buy 1 tick	-0.77	0.44	1.04	$\boxed{2.25}$	$\boxed{2.86}$	4.07	3.72	4.12	4.32	4.72	4.92	5.32	-0.77	0.38	0.96	2.11	2.68	3.83
Buy marginal	1.50	1.63	1.70	1.83	1.90	2.03	4.24	4.38	4.45	4.58	4.65	4.79	1.25	1.40	1.47	1.62	1.70	1.84
No order	1.79	1.79	$\boxed{1.79}$	1.79	1.79	1.79	4.61	4.61	$\boxed{4.61}$	4.61	4.61	4.61	1.54	1.54	$\boxed{1.54}$	1.54	1.54	1.54
Sell marginal	$\boxed{2.50}$	$\boxed{1.97}$	1.71	1.18	0.92	0.40	$\boxed{6.22}$	$\boxed{4.65}$	3.87	2.30	1.52	-0.05	2.53	$\boxed{1.85}$	1.51	0.83	0.49	-0.19
Sell 1 tick	2.35	0.77	-0.03	-1.61	-2.40	-3.99	2.48	-1.08	-2.86	-6.42	-8.20	-11.75	$\boxed{2.84}$	0.99	0.06	-1.78	-2.71	-4.56
Sell market	1.51	-0.66	-1.74	-3.91	-4.99	-7.16	-0.87	-5.91	-8.43	-13.47	-15.99	-21.03	2.36	-1.08	-2.81	-6.25	-7.97	-11.41

The table reports, for traders with different private values, the expected utilities for alternative order submissions. The expected utilities are computed for three different states; a low liquidity state where the bid-ask spread is one standard deviation above its mean and the depth variables are one standard deviation below their means; a high liquidity state where the bid-ask spread is one standard deviation below its mean and the depth variables are one standard deviation above their mean values; and a moving market state where the common value proxy divided by the mid-quote is one standard deviation above its mean. All other variables are at their in-sample mean values. All utilities are reported in cents per share.

Table 9: **Order Submission Probabilities by Trader Private Value**

Order submission		Private value $(-\infty, -5\%]$	$(-5\%, -2.5\%]$	$(-2.5\%, +2.5\%)$	$[2.5\%, +5\%)$	$[+5\%, +\infty)$
				BHO		
Mean probability of private value in interval		0.21	0.13	0.32	0.13	0.21
				Order submission probabilities		
Sell market	Mean	0.74	0.25	0.01	0.00	0.00
	Standard deviation	0.30	0.36	0.07	0.01	0.00
Sell limit	Mean	0.24	0.64	0.25	0.00	0.00
	Standard deviation	0.29	0.39	0.20	0.04	0.01
No order	Mean	0.02	0.09	0.36	0.07	0.01
	Standard deviation	0.09	0.25	0.28	0.22	0.07
Buy limit	Mean	0.00	0.02	0.37	0.74	0.30
	Standard deviation	0.03	0.13	0.27	0.36	0.31
Buy market	Mean	0.00	0.00	0.01	0.19	0.69
	Standard deviation	0.00	0.01	0.05	0.32	0.31
				ERR		
Mean probability of private value in interval		0.12	0.14	0.48	0.14	0.12
				Order submission probabilities		
Sell market	Mean	0.85	0.32	0.01	0.00	0.00
	Standard deviation	0.23	0.35	0.06	0.03	0.01
Sell limit	Mean	0.15	0.66	0.16	0.00	0.00
	Standard deviation	0.23	0.36	0.12	0.03	0.01
No order	Mean	0.00	0.03	0.61	0.01	0.00
	Standard deviation	0.03	0.12	0.15	0.07	0.02
Buy limit	Mean	0.00	0.00	0.21	0.75	0.22
	Standard deviation	0.00	0.02	0.12	0.33	0.26
Buy market	Mean	0.00	0.00	0.01	0.24	0.78
	Standard deviation	0.00	0.00	0.04	0.33	0.26
				WEM		
Mean probability of private value in interval		0.11	0.13	0.52	0.13	0.11
				Order submission probabilities		
Sell market	Mean	0.92	0.48	0.04	0.00	0.00
	Standard deviation	0.16	0.41	0.09	0.00	0.00
Sell limit	Mean	0.08	0.51	0.41	0.00	0.00
	Standard deviation	0.16	0.41	0.17	0.05	0.01
No order	Mean	0.00	0.01	0.12	0.01	0.00
	Standard deviation	0.01	0.05	0.18	0.09	0.02
Buy limit	Mean	0.00	0.00	0.40	0.62	0.16
	Standard deviation	0.00	0.02	0.12	0.42	0.24
Buy market	Mean	0.00	0.00	0.03	0.37	0.84
	Standard deviation	0.00	0.01	0.09	0.41	0.24

The table reports the in-sample mean of the probability of drawing a private value (u) from five different intervals. For each interval and stock the table reports the in-sample mean and standard deviation of the probability of a trader optimally submitting a sell market order, a sell limit order, no order, a buy limit order, or a buy market order, conditional on the trader's private value being in that interval.

Table 10: **Estimates of the Gains from Trade**

	BHO	ERR	WEM
Gains			
Maximum gains as a % of the common value			
	9.07	8.61	6.75
Current gains as a % of the common value			
Lower bound	7.88	8.09	6.08
Upper bound	8.45	8.31	6.40
Average	8.16	8.20	6.24
Maximum gains minus current gains			
Lower bound	0.62	0.30	0.35
Upper bound	1.20	0.52	0.67
Average	0.91	0.41	0.51
Current gains as a % of maximum gains			
Lower bound	86.79	93.97	90.07
Upper bound	93.13	96.57	94.81
Average	89.96	95.27	92.44
Decomposition of losses			
No execution as a % of total losses			
Sell side	32.32	31.20	33.05
Buy side	40.10	39.01	41.85
Subtotal	72.42	70.21	74.90
No submission as a % of total losses			
Sell side	2.24	0.62	0.41
Buy side	1.98	0.15	0.71
Subtotal	4.22	0.77	1.12
Wrong direction as a % of total losses			
Sell side	0.86	0.02	0.39
Buy side	0.20	0.05	0.63
Subtotal	1.06	0.07	1.02
Crowding out as a % of total losses			
Sell side	9.81	11.87	10.30
Buy side	12.49	17.07	12.66
Subtotal	22.30	28.94	22.96
Total	100.00	100.00	100.00

The first row of the top panel reports the estimates of the maximum gains from trade, measured as a percent of the common value. The next three rows report the estimates of the lower and upper bound and the average current gains from trade; details are provided in Appendix D. The next six rows report the lower and upper bounds and the average for the difference between the maximum and the current gains from trade, and the current gains from trade as a percentage of the maximum gains from trade. The bottom panel reports the average percentage of the efficiency loss associated with no execution, no submission, wrong direction, and crowding out computed for the average current gains from trade.

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